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H. D. Vinod
Fordham University, VINOD@FORDHAM.EDU

P. M. Rao
Long Island University, pmrao@liu.edu

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H D. Vinod* and P. M. Rao

Vinod is Professor of Economics, Fordham University, Bronx, New York, USA 10458. E-mail: vinod@fordham.edu and Rao is Professor of Marketing & International Business, Long Island University, Post Campus. E-mail: pmrao@liu.edu.

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Abstract

Production functions often study the output of physical products with capital and labor inputs. Instead, we use 2004 to 2016 data for 55 Indian multinational companies to assess the production of services. Our estimates of flexible production functions yield estimates of scale elasticity (SCE) and elasticity of substitution (EOS) for pooled data. A subset of 31 companies with relatively complete data yields their individual SCE and EOS values, revealing their heterogeneity. Sorting the 31 companies by their SCE help name scale-efficient (high SCE) and scale inefficient (low SCE) multinationals. Similarly, a listing of 31 companies sorted by EOS allows us to name companies that are (and are not) robust to input price shocks. Using stock market data on these publicly traded companies, we report the values of three stock market criteria for top ranking companies by SCE. We also study empirical causal paths from the market criteria to EOS and SCE, suggesting that SCE and EOS do drive stock market indicators implying efficient markets. Our pooled and detailed results are relevant for government policy toward the IT sector and corporate governance issues.

1 Introduction

A production function helps measure the contribution of individual inputs to the creation of output. Consider the production of a tangible product such as bushels of corn by a farm. If we have data on farmer's revenue (quantity × price) and the price of corn per bushel, a deflated revenue [(quantity × price) /(price)], readily gives the desired output quantity. A modern firm produces several products and services whose quantities cannot be directly aggregated.

Hence, economists define a firm's 'value-added' (Va) as the difference between its total sales revenue and the total cost of components, materials, and services purchased from other firms. Now, a firm's high Va in a year might have been so because the firm raised the prices. If a firm produces mostly tangible products, it is customary to remove the effect of output price inflation by deflating the Va by a price index as specific to the firm's output as possible to yield its output quantity. Unfortunately, service-output-specific price index for deflating value-added numbers is unavailable.

In this paper, we are concerned with the production of intangible services by Indian multinational firms who sell those services mostly to developed countries. Using a suffix R for rupees, we let VaR stand for the value-added in millions of rupees, deflated by the consumer price index (CPI) in India. Now we define logs of capital and labor inputs, K = log(Asst), where Asst represents total assets in millions of Rupees, making K the log of capital input. Similarly, L = log(Emp), where Emp means the number of employees, making L the log of labor input.

Three popular functional forms for a production function with two inputs include the Cobb-Douglas,

$$y = \alpha_0 + \alpha_1 K + \alpha_2 L, \tag{1}$$

which is linear in parameters. The trans-log functional form with cross product and square terms is

$$y = \alpha_0 + \alpha_1 K + \alpha_2 L + \alpha_3 K\, L + \alpha_4 K^2 + \alpha_5 L^2. \qquad (2)$$

The constant elasticity of substitution (CES) production function is defined as

$$y = \gamma[\delta K^{-\rho} + (1 - \delta)L^{-\rho}]^{-\nu/\rho}, \qquad (3)$$

where γ measures the efficiency, δ measures input intensity, ν measures the scale elasticity (SCE) and ρ measures elasticity of substitution (EOS).

The highly nonlinear functional form (3) makes the estimates depend on the optimization algorithm used, which searches in the neighborhood of some starting values for unknown coefficients. In addition to such nonlinear estimation difficulties, there is little theoretical or empirical justification for assuming, as the functional form (3) does, that SCE= ν and EOS= ρ should be fixed constants for all firms and years.

Unfortunately, all three functional forms listed above are not suitable for our purposes. The Cobb-Douglas is too restrictive in assuming without evidence that SCE is a fixed constant and EOS=1. For our data the matrix of correlation coefficients $\{r_{ij}\}$ reported in Table 1 between (K, L, KL, K^2, L^2) has all correlations large, $\{r_{ij}\} > 0.858$, suggesting collinearity. The ordered vector of eigenvalues $\lambda_i, i = 1, 2, \ldots 5$, for the regressors in (2) are: (4.710, 0.269, 0.018, 0.002, 0.000). Note that the smallest eigenvalue $\min(\lambda) \approx 0$ for these data suggest a near-zero determinant and a very high condition number, $(\max(\lambda_i)/\min(\lambda_i)) \to \infty$, implying a very ill-conditioned matrix of regressors. Numerical mathematicians have long warned against inverting ill-conditioned matrices.

Table 1: Correlation matrix between variables in a translog production function

	y	K	L	KL	K^2	L^2
y	1.000	0.938	0.928	0.957	0.931	0.928
K	0.938	1.000	0.858	0.952	0.991	0.867
L	0.928	0.858	1.000	0.960	0.860	0.991
KL	0.957	0.952	0.960	1.000	0.964	0.974
K^2	0.931	0.991	0.860	0.964	1.000	0.880
L^2	0.928	0.867	0.991	0.974	0.880	1.000

The mean squared error, $MSE(\hat{\alpha}) = E(\hat{\alpha} - \alpha)^2$, is the expected value of the squared Euclidean distance between the $p \times 1$ vector of estimates and true values (α). When ($E(\hat{\alpha}) = \alpha$), we have an unbiased estimator. Since the ordinary least squares (OLS) estimator is unbiased, it can be shown that the $MSE(\hat{\alpha}) = Var(\hat{\alpha}) = \sigma^2(\lambda_1^{-1} + \lambda_2^{-1} \ldots + \lambda_p^{-1})$. When we have collinear data, the smallest eigenvalue ($\min(\lambda_i)$) of the covariance (or correlation) matrix is very small, close to zero. Then, $1/\min(\lambda_i) \rightarrow \infty$ implying a very high MSE suggesting a highly unreliable OLS estimate ($\hat{\alpha}$). Our data analysis reveals that the full trans-log form of equation (2) has $\min(\lambda_i) \approx 0$. We also find that omitting the square terms (K^2, L^2) of the trans- log helps reduce the condition number from near infinite to 16.073, solving the collinearity problem. This avoids the use of ridge regressions or other biased estimators, Vinod (1976).

We choose a version of the trans-log production function, which remains linear in parameters but includes KL term allowing for limited nonlinearity while omitting (K^2, L^2) causing the collinearity. It can be proved that the following form is non-homogenous and exhibits variable EOS, (VES). We estimate:

$$y = \alpha_0 + \alpha_1 K + \alpha_2 L + \alpha_3 (K L) + , \tag{4}$$

where the notation already uses logs of output, capital, and labor. The Cobb-Douglas production function (1) is a particular case when the coefficient of the cross-product term $\alpha_3 = 0$ in (4).

This production function is non-homogeneous and hence flexible in the sense that it lets data determine various elasticities described in the sequel, rather than simply fixing their values as known constants. For example, the Cobb-Douglas form sets the elasticity of substitution (EOS) to be unity, irrespective of the data. Similarly, constant elasticity of substitution (CES) production function sets EOS = η, some constant. Instead, the formulation in equation (4) lets the data estimate variable EOS. If the estimated EOS values are indeed nearly constant, one can always simplify the specification and use a CES functional form. We shall see later that our data on Indian multinationals exhibit a considerable range of EOS values, far from being a constant.

1.1 Marginal and scale elasticity

Output per unit of labor (per employee) VaR/Emp, is a crude but straightforward measure of labor productivity. The limitation of (VaR/Emp) ratio is that it ignores the contribution of capital, falsely pretending that all output can credited to the labor input. Similarly, one can define ($VaR/Asst$) ratio as a crude measure of capital efficiency. The input productivity is better measured by the marginal productivity of capital and labor (MP_K, MP_L), defined by the partial derivative of the output with respect to an input. Unfortunately, the MP_K, MP_L values are sensitive to the units of measurement for assets and employees and we cannot conveniently compare MP_K with MP_L over time, nor can we compare their values across firms.

The marginal elasticities of capital and labor, denoted by MEK, MEL, respectively measure the percent change in the output as an effect of a 1% change in capital or labor input, one at a time. Now, percent changes are not sensitive to units of measurement and are readily compared over time and across firms. In fact, we can compute MEK, MEL as the partial derivatives of the log of output with respect to (wrt) the log of one input. For the specification in equation (4) the MEK ($=\partial y/\partial K$) values are defined as:

$$MEK \quad = \alpha_1 + \alpha_3(L) \tag{5}$$

An empirical estimate of these marginal elasticities requires regression coefficient estimates. The term (L) or log of labor appearing in the last right-hand side terms of (5) represent data on the input, which varies with each observation. Hence, MEK estimates will vary with each L. Since one cannot report so many (750 values for 55-firm 13-year data) pooled data MEK estimates, it is customary to estimate MEK values at the sample mean or L That is, one replaces L in the formula (5) by a single number L for average L in the data.

Analogous marginal elasticities of labor (MEL) are also readily computed from estimated coefficients and the known sample mean, K The scale elasticity SCE measures the percentage change in output from a one percent increase in the scale of operations measured by a one percent change in all inputs. Thus, we have:

$$SCE = MEK + MEL, \tag{6}$$

which is a summary measure of the efficiency of the Indian multinational entity.

1.2 Elasticity of substitution

The ratio of marginal productivities of capital and labor, MP_K/MP_L, is called the marginal rate of transformation (MRT). J. R. Hicks first defined elasticity of substitution (EOS) as the percent change in MRT in response to a one percent change in input ratio K/L. It is best expounded in Ferguson (1971) and partially covered in many economics textbooks. Here we are interested in the sign of EOS(K,L) between inputs K and L. Assuming the corporate entity producing the output is on an output-maximizing ‘expansion path,’ Hicksian theory implies that whenever the EOS(K,L) is positive, the gains in output remain positive despite adverse input price changes. There is, however, no reason to assume that EOS remains constant, EOS = η, for each observation, which is done by constant elasticity of substitution (CES) production functions. If our marginal elasticities already vary with each observation, CES' constancy assumption is all the more unrealistic, and hence avoided here.

The non-homogeneous production function (4) has the advantage that its elasticity of substitution is given by a simple expression

$$EOS = SCE/(SCE + 2\alpha_3). \tag{7}$$

Since the scale elasticity SCE defined in (6) is easy to estimate, estimating EOS merely needs one to plug in $\hat{\alpha}_3$, an already estimated regression coefficient.

The Cobb-Douglas production function is obtained by setting the coefficients of the cross-product term ($\alpha_3 = 0$). A quick check on the validity of (7) is that EOS for Cobb-Douglas should be unity. Now, EOS is indeed unity, since it becomes $EOS = SCE/SCE = 1$ for the special case when $\alpha_3 = 0$.

2 Pooled data production function estimation

Using the data pooled together for all 55 Indian multinational companies for years 2004 to 2016 on logs of outputs and inputs, we first estimate a few versions of the non-homogeneous production function:

$$y = \alpha_0 + \alpha_1 K + \alpha_2 L + \alpha_3 (K\,L) + \alpha_4 Year + \varepsilon, \tag{8}$$

where we have an additional regressor for time ($Year = 2004, 2005 \ldots 2016$) represents a proxy for technological change, commonly used in the related literature. Our results in Table 2 use pooled data, which makes no distinction between individual companies. One way to assess whether pooling is appropriate is to check the quality of its statistical fit. We use the usual F test and a newer exogeneity test for this purpose here.

Recall that our value-added output variable VaR uses CPI to correct for inflation. Since the Cobb-Douglas scale elasticity, $SCE = \alpha_1 + \alpha_2$, is 0.596 + 0.531 = 1.127, is above unity, these corporations as a whole in pooled data enjoy increasing returns to scale, implying general efficiency. All models are statistically significant from the reported large F statistics.

The statistically significant estimates of ($\hat{\alpha}_4 = -0.017$) in columns (2) and (3) suggests that as time increases the output mildly decreases. Since the coefficients of Y ear is rather small it is convenient to exclude the Year as a regressor in the sequel. Although the coefficient of the cross-product, K L, is even smaller,we need to retain it in our VES specification. Otherwise, we are implicitly using a Cobb-Douglas functional specification, which fixes the elasticity of substitution to be unity and SCE to be a fixed constant. The pooled data estimates in Table 3 continue to support our model specifications and continue to suggest

Table 2: Cobb-Douglas and non-homogeneous production functions: Indian multinationals producing services output measured by 'Value added' deflated by CPI using pooled data, eq. (8)

	Dependent variable:		
	y		
	(1)	(2)	(3)
K	0.596_{***}	0.604_{***}	0.677_{***}
	(0.028)	(0.028)	(0.062)
L	0.531_{***}	0.526_{***}	0.609_{***}
	(0.029)	(0.029)	(0.069)
K L			−0.009
			(0.007)
Year		-0.017_{**}	-0.017_{**}
		(0.008)	(0.008)
Constant	-1.816_{***}	32.006_{**}	31.691_{**}
	(0.126)	(15.367)	(15.353)
Observations	374	374	374
R^2	0.937	0.937	0.938
Adjusted R^2	0.936	0.937	0.937
Resid. Std. Err	0.543 (df = 371)	0.540 (df = 370)	0.539 (df = 369)
F Statistic	$2,736_{***}$(df=2; 371)	$1,844_{***}$(df=3; 370)	$1,387_{***}$(df=4; 369)

Note: $_{*}$p<0.1; $_{**}$p<0.05; $_{***}$p<0.01

that Indian multinationals are generally efficient (all $SCE \approx 1$) and robust with respect to price shocks ($EOS > 0$). The estimates along rows 1 to 3 in Table 3 are computed from the coefficients reported in columns 1 to 3 of Table 2.

Table 3: Pooled data various elasticities

Row	Type	ME-K	ME-L	SCE	EOS
1	CobbD	0.596	0.531	1.127	1.000
2	CobbD-Yr	0.604	0.526	1.130	1.000
3	VES-Yr	0.607	0.537	1.144	1.016

2.1 Exogeneity assessment of model regressors

This subsection hopes to learn whether the VES model specification (4) might suffer from the so-called "endogeneity problem," Koopmans (1950). Generally speaking, we want the right-hand side (RHS) variables in our models illustrated by eq. (4) to be exogenous, in the sense that they should have independent variation and drive the variation in the dependent variable. Correlation coefficients between output variable (y) on the left-hand and inputs (K, L) are fairly large and positive, implying that when the companies increase the number of employees, their 'value-added' output increases. However, exogeneity assessment requires deeper analysis discussed next.

We use the R package called 'generalCorr' from Vinod (2016), which contains decision rules based on a unanimity index summarizing three criteria (Cr1 to Cr3). The theory is explained in software vignettes and Vinod (2019). The software readily compares flipped models where one model has an output (y) on the one hand and inputs K, L on the other. The decision rules help name the variable in the column entitled 'cause' and also name the flipped variable in the column entitled 'response.' Koopmans showed long ago that regressors should approximately 'cause' the dependent variable. Otherwise, the model suffers from the so-called 'endogeneity problem.'

2.2 Description of headings in causal path tables

We describe the headings of all causal path tables in this paper in this subsection for easy reference based on the theory in Vinod (2019).

1. cause = name of the causal variable.

2. response = name of the response variable.

3. strength = absolute value of the unanimity index (UI) in the range [0,100]. If strength is less than 5% the causal path is bidirectional.

4. corr = the usual correlation coefficient between the two flipped variables.

5. p-value = p-values for testing the null hypothesis that the population correlation coefficient is zero. We generally reject the null if p-value < .05.

Table 4 has our causal path results relevant for assessment of exogeneity of variables on the right-hand side of (4). The p-values are all near zero in Table 4, suggesting highly significant correlations. Since the input variables K and L are in the 'response' column, we do have the endogeneity problem for pooled data problems.

Table 4: Causal paths for exogeneity assessment for pooled data

	cause	response	strength	corr.	p-value
1	y	K	37.008	0.9076	0
2	y	L	100	0.9277	0

We conclude subsection 2.2 by noting that Indian multinationals appear to hire their inputs of K, L in response to their profitability measured by their value-added output. Since three regression models reported in Tables 2 are statistically significant with plausible coefficients, we can conclude Section 2 by noting that pooled data are providing a reasonable picture of the production of services by Indian multinationals. Despite the endogeneity problem, the overall conclusion is that $SCE \approx EOS \approx 1$ along all rows of Table 3. The pooled results refer to multi- nationals as a group and are of interest for economic policy toward them as a group. We study individual company performance in the next section.

3 Cobb-Douglas function estimates for individual companies

We estimate separate Cobb-Douglas production functions for each Indian multinational to assess the nature of heterogeneity between the Indian multinationals. Only 31 companies out of 55 are selected for reporting. They have at least five years of non-missing data for all variables.

Table 5 reports two marginal elasticities and their sum as scale elasticity using the formulas given in (5) to (6) earlier. Of course, in the absence of the cross-product term in a Cobb-Douglas specification, we are implicitly fixing $\alpha_3 = 0$. That is, the scale elasticity for Cobb-Douglas is analytically known to be $SCE = \alpha_1 + \alpha_2$.

The column entitled 'rank' reports the sorted rank of the company measured by its scale elasticity, with rank=1 indicating the company with the largest SCE. Since we have some negative slope estimates, $MEK=\alpha_1 < 0$, and $MEL=\alpha_2 < 0$, in Table 5, it is tempting to conclude that measured capital or labor is an unproductive input in producing services of those companies. We need a deeper review of possible data errors, nonlinearities missed by the Cobb-Douglas form, and of circumstances faced by these companies before we can say that some inputs are "unproductive." Any identification of relatively inefficient firms based on $SCE = \alpha_1 + \alpha_2$ alone is subject to limitations, simply because the Cobb-Douglas form arbitrarily assumes that SCE is a constant and EOS=1.

Table 5: Cobb-Douglas Model elasticities for 31 firms sorted by SCE

rank	n	MEK	MEL	SCE	EOS	R^2	Name
1	6	4.9527	0.6792	5.6319	1	0.9894	TRIG
2	11	1.622	0.5158	2.1378	1	0.3624	APTE
3	8	-0.0362	2.095	2.0588	1	0.3837	FINT
4	7	0.4079	1.2289	1.6368	1	0.9126	NITT
5	13	-0.0412	1.5722	1.531	1	0.1225	TIMK
6	10	-0.5765	2.0469	1.4704	1	0.905	MPHA
7	12	0.3776	1.0316	1.4093	1	0.6425	CSS
8	9	1.076	0.2561	1.3321	1	0.9468	ALLS
9	12	0.4343	0.8681	1.3024	1	0.4403	MIT
10	13	0.2167	0.9983	1.215	1	0.9069	ASMT
11	12	-0.1443	1.2473	1.103	1	0.9505	SONA
12	6	0.8365	0.1952	1.0317	1	0.89	MAST

13	7	0.2394	0.7518	0.9912	1	0.9359	OMNI
14	13	0.3276	0.5802	0.9078	1	0.9271	HCLT
15	8	0.2032	0.7043	0.9075	1	0.9923	MINT
16	10	0.2241	0.6357	0.8598	1	0.7445	MIND
17	13	-0.2418	1.0919	0.8501	1	0.9622	KPI
18	11	0.1886	0.6455	0.8341	1	0.9869	INFS
19	8	0.5468	0.2384	0.7853	1	0.9531	ECLE
20	13	0.241	0.5301	0.7711	1	0.6498	HEXA
21	6	0.0992	0.6476	0.7468	1	0.9904	IGS
22	10	0.8599	-0.1896	0.6703	1	0.9944	TCS
23	12	0.4479	0.1747	0.6226	1	0.964	WPRO
24	8	0.7434	-0.1372	0.6061	1	0.2558	RSI
25	12	0.0374	0.545	0.5825	1	0.9043	PFT
26	7	0.4723	0.0269	0.4992	1	0.9368	CAMB
27	9	-0.2292	0.6554	0.4261	1	0.3915	ONTR
28	12	-0.1332	0.457	0.3238	1	0.8649	CMC
29	7	0.5478	-0.2988	0.249	1	0.5225	PANO
30	11	0.2114	-0.1832	0.0283	1	0.2186	TATA
31	8	-0.1506	0.1615	0.0108	1	0.2742	ICRA

The Table 5 clearly reveals the heterogeneity of Indian multinationals, when one considers the great variability in the range of estimated marginal and scale elasticities. These confirm that the companies are heterogeneous with distinct production function coefficient estimates. The Cobb-Douglas SCE estimates range from 5.6319 for rank 1 Trigyn Tech denoted as 'TRIG' to 0.0108 for rank 31 'ICRA' in Table 5.

We can surmise that a more general non-homogenous production function will also suggest that different Indian multinationals are distinct from one another. We report estimates after including the cross-product term to incorporate nonlinearities arising from the input interactions in the next section.

4 Percent Compound Growth Rates Compared

Our data set contains several missing values, especially with stock market indicators. Our growth rate study in this section considers 31 companies having an adequate number of non-missing data. Accordingly, our data covers years representing (2004 to 2016). Let X denote the values for a particular company from the list of variables (Emp, Asst, VaR, RoaR, ShPr, MktK). We have already encountered the notation Emp for employees, Asst for assets, and VaR for value-added in Rupees. Newer variables are 'rate of return on assets in Rupees,' (RoaR), share price (ShPr), and market capitalization (MktK).

Let the available data range be $X(t_1)$ to $X(t_2)$. For example, VaR data for KPIT Tech is available from 2004 to 2016. Hence, X(2004), X(2016) will denote VaR values for those years. Now the overall percent compound growth rate r over the included period is given by solving the following equation for the rate

$$X(t_2) = X(t_1)(1 + r)^{\tau}, \quad \text{where} \quad \tau = (t_2 - t_1 + 1). \tag{9}$$

The percent growth rate is $r = 100[(X(t_2)/X(t_1))^{(1/\tau)} - 1]$ assuming that the denominators τ and $X(t_1)$ are nonzero.

Table 6 reports growth rates of various companies included in the study

sorted by their asset growth rates ranging from −7.97% for Melstar

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Tech (MIT) to 45.17% for Eclerx Services (ECLE). See Tables 14 to 16 in the Appendix for more detailed names of companies.

Table 6: Percent Growth Rates sorted by Asset growth

	Emp	Asst	VaR	RoaR	ShPr	MktK
MIT	0.69	-7.97	-3.50	-13.60	-4.48	-4.91
APTE	-3.27	-2.90	-6.00	-1.20	1.50	3.85
PFT	1.77	1.01	2.89	-2.18	0.57	0.91
TATA	6.23	1.32	0.01	-4.29	4.81	4.89
MAST	-2.58	1.91	1.27	-4.23	12.80	-0.88
CMC	12.80	6.20	3.55	2.39	11.95	18.64
XCS	-14.47	7.15	1.64	5.53	39.19	1.86
RSI	-5.07	7.30	13.61	31.28	-7.47	9.83
SONA	10.68	7.45	12.62	7.77	22.79	22.78
AKS	9.69	8.40	9.60	14.42	28.58	33.89
SCT	-5.20	8.68	5.71	7.27	-0.05	-4.11
GEO	1.79	9.69	7.94	1.25	-6.55	13.07
INTE	2.10	10.52	24.05	-23.62	8.69	1.21
HEXA	7.91	10.60	10.80	5.10	-1.84	12.19
TIMK	0.95	10.94	3.39	-3.10	19.50	15.63
SAKS	1.85	12.65	4.08	4.12	8.02	3.91
ICRA	14.52	13.37	7.69	-2.65	21.72	21.14
HCLT	18.62	14.29	14.87	3.91	9.58	21.82
ABMK	2.12	14.70	19.51	15.00	39.97	39.97
PANO	7.31	16.03	4.29	-10.57	-20.20	14.48
WPRO	9.44	16.32	11.72	-1.60	-6.55	12.07
MPHA	14.92	17.50	11.86	-6.74	10.40	13.76

CAMB	-9.07	18.74	4.88	-5.80	12.49	10.14
INFS	16.65	18.85	13.06	-2.74	-10.21	17.90
MINT	16.33	20.36	15.07	-0.26	-2.38	13.45
KPI	21.01	21.63	19.42	-1.60	-2.60	27.61
INTR	8.31	23.58	14.75	0.95	33.51	33.64
VAKR	13.79	26.12	33.67	20.62	3.81	61.22
INFE	13.98	33.13	15.07	-9.86	1.25	17.51
TCS	19.73	39.60	77.82	21.69	4.82	17.89
ECLE	20.12	45.17	34.74	-6.25	20.29	31.08

Scatterplots of various pairs of columns in Table 6 (omitted for brevity) show that the relations are not linear. Hence we report generalized correlation coefficients in Table 7 based on the R function gmcmtc0() in R package 'generalCorr.' The matrix entries are non-symmetric in that the entry at location $[i, j]$ along row i and column j does not, in general, equal the across-diagonal entry at location $[j, i]$.

Comparing across-diagonal entry pairs, the one with the larger magnitude is identified by the superscript 'L' for larger. According to the theory described in Vinod (2019) this is Cr3 of three criteria (Cr1 to Cr3) for determination of causal paths. That is, the variable named in the column is 33% likely to be the cause, since the other two criteria based on residuals of flipped kernel regressions may well suggest the opposite causal path.

For example, consider Table 7 row 2 for Asst and column 1 for Emp has 0.8833^L, which implies that the generalized correlation between the two variables is 0.8833. Moreover, the superscript suggests that the column heading Emp for employee growth is at least 33% likely to be the 'cause' of the row heading Asst for growth in assets. The usual Pearson correlation coefficient between (Emp, Asst) 0.6015 is a bit smaller than 0.8833. Not surprisingly, Pearson correlations (assuming linear relations) are almost always smaller in magnitude than generalized correlation coefficients based on nonparametric, nonlinear kernel regressions.

Table 7: Generalized Correlations Between Percent Growth Rates, superscript L indicates larger absolute value where column name has the potential cause

	Emp	Asst	VaR	RoaR	ShPr	MktK
Emp	1	0.6279	0.7719^{L}	-0.1241^{L}	0.7776^{L}	0.5651
Asst	0.8833^{L}	1	0.7363	0.2393^{L}	-0.1584	0.5819
VaR	0.6011	0.9014^{L}	1	0.35^{L}	-0.0083	0.3996
RoaR	-0.079	0.2041	0.3303	1	0.1665	0.3609^{L}
ShPr	0.0427	-0.4346^{L}	-0.0603^{L}	0.6622^{L}	1	0.8033^{L}
MktK	0.6322^{L}	0.8818^{L}	0.4145^{L}	0.3265	0.4658	1

Table 7 reveals positive correlations between MktK growth and all other variables in the bottom row. However, growth in ShPr has a negative correlation with Asst and VaR. The negative correlation between RoaR and Emp suggests that increasing return on assets negatively impacts employee count growth.

Instead of focusing causal paths suggested by only one (Cr3) of the three criteria (superscript L) as in Table 7, it is better to consider comprehensive causal paths (based on unanimity strength index UI) between all pairs of growth rates reported in Table 8 with column headings described in Section 2.2. The Table 8 reports approximate causal paths based on a (UI) using all three criteria (Cr1 to Cr3) for all possible (=15) pairs of growth rates among the six variable.

Table 8: Causal paths between growth rates of variables

	cause	response	strength	corr.	p-value
1	Emp	Asst	100	0.5655	2e-05
2	Emp	VaR	4.724	0.5297	9e-05
3	RoaR	Emp	31.496	-0.0237	0.88945
4	ShPr	Emp	100	0.0041	0.97927
5	Emp	MktK	31.496	0.4425	0.00335
6	Asst	VaR	100	0.7044	0
7	RoaR	Asst	31.496	0.0436	0.78409
8	ShPr	Asst	37.008	-0.1616	0.27243
9	Asst	MktK	100	0.3367	0.02064
10	RoaR	VaR	100	0.3361	0.02955
11	ShPr	VaR	37.008	-0.0036	0.98066
12	VaR	MktK	50.394	0.3228	0.02687
13	RoaR	ShPr	100	0.1863	0.2697
14	RoaR	MktK	21.26	0.2779	0.09589
15	MktK	ShPr	31.496	0.3735	0.01149

First, we list the following plausible causal paths between growth rates: Emp→ Asst, ShPr → Emp, Emp→ MktK, Asst→ VaR, RoaR → Asst,

Asst →MktK, RoaR → VaR, VaR→ MktK, RoaR → ShPr, RoaR→ MktK, and MktK → ShPr.

The following causal paths are based on negative correlations suggesting that growth in the 'cause' reduces the growth in the response variable. RoaR [Neg]→ Emp, ShPr [Neg] → Asst, and ShPr [Neg] → VaR. Their relatively low 'strength' values may explain why they are intuitively less plausible. We also find that the following path is most likely bi-directional: Emp↔ VaR, because the unanimity strength index is less than 5%.

5 Individual company non-homogenous production function estimates

This section reports estimates of the non-homogenous (or variable elasticity of substitution, VES) production function (4) having the cross-product term. Similar to Table 5, we report various elasticities evaluated at the data means in Tables 9 and 10 using the VES. Both tables have a column for EOS, except that the EOS is always unity in Table 9 for the Cobb-Douglas specification. Note that a large firm like TCS with the highest market capitalization (table 11) ranks low in terms of SCE (table 9), suggesting perhaps it is farther down the cost curve with fewer opportunities to exploit scale economies. On the other hand, TCS ranks among the top (table 10) in terms of EOS suggesting that the firm has greater flexibility to substitute capital for labor and vice versa.

Table 9 reports the top 31 companies ranked by their SCE, whereas Table 10 reports the top 31 companies ranked by their EOS. In addition to elasticities, we report the R^2 and a four-character abridged name for the company. The reader can know the corresponding long names of any company from alphabetic lists in Tables 14 to 16 in the Appendix.

Table 9: Marginal elasticities, scale and substitution elasticities with R^2 and n for non-missing observation count, sorted by SCE

rank	n	MEK	MEL	SCE	EOS	R^2	Nam
1	6	4.8462	0.4395	5.2858	0.5675	0.9924	TRIG
2	12	2.0759	0.6136	2.6895	-0.1578	0.9274	CSS
3	8	0.1728	2.0069	2.1797	0.5909	0.3848	FINT
4	10	-0.9057	2.8049	1.8993	0.5464	0.9488	MPHA
5	11	1.905	-0.0219	1.8831	-0.2973	0.4043	APTE
6	13	-0.0268	1.4836	1.4568	0.489	0.1271	TIMK
7	7	0.5812	0.8206	1.4018	-0.4972	0.9747	NITT
8	12	0.4735	0.9156	1.3891	0.6505	0.4434	MIT
9	9	0.9262	0.3106	1.2368	28.6707	0.9601	ALLS
10	13	0.2277	0.9819	1.2097	1.0603	0.907	ASMT
11	12	-0.6492	1.7555	1.1063	0.3374	0.9731	SONA
12	6	0.8585	0.2357	1.0942	0.5913	0.8906	MAST
13	7	0.2712	0.6708	0.942	1.1151	0.9365	OMNI
14	8	0.1662	0.767	0.9332	0.7955	0.9971	MINT
15	13	0.3881	0.5173	0.9055	1.1612	0.9273	HCLT
16	13	0.4149	0.4868	0.9017	0.5786	0.6641	HEXA
17	10	1.2212	-0.4075	0.8138	0.1052	0.6344	SAKS
18	13	-0.1525	0.9306	0.7781	1.5469	0.9776	KPI
19	8	1.6482	-0.8881	0.7601	-1.1136	0.9888	ECLE
20	6	0.2511	0.4764	0.7275	0.8599	0.9939	IGS
21	11	0.6198	0.1078	0.7275	2.3843	0.9943	INFS
22	10	1.1223	-0.4933	0.629	1.3125	0.997	TCS
23	12	0.1836	0.4445	0.6282	-2.5316	0.9113	PFT
24	12	0.4174	0.1321	0.5496	2.7816	0.9743	WPRO
25	10	-0.0453	0.5945	0.5492	-0.3335	0.9444	MIND
26	8	0.2419	0.2379	0.4798	-0.0485	0.402	RSI
27	7	0.439	0.0179	0.4569	2.3336	0.9402	CAMB
28	9	-0.2101	0.6299	0.4198	0.4706	0.3949	ONTR
29	7	-0.0774	0.3876	0.3101	-0.0836	0.7056	PANO
30	12	-0.2096	0.4891	0.2795	-0.7526	0.8833	CMC
31	8	0.5712	-0.4029	0.1683	-0.0565	0.8589	ICRA

Equation (6) assures us that SCE is an indicator of the productive efficiency of inputs. Since the last column of the table contains an abridged company name, referring to Tables 14 to 16, the reader can know the full names of relatively inefficient companies from the bottom parts of the table. The names of relatively efficient companies are found in the top part of the table where SCE values are positive and large.

Under assumptions of neoclassical production theory, equation (7) assures us that EOS measures the robustness of a company's input mix when the company is faced with input price shocks. Positive EOS values are known to be more desirable.

Table 10 has identified some Indian multinationals with negative EOS estimates, which appear to be too sensitive to input price shocks. Since the last column of the table contains an abridged company name, the reader can identify relatively non-robust input price shock companies from negative EOS values near the bottom of the table. Indian companies that are robust against input price shocks are named in the top part of Table 10 where the EOS values are relatively large.

Our marginal elasticity estimates assume that the VES functional form of the production function is valid. The coefficient estimates allow us to do thought experiments on what happens to output when input increases by one percent. One can imagine individual company situations where these thought experiments are inappropriate. Readers interested in productive efficiency are encouraged to supplement our production function-based comparisons with the traditional ratio comparisons discussed next.

5.1 Traditional stock market and productivity ratios

Stock market analysts consider 'rate of return on assets in Rupees' (RoaR), share price (ShPr), and market capitalization (MktK). Growth rates of these variables were already discussed in Section 4. Traditional productivity ratios are value-added output per unit of total assets (y/K) and value-added output per employee (y/L). We report in Table 11 the above values for selected 31 publicly traded Indian multinationals included in our data set. The 31 companies are chosen because they have relatively large-scale elasticity (SCE) values, as reported earlier in Table 9. The reader is referred to alphabetically listed long company names and their abbreviations Tables 14 to 16 in the Appendix.

The reported numbers are simple averages over the set of 13 years covered in our data set. In some years, some multinationals appear to have suffered net accounting losses resulting in negative rates of return (RoaR). Tables for the remaining companies are omitted for brevity.

Table 10: Marginal elasticities, scale and substitution elasticities with R^2 and n for non-missing observation count, sorted by EOS

rank	n	MEK	MEL	SCE	EOS	R^2	Nam
1	9	0.9262	0.3106	1.2368	28.6707	0.9601	ALLS
2	12	0.4174	0.1321	0.5496	2.7816	0.9743	WPRO
3	11	0.6198	0.1078	0.7275	2.3843	0.9943	INFS
4	7	0.439	0.0179	0.4569	2.3336	0.9402	CAMB
5	13	-0.1525	0.9306	0.7781	1.5469	0.9776	KPI
6	10	1.1223	-0.4933	0.629	1.3125	0.997	TCS
7	13	0.3881	0.5173	0.9055	1.1612	0.9273	HCLT
8	7	0.2712	0.6708	0.942	1.1151	0.9365	OMNI
9	13	0.2277	0.9819	1.2097	1.0603	0.907	ASMT
10	6	0.2511	0.4764	0.7275	0.8599	0.9939	IGS
11	8	0.1662	0.767	0.9332	0.7955	0.9971	MINT
12	12	0.4735	0.9156	1.3891	0.6505	0.4434	MIT
13	6	0.8585	0.2357	1.0942	0.5913	0.8906	MAST
14	8	0.1728	2.0069	2.1797	0.5909	0.3848	FINT
15	13	0.4149	0.4868	0.9017	0.5786	0.6641	HEXA
16	6	4.8462	0.4395	5.2858	0.5675	0.9924	TRIG
17	10	-0.9057	2.8049	1.8993	0.5464	0.9488	MPHA
18	13	-0.0268	1.4836	1.4568	0.489	0.1271	TIMK
19	9	-0.2101	0.6299	0.4198	0.4706	0.3949	ONTR
20	12	-0.6492	1.7555	1.1063	0.3374	0.9731	SONA
21	10	1.2212	-0.4075	0.8138	0.1052	0.6344	SAKS
22	10	-0.3789	0.288	-0.091	0.0212	0.7163	ACCL
23	8	0.2419	0.2379	0.4798	-0.0485	0.402	RSI
24	8	0.5712	-0.4029	0.1683	-0.0565	0.8589	ICRA
25	7	-0.0774	0.3876	0.3101	-0.0836	0.7056	PANO
26	12	2.0759	0.6136	2.6895	-0.1578	0.9274	CSS
27	11	1.905	-0.0219	1.8831	-0.2973	0.4043	APTE
28	11	0.2146	-0.1863	0.0283	-0.3029	0.2196	TATA
29	10	-0.0453	0.5945	0.5492	-0.3335	0.9444	MIND
30	7	0.5812	0.8206	1.4018	-0.4972	0.9747	NITT
31	12	-0.2096	0.4891	0.2795	-0.7526	0.8833	CMC

Table 11: Rate of return on assets, share price, market capitalization, Value added output per unit of asset, y/K and output per employee y/L sorted by SCE as in Table 9

rank	RoaR	ShPr	MktK	y/K	y/L	Name
1	0.0473	21	57	0.28308	0.52435	TRIG
2	0.0494	16	43	0.19487	0.42958	CSS
3	0.1308	930		0.21625	3.78306	FINT
4	0.1449	328	7148	0.41391	0.32088	MPHA
5	0.0209	101	516	0.16953	0.71148	APTE
6	0.1048	186	1358	0.24615	1.37531	TIMK
7	0.1479	259		0.51492	0.63205	NITT
8	-0.0236	8	12	0.45185	0.27545	MIT
9	-0.0111	84	120	0.48903	0.13289	ALLS
10	0.1068	59	29	0.67396	0.35541	ASMT

Table 11 (Continued)

rank	RoaR	ShPr	MktK	y/K	y/L	Name
11	0.1313	53	557	0.48907	0.61377	SONA
12	0.1188	303	589	0.79449	1.21776	MAST
13	0.0972	87	154	0.27178	1.69991	OMNI
14	0.164	706	4220	0.86683	0.90282	MINT
15	0.16	555	44953	0.40239	0.71241	HCLT
16	0.1461	198	2958	0.43583	0.4838	HEXA
17	0.0851	90	91	0.37515	0.47064	SAKS
18	0.1102	163	1409	0.45239	0.45108	KPI
19	0.4481	755	2310	0.86843	0.41254	ECLE
20	0.1052			0.73792	0.75912	IGS
21	0.2363	2561	141079	0.69068	1.5233	INFS
22	0.2874	1487	228926	0.82199	1.1119	TCS
23	0.1012	145	1435	0.88645	0.75961	PFT
24	0.1608	587	92470	0.53045	1.04123	WPRO
25	0.0336	32	67	0.26063	0.42662	MIND
26	0.0921	106	410	0.49784	0.4901	RSI
27	-0.0151	35	51	0.66482	0.36499	CAMB
28	0.0047		24	0.13555	0.2579	ONTR
29	0.0922	72	115	0.12996	2.89643	PANO
30	0.1122	1088	2360	0.3488	0.43895	CMC
31	0.1105	1671	1670	0.25241	2.78426	ICRA

6 Causal paths between SCE or EOS and stock market indicators

This section reports causal path estimates between RoaR, ShPr, and MktK separately paired with SCE in Table 12 and with EOS in Table 13. We use column headings described in Section 2.2.

Table 12: causal paths paired with SCE

	cause	response	strength	corr.	p-value
1	SCE	RoaR	31.496	-0.093	0.60666
2	SCE	ShPr	31.496	-0.2518	0.17945
3	SCE	MktK	100	-0.1264	0.50584

Table 12 reports causal paths between scale elasticity SCE and stock market evaluations, RoaR, ShPr and MktK. The table is similar to our earlier pooled data causality Table 4. We find that the scale of the firm measured by SCE drives the stock market evaluations.

Table 13 reports causal paths between elasticity of substitution EOS and stock market evaluations, RoaR, ShPr and MktK. The column headings are described in Section 2.2. We find that RoaR drives the EOS, suggesting that the flexibility to input price shocks does not much affect accounting rate of return. By contrast, independent variation in EOS does cause ShPr and MktK.

Table 13: causal paths paired with EOS

	cause	response	strength	corr.	p-value
1	RoaR	EOS	100	-0.219	0.22069
2	EOS	ShPr	31.496	-0.06	0.75284
3	EOS	MktK	100	0.025	0.89577

7 Final Remarks

Publicly traded Indian multinationals have clients around the world, mostly in advanced countries. They 'produce' mostly information and communication technology services (ICT). This paper studies certain publicly available data on them to look for patterns not only in data levels, but also in percent rates of growth. There is a significant number of missing data regarding the number of employees and value-added outputs.

There are two standard measures of productive efficiency called scale elasticity or SCE and sensitivity to price shocks by the elasticity of substitution (EOS) developed by Hicks and explained by Ferguson (1971). We estimate SCE and EOS values for a non-homogeneous VES production function with a cross-product term. The analysis using SCE assumes "thought experiments" to assess what might happen to the output when one or both inputs are increased by one percent. The EOS analysis considers even more sophisticated "thought experiments" to assess what might happen to marginal rates of transformation between the two inputs when relative prices of inputs are changed.

Despite heterogeneous firms and missing data, we find plausible estimates revealing overall patterns based on pooled data in Table 9 sorted by SCE reveals names of efficient firms along the top rows and inefficient firms along bottom rows. Similarly, Table 10 sorted by EOS identifies firms with poor flexibility against input price shocks in the ICT sector along the bottom rows, especially where EOS values are negative, whereas flexiable ones are named along top rows.

Our individual company estimation reveals great heterogeneity among the firms. The heterogeneity leads to a variety of marginal elasticities, scale elasticities (SCE), and elasticity of substitution (EOS) among these firms. Focusing on 31 companies with relatively complete data, we are able to rank and identify by name efficient and price-shock-robust companies in two separate tables.

A novel study of causal paths between SCE, the scale elasticity, and stock market valuations in terms of individual company rate of return, share price, and market capitalization (RoaR, ShPr, and MktK) shows that SCE does drive stock market values, supporting the Hicksian production theory and rational ranking of these companies by the Indian stock market. We also find that most causal paths between growth rates of data are plausible.

Among the limitations of our research, we must mention that efficiency and price-shock sensitivities can have many unmeasured aspects. Our naming of companies should be treated as indicating a need for a further focus on why certain companies have high (low) estimated values of SCE and EOS. Our estimates remain subject to sampling variation. We also report traditional output per employee and output per unit of capital. Thus we provide a wealth of information regarding the overall health of Indian multinationals, as well as detailed estimates for several individual companies for potential use by academics, investment analysts, and policy-makers.

8 Appendix

We report the long names of the ICT multinationals with their abbreviations in the Appendix. Since we have a great many missing data values, only a subset of these companies is included in the present study, which will be extended to include additional companies at a future date if and when more complete data become available.

Table 14: Alphabetic (A to H) table of abbreviated company names and their longer forms

Row	Short Name	Long Name
1	ABMK	A B M Knowledgeware
2	ACCL	Accel
3	AKS	Accelya Kale Solutions
4	ALLS	Allsec Tech
5	APTE	Aptech
6	ASMT	A S M Tech
7	BIR	Birlasoft
8	BRIS	Bristlecone India
9	CAMB	Cambridge Tech
10	CIGN	Cigniti Tech
11	CMC	C M C
12	CSS	Cybertech Systems and Software
13	DFIN	Datamatics Fin
14	DGS	Datamatics Global Services
15	DPLM	3D P L M Software Solutions
16	ECLE	Eclerx Services
17	FINT	Financial Tech
18	GEO	Geometric
19	HACK	Hackett
20	HCLT	H C L Tech
21	HEAL	Healthfore Tech
22	HEXA	Hexaware Tech
23	HIGH	Highbar Tech
24	HIND	Hinduja Global
25	HMIT	Helios and Matheson Info Tech
26	HOV	H O V

Table 15: Alphabetic (I to O) table of abbreviated company names and their longer forms

Row	Short Name	Long Name
27	IBPO	Infosys B P O
28	ICRA	I C R A
29	IGS	Igate Global Solutions
30	IINF	3I Infotech
31	INCO	Infinite Computer
32	INFE	Info Edge
33	INFS	Infosys
34	INTE	Intense Tech
35	INTR	Intrasoft Tech
36	ITCI	I T C Infotech
37	KPI	KPIT Tech
38	LTI	Larsen and Toubro Infotech
39	MAH	Mahindra Eng
40	MAST	Mastek
41	MIND	Mindteck
42	MINT	Mindtree
43	MIT	Melstar Information Tech
44	MPHA	Mphasis
45	NEIL	Neilsoft

46	NIIT	N I I T Smartserve
47	NITT	N I I T Tech
48	OBPO	Oracle B P O
49	OMNI	Omnitech Infosolutions
50	ONTR	Ontrack Systems
51	ONW	Onward Tech

Table 16: Alphabetic (P to Z) table of abbreviated company names and their longer forms

Row	Short Name	Long Name
52	PANO	Panoramic Universal
53	PFT	Polaris Fin Tech
54	REL	Reliance Mediaworks
55	RSI	R Systems International
56	RSS	R S Software
57	SAKS	Saksoft
58	SCT	Sasken Communication Tech
59	SONA	Sonata Software
60	STER	Steria
61	SUND	Sundaram Infotech
62	SYNT	Syntel
63	TATA	Tata Communications
64	TCS	TCS
65	TIMK	Timken India
66	TMAH	Tech Mahindra
67	TRIG	Trigyn Tech
68	USH	Unisys Softwares and Holding
69	VAKR	Vakrangee
70	WINF	Winfoware Tech
71	WINF	Cades Digitech Pvt
72	WPRO	Wipro
73	XCS	Xchanging Solutions
74	XRXI	Xerox India
75	ZEN	Zensar Tech

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