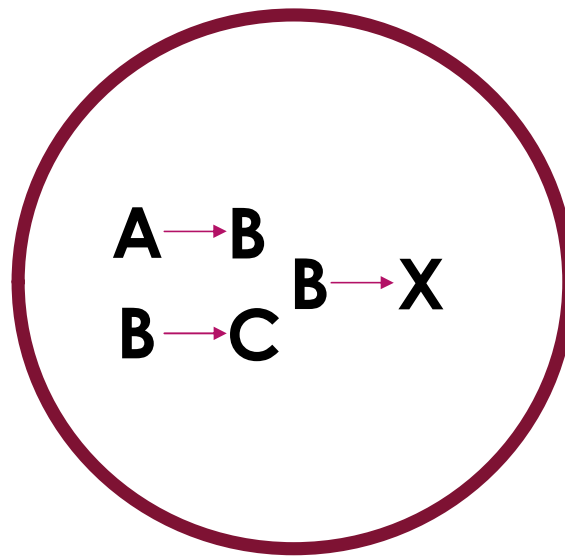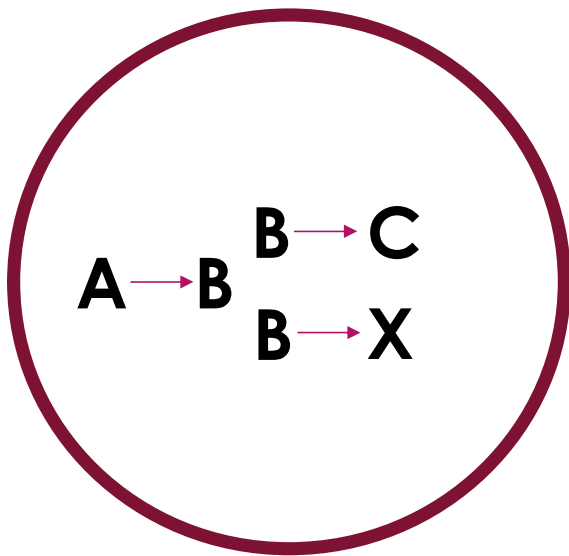# Spatial Metabolic Modeling and Analysis

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General Reaction Pathways

A →B $\quad$ B →C $\quad$ B →X

A →B $\quad$ B →C $\quad$ B →X

Letters are arbitrary chemical compounds

Reactions happen throughout

Assumption: our cell is a perfect circle

Why this specific set of reactions?

Equations Utilized

Michaelis-Menten

$$V_0 = \frac{V_{max}A}{K_m + A}$$

Diffusion

$$D\Delta A = D\left(\frac{\partial A^2}{\partial^2 r} - \frac{2\partial A}{r\,\partial r}\right)$$

Rate Law

$$\frac{\partial A}{\partial t} = D\Delta A - \frac{V_{max}A}{K_m + A}$$

Governing Equations

$$\frac{\partial A}{\partial t} = D\Delta A - \frac{V_b A}{k_b + A}$$

$$\frac{\partial B}{\partial t} = D\Delta B + \frac{V_b A}{k_b + A} - \frac{V_c B}{k_c + B} - \frac{V_x B}{k_x + B}$$

$$\frac{\partial C}{\partial t} = D\Delta C + \frac{V_c B}{k_c + B}$$

$$\frac{\partial X}{\partial t} = D\Delta X + \frac{V_x B}{k_x + B}$$

$$D\partial_r A = k^A (A_{ext} - A)$$

Variables:

A, B, C, X = Concentration
k = Rate Constant
D = Diffusion Constant
V = Maximum Rate of Reaction

Rate Laws:
Change of concentration with respect to time / Diffusion of concentration with respect to space (radius)

Boundary Condition: Concentration of A at the cell's boundary (radius)

Assumptions: Analytical Solutions

1) Steady State Assumption: $\dfrac{\partial A}{\partial t} = 0$

2a) A >> k: $\dfrac{V_b A}{k_b + A} \rightarrow \dfrac{V_b A}{A} = V_b$

2b) A << k: $\dfrac{V_b A}{k_b + A} \rightarrow \dfrac{V_b A}{k_b}$

Simplifications: Analytical Solutions

A >> k (constant):

$$D\left(\frac{\partial^2 A}{\partial r^2} + \frac{2}{r} * \frac{\partial A}{\partial r}\right) = V_b$$

A << k (linear):

$$D\left(\frac{\partial^2 A}{\partial r^2} + \frac{2}{r} * \frac{\partial A}{\partial r}\right) = \frac{V_b}{k_b}A$$

General Analytical Solutions

A >> k (constant):

$$A(r) = C_1 + \frac{C_2}{r} + \frac{V_b r^2}{6D}$$

C_2 = 0

A << k (linear):

$$A(r) = \frac{C_1}{r} \cosh(\lambda r) + \frac{C_2}{r} \sinh(\lambda r)$$

C_1 = 0

$$\lambda = \sqrt{\frac{V_b}{D k_b}}$$

Exact Solutions

A >> k (constant):

$$A(r) = A_{ext} - \frac{V_b R}{6D}\left(1 + \frac{2D}{Rk^A}\right) + \frac{V_b r^2}{6D}$$
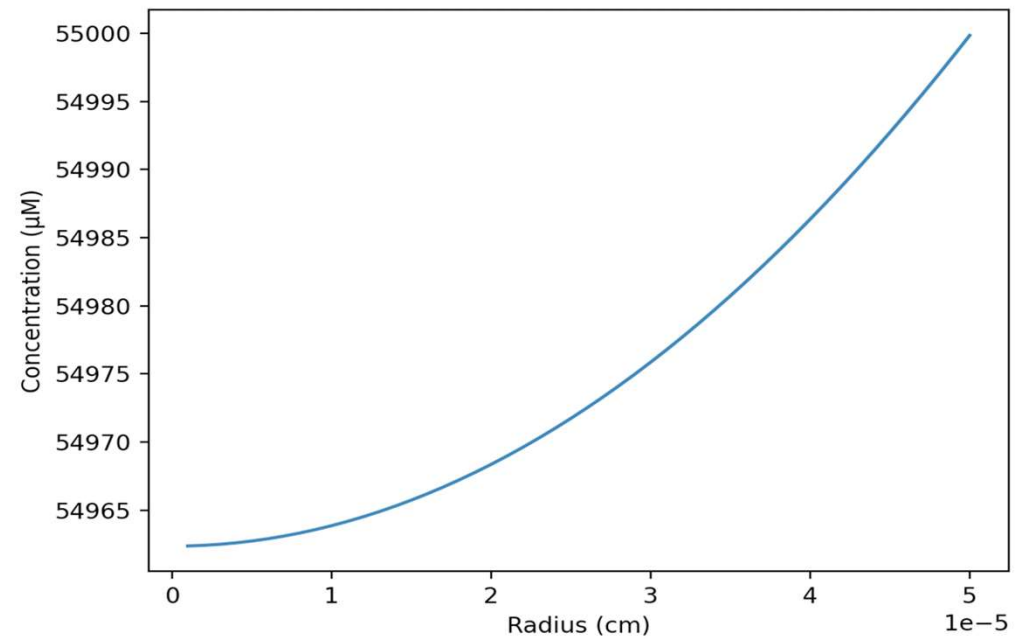
Exact Solutions

A << k (linear):

$$A(r) = \frac{R^2 k^A A_{ext} \sinh(\lambda r)/r}{RD\lambda \cosh(\lambda R) + (k^A R - D)\sinh(\lambda R)}$$
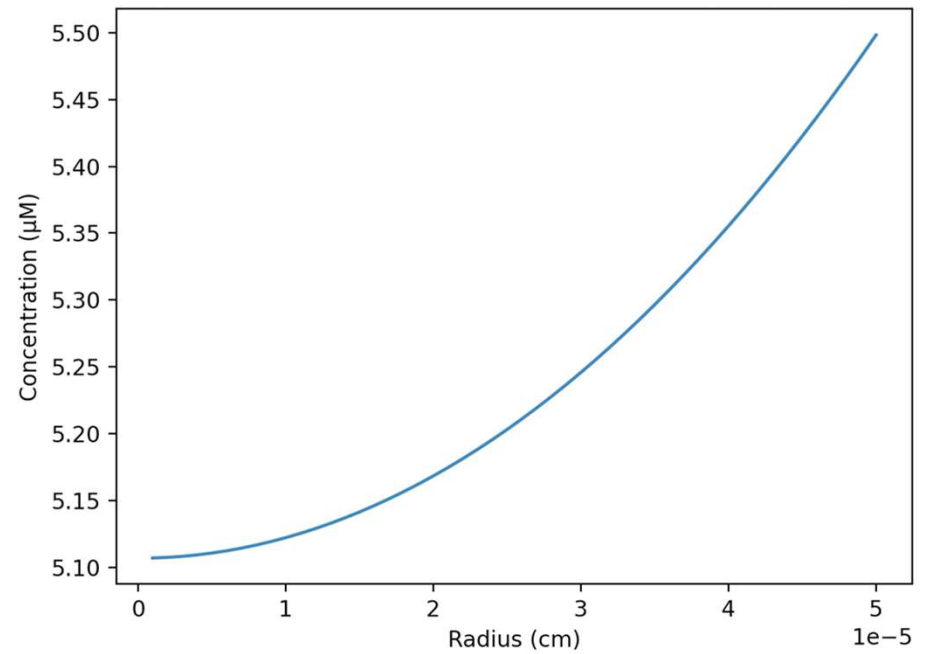
What About a Numeri-cal Method?

Makes it easier to solves these PDE's

No need for the assumptions

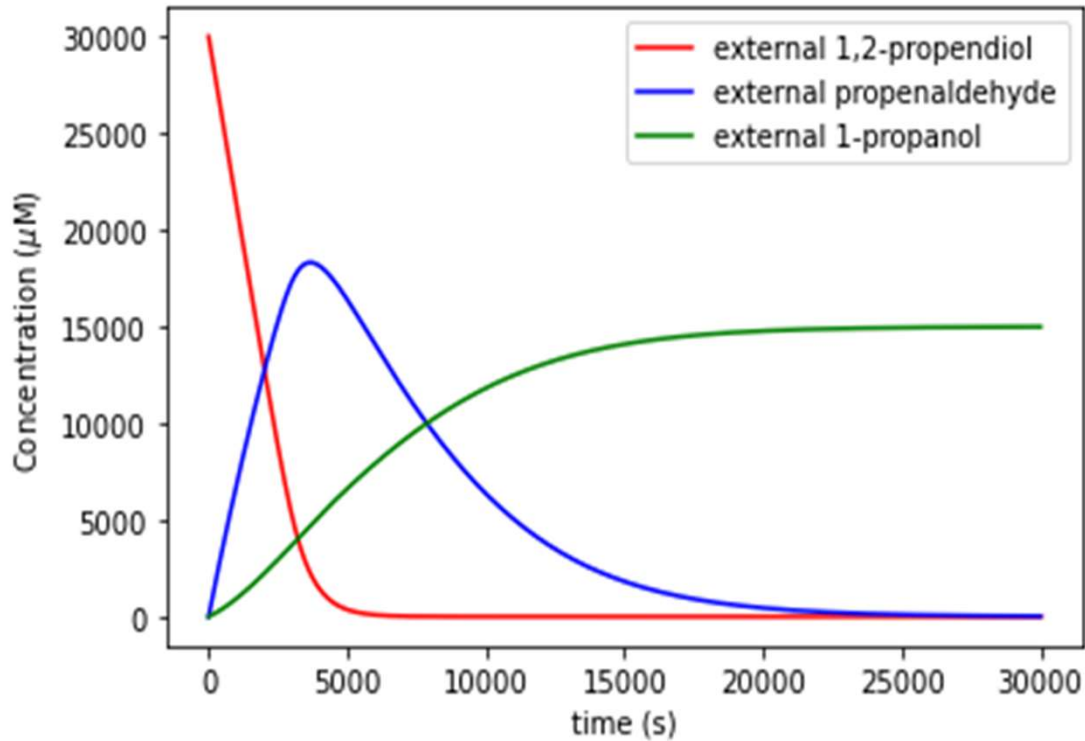- Extremely accurate guesses

Let's look at a specific set of reactions…

What The Original Code Looked Like

Models 2 Sets of Reactions:
- Varying external concentrations
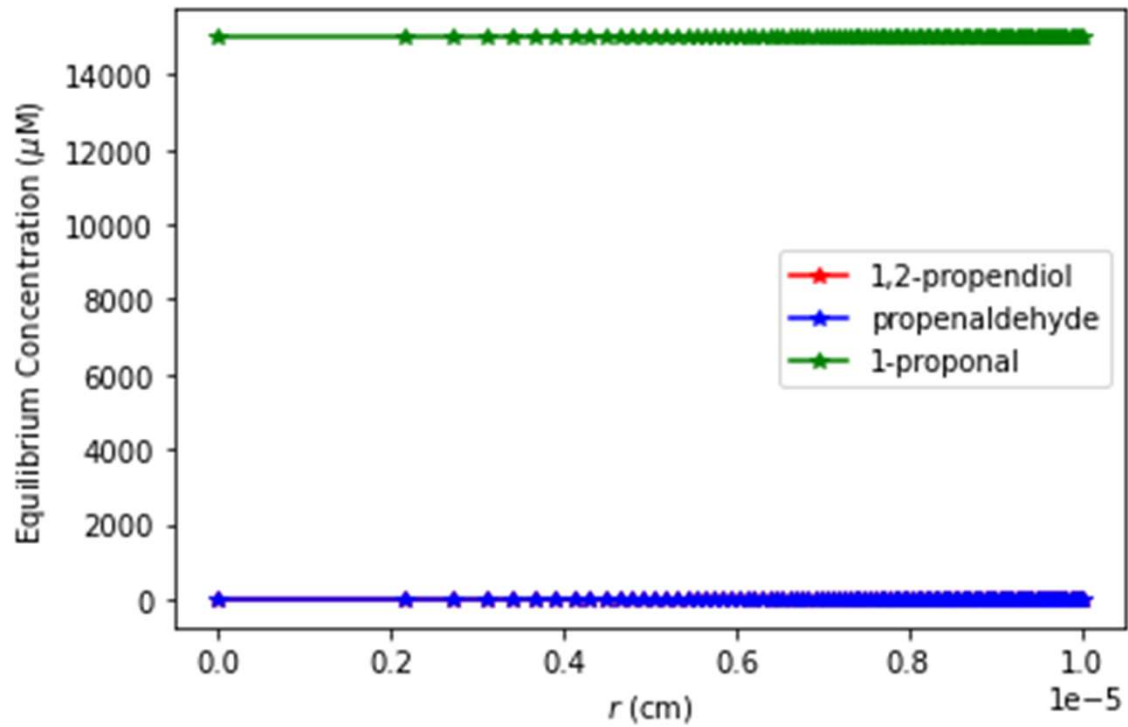
1,2-propendiol ⟶ propionaldehyde

propionaldehyde ⟶ 1-propanol

Concentration vs. Time Graph

Steady-State Graph

What If We Added Another Reaction to Our System?

Note: X is a stand-in for a competitive reaction

1,2-propendiol → propenaldehyde

1-propanol
↑
propenaldehyde —|
↓
X
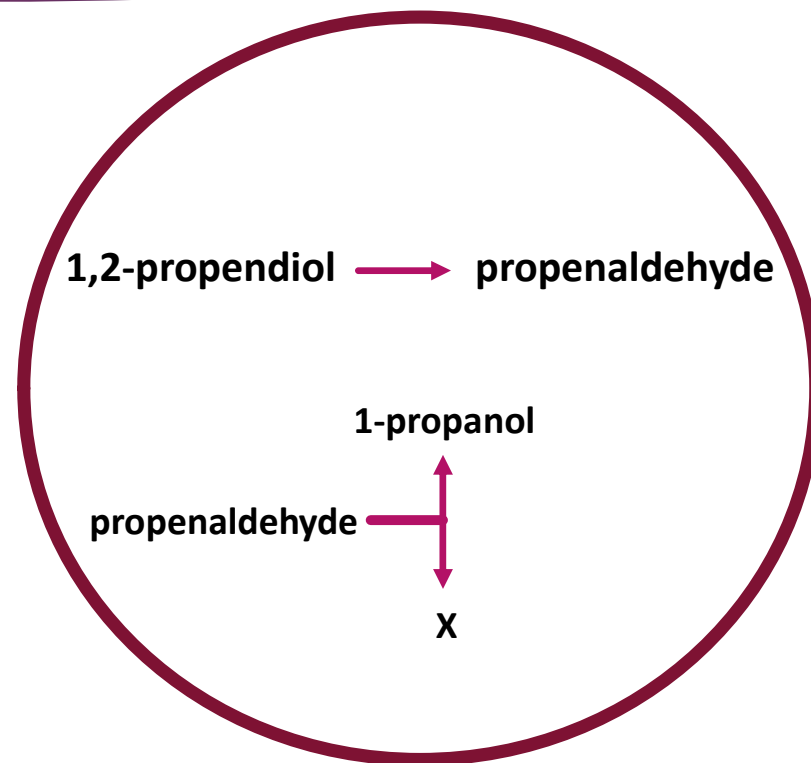
The Purpose of the Programs

Use complex numerical methods and systems as a means of solving the rate laws for every product/reactant modeled

- Typically, can't be solved analytically

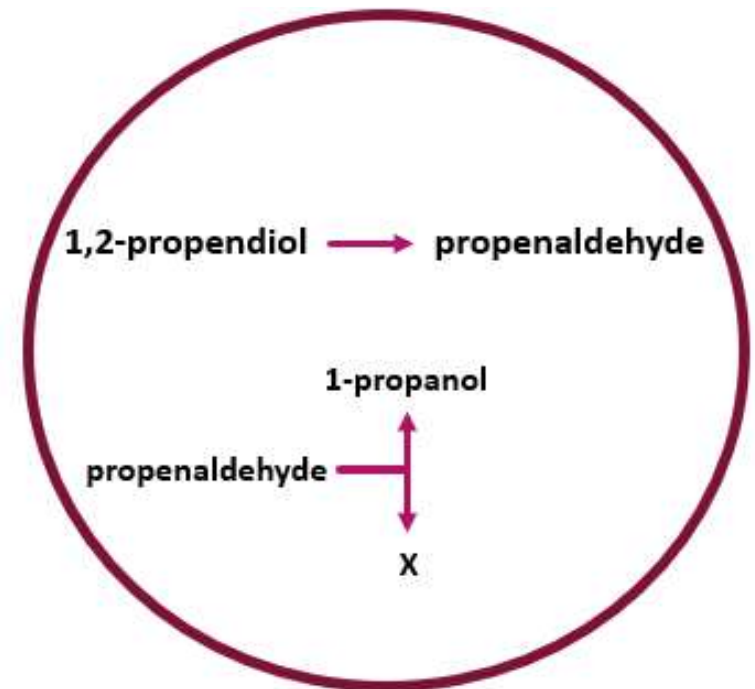**Turns the PDE's into systems of ODE's**

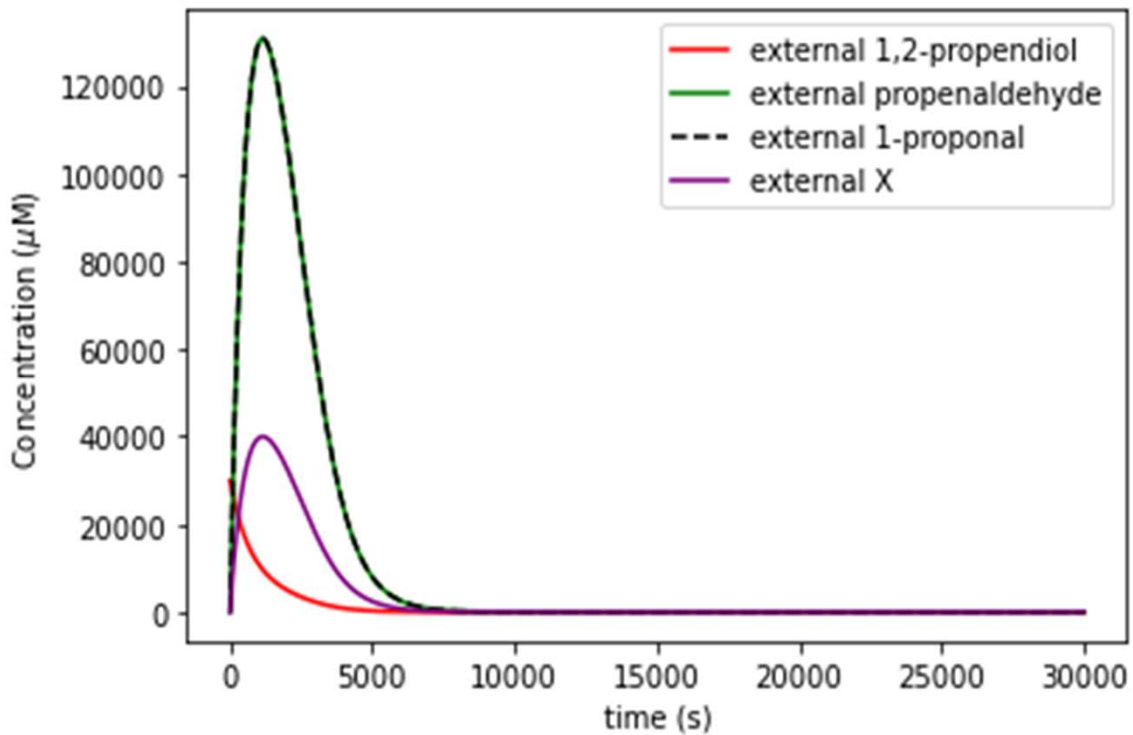- Segments of space

- "odeint"

Model Development

Nondimensionalization

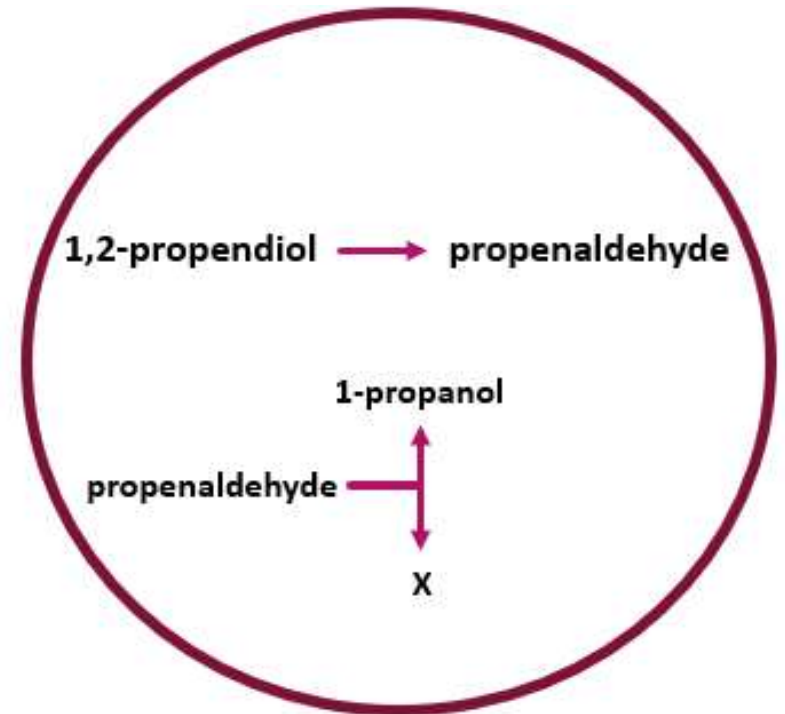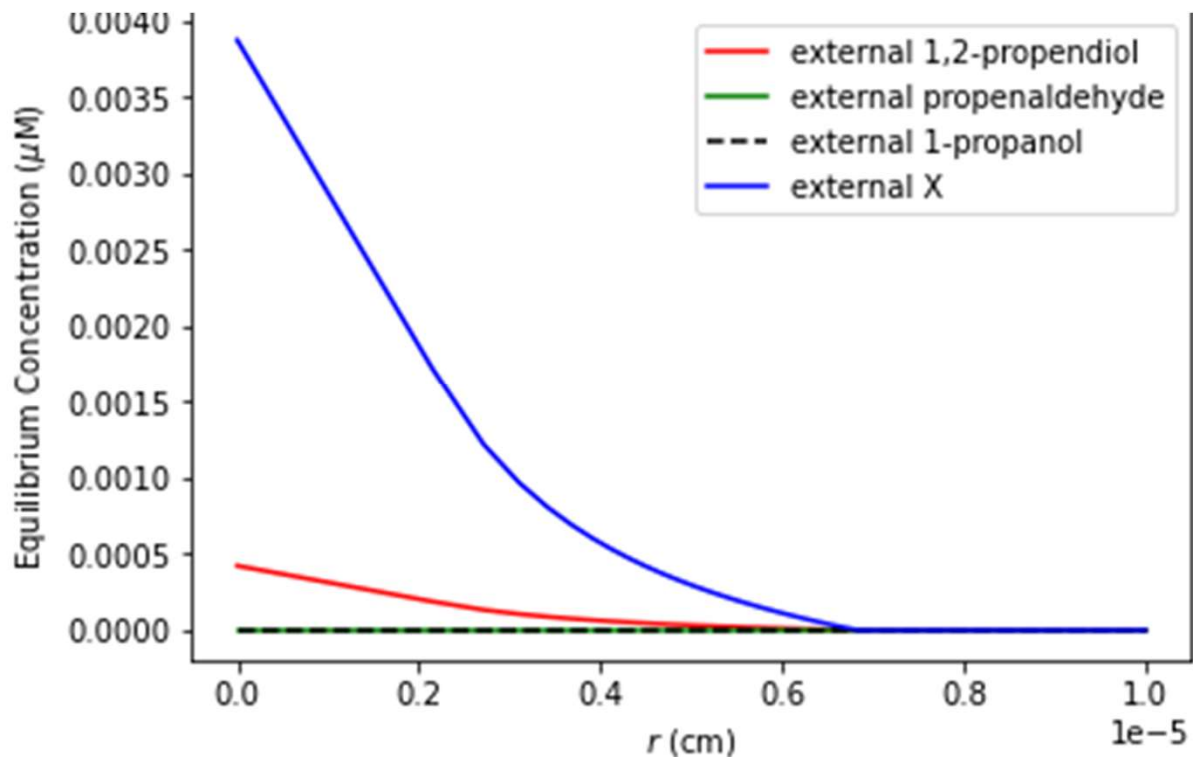Removes physical dimensions from equations then rescales to appropriate values/units

Functions

Created to model sets of reactions using rate laws

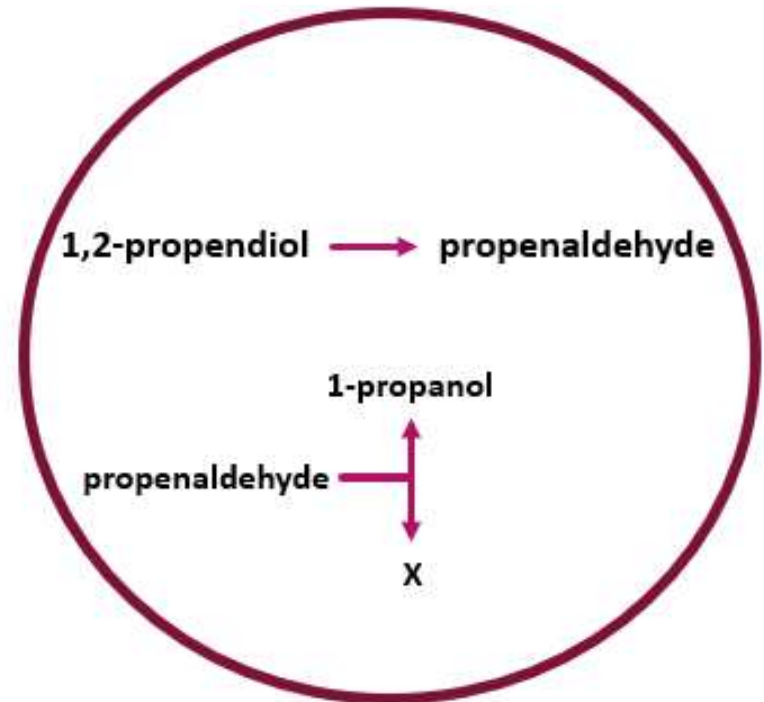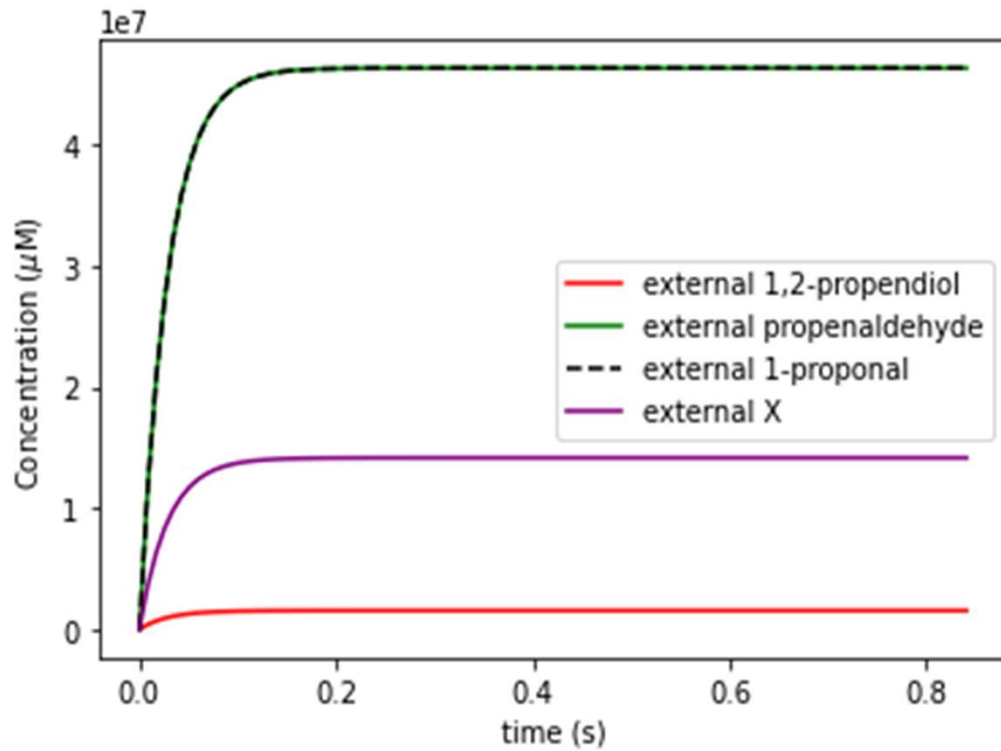# Results (Concentration vs. Time Graph)
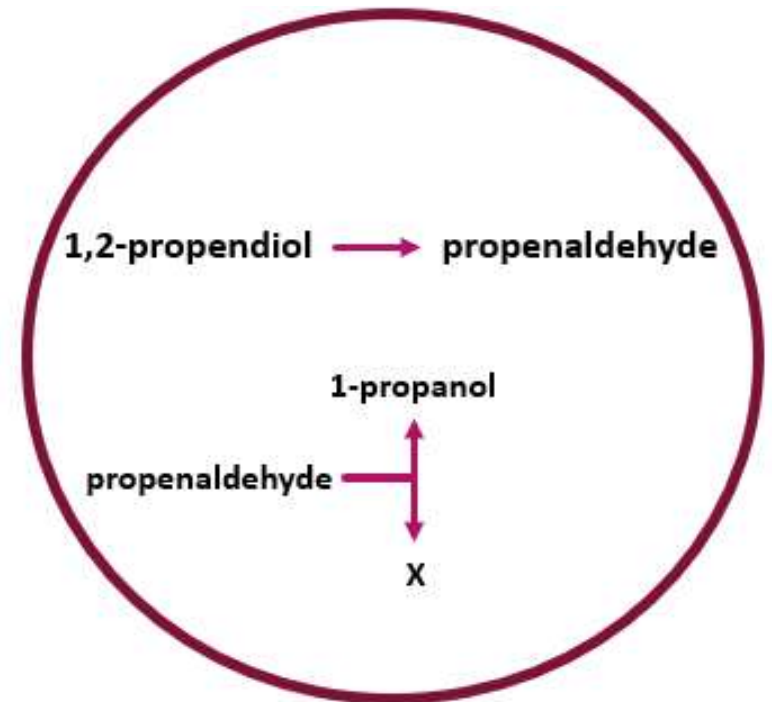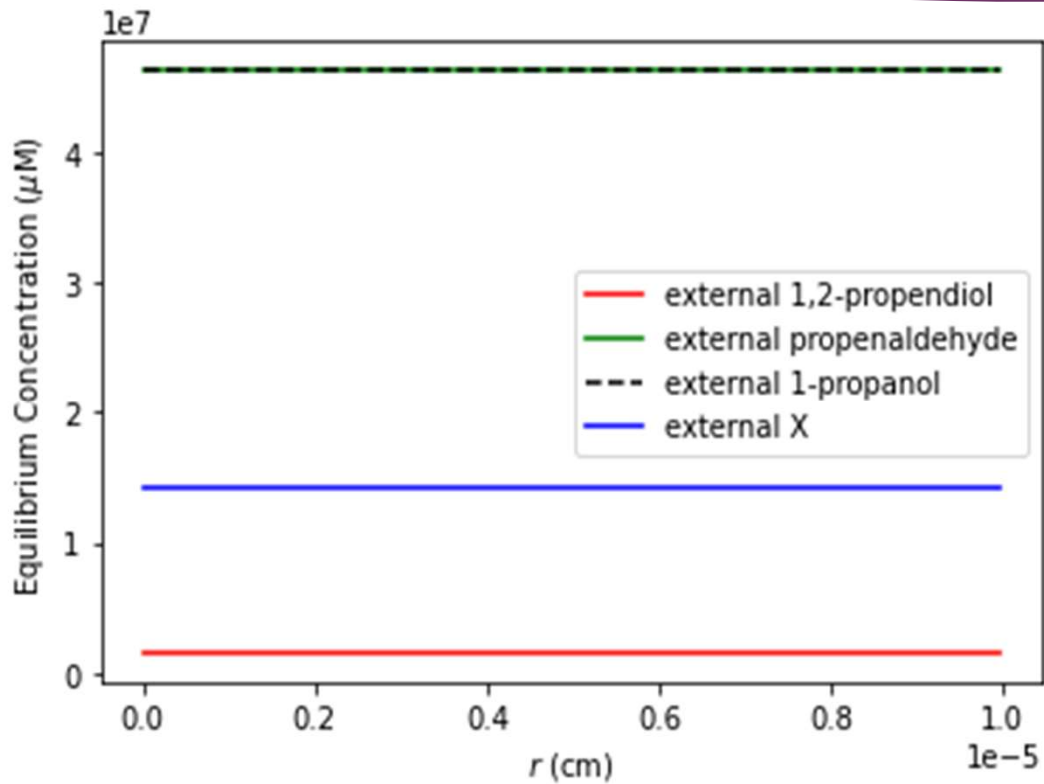
Steady-State Graph

Next Step…

We investigated how constant external concentrations of each molecule would affect the system

Concentration vs. Time Graph

Steady-State Graph

Questions?

Acknowledgements

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