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Editorial Remembering Frank Harary

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Editorial

Remembering Frank Harary

On March 11, 2005, at a special session of the 36th Southeastern International Conference on Combinatorics, Graph Theory, and Computing held at Florida Atlantic University in Boca Raton, the famous mathematician Ralph Stanton, known for his work in combinatorics and founder of the Institute of Combinatorics and Its Applications, stated that the three mathematicians who had the greatest impact on modern graph theory have now all passed away. Stanton was referring to Frenchman Claude Berge of France, Canadian William Tutte, originally from England, and a third mathematician, an American, who had died only 66 days earlier and to whom this special session was being dedicated. Indeed, that day, March 11, 2005, would have been his 84th birthday. This third mathematician was Frank Harary. Let's see what led Harary to be so recognized by Stanton.

Frank Harary was born in New York City on March 11, 1921. He was the oldest child of Jewish immigrants from Syria and Russia. He earned a B.A. degree from Brooklyn College in 1941, spent a graduate year at Princeton University from 1943 to 1944 in theoretical physics, earned an M.A. degree from Brooklyn College in 1945, spent a year at New York University from 1945 to 1946 in applied mathematics, and then moved to the University of California at Berkeley, where he wrote his Ph.D. Thesis on *The Structure of Boolean-like Rings*, in 1949, under Alfred Foster. Foster was, in turn, a Ph.D. student of the famous Princeton logician Alonzo Church.

After this, Harary became a research assistant in the Institute for Social Research at the University of Michigan in 1950, and then joined the Department of Mathematics of the University of Michigan, first as an instructor, then an assistant professor in 1953, an associate professor in 1959, and a professor in 1964. Harary remained with the Institute for Social Research until 1982, and with the Department of Mathematics until 1986.

In 1953, he co-authored the paper "On the number of Husimi trees" with the physicist George Uhlenbeck. (While a tree is a connected graph every block of which is K_2, a *Husimi tree* was initially defined as a connected graph every block of which is K_2 or a cycle. Such a graph is now called a *cactus*.)

As the title of the paper with Uhlenbeck indicates, Harary had become interested in enumeration problems in graph theory. It was the research of George Polyá that had a major impact on combinatorial enumeration. Using methods developed by Polyá, Harary was successful in counting graphs, connected graphs, rooted graphs, and digraphs. These results all appeared in a single research paper:

F. Harary, The number of linear, directed, rooted, and connected graphs, *Trans. Amer. Math. Soc.* **78** (1955) 445–463.

Harary continued to work on graphical enumeration with a student who would become his first of sixteen doctoral students, Robert Z. Norman. These sixteen doctoral students are listed below, and a 1972 photograph taken with six of them is shown in Figure 1.

Robert Norman (1954)
Geert Prins (1957)
Joseph Battle (1963)
Charles Trauth, Jr. (1963)
Lowell Beineke (1965)
Edgar Palmer (1965)
Stephen Hedetniemi (1966)
Michael Plummer (1966)
Bennet Manvel (1970)
Paul Stockmeyer (1971)
Allen Schwenk (1973)
Zevi Miller (1979)
Geoffrey Exoo (1980)
Jerald Kabell (1980)
Michael Plantholt (1982)
Niall Graham (1989)

Figure 1: Geert Prins, Lowell Beineke, Edgar Palmer, Frank Harary
Allen Schwenk, Michael Plummer, Stephen Hedetniemi.

After his early work on enumeration of graphs, Harary's interest turned totally to graph theory, which was the beginning of a worldwide reputation he was to obtain in this area. There were periods while he was a faculty member in the Department of Mathematics that he also held a research position with the Research Center for Group Dynamics, housed in the Institute for Social Research. There, he would often meet with his doctoral students and other visitors during 10-11 in the morning, discussing research, covering a wide range of topics in graph theory, asking interesting questions, and often introducing new concepts. The primary social scientist with whom Harary worked while at the Research Center for Group Dynamics was Dorwin Cartwright referred to as "Doc," an always exquisitely dressed scholar. Cartwright became the director of the Research Center for Group Dynamics and initiated a collaboration with Harary to evaluate the potential usefulness of what is called "hodological space" in research on social systems. Harary found that basic concepts in hodological spaces were equivalent to those in graph theory. This discovery put these spaces in a broader intellectual context and stimulated the use of these models in research on a variety of social phenomena. Harary and Cartwright then worked on applications of graph theory to social psychology. They, with Robert Norman, then co-authored the book:

> F. Harary, R. Z. Norman D. Cartwright, *Structural Models: An Introduction to the Theory of Directed Graphs*, Wiley, New York, 1965.

If you were to walk into his office on the third floor of Angell Hall, home of the Department of Mathematics at the University of Michigan, you would see papers scattered every which way throughout his office. Yet, the occupant of this office always seemed to be able to locate exactly what he was looking for. Among the many papers were lists – a list of the research topics he was working on, a list of journals in which he had published papers, a list of the cities in which he had lectured, a list of the mathematicians with whom he had co-authored a paper, and so on.

Harary's homepage at https://www.cs.nmsu.edu/fnh/curric.html lists 72 countries, 166 American cities, and 274 international cities in which he gave lectures. This includes lectures in all 50 U.S. states, all 10 Canadian provinces and all 6 Australian states, as well as all 8 universities in Denmark, all 8 in Israel, all 6 in New Zealand, all 4 in Norway and both in Papua, New Guinea.

Harary's research in graph theory goes far beyond his work in enumeration of graphs and applications to social psychology. For example, he had much interest in applications of graph theory to chemistry, namely the field of chemical graphs. There are various topological indices derived from graph theory that model the geometric structure of chemical compounds. One of these, named for Harary, is the Harary index $H(G)$ of a nontrivial connected graph G, defined as the

sum of the reciprocals of the distances between all pairs of distinct vertices of G, that is,

$$H(G) = \sum_{u,v \in V(G):\ u \neq v} \frac{1}{d(u,v)}.$$

The book *The Harary Index of a Graph*, written by K. Xu, K. C. Das, and N. Trinajstić, and published by Springer in 2015, deals exclusively with this index.

Among the many areas of graph theory in which Harary worked are two in which he authored or co-authored sequences of papers. One of these, titled Generalized Ramsey Theory for Graphs (often co-written with Vašek Chvátal) describes numerous ways in which the Ramsey number of a graph can be generalized. Another, Isomorphic Factorizations of Graphs (often co-written with Robert Robinson and Nicholas Wormald) describes numerous ways that certain graphs can be decomposed into particular spanning subgraphs, all of which are isomorphic.

While MathSciNet of the American Mathematical Society lists more than 650 papers authored by Frank Harary, even this large number does not give an indication of the breadth of his research, as more than 100 of his papers dealt with topics other than mathematics. Harary had more than 300 co-authors. He was in great demand as a speaker, lecturing in approximately 440 cities, over 60% outside the United States, and some 72 countries in all. Harary gave lectures in cities with names beginning with every letter of the alphabet. For example, he lectured in Xenia, Ohio, in the United States and in Xanten, Germany, where he lectured in the excavated Roman amphitheatre.

Harary had a great sense of humor and interest in the arts. Shown in Figure 2 is a self-portrait. He appreciated the regular geometric designs in the work of the famous artist M. C. Escher and had a personal collection of his art.

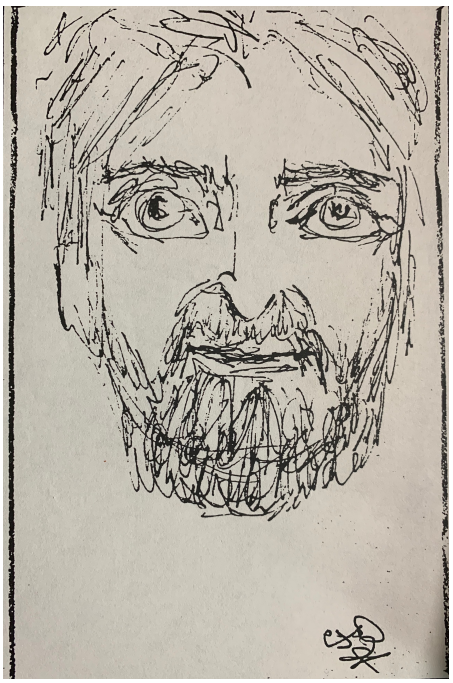

Figure 2: A self-portrait of Frank Harary.

He had great enthusiasm for graph theory. He lectured on graph theory around the world, often introducing this area of mathematics to people. His lectures were not only amusing but he was known for his writing style. He took great care with the terminology and notation he used. Indeed, he influenced much of the terminology and notation commonly used in graph theory today. It would not be unusual for Harary to meet a fellow graph theorist and say, "Aren't we lucky to be working in graph theory?" While he had great fondness for graph theory, he felt that only parts of it were attractive, the part he referred to as BGT (Beautiful Graph Theory). He would say that there is only so much time to work on research in graph theory and it's important to spend this time on BGT. At times, Harary was criticized for working on research that wasn't deep enough – when, in fact, he was able to describe much of this so clearly as to make it appear easier than it actually was. He was often 'comically' annoyed by research papers that were not clearly written. When he asked the authors what they were referring to in their paper, and the authors explained this, Harary would reply, "So that's what you meant. Then why didn't you say so?". He would, once again, smile. Harary stated that some research papers appeared deeper than they actually were, often because details of the arguments were omitted, illustrations were not provided, unnecessarily complicated notation was used, and non-suggestive terminology was employed. This made these papers less interesting and more difficult to read. While Harary was considered opinionated by some, his opinions were based upon sound reasoning.

In addition to his own PhD students, Harary unselfishly took several other graph theorists under his wing and mentored them. To those fortunate enough to be able to be "adopted" by him, he stressed respect for beautiful graph theory and emphasized how to write clearly and revise papers several times before submitting them for publication. He was very encouraging to these mathematicians, even when they presented him with something to read that wasn't written that

Figure 3: Frank Harary, New Mexico State University 1994.

well. He would always say the same thing, "Not to worry." He would then proceed to demonstrate how to write it clearly. He also shared with them many new ideas for research.

However, many mathematicians with an interest in graph theory never knew Frank Harary. They only encountered his name in a variety of ways. While research mathematicians may have seen books or papers authored by Harary or seen papers referencing papers by Harary, what are other reasons that graph theorists should be aware of contributions made by him?

It is well recognized that the first book on the theory of graphs was written by the Hungarian mathematician Dénes König in 1936. Notable were his students Paul Turán, Paul Erdős, and Tibor Gallai, his only doctoral student.

D. König, *Theorie der endlichen und unendlichen Graphen*, (German) Leipzig, 1936.

The second book on the theory of graphs was written by the French mathematician Claude Berge in 1958.

C. Berge, *Théorie des graphes et ses applications*, (French) Collection Universitaire de Mathématiques, II. Dunod, Paris, 1958.

While König's book was written in German and Berge's in French, the first book on the theory of graphs written in English, was by Yale University's mathematician Oystein Ore and was published in 1962.

O. Ore, *Theory of Graphs*, Amer. Math. Soc. Colloq. Publ. Vol. XXXVIII Amer. Math. Soc., Providence, 1962.

Seven years later, in 1969, Frank Harary published his graph theory book.

F. Harary, *Graph Theory*, Addison-Wesley, Reading, 1969.

The following excerpt is from a review of *Graph Theory*, written by Robin J. Wilson, which appeared in the *American Mathematical Monthly* in 1972:

> *Here, at last, is the sort of book that graph theorists have been waiting for! For far too long, various mathematicians, too snobbish or ignorant to be able to peer beyond the realms of their own little 'pet' problems, have been content to dismiss the subject by describing it as "little more than recreational mathematics", or even "all right for economists and biologists, but not really 'proper' mathematics". In spite of the pioneering books of König (1936), Berge (1958) and Ore (1962), such comments are still occasionally heard; however, it is now generally realized that the tide has turned and graph theory is becoming universally recognized as 'respectable'.*

In 1969 another book was written in graph theory, this one in Russian, by Alexandr Zykov.

A. A. Zykov, *Theory of Finite Graphs*, Vol. I, (Russian) Izdat. "Nauka" Sibirsk. Otdel., Novosibirsk, 1969. *Fundamentals of Graph Theory*, Translated from the Russian and edited by L. Boron, C. Christenson and B. Smith, BCS Associates, Moscow, 1990.

Zykov once gave a gift of three photographs to Harary, who proudly displayed these on a wall in his house. These were photographs of three lions. One photograph showed a lion sleeping, the second a lion awakening, and the third a lion roaring. Under these were written: *Graph Theory yesterday, Graph Theory today, Graph Theory tomorrow*.

After the book *Structural Models* with Norman and Cartwright and his book *Graph Theory*, Harary went on to co-author the following books.

F. Harary, E. Palmer, *Graphical Enumeration*, Academic Press, New York, 1973.

P. Hage, F. Harary, *Structural Models in Anthropology*, Cambridge Univ. Press, Cambridge, 1983.

F. Buckley, F. Harary, *Distance in Graphs*, Addison-Wesley, Reading, 1990.

P. Hage, F. Harary, *Exchange in Oceania*, Clarendon Press, Oxford, 1991.

P. Hage, F. Harary, *Island Networks*, Cambridge Univ. Press, Cambridge, 1996.

S. L. Arlinghaus, W. C. Arlinghaus, F. Harary, *Graph Theory and Geography: An Interactive View*, John Wiley & Sons, New York, 2002.

Figure 4: Frank Harary.

In addition, Harary contributed tirelessly to the ever-growing graph theory literature by either editing or co-editing the following books.

A Seminar on Graph Theory, Holt, Reinhart and Winston, New York, 1967.

Graph Theory and Theoretical Physics, Academic Press, London, 1967.

Proof Techniques in Graph Theory, Academic Press, New York, 1969.

(Edited with H.S. Wilf) *Mathematical Aspects of Electrical Network Analysis*, SIAM-AMS Proceedings 3, 1971.

New Directions in the Theory of Graphs, Academic Press, New York, 1973.

(Edited with R. Bari) *Graphs and Combinatorics*, Lecture Notes in Mathematics, Vol. 406, Springer, Berlin, 1974.

(Edited with J. Maybee) *Graphs and Applications*, Wiley, New York, 1985.

(Edited with K. Phillips) *Graph Theoretic Models in Computer Science I, Comput. Math. Appl.*, Vol. 15, 1988.

(Edited with Z. Tuza) *Graph Theoretic Models in Computer Science II, Math. Comput. Modelling*, Vol. 17, 1993.

While the number of papers dealing with graph theory has grown significantly for more than half a century, the number of journals publishing papers primarily in graph theory has grown steadily. In 1966 Frank Harary, along with Gian-Carlo Rota, founded the *Journal of Combinatorial Theory*, publishing papers in all areas of combinatorics. With the growth of combinatorics, especially graph theory, the *Journal of Combinatorial Theory* was divided into two journals, Series A and Series B, in 1971 with the latter emphasizing graph theory. Three years later, in 1974, Harary obtained the approval of Wiley Publishers to introduce a new journal that published papers exclusively in graph theory. He named this the *Journal of Graph Theory*, and became its first Editor-in-Chief. For more than two years, work was done developing the design of the journal, establishing an Editorial Board, and, most challenging, introducing researchers to the journal and encouraging them to submit their research papers to the journal. With his great fondness for graph theory and interest in those whose work had led to the development of the theory, Harary wanted to emphasize papers that dealt with interesting topics in graph theory, surveys of special topics in graph theory, articles dealing with graph theory pioneers, and papers describing how certain topics and problems came about. He especially wanted the first issue of the journal to be published in 1976 (the

year that the Four Color Problem was solved), but Wiley said that this wasn't possible, and the first issue was published in 1977. In 1978 the Association of American Publishers recognized the *Journal of Graph Theory* as the Best New Journal in 1977 in Science, Technology and Medicine. Later, Harary stepped down as the Editor-in-Chief, after which the emphasis of the journal changed.

Because of this, Harary thought of developing a new journal that would return to the original emphasis of the *Journal of Graph Theory*. He planned on calling this new journal the *Graph Theory Monthly*, and later, the *Graph Theory Quarterly* when it became clear that publishing such a journal on a monthly basis was not practical. This dream of his never materialized.

Figure 5: Frank Harary.

In addition to the emergence of books and journals emphasizing graph theory in the 1960s and 1970s, there has been significant growth in the number of conferences and meetings emphasizing graph theory, the first of which were held in the late 1950s. An early conference was the *Symposium of the Theory of Graphs and Its Applications*, which took place in Smolenice, Czechoslovakia during June 17–20, 1963. The participants of this conference consisted of many of the best known graph theorists of the time, including such notable mathematicians as Gabriel Dirac, Paul Erdős, Tibor Gallai, Crispin Nash-Williams, Gerhard Ringel, Alexander Zykov, Anton Kotzig, Alexander Rosa, Jiri Sedláček – and Frank Harary. Other conferences on the theory of graphs followed soon afterward, including one in Rome during July 5–9, 1966 and another in the United States in Kalamazoo Michigan in 1968. Graph theory conferences became commonplace in the 1970s and thereafter. Regional graph theory conferences began as well. Among the most popular of these were the MIGHTY conferences (http://www.cs.bsu.edu/homepages/mighty/), which were started by Harary at the University of Michigan. Because of the strong interest in graph theory at several universities in the State of Michigan, these were called the Michigan Graph Theory Conferences, that is, MIchigan GrapH TheorY, or MIGHTY meetings. When they expanded to Indiana, they became Michigan–Indiana GrapH TheorY Conferences, and still later the MIdwest GrapH TheorY Conferences. There have been scores of these conferences since 1978. In the first such conference, the participants were asked to describe what they were working on. Harary followed each speaker's presentation with a suggestion on a related new problem to consider.

Harary received many honors for his contributions, including honorary degrees from Brooklyn College, the University of Aberdeen, Scotland, the University of Lund, Sweden, the University of Exeter, England, the University of Macedonia, Thessaloniki, and the University of Louisville.

Frank Harary passed away in Las Cruces, New Mexico on January 4, 2005 from a postoperative infection. His passing was recognized by the *New York Times* on January 25, 2005:

> **Frank Harary, 83, a Top Scholar on a Mathematical Specialty, Dies**
>
> *Dr. Frank Harary, a mathematician who wrote and lectured extensively on graph theory, a mathematical specialty often applied in computer science and other fields, died on Jan. 4 in Las Cruces, N.M. He was 83. Dr. Harary's 1969 book "Graph Theory" has been credited with giving the field a broader relevance. The theory, which dates from the 18th century or earlier, is concerned with the edges and vertices found in graphs. It is frequently used to model physical or abstract problems in chemistry, computer networks, transportation lines and even sociology, as a way to express mathematically the relationships among individuals. Solutions to problems can appear as theorems or algorithms. Dr. Stephen T. Hedetniemi, a professor of computer science at Clemson University and former student of Dr. Harary's, said the elegance of the writing in "Graph Theory" had been crucial to the specialty's acceptance. "Harary made a beautiful presentation of the theory that hasn't been equaled since," Dr. Hedetniemi said. "The book then caught on as a text."*

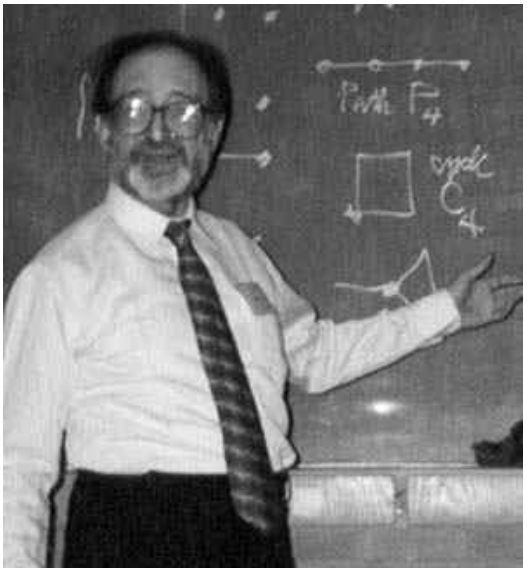

Figure 6: Frank Harary.

At the time of Harary's death, Desh Ranjan, Chair of the Department of Computer Science at New Mexico State University said:

> *Dr. Harary was a true scholar with a genuine love for graph theory which was an endless source of new discoveries, beauty, curiosity, surprises and joy for him till the very end of his life.*

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