Investigating Properties of the Hitomezashi Sashiko Pattern

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Hitomezashi Sashiko pattern is an embroidery practice from Japan. It is made by stitching discrete line segments on a piece of cloth (see Fig. 1; all figures are at the end of the document). The rows and columns are stitched separately. The stitches alternate between the front and the back of the cloth, thus leaving alternating gaps and visible stitches on either side of the cloth.

Mathematically, this pattern can be encoded by two binary strings: one that describes the horizontal pattern, and the other that describes the vertical pattern. The rules of conversion from binary string to pattern are as follows:

- For any '0' in the string, the first line segment corresponding to that position must go through the back of the cloth, leaving the space in the front blank. This then alternates.
- For any '1' in the string, the first line segment corresponding to that position must go through the front of the cloth, leaving the space in the back blank. This then alternates.

I coded these rules from the perspective of the "front" of the cloth. An example of this is shown in Fig. 2.

Then, by defining the sequence of 1s and 0s that encode the horizontal stitches and vertical stitches separately, I determined what kind of sequences lead to which kind of symmetries in the resulting pattern. These include patterns symmetric about the middle horizontal line, the middle vertical line, both diagonals (for square grids), and rotational symmetries. For example, the pattern in Fig. 2 is symmetric about the horizontal and the vertical. It is also symmetric when it is rotated by 180°. **The main result of my work is classifying symmetries of the Sashiko pattern based solely on the properties of the binary sequences.** In this example, the special property of the horizontal and vertical binary sequences is that they are both palindromic. I worked out several such arguments for various symmetries of the finite Sashiko patterns.

I then used these results to answer the following question: "What would happen if I physically flip the fabric over to see the back?" It's interesting that flipping a pattern isn't the same as just looking through to the other side. Imagine that you are holding a piece of cloth with your left hand gripping the middle of the left edge and your right hand holding the middle of the right edge. Now, imagine flipping it over like you would the page of a book. This then combines a reflection and also turns over the fabric – it is a compound motion!

Once I had intuition about the physical process undertaken for each transformation, I was able to use geometric arguments to determine how these would affect an arbitrary Sashiko pattern. I am currently working on using these ideas to generate rules for how finite Sashiko grids can be used to tile the infinite plane. Then, the ultimate goal is to classify the resulting infinite patterns into one of 17 possible wallpaper groups (Williams 1987) based on the binary strings which are repeated infinitely.

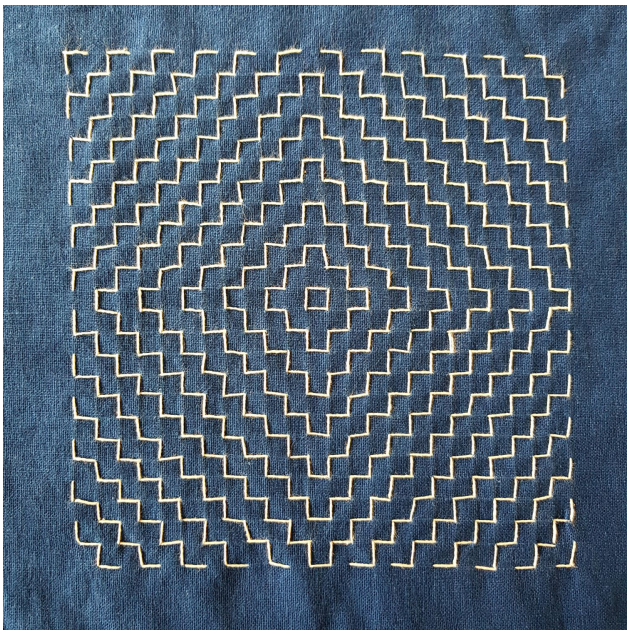

Figure 1: Example of the Hitomezashi Sashiko pattern. Image credit: Romor Designs.

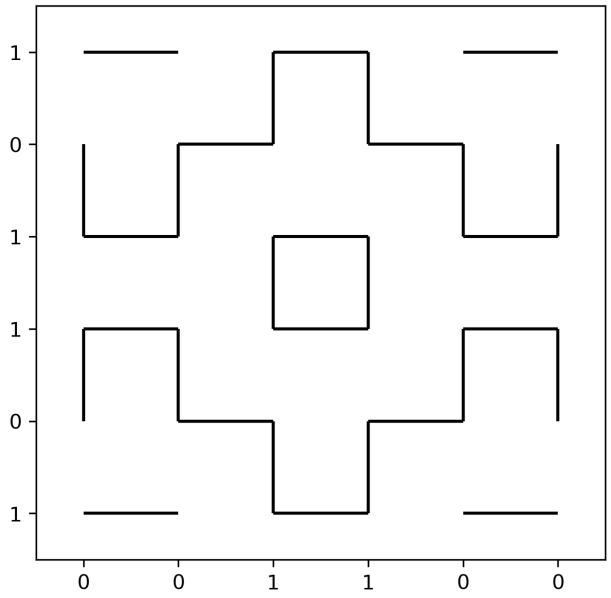

Figure 2: Example of the Hitomezashi Sashiko pattern generated using binary strings.

References

Williams, H. (1987), 'Tilings and patterns, by b. grunbaum and gc shephard. pp 700.£ 54· 95. isbn 0-7167-1193-1 (hardback)(freeman)', *The Mathematical Gazette* **71**(458), 347–348.