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Forecasting Demand for Optimal Inventory with Long Lead Times: An Automotive Aftermarket Case Study

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An Automotive Aftermarket Case Study

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Abstract

Accuracy in predicting customer demand is essential to building an economic inventory policy under periodic review, long lead-time, and a target fill rate. This study uses inventory and customer service level as a stock control metric to evaluate the forecast accuracy of different simple to more complex predictive analytical techniques. We show how traditional forecast error measures are inappropriate for inventory control, despite their consistent usage in many studies, by evaluating demand forecast performance dynamically with customer service level as a stock control metric that includes inventory holdings costs, stock out costs, and fill rate service levels. A second contribution includes evaluating the utility of introducing more complexity into the forecasting process for an automotive aftermarket parts manufacturer and the superior inventory control results using the Prais-Winsten, an econometric method, for non-intermittent demand forecasting with long-lead times. This study will add to the limited case study research on demand forecasting under long lead times using stock control metrics, dynamic model updating, and the Prais-Winsten method for inventory control.

Keywords: inventory control, Prais-Winsten, automotive parts, customer service level, stock control, rolling origin cross-validation, dynamic model updating.

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Chapter 1: Introduction

Many businesses that carry inventory either have too much of it, which is expensive, or too little, leading to stockouts and lost sales. Inventory constitutes the most significant portion of current assets for most manufacturing firms tying up significant organizational capital (Singh & Verma, 2018). An essential factor in firms achieving optimal inventory is demand forecasting (Kocer, 2013), part of the sales and operational planning process. Managers need to identify suitable sources of external data and simple analytical tools that are easy to use to reliably gauge the effectiveness of demand forecasts and draw conclusions on what inventory to order (Blackburn, Lurz, Priese, Göb, & Darkow, 2015). Long lead times can lead to inaccurate forecasts caused by delays in replenishment, which is one of the many reasons for poor inventory management (i.e., supplier delivery performance, poor material yields, poor supplier quality, inappropriate order quantities). Accuracy in forecasting demand is crucial to developing a good inventory policy and managing an effective supply chain. The use of complex forecasting methods increases the opportunities for errors in judgment, understanding, prediction, and explanatory power (Green & Armstrong, 2015), so simple analytical methods are essential for practical use and easier assimilation. Simple forecasting and accurate demand planning are large factors in appropriately managing optimal inventory.

This study focuses on demand forecasting under long lead times using dynamic model parameter updating for exponential smoothing (ES), its variants double ES, triple ES; linear regression, autocorrelation using the Prais-Winsten transformation, and some more straightforward time series methods naïve and simple moving average (SMA) as we seek to answer the question: do complex forecasting methods increase forecasting accuracy? The study will also add exogenous data from Google Trends to answer: Can exogenous data improve demand forecasting?

The related literature typically addresses optimal inventory and demand forecasting as separate questions. However, the availability of cost information will estimate the economic effects of changing forecast parameters on inventory. Finally, the study will examine the relationship between traditional forecast measures of accuracy, such as Mean Error (ME), Root Mean Squared Error (RMSE) or Mean Absolute Error (MAE), and customer service level (CSL) as stock control metric in the calculation of economic order quantity (EOQ), which consider measures like holdings costs, ordering costs, and service levels based on fill rate. All to answer the question: how can CSL stock control metrics be used to evaluate forecast accuracy?

The overall goal is to determine the best use of historical data to make ordering decisions with long lead times and find a relatively easy to use optimal inventory policy with periodic review and a target fill rate using CSL stock control metrics for an automobile aftermarket parts company without introducing too much complexity into the forecasting process. Moreover, it adds to the limited empirical research on demand forecasting under long lead times, CSL stock control metrics, and dynamic model parameter updating. This study will try to answer the question: Can a procedure be developed that is likely to be adopted?

Analytics

Big data and predictive analytic (BDPA) tools used to improve decision making and material flow are rapidly evolving within supply chain analytics, growing in response to the volumes of data made available in the Internet age. The analytical methods used in supply chain analytics fall into three types: descriptive analytics, predictive analytics, or prescriptive analytics (Souza, 2014). Descriptive analytics looks at past events up to the present (real-time) and tries to answer what happened or what is happening. For example, analyzing radio frequency identification (RFID) location data to understand how material flowed through the plant to streamline material flow, optimize material handling, or track inventory. Predictive analytics evaluates the output from descriptive analytics to forecast or predict the likelihood of what will happen at a future time. For example, predictive maintenance uses past machine failure data to estimate the likelihood of critical components failing to schedule machine maintenance cycles.

Prescriptive analytics builds on both the descriptive and predictive analytics outputs to determine the best course of action or what should happen, or how it can be made to happen. For example, using data on past deliveries to determine how long it will take (lead time) for the supplier deliveries, or using past production data to determine the turnaround time to fill the order, or using both to prescribe the order quantity, given the variability in demand, supplier lead time, and production lead time. Another example is using past sales ordering or demand data from previous customers to determine how much stock was needed (order quantity) to meet the demand to determine the stocking order for a new customer account. New customers have no previous sales data to calibrate the stocking order, but there is data on similar customers and their order patterns to estimate new account ordering. Predictive and prescriptive analytics are very similar. The difference is that predictive analytics is focused on the outcome, while prescriptive analytics considers various future situations to prescribe a course of action.

In the last few decades, interest in data science, machine learning, and the use of big data has exploded, bringing with it a host of new conferences, journals, software companies, and even prizes for forecasting research (R. Fildes, Nikolopoulos, Crone, & Syntetos, 2008b). Interest in supply chain analytics has grown alongside data science, emphasizing predictive analytics for demand forecasting.

Predictive Analytics Assimilation

Supply chain organizations have collected and stored vast amounts of digital data for years (Dekker, Pinçe, Zuidwijk, & Jalil, 2013). These datasets have enabled the growth in supply chain management (SCM) data analytics techniques involving data mining and statistical analysis to develop more accurate predictive analytics to forecast behavior. The collection and sharing of information along the supply chain result in a more intelligent supply chain armed with analytical tools and techniques to be more efficient and allow more data-driven decisions (Govindan, Cheng, Mishra, & Shukla, 2018), and improve profitability. Effectively utilizing vast amounts of historical and real-time data to improve the organizations' performance and their supply chains are what big data and predictive analytics (BDPA) promises. Assimilating predictive analytic methods into the organization is one area of sustaining and disruptive technology research growing in importance within both academics and practitioners of SCM.

BDPA is used to solve complex supply chain problems and improve overall business process performance (Wang, Gunasekaran, Ngai, & Papadopoulos, 2016). Problems like the bullwhip effect, demand forecasting, order flow along the supply chain, and optimizing flow use analytics to improve business performance. There are many opportunities inside existing processes using descriptive analytics, forecasting future demand with predictive analytics, and making better decisions using prescriptive analytics. Although, companies struggle with assimilating methods with a sufficient understanding to utilize the forecasts to make better decisions.

BDPA assimilation across the organization and occurs in three phases starting with acceptance, moving on to a routine, and ending in the assimilation of BDPA (Hazen, Overstreet, & Cegielski, 2012). The acceptance stage encompasses the growing awareness of BDPA and how well stakeholders understand the scope of BDPA in their job. The routine phase begins when the organization's systems of governance are altered to incorporate BDPA. Furthermore, assimilation occurs when BDPA has spread through all affected business processes. BDPA assimilation research has found a positive association with both organizational performance (OP) and supply chain performance (SCP) (Gunasekaran et al., 2017). Once assimilated, data is parsed into actionable knowledge items displayed with visual dashboards to identify problems (Bumblauskas, Nold, Bumblauskas, & Igou, 2017). While some have found data quality or data security a significant barrier (Verma, Bhattacharyya, & Kumar, 2018), others believe the most significant barriers to BDPA adoption are learning how to use BDPA to improve performance (LaValle, Lesser, Shockley, Hopkins, & Kruschwitz, 2011). Some have suggested a tiered model involving the three primary elements of management, technology, and human capability (Akter, Wamba, Gunasekaran, Dubey, & Childe, 2016). BDA skills, talent, and management capability are emerging as the strongest indicator of BDA success, suggesting a well-developed approach to recruiting analytics talent (Court, 2015) will help achieve a sustainable competitive advantage with BDA. To assist in overcoming these barriers, Lamba and Singh (2018) found the most significant driving enablers for big data deployment and use are top management commitment, financial backing, BDPA skills, organizational BDPA infrastructure, and a change management program.

There are many positive SCM effects to using BDPA, ranging from increased supply chain visibility, efficiency, and maintenance to improved integration, collaboration, and product design (Kache & Seuring, 2017). BDPA has evolved into a vital strategic component adding a new competitive advantage for those companies that fully embrace its' assimilation into the organization. Companies that have learned to sift through substantial amounts of historical supply chain and public data have positioned themselves to improve decision making and deliver the efficiency and effectiveness they desire, with lower costs, greater global SCM capabilities, increased BDPA skills, and a sustainable competitive advantage. Assimilating BDPA technology and methods into the organization is the key to securing a competitive advantage.

The evaluation process starts with the baseline forecast that the company uses now compared with various forecasting methods and the additional predictor variables to determine the new improvement level. The intent is to compare what the automotive parts manufacturer is currently doing for forecasting to methods for demand forecasting based on predictive analytics research that combines exogenous data. We will use stochastic inventory models, stock control, and customer service level metrics to evaluate the performance of the demand forecasts. This study intends to address uncertain demand using a prescriptive analytics approach to determine optimal inventory. Although data from the automotive aftermarket space is the primary focus, the study methods apply to other companies or markets seeking to solve the uncertain demand problem.

Optimal Inventory

Previous research into optimal inventory policy has focused almost exclusively on customer demand using various forms of the Economic Order Quantity (EOQ) model to determine the inventory level (Harris, 1990; Li & Arreola-Risa, 2017; Napier, 2014; Souza, 2014). These studies often assume that demand is stable or easily estimated from historical demand data, and the lead times are shorter. In the automotive sector, they ignore the highly irregular stochastic demand patterns and many external variables that influence them, including customer forecasts, the number of registered cars on the road, or Google searches by potential customers, and the long lead times of foreign suppliers. Other studies on spare parts demand suggest integrating automotive data from failure rates or installed base information with a combination of forecasting techniques (Van Der Auweraer, Boute, & Syntetos, 2019).

Increasing forecast accuracy has been the focus of countless studies (Danese & Kalchschmidt, 2011; Robert Fildes, 2006; Peidro, Mula, Poler, & Lario, 2009). Some inventory management research on intermittent demand suggests using stock control metrics to evaluate performance instead of traditional measures of error dispersion for forecast accuracy (Sagaert, Kourentzes, Vuyst, Aghezzafa, & Desmet, 2018; Syntetos & Boylan, 2005, 2006; Teunter & Duncan, 2009; Tiacci & Saetta, 2009). Kourentzes, Trapero, and Barrow (2020) propose using stock control metrics of service level (turnaround time) and fill rate.

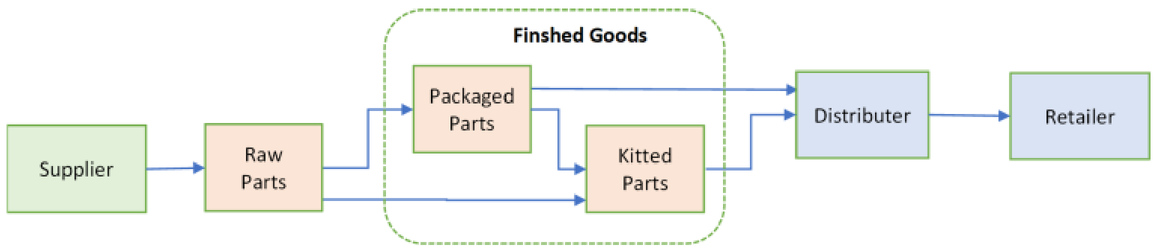
Case: Automotive Aftermarket Manufacturer

Only a limited number of case studies develop and implement solutions to inventory control problems using real data, a recurring topic in the advancement of inventory theory, using more realistic demand assumptions into inventory models. This case study involves a monthly periodic review inventory system with non-stationary stochastic demand, fixed replenishment setup costs (distant offshore supplier freight fees), linear holding and penalty costs over a fixed planning horizon, and a deterministic lead time of 120 days. The problem focuses upon how much inventory to replace to minimize total expected costs while maintaining a 95% customer service level (CSL), which is also tied into their customer contracts.

The data is provided by a business-to-business automotive aftermarket manufacturer that provides parts to large auto parts distributors and retailers. The company is close to outgrowing its current warehouse space and is interested in alternatives to increasing inventory stocking levels. The company's marketing strategy focuses on maintaining sufficient inventory coverage of parts to guarantee a target fill rate of 95% and a customer order fill rate of one week. Otherwise, the company faces significant contractual penalties with some of its largest customers. This 95% CSL, along with inventory, will be used as boundary conditions to evaluate the demand forecast accuracy of the various models.

Due to the variety of parts the company makes, the focus will be on clutch parts made of individual component parts and used as replacement parts for manual transmissions for trucks, sport utility vehicles (SUV), and sport model performance cars. The clutch conveys power from the engine to the gearbox without disrupting the engine transmission while a gear is selected. The engine must be disengaged from the automobile's wheels since the engine is continually rotating, whether the wheels are spinning or not. Common parts that are included in the clutch kits include the pressure plate, clutch or friction disc with a friction material, release bearings, flywheel, associated hydraulic components, and alignment tools. Clutch parts are sold both separately as packaged parts or bundled together as clutch kits (kitted parts). Replacing a clutch can be an expensive, labor-intensive operation requiring the complete disassembly of the clutch itself. Providing clutch kits can save the customer from replacing a related clutch part in the future while also assisting the installer with standard replacement parts to complete the work.

Figure 1. *Flow of Parts*



The company receives raw material parts that are converted into packaged parts and kitted parts (see Figure 1. *Flow of Parts*) before they are combined into orders for distributors. A single raw material part can be used in several different clutch kits. Once the raw part is committed to a clutch kit, it is not available for another kit without significant additional rework and handling costs. They use a batch production scheduling process that targets 30 days of finished goods inventory, consisting of packaged parts and kitted parts, plus 30 days of raw parts inventory for a target total of 60 days of inventory. They focus on carrying sufficient raw material and finished goods inventory to prevent stockouts from their Asian suppliers (those with extended lead times) and prevent costly fulfillment penalties that are imposed by large distributors for failing to meet service level agreements. Extended lead times have resulted in carrying excess raw materials inventory of one year or more on many items, which is well beyond their stated goal.

The company places new orders each month for new materials from various suppliers located from the U.S. to Asia. The lead-time from Asian suppliers includes waiting and transport time (via the Pacific Ocean, into a west coast port and continues by land-based trucking), averaging 16 weeks, with an order review period of 4 weeks of lead-time, 12 weeks of sea travel transport, port clearance, and domestic transportation. The company has provided 54 months of historical manual transmission clutch data that consists of customer demand orders, supplier purchase orders, and the resulting monthly inventory levels. We will also investigate correlations with external data sources to improve demand forecast accuracy and establish the optimum inventory policy for the inventory's highest cost items.

The company’s demand forecasting is performed using four inputs. First, a linear time trend (Excel 'forecast' function) predicts demand in the next four months. Second, salespeople provide input on account changes like retail store openings, closing, and new accounts. Third, some large accounts provide a forecast of stocking changes or request to stock balance inventory from various stores. Finally, adjustments are made based on sales, marketing, or economic conditions, plus the purchasing manager's judgment to inform the reorder quantity. The firm believes there may be better forecasting approaches that would allow more efficient use of their current warehouse space and inventory of parts on hand. The company would be interested in those methods, provided they are not too cumbersome or difficult to utilize with existing personnel.

The remainder of this paper is structured as follows: Chapter 2, the critical literature is reviewed that relates to determining optimal inventory, forecasting methods used for demand forecasting, the criteria for evaluating forecast accuracy, and ending with the use of intermittent data and exogenous variables. In Chapter 3, the measures and details of the proposed solution's experimental structure are presented, followed, in Chapter 4, by the intended results, contribution, and conclusion of the paper.

Chapter 2: Literature Review

The concepts underlying optimal inventory can be considered either in the simple case where demand is constant amongst other known quantities or the more complex case where demand is dynamic, random, and less specific (Arrow, Harris, & Marschak, 1951). Research into the simple case dates back to Ford W. Harris, a production engineer back in 1913, struggling with production lot sizes and determining the number of parts to make (Harris, 1913). He determined the economic lot size by balancing the setup costs with the stocking or holding cost. If one makes too little (or bought), then the order frequency increases, and set up (or ordering) costs rise, but if one makes too much, then the order frequency drops, and holding costs rise. This balance became known as the economic order quantity (EOQ) Equation (1), a constant demand, continuous time scale, and infinite time horizon model, frequently used to resolve inventory purchasing and planning problems under an assumed deterministic demand (Wilson, 1934).

The basic formula for EOQ $= \sqrt{\dfrac{2 \times K \times D}{h}}.$ (1)

The parameters used in the formula for EOQ are K, D, and h and represent the fixed ordering cost, constant demand, and holding cost per unit of time (usually over a year), respectively. The order cost (K) includes ordering administration, receiving inspection, material handling, and any equipment set up (required for manufacturing). The demand (D) denotes the constant deterministic demand. The inventory-holding cost (h) takes into account the cost of capital (i.e., the weighted average cost of capital, which includes both equity and debt) invested in inventory units, the cost of warehouse space, taxes, insurance, scrap, obsolescence, or "shrinkage," and even opportunity cost of retaining old inventory. Typical inventory-holding costs average around 20% of the cost of total inventory held (Waters, 2008). Due to differences in product cost per unit weight or unit area or space, inventory holding costs can vary significantly (Gurtu, 2021).

Research into inventory management has led to many variations of the EOQ model (Cárdenas-Barrón, Chung, & Treviño-Garza, 2014), which have been developed to account for price-dependent supply and demand (Teksan & Geunes, 2016), supply disruptions (Snyder, 2014), back-ordering (Sphicas, 2014), quantity discounts (Taleizadeh & Pentico, 2014), living items ((Rezaei, 2014), cold items (Bozorgi, Pazour, & Nazzal, 2014), deteriorating items (Sicilia, González-De-La-Rosa, Febles-Acosta, & Alcaide-López-De-Pablo, 2014), and continuous improvement (Sarkar & Moon, 2014), to name a few of the different supply chain situations. The EOQ model delivers a near-optimal solution if demand is mainly constant with slight variation (Schwarz, 2008), but demand is frequently not deterministic, often it is stochastic and non-stationary. The simple case assumed stationary demand due to the computational complexity involved in identifying other demand patterns. Extended supply chains consisting of multiple firms exacerbate forecasting errors leading to exaggerated order swings, this is known as the bullwhip effect (H. L. Lee, Padmanabhan, & Whang, 1997) where uncertainty increases as lead time increases between firms. Information sharing is necessary to reduce order variation at the highest level of a multi-level supply chain (Dejonckheere, Disney, Lambrecht, & Towill, 2004).

When demand is random and less certain, we use the (Q, r) stochastic model, where Q represents the fixed quantity ordered (current inventory level + on order inventory – any backorder amount) when inventory decreases below r a fixed reorder point (Zheng, 1992). When using stochastic demand, the upper bound for relative error was determined to be 11.8 percent (Axsäter, 1996), whereas there is no boundary for the deterministic EOQ. The dynamic version of EOQ, first derived by Wagner and Whitin (1958), provides mean demand estimates for EOQ. The reorder point r must account for the uncertain demand while awaiting resupply. It includes a buffer known as safety stock needed to prevent stockouts due to errors in forecasting and lead time expectations. It works well for calculating the next order. However, it does not work well for a series of forecasted orders over a determined planning horizon (Vargas, 2009) or when future orders occur at a random price (Sana, 2011). These stochastic models assume a known probability distribution to simplify the problem, but if it is unknown, Bertsimas and Thiele (2006) provide a more robust optimization approach.

Demand can be stochastic and non-stationary, typical for many component parts and subassembly providers, requiring considerably more safety stock than within stationary demand situations (Graves, 1999; Strijbosch, Syntetos, Boylan, & Janssen, 2011). The (s, S) inventory policy is used both in stationary and non-stationary demand cases and has proven optimal when the holding and shortage costs are linear (Scarf, 1959). A periodic review control system for stochastic demand is widely used in inventory management situations where a continuous review is not practical. Inventory is controlled by ordering on fixed periodic review intervals (R) with fluctuating order quantities placed to bring the inventory position up to a certain level (S) (Hadley & Whitin, 1963). The Periodic-Review, order-up-to-level systems (R, S) (Silver, Pyke, & Peterson, 1998) is a standard replenishment method, although not as responsive and more expensive than the (Q, r) policy, ranging from a few percent to as much as 41% (Rao, 2003). However, it is simpler to operate and is frequently used when coordinating shipping containers from overseas suppliers with constant lead time (L). Suppliers also prefer periodic review systems because of the lower uncertainty of order timing.

Silver and Bischak (2011) derived a simple expression for safety stock in (R, S) systems based on the fill rate (under normally distributed demand) and standard deviation of the demand forecast errors over the replenishment period R+L, Equation (2).

$$SS = k * \sigma_{L+R} \qquad (2)$$

Where k is a safety factor (i.e., NORMSINV(fill rate) function in excel for the desired fill rate) and σ_{L+R} is the standard deviation of demand forecast errors over the replenishment period R+L. Using forecast errors, instead of the more popular demand variance to calculate safety stock, results in 15% lower safety stock at the same level of customer service (Zinn & Marmorstein, 1990) for shorter lead times.

Forecasting Methods

The demand process is the primary source of uncertainty, leading us to the next critical factor in inventory costs, selecting the correct forecasting method (R Fildes & Kingsman, 2011). For example, using a moving average can cause the bullwhip effect (Dejonckheere, Disney, Lambrecht, & Towill, 2003), whereas choosing an autoregressive method outperforms the exponential smoothing approach (Chandra & Grabis, 2005) and reduces the bullwhip effect. Some methods are chosen for the type of data available, forecasting simplicity, error, and utility of the results (R. Fildes, Nikolopoulos, Crone, & Syntetos, 2008a).

Table 1. *Forecasting Methods*

FORECAST METHOD	ACRONYM	DESCRIPTION	INPUTS
Naive Method	RFW	Uses the previous data point in the sequence as the forecast.	- Data series
Naive Drift Method	RFWD	Uses the previous data point in the sequence plus average change over time (drift) as the forecast.	- Data series
Linear Regression	LM	Based on the regression of a certain number of previous data points (i.e., 12 or 18).	- Data series
Simple Moving Average	SMA	Based on the average of a certain number of previous data points (i.e., 12 or 18).	- Data series
Brown's Method of Single Exponential Smoothing	SES	Utilizes a weighted average of historical data and alpha as a smoothing constant to assign exponentially smaller weights to previous data.	- Data series - Alpha
Holt's Method of Double Exponential Smoothing	DES	Utilizes SES applied to both level and trend using alpha as a smoothing constant and beta as a trend constant.	- Data series - Alpha - Beta
Holt-Winters Method automatically selecting Single or Double smoothing parameters	DESZ	Utilizes SES applied to level, trend, and season using alpha as a smoothing constant, Beta as a trend constant.	- Data series - Alpha - Beta
Prais-Winsten Regression	PW	Uses an iterative ordinary least squares (OLS) method to recursively estimate beta and error autocorrelation rho at convergence.	- Data series - Rho - Beta0 - Beta1 - Beta2

Strasheim (1992) performed a study of the 17 most popular forecasting techniques at the time using traditional statistical measures (mean error, mean absolute error, sum of squared error, mean squared error, and standard deviation of errors), for automotive spare parts and concluded that Brown's method of Exponential Smoothing consistently provided the most acceptable forecasts, was stable, insensitive to the smoothing constant chosen, the lowest cost variance was reliable for limited demand

history, and was easy to understand.

In this study, we will focus on Brown's Method of Exponential Smoothing (ES), and its variant double exponential smoothing. The data was found to be non-stationary and did not possess any seasonality, so seasonality models like triple exponential smoothing were ruled out. The primary forecasting methodologies are summarized in Table 1. *Forecasting Methods*.

Naïve Method (RFW) Uses the previous data point in the sequence as the forecast. The Naïve Method with drift is a variant of the Naïve Method, which uses the previous data point in the sequence plus the average change over time (drift) as the forecast.

Linear Regression (LM) is a linear approach for modeling the relationship between a certain number of previous data points (i.e., 12 or 18) known as the independent or explanatory variables using a linear predictor function with estimated model parameters to determine the dependant variable.

Simple Moving Average (SMA) is based on the mean or average of a certain number of previous data points (i.e., 12 or 18). There are no model parameters to calculate, so the model is very simple.

Exponential Smoothing (**ES**), introduced by Brown (1959), is a standard method of demand forecasting with a smoothing constant (alpha) used for inventory management within various enterprise resource planning applications. Brown worked as an analyst for the US Navy and first introduced ES demand forecasting as a method for inventorying spare parts (Gass & Harris, 2000) as an improvement over SMA. ES is also called **Single Exponential Smoothing** (SES) or exponential moving average, where alpha is derived from the weighted mean or SMA allocating more weight to recent data while applying an exponentially decaying weight to past events.

Charles Holt modified ES to include support for trends (beta), now called **Double Exponential Smoothing** (DES) or Holt-method. Charles Holt and Peter Winters developed **Triple Exponential Smoothing** (ETS in Excel) as an extension of the ES model to use both trend and seasonality (gamma). The level (magnitude), trend (direction), seasonality (recurring pattern length), and residuals of the model are easily calculated with a minimum amount of data (Holt, 1957). Change in seasonality can be selected as additive or multiplicative, representing either linear or exponential changes. ES is popular because it does not require the time series to be stationary, and it is mainly robust when the appropriate model is chosen (Gardner, 1985, 2006). There are 30 possible ES parameter combinations to select to minimize the forecast error using arithmetic, multiplicative, or damping for the error, trend, or seasonality parameters.

The **Prais-Winsten** model (Prais & Winsten, 1954) is an econometric model that accounts for autoregressive AR(1) serial correlation of errors in a linear regression model. The autoregressive model specifies that the dependant variable is linearly based on its values and some additional precise terms. Prais-Winsten is a variant of the Cochrane–Orcutt estimation, which deletes the initial observation. The model recursively estimates the coefficients and the error autocorrelation until sufficient convergence of the AR(1) coefficient is accomplished.

Intermittent Data

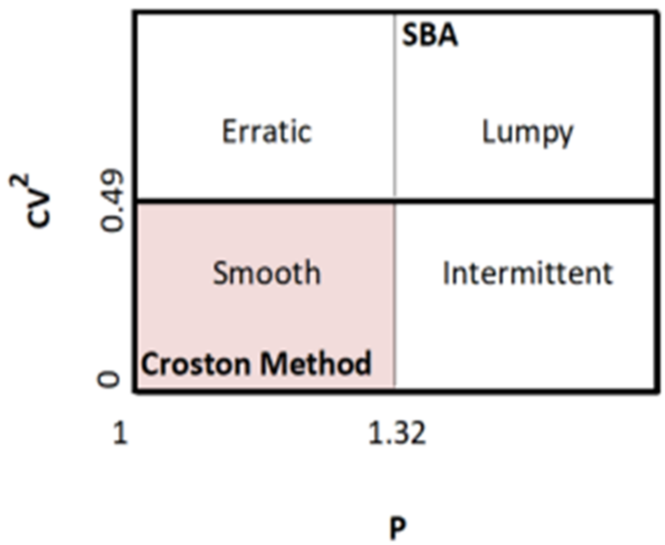
Intermittent data is expected in inventory control situations with less popular selling or used parts. Syntetos, Boylan, and Croston (2005) defined intermittent spare parts demand based on the count of zero demand periods occurring over a given number of time periods. The Average Demand Interval (ADI) is the average interval between two consecutive periods, with non-zero demand (Costantino, Di Gravio, Patriarca, & Petrella, 2018). Johnston and Boylan (1996) suggest using an ADI greater than 1.25 for intermittent demand.

$$\text{ADI} = \frac{\text{Average time interval between two demand occurrences}}{\text{Total number of periods}} \qquad (3)$$

$$\text{or} \quad = \frac{\text{Total number of periods}}{\text{Total number of non-zero periods}} \qquad (4)$$

Syntetos et al. (2005) further categorized intermittent demand based on ADI (P) and the Coefficient of Variation squared (CV^2 = (Standard Deviation / Mean)^2). They determined the cut-off values as 1.32 and 0.49 for P and CV, respectively, which leads to parts classified into four groups: Erratic, Lumpy Smooth, and Intermittent, which is illustrated in Figure 2. *SBA Classification*. The SBA method is commonly used for all but smooth, which uses Croston's method.

Figure 2. *SBA Classification*



Exogenous Data

There has been a lot of research into using exogenous variables for predictive analytics within the econometrics field to develop theories on the economy's economic modeling. Many BDPA methods are now being applied to supply chain analytics' evolving field because of the Internet age's widely available data.

Some studies have looked into the problem of forecasting demand (see Table 2. Exogenous Automotive Research), choosing many different types of forecasting methods. Some of the methods are chosen for the type of data available, simplicity, error, and utility of the results (R. Fildes et al., 2008a). Simple models like the Naïve or SMA are unable to use exogenous data. Complex forecasting methods have been used to integrate exogenous variables as predictors or covariates such as Autoregressive

Integrated Moving Average (ARIMA) with Seasonality (SARIMA), Exponential Smoothing with Covariates (ESCov), Variable Mean Response (VMR), Vector Autoregressive (VAR) with exogenous variables (VARX), and finally, Vector Error Correction (VECM) with exogenous variables (VECMX).

Table 2. *Exogenous Automotive Research*

Study	**Method**	**Measure**	**Exogenous Data**
Blackburn et al. (2015)	ESCov	MAPE	BASF-process industry
Chuang and Chiang (2016)	VMR	Fit Statistic	Days Supply, Personal Income, Inventory
Cortés and Borrego	Croston's method	MAD, MSE, MAPE	Service parts
Fantazzini and Toktamysova (2015)	VECM, VECMX, VAR, BVAR	MSPE	Google data and economic variables: BC, CCI, CPI, EURIBOR, GDP, PI, UR, PP for car sales.
Gao, Xie, Cui, Yu, and Gu (2018)	VAR, VECM, ARMA	RMSE, MAPE,	Exogenous variables: consumer confidence index (CCI), steel production, CPI, and 95# unleaded gasoline price on car sales
Wayne Smith, Coleman, Bacardit, and Coxon (2019)	Expectation-Maximisation (EM) algorithm	Replacement %	Invoice, mileage, make, model, brake disc
W Smith, Coleman, Bacardit, and Coxon (2018)	empirical cumulative density function (ECDF)	Replacement %	Invoice, mileage, make, model,
Qin and Yun (2012)	SVR, ARIMA, Multiple Regression, Combined Model	MAPE, Variance	Auto Parts

Evaluation Criteria

An underlying principle of demand forecasting is the proposition that a good fit using past data will lead to a realistic future forecast. For this to be true, there must be a discernible pattern, even an irregular pattern, that can be discerned from the data and relied on to repeat in the future. Model fit is usually determined by minimizing forecast error using either Root Mean Square Error (RMSE), Mean Squared Error (MSE), or Mean Absolute Error (MAE), to name a few. Gneiting (2011) found that demand forecasting methods optimized for the in-sample mean errors (absolute error and squared error) produce optimal predictions based on mean demand. Using maximum likelihood estimation will result in optimal mean demand predictions ensuring unbiased in-sample forecasts. However, there is no later guarantee of an unbiased or accurate prediction out-of-sample (Barrow & Kourentzes, 2016). Gardner (2006) found more robustness with MAE against demand changes resulting in optimal median demand forecasts (Gneiting, 2011). However, inventory management does not require optimality based on a median or mean demand forecast. Inventory management uses demand forecasts to determine the reorder frequency, order quantity, and safety stock level.

Forecasting methods based on time series analysis are used to forecast demand in a future period. Demand model parameters are calculated using a forecast performance metric such as MSE, which penalizes overestimating and underestimating demand equally. In stock control situations, backorders or stockouts can be more costly than holding inventory. This results in a bias with the MSE optimization model penalizing under- and over-predictions unequally. Orders are made in each interval based on a dynamic forecasting model prediction of demand, where model parameters are recalculated at each interval; however, the forecasting model projections do not account for the optimization process bias, instead of minimizing the (symmetric) error between the forecasts and the actual demand.

Customer service levels (CSL), made of the ratio of filled demand (demand not including backorders) to total demand, have been used to measure forecast performance (Boylan, Syntetos, & Karakostas, 2008) but must be constrained by another measure; otherwise, a 100% CSL can be achieved given enough inventory.

Some inventory management research on intermittent demand suggests using stock control metrics to evaluate performance instead of traditional mean error calculations (Sagaert et al., 2018; Syntetos & Boylan, 2005, 2006; Teunter & Duncan, 2009; Tiacci & Saetta, 2009) or demand rates (Kourentzes, 2014). One recurring theme in the research is that accurate, unbiased in-sample forecasts using MSE may over-fit the out-of-sample prediction resulting in a low-performing forecast. For example, an exact forecast (based on optimal MSE) with a lot of daily variances may exhibit greater operational difficulty in scheduling production than a consistent but less accurate forecast (R Fildes & Kingsman, 2011; Sagaert et al., 2018). Others have found that decreasing forecast bias may be more important than forecast accuracy (Sanders & Graman, 2009; Syntetos & Boylan, 2001). Kourentzes et al. (2020) propose using stock control metrics based on service level (turnaround time) and fill rate and combining them into a signal variable, which mixes the order error cost into one metric and simplifies the multivariate problem into single optimization. This is in line with the business' contractual service level and fill rate requirements. Others have focused on automotive parts using a simulation to find the forecast stock control parameters that would lead to optimal inventory stocking (Bruzda, 2020; Kourentzes et al., 2020; Rego & Mesquita, 2015). Some researchers have found that combining several forecasts into a single model has been shown to reduce forecasting errors and the constraints inherent in a single model (Barrow & Kourentzes, 2016).

Rolling Origin Cross-Validation

One method of evaluating forecast models is to split the data into two data sets; the first is the in-sample data or training set, and the second is the out-sample data, holdout, or test set. Forecast models are applied to the training set, the model parameters are calculated, and the models are evaluated based on errors measures like MSE. This is a "fixed origin" method and is useful for time series forecasting, but in inventory control situations, decisions are made in every interval.

An alternative method used in time-series forecasting is rolling origin cross-validation. The forecast origin is updated at each interval as new data is incorporated into the forecast and new smoothing parameters are estimated (Tashman, 2000). The last interval in the training set is known as the forecast origin, which changes at each new interval. The lead time intervals, made up of the time between the forecast origin and the forecast, comprise the forecast horizon or prediction interval. A rolling origin evaluation averages multiple forecast errors providing a better understanding of model performance (Hyndman & Athanasopoulos, 2018). A fixed-sized rolling window of constant length may be added, which replaces the oldest data with the latest data to consider changes in the environment. Figure 3. *Rolling Origin with Constant In-Sample Window* illustrates a rolling origin from 29 observations with a fixed-sized window representing 17 origins starting at origin 12. Hyndman and Athanasopoulos (2018) suggest using the lowest RMSE when evaluating the best forecasting model on a rolling forecasting origin.

Figure 3 *Rolling Origin with Constant In-Sample Window*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
O12	1	2	3	4	5	6	7	8	9	10	11	12	h1	h2	h3	h4													
O13		1	2	3	4	5	6	7	8	9	10	11	12	h1	h2	h3	h4												
O14			1	2	3	4	5	6	7	8	9	10	11	12	h1	h2	h3	h4											
O15				1	2	3	4	5	6	7	8	9	10	11	12	h1	h2	h3	h4										
O16					1	2	3	4	5	6	7	8	9	10	11	12	h1	h2	h3	h4									
O17						1	2	3	4	5	6	7	8	9	10	11	12	h1	h2	h3	h4								
O18							1	2	3	4	5	6	7	8	9	10	11	12	h1	h2	h3	h4							
O19								1	2	3	4	5	6	7	8	9	10	11	12	h1	h2	h3	h4						
O20									1	2	3	4	5	6	7	8	9	10	11	12	h1	h2	h3	h4					
O21										1	2	3	4	5	6	7	8	9	10	11	12	h1	h2	h3	h4				
O22											1	2	3	4	5	6	7	8	9	10	11	12	h1	h2	h3	h4			
O23												1	2	3	4	5	6	7	8	9	10	11	12	h1	h2	h3	h4		
O24													1	2	3	4	5	6	7	8	9	10	11	12	h1	h2	h3	h4	
O25														1	2	3	4	5	6	7	8	9	10	11	12	h1	h2	h3	h4
O26															1	2	3	4	5	6	7	8	9	10	11	12	h1	h2	h3
O27																1	2	3	4	5	6	7	8	9	10	11	12	h1	h2
O28																	1	2	3	4	5	6	7	8	9	10	11	12	h1

Company Adoption

BDPA can be used to solve supply chain performance problems that may include high inventories, stockouts, late deliveries, and expedited fulfillment costs. However, to realize the promise of BDPA, it is not just a matter of introducing the technology and methods into the organization. One must fully understand and embrace the new methods. Green and Armstrong (2015) reviewed research comparing simple and complex forecasting methods. They concluded there was little support for the proposition that complexity enhances forecast accuracy. However, as complexity is introduced into the organization, it becomes harder to understand or explain the models, inhibiting adoption. The effective assimilation of any new information technology (IT) methods must incorporate changes in the organization's procedures, practices, and technology (Leonard, 1988). Mu, Kirsch, and Butler (2015) highlighted the importance of identifying the organization's need for new methods and technology and then actively managing the technology change after implementation to increase assimilation. According to the task-technology fit (TTF) model, user adoption occurs when the technology meets the requirements of the task assigned (Goodhue & Thompson, 1995) and the user recognizes the usefulness and ease of technology, but not if it fails to enhance their job performance (C.-C. Lee, Cheng, & Cheng, 2007). The technology acceptance model (TAM) is similar in asserting that the perceived usefulness positively influences the assimilation and adoption of BDA (Verma et al., 2018). Organizations must link BDPA to business strategy, make it easy for the users, and insert it into their organizational processes so that the right decisions can be made at the right time.

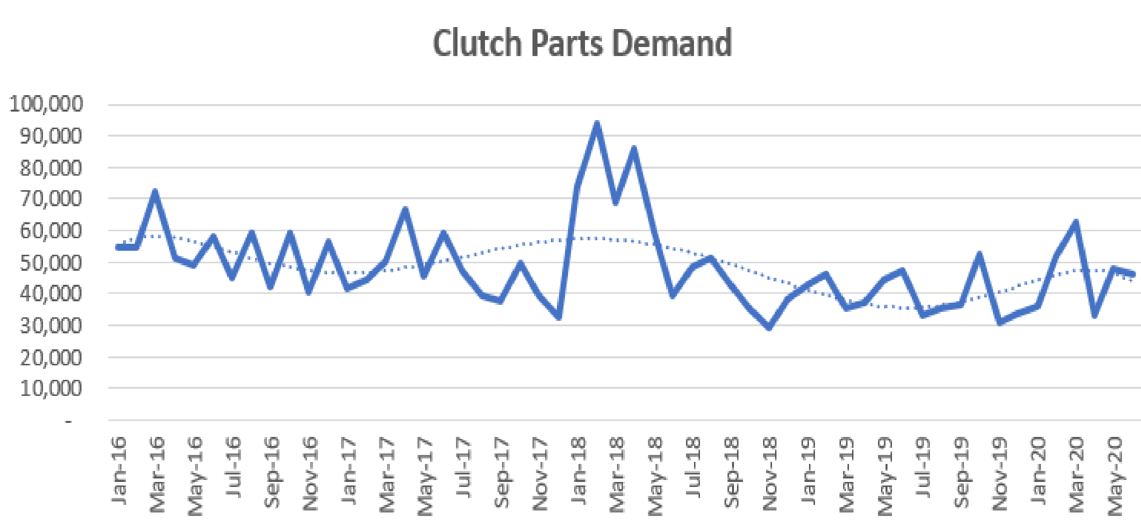
Chapter 3: Research Methodology

The research methodology used historical data from an automotive company and external data from Google Trends combined with several forecasting methods to determine their accuracy for inventory control under long lead times. First, a description of the measures used followed by the methods and procedure observed to obtain the study results.

Measures

An automotive aftermarket manufacturer that provides parts to large parts distributors and retailers has provided ten years of historical data on clutch parts and clutch kits that consist of customer purchases constrained by their purchase agreements. The data includes request date, date received, price, item, quantity, and ship date. The second set of data includes monthly on-hand inventory levels for each stock-keeping unit (SKU). Reliable demand predictions could effectively lower inventory costs, increase

Figure 4. *Clutch Part Aggregate Demand*



available warehouse space, and improve cash flow. Accurate demand forecasts, both for the next quarter and the next month, would help control inventory.

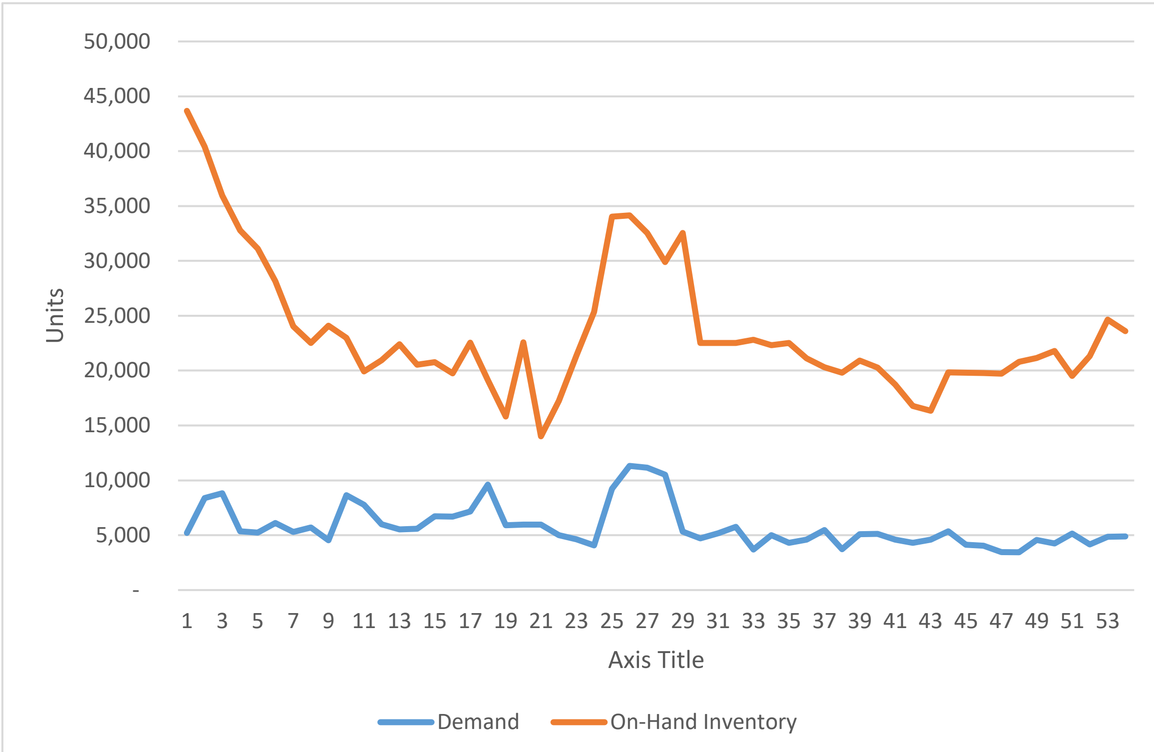
This study examines the forecasts obtained by evaluating demand over 54 months (see Figure 4. *Clutch Part Aggregate Demand*). The data is comprised of 1,111,111 rows of invoice data covering the period January 2016 to July 2020, plus monthly onhand inventory levels for all clutch parts. The sales demand is primarily for clutch kits or kitted parts, comprised of multiple component parts, and each component is used in one or more kits. The company utilizes a bill of materials (BOM) defining the components used in each kit. There are 128 raw material suppliers, but the study focuses on 16 suppliers that require 120 days (predication interval of 4 months) of lead time for materials delivered from Asia. All kits and component packaged parts break down into 1,033 raw material parts that are ordered from the 16 suppliers. Table 3. Shows the parts breakdown. 899 of the parts are uncommon, new, or old representing intermittent demand, leaving 134 that have regular order flow (ADI = 1) or non-intermittent demand.

Table 3. *SBA Classification*

SBA Classification	CV^2
Total Intermittent	899
-- Smooth	48
-- Intermittent	76
-- Erratic	290
-- Lumpy	485
Non-intermittent	134
Total	*1,033*

The study will focus on a sample of 100 non-intermittent parts (see Figure 5. *Smooth Parts Sample*), representing 9.7% of the 1,033 parts from the 16 Asian suppliers. When inventory levels are compared to sales, they appear to follow two different patterns. Improved forecasting methods coupled with a new inventory policy that moves with sales would save the company money.

Figure 5. *Smooth Parts Sample*



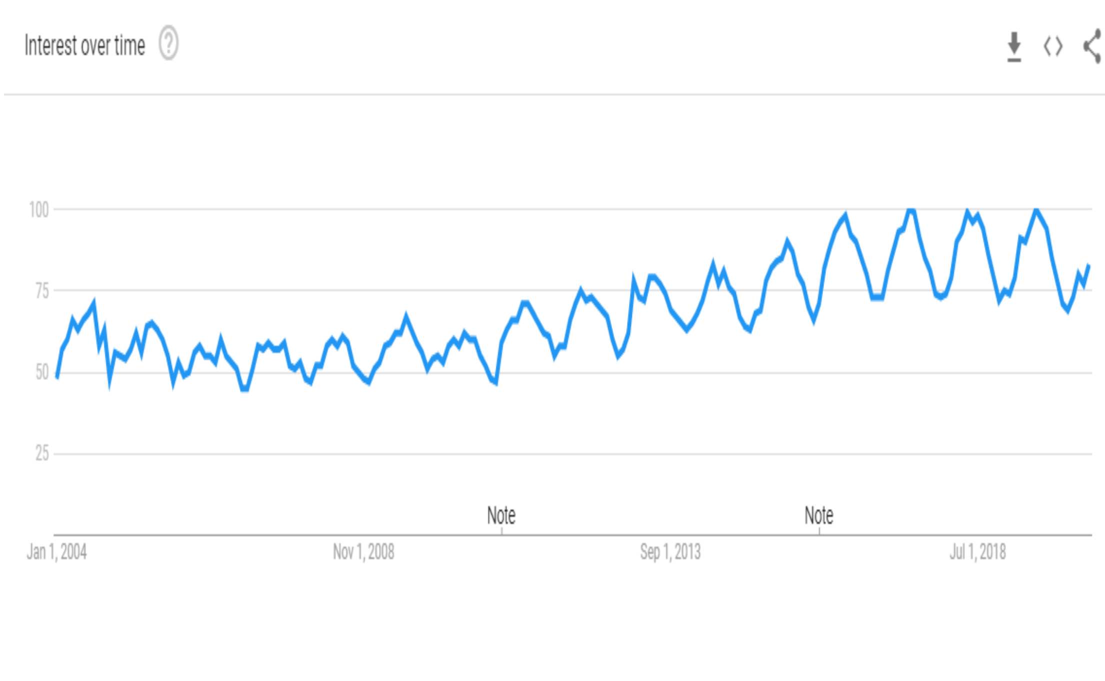
Exogenous Variables

An essential aspect to improving forecast accuracy will be using big data analytics to look ahead and get as close to the customer as possible (Cachon & Fisher, 2000) by using customer interaction data, website page views, point of sale data, or other proxy variables as covariates to predict demand (Cohen, 2015). Internet search engines are a convenient choice to use as a proxy for demand. External variables under investigation include Google searches, clutch failure rate assumptions, and past automotive registration data. Google has become a leading search engine with an 87% market share (Chris, 2020). The automotive manufacturer also uses customer communications or industry knowledge to adjust the forecasts.

Google Trends Search Data

We propose using Google Trends search data, which provides information on users' relative searches at a given geographic region and time (monthly, weekly, or daily). Google Trends data is 'broad matched,' meaning keyword strings are reduced to popular searches for the most meaningful words in the string. Search results are calculated using an anonymized unbiased sample. The Google Index (Google, 2020) provides an estimate using the ratio of the number of queries relative to a particular category (clutches) concerning all queries in the selected region (United States) at a given point of time (See Figure 6. *Google Trends for "clutch"*) and then the data is indexed to 100 (the maximum search interest for that time and location).

Figure 6. *Google Trends for "clutch"*



Therefore results may vary from day to day, and only searches with significant volume are tracked. Moreover, researchers can use Google Trends to produce real-time forecasts, but we will use it as a forecasting indicator of clutch demand. It is unclear whether the Google data is stationary because Google divides the searches by the total searches in the week and geographic area.

The study believed there was a first-order positive autocorrelation in the Google Trends data based on observing the regular pattern in the data series. It also exhibits first-order positive autocorrelation meaning the time series errors are correlated with their past values. Equation 8 represents the relationship of the forecasted demand of a part number with the formula for Google trend (equation 9) with respect to time. Equation 10 results from inserting Eq. 9 into Eq. 8.

$$Y_t = \beta_0 + \beta_1 t + \beta_2 G_t + \epsilon_t \qquad (8)$$

$$G_t = \Upsilon_0 + \Upsilon_1 t + \Upsilon_2 M + \Upsilon_3 M^2 + \delta_t \qquad (9)$$

$$Y_t = \beta_0 + \beta_1 t + \beta_2 \Upsilon_0 + \beta_2 \Upsilon_1 t + \beta_2 \Upsilon_2 M + \beta_2 \Upsilon_3 M^2 + \varepsilon_t + \delta_t \qquad (10)$$

Where:

$Y_{t\,=}$ Forecasted SKU demand for the current period

G_t = Google Trends for the current period

t = Time period, months since the first observation

β = vector of coefficients

ϵ_t = residual error term

Υ = vector of coefficients

M = current calendar month

δ_t = residual error term

This led to adding the econometric Praise Winston method to account for the autoregressive AR(1) serial correlation of errors in relation to time (see Step Three – Analyze Data).

Methods

The automotive company is currently using the forecast function within Microsoft Excel software to create and manage its demand forecasts. The study will use the Excel forecasting method as a baseline indicating the actual performance of the company. The Excel forecast function provides six different outputs. The new FORECAST has replaced the old FORECAST function.LINEAR, which predicts future values using a simple time trend of historical data. The FORECAST.ETS variant predicts future values based on *Exponential Triple Smoothing* (ETS), which considers error, trend, and seasonality components.

The FORECAST.ETS function requires consistent intervals, but it will work with up to 30% of the periods with no demand (to consider intermittent data) before reverting to a linear time trend, which then becomes the same as FORECAST.LINEAR. The confidence interval (+/- offset for the upper and lower bounds) is output using FORECAST.ETS.CONFINT function. The recurring pattern length or seasonal interval is output using FORECAST.ETS.SEASONALITY function. The remaining forecast parameters and the error statistics of the forecast are output using FORECAST.ETS.STAT and includes:

- Alpha is the weighting component used for smoothing recent data points.
- Beta is the trend component detected in the time series.
- Gamma is the seasonality component detected in the time-series.
- MASE is a forecast accuracy measure.
- SMAPE (symmetric mean absolute percentage error) is a percentage or relative error measure of forecast accuracy.
- MAE is a measure for the average size of the prediction errors.
- RMSE is a measure of the predicted and observed differences.
- Step size detected in the time series.

The study uses the forecast error for evaluating the forecasting model performance through rolling origin cross-validation. Safety stock is based on the variance of forecasts error, which is the forecasted demand minus the actual demand. A positive error means we forecasted too high, and negative means we did not forecast enough. The estimated inventory expected in the current period is a function of the inventory on hand at the end of the last period plus the sum of the next four months of expected incoming deliveries (which are the orders placed over the last four months) minus the four times the forecasted demand (which is also the naive estimate of expected demand in the next four months). The actual order placed is either the calculated demand forecast from the model plus safety stock minus the estimated inventory expected to be on hand or zero (because enough expected inventory is available or on order over the next four periods).

The naive forecast assumes the last month's actual demand is the only important one, so all future forecasts are equal to the last period's actual demand. The SES model is similar; producing a forecast without a trend will have a constant value in the prediction intervals. The models are all based on forecasting four months ahead to consider the four-month lead time. Actual demand is collected at the end of each month and used to estimate demand and inventory for each of the next four months in all models. Therefore, the formula for each model needs to be adjusted accordingly. The standard SES formula (equation 5) is a weighted average using a smoothing constant α, of the form:

$$F_{t+1} = \alpha * A_t + (1- \alpha) * F_t \qquad (5)$$

Where:

F_{t+1} = Forecast for the current period,

F_t = Forecast demand for the last period,

A_t = Actual demand for the last period,

α = smoothing constant (between 0 and 1).

Since the forecast is for four months ahead (F_{t+4}), A_{t+3}, A_{t+2}, and A_{t+1} are not known. Therefore, the actual formula used is of the form

$$F_{t+4} = \alpha * F_{t+3} + (1- \alpha) * F_{t+2}$$

$$F_{t+3} = \alpha * F_{t+2} + (1- \alpha) * F_{t+1}$$

$$F_{t+2} = \alpha * F_{t+1} + (1- \alpha) * A_{t+1}$$

$$F_{t+1} = \alpha * A_t + (1- \alpha) * F_t$$

The SES method ends up forecasting a constant level, which means that subsequent forecasts become the value of F_{t+1} in the future. Additional parameters are added to determine the trend (DES) or seasonality (TES). The standard DES formula (equation 6) adds a second equation to calculate the trend (equation 7) using a coefficient β, of the form:

$$F_{t+1} = \alpha * A_t + (1- \alpha) * (F_t + B_t) \qquad (6)$$

$$B_{t+1} = \beta * (F_{t+1} - F_t) + (1 - \beta) * B_t \qquad (7)$$

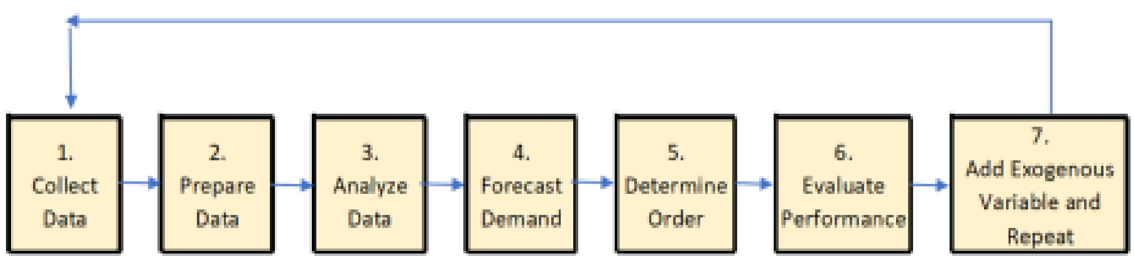
Where:

- B_{t+1} = forecast for the current period,
- B_t = forecast for the previous period,
- β = trend smoothing coefficient (between 0 and 1).

Procedure

A hierarchical test process will evaluate the baseline forecast that the company uses now and then test adding methods and data to see the level of improvement gained over the baseline. The plan is to explore the performance of different forecasting models using a seven-step process illustrated in Figure 7. *Procedure Steps*.

Figure 7. *Procedure Steps*



The first three steps are to collect, prepare, and analyze the data to be used in forecasting. The next two steps are to estimate demand and determine the next order quantity. The second to last step is to evaluate the performance of the various forecasting models. The last step is to add an exogenous variable to the models that accept an external regressor and repeat the process to determine the impact of external data.

Step One – Collect Data

Several meetings with the automotive company supported the data gathering process. These sessions enabled the creation of a historical image of how inventory management and the ordering steps are performed, which led to identifying appropriate variables for the project from existing computer applications and data sources. The variables collected for the project included: customer orders (including date, part, unit cost, price, quantity, and notices of future order events), suppliers, supplier purchase orders, monthly on-hand inventory, and the bill of materials used for making both packaged and kitted parts.

Step Two – Prepare Data

Daily customer order demand data was obtained, indicating orders for packaged parts and kitted parts. However, raw material parts are ordered monthly from suppliers (see Figure 1. *Flow of Parts*), requiring a conversion of finished goods into monthly totals of raw material parts. Many of the individual raw materials are identified as being used in multiple finished goods. The customer order data was imported into MS-Access, converted into a time series for monthly demand by part number or SKU, and then converted into monthly raw material part totals using the bill of materials to create the raw demand that is ordered from the suppliers.

A monthly time series of customer notices by raw part was created to consider any advanced notices that were obtained in advance for stock balancing, returns, or inventory changes at the retail customer sites. Finally, A time series was created out of monthly on-hand inventory by raw material part.

Step Three – Analyze Data

The time-series data was imported into MS-Excel to be examined, sorted, and classified. A total of 1,033 raw material part numbers resulted from the data preparation that was classified into 899 intermittent and 134 non- intermittent parts. The data was checked for outliers and used an ADI = 1 to select the non-intermittent parts from the intermittent parts (Boylan et al., 2008 Syntetos et al., 2005) to exclude irregular data, intermittent data, inactive (zero demand), newer, and older negative demand parts. Older parts tend to be at the end of the life cycle, which results in more returns than sales. The study focused on a sample of 100 non-intermittent parts out of a total of 433 non-intermittent parts. Upon inspection of the data, it was determined that seasonality could be excluded due to its poor performance, which led to the selection of some classical nonseasonal forecasting methods.

The Google Trends data appeared to exhibit a clear pattern suggesting that it increases with time and that it cycles with the month of the year. Rewriting Eq. 10 into the reduced form of the structural equation (a recursive system) results in equation 11.

$$Y_t = \alpha_0 + \alpha_1 t + \alpha_2 M + \alpha_3 M^2 + \varphi_t \qquad (11)$$

Equation 12 defines the first-order autocorrelation in the error term φ_t. To get what was estimated in Eq. 10, we solve Eq. 11 for $\varphi_{t-1.}$ Substituting the result into Eq. 12 and then substituting that into Eq. 11 at time t and rewriting again. That is the equation that is estimated for all but the first time period of the Praise-Winston transformation.

$\varphi_t = \gamma_t + \rho\varphi_{t-1}$ (12)

Where:

$Y_{t\,=}$ Forecasted SKU demand for the current period

M = current calendar month

φ_t = residual autocorrelated error term $\varepsilon_t + \delta_t$

γ_t = residual error term

ρ = estimated AR(1) model errors coefficient

The Google Trends exogenous data had a first-order positive autocorrelation meaning the time series is correlated with its past values. The Autocorrelation (ACF) bar chart depicts the correlation coefficients between the Google Trends time series and its lagged values. Figure 8. *Google Trends Autocorrelation Plot* shows a significant spike (correlation of 1) at lag 0 followed by a decreasing wave alternating between statistically insignificant (or close to it) positive and negative correlations indicating a higher-order autoregressive term may not by in the data. The corresponding partial autocorrelation (PACF) in Figure 9. *Google Trends Partial Autocorrelation Plot* shows a steady decay toward zero after the first significant lag indicating a first-order moving average process with autocorrelation. Figures 8 and 9 indicate Google Trends data differences follow an autoregressive AR(1) process with a first-order moving average. This indicated using the Prais-Winsten method, which is an iterative process designed for producing unbiased and efficient estimates that account for error autocorrelation. By comparison, the company data did not appear to exhibit an AR(1) process.

Figure 8. *Google Trends Autocorrelation Plot*

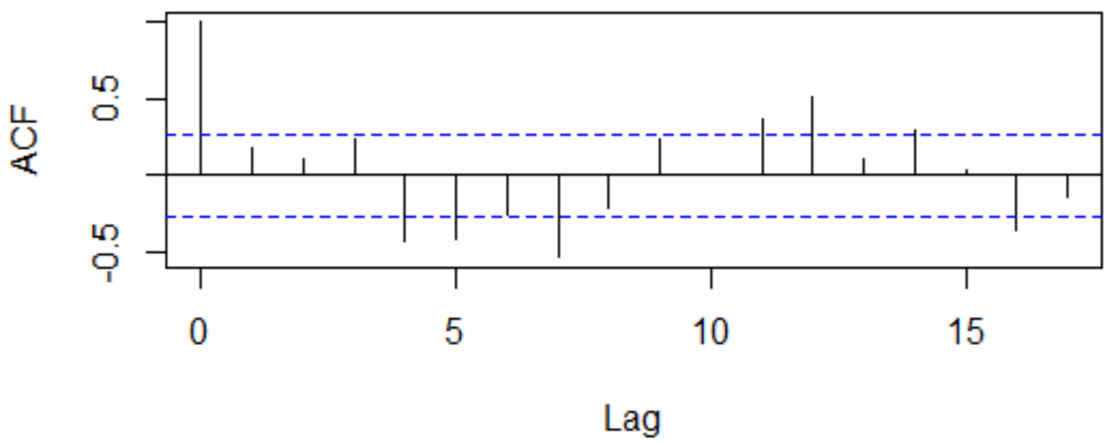
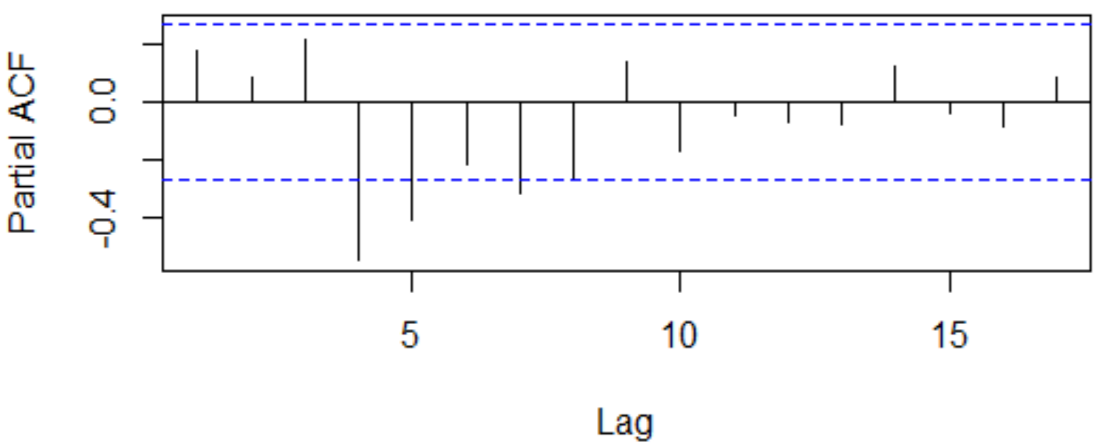


Figure 9. *Google Trends Partial Autocorrelation Plot*



Step Four – Forecast Demand

A series of forecasting models will then be used to determine the best fit using in-sample data (years 1-5) to train and test the models, adjust model parameters, and determine covariance relationships with the exogenous variables. The same initial starting conditions were used to initialize the models, and then the study ignored the first twelve months to evaluate the performance, thereby excluding the starting conditions.

Table 4. *R Model Calls*

FORECAST METHOD	**ACRONYM**	**R Function Call**	**R Library**
Naive Method (random walk)	RWF	rwf(fwin)	Forecast (version 8.15)
Naive Drift Method (random walk with drift)	RFWD	rwf(fwin, drift=TRUE)	Forecast (version 8.15)
Linear Forecast	LM	alm(fwin~Mwin,df,distribution="dnorm")	Greybox (version 1.0.0)
Simple Moving Average	SMA	sma(fwin, num, h=hz)	Smooth (version 3.1.2)
Brown's Method using addititive errors with no trend or seasonaility	SES	es(fwin, model="ANN",h=hz,holdout=FALSE)	Smooth (version 3.1.2)
Holt's Method using addititive errors with trend and no seasonality	DES	es(fwin, model="AAN",h=hz,holdout=FALSE)	Smooth (version 3.1.2)
Holt's Method using the best additive, multiplicative, or damping errors with trend and no seasonality	DESZ	es(fwin, model="ZZN",h=hz,holdout=FALSE)	Smooth (version 3.1.2)
Prais-Winsten	PW	prais_winsten(demand.v[1:n] ~ mo.v[1:n]+mo_sq.v[1:n], data=demand.df)	Prais (version 0.1.1)

Given the initial prediction interval of four, the first four forecasts and the first order was prepopulated with the mean demand over the first 12 months. The starting inventory was based on the actual on-hand inventory, and the first four deliveries were based on the actual deliveries received for each part.

The programming environment R was used to construct the models, estimate all of the parameters, and analyze the results (R Core Team, 2013). Table 4. *R Model Calls* lists the exact R function calls and libraries used for the chosen forecasting methods. The ES models were used from the Smooth package because of its support of external regressors (xreg). ES function call options were limited to additive errors to simplify the study. There are 30 potential models, but with no seasonality, there are only ten models that remain. The “Z” option was used (DESZ) to check alternative models' multiplicative errors and dampened trend, and then the model with the lowest Akaike information criteria corrected (AICc) was selected by the ES function.

Step Five – Determine Order

Orders are determined using the (R, S) policy proposed by Silver, Pyke, and Peterson (1998, p. 275), in which the inventory position is assessed over the time horizon (R+L), and ff R is less than S, the order-up-to-level at the end of each order review period R, a new order is issued to replenish the stock, bringing the inventory level up to S. In this study, any unmet demand is backlogged, and supply capacity is not constrained. The R value in this study is determined by the fixed monthly shipping schedules for delivery instead of being selected by the EOQ cost optimization approach. For a given target service level α, the forecasted quantity S must cover the demand over the order review interval R and the purchase delivery lead time L. Safety stock *SS* Equation (2) is based on the target service level α converted into a safety factor k, making the inverse cumulative distribution function of demand over the (R+L) interval plus the demand forecast errors standard deviation for the replenishment period (R+L).

The order (O) placed (equation 13) in this case is the forecasted demand (S) plus safety stock (SS) minus the expected inventory (I) over (R+L). The value $I_{(R+L)}$ (equation 14) includes the current on-hand inventory (H), but also needs to account for the expected deliveries (orders placed R+L periods in the past) minus the expected demand over (R+L) minus the customer notices (N) regarding known future demand.

$$O = S + SS - I_{(R+L)} \tag{13}$$

$$I_{(R+L)} = H + \sum_{t}^{t-R-L} O - \sum_{t}^{t+R+L} F(S) - \sum_{t}^{t+R+L} N \tag{14}$$

Where:

O = Order placed

S = forecasted demand

I = Expected deliveries $\sum_{t}^{t-R-L} O$ (orders placed R+L periods in the past)

H = End of last period inventory

SS = Safety Stock (equation 2)

F(S) = Order up to level based on forecasted demand

N = Customer notices $\sum_{t}^{t+R+L} N$ (known orders placed R+L periods in the past)

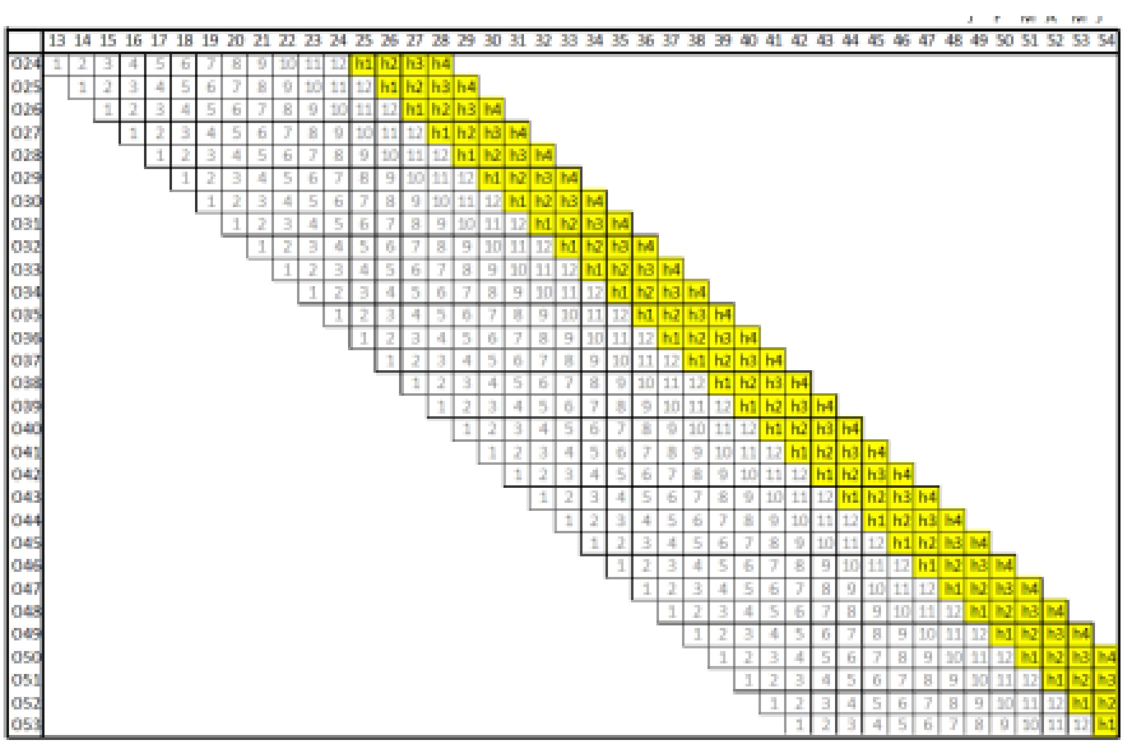
Step Six – Evaluate Performance

The evaluation of forecasting methods generally consists of examining a sample of forecast errors, testing the starting assumptions, testing the in-sample data fit, and then assessing outputs or out-of-sample data fit. Mathematically, this can be done using scale-dependent, scale-independent measures, or, in the case of demand forecasting for inventory, stock control metrics can be used. Scale-dependent measure mean error (ME), Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Mean Absolute Error (MAE), among others. ME should be near to 0 to show whether forecasts are skewed high or low. MSE averages the square of the errors and defines how close the forecast is to the actual data. It is used to represent demand variability. RMSE is the square root of the MSE and one of the most popular goodness-of-fit measures. It penalizes large errors more than small ones because it squares them first; if the mean error is near to zero, it is approximately the standard deviation of the errors. MAE averages the absolute values of all errors instead of squaring the errors resulting in greater tolerance of intermittent shocks or large errors.

The scale-independent measures Mean Absolute Percentage Error (MAPE) and Mean Scaled Percentage Error (MSPE), which help compare multiple series. MAPE measures the errors in percentage terms, which is better for data with a large variance like compound growth, inflation, or seasonality. Furthermore, MSPE averages each error as a standard average error ratio, preventing undefined or infinite values generated for intermittent demand (periods of zero demand). Armstrong (2001) concluded that using scale-independent measures to evaluate in-sample fit was not helpful in evaluating predictive performance. Some error measures are believed to be more effective than others in assessing time-series forecasts. He rated them as fair or reasonable in terms of dependability, construct validity, outlier protection, and whether or not they control for difficulty. Therefore, no scale-independent measures are used.

The calculation of the error will be done through rolling origin cross-validation, which produces 30 rolling origins starting at origin 24 from 54 observations using a fixed-sized window of 12. As new data is integrated into the prediction, new smoothing parameters are computed. The forecast origin is updated at each interval resulting in 30 training sets of 12 observations and an average error of 30 test sets of four predictions each, one for each month in the prediction interval. This average error figure is then used to compute the ME, RMSE, and MEA for the models. Figure 10. *Rolling Origin Cross-Validation with Constant In-Sample Window* depicts the rolling origin plan. The study will use the lowest RMSE to evaluate the error using cross-validation with a rolling forecasting origin (Hyndman & Athanasopoulos, 2018) and then compare it to the stock control metrics.

Figure 10. *Rolling Origin Cross Validation with Constant In-Sample Window*



Financial, operational, and service-related indicators are used to assess ordering and inventory performance (Petropoulos, Wang, & Disney, 2019) in business operations. System expenses such as inventory holding, backlogs, and orders are examples of financial metrics, whereas order and inventory variance are examples of operational measures. Finally, customer service level and fill rate are service-related measures. Instead of minimizing the historical demand forecast error, Kourentzes et al. (2020) used cost derived from inventory evaluations that resulted in reduced forecast accuracy but substantial increases in forecast bias (up to 62 percent) for the out-of-sample portion. However, they improved on the out-of-stock or inventory on hand performance.

The forecast accuracy criteria will be determined using the holding cost of capital of 5% per year and stockout cost of 90% of the potential profit of the part (part list price minus part cost) combined with the customer service-level goal of 95% for understanding the impact on operating performance for a given forecast model. The stockout cost is much higher than inventory holding costs due to the large contractual penalties that some of the largest distributors impose for unfilled orders. The best fit will be determined using the lowest inventory value overall and over the last thirteen periods. The study will compare the performance of stock control metrics against forecasting error measures ME, RMSE, and MAE.

Step Seven – Add Exogenous Variable and Repeat

Exogenous data from Google Trends will be used with the individual forecast models to investigate whether it will lead to an increase in forecast accuracy. Table 5. *R Xreg Model Calls* exhibits the function call with the integration of the xreg variable. The accuracy of each forecast model will then be compared against the baseline forecasts obtained in the first iteration, and the results will be analyzed to determine the level of improvement. Some of the simpler models RWF, RWFD, and SMA do not support external variables. The regression-based models LM and PW are statistical techniques that predict the result of a response variable using one or more explanatory factors.

Table 5. *R Xreg Model Calls*

FORECAST METHOD	ACRONYM	R Function Call	R Library
Naive Method (random walk)	RWF	No support of external regressors	NA
Naive Drift Method (random walk with drift)	RFWD	No support of external regressors	NA
Linear Forecast	LM	alm(fwin~Mwin+Gwin, df,distribution="dnorm")	Greybox (version 1.0.0)
Simple Moving Average	SMA	No support of external regressors	NA
Brown's Method using addititive errors with no trend or seasonaility	SES	es(fwin,model="ANN",h=hz,holdout =FALSE, xreg=Gwin)	Smooth (version 3.1.2)
Holt's Method using addititive errors with trend and no seasonality	DES	es(fwin,model="AAN",h=hz,holdout =FALSE, xreg=Gwin)	Smooth (version 3.1.2)
Holt's Method using the best addititive, multiplicative, or damping errors with trend and no seasonaility	DESZ	es(fwin,model="ZZN",h=hz,holdout= FALSE, xreg=Gwin)	Smooth (version 3.1.2)
Prais-Winsten	PW	prais_winsten(demand.v[1:n] ~ mo.v[1:n]+mo_sq.v[1:n]+Gtrends.v[1 :n], data=demand.df)	Prais (version 0.1.1)

However, the ES models have some theoretical issues supporting external data using the state-space framework. The smooth library package es() function does support external regressors. It first estimates the parameters for the primary variable and then estimates constant parameters for the exogenous variable for all the exogenous observations. Predictions for each variable are made and then utilized in the final forecast, with more weight placed on the ES model than on the exogenous variables.

In conclusion, our procedure enabled us to answer our study question and assess the performance of our proposed forecasting methods. The data, results, and analysis after following the procedures are presented in the following section.

Chapter 4: Results

The impact of different forecasting methods with long lead times for inventory control was evaluated. First, a discussion of the results of the different forecasting methods is followed by an examination of the factors that are influencing the results of the study.

Single Forecasting Model Results

The study averaged the results of the eight Individual forecasting model performances of resulting from the 100 non-intermittent part numbers sampled under study. The results are shown in Table 6. *Average Forecasting Model Performance*. The usage of a single forecasting method turned out to be less than ideal as none of the forecasting methods achieved an average target CSL of 95% over all time periods and part numbers. The Naïve method with drift (RWFD) performed the best over all the time periods at 89.6% but with a very high average model inventory cost of $37,917, as compared to the average actual on-hand inventory costs of $248 that the company experienced over the same time period using their linear method combined with their experience and judgment.

The traditional forecasting error measures (using cross-validation) with the lowest AIC or MAE came in at 482.6, and 23.79 respectively, using the Linear method in the study. The lowest RMSE of 32.67 was from the Simple Moving Average (SMA) method. Yet, the linear method had the fifth-highest cost of $52,504, and SMA had the fourth-highest cost of $53,149. Cross-Validation could not be calculated for the Prais-Winsten model because the Forecast library in R did not work with or support the use of

Table 6. *Average Forecasting Model Performance*

	Average					Average Inventory Cost	
Model Name	**CSL**	**cvME**	**cvRMSE**	**cvMAE**	**AIC**	**Model**	**Actual**
Naive	84.3%	(3.04)	40.89	29.03	NA	$ 37,917	$ 911
Naive Drift	89.6%	(1.38)	46.87	33.17	NA	$ 27,602	$ 911
Brown SES	40.6%	(5.17)	35.70	26.39	484.29	$ 113,805	$ 911
Holt DES	43.0%	(0.39)	46.21	32.56	483.77	$ 107,802	$ 911
Holt DESZ	40.0%	(3.75)	37.01	27.43	484.29	$ 113,502	$ 911
Linear	70.3%	14.46	35.18	23.79	482.60	$ 52,504	$ 911
Moving Average	72.1%	(6.00)	32.67	25.10	485.94	$ 53,149	$ 911
Prais Winsten	75.8%	NA	NA	NA	NA	$ 45,615	$ 911

	Average WITH Google Trends					Average Inventory Cost	
Model Name	**CSL**	**cvME**	**cvRMSE**	**cvMAE**	**AIC**	**Model**	**Actual**
Naive	84.3%	(3.04)	40.89	29.03	NA	$ 37,917	$ 911
Naive Drift	89.6%	(1.38)	46.87	33.17	NA	$ 27,602	$ 911
Brown SES	40.6%	(4.13)	36.49	26.45	485.18	$ 113,282	$ 911
Holt DES	44.2%	(2.67)	50.38	34.42	484.58	$ 107,196	$ 911
Holt DESZ	40.0%	(4.11)	38.07	27.42	485.18	$ 113,824	$ 911
Linear	71.3%	NA	NA	NA	483.50	$ 51,416	$ 911
Moving Average	72.1%	(6.00)	32.67	25.10	485.94	$ 53,149	$ 911
Prais Winsten	78.6%	NA	NA	NA	NA	$ 41,180	$ 911

the Prais-Winsten function from the Prais package leading to NA values for the cross-validation calculations. The addition of Google Trends as an external regressor improved performance only slightly, but not significantly enough, ranging from no change to 0.02% difference, to confirm that it should not be used in any forecasting method. In addition, the linear function with exogenous regressors was not supported by the Forecast library and the cross-validation function.

The performance in the last 13 months provides the greatest value since we are looking at the ability of fully trained forecasting models (over a range of 12 to 29 periods) to predict future performance over the last 13 months. Table 7. *Average*

Forecast Model Performance in the Last 13 Months shows The Prais-Winsten model performed the best over the last 13 months. It was the only model to achieve an average CSL of 95.9%, beating the target CLS by 0.9%, and with a much lower average inventory cost of $2,858, the lowest of the eight models. The Naïve models and the Linear model came close, but with larger inventories of almost twice as much as the Prais-Winsten method.

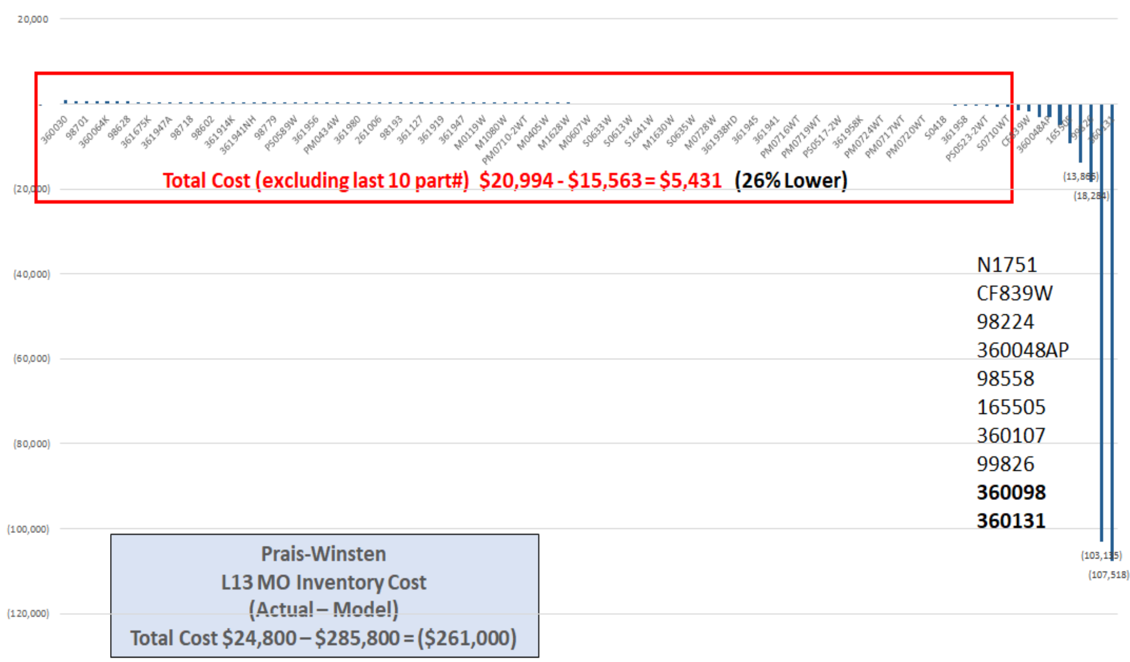
Table 7. *Average Forecast Model Performance in the Last 13 Months*

	Last 13 Month Average Inventory Cost		
Model Name	**CSL**	**Model**	**Actual**
Naive	92.8%	$ 4,758	$ 248
Naive Drift	92.8%	$ 4,495	$ 248
Brown SES	65.1%	$ 13,121	$ 248
Holt DES	60.6%	$ 14,755	$ 248
Holt DESZ	61.7%	$ 13,773	$ 248
Linear	92.9%	$ 5,017	$ 248
Moving Average	85.6%	$ 7,188	$ 248
Prais Winsten	95.9%	$ 2,858	$ 248

	Last 13 Month Average Inventory Cost		
Model Name	**CSL**	**Model**	**Actual**
Naive	92.8%	$ 4,758	$ 248
Naive Drift	92.8%	$ 4,495	$ 248
Brown SES	64.9%	$ 13,184	$ 248
Holt DES	60.5%	$ 14,534	$ 248
Holt DESZ	62.7%	$ 13,817	$ 248
Linear	93.0%	$ 4,800	$ 248
Moving Average	85.6%	$ 7,188	$ 248
Prais Winsten	96.1%	$ 2,821	$ 248

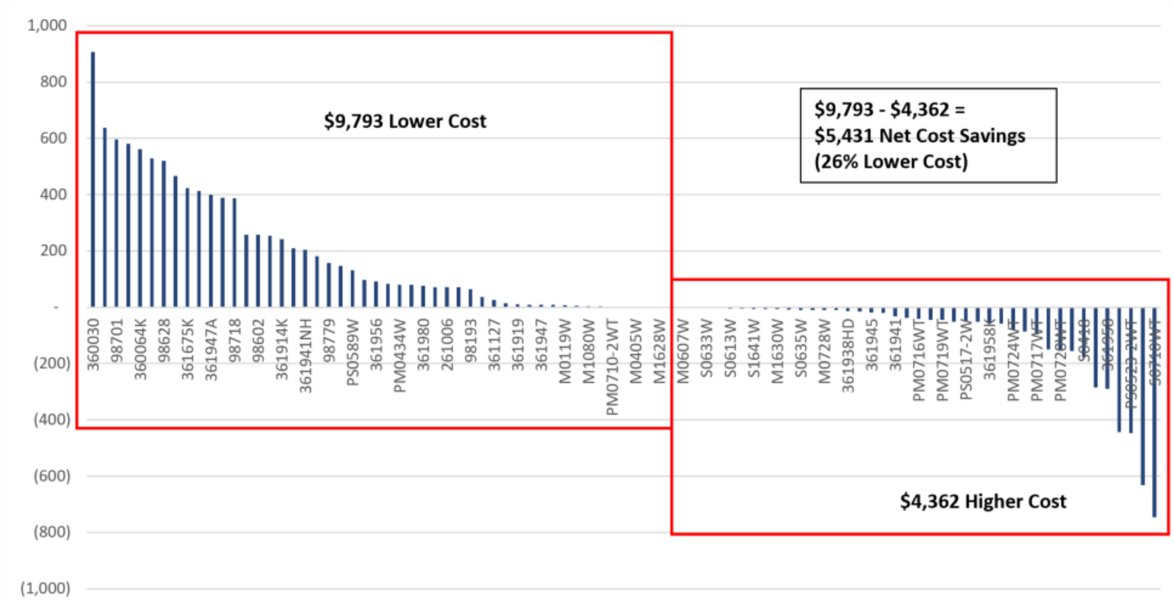
Upon a closer examination of the single Prais-Winsten model inventory cost, shown in Figure 11. *Prais-Winsten Last 13 Month Inventory Cost*, a dramatic drop-off performance occurs after 90% of the part numbers with a subset of 10% of the part numbers performing poorly. It can be seen that only two of the part numbers accounted for 74% of the total overage, which is $210,653 of the $285,800 total inventory cost.

Figure 11. Prais-Winsten Last 13 Month Inventory Cost



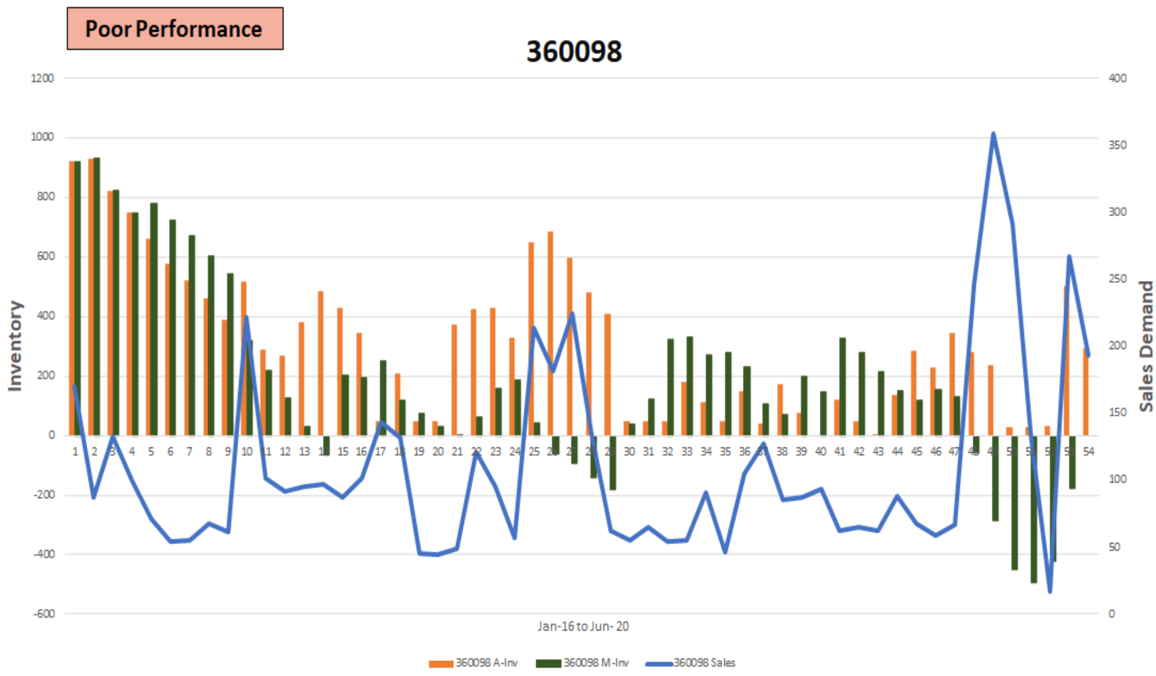
By removing the last 10 of the part numbers, then it can be seen that the total model cost of inventory is $15,431 compared with the actual on-hand cost of $20,994. Figure12. *Prais-Winsten Last 13 Month Inventory Cost Excluding the Last 10 Part Numbers* shows 49 of the part numbers with lower cost compared to 42 of the part numbers with somewhat negative performance, but overall producing $5,431 in total cost savings. The Prais-Winsten model produced a 26% lower cost using 90% of the parts numbers. The ten parts that were excluded appeared to exhibit non-normal behavior with various spikes in demand. This was determined to be because of a new major account.

Figure 12. *Prais-Winsten Last 13 Month Inventory Cost Excluding the Last 10 Part Numbers*



These spikes are illustrated in Figure 13. *Part 360098 Sales Demand and Inventory* shows the non-normal demand (blue line) causing the forecasting models to have difficulty ordering enough inventory, given the four-month lead time, to prevent stockouts. The green line shows the model's inventory results compared to the actual results the company experienced. The company tends to keep more inventory on hand because of the potential stockout penalties, and it was noticed that the company did not stock out during any of these spikes in demand.

Figure 13. *Part 360098 Sales Demand and Inventory*



The company explained that the spikes are caused by sales and marketing lift promotional programs when bringing on a new account. When a major new account is brought onboard, the company might agree to accept stock from the distributor to refurbish or rework. Then some of this stock is sent back out to the account's other distribution locations. This stock is not ordered from their Asian suppliers, and the rework is mostly repackaging, which can be done quickly to ship rapidly. The parts that are returned may not be the same quantity as those that are shipped out, resulting in a stock rebalancing with no significant change in total inventory because they immediately had the stock ready to ship, and they did not have to wait four months for resupply.

In addition, the order data that was sent included a request date and a ship date. Sometimes the customer will hold an order for four to eight weeks because their store is not ready to receive the stock. This delay was not accounted for in the data, which means the company did have advanced notice of an order and reacted accordingly by ordering enough inventory. The demand data used for this project included this rebalancing and held orders, but the advanced stocking orders were not included in the prior notice data table. Since the lift adjustment could not be made, it caused these spikes in the demand data picture.

The use of a single forecasting model to predict future demand across several different parts numbers can perform poorly and appears to be less robust or agile to respond to the variation in demand even when using a sample of 100 non-intermittent and low CV^2 classified parts. It is possible that using different forms of classification, data clustering, and demand prediction (Steuer, Hutterer, Korevaar, & Fromm, 2018) could lead to better results. Clearly, each forecasting method is different, just like the differences found in each of the individual part number demand patterns.

Multiple Forecasting Model Results

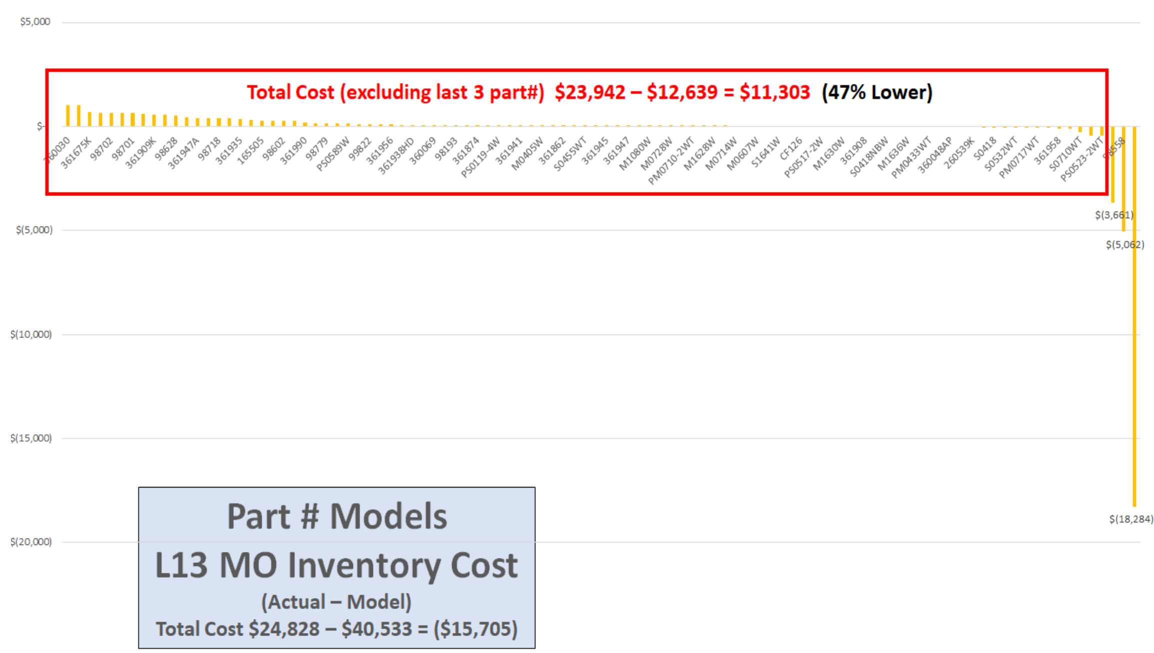
Choosing the optimal forecasting model for each part number produced much better results. Table 8. *Select Forecasting Model Performance* shows the total inventory cost performance of each model optimized around individual raw part numbers leading to a significant improvement over the one-size-fits-all approach. Six of the eight or 75% forecasting models (highlighted in red) did better than their actual performance. Again, the few non-normal demand part numbers caused the Prais-Winsten model to perform poorly, just like before.

Table 8. *Select Forecasting Model Performance*

	Last 13 Month Total Inventory Cost			
Model Name	**Count**	**CSL**	**Model**	**Actual**
Brown SES	8	99.9%	$ 1,439	$ 2,522
Holt DES	10	100.0%	$ 682	$ 1,131
Holt DESZ	4	100.0%	$ 509	$ 724
Linear	14	99.4%	$ 6,290	$ 3,900
Moving Average	24	100.0%	$ 2,379	$ 6,662
Naive	5	100.0%	$ 846	$ 1,457
Naive Drift	2	100.0%	$ 158	$ 289
Prais Winsten	33	98.3%	$ 28,229	$ 8,144
Grand Total	**100**	**99.4%**	**$ 40,533**	**$ 24,828**

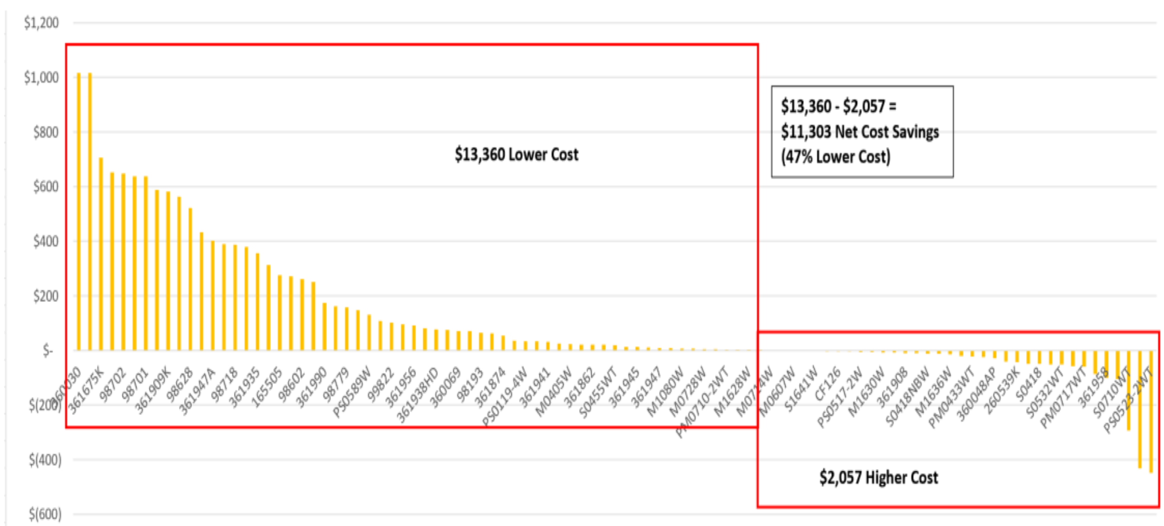
with Google Trends	Last 13 Month Total Inventory Cost			
Model Name	**Count**	**CSL**	**Model**	**Actual**
Brown SES	4	100.0%	$ 551	$ 1,617
Holt DES	11	100.0%	$ 948	$ 755
Holt DESZ	6	100.0%	$ 830	$ 1,094
Linear	16	99.4%	$ 7,511	$ 5,008
Moving Average	23	100.0%	$ 2,115	$ 6,441
Naive	11	100.0%	$ 1,589	$ 2,747
Naive Drift	2	89.3%	$ 5,407	$ 385
Prais Winsten	27	98.7%	$ 22,940	$ 6,780
Xreg Total	**100**	**99.3%**	**$ 41,891**	**$ 24,828**

Figure 14. *Part Number Models over the Last 13 Month Inventory Cost*



The inventory cost resulting from the performance of the individual models is shown in Figure 14. *Part Number Models over the Last 13 Month Inventory Cost*. Each part number is sorted from those that did the best to worst. The total cost excluding the last three of the part numbers is $12,639 compared to the actual cost of $23,942, resulting in a difference of $11,303 in savings (see Figure 15. *Part Number Models Last 13 Month Inventory Cost Excluding 3 Part Numbers*) made up of 62 of the part numbers with $13,360 lower cost and 35 of the part numbers with $2,057 in higher cost. The total is 47% lower than the company's actual performance and almost twice the performance of a single Prais-Winsten model, which was 26%.

Figure 15. *Part Number Models Last 13 Month Inventory Cost Excluding 3 Part Numbers*



Using a different model for each part number is slightly more computationally intensive. However, if this were done every month, it would not be necessary to recompute the past months. Only the new data would need to be added each month, and the new parameters could be optimized, and possibly a new model could be selected based on its performance that month.

Results Summary

In answer to the first research question: Do complex forecasting methods increase forecasting accuracy? Yes, using an optimal model for individual part numbers worked even better. Selecting the complex model Prais-Winsten as a single model over the whole time series performed best at 96%, but it incurred 11x greater cost. If we exclude last 10 of the most volatile part numbers due to non-normal behavior, then the total model inventory cost would be $5,431 or 26% lower than the company's current method of inventory control. Using the forecasting model that performed the best for an individual part number created a total model inventory cost of $11,303 or 47% lower than their current method of inventory control, when we exclude the last three of the part numbers due to non-normal behavior.

In answer to the second research question: Can exogenous data improve demand forecasting? Maybe, but using Google Trends was not significantly better. The Google data appeared promising at the start, but after analysis, it turned out to be insignificant. However, the Google Trends data was helpful in finding an autoregressive AR(1) process with a first-order moving average. This indicated the use of the Prais-Winsten method, an iterative process designed for dealing with AR(1) errors., which led to it being added as a forecasting method in this study.

In answer to the third research question: Can stock control metrics be used to evaluate forecast accuracy? Yes, using inventory costs is better than forecast error because it is in dollar terms, so everyone can easily understand the impact to the bottom line. Imprecise forecasting projections are costly to businesses, resulting in stockouts and lost revenues, as well as overstocking and failing to fulfill service level agreements resulting in incurring penalties (Kourentzes et al., 2020). Stock control includes more factors than looking at forecast error in terms like RMSE or MAE. Syntetos, Nikolopoulos, and Boylan (2010) note that forecasting methods utilized as an input to inventory control should be assessed on their effects on inventory control.

There is also the effect of safety stock, customer service expectations, and current levels of inventory on hand that all inform the order. A business is more concerned with the effects of forecast error on CSL and inventory levels than the value of forecast error. Translating the forecast error into an inventory cost number provides better feedback for stock control. The company currently tracks monthly fill rates, but it does not have a method for order accuracy beyond their fill rate. The research results support the use of the Prais-Winsten method for non-intermittent demand as an improved method over the linear model they are using today. The company believes they could save even more using optimized models for each part number.

In answer to the fourth research question: Can a procedure be developed that is likely to be adopted? Yes. The purchasing manager currently aggregates different inputs to inform his judgment on the final order quantity, including using the linear time trend in an Excel spreadsheet to estimate demand in the next four months. A routine can be developed from within MS Excel or Power BI calling an R or Python program to perform the complex forecasting. The selected model that performs the best could be deployed into the organization and validated against future data (periods +1, 2, 3). In each future month, the parameters of the forecast model would be updated with the new data.

Chapter 5: Discussion

The study provided a solution to demand forecasting based on predictive analytics of automotive aftermarket company data combined with exogenous data from Google Trends. The contribution to practice includes stock control metrics to determine forecast accuracy and evaluate whether forecast model complexity leads to better results. Although an improvement in forecast accuracy was not obtained by using external Google Trends, it is still possible that other data sources could lead to such improvement.

The resulting optimal inventory policy should not be difficult to implement and produce cost savings over the company's current method. An optimal inventory policy would lead to lower inventory uncertainty and a significant rise in warehouse space utilization while maintaining a high CSL demanded by customers and the company's marketing strategy.

The study illustrates the inadequacy of simple univariate models used for forecasting that did not perform better than the complex models like Prais-Winsten, an econometric model rarely used in demand forecasting for inventory control or automotive parts. The study shows how forecast models that consider stock control metrics can provide more significant inventory optimization over traditional accuracy measures. Moreover, the study adds to the limited empirical research on demand forecasting using predictive analytics with long lead times, exogenous variables, stock control metrics, and dynamic model updating, and the use of Prais-Winsten for demand forecasting for automotive parts.

Limitations

There were a few limitations of the study based on the research design and provided data. The company only provided 54 months of data the was usable due to a change in accounting systems that made the older data difficult to reconcile with the new accounting system. Utilizing more data would have allowed larger training sets for the forecasts.

The study design focused on 100 nonintermittent sample parts in order to reduce computational complexity; therefore, none of the intermittent parts were included in the study. Adding intermittent part numbers would have provided greater certainty regarding the overall inventory performance for the whole company.

Data governance was an issue with the company. There did not appear to be clear historical records of the advanced notice data making it difficult to break out the non-normal behavior that was known to be occurring. Also, the historical stockout costs were estimated at 90% of profit, but the actual costs were estimated to vary by more due to changes in customer discounts, promotions, supplier costs, and quantity purchase discounts.

The study's final limitation stemmed from the constraints of the ‘R’ language libraries used for computation. None of the libraries supported stock control metrics for evaluation. The study was also unable to overcome computing cross-validation numbers using exogenous regressors for unsupported forecasting models. Given more time, the study could overcome some of these limitations in ‘R’ libraries or data.

Extensions and Future Research

This study provides the opportunity for several future possibilities to extend these research findings. Further research into how easily the new forecast method is adopted, used, and performed in the future could be explored better to understand continuous utilization, adaptation, and changes.

There were 100 non-intermittent and low CV2 classified parts. Further exploration of the remaining intermittent and non-intermittent parts could be investigated to determine if better performance can be obtained with new, more sophisticated, and different models that could produce even better results.

The company's judgment worked quite well in the past without using safety stock in their ordering policy. Although research studies have shown the advantages of both safety stock (Chu & Shen, 2010; Kang, Ullah, & Sarkar, 2018) and statistical forecasting methods, businesses continue to rely on their judgment integrated with demand forecasting, frequently described as "integrating forecasting" methods (Arvan, Fahimnia, Reisi, & Siemsen, 2019; Baecke, De Baets, & Vanderheyden, 2017; Syntetos, Nikolopoulos, Boylan, Fildes, & Goodwin, 2009). A model could be developed to understand how well their current judgment performance works, both with and without safety stock, and then integrate the existing heuristics used for forecasting judgment into a standard ordering policy while also measuring effectiveness.

Combining several forecasts into a single model has been shown to reduce forecasting errors and reduce the constraints inherent in a single model (Barrow & Kourentzes, 2016). Additional research into different sources of exogenous data like using the companies own website searches, hits, or individual customer ordering patterns might lead to additional improvement.

Understanding how automotive parts age and the production cycle is also crucial to raw material purchasing. As automobiles age, they become less popular and expensive to maintain, which results in demand for automotive parts declining. This results in retailers using stock balancing of older inventory or negative demand to the manufacturer. At the same time, as the purchasing cycle improves, it is essential for production planning to improve and become more efficient. Significant opportunities for improvement would exist for production planning, material handling, and inventory usage to understand how this negative demand due to aging, along with committed raw material parts, affects finished goods inventory. Furthermore, a replication of this study in other firms and industries would be valuable.

There is also the use of clustering analysis to break down large groupings of data items into smaller groups based on their similarities while considering the complex bill of material relationships between items (Srinivasan & Moon, 1999). There is also the use of K-nearest-neighbor (KNN), which Nikolopoulos, Babai, and Bozos (2016) used to forecast intermittent automotive spare parts demand. KNN is a classification method that identifies similarities in each object to nearby objects (named tuples) with a similarity index. These tuples are explained by n characteristics corresponding to a place in an n-dimensional space. The KNN method finds k tuples most similar to a particular tuple. These classifications result in the development of clusters of objects that are comparable. KNN can also be used to reduce the dimensionality of data in regression analysis situations.

Conclusions

Demand forecasting with long lead times is challenging to obtain an accurate forecast. In this paper, we use the data from an automotive clutch manufacturing company, which consisted of over 1,033 imported raw parts with lead times of four months. Monthly forecasting of part demand resulted in an assessment of various simple and complex forecasting methods. The comparison results show that the forecasts obtained using a single Prais-Winsten econometric method inventory order policy with safety stock were more accurate than those obtained by other classical forecasting methods and produced a 26% improvement over the current company use of the linear method that is coupled with judgment and does not use safety stock.

In addition, the forecasts obtained using the best method for individual part numbers produced a 47% improvement over the current linear method used by the company. The paper also shows how traditional forecast error measures were inappropriate and that using the stock control metrics, CSL, and inventory cost is superior for non-intermittent demand, despite their consistent usage for evaluating forecast error in many forecasting studies. The research results support the use of the Prais-Winsten method for non-intermittent demand with long-lead times and using multiple forecasting models that are optimized to individual part numbers for an inventory order policy.

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Appendix – R Program Code

```
# Load Libraries
library(forecast)
library(prais)
library(smooth)

###START

# LOOP all SKUs using Exogenous regressors
xreg_sw <- "FALSE"

for (sw in (1:2)) { # xreg_sw loop

#load company dataframe from excel delimited
Company.df = read.delim("Data_inv_sales_not100cv-smooth.txt", header = TRUE)
last <- 54

#Replace missing data with "0"
Company.df[Company.df==""] <- 0
demand_o.v     <<- vector("integer", length = last)
onhand_inv.v  <<- vector("integer", length = last)
notice.v  <<- vector("integer", length = last)
date.v    <<- seq(as.Date("2016-1-1"), as.Date("2020-6-1"), by = "months")

#load Google dataframe from excel delimited
Google.df = read.delim("GoogleTrends.txt", header = TRUE)
Gtrends.v <- vector("integer", length = last)
Gtrends.v <- as.integer(Google.df[1:last])
Gwin.ts <<- ts(Gtrends.v, start=c(2016,1), freq=12)
eXreg  <- "GoogleTrends"

N <- 100 # total SKU 100
results.df  = data.frame(
   sku         = character(N),
   cost        = numeric(N),
   price       = numeric(N),
   Hcsl       = numeric(N),
   MHcsl       = character(N),
   MCSLmethod  = character(N),
   MTinvC      = numeric(N),
   LcvRMSE     = numeric(N),
   McvRMSE     = character(N),
   RMSETinvC   = numeric(N),
   Laic        = numeric(N),
   Maic        = character(N),
```

```
  aicTinvC    = numeric(N),
  MTinv_Tgtcsl= numeric(N),
  MTinv_Tgt   = character(N), #Model of Lowest Inv to meet Target CSL
  MCSLmethTgt = character(N),
  MTinv_TgtC  = numeric(N),
  Act_invC    = numeric(N),
  L13_CSL     = numeric(N),
  L13_invC    = numeric(N),
  L13_ActinvC = numeric(N),
  L13_MTinv   = character(N),
  L13_avg     = numeric(N),
  L13_sd      = numeric(N),
  L13_cv2     = numeric(N),
  L_avg       = numeric(N),
  L_sd        = numeric(N),
  L_cv2       = numeric(N),
  hz          = integer(N),
  win         = integer(N),
  eXreg       = character(N),
  fill_rate   = numeric(N)
)
irate  <- 0.05/12 # annual interest rate
discount  <- -0.9 # % of profit for backorder
fill_rate <- 0.99
win       <<- 12  # 12-period rolling window
hz        <<- 4   # 4-period forecast lead time
stwin     <<- 12  # start window ignores first 12 mo
mo.v      <<- rep(1:12,5)
mo.v    <<- mo.v[1:54]
mo_sq.v   <<- mo.v * mo.v
Nmo.v     <<- seq(54)
Nmo.ts    <<- ts(Nmo.v, start=c(2016,1), freq=12)

#model "N"=none, "A"=additive, "M"=multiplicative, Ad=AdditiveDamped,
"Z"=automatically selected and "C"=combine

# CV Forecast Methods
flm  <- function(y, h) {
  df <- data.frame(fwin.cv, Mwin.cv)
  mod <- alm(fwin.cv ~ Mwin.cv, df, distribution="dnorm")
  forecast(mod,h=hz,interval="p")
}
flmp <- function(y, h) {
  df <- data.frame(y, Mwin.cv)
  mod <- alm(demand.ts ~ Mwin.cv, df, distribution="dpois")
    forecast(mod,h=hz,interval="p")
```

```
  }

  fses <- function(y, h) { forecast(es(y, model="ANN"), h=h) }
  fdes <- function(y, h) { forecast(es(y, model="AAN"), h=h) }
  ftes <- function(y, h) { forecast(es(y, model="ZZN"), h=h) }

  fma  <- function(y, h) { forecast(sma(y, win), h=h) }
  fpw  <- function(y, h, x1, x2, x3) { forecast(prais_winsten(y ~ x1 + x2, data=x3), h=h) }

  fgpw <- function(y, h, x1, x2, x3, xreg) { forecast(prais_winsten(y ~ x1 + x2 + xreg,
data=x3), h=h,interval="p") }
  fglm <- function(y, h, xreg) {
     df <- data.frame(fwin.cv, Mwin.cv, Gwin.cv)
     mod <- alm(fwin.cv ~ Mwin.cv+Gwin.cv, df, distribution="dnorm")
     forecast(mod,h=hz,interval="p")
  }
  fglmp <- function(y, h, xreg) {
     df <- data.frame(y, Mwin.cv, xreg)
     mod <- alm(y ~ Mwin.cv+xreg, df, distribution="dpois")
     forecast(mod,h=hz,interval="p")
  }
  fgses <- function(y, h, xreg, newxreg) { forecast(es(y, model="ANN", holdout=FALSE,
xreg=xreg), h=h) }
  fgdes <- function(y, h, xreg, newxreg) { forecast(es(y, model="AAN", holdout=FALSE,
xreg=xreg), h=h) }
  fgtes <- function(y, h, xreg, newxreg) { forecast(es(y, model="ZZN", holdout=FALSE,
xreg=xreg), h=h) }

mean_fcst  <- function(y, h, n) {
  fcst <<- predict(y,h=h,interval="p")
  forecast.v[n+1] <<- round(fcst$mean[h], digits=0)
  Pforecast.v[n+1]<<- round(sum(fcst$mean[1:h]), digits=0) # 4-period forecast sum
  for (i in seq(h)) { Pfcst.mat[n+1,i] <<- fcst$mean[i] }
  return(round(fcst$mean[h], digits=0))
}

#LOOP through part number SKUs
for (f in (1:N)) {

 #load vectors with company SKU data
 RM   <<- as.character(Company.df[f,1])
 cost   <<- as.numeric(Company.df[f,2])
 price   <<- as.numeric(Company.df[f,3])
 demand_o.v  <<- as.integer(Company.df[f,4:57])
 notice.v  <<- as.integer(Company.df[f,58:111])
 onhand_inv.v  <<- as.integer(Company.df[f,112:165])
```

```
profit  <- price - cost
part.v  <<- c(RM, cost, price, profit)


CSL.df <<- data.frame(
   model_name.v = c("Naive", "Naive Drift", "Brown SES","Holt DES","Holt
DESZ","Linear","Moving Average","Prais Winsten"),
   CSL     = 0.0,
   inv_low  = 0,
   inv_high = 0,
   inv_rang = 0,
   cvME     = 0.0,
   cvRMSE   = 0.0,
   cvMAE    = 0.0,
   aic      = 0.0,
   method   = "NA",
   Tinv_neg = 0.0,
   Tinv_pos = 0.0,
   Tinv     = 0.0,
   Tinv_negC= 0.0,
   Tinv_posC= 0.0,
   TinvC    = 0.0,
   ActinvC  = 0.0,
   L13_CSL  = 0.0,
   L13_invC = 0.0,
   L13_ActinvC = 0.0,
   L13_MTinv_Tgt = " "
)

# Initialize Start Conditions
demand.v <<- demand_o.v - notice.v
start    <- round(mean(demand.v[1:12]), digits=0)
demand.ts   <- ts(demand.v, start=c(2016,1), freq=12)

#LOOP through Methods
write.table(RM, "results.csv", sep = ",", col.names = TRUE, append = T)
for (fn in (1:8)) {

 # Initialize Vectors
 forecast.v   <<- vector("integer", length = last)
 level.v   <<- vector("integer", length = last)
 trend.v   <<- vector("integer", length = last)
 season.v   <<- vector("integer", length = last)
 alpha.v   <<- vector("integer", length = last)
 beta.v    <<- vector("integer", length = last)
 method.v    <<- vector("character", length = last)
```

```
  SS.v    <<- vector("integer", length = last)
  order.v   <<- vector("integer", length = last)
  ExpInvB4_order.v <<- vector("integer", length = last)
  delivery.v   <<- vector("integer", length = last)
  EOP_Inv.v   <<- vector("integer", length = last)
  forecast_err.v   <<- vector("integer", length = last)
  order_err.v   <<- vector("integer", length = last)
  Inv_err.v   <<- vector("integer", length = last)
  Psim_fc.v   <<- vector("integer", length = last)
  Pforecast.v  <<- vector("integer", length = last)
  Pfcst.mat   <<- matrix(0, nrow = last, ncol = 5)

  forecast.v[1]  <- start
  Pforecast.v[1]  <- forecast.v[1]*hz
  SS.v[1:5] <- round(qnorm(fill_rate) * sd(demand.v[1:12]), digits=0)
  order.v[1]  <- start
  ExpInvB4_order.v[1]  <- start
  delivery.v[1]   <- start
 # delivery.v[1:(hz+1)] <- start
  for (d in (1:3)) { delivery.v[d+1] <- onhand_inv.v[d+1] - (onhand_inv.v[d] -
demand_o.v[d+1]) }
  EOP_Inv.v[1]  <- onhand_inv.v[1]
  order_err.v[1]  <- order.v[1] - demand.v[1]
  Inv_err.v[1]  <- EOP_Inv.v[1] - demand.v[1]

        message(paste("SKU", f, RM, "method loop", fn, CSL.df$model_name.v[fn],
"Xreg=", xreg_sw, "start=", start, "Fill Rate:", fill_rate))
        write.table(paste(RM, "method loop", fn, CSL.df$model_name.v[fn]), "results.csv",
sep = ",", col.names = TRUE, append = T)

  # Loop through orders(n)
  for (n in (1:(last-1))) {
     fwin <<- window(demand.ts, end=2016+(n-1)/12)
     Gwin <<- window(Gwin.ts,end=2016+(n-1)/12)
     Mwin <<- window(Nmo.ts, end=2016+(n-1)/12)
     if (xreg_sw == "TRUE") demand.df <<- data.frame(demand.v[1:n], mo.v[1:n],
mo_sq.v[1:n], Gtrends.v[1:n])
     else demand.df <<- data.frame(demand.v[1:n], mo.v[1:n], mo_sq.v[1:n])

     if (fn == "1") {  # Naive
   fit1 <- rwf(fwin)
   mean_fcst(fit1, hz, n)
     } else if (fn == "2") {  #Niave w/Drift
   if (n == "1") {   # cannot forecast a single period
      forecast.v[n+1]  <- start
      Pforecast.v[n+1] <- forecast.v[n+1]*hz
```

```
  } else {
     fit1 <- rwf(fwin, drift=TRUE)
     mean_fcst(fit1, hz, n)
  }
    } else if (fn == "3") {  # "Brown SES ANN"
  if (n < stwin) {   # cannot forecast a single period
    forecast.v[n+1]  <- start
    Pforecast.v[n+1] <- forecast.v[n+1]*hz
  } else {
    if (xreg_sw == "TRUE") { fit1 <- es(fwin,model="ANN",h=hz,holdout=FALSE,
xreg=Gwin) }
    else { fit1 <- es(fwin, model="ANN",h=hz,holdout=FALSE) } #Additive errors w/no
trend or season
    forecast.v[n+1]<- round(fit1$forecast[hz], digits=0)
    Pforecast.v[n+1]<- round(sum(fcst$mean[1:hz]), digits=0) # 4-period forecast sum
                for (i in seq(hz)) { Pfcst.mat[n+1,i] <- fcst$mean[i] }
    alpha.v[n+1] <- fit1$persistence[1]
    level.v[n+1] <- fit1$states[1]
    method.v[n+1]<- fit1$model
  }
    } else if (fn == "4") {   # "Holt DES AAN"
  if (n < stwin) {   # cannot forecast a single period
    forecast.v[n+1]  <- start
    Pforecast.v[n+1] <- forecast.v[n+1]*hz
  } else {
    if (xreg_sw == "TRUE") { fit1 <- es(fwin,model="AAN",h=hz,holdout=FALSE,
xreg=Gwin) }
    else { fit1 <- es(fwin, model="AAN",h=hz,holdout=FALSE) } #Additive errors
w/trend
    forecast.v[n+1]<- round(fit1$forecast[hz], digits=0)
    Pforecast.v[n+1]<- round(sum(fcst$mean[1:hz]), digits=0) # 4-period forecast sum
                for (i in seq(hz)) { Pfcst.mat[n+1,i] <- fcst$mean[i] }
    alpha.v[n+1] <- fit1$persistence[1]
    level.v[n+1] <- fit1$states[1]
    method.v[n+1]<- fit1$model
  }
    } else if (fn == "5") {  # "Holt DES ZZN"
  if (n < stwin) {   # cannot forecast a single period
    forecast.v[n+1]  <- start
    Pforecast.v[n+1] <- forecast.v[n+1]*hz
  } else {
    if (xreg_sw == "TRUE") { fit1 <- es(fwin,model="ZZN",h=hz,holdout=FALSE,
xreg=Gwin) }
    else { fit1 <- es(fwin, model="ZZN",h=hz,holdout=FALSE) } #Additive errors
w/trend
    forecast.v[n+1]<- round(fit1$forecast[hz], digits=0)
```

```
    Pforecast.v[n+1]<- round(sum(fcst$mean[1:hz]), digits=0) # 4-period forecast sum
   for (i in seq(hz)) { Pfcst.mat[n+1,i] <- fcst$mean[i] }
    alpha.v[n+1] <- fit1$persistence[1]
    level.v[n+1] <- fit1$states[1]
    method.v[n+1]<- fit1$model
  }
    } else if (fn == "6") {  # "Linear"
  if (n < stwin) {   # cannot forecast a single period
    forecast.v[n+1]  <- start
    Pforecast.v[n+1] <- forecast.v[n+1]*hz
    Pfcst.mat[ n+1,1:(hz+1)] <- forecast.v[n+1]
  } else {
    if (xreg_sw == "TRUE") {
      df   <- data.frame(fwin, Mwin, Gwin)
      fit1 <- alm(fwin~Mwin+Gwin, df,distribution="dnorm")
    } else {
      df   <- data.frame(fwin, Mwin)
      fit1 <- alm(fwin~Mwin,df,distribution="dnorm")
    }
    mean_fcst(fit1, hz, n)
  }
    } else if (fn == "7") {  # SMA
  if (n < stwin) {   # cannot forecast a single period
    forecast.v[n+1]  <- start
    Pforecast.v[n+1] <- forecast.v[n+1]*hz
  } else {
    if (n < win) { num = n } else { num = win }
    fit1 <- sma(fwin, num, h=hz)
    forecast.v[n+1]<- round(fit1$forecast[hz], digits=0)
    Pforecast.v[n+1]<- round(sum(fit1$forecast[1:hz]), digits=0) # 4-period forecast sum
    for (i in seq(hz)) { Pfcst.mat[n+1,i] <- fcst$mean[i] }
  }
    } else if (fn == "8") { # "Prais Winsten"
  if (n < stwin) {   # cannot forecast a single period
    forecast.v[n+1]  <- start
    Pforecast.v[n+1] <- forecast.v[n+1]*hz
  } else {
    if (xreg_sw == "TRUE") {
      fit1 <- prais_winsten(demand.v[1:n] ~ mo.v[1:n]+mo_sq.v[1:n]+ Gtrends.v[1:n],
data=demand.df)
      b4  <- fit1$coefficients[4] # Gtrends.v
    } else {
      fit1 <- prais_winsten(demand.v[1:n] ~ mo.v[1:n]+mo_sq.v[1:n], data=demand.df)
    }
    e  <- summary(fit1)
```

```
    dw <- e$dw[1] # Durbin-Watson statistic  2< autocorrelation (AC), 0=No AC, >2
Neg AC
    method.v[n+1] <- paste("DW=", round(dw, digits=2))

    rho<- fit1$rho[length(fit1$rho)]
    b1 <- fit1$coefficients[1] # intercept
    b2 <- fit1$coefficients[2] # mo.v
    b3 <- fit1$coefficients[3] # mo_sq.v

    if ( n > 53) {
      forecast.v[n+1] <- 0
      Pfcst.mat[ n+1,] <- 0
    } else if (xreg_sw == "TRUE") {
      err <- demand.v[n] - (rho * demand.v[n-1]) - ((1-rho) * b1) - (b2 * (mo.v[n] - rho *
mo.v[n-1])) - (b3 * (mo_sq.v[n] - rho * mo_sq.v[n-1])) - (b4 * (Gtrends.v[n] - rho *
Gtrends.v[n-1]))
      forecast.v[n+1] <- round(I(rho^hz)*err + b1 + b2 * mo.v[n+1] + b3 * mo_sq.v[n+1]
+ b4 * Gtrends.v[n+1] + 0.5, digits=0)
      for (i in seq(hz)) {
        Pfcst.mat[ n+1,i] <- round(I(rho^i)*err + b1 + b2 * mo.v[n+i] + b3 *
mo_sq.v[n+i] + (b4 * Gtrends.v[n+1]) + 0.5, digits=0)
      }
    } else {
      err <- demand.v[n] - (rho * demand.v[n-1]) - ((1-rho) * b1) - (b2 * (mo.v[n] - rho *
mo.v[n-1])) - (b3 * (mo_sq.v[n] - rho * mo_sq.v[n-1]))
      forecast.v[n+1] <- round(I(rho^hz)*err + b1 + b2 * mo.v[n+1] + b3 * mo_sq.v[n+1]
+ 0.5, digits=0)
      for (i in seq(hz)) {
        Pfcst.mat[ n+1,i] <- round(I(rho^i)*err + b1 + b2 * mo.v[n+i] + b3 *
mo_sq.v[n+i] + 0.5, digits=0)
      }
    }
    Pforecast.v[n+1] <- round(sum(Pfcst.mat[ n+1,1:hz]), digits=0)
  }
    } # end model IF
    if (n > hz) { forecast_err.v[n] <- forecast.v[n-hz] - demand.v[n]} # cannot determine
error of last data points beyound hz (no demand)
    else { forecast_err.v[n] <- forecast.v[n] - demand.v[n+hz] }
    order_err.v[1]    <- order.v[n] - demand.v[n]

    if (n > hz) {
     SS.v[n+1] <-  round(qnorm(fill_rate) * sd(forecast_err.v[(n-hz):n]) + .5, digits=0)
     delivery.v[n+1] <- order.v[n-(hz-1)]
     ExpInvB4_order.v[n+1] <- EOP_Inv.v[n] + sum(order.v[(n-(hz-1)):n]) -
Pforecast.v[n+1] - sum(notice.v[(n-(hz-1)):n])
```

```
    } else {  # n < hz
      delivery.v[(n+hz)] <- order.v[n]
      ExpInvB4_order.v[n+1] <- EOP_Inv.v[n] + sum(delivery.v[(n+1):(n+hz)]) -
Pforecast.v[n+1] - sum(notice.v[(n+1):(n+hz)])
    }
    order.v[n+1]  <- max((forecast.v[n+1] + SS.v[n+1] - ExpInvB4_order.v[n+1]),0)
    order_err.v[n+1] <- order.v[n+1] - demand.v[n+1]
    EOP_Inv.v[n+1] <- EOP_Inv.v[n] + delivery.v[n+1] - demand_o.v[n+1]
    Inv_err.v[n+1] <- EOP_Inv.v[n+1] - demand.v[n+1]

  } # End order loop
  forecast_err.v[n+1] <-  0

  #Output Model Detail
  print(part.v)
  if (xreg_sw == "TRUE") {
    ordering.df <- data.frame(date.v, demand_o.v, notice.v, forecast.v, SS.v, order.v,
ExpInvB4_order.v, delivery.v, EOP_Inv.v, forecast_err.v, Pforecast.v, alpha.v, beta.v,
method.v, level.v, trend.v, onhand_inv.v, Gtrends.v)
  } else {
    ordering.df <- data.frame(date.v, demand_o.v, notice.v, forecast.v, SS.v, order.v,
ExpInvB4_order.v, delivery.v, EOP_Inv.v, forecast_err.v, Pforecast.v, alpha.v, beta.v,
method.v, level.v, trend.v, onhand_inv.v)
  }
  print(ordering.df)
  write.table(ordering.df, "results.csv", sep = ",", col.names = TRUE, row.names=FALSE,
append = T)

  # Determine CV Errors
  fwin.cv <<- window(demand.ts, start=2017, end=2016+last/12)
  Gwin.cv <<- window(Gwin.ts,start=2017, end=2016+last/12)
  Mwin.cv <<- window(Nmo.ts, start=2017, end=2016+last/12)
  mo_sq.cv<<- Mwin.cv * Mwin.cv
  if (xreg_sw == "TRUE") demand.df <<- data.frame(fwin.cv[13:n], Mwin.cv[13:n],
mo_sq.cv[13:n], Gwin.cv[13:n])
  else demand.df <<- data.frame(fwin.cv[13:n], Mwin.cv[13:n], mo_sq.cv[13:n])

  if (fn == "1") {
    err  <- tsCV(fwin.cv, rwf, h=hz, window=win) # Naive
  } else if (fn == "2") {
    err  <- tsCV(fwin.cv, rwf, drift=TRUE, h=hz, window=win) #Naive Drift
  } else if (fn == "3") {
    if (xreg_sw == "TRUE") { err <- tsCV(fwin.cv, fgses, h=hz, window=win,
xreg=Gwin.cv) }
    else { err <- tsCV(fwin.cv, fses, h=hz, window=win) } #Brown SES
  } else if (fn == "4") {
```

```
    if (xreg_sw == "TRUE") { err <- tsCV(fwin.cv, fgdes, h=hz, window=win,
xreg=Gwin.cv) }
    else { err <- tsCV(fwin.cv, fdes, h=hz, window=win) } #Holt DES
  } else if (fn == "5") {
    if (xreg_sw == "TRUE") { err <- tsCV(fwin.cv, fgtes, h=hz, window=win,
xreg=Gwin.cv) }
    else { err <- tsCV(fwin.cv, ftes, h=hz, window=win) } #Holt DESZ
  } else if (fn == "6") {
    if (xreg_sw == "TRUE") { err <- tsCV(fwin.cv, fglm, h=hz, window=win,
xreg=Gwin.cv) }
    else { err <- tsCV(fwin.cv, flm, h=hz, window=win) } #Linear
  } else if (fn == "7") {
    err <- tsCV(fwin.cv, fma, h=hz, window=win) #Moving AVG
  } else if (fn == "8") {
    if (xreg_sw == "TRUE") { err <- tsCV(fwin.cv, fgpw, h=hz, window=win,
x1=Mwin.cv, x2=mo_sq.cv, x3=demand.df, xreg=Gwin.cv) }
    else { err <- tsCV(fwin.cv, fgpw, h=hz, window=win, x1=Mwin.cv, x2=mo_sq.cv,
x3=demand.df) } #PW
  }

  Arange.v  <- onhand_inv.v[win:last]
  ActinvC <- (sum(Arange.v[Arange.v>0]) * cost * irate) + (sum(Arange.v[Arange.v<0])
* profit * discount)
  results.df$Act_invC[f] <- ActinvC
  CSL.df$ActinvC[fn]     <- ActinvC

  CSL.df$method[fn] <- method.v[last]

  CSL.df$cvRMSE[fn] <- signif(sqrt(mean(err^2, na.rm=TRUE)),digits=6)
  CSL.df$cvMAE[fn]  <- signif(mean(abs(err), na.rm=TRUE),digits=6)
  CSL.df$cvME[fn]   <- signif(mean(err, na.rm=TRUE),digits=6)

  idemand.v <- demand.v[win:last]
  irange.v  <- EOP_Inv.v[win:last]
  CSL.df$inv_low[fn] <- min(irange.v)
  CSL.df$inv_high[fn]<- max(irange.v)

  CSL.df$Tinv_neg[fn] <- sum(irange.v[irange.v<0])
  CSL.df$Tinv_pos[fn] <- sum(irange.v[irange.v>0])
  CSL.df$Tinv[fn]     <- CSL.df$Tinv_pos[fn] - CSL.df$Tinv_neg[fn]

  CSL.df$Tinv_negC[fn] <- CSL.df$Tinv_neg[fn] * profit * discount
  CSL.df$Tinv_posC[fn] <- CSL.df$Tinv_pos[fn] * cost * irate
  CSL.df$TinvC[fn]     <- CSL.df$Tinv_posC[fn] + CSL.df$Tinv_negC[fn]

  CSL.df$inv_rang[fn]<- abs(min(irange.v) - max(irange.v))
```

```
  CSL.df$CSL[fn] <- (sum(idemand.v) + sum(irange.v[irange.v<0], na.rm = TRUE)) /
sum(idemand.v)

  if (fn < 3) {   # Naive
    CSL.df$aic[fn] <- 0
  } else if (fn < 6) {  # ES
    CSL.df$aic[fn] <- signif(fit1$ICs[1],digits=5)
  } else if (fn < 7) {  # Linear
    CSL.df$aic[fn] <- signif(AIC(fit1),digits=5)
  } else if (fn < 8) {  # SMA
    CSL.df$aic[fn] <- signif(fit1$ICs[1],digits=5)
  } else if (fn < 9) {  # Prais-Winsten
    CSL.df$aic[fn] <- c("NA")
  } else if (fn < 10) {  # lm w/Poisson Distribution
    CSL.df$aic[fn] <- signif(AIC(fit1),digits=5)
  }

  irange_len     <- length(irange.v)
  sub_irange.v        <- irange.v[(irange_len-12):irange_len]

  CSL.df$L13_CSL[fn]  <- (sum(idemand.v[(irange_len-12):irange_len], na.rm = TRUE)
+ sum(sub_irange.v[sub_irange.v<0], na.rm = TRUE)) / sum(idemand.v[(irange_len-
12):irange_len], na.rm = TRUE)
    CSL.df$L13_invC[fn] <- (sum(sub_irange.v[sub_irange.v>0], na.rm = TRUE) * cost *
irate) + (sum(sub_irange.v[sub_irange.v<0], na.rm = TRUE) * profit * discount)

        sub_onhand_inv.v    <- onhand_inv.v[(last-12):last]
        L13_ActinvC      <- (sum(sub_onhand_inv.v[sub_onhand_inv.v>0], na.rm = TRUE)
* cost * irate) + (sum(sub_onhand_inv.v[sub_onhand_inv.v<0], na.rm = TRUE) * profit
* discount)
   CSL.df$L13_ActinvC[fn] <- L13_ActinvC

 } # End method loop

 #Descriptive Statistics
 results.df$sku[f]   <- RM
 results.df$cost[f]  <- cost
 results.df$price[f] <- price
 results.df$fill_rate[f] <- fill_rate

 maxCSL <- which.max(CSL.df$CSL)
 results.df$Hcsl[f]      <- max(CSL.df$CSL)
 results.df$MHcsl[f]     <- CSL.df$model_name[maxCSL]
 results.df$MCSLmethod[f]<- CSL.df$method[maxCSL]
 results.df$MTinvC[f]    <- CSL.df$TinvC[maxCSL]
```

```
minRMSE <- which.min(CSL.df$cvRMSE)
results.df$LcvRMSE[f]   <- CSL.df$cvRMSE[minRMSE]
results.df$McvRMSE[f]   <- CSL.df$model_name[minRMSE]
results.df$RMSETinvC[f] <- CSL.df$TinvC[minRMSE]

minaic <- which.min(CSL.df$aic[3:7])
results.df$Laic[f]      <- min(CSL.df$aic[3:7])
results.df$Maic[f]      <- CSL.df$model_name[minaic]
results.df$aicTinvC[f]  <- CSL.df$TinvC[minaic]

results.df$HTinv_neg[f] <- max(CSL.df$Tinv_neg)
results.df$LTinv_pos[f] <- min(CSL.df$Tinv_pos)
results.df$LTinv[f]     <- min(CSL.df$Tinv)

Tinv_sort <- order(CSL.df$TinvC)
target    <- 0.944
for (x in (1:length(Tinv_sort))){
  if (CSL.df$CSL[Tinv_sort[x]] > target) {
    results.df$MTinv_Tgtcsl[f] <- CSL.df$CSL[Tinv_sort[x]]
    results.df$MTinv_Tgt[f]    <- CSL.df$model_name[Tinv_sort[x]]
    results.df$MCSLmethTgt[f]  <- CSL.df$method[Tinv_sort[x]]
    results.df$MTinv_TgtC[f]   <- CSL.df$TinvC[Tinv_sort[x]]
    results.df$Mbought_Tgt[f]  <- CSL.df$bought[Tinv_sort[x]]
    break
  } else {
    results.df$MTinv_Tgtcsl[f] <- results.df$Hcsl[f]
    results.df$MTinv_Tgt[f]    <- results.df$MHcsl[f]
    results.df$MCSLmethTgt[f]  <- results.df$MCSLmethod[f]
    results.df$MTinv_TgtC[f]   <- results.df$MTinvC[f]
    results.df$Mbought_Tgt[f]  <- results.df$bought[f]
  }
}

Tinv_sort <- order(CSL.df$L13_invC)
results.df$L13_CSL[f]     <- CSL.df$L13_CSL[Tinv_sort[1]]
results.df$L13_invC[f]    <- CSL.df$L13_invC[Tinv_sort[1]]
results.df$L13_ActinvC[f] <- CSL.df$L13_ActinvC[Tinv_sort[1]]
results.df$L13_MTinv[f]   <- CSL.df$model_name.v[Tinv_sort[1]]
for (x in (1:length(Tinv_sort))) {
  if (CSL.df$L13_CSL[Tinv_sort[x]] > target) {
    results.df$L13_CSL[f]   <- CSL.df$L13_CSL[Tinv_sort[x]]
    results.df$L13_invC[f]  <- CSL.df$L13_invC[Tinv_sort[x]]
    results.df$L13_MTinv[f] <- CSL.df$model_name.v[Tinv_sort[x]]
    break
  }
}
```

```
 results.df$L13_avg[f] <- mean(idemand.v[(irange_len-12):irange_len], na.rm = TRUE)
 results.df$L13_sd[f]  <- sd(idemand.v[(irange_len-12):irange_len], na.rm = TRUE)
 results.df$L13_cv2[f] <- (results.df$L13_sd[f] / results.df$L13_avg[f])^2

 results.df$L_avg[f] <- mean(idemand.v, na.rm = TRUE)
 results.df$L_sd[f]  <- sd(idemand.v)
 results.df$L_cv2[f] <- (results.df$L_sd[f] / results.df$L_avg[f])^2

 results.df$win[f]   <- win
 results.df$eXreg[f] <- xreg_sw
 results.df$hz[f]    <- hz

 print(RM)
 print(CSL.df)

 # Write results to a file
 #write.table(descriptive.list, "results.csv", sep = ",", col.names = TRUE,
row.names=FALSE, append = T)
 write.table(RM, "results.csv", sep = ",", col.names = TRUE, row.names=FALSE, append
= T)
 write.table(CSL.df, "results.csv", sep = ",", col.names = TRUE, row.names=FALSE,
append = T)

 #write.table(descriptive.list, "results_sum.csv", sep = ",", col.names = TRUE,
row.names=FALSE, append = T)
 rmout.df <- data.frame(RM, cost, price, ActinvC)
 write.table(rmout.df, "results_sum.csv", sep = ",", col.names = TRUE,
row.names=FALSE, append = T)
 write.table(CSL.df, "results_sum.csv", sep = ",", col.names = TRUE,
row.names=FALSE, append = T)
 write.table(results.df[f,], "results_sum.csv", sep = ",", col.names = TRUE,
row.names=FALSE, append = T)

} # End SKU loop of part numbers

write.table(results.df, "results_SKU.csv", sep = ",", col.names = TRUE,
row.names=FALSE, append = T)

# LOOP again using Exogenous regressors
xreg_sw <- "TRUE"

} # END xreg_sw LOOP
```