A Solution To An Open Problem In Random Graph Theory

John Finlay
jf142583@umconnect.umt.edu

A Solution To An Open Problem In Random Graph Theory

John Finlay

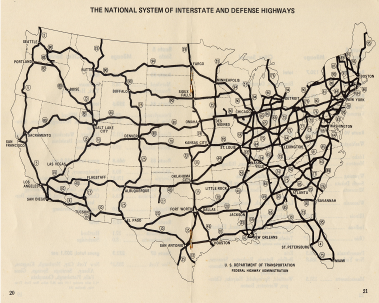
University Of Montana

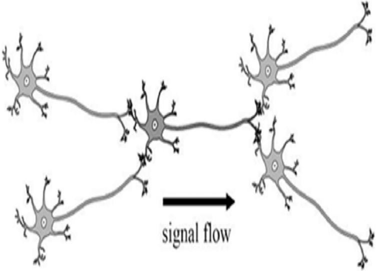
March 4, 2022

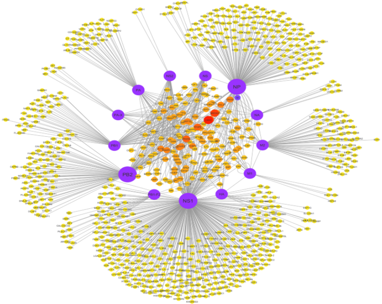
Rainbow connectivity of randomly perturbed graphs

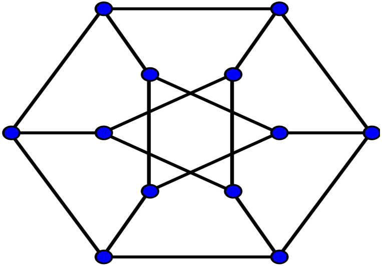
József Balogh* John Finlay† Cory Palmer‡

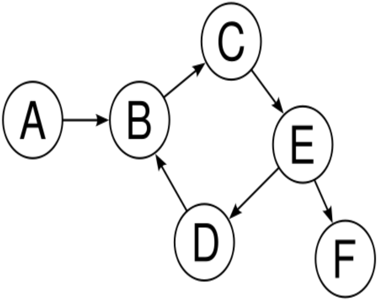
December 28, 2021

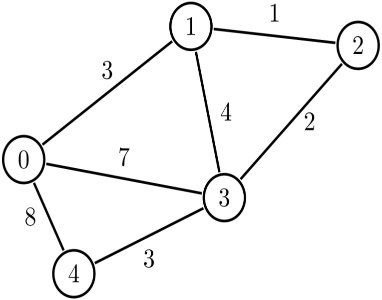

THE NATIONAL SYSTEM OF INTERSTATE AND DEFENSE HIGHWAYS
SEATTLE
PORTLAND
BUTTE
BOISE
FARGO
MINNEAPOLIS
SIOUX FALLS
BUFFALO
DETROIT
BOSTON
NEW YORK
CHICAGO
DES MOINES
OMAHA
SALT LAKE CITY
DENVER
SACRAMENTO
SAN FRANCISCO
KANSAS CITY
ST. LOUIS
WASHINGTON
LEXINGTON
NASHVILLE
LAS VEGAS
FLAGSTAFF
LOS ANGELES
SAN DIEGO
ALBUQUERQUE
OKLAHOMA CITY
LITTLE ROCK
ATLANTA
SAVANNAH
TUCSON
EL PASO
FORT WORTH
DALLAS
JACKSON
NEW ORLEANS
SAN ANTONIO
HOUSTON
ST. PETERSBURG
MIAMI
U. S. DEPARTMENT OF TRANSPORTATION
FEDERAL HIGHWAY ADMINISTRATION
20
21

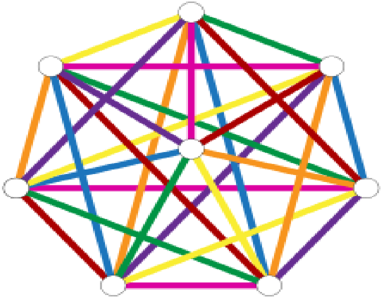

signal flow


NP
PB2
NS1




A
B
C
D
E
F


0
1
2
3
4
1
3
4
2
7
8
3



What Is A Graph

Graph

We define a graph G to be the two disjoint sets $V(G)$ and $E(G)$. We call $V(G)$ the vertex set of G or the vertices. Informally, these are the "points" in an illustration. We call $E(G)$ the edge set of G. $E(G)$ is made up of two element subsets of $V(G)$. Informally, these are the "lines" that connect the points in an illustration.

Graph Definitions

Definitions Related to Graph

- If u and v are vertices in G such that they have an edge between them, we denote that edge by uv and say that u is **adjacent** to v. Throughout this presentation we will only be using simple graph (no direction) so $vu = uv$.
- The edge uv is said to be **incident** with both u and v.
- A **path** is a sequence of vertices where each vertex is adjacent to the next one in the sequence.
- A graph G is **connected** if there exists a path between any two vertices of G.
- We denote a path whose endpoints are u and v as $u - v$.
- The number of edges incident to a vertex is the **degree** of the vertex. We denote the minimum degree of G by δ.

Rainbow Connectivity

What is rainbow connected?

- Suppose that we color every edge of a graph G using $[r] = \{1, 2, ..., r\}$ colors.
- A **rainbow path** is a path where no edge color is repeated.
- A graph G is **rainbow connected** if between any two vertices in G there exists a rainbow path.
- For a graph G the **rainbow connected number** is the smallest r such that G can be rainbow connected.

Randomly Perturbed Graphs

How to make a randomly perturbed graph

- Fix some $\delta > 0$.
- Choose some graph from the set of all graphs on n vertices with minimum degree at least δn arbitrarily, call this graph H.
- Let M be a set of edges of size m chosen at random from all of the $\binom{n}{2}$ possible edges.
- Insert the m edges into H, ignoring double edges, the resulting graph is $G_{H,m}$.
- Randomly color the edges with r colors, producing $G^r_{H,m}$

The Probabilistic Method

Recall

In a probability space, all of the event probabilities must sum to exactly 1.

The Probabilistic Method

In order to prove that something exists, rather than using a constructive argument, we define an appropriate probability space of structures and prove that a structure with the desired properties exists with positive probability. Or that the probability that it does not exist does not have probability 1.

The Union Bound

F

or any events $E_1, E_2, \ldots$ we have,

$$Pr\left(\bigcup_{i=1}^{\infty} E_i\right) \leq \sum_{i=1}^{\infty} Pr(E_i)$$