Spring 5-1-2022

Analyzing the Effectiveness of Different Media to Enhance Mathematical Discourse

Kyle Och
sd3399hi@go.minnstate.edu

Analyzing the Effectiveness of Different Media
to Enhance Mathematical Discourse

A Project Presented to
The Graduate Faculty of
Minnesota State University Moorhead

By

Kyle Och

In Partial Fulfillment of the
Requirements for the Degree of
Master of Education in
Curriculum and Instruction
with Emphasis in Mathematics Education

May 2022

Moorhead, Minnesota

Abstract

This qualitative and quantitative study compared the effect of paper-pencil work in a mathematical discourse with problems and prompts presented virtually through various technological tools. The participants of this study were ninth and tenth grade students in two separate blocks of geometry class. These classes consisted of predominantly White students split into class sizes of 26 and 21. Since students were separated into two different geometry classes, one class was given paper problems and discussion prompts while the other received problems and prompts through Desmos Classroom Activities. This way, a quantitative analysis of three data points could be compared between the two classes to analyze the effect the different media had on student to student mathematical discourse. After weeks of analysis, data showed that paper prompting was slightly more effective at stimulating mathematical discussion. In fact, most of the discussion came while students worked through discrepancies in answers. Meaning, even though technology helped students learn with immediate feedback and learning aids, it was the mistakes in students' work, without immediate feedback, that helped students collaborate verbally and work to understand their mistakes.

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

Introduction

Over the past few years, the world of education has shown teachers that the learning process continues forever. Due to the COVID-19 pandemic, numerous teachers were forced to discover new ways to connect to students through virtual platforms. Similarly, many mathematics teachers have moved away from traditional teaching methods. This movement had a push for more problem solving and communicative skills. As a result, classroom discussion and collaboration had become the primary focus to grow these skills. However, there are many factors that affect classroom discussion and collaboration, especially when students are learning remotely. So, this research has analyzed factors of mathematical discussion in order to determine which methods of prompting and questioning promote discussion and problem solving in the classroom.

Brief Literature Review

There has been important legislation surrounding the importance of mathematical discourse between students in the classroom. Both the Every Student Succeed Act of 2015 and the National Council for Teachers of Mathematics (NCTM) have pushed teachers to implement various instructional strategies that support student to student discussion and collaboration (NCTM, 2014). When teachers focus on the social aspect of learning, there is more opportunity for student to student discussion, and this sort of discussion can be extremely beneficial in a mathematics classroom.

Two pieces of literature discussed the importance of social interaction and student context in a mathematics discussion. Christopher Danielson and Dan Meyer (2016) praise the creation of learning environments where students are forced to talk about math concepts before the teacher tells them what the concepts are. This way, students discuss them informally in their own context so they develop their own understanding of the context (Danielson and Meyer, 2016). Similarly, King and Cambell (2019) present the idea of "exploratory talk" which is when students explore and critique each other's ideas about mathematical concepts. Meaning, teachers should try to inspire students to determine their own ideas and representations about concepts rather than simply instruct students directly from teacher to student.

This is fascinating because every student has a different context that they can apply to a discussion. By discussing, comparing, and connecting the different thoughts in a group, students are able to develop their own understanding of a problem. So, it was intriguing being able to analyze the student to student discussions and the different thoughts they uncover because these results may not only impact a mathematics classroom, but other subject areas as well.

Statement of the Problem

The goal of this study was to determine if technology is not always as beneficial at promoting discussion and problem solving in the mathematics classroom. Sometimes, technology is not as beneficial as using traditional teaching strategies and resources. There are often distractions that come with the use of technology. Often, technology is implemented without supporting the learning outcomes and skills of the lesson. Because of this, it is important we analyze the effectiveness of using technology to enhance discussion in the classroom. To do this, this research has conducted a study that created two identical learning environments. So, both how well virtual prompting using a Desmos Activities enhanced discussion and problem solving with how well prompting on paper enhanced discussion and problem solving in a mathematics classroom can easily be prepared.

Purpose of the Study

The purpose of this study is to examine the implementation of technological tools rather than paper usage. Since COVID-19 forced many teachers to explore alternative modes of teaching, many teachers used remote learning platforms to initiate student to student discussion. However, it is not well known how beneficial or detrimental these alternative modes of discussion have been. This study will determine whether or not alternative technological resources can or should be sustained in the classroom even after schools, teachers, and students return to full-time in-person learning.

Research Question

How do paper and virtual prompts impact mathematical discourse in a high school classroom?

Definition of Variables

The following are the variables of study:

The independent variable of the study is the medium in which students will receive questions, prompts, and diagrams. One group has received questions, prompts, and problems virtually in a Desmos Activity. Another group has received questions, prompts, and problems on a paper worksheet. Both provided students the opportunity to respond to questions and prompts, interact with other students, and mark up diagrams relating to the concepts of each lesson.

The dependent variable of the study is the quality of mathematical discourse. This will be measured in multiple ways. First, key vocabulary terms and concepts will be identified for each lesson. This way, during each lesson, a tally can be kept to see how often students are using these concepts. So, the more students use these words in discussion, the higher the quality of mathematical discourse. Additionally, the teacher will collect student work on problems, explanations of work, and responses to prompts. Last, field notes will be collected during each lesson so that the teacher can review the notes after each lesson.

Several control variables have been identified and implemented. The control variables, such as classroom setup, questions to ask, instructional strategies will be the same in both groups. These control variables will be in place to encourage student to student discussion so that this study could focus on determining how the medium of prompting has an effect on the quality of mathematical discourse between students.

Significance of the Study

Since the participants of this study were students in the classroom, students have been put into learning situations where discussion and problem solving have taken place. Subsequently, instructional strategies have been implemented so that students have been cooperatively learning through social interaction. This learning environment, of positive student to student discussion, is important and was supported from the literature we reviewed. In addition, through this cooperative learning environment, students were developing career and life skills about valuable communication, critical thinking, and problem solving. Therefore, this study gave students accommodating learning situations and valuable career and life experiences.

Research Ethics

Permission and IRB Approval

In order to conduct this study, the researcher will seek MSUM's Institutional Review Board (IRB) approval to ensure the ethical conduct of research involving human subjects (Mills & Gay, 2019). Likewise, authorization to conduct this study will be sought from the school district where the research project will take place (See Appendix X and X).

Informed Consent

Protection of human subjects participating in research will be assured. Participant minors will be informed of the purpose of the study via the Method of Assent (See Appendix X) that the researcher will read to participants before the beginning of the study. Participants will be aware that this study is conducted as part of the researcher's Master Degree Program and that it will benefit his/her teaching practice. Informed consent means that the parents of participants have been fully informed of the purpose and procedures of the study for which consent is sought and that parents understand and agree, in writing, to their child participating in the study (Rothstein & Johnson, 2014). Confidentiality will be protected through the use of pseudonyms (e.g., Student 1) without the utilization of any identifying information. The choice to participate or withdraw at any time will be outlined both, verbally and in writing.

Limitations

One major limitation for this study to truly determine whether virtual prompting is better than paper prompting was the differences in student behavior between the two groups of students. That is, different groups of students may be more motivated and more capable in a class discussion than other groups. So, since evaluating the quality of mathematical discourse was vague, it would have been important to understand a baseline for how well each group of students discusses concepts as a group. Then, after that baseline is understood, we can examine how well the different media enhanced their class's mathematical discourse. Consequently, it may seem like one group of students is more engaged in a discussion than the other group. Unfortunately, this limitation may lead to inconclusive results in the study.

Conclusions

This chapter outlined the value of mathematical discourse and why mathematical teachers should be promoting student to student discussion in the classroom. As a result, this study has researched the effectiveness of different media to promote mathematical discourse in a high school classroom. However, no matter how important high quality mathematical discourse is, analyzing the effectiveness has had limitations. In this next section, research and literature is reviewed in order to maximize the opportunities for high quality discourse and minimize the limitation of this study.

CHAPTER 2

LITERATURE REVIEW

Introduction

The goal of this study was to determine how different forms of prompting impact mathematical discussion in the classroom. The research in this section guides the desired outcomes for successful mathematical discourse in the study. Since educators strive for new methods, especially in the time of a global pandemic, this research was important to the growth of educational strategies and technologies. That is, this research was intended not only for widespread growth, but growth for educators in districts that look to provide students opportunity to grow skills and characteristics relevant to the world today. This section will analyze the context, the variables, and the educational theory important to the implementation of mathematical discourse.

Context

As the world of education continues to change based on research, mathematics education is changing as well. Researchers have continued to analyze the effectiveness of various instructional strategies. For instance, more traditional instructional strategies, such as direct-instruction, are less impactful on student learning as strategies like classroom discussion, providing feedback and summarization (Hattie, 2009). Because of this, many educators are moving away from a more traditional, teacher-centered classroom toward a more collaborative, student-centered learning.

At the national level, legislation has pushed for a new educational focus, especially in math classrooms. This was seen throughout several articles and journals found throughout the research process. For instance, the Every Student Succeed Act (2015) "draws on expectations in education reform movements that all students learn to reason by constructing and communicating mathematical arguments while expressing and evolving mathematical ideas" (Xin Chiu Tzur Park and Yang, 2019, p. 43). Additionally, the National Council of Teachers of Mathematics (NCTM) urged for change in mathematics classrooms by challenging teachers to create classroom strategies that allow student-centered learning and student to student discourse (NCTM, 2014). This national push for reform stems from prior research regarding discussion-based and cooperative learning.

According to Anderson-Pence (2017, p. 2) "higher level learning first occurs on the social plane." This is why simply having teacher to student social interaction, as many would experience in a lecture or direct instruction lesson, is less effective at developing concepts and understanding; whereas, classroom discussion between students allows for greater social interaction. So, it is more important that educators promote student to student interaction. Not to say the teacher should have no influence on the interaction, but, according to Doolittle (1995) student to student cooperative learning supports equity and inclusion following Vygotsky's zone of proximal development. This student to student cooperative learning may come in the form of collaborative practice and classroom discussion.

Prominent minds in the world of mathematics education, Christopher Danielson and Dan Meyer, wrote about collaboration and discussion. They contend that mathematics teachers should not start by telling students formal definitions of concepts they are teaching. Rather, teachers should attempt to develop environments that create "a need for students to talk about properties of objects—properties for which they may not yet have names. This environment allows students to develop rich informal language, which is captured for teachers to use later in discussion and formalizing" (Danielson and Meyer, 2016, p. 262). Since all learners have their own zone of proximal development, activities and practices that stimulate each student's learning must begin at a basic sociable level (Doolittle, 1995). This way, each student is able to use their prior knowledge to apply to the social context of the learning scenario.

This leads to students using their own understanding and information by connecting different representations and thought processes. This is an idea, discussed by King and Campbell (2019), called "exploratory talk." Exploratory talk is when students critique and analyze each others' representations, explanations, and justifications. That is, students are inspired to determine their own ideas about concepts rather than simply be instructed directly by the teacher. In this atmosphere, the goal of student to student discussion aims to grow student competence by putting "the authority to determine the answer to mathematical questions within students," not the teacher (Munson, 2018 p. 246). Then, the conversations and ideas shared during the discussion can be observed, guided, and later summarized by the teacher.

Quality Mathematical Discourse

In order for high quality mathematical discourse to occur, students must have the opportunity to construct their own representation of concepts. Students' representations could come logically, contended Bennett (2014), engaging discourse is students evaluating and interpreting "perspectives, ideas, and mathematical arguments of others as well as construct valid arguments of their own. That is, students develop deeper understandings of mathematics when they engage in meaningful social interactions" (p. 20). Even though logical arguments are a centerpiece to rich mathematical arguments, visualization is also important According to Berenger (2018), "visualisation is vital for communicating geometric concepts, both verbally and non-verbally at all levels" (p. 170). In fact, the journal continues to discuss the connectedness of visualization and logic. Berenger discusses how the practice of processing imagery, whether or not there's an image or diagram, forces students to organize information logically and analyze the meaning of terms and concepts.

Even though classroom discussion is meant to be interpersonal, that doesn't mean it is always *verbal* communication. Discourse is just a term describing communication between two or more people.

Communication is far more than just the exchange of words. It is the exchange of gestures and movements. That is, it is nonverbal communication is also important to analyze. According to Guefre (2018), “gestures are not only a simple hand gesture but also play an important role in cognition” and “are shown as a means of understanding” of mathematical data, concepts, and relationships (p. 126). That is, classroom mathematical discussion can be *seen* as students pointing at pictures, marking up diagrams, sketching their visual thinking, and other gestures. So, in the methodology, it is important to be aware of the different types of communication. Since the mathematical discussion does not only need to take place verbally, aspects of student to student communication can be written on paper or typed virtually in Desmos. So, analysis of responses on both media will shape the methodology of the study.

Prompting Discussion on Paper and Virtually

Technological innovations impacted career and professional workplaces, but, consequently, affected classroom procedures and practices. Thomas (2016) argued that “although education is about more than serving the needs of industry, it is interesting to note that an observable shift in the role of mathematics plays in the professional lives of non-mathematics could be seen twenty-five years ago, yet many mathematics classrooms still devote a great deal of effort to paper-and-pencil mechanics” (p. 14). This stems the purpose of this research: are virtual prompts or paper prompts more effective at enhancing mathematical discourse?

Paper prompts and notes have been used with several classroom discussion strategies (Weih, 2015). In his writing, Weih outlines several discussion strategies, such as Think-Pair-Share, Quick Writes, Recorder-Reporter, and K-W-L, that provide opportunities for all students to engage in discussion by prompting through pencil-and-paper writing. However, the most intriguing notation Weih makes concerns the way in which classroom discussion should be orchestrated. That is, when teachers pose questions and prompts to the *whole class*, only a few students respond, and they respond more than once. This teacher to student discussion is not an effective classroom strategy; whereas, the strategies listed above not only follow the desired student to student discussion, but give guidance for the type of prompting that was included to enhance discussion.

Similarly, Desmos Activity Builder is a virtual tool for prompting discussion where “teachers can configure to meet their curricular needs in a variety of topic areas (Danielson and Meyer, 2016, p. 259). Since both paper prompting and virtual prompting must require similar tools and technology, there are several factors to consider.

For instance, one tool, which can be used in conjunction with Desmos Activity Builder, is the Desmos Graphing Calculator. Even if students are being prompted with paper, they will have a hand-held graphing calculator available. However, the Desmos Graphing Calculator is “able to complete a variety of other computational features such as lists, plots, regressions, graph restriction, and simultaneous graphing tasks” (King and Campbell, 2019, p. 33). Now, when this study was conducted, students who did not have access to Desmos still had access to a handheld graphing calculator. Nonetheless, the graphing and imaging tools on Desmos, because of its variety of features and easy accessibility, poses interesting possibilities in the study.

Another feature, that can affect the quality of discourse, is the text input option in Desmos Activity Builder. That is, when completing a Desmos Activity, students can see other students’ responses on their screen. Anderson-Pence (2017) found contrasting ideas that are particular to the study. First, she found that graphing software programs' capability to provide dynamic feedback often had a positive impact on student collaboration when problem-solving. This idea is shared by Danielson and Meyer (2016) who assert that technology provided feedback allows students to self-evaluate their own mistakes and adjust their work accordingly. However, when students worked out problems on paper and then compared them to a dynamic graphing technology, “discussions ensued regarding the discrepancies and how to reconcile them” (Anderson-Pence, 2017, p. 6).

Since both paper prompting and virtual prompting tools can enhance classroom discussion on their own, the next step is to determine which may be more effective. That said, it was difficult to find any resources comparing the virtual prompting to paper prompts. There are several factors that may limit the effectiveness of the study, the biggest one being computers being distracting. Several small variables may negatively affect the mathematical discourse in the study. So, keeping in mind the similar uses of Desmos and paper prompting, it is important to determine a framework of each discussion-based lesson so that best practices can be implemented.

Theoretical Framework

Before analyzing whether Desmos or paper prompts are more effective at enhancing mathematical discourse in the classroom, there was a methodical approach when preparing each discussion-based lesson. As stated previously, Vygotsky's learning theory about the Zone of Proximal Development shows valuable student to student discussion has a significant impact on student learning. So, the study must include practices, procedures, and techniques that stimulate mathematical discourse. This way, the chances of high quality mathematical discussion are likely to occur. Sullivan emphasizes three practices required for high quality classroom discourse. These practices include “1) implementing tasks that promote reasoning and problem solving, 2) facilitating meaningful mathematical discourse, and 3) posing purposeful questions (Sullivan, 2019; p. 1576). Meaning, it is important to examine how these three practices play a significant role in shaping the structure of the lessons that were planned to be implemented.

The first practice, implementing tasks that promote reasoning and problem solving, relies heavily on teacher efficacy of technological use. Since this study will be assessing the effectiveness of stimulating discussion from technological resources, lesson preparation relies on the implementation of the technology in the classroom. For instance, using a Geogebra or using a resource from a school's curriculum, may have had adverse effects towards stimulating mathematical discourse. So, the use of that technology impacts the classroom discussion. Thomas (2016) contends that "simply providing access to technology is not sufficient for change to occur," and the teacher's use of a technological or online tool "can heavily influence potential student outcomes" (p. 15). In one study analyzing the use of manipulatives, both concrete and virtual, researchers ascertained that teachers depended heavily on manipulatives to promote student learning; however, these teachers were not always aware of the implementation of the manipulatives for student learning (Mntunjani, 2018). Similarly, this research must be mindful about the implementation of technology and virtual resources used in each lesson.

Also, the teacher must move to initiate and facilitate meaningful discussion. More specifically, the teacher still needs to instigate the tasks and establish a foundation to each lesson. For instance, in any lesson where learning should occur as a conversation between two learners, through the theory of Vygostky's ZPD," Hayward (1995) establishes key elements for lessons. One element, for students and adults to be collaborating on an "experimental activity, there should be a clear goal to the activity…There should be a definite direction outlines for the activity portion."

Lastly, the structure of questioning is crucial to any classroom discussion. So, being deliberate when posing questions and determining whether or not they are purposeful to the discussion has a large impact on the classroom discussion. Jen Munson of NCTM (2018) argues the interactions learners have with mathematics and with their mathematics teachers promotes students competency and interactions. She suggests two questioning techniques in the classroom--the "state-and-inquire move," and the "what could you do?" move. These two moves elicit student responses, without the feeling of being right or wrong, and urge students to further their own thinking. Meaning, the study needed to include prompts and questioning that promote this sort of student behavior.

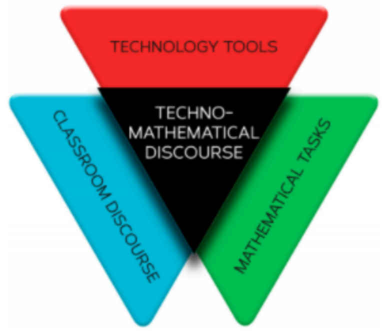

Figure 1: Techno-Mathematical Discourse (TMD) Framework. (Anderson-Pence, 2017, p. 2)

Coinciding with the ideas presented above, Anderson-Pence (2017, p. 1) presents the idea for analyzing mathematical discourse using the Techno-Mathematical Discourse Conceptual Framework (TMD) that "considers three components of the learning environment that impact mathematical discussion: classroom discourse, technology tools, and mathematical tasks" (See Figure 1).

Using these theories as a framework for the study, the careful planning and preparation of elements of the study helps identify the control variables. That is, by planning to maintain all important variables needed to create a mathematical discussion, outside factors are less likely to impact the research question. More specifically, when planning, the study has kept all mathematical tasks, all questions or prompts, and any technological tools used in the groups of this study identical, and, thus, reducing the chances that underlying variables may impact student engagement in mathematical discourse throughout the study.

Research Question

How do paper and virtual prompts impact mathematical discourse in a high school classroom?

Conclusions

This chapter reviewed literature that supported the study in determining different strategies that have been used to promote discussion of mathematical concepts in the classroom. It was shown that there are several factors that affect quality mathematical discourse. One of the most important factors is *how* the facilitator implements classroom discussion. This section discussed that teacher led discussions are not as impactful when compared with discussions with student to student interaction. Hence, this study must be mindful about the details of classroom discussion in order for high quality discourse to be analyzed. Encouraging strategies for high quality discourse impacts the methodology used to collect and analyze these discussions. These are the reasons for the mixed methods design used to collect and analyze both quantitative and qualitative data in order to determine whether paper or virtual prompting affects mathematical discourse. The following chapter discusses the methods used in the study.

CHAPTER 3

METHODS

Introduction

Using the findings from the literature review, this study has researched the implementation of practices and strategies that promote discussion in the classroom. Since the focus of this study is to examine classroom discussion, both a qualitative and quantitative observation of the mathematical discourse were used. This way, the researcher could easily and effectively analyze the quality of mathematical discourse in each lesson. This chapter discusses how the consideration of certain procedures and data impacted the study.

Research Question

How do paper and virtual prompts impact mathematical discourse in a high school classroom?

Research Design

The approach used will utilize a mixed methods experimental design. The plan is to analyze if an independent variable (i.e. virtual prompts, questions and diagrams or paper prompts, questions, and diagrams) has an effect on a dependent variable. Since the students participating in this study are assigned to specific classes, it must be noted that these students are not random, but rather assigned. Several factors have helped determine that the quantitative and qualitative experimental design would be the most effective for analyzing the results of the research study. These factors are discussed further in the following sections of this paper.

Setting

The study is taking place in a suburban-rural high school in Minnesota. The community surrounding the school district comprises of two small towns whose population totals about 6,000. While the district consists mostly of farmland, several suburban developments are scattered within and on the borders of the district lines. Most people of this community live here because it is in close proximity to the Twin Cities. So, many students have parents who commute to work in the metropolitan area.

The high school of the district consists of approximately 475 students. Out of these 475, approximately 450 students are White (95%), 12 are Hispanic or Latino (2.5%), three are Black/African-American (0.6%), 3 are Asian (0.6%), and seven are of two or more races (1.4%). About 10% of the student population is enrolled in special education. Also, around 15% of the student population has Free/Reduced Lunch.

Participants

The participants of this study were 9th and 10th grade Geometry students. There were 47, varied achieving students from two different Geometry classes who participated. These two classes each received a different medium for prompting. That is, one class that consisted of 26 students, received prompting and problems digitally. Consequently, the other class, which consisted of 21 students, received prompting and problems via paper. In the first class, 50% of the students were male and 50% of them were female. In the second class, 50% of the students were male and 50% of them were female. In total, out of the 46 participants, 44 students (96%) were White and two students (4%) were of two or more races. four of the 50 students in the study are in special education. Lastly, five of the participants have free and reduced lunch.

Sampling

This is a convenience sample since the two groups of students analyzed were put into these classes by the dean of students based on the required classes of each individual's schedule. There was no criteria for selecting which class would receive prompting virtually and which class would receive prompting via paper. These two classes varied in math performance, as seen in their middle school math MCA scores and there are students from both 9th and 10th grade.

Instrumentation

Three data points have been collected and analyzed in this study. All three data points have been collected and eventually organized in a spreadsheet using Google Sheets (see Appendix A). The three data points include field notes, tallies of kept concepts, and student responses. Each observable day, field notes were kept on paper. At the same time, a tally was kept for how often students spoke of the key concepts from each lesson. Third, students' responses were collected from the Desmos prompts and diagrams, from Schoology assessments, and in the data found on MathXL assignments. Also, for all paper prompts and problems, students . Since Google Sheets allows for multiple spreadsheets, each observable day has its own spreadsheet. Then, when all data collection was finished, the data was reorganized so that it could be compared more easily.

Data Collection

Data was collected in three ways. First off, during each lesson, field notes were kept in a notebook or on a piece of paper. That is, while discussions were taking place, the researcher observed and wrote down notes about the students' interaction in the room. These field notes have included any discussion, concepts, or conversations observed during the lessons. Similarly, the number of times that students discussed desired responses were tallied to quantitatively examine the discussion (see Appendix B). Before each lesson, the researcher identified key words and concepts that should be discussed within the lesson. So, while discussions were taking place, the researcher observed and tallied the number of times students spoke about these keywords or ideas that related to the key words. Additionally, students have recorded responses and drawings either on paper or in Desmos. So, after each lesson, these responses were collected. The goal when collecting these was to find valuable student responses and drawings. In the context of this study, valuable responses and drawings are responses either directly related to the concepts of the lesson or the responses and drawings that are not connected and considered off task. This showed us whether the prompting kept students engaged in the lesson and with the lesson. After all, since the goal of the study was to analyze how paper and virtual prompts impact mathematical discourse, evaluating the engagement of the students during each lesson is crucial to answering that question.

Data Analysis

After one week of observed lessons, all data was sorted into Google Sheets (see Appendix A). The desired words and concepts were tallied quantitatively. Additionally, throughout each lesson, the teacher observed and recorded field notes regarding their sense of the quality of mathematical discourse at parts of the lesson. These field notes were typed into the same spreadsheet as well. Finally, valuable responses from Desmos and paper responses will be typed into Google Sheets. This gave the study three data points that are all summarized in a Google Sheet. Since Google Sheets allows for several spreadsheets, each day has its own spreadsheet. This way, after all data was collected, all the data was easily synthesized using all the spreadsheets.

Research Question and System Alignment

Table 3.1.

Research Question Alignment

Research Question	Variables	Design	Instrument	Validity & Reliability	Technique (e.g., interview)	Source
How do paper and virtual prompts impact mathematical discourse in a high school classroom?	IV: Medium used for prompting and problem solving DV: Quality of Mathematical Discourse	Qualitative and Quantitative Experimental Design	Spreadsheet	Both prompting and observing allowed for accurate number of uses of the desired terms Field notes kept by the teacher limits the facilitation of classroom discussion done by the teacher. Both Desmos and paper responses will be collected after or during the lesson to validate the desired response.	Observing and Talling Observing and field notes Collecting Responses	2 different High School Geometry classes Sample Size: 50 students, with 26 receiving Desmos prompts and 21 receiving paper prompts

Procedures

Over a 4 week period, this study has implemented classroom discussion in two different high school Geometry classes. One class received prompts, questions, and diagrams in the form of paper while the other received them in a Desmos Activity. Prior to each lesson, instruction was planned so that all students, from both groups, receive identical prompts, questions, and diagrams. For each lesson, 3 data points were collected. During the lessons, tallies have been used to quantify how often mathematical concepts were discussed. Also, field notes were used to generalize the amount and quality of mathematical discourse of each lesson. Then, following each lesson, student responses were collected, on paper or in Desmos, and analyzed as a third data point. After collecting data, the data was organized into a spreadsheet of Google Sheets. This way, each lesson's data can be easily compared and tracked over the 4 week period. Hopefully, using these data points, the experimental study has determined if one medium of prompting has a greater impact on the quality of mathematical discourse.

Ethical Considerations

Students were able to choose how they want to work (individually, with a partner, in a group). This accommodates all learners learning through a classroom discussion. Since tallies, to keep track of the number of desired responses, these values will only be counted quantities that recorded field notes will use alternative names (Student A, Student B, etc.). Similarly, on Desmos, all responses remained anonymous since each Desmos activity can be anonymized. So, when screenshots were taken regarding conversations and responses were recorded.

Conclusions

This qualitative and quantitative designed study intended to gather data to analyze the impact different mediums of prompting and problem solving has on mathematical discourse in a high school classroom. Data was gathered throughout lessons in two different classes using tallies and field notes to measure the quality of mathematical discourse. The following chapter outlines the results of the study.

CHAPTER 4

DATA ANALYSIS AND INTERPRETATION

Introduction

In the beginning, this study questioned the best practices for secondary level math teachers. Inquiries were made about whether a teacher-led classroom or a student-centered classroom promoted the most valuable skills for young learners growing from adolescence to adulthood. This led to understanding the role of student to student discussion and the importance of mathematical discourse in classrooms today. This helped the study determine a specific goal for the research.

The purpose of this study was to examine the best ways to move away from teacher-led instruction into a classroom that focuses on student centered learning. It is important for educators to understand the best methods to use when trying to foster a mathematical discussion in a classroom because they can choose methods and materials for instruction from evidence-based best practices. In this chapter, all data collected will be presented to explain whether one medium of prompting had a strong impact on mathematical discussion. First, this study will present the data collected during its observation of two high school mathematics classrooms.

Data Collection

The desire to create a more student centered classroom led to the facilitation of mathematical discussion in two geometry classrooms. Over the course of 10 lessons that included classroom discussion, one geometry class received prompts and problems in the form of paper. The other class received prompts and problems virtually where students worked through Desmos Activities, completed concept explorations using GeoGebra, MathXL assignments and solved problems in Schoology assessments. These two classes were observed by their teacher, the facilitator of the classroom discussions, who collected three data points. The first two data points collected were tallies of spoken vocabulary and key terms from each lesson as well as responses collected from paper or virtual prompts given to the students. While these two data points measure quantitative data, the final data point, field notes from the facilitator, collects the qualitative data about students' understanding, their discussion, and the overall classroom environment. Next, these three data points will be analyzed to answer the research question.

Results

Research Question

How do paper and virtual prompts impact mathematical discourse in a high school classroom?

When observing the students during class discussions, there was greater student to student *verbal* discussion. This was seen in both the verbal discussion tallies and in the field notes. Out of the ten lessons where discussion was used, there were key words, phrases, and concepts identified for each lesson. In all of the discussions with the group given paper prompts and problems, the mean number of tallies was 82 discussed words with a standard deviation of 27.64 tallies. On the other hand, in all of the discussions with the group of students given prompts and problems virtually, there was a mean of 57.2 tallies for the discussed key words with a standard deviation of 15.17 tallies. Now, the two groups of students were slightly different sizes because the paper group had 26 students while the virtual group had 21 students. However, this difference in size is not significant enough to diminish the difference in the data. So, quantitatively, the number of key words, phrases, and concepts are more frequently discussed between students when they are given paper prompts.

Synthesizing the field notes, there were several indications that the discussions with paper prompts produced higher quality discourse between students. The quality of discourse was discussed in two contrasting ways in the fieldnotes. First, there were a number of remarks written about how the group with paper was more engaged, asked more questions, or referenced each other more often. This is evident when the facilitator writes, “Many students struggled with the initial triangle problems (isosceles and equilateral triangle properties). There were several questions/ lot of discussion at this time,” “Lots of confusion (but also discussion) about which letter combination tells congruency (SAS, ASA, AAS, HL),” and “When we got to using distance/Pythagorean Theorem, discussion picked up because students were constantly checking their work with others.” On the other hand, the facilitator also makes comments regarding the lack of discussion between students in the group given prompts and problems virtually. The facilitator writes, “Not many students conversing with each other or asking other students questions,” “High engagement with the Schoology assignments, but only some students conversing with each other or asking other students questions,” and “Many students concentrating hard on their devices, not with classmates.” These field notes, along with the tallies, show that paper prompting and problem solving impacts discussion more effectively.

Whereas, the students who engage in discussion using the virtual prompts, on platforms of Desmos exclusively, shared and responded to many more mathematical concepts than the group of students using paper prompts and problems. With Schoology and MathXL tools and problems, there was very little discourse between students. In fact, comparing the lessons that included a Desmos Activity with the lessons that only used Schoology or MathXL, there was a significant difference in both discussions and responses. In the lessons where students were engaged with a Desmos Activity, the mean amount of discussed terms and vocabulary was 10.12. In the lessons where students were engaged with a Schoology or MathXL, the mean amount of discussed terms and vocabulary was 7.80.

Data Analysis

Looking back at how the field notes corresponded with the literature review, there was a connection between the discussion between students and using technology. First, it was discussed that with Thomas that there was an "observable shift in the role of mathematics plays in the professional lives of non-mathematics could be seen twenty-five years ago, yet many mathematics classrooms still devote a great deal of effort to paper-and-pencil mechanics" (p. 14). This quote suggests that it is important for young learners to not just learn mathematics, but to understand technological skills so that they will be more prepared in the future, even if they are not in a specifically mathematical profession. This is fascinating because the focus of this study was to investigate whether one medium for prompting discussion was more effective than another, not about whether technological skills improved. So, looking at the field notes, there were several occurrences where technological fluency was being developed. In the field notes, the facilitator noted that, "There was a gap in tech literacy on Desmos," and "The feedback given by Desmos during the lesson helped several students. Meaning, students typed in equations and saw their linear equation graphed."

This quote about technological literacy from Thomas contrasts seemingly with a quote from Berenger. Since many students were learning how to use the technology, some of their discussion was not mathematical, but technological. As a result, because mathematics is so visual, the gap in technological literacy may have impacted how students visualize the data. This idea connects back to the literature review. That is, Berenger (2018) states that "visualisation is vital for communicating geometry concepts, both verbally and non-verbally" (p. 170). This may explain why students in the paper group more frequently discussed the geometry concepts while the virtual group did not talk to each other as much. The more students had to learn *how* technology can help them visualize concepts, the less students actually understood how to communicate with each other. When the students with technology did discuss their work and their thinking, they often did so in the form of answers.

Throughout the set of lessons where students worked together while problem solving, students who worked on paper more commonly explained their processes while students who worked king virtually. For the group of students who worked through the virtual prompts and problem solving, some of the online problem solving was done through the curriculum with a platform called MathXL. Similar to IXL, MathXL assignments are filled with questions where students attempt one at a time. The field notes suggested in different lessons that many students did not discuss the mathematical content while working on these MathXL assignments. Instead, many students only shared what their solutions were rather than showing others how they solved the problem or where they derived their solution from. In the field notes that the group with paper prompts and problems, the facilitator wrote, “When using Pythagorean Theorem, discussion picked up because students were constantly checking or adjusting their calculations with others,” and “Lots of confusion (but also discussion) about which letter combination tells congruency (SAS, ASA, AAS, HL). Referred to diagrams often.” Both of these quotes reference how students worked with one another to explain how they arrived at their solution or discuss *why* they know they are correct rather than simply sharing their answers and moving along.

However, another reason the data showed a lack of mathematical discussion with the group working onle could be because Schoology and MathXL presented problems that only allowed for the answer without access to workspace. In the literature review, it was stated that Danielson and Meyer (2016) share that technology tools that provide feedback allow students to self-evaluate their own mistakes and adjust their work accordingly. However, when working on paper, as opposed to using technological tools, “discussions ensued regarding the discrepancies and how to reconcile them” (Anderson-Pence, 2017, p. 6). This quote may indicate why there were very small amounts of mathematical discourse during the times when students were working to understand problems on Schoology and MathXL. It was clear from the field notes that students working on paper were more likely to explain their process to solving problems or uncover why their answers are different. This is because, while working on paper, more students wrote out their work. They had open space where they could work out the steps to solving a problem. Whereas, most of the students working online, attempted to do all of the work in their heads or all on a calculator. As a result, they had nothing to reference back to when communicating with the people around them. Anderson-Pence explains that when a student has their work written down on the paper, a diagram marked up, or a picture drawn, they can explain themselves more clearly or ask clarifying questions more easily. It is because of these factors, which students who work on paper have, that answering questions on paper is more likely to result in quality mathematical discourse.

Conclusion

Initially, it appeared that paper prompts were more effective at generating quality mathematical discourse. Throughout the lessons with paper prompts, there was more verbal discussion of key words and the field notes revealed how high the engagement was, not only with the content, but between students. However, there were still benefits integrating technology into a mathematical discussion. After collecting responses from students on both paper and virtual prompts, there were slightly more responses that included the key words and terms from several lessons. There was strong evidence that suggested Schoology and MathXL assignments did not foster much discussion during the lessons, but when students interacted with a Desmos Activity, they were actually more engaged with the content and their peers when compared to the paper prompts. This shows that some virtual mediums still created a high level of engagement and were effective at fostering quality mathematical discourse. Now, with this data, we can modify strategies for future implementation. These strategies will include actions for future teachers as well as considerations for district-wide implementation.

CHAPTER 5

IMPLICATIONS FOR PRACTICE

Introduction

In summary, this study intended to analyze the ways in which students can collaborate and communicate in the classroom. More specifically, the goal was to determine what prompts, learning techniques, and methods of facilitating discussion were most effective at fostering quality mathematical discourse in a high school classroom. In chapter 4 of the study, it was concluded that paper prompts were slightly more effective at fostering quality mathematical discourse during lessons and problem solving activities when compared to prompts and problems given virtually. This means, discussions used to promote mathematical literacy and conceptual understanding were best when students were given problems and prompts on paper. When problems and prompts were given virtually, via Desmos Activities, GeoGebra explorations, MathXL assignments, and Schoology assignments, students did not engage in mathematical discourse as much out loud, nor in their submitted work. Now, this data must be used to decide what teaching implications must occur going forward.

Action Plan

In this section, this research will discuss two implications for future practice. The first being that educators must create paper assignments that foster discussion and problem solving, not just getting the answer right or wrong. This research showed that this is not only best practice, but this practice promotes cooperative learning that deepens students' understanding of mathematical concepts. This section will also discuss how feedback plays a role in the future implementation of paper and virtual prompting. It was found that feedback was provided by both prompting techniques. It may be fascinating to educators how feedback not only impacted student learning, but helped the classroom facilitator assess student achievement as well.

The implementation of paper problem solving must create a challenging environment for students to discuss mathematical concepts. In the district in which this research was conducted, the facilitator of these classroom discussions, for the most part, moved away from the curriculum in order to foster mathematical discourse among the students in both geometry classes. That is, students are typically given direct instruction using vocabulary, diagrams, and examples provided by the curriculum. After this direct instruction, students often complete a set of online problems, called MathXL, with only right or wrong answers. In general, with the facilitator of this study, students often, following the lesson and discussions, practiced solving problems. In this study, one group solved problems on paper while the other group solved problems multiple times in MathXL assignments. Looking back at the field notes kept during the study, many students did not discuss the mathematical concepts and procedural literacy while engaged with these MathXL assignments. Instead, many students simply shared answers rather than explaining how they solved the problem or where they derived their solution from. This is the opposite of what was desired. When creating a classroom for quality discourse between students, the goal of the facilitator is to generate higher order thinking, not just a sharing of solutions.When students are forced into higher order thinking, they develop a deeper level of understanding of concepts, develop skills that dissect problems and distinguish valuable patterns for problem solving. So, in the earlier parts of discussion, it is okay to include simpler problems for students to discuss on a correct or incorrect basis. But, by the end of the lesson, students should be working through complex problems where they are sharing *ideas* for how to solve problems and multiple pieces of a problem.

This is how many math lessons should be created. It begins with objectives and an introduction to the content. Next, at a simple level, students must do skills such as recall ideas or recognize patterns. Following that, students grow to understand where these concepts come from, when they are used, and how to apply them in certain situations. It is after this that students should be challenged with complex problems. These may be word or diagram problems where students must identify important information, connect the information with what they have learned, and evaluate a best plan for solving. All of these actions require a high level of thinking. Best of all, all of these actions can be discussed with others. So, while solving difficult problems, students have the support of their peers to fall back on when in need.

In addition to this, the implementation of paper prompting strengthens the use of feedback for both students and educators. As quoted in the fieldnotes of the study, both the paper and virtual prompts and problems provided feedback. Sometimes, students received feedback in their work and in the responses from their peers. When solving problems on the computer, students got feedback from their learning platform, Schoology and MathXL, about whether their thinking was accurate. That is, they would know during and after their problem solving if they answered problems correctly because both Schoology and MathXL have a built in answer key. Conversely, the group working through problems on paper did not share answers as frequently. The field notes showed that this group of students explained their work much more rather than only sharing what the answer was. Additionally, analyzing the collected responses, most students showed their work in each problem when solving. This shows two important features of feedback with paper prompts and problems. First, it makes it clear how feedback, in the form of cooperative learning and explanations from peers guides students to greater understanding of the content. Also, when looking at the collected responses, it was clear to the facilitator whether students understood the content or not. This means, when the students worked on paper, all of their writing and work could be analyzed to see if they truly understood what was being taught. To be clear, online assessments still help teachers understand if their students understand the content or not. However, this is often only a quantitative result based on whether students answered a question correctly or incorrectly. As mentioned previously, sometimes students don't do the work and simply get the answers from someone else. If students do this with an online assignment, such as a Schoology or MathXL assignment in this study, the teacher cannot determine whether the student answered on their own or from a peer. The teacher will only know if the problem is correct or incorrect. However, when collecting student work done on paper, there are multiple pieces of information that show the teacher whether a student understood the assignment or not. The teacher can see the steps a student took, what numbers they used and calculated, how they were applied, and the final solution to a problem. All of these pieces are valuable information that provide feedback for the *teacher*, not just the student. Hence, this study proves the value of implementing paper prompting and paper practice in order to provide feedback.

Plan for Sharing

It is important to understand that this research is another account of generating engagement through classroom discussion. A majority of math teachers across the country teach using lecture and direct instruction. This is not as effective at generating classroom engagement with all learners as cooperative learning and classroom discussion strategies often generate. That said, the findings of this study should be shared with local educators. That is, educators within the district should hear about the overall research behind discussion in the classroom. More importantly, this information should be shared with *all* local educators, not just math teachers. Hearing this information district wide would benefit all levels of teachers in a variety of subject areas.

This research may help lead to more students comfortable discussing and engaging with other students. At the high school, several teachers in the district in which this study was conducted admit that they teach using direct instruction because that is the method students are the most familiar with. Meaning, when students are taught year after year by direct instruction, it is harder for secondary teachers to implement more variety of practices. Likewise, students at a younger age miss out on the opportunity to build interpersonal communication and collaborative skills. So, in this district, the findings of this research must be shared district-wide in order to build these skills at all grade levels.

After all, without constant practice of communication and problem solving skills, students are not developing the skills desired by the district. That is, over the last 10 years, this district has made it a goal to develop 6 types of skills that are valuable for all students beyond graduation from high school. Three of these skills include creative problem solving, critical thinking, and effective communication. In sharing the findings of this study, members from all parts of the district should see its connection to the problem solving thinking and interpersonal communications skills that are developed in a classroom setting where discussion and collaboration are valued.

APPENDIX A

Lesson: 4.2 Isosceles and Equilateral Triangles						
Prompting: Paper	**Date: 2/23**			**Prompting: Virtual**	**Date: 2/23**	
Key Terms/Phrases:	**Discussed**	**Collected**		**Key Terms/Phrases:**	**Discussed**	**Collected**
Same sides/2 sides that are congruent	12	5		Same sides/2 sides that are congruent	5	17
Same angles/2 congruent angles	2	3		Same angles/2 congruent angles	2	1
3 *sides* the same/congruent/all the same	16	24		3 *sides* the same/congruent/all the same	12	19
3 *angles* the same/congruent/all the same	4	2		3 *angles* the same/congruent/all the same	0	3
Equilateral	5	12		Equilateral	3	9
Isosceles	6	9		Isosceles	1	10
Equiangular	0	0		Equiangular	0	1

Fieldnotes:		Fieldnotes:
Students measured using different units		On Desmos, students struggled with the objective of the manipulatives.
- Some measured with inches, some used cm		I think most students knew what an equilateral triangle was, but I could not tell for certain
Many students knew what an equilateral triangle was prior to the lesson.		
- They asked something like "Is that called an equilateral triangle?"		Some students marked up the pictures as they solved for missing angles.
Many students easily completed the first few page of problems		When solving more difficult problems, a few students gave up quickly.
- These problems were much simpler than the later pages		There was a gap in tech literacy on Desmos
In the later pages, many students marked angles with their pencils		- Some students knew more of the tools than others, helping them mark up, label parts, and calculate numbers more easily
Many students pointed/marked up the pictures when talking with others		

REFERENCES

Anderson-Pence, K. L. (2017). Techno-Mathematical Discourse: A Conceptual Framework for Analyzing Classroom Discussions. *Education Sciences*, *7*.

Bennett, C. A. (2014). Creating Cultures of Participation to Promote Mathematical Discourse. *Middle School Journal*, *46*(2), 20–25.

Berenger, A., & Mathematics Education Research Group of Australasia. (2018). Changes in Students' Mathematical Discourse When Describing a Square. *Mathematics Education Research Group of Australasia*.

Danielson, C., & Meyer, D. (2016). Increased participation and conversation using networked devices. *The Mathematics Teacher, 110*(4), 258-264. doi:10.5951/mathteacher.110.4.025

Doolittle, P. E. (1995). *Understanding Cooperative Learning through Vygotsky's Zone of Proximal Development*.

Gürefe, N. (2018). The Role of Gestures in Mathematical Discourse of Hard-Hearing Students: Prism Example. *Acta Didactica Napocensia*, *11*(3–4), 125–140.

Hayward, P. A. (1995). *The Use of Vygotsky's Theory of the Zone of Proximal Development in Quantitative Research: A Critical Review*.

King, S., & Campbell, T. (2019). Using Interpersonal Discourse in Small Group Development of Mathematical Arguments. *North American Chapter of the International Group for the Psychology of Mathematics Education*.

Mntunjani, L. M., Adendorf, S. A., & Siyepu, S. W. (2018). Foundation Phase Teachers' Use of Manipulatives to Teach Number Concepts: A Critical Analysis. *South African Journal of Childhood Education*, *8*(1).

Munson, J. (2018). Two instructional moves to promote STUDENT COMPETENCE. *Teaching Children Mathematics, 24*(4), 244-249. doi:10.5951/teacchilmath.24.4.0244

National Council of Teachers of Mathematics (NCTM), I. . R. V. (1997). *Fostering Algebraic and Geometric Thinking: Selections from the NCTM Standards*.

Shilo, A., & Kramarski, B. (2019). Mathematical-Metacognitive Discourse: How Can It Be Developed among Teachers and Their Students? Empirical Evidence from a Videotaped Lesson and Two Case Studies. *ZDM: The International Journal on Mathematics Education*, *51*(4), 625–640.

Sullivan, P. (2019). Examining the Tensions between High Quality Discourse and Sharing Mathematical Authority with Students. *North American Chapter of the International Group for the Psychology of Mathematics Education*.

Thomas, R. V. (2016). The Effects of Dynamic Graphing Utilities on Student Attitudes and Conceptual Understanding in College Algebra [ProQuest LLC]. In *ProQuest LLC*.

Weih, T. G. (2015). Discussion Strategies for the Inclusion of ALL Students. In *Online Submission*. Online Submission.

Xin, Y. P., Chiu, M. M., Tzur, R., Ma, X., Park, J. Y., & Yang, X. (2020). Linking Teacher-Learner Discourse with Mathematical Reasoning of Students with Learning Disabilities: An Exploratory Study. *Learning Disability Quarterly*, *43*(1), 43–56.