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Brooke Krejci
Winona State University

Elana Joram
University of Northern Iowa

See next page for additional authors

Authors
Brooke Krejci, Elana Joram, and Anthony J. Gabriele

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Shifting Preservice Teachers' Sources of Mathematics Teaching Efficacy Through Scaffolded Reflection:

Fostering Commitment to Reform-based Mathematics

Brooke Krejci, Ed.D.
Winona State University, USA

Elana Joram, Ph.D.
University of Northern Iowa, USA

Anthony J. Gabriele, Ph.D.
University of Northern Iowa, USA

ABSTRACT

Efforts to promote reform-based mathematics instruction in schools are pervasive, yet many teachers continue to use more traditional, procedural methods (Paolucci, 2015). In this chapter, the authors suggest that to achieve sustained adoption of reform-based instruction in mathematics, it is essential for professional development programs to draw teachers' attention to sources of mathematics teaching efficacy afforded by reform-based teaching, such as enhanced student thinking. Educator preparation programs are ideal settings in which to begin this process, and the authors describe how scaffolded reflection activities within a mathematics methods course were used to focus preservice teachers' attention on sources of mathematics teaching efficacy that are congruent with reform-based mathematics instruction. The authors suggest that engendering such a shift in the sources for mathematics teaching efficacy may lead to more widespread and sustained use of reform-based teaching methods by preservice teachers in their future practice.

Keywords: Elementary, Mathematics Methods Courses, Scaffolded Reflection, Teacher Education

INTRODUCTION

Despite widespread efforts by teacher educators and those providing professional development to help teachers integrate reform-based mathematics approaches into their everyday instruction (McGee, Wang & Polly, 2013; Paolucci, 2015), traditional methods of teaching mathematics are still common (Hill et al. 2018). Teachers may focus classroom instruction on procedural computations without making deeper connections, and emphasize fact and skill recall instead of reasoning and problem solving even after professional development in reform-based mathematics instruction (NCTM, 2014; Paolucci, 2015). Indeed, research reveals that sometimes even teachers who appear to embrace reform-based mathematics principles may utilize more traditional approaches of instruction in their practice (e.g., Lane & Ríordáin, 2020).

Although many factors—including institutional, socio-economic, and cultural—may contribute to this puzzling pattern, the authors suggest that one important element largely missing from the discussion is "mathematics teaching efficacy," defined as teachers' or preservice teachers' beliefs that they can effectively teach mathematics (Briley, 2012). This chapter explores mathematics teaching efficacy, explains why it is central to supporting and sustaining reform-based mathematics teaching, and describes some of the special challenges faced by mathematics educators who attempt to support its development. The authors argue that an opportune time to support and facilitate the development of mathematics teaching efficacy is during teacher preparation (Giles et al., 2016), when teacher beliefs may be more malleable than after years of classroom practice. This chapter describes the unique opportunities and challenges of encouraging the development of mathematics teaching efficacy beliefs for teaching reform-based mathematics with preservice elementary teachers. Examples of one of the author's initial efforts to do so in an undergraduate mathematics methods course are included. The purpose of this description and accompanying examples is two-fold: it is intended to help focus the attention of both educational researchers and teacher educators on a potential roadblock to the adoption and sustained use of reform-based instructional methods in mathematics, and also to provide an illustration of how mathematics teaching efficacy can be facilitated within teacher education programs.

BACKGROUND

Since the late 1980s, marked by the introduction of the National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for Mathematics* (Romberg, 1989), mathematics education in the United States changed from emphasizing teacher-centered instruction to promoting more student-centered approaches (Hill, 2021; NCTM, 2014). More recently, the NCTM updated their recommendations of student-centered and inquiry-based teaching practices ("reform-based") in *Principles and Standards for School Mathematics* (NCTM, 2000). Further, in the *Common Core State Standards Initiative* (CCSSI), eight standards for mathematical practice are provided that describe the ways in which students should engage with mathematical content standards (CCSSI, 2010). As noted in the description of each mathematical practice standard, students play an active role in learning content rather than replicating the action of the teacher (CCSSI, 2010). In line with these student-centered practice standards and specifically targeting effective mathematics teaching practices for student-centered mathematics, NCTM (2014) released *Principles to Actions: Ensuring Mathematical Success for All.* A common thread in these practices is for teachers to incorporate student thinking and allow it to guide their instructional moves, with the goal of developing deep conceptual understanding in students (NCTM, 2014).

The acknowledged benefits and deepened understanding that students gain from reform-based mathematics instruction raises the question of why these standards are not currently implemented everywhere. This is the question addressed in this chapter, and the authors suggest that one factor that may play a role in the implementation and sustained practice of reform-based mathematics instruction is "mathematics teaching efficacy." More generally, "teaching efficacy" refers to a teacher's belief in their ability to effectively teach and positively influence their students' learning (Tschannen-Moran & Hoy, 2001), which has been credited with increased teacher effectiveness (von Suchodoletz et al., 2018). In a general sense, higher efficacy is linked to increasing likelihood of teachers' willingness to try new instructional strategies, according to a meta-analyses over 40 years of literature (Zee & Koomen, 2016); for this reason, "teacher efficacy" is relevant to those hoping to promote reform-based mathematics instruction.

Beliefs about one's ability to teach can be further specified according to content domain (Tschannen-Moran & Hoy, 2001); thus "mathematics teaching efficacy" is based on the beliefs a teacher holds about their ability to effectively teach mathematics (Briley, 2012). With respect to mathematics teaching efficacy, along with others, the authors argue that shifting to teaching reform-based mathematics requires significant changes to mathematics teaching efficacy, and that this shift is necessary for the sustained adoption of reform-based mathematics instruction (Gabriele & Joram, 2007; Smith III, 1996). Below, specific aspects of these changes to mathematics teaching efficacy are described.

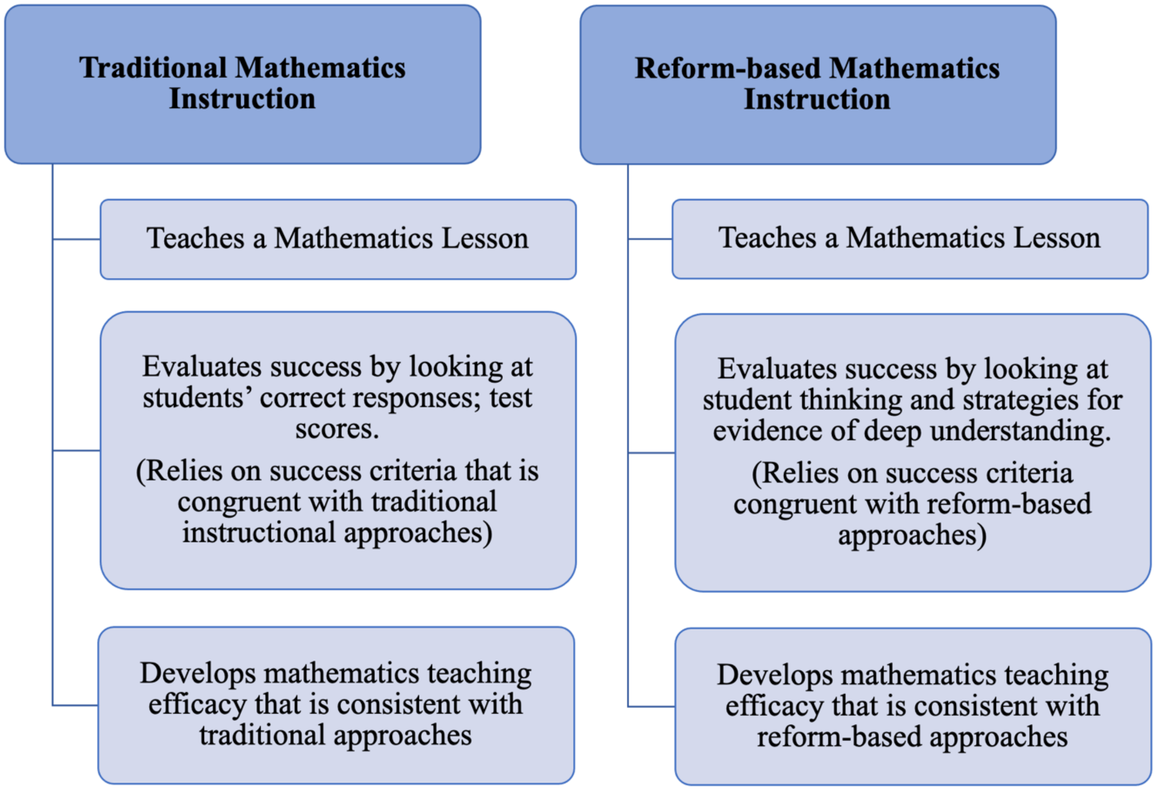
Sources of Mathematics Teaching Efficacy and Its Role in the Shift to Reform-Based Teaching

In the mathematics education literature, mathematics teaching efficacy has most often been discussed with respect to enhancing teachers' content and pedagogical content knowledge, with the underlying assumption that mathematics teaching efficacy naturally grows as teachers gain confidence in their own mastery of mathematical content and knowledge of teaching strategies (e.g., Kleickmann et al., 2017; Thomson et al., 2017). Although this assumption intuitively makes sense, and may in fact be true with some mathematical teaching practices linked to traditional mathematics instruction, prior research suggests that this is likely not the case with reform-based mathematics instruction. Instead, the authors have previously argued that active, intentional support is required for its development (Gabriele & Joram, 2007). In part, this may be because the sources of mathematics teaching efficacy related to traditional and reform-based mathematics instruction are different and require distinct kinds of support (Althauser, 2018; Gabriele & Joram, 2007; Smith III, 1996).

In a study comparing teachers with extensive experience using reform-based mathematics instruction (*veteran*) to those with less experience (*novice*), teachers who had used reform-based methods for five or more years reported pleasure and enjoyment when they observed a student showing partial or deep understanding of a problem or concept (Gabriele & Joram, 2007). In contrast, those teachers who had used reform-based methods for fewer than five years tended to evaluate their own success as a mathematics teacher with respect to their lesson coverage and students getting correct answers to problems, suggesting they were still transitioning to fully embracing reform-based methods. In other words, with respect to reform-based mathematics instruction, the "veteran" and "novice" teachers in this study drew on different criteria to judge the success of their teaching practices even though both were implementing mathematical tasks considered to be reform-based in nature. This study suggests, as illustrated in Figure 1, that important distinctions need to be made with respect to how mathematics teaching efficacy develops and is sustained in reform-based and more traditional mathematics instructional contexts (Gabriele & Joram, 2007; Smith III, 1996). This, in turn, points to the critical role that sources of mathematics teaching efficacy may play in the shift to reform-based mathematics instruction.

Thus, this chapter focuses on the relatively unexplored dimension of teachers shifting to a greater focus on student thinking as evidence for their mathematics teaching efficacy and valuing the observed changes in student thinking that align with reform-based teaching. In this sense, the authors suggest that the very concept of "mathematics teaching efficacy" needs to be understood within the context of reform-based mathematics instruction as continuously cultivated through specific observations of student thinking that serve as evidence for teachers that their teaching methods result in deeper understanding of mathematics by their students. This evidence can then serve as an important source of their mathematics teaching efficacy for teachers using reform-based methods (see Figure 1).

Figure 1. Congruence within different mathematics teaching approaches and success criteria drawn on as sources of mathematics teaching efficacy



Preservice Teacher Education as an Ideal Context to Foster Mathematics Teaching Efficacy

Teacher beliefs about effective instruction are often developed through their own past experiences (Althauser, 2018; Fives et al., 2015). Many teachers' past experiences as students of mathematics likely involved primarily traditional and/or procedural methods (e.g., Jao, 2017; Paolucci, 2015), and the authors suggest teacher education programs may present an opportune time in which to target beliefs about the nature of effective mathematics instruction (Giles et al., 2016). Wyatt (2016) notes that teacher self-efficacy beliefs may be easily impacted during teacher education, and recommends that teacher educators target the growth of teacher self-efficacy in relation to specific tasks rather than more generally. Thus, methods courses within teacher education programs seem well suited to doing so.

In U.S. educator preparation programs, mathematics methods courses are typically a primary context in which preservice teachers (referred to hereafter as "*PSTs*") encounter reform-based approaches to mathematics instruction. Exposing PSTs to reform-based mathematics instruction in these methods courses and encouraging reflection on these instructional methods gives them an opportunity to embrace these approaches and shift their beliefs about effective instruction. As suggested above, it may be that in the absence of efforts by instructors to intentionally scaffold PSTs' observations and reflections, mere exposure to reform-based teaching methods is not sufficient to shift PSTs' beliefs about effective instruction in mathematics. Therefore, to draw attention to salient aspects of reform-based methods, the authors propose that teacher educators make intentional use of scaffolds when working with PSTs in mathematics methods courses to guide their observation and reflection. Later in this chapter the authors provide a description and examples of the use of such scaffolds, as well as PSTs' responses to them.

Reflection as a Catalyst for the Development of Mathematics Teaching Efficacy

The process of reflection or "...the kind of thinking that consists in turning a subject over in the mind and giving it serious and consecutive consideration" (Dewey, 1933, p. 3) has been recognized as critical for the development of self-efficacy (Wyatt, 2016). Uzun et al. (2013) suggest that an increase in PSTs' reflective thinking tendencies will result in growth in their self-efficacy beliefs, which in turn may lead to enhanced teaching performance. Extending this line of reasoning to the development of mathematics teaching efficacy, the authors explored whether promoting reflection in the context of reform-based instruction would lead to a greater focus on sources of mathematics teaching efficacy that are congruent with reform-based mathematics.

Reflective thought that is facilitated by professional development for teachers and through educator preparation courses for PSTs has been examined in relation to teaching efficacy in general (e.g., Noormohammadi, 2014; Phan, 2014; Tavil, 2014; Uzun et al., 2013). Reflection can be promoted through a variety of methods: journals, videos, talk-alouds, debriefing sessions, frameworks, etc. (e.g., Gabriele & Joram, 2007; van Es et al., 2017; Wilkerson et al., 2018). In some cases, teachers or PSTs reflect on their own teaching or mastery experiences; in others they focus on the teaching of others, thus building self-efficacy through vicarious experiences (Bandura, 1977).

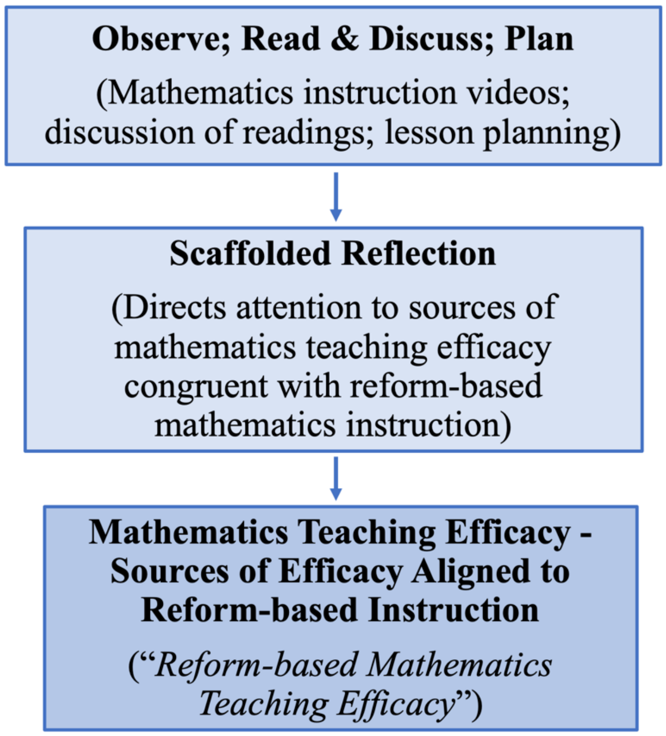
Because there is a paucity of research investigating the influence of reflective activities on the growth of mathematics teaching efficacy, the authors looked to other fields also undergoing educational reforms in which researchers have examined the relationship between reflective activities and growth in teaching efficacy. For example, in science education, the action-reflection cycle was found to be productive for promoting PSTs' science teaching efficacy across instructional activities (Naidoo & Naidoo, 2021). Specifically, Naidoo and Naidoo (2021) recommend the use of successful teaching episodes to develop PSTs' perceived abilities, reflection in a collaborative environment, and recognition that challenges are part of the development of PSTs' own teaching competence. These studies suggest that similar intentional reflective opportunities following instructional tasks for PSTs in the context of reform-based mathematics teaching could be beneficial for increasing their mathematics teaching efficacy.

Using Reflective Prompts to Shift PSTs' Attention to Student Thinking and Understanding

Although researchers have focused on professional development for practicing teachers to encourage a greater focus on student thinking, less attention has been directed to such efforts for PSTs (Coffey & Edwards, 2015). There is an indication in the literature, however, that prompting and focusing questions in reflection activities can guide PSTs toward focusing on specific aspects of instruction such as student learning and understanding (e.g., Santagata & Yeh, 2014; van Es et al., 2017). This could be particularly beneficial for PSTs because they tend to focus on teaching rather than on learning (e.g., Chamoso et al., 2012; Gelfuso & Dennis, 2014; Seung et al., 2014); in other words, on a teacher's actions rather than students' thinking and strategies. This focus conflicts with the goals and aims of reform-based mathematics instruction which places greater emphasis on students' learning and understanding. Reflective prompts can guide PSTs away from an exclusive focus on the teacher, and more towards attending to student learning and understanding (Santagata & Yeh, 2014; van Es et al., 2017). Through reflective activities, it may be possible to draw PSTs' attention to student thinking and understanding as a way of gauging a teacher's (and eventually their own) success in teaching mathematics. This would also shift them away from looking to summative test scores and number of correct responses by students as the most important (or sole) indicators of a teacher's impact.

To summarize, the authors suggest that teacher education provides an ideal context in which to scaffold the development of mathematics teaching efficacy through the provision of opportunities to reflect on reform-based instructional tasks that PSTs either observe or carry out. More specifically, the authors suggest that it will be critical during teacher education courses to provide activities that direct PSTs to shift to relying on sources of mathematics teacher efficacy that align with reform-based mathematics instruction (see Figure 2). This contrasts with those sources that have typically been relied on to build mathematics teaching efficacy in traditional contexts. The authors argue that this shift to drawing on sources that are consistent with the culture of reform-based mathematics instruction is key to building mathematics teaching efficacy in a reform-based context. In turn, this should, lead to sustained and consistent use of reform-based mathematics instruction.

Figure 2. Instructional activities in a mathematics methods course designed to facilitate mathematics teaching efficacy in the context of reform-based instruction



THEORETICALLY MOTIVATED ACTIVITIES DESIGNED TO PROMOTE REFORM-BASED MATHEMATICS INSTRUCTION

As suggested above, in a reform-based mathematics culture, it is critical for teachers to seek out windows into their students' thinking about mathematical concepts and problems, value the insights their students may have, and take satisfaction in these. These sources provide evidence for the teacher's own positive impact on student mathematical thinking and understanding; in other words, their mathematics teaching efficacy. To achieve these goals, one of the authors designed a series of instructional tasks that were introduced in an elementary mathematics methods course, described below.

Prior to engaging in the instructional activities outlined in this methods course, PSTs had completed a mathematical content course for elementary teachers, typical of the sequence of coursework in many teacher preparation programs in the United States. PSTs were enrolled in a four-year undergraduate teacher education program in a medium-size university in the midwestern United States. The sequence of courses in this program consisted of general education courses (e.g., English composition, elective humanities), followed by coursework for developing content knowledge and pedagogical practices across main subject areas and core professional knowledge courses such as assessment, diverse learners, etc. In addition, PSTs engaged in field experiences throughout the program, culminating with student teaching.

The design of the mathematics methods course described here was based on the assumption that the intentional facilitation of mathematics teaching efficacy in PSTs should be integrated into opportunities to further develop their content and pedagogical knowledge in mathematics. This contrasts with the assumption described above that it will emerge naturally on its own after PSTs acquire content and pedagogical content knowledge. Another rationale for targeting PSTs in a methods course is that they can be given tasks in which they comment on someone else's teaching, thus freeing up their attention to focus on the task at hand. If they were delivering a lesson in a field experience or student teaching, PSTs' attention would instead likely be devoted to the logistics of delivery, making it difficult to introduce another focus of attention.

Throughout the eight-week long mathematics methods course, PSTs' mathematics teaching efficacy was intentionally facilitated through activities including watching instructional videos, planning lessons, and discussing course readings. The lesson planning activity was structured around *Thinking Through a Lesson: Successfully Implementing High-Level Tasks* (Smith et al., 2008), which encourages the use of student-centered instruction. All tasks included were immediately followed by reflective prompts (see Table 1 for examples).

Table 1. Instructional tasks and examples of reflection prompts

Scaffolded Reflection on Reform-Based Teaching (videos)	Weeks 1 and 8 (General reflection prompts): 1. "Please reflect on the lesson you just observed." 2. "How do you think the lesson went today? What stood out for you?" Weeks 2-7 (Prompts targeting aspects of reform-based instruction): 1. "Specific to the mathematical concept of ________, how did the student(s) in the video demonstrate their understanding? Provide specific examples." 2. "What did the teacher say or do to develop student understanding of ______? Provide specific examples." 3. "What could the teacher do, additionally, to increase student understanding of ______?" 4. "How does reflecting on this video inform your own practice? What will you take away from this video, or what connections can you make to your own teaching or future teaching?"
Scaffolded Reflection on Reform-Based Lesson Planning	1. "What are the mathematical concepts you are teaching in this lesson?" 2. "How confident are you that this lesson would be effective in helping students understand the mathematical concepts? Please explain your reasoning for your response." 3. "How will you know that students understand the mathematical concept targeted in your lesson?" 4. "How confident are you that you could successfully teach this lesson? Please explain your reasoning for your response."
Scaffolded Reflection on Reform-Based Readings (Example)	After reading "Building a Discourse Community: Initial Practices" by Hodge and Walther (2017): 1. "As you think about your own classroom, how will you incorporate the '4 Initial Practices to Build a Foundation of Productive Discourse' in your teaching?"

Task 1: Scaffolded Reflection of Reform-Based Teaching

In the first instructional task, PSTs were asked to observe videos showing a teacher demonstrating reform-based mathematics instruction. They were then prompted with reflection questions focused on student thinking and understanding as evident in the video, with the intent of drawing the PSTs' attention to student thinking. Researchers have conducted studies centered around the use of videos for reflection, demonstrating the potential for PSTs to shift the focus of their reflection in this context (e.g., Chamoso et al., 2012; van Es et al., 2017; Wilkerson et al., 2018). A common theme in these studies is the use of intentional prompts focused on student learning and thinking, and this was incorporated into the design of the activities in the present course. As suggested by van Es and colleagues (2017), videos should include cognitively demanding tasks that lend themselves to reflective prompts focused on student understanding, consistent with reform-based instructional practices. Thus, videos were selected for the methods course that showed teachers using reform-based instructional methods that involved tasks with a scope that allowed for reflective prompts about student understanding.

Video reflection prompts from weeks 1 and 8 were general in nature to allow any shifts in PSTs' focus that may have occurred throughout the course to be observed. In contrast, prompts used in weeks 2 through 7 directed PSTs' attention to specific aspects of instruction. These scaffolded reflection prompts were adapted from two different sources. The first three scaffolded prompts (see Table 1) were drawn from a pilot study conducted by one of the authors of a small number of PSTs in a mathematics methods course, and focused PSTs' attention on student understanding and the teachers' role in this development during the intervention. As recommended by Lee and Ertmer (2006), prompts did not narrowly focus PSTs' attention (which could potentially limit the scope of their reflections). Instead, PSTs were invited to process the information and select examples they felt demonstrated student understanding. The fourth prompt, which encouraged PSTs to focus on making connections to their own practice, was adapted from a framework used by Wilkerson et al. (2018) in their study of teachers reflecting on written vignettes.

Reflections were prompted by questions tailored to each mathematical concept exhibited in the video (see Table 1, weeks 2-7, prompts 1 & 2). For example, one prompt was: "Specific to the mathematical concept of fraction representation of equal parts, how did the student(s) in the video demonstrate their understanding? Provide specific examples." In addition, to direct PSTs to attend to student understanding as a source of self-efficacy, another prompt focused on the observed teacher's actions as related to aspects of reform-based mathematics, "What did the teacher say or do to develop student understanding of equivalent fraction representation? Provide specific examples."

The prompts provided an opportunity to evaluate an important component of reform-based mathematics instruction: developing student understanding and recognizing the teacher's actions that promoted this understanding. Scaffolded reflection following observation provides an opportunity for PSTs to engage in observing an instructor model reform-based mathematics. It also invites PSTs to take the time to reflect on important characteristics of reform-based mathematics instruction to inform their future teaching (e.g., Wilkerson et al., 2018), which may then facilitate the development of their beliefs in their ability to teach mathematics in a student-centered way.

Task 2: Scaffolded Reflection of Reform-Based Lesson Planning

Lesson planning activities directed PSTs' attention to important components of reform-based mathematics and also provided insights into what PSTs intend to enact in their instruction. Similar to recommendations by van Es and colleagues (2017) regarding video selection, lesson plan templates were designed to be consistent with the aims of reform-based mathematics, specifically based on components of student-centered instruction outlined in *Thinking Through a Lesson: Successfully Implementing High-Level Tasks* (Smith et al., 2008). In contrast to more traditional methods of mathematics instruction in which the teacher models procedures for mathematical computations, Smith et al.'s (2008) protocol encourages PSTs to support student understanding by implementing tasks with multiple entry or access points, asking questions to develop and advance student thinking, and facilitating a classroom discussion by sequencing predicted student strategies. "Multiple entry/access points" is a term commonly used in reform-based mathematics instruction, and means that students can begin or access a problem in many different ways, regardless of different levels of prior knowledge.

After PSTs completed their written lesson plans both at the beginning and end of the course, they engaged in reflection using prompts designed to focus their attention on specific aspects of reform-based teaching such as developing and assessing student understanding. Specifically, the third lesson plan reflection prompt asked PSTs to describe student understanding based on their anticipated instruction, allowing PSTs to identify their own examples based on their interpretation of student understanding (see Table 1). The focus on student understanding can be observed in several of the prompts used in the lesson planning task, for example, "How confident are you that this lesson would be effective in helping students understand the mathematical concepts? Please explain your reasoning for your response." By directing PSTs' attention to key themes and aspects of reform-based instruction such as the development student understanding as a process, these prompts encouraged PSTs to reflect on their anticipated instruction and its alignment with reform-based instruction.

Task 3: Scaffolded Reflection of Reform-Based Readings

The final instructional task designed to shift PSTs' sources of efficacy was scaffolded reflection following course readings that embody reform-based mathematics teaching principles and give examples of reform-based practices. Similar to video selections, readings that are aligned with reform-based mathematics instruction offer PSTs an opportunity to learn about key aspects of reform-based instruction that may be unfamiliar to them (van Es et al., 2017). Readings were selected from the NCTM's practitioner journals and reflected the aims of the eight *Mathematics Teaching Practices* outlined in NCTM's *Principles to Actions: Ensuring Mathematical Success for All* (2014). They included questioning strategies teachers could use to move their students' thinking forward, developing productive classroom discourse, sequencing and connecting strategies to develop student understanding, etc. Readings were selected with the intent to both inform PSTs about important components and examples of reform-based instructional practices and to provide PSTs with opportunities to reflect on how they might integrate components of reform-based instruction into their future classrooms.

As noted above, one important aspect of reform-based instruction is using tasks with multiple entry points to allow all students access and to engage them in the development of their own thinking and understanding. To focus on this aspect, the following was used as a course reading: *Building A Discourse Community: Initial Practices* (Hodge & Walther, 2017) which discusses how to encourage student participation through discourse in the mathematics classroom. Consistent with the other instructional activities, PSTs' reflection on course readings was prompted, this time in an online forum. Prompts were also tailored to the selected readings to direct PSTs' attention to specific instructional components that embody reform-based mathematics instruction (van Es et al., 2017) and their implementation in their future classroom (Wilkerson et al., 2018) (see Table 1).

OBSERVED SHIFTS ACROSS INSTRUCTIONAL TASKS: CONSIDERATIONS FOR TEACHER EDUCATORS

In this section, the authors explicitly link the theoretical model of mathematics teaching efficacy outlined above with a description of PSTs' responses to the instructional activities in the mathematics methods course. The discussion of these responses is organized according to three shifts observed across PSTs' observations and reflections that the authors suggest may constitute new sources of mathematics teaching efficacy in the context of reform-based mathematics instruction.

Increased Focus on Teacher Actions to Facilitate Student Thinking

At the beginning of the course, PSTs tended to focus on traditional aspects of instruction, that is, teacher modeling and student performance of procedural steps. Following the course and instructional activities, in contrast, PSTs' reflections suggest that they came to view the teacher as more of a facilitator of the classroom and developer of student understanding. Although PSTs still focused on the teacher near the end of the course, the role they saw teachers playing in mathematics instruction better reflects the goals of reform-based instruction. This change in the perceived role of the teacher was observed across all instructional tasks.

For example, prior to the use of scaffolded video prompts, when asked to reflect on a video-taped lesson, PSTs focused on general student engagement and on the teacher's overall management of the classroom, as characteristic of traditional mathematics instruction. In contrast to reflections at beginning of the course, by the end of the methods course, PSTs focused more often on specific examples of student understanding and what the teacher did in the video to address or further student understanding. This shift in focus when watching video-taped lessons was noted in all PSTs who were enrolled in the course, and is illustrated below by one PST when prompted generally about the video-taped lesson they observed at the end of the course:

> *As the lesson progressed, the students were modeling their thinking through different strategies. The teacher highlighted many different opinions throughout the classroom and those students either discussed or lead their thinking at the front of the classroom. The teacher never discussed the idea of the commutative property but the students highlighted the idea of the concept through their ideas.*

Similarly, a change in the perceived role of the teacher in mathematics instruction was also observed in PSTs' lesson plans and reflections on course readings. In lesson plans constructed prior to the intervention, PSTs described the teacher's role as central; specifically, as a modeler of the mathematics students were intended to replicate. For example, one PST launched a task, assuming students knew multiplication, and used the exploratory phase as a way for the teacher to model a single strategy and have students practice that one strategy, stating, "The strategy for solving multiplication by 8 is the double, double, double strategy. The teacher will use 6 x 8 for example. We take the factor (that is not 8) and we are going to double it." The lesson continued by asking students to replicate this specific strategy. Despite the nature of the lesson planning template, PSTs' initial lesson plans appeared to be designed according to traditional approaches to mathematics instruction.

At the end of the course when PSTs wrote their final lesson plans, the role of the teacher depicted in these plans seemed to shift from that of a central lecturer to more of a facilitator of instructional conversations. In these final lesson plans, teachers' actions, as reflected in the lesson plans, appeared to be designed to support the development of student thinking and understanding, thus more clearly embodying the goals of reform-based mathematics instruction. As evident in the following example, the actions of the teacher in PSTs' lesson plans shifted from a modeler of mathematics to a facilitator of student thinking, as would be expected in the actions of a teacher in a reform-based mathematics instructional setting. The PST below did not funnel students to a singular method of solving the task, but instead incorporated students' methods to orchestrate a whole class discussion:

> *The teacher will make notes on each student/group and the strategy they are using. This information will help rank the strategies used, clear up common misconceptions, and will help when students are asked to share their strategies under the document camera to the class and participate in whole group discussion.*

After outlining the use of student strategies to guide the classroom discussion, the PST included the order in which they would present anticipated students' responses and strategies, building in complexity:

> *I would first share the misconception, only if others are also making similar mistake. I would then have students who solved the problem by drawing a picture, array, or table share first. All students could connect to this thinking as we have worked with arrays in the past. All students could clearly understand the problem and clear up misconceptions if seeing it displayed as a visual representation.*

This continued with three more strategies, leading to the desired strategy of using an "unknown" factor:

> *Last, I would have the students who knew that 12 x 8 =96 go last because they set up their division problem by using the unknown factor in a multiplication problem which is what we want to focus on. I would then go back to strategy 3 and show how division and multiplication can both help solve the same problem*

As demonstrated by this progression, the PST became the facilitator of student strategies, connecting them in a way that helped advance student thinking from their current level of understanding, and orchestrating students' ideas in a way that was mathematically productive.

Following the course reading, *Building A Discourse Community: Initial Practices* (Hodge & Walther, 2017), students were prompted to reflect, "As you think about your own classroom, how will you incorporate the '4 Initial Practices to Build a Foundation of Productive Discourse' in your teaching?" In response to this prompt, one PST focused on facilitating a classroom discussion, specifically centered around students' strategies and understanding:

> *Building this discourse in the classroom begins by providing the students with more opportunities to develop knowledge. In my classroom, engaging students in a more broad task seems beneficial… As the discussion builds, the students can discuss the strategies and techniques they used to solve their problems and learn from other students.*

Here, the PST is clearly focusing on student thinking that is facilitated by a class discussion and further developed by engaging with other students.

Similarly, when reflecting on another course reading, *Differentiating Instruction in Mathematics for the English Language Learner* (Murrey, 2008), several PSTs mentioned the role of the teacher as the facilitator of the classroom and in moving all students' understanding forward. Specifically, one PST mentioned: "When solving a math task the teacher should encourage students to further their understanding by listening, reading, writing and discussing. It may be helpful for students to be in groups to really discuss the math with peers…" Again, the PST's focus here is on the teacher providing opportunities for students to develop understanding of concepts rather than on their ability to lead students through step-by-step procedures. These examples of PSTs' reflections suggest that the reflective activities in the methods course provided an opportunity for PSTs to consider some of the key elements of reform-based mathematics instruction to inform their future teaching.

Evidence of Student Thinking and Understanding

Another change noted in PSTs' reflections, lesson plan designs, and comments about readings was that they placed a greater emphasis on student thinking and understanding following the use of these instructional tasks than at the beginning of the course. Prior to the intervention, PSTs tended to focus on students' ability to replicate procedures or their general engagement. However, following the implementation of the instructional tasks in the course, PSTs focused more on student thinking as illustrated by their written comments, use of multiple strategies, and perseverance when solving tasks.

Evidence of student thinking was lacking in PSTs' initial observations and reflections across the three instructional tasks. However, consistent with the aims of reform-based instruction, PSTs came to emphasize the idea of embracing productive struggle and connecting to students' current level of understanding throughout the instructional tasks in the course. At times, PSTs' comments seemed to suggest a growing sense confidence in their ability to successfully deliver the lesson described in their plan in relation to their ability to successfully develop student understanding. For example, one PST stated: "I feel very confident if I were to teach this lesson. I feel the task is cognitively demanding but as we have learned-- we want students to struggle... I believe I set up assessing and advancing questions to benefit students' understanding." Another PST focused on the use of multiple strategies and class discussion to model student thinking when prompted about a lesson: "As the lesson progressed, the students were modeling their thinking through different strategies. The teacher highlighted many different opinions throughout the classroom and those students either discussed or lead their thinking at the front of the classroom."

This shift is also illustrated by the following PST when reflecting on a course reading, *Strategies to Support the Productive Struggle* (Warshauer, 2015). PSTs were prompted to think about how one of the four strategies mentioned in the article would look in their future classroom. In response, one PST focused on questioning and student understanding:

> *When you question students you are encouraging them to use their voice and explain... When you question students you [are] getting them to think about the why and how. When students start thinking about why and how they can ask questions and have the potential of full understanding…*

Understanding, according to this PST's reflection, includes the ability to explain, not just replicate procedures, as recommended by reform-based mathematics approaches. As illustrated in both examples above, following their experiences with the instructional tasks, PSTs tended to focus more on student thinking and understanding, specifically on the ways in which students can demonstrate their understanding. The changes observed across PSTs' reflections and observations suggest that these activities helped focus their attention on student thinking about mathematical content.

Evaluating Student Thinking and Understanding

What is considered to be evidence of student understanding varies with the instructional styles used. In traditional mathematics instruction, student understanding is often considered to be evident in students' ability to replicate procedures or compute correct answers. However, in the context of reform-based mathematics instruction, evidence for student understanding requires more than a demonstration of a correct procedure; instead, teachers must look for evidence of deeper, conceptual understanding such as explanation, application, or interpretation. To develop conceptual understanding, teachers need to devote attention to students' current level of understanding, and allow them to make meaningful connections. Across all tasks, PSTs demonstrated a shift in their ability to gauge student understanding through a variety of instructional actions in contrast to their initial reflections.

Prior to engaging in the instructional activities, the PSTs designed their first lesson plan, and were then asked "How will you know that students understand the mathematical concept targeted in your lesson?". One PST responded: "the students are able to compute the answer and show their work by using the new strategy [as demonstrated by the teacher]." This PST's comment suggests they initially believed that student understanding is demonstrated through computations or written work, consistent with the intentions of traditional mathematics teaching which emphasizes the accuracy of a solution only. Similarly, another PST stated that: "The 'exit ticket' assessment piece of this lesson will show me if students are successfully understanding this concept," indicating that the exit ticket would serve as a summative assessment. At the end of the course, however, one PST also mentioned the exit ticket but also included a description of how they would check for understanding throughout the lesson in order to prepare them for the class discussion:

> *The exit ticket is the grand summation of the lesson, but it's at the end. I would walk around during the explore phase to see which of my students was A) understanding the information and B) using the different strategies. The basic to more complex strategies offers some insight into student understanding and helps me know where I need to funnel my emphasis for later.*

The PST in the quotation directly above voices their sense of the importance of evaluating the level of student understanding to develop their thinking throughout the lesson versus assessing only the students' ability to compute the correct answer. This contrasts with the earlier the example in which the PST uses the exit ticket only as a summative assessment, Another PST also referred to evaluating students' level of understanding throughout the lesson: "The variety of different assessments I add to the lesson will show if they are understanding the concept. Discussion and explanation of their strategies is where I will learn the most about their understanding during the lesson…" Again, this PST emphasizes evaluating student understanding through the use of multiple methods throughout the lesson, not only by means of a summative assessment at the end of the lesson that focuses on the attainment of correct answers.

Similarly, at the end of the course, when asked "How do you think this lesson went?" following a video observation, one PST focused on the development of student understanding throughout the observed lesson. This PST evaluated students' level of understanding through their verbal responses and ability to explain their thinking: "The students were verbal with their understanding of the concept and explained their thinking when asked too. The teacher highlighted several strategies that the students used and the class acknowledged their understanding of those strategies."

In placing a greater emphasis on the use of multiple sources to evaluate student understanding throughout a lesson, PSTs' responses better reflect the goals and aims of reform-based instruction, which center on student thinking and understanding. In the next section, the authors discuss how shifting PSTs to an increased focus on student understanding may promote the development of their mathematical teaching efficacy.

IMPLICATIONS FOR TEACHER EDUCATORS

The three themes across instructional tasks demonstrate PSTs' increased ability to focus on key components of reform-based instruction, serving as potential sources to draw on for their mathematics teaching efficacy. Instead of gauging their ability to teach mathematics effectively by means of examining students' performance on standardized assessments and their ability to replicate procedures for correct answers, PSTs can develop beliefs about their own mathematics instruction in light of how it supports their students' growing understanding. In addition, the instructional tasks help focus PSTs' attention on their own role in facilitating this understanding. As observed across the instructional activities described above, shifting PSTs' focus to student understanding offers a potential new source to draw on for developing their mathematics teaching efficacy that fits with the culture of reform-based mathematics instruction.

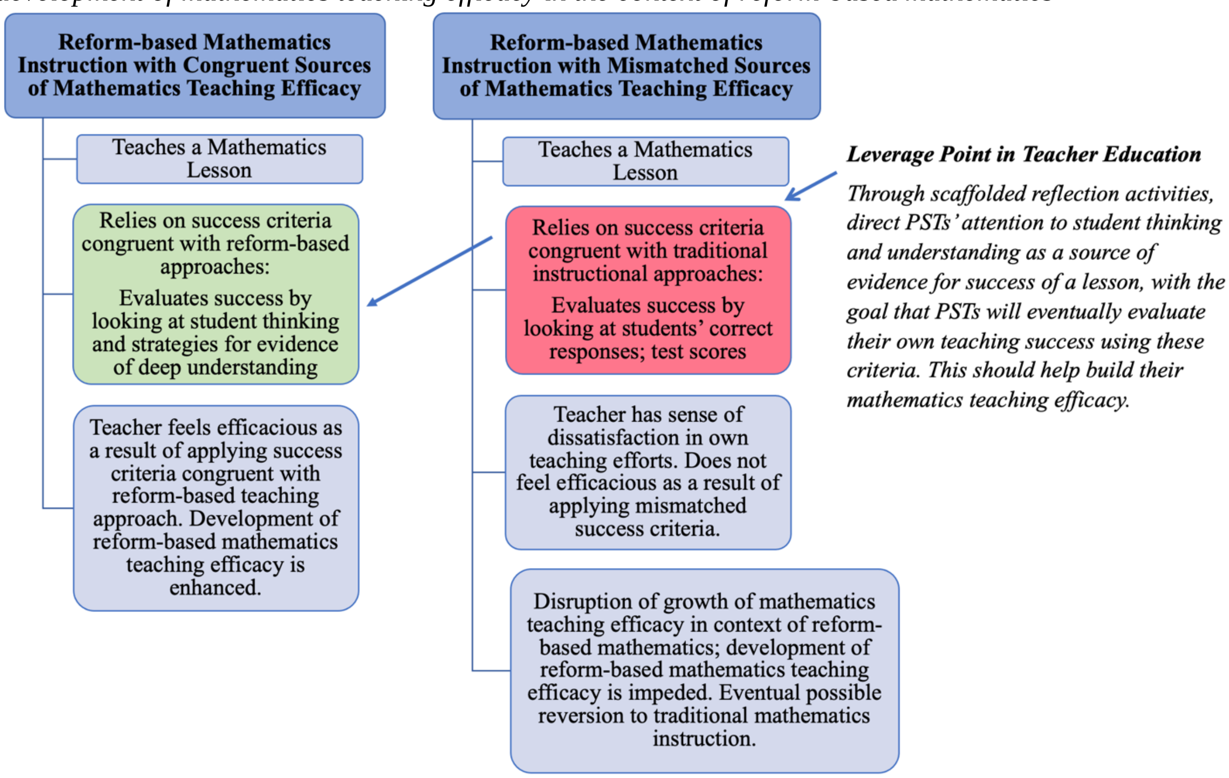
In making this transition, confidence in PSTs' own mathematics teaching becomes redefined. For example, prior to the mathematics methods course, one PST referenced a single strategy they would use in teaching their lesson plan as the sole basis for their confidence in teaching it effectively, stating: "I am fairly confident because I have taught this lesson while subbing 4th grade before. This helps students with figuring out their x8 math facts." This confidence was based on their prior experience (substitute teaching) with this specific content, using a single strategy (double-double-double). This limited prior experience seems to fit with traditional approaches of mathematics instruction, placing greater emphasis on procedural computations without facilitating students making deeper connections to the concept, similar to findings of another study with a larger group of PSTs (Althauser, 2018).

In contrast, following the course, PSTs expressed confidence in their ability to teach mathematics that better reflects the goals of reform-based instruction. This entailed gauging their own ability to effectively facilitate a reform-based mathematics classroom: predicting student strategies, developing questions for students in a way that attended to their level of thinking and understanding, and using anticipated student strategies to orchestrate discussions.

CONCLUSION

In summary, throughout this chapter the authors argue that the lack of uptake and consistent, sustained use of reform-based methods in mathematics (Hill, 2021) may stem to some extent from the mismatch between use of reform-based methods and the criteria teachers seek out to evaluate their own teaching success. As depicted in Figure 3, when a teacher adopts reform-based teaching methods yet continues to evaluate their success by drawing on sources of mathematics teaching efficacy that are more suited to traditional teaching methods, this may lead them to feel a sense of failure and revert back to traditional methods of teaching mathematics. The presence of this kind of mismatch reflects gaps in professional development programs aimed at reform-based mathematics instruction, as well as in mathematics methods courses in teacher education programs.

Figure 3. Congruent and incongruent sources of evidence for lesson success with subsequent impact on development of mathematics teaching efficacy in the context of reform-based mathematics



As shown in Figure 3, a key leverage point for change may lie in shifting PSTs to draw on those sources of mathematics teaching efficacy that match the goals of reform-based mathematics. The authors have suggested that teacher education is an opportune time to target the development of PSTs' mathematics teaching efficacy, and the examples in this chapter of activities in a mathematics methods course illustrate one way that teacher educators can facilitate a fuller adoption of reform-based methods. Each of the three reflective components of this mathematics methods course were designed to facilitate PSTs' developing ability to attend to student thinking and understanding and use it to evaluate the effectiveness of their mathematics teaching. This may ultimately lead to increased mathematics teaching efficacy and continued, lasting use of reform-based mathematics instruction. The authors have argued here that it is only when teachers look to these sources to evaluate their own success in teaching, will they fully embrace reform-based teaching by building the kind of mathematics teaching efficacy that sustains the use of reform-based mathematics teaching.

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ADDITIONAL READING

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KEY TERMS AND DEFINITIONS

Mathematics Teaching Efficacy: One's belief in their ability to successfully teach mathematics and influence their students' learning of mathematics.

Pedagogical Content Knowledge: Specialized knowledge around the how to instruct in a particular subject area.

Reflection: Conscientious thought about actions or ideas.

Scaffolded Reflection: Directing attention to specific considerations for reflection.

Self-Efficacy: One's belief in their ability to successfully perform a task.

Student-Centered Mathematics: Mathematics instruction centered around students' thinking and current level of understanding, often explorative in nature.

Teaching Efficacy: One's belief in their ability to successfully teach and influence their students' learning/performance.

Traditional Mathematics Instruction: Mathematics instruction centered around direct and often, procedural instruction from the teacher who disseminates this knowledge to students.

Vicarious Experiences: Experience through observation of others.