1-1-1986

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Occasional Paper No. 8607
December, 1986

Occasional Papers

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by

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This paper was presented at the Atlantic Economic Society meetings on December 28, 1986 in New Orleans.

*Research support for this paper has been provided by the Center for International Studies, University of Missouri-St. Louis.

ABSTRACT

An Extended Dynamic Asset Model Revisited

The current account has played a major role in earlier theories of balance of payments and exchange-rate determination. Contemporary theories explain exchange rates in the asset markets. But the current account, through asset accumulation or decumulation, changes the portfolio positions and, hence, exchange rates over time. This paper looks at the dynamic effects of wealth transfers on exchange rates in an extended asset model. A less restrictive stability condition under adaptive and rational expectations is applied.

An Extended Dynamic Asset Model Revisited

1. Introduction

This paper looks at the dynamic effects of wealth transfers (due to current account imbalances) on the dynamic behavior of state variables. The model developed here extends the work of Dornbusch and Fischer (1980) in one respect. We depart from the perfect substitutability of domestic and foreign currency denominated financial securities. This establishes a more general setting of the model in a Mundell-Fleming framework. It also departs from the Masson (1981) model by introducing a goods market adjustment process. The rationale for this introduction is, that the current account not only responds to exchange rate and stock of foreign assets (Masson), but also to real income. The adjustment process of the goods market guarantees the endogenity of both real income and hence the current account.[1]

In the financial sector of this model, private citizens are allowed to hold a composite portfolio of domestic money balances, home bonds, and foreign denominated securities.[2]

$$w = M + B^d + EF$$

where w, M, B^d, E, and F denote real wealth, real cash balances, portion of domestic securities held by home residents, home currency price of foreign exchange, and foreign currency denominated securities held by home residents, respectively.

This single-country economy solves the equilibrium values for real income (Y) in the goods market, exchange and interest rates (E and r) in the financial markets.

Section Two depicts the dynamic structure of the model. The stability properties under both static and adaptive-rational expectations are discussed. Conclusions are described in section Three.

2. The Dynamical Model

Let X denote the state space of all the feasible states of the model. Stating it differently, X can be described as a three-dimensional vector: X = (E, Y, r). A reaction function for the model is depicted by a set of ordinary differential equations.

$$\dot{x} = dx_i(t)/dt = f(x(t),a) \qquad (1)$$

where t is time and a is a multi-dimensional vector of parameters.[3]

The function f defines a three dimensional vector of excess demand for foreign currency denominated bonds, domestic goods, and domestic bonds in terms of the same state variables and vector parameters. An excess demand for foreign bonds would cause home private wealth holders to realign their portfolio by replacing real cash balances and/or home bonds with foreign bonds in the foreign exchange market. Home currency depreciates, and E rises. The basic underlying hypothesis for excess demand in the goods market is the producers' response to unexpected inventory changes: as firms find their inventories unexpectedly decreasing, they increase output to meet the demand for goods. The same equation (1) indicates that change in home interest rate, r, is proportional to excess supply of home bonds. If the stock of home bonds held by home private wealth holders rises relative to its demand, the Treasury must offer a higher interest rate to borrow from private wealth holders.

The desired demand for foreign bonds is:

$$b^*(r,\ e,\ Y)w, \qquad b^*_r < 0, \quad b^*_e > 0, \quad b^*_Y > 0$$

where e is the expected future domestic currency price of foreign exchange.

The actual supply of foreign bonds is:

$$E(F+\dot{F})$$

A linear version of equation (1) for x_1, i.e., a reaction function for E, is:

$$\dot{E} = \alpha\,[b^{*}(r,e,Y)w - E(F+\dot{F})] \qquad (2)$$

$$0 < \alpha < 1$$

The desired demand for domestic goods is:

$$c[1-t][Y+rB^{d}+Er^{*}F] + I(r) + G + x([1-t][Y+rB^{d}+Er^{*}F],E) + f$$

where c and t are marginal propensity to consume and tax, respectively. I, G, x, and f denote investment, government expenditure, and autonomous consumption respectively.

$$I_r < 0, \quad x_E > 0, \quad x_Y < 0$$

The reaction function for the goods market is given as:

$$\dot{Y} = \beta\{\, c[1-t][Y+rB^{d}+Er^{*}F] + I(r) + G + x(.) + f - Y \} \qquad (3)$$

$$0 < \beta < 1$$

The desired demand for domestic bond is:

$$b^{d}(r,e,Y)w + b^{f}(r,e)w^{*}$$

$$b^{d}_{r} > 0,\ b^{d}_{e} < 0,\ b^{d}_{Y} > 0; \quad b^{f}_{r} > 0,\ b^{f}_{e} < 0; \ 0 < b^{d},\ b^{f} < 1$$

where the first (second) term in the above formulation describes the home (foreign) demand for domestic bonds.

The actual change in domestic bond is:

$$B + \dot{B}$$

The reaction function, change over time in r, is given as:

$$\dot{r} = \lambda\,[B + \dot{B} - b^{d}(r,e,Y)w - b^{f}(r,e)w^{*}] \qquad (4)$$

$$0 < \lambda < 1$$

The government budget constraint is described as:

$$\dot{M} + \dot{B} = g = G + rB - t[Y + rB^d + Er^*F] \qquad (5)$$

where B^d describes the portion of B held by home residents.

The wealth constraint is described as:

$$w = M + B^d + EF$$

The balance of payments constraint is denoted by:

$$x([1-t][Y+rB^d+Er^*F], E) + [\dot{B}^f-E\dot{F}] + [Er^*F-rB^f] = 0 \qquad (6)$$

Rearranging equation (6), a capital outflow can be specified as:

$$[E\dot{F}-\dot{B}^f] = x(.) + Er^*F - rB^f$$

Or

$$E\dot{F} = x(.) + Er^*F - rB^f + \dot{B}^f \qquad (7)$$

Assuming that the outstanding domestic bonds can be accumulated by both domestic and foreign wealth holders, we have:

$$\dot{B} = \dot{B}^d + \dot{B}^f$$

$$\dot{B}^f = \psi\dot{B}, \qquad \psi_r > 0 \qquad (8)$$

Equation (8) assumes that, <u>ceteris</u> <u>paribus</u>, the domestic and foreign portfolio ratio of domestic bonds is a function of home interest rate.

Using equations (5 and 8), $\psi\theta g$ is substituted for $\dot{B}^f$ in equation (7) to get:

$$E\dot{F} = x(.) + Er^*F - rb^f + \psi\theta g \qquad (9)$$

where $0 < \theta < 1$ describes that portion of budget deficits financed by issuing new domestic securities.[4]

The dynamic structure of the model can be reduced to the following equations:

$$\dot{E} = \alpha[b^*(r,e,Y)w - EF - x(.) + Er^*F - rB^f + \theta\psi g] \qquad (10)$$

$$\dot{Y} = \beta[c[1-t][Y+rB^d+Er^*F] + I(r) + G + x(.) + f - Y] \qquad (11)$$

$$\dot{r} = \lambda[B + \theta g - b^d(r,e,Y)w - b^f(r,e)w^*] \qquad (12)$$

$$g = G + rB - t[Y + rB^d + Er^*F] \quad (13)$$

$$w = M + B^d + EF \quad (14)$$

To get equation (10), we substituted $E\dot{F}$ from equation (9) into the reaction function (2). Equation (11) is the same as equation (3). Equation (12) is derived by substituting θg for B in equation (4). Equations (13) and (14) are simply the budget deficit and wealth constraint, respectively.

The set of ordinary differential equations (10), (11), and (12) describe the state of the reaction function of equation (1) including budget, balance of payments, and wealth constraints. This reaction function gives the state of the economy at time t if the economy was in the state space of $x(x_1=E,\ x_2=Y,\ x_3=r)$ at time 0.

2a. Steady State Solution and Stability

With home price level, P, and foreign rate of interest, r^*, given, equilibrium in the asset market requires that the existing stock supply of financial assets be equal to their respective demands.[5] Using the wealth constraint implication, we drop the market clearing condition for real balances.[6] Thus, a steady-state solution in this model requires equilibrium in bonds and goods markets. These conditions are met when we set equation (1) or equations (10, 3, and 11) equal to zero.

The local stability of this model is insured if:

$$\sum_{i=1}^{3} f_{ii} f(x(t),a) < 0 \quad (15)$$

The above condition asserts the fact that the scalar summation of the principal diagonal of the Jacobian matrix of equation (1) (the trace of the matrix) must be negative in order for the model to return to equilibrium, after it was disturbed. The literature has treated this as a necessary but not sufficient condition. A positive determinant of the same matrix is the sufficient condition.[7] However, Olech (1963) has proved that, given the above condition (15), the system is asymptotically stable if a boundary condition applies. Thus the stability condition (15), in terms of equations (10, 11, and 12), can be rewritten as:

$$[\partial\dot{E}/\partial E] + [\partial\dot{Y}/\partial Y] + [\partial\dot{r}/\partial r] < 0$$

which is equal to:

$$\alpha\{-[1-b^*]F - [1-t]r^*Fx_E - x_E - \theta\Psi tr^*F\} + \beta\{[1-c[1-t]] + [1-t]x_Y\} + \lambda\{\theta[B-tB^d] - wb_r^d - w^*b_r^f\} \qquad (16)$$

Literature has conferred that the stability condition requires the country to be a net debtor ($F < 0$).[8] Studying condition (16) reveals that stability may not necessarily require F to be negative. In the above stability condition all terms except the first $-[1-b^*]F$ and seventh $\theta[B-tB^d]$ are negative. If we assume a balanced budget in the steady state solution, the latter vanishes. Thus, even if $F < 0$, other terms may either individually or collectively offset the former, and the economy may still achieve the stability condition of (15) or (16).

2b. Expectations

The stability condition was studied in the above subsection in a context of static expectations. In other words, the model assumed that current exchange rate should be used as the best predictor of its future value. Here we depart from this assumption by introducing some form of an expectation formation mechanism. The starting point is an adaptive expectation formation:

$$\dot{e} = \gamma[\dot{E}-e] \qquad (17)$$

where γ is a positive adjustment parameter. Scarth (1977) argues that as γ approaches infinity adaptive expectations become rational.[9] We incorporate this concept into our model by replacing $\dot{E}$ in the expectation formation (17) with the reaction function (10). Consequently, the dynamic structure of our model in the context of expectations is given by equations (11), (12), and (17). Equations

(13) and (14) are constraints.

Stability of this model, using condition (15), gives a trace of the Jacobian matrix where:

$$[\partial\dot{e}/\partial e] = \gamma\alpha[b^{*}w - 1] \tag{18a}$$

$$[\partial\dot{Y}/\partial Y] = \beta[1-c[1-t]] + [1-t]x_{Y} \tag{18b}$$

$$[\partial\dot{r}/\partial r] = \lambda[\theta[B - tB^{d}] - wb_{r}^{d} - w^{*}b_{r}^{f}] \tag{18c}$$

Taking the value of wealth into account, $[\partial\dot{e}/\partial e]$ is positive while those of $[\partial\dot{Y}/\partial Y]$ and $[\partial\dot{r}/\partial r]$ are negative. Stability of this model would be achieved if the first term is offset by the remaining two terms. This raises an interesting question. Can the well known concept of unstability of expectational behavior in the foreign bond market be offset through interaction with domestic bond and goods markets?

As long as expectation formation is taking an adaptive form in interacting with the rest of economy, it does produce an overall stability of the model. But, if expectation formations approach rationality (γ approaches infinity), the trace of the Jacobian matrix would become positive and the model will be unstable.

3. Conclusions

The current account has been introduced in dynamic asset models to capture the effect of wealth accumulation on state variables. This paper departed from the previous work in many respects. First, by introducing a goods market reaction function, income became endogenous. This established a formal link between wealth transfer (capital flows) and net exports through an important factor, real income. Second, it was argued that a less restrictive local stability can be achieved. Third, net debting may not necessarily lead to instability of the model. This was examined both under static and adaptive/rational expectations.

Footnotes

1- For recent work on the current account and exchange rates see also Dornbusch (1974), Frenkel and Rodriguez (1975), Lapan and Enders (1978), Obstfeld (1980) and Razin and Svensson (1983).

2- For a survey of asset models, see Dornbusch (1980).

3- In this paper, the symbols [.] and (.) denote scalar multiplication and functional relations, respectively.

4- Real cash balances finances [1-θ] portion of budget deficits.

5- Relaxing this assumption would not change the results substantially. Gylfason and Helliwell (1984) incorporate aggregate supply, among others, into their models.

6- Foley (1975).

7- See Barnson, Halttunen, and Masson (1977) among others.

8- Ibid.

9- Evans and Yarrow (1979) conclude that some care must be given in considering rational expectation as a limiting case of adaptive expectation.

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