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# Whistler Waves: Modeling and Observations 

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# WHISTLER WAVES: MODELING AND OBSERVATIONS 

## By

Daniel Williams

Submitted to the Department of Physical Sciences and the Committee on Graduate Studies In Partial Fulfillment of the Requirements for the Degree of Master of Science in Engineering Physics

Embry-Riddle Aeronautical University
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## WHISTLER WAVES: MODELING AND OBSERVATIONS



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#### Abstract

The thesis presents the results of all the research from the published and in publication process research in the Journal of Geophysical Research [1]. This research focuses on whistler wave ducting events in the equatorial magnetosphere. High-density ducts are the main focus of whistler study in both studies as they are commonly observed by the Van Allen Probe satellites. A three-step procedure based on the analysis of the whistler wave dispersion relation and numerical simulations of the electron magnetohydrodynamics model. We use this model to identify the parallel and perpendicular wave numbers of the "most trapped" wave in an attempt to understand the filtering aspects of HDD. Statistical analysis was done in a separate study to determine common parameters and characteristics of narrow and wide ducting events. A total of 213 events were cataloged where 164 were narrow events. The narrow HDD were found to most commonly have a duct size between $0-50 \mathrm{~km}$ with a density increase of $161 \%$ in the background magnetic field range of $100-300 \mathrm{nT}$. Wide events are events were the background magnetic field and the density gradient vary in the same direction, either both increases or decreases. The results from the studies show a robust three-step procedure that was applied to an observed event as well as the common characteristics and catalog of events in which this procedure is applicable.


## CHAPTER 1 INTRODUCTION AND BACKGROUND

Very-low-frequency (VLF) whistler-mode waves (whistlers) are a driving force in the distribution of the electromagnetic power and wave-particle interactions in the Earth's magnetosphere. They are particularly important for many processes occurring in the Earth's radiation belts. Whistlers interact with energetic electrons and can cause energization, transport, and precipitation of these particles via cyclotron resonance [2,3]. Whistlers are frequently observed by satellites in the equatorial magnetosphere, predominately near the plasmapause $[4,5,6,7]$. The plasmapause is a transitional region binding the cold plasma torus that forms the Earth's plasmasphere. The plasmapause shares a boundry with the electron radiation belt which leads to explain why whistlers are so commonly observed in this region of the magnetosphere. Energetic electrons in the radiation belt are an energy source for generating whistlers $[3,8,9,10]$. These whistler then can become trapped in the plasmapause as the plasmapause contains many small-scale, localized density gradients which can trap and guide the VLF waves. This leads to one of the main questions about whistlers, how they can propagate over significant distance through inhomogeneous, magnetized plasma, with little attenuation.

This question has been studied extensively since the pioneering paper by Storey [11]. Many experimental studies devoted to this problem were conducted at the Siple station in Antarctica (e.g., [12]) and the High-frequency Active Auroral Research Program (HAARP) facility in Gakona, Alaska [13]. Propagation of the whistler-mode waves through plasma with varying parameters has also been studied in laboratory plasma devices $[14,15,16,17$, 18].

These theoretical and experimental studies found that field aligned density inhomogeneities are one of the main mechanisms that allow for whistlers to propagate significant distances around the magnetosphere $[12,19,20,21,22]$. These inhomogeneities can be formed by the field density enhancements, known as high-density ducts (HDD), and density depletions, known as low-density ducts (LDD) [23, 24, 25, 26]. There are currently two forms of ducts, these are known as narrow or wide ducts. Narrow ducts are events where the transverse size of the inhomogeneity is small enough that the background magnetic field does not change significantly over the distance of the duct. These ducts are formed due to the behavior of the density only. Wide ducts differ from narrow ducts as the transverse size of the duct is large enough for a significant change to occur in the background magnetic field. This signifies that the gradient of the magnetic field and the gradient of the density must be taken into account when determining the parameters of the duct [27, 28, 29, 30]. The important difference between LDD and HDD are that, LDD can guide waves without any leakage, while HDD always have some leakage of the wave's energy. This leakage depends on the parameters of the wave and the duct. A duct that can guide one whistler a significant distance with almost no leakage, will not trap a different whistler at all leaking away all the energy[31, 32, 1]. This suggest that HDD functions like a filter for whistler waves.

The first study in which I will be discussing was based on the idea that HDD behave like a filter. By observing ducting events in the magnetosphere and retrieving the parameters of the ducts, the optimal ducting parameters of the whistler based on the dispersion relation derived from electron magnetohydrodynamics (EMHD) were calculated. The range of these parameters were then simulated to determine what value in the range of the possible ducting parameters were the most trapped in the duct and therefore leaked the least amount of energy out of the duct [1]. These events were found and the parameters of the ducts recorded using the NASA Van Allen Probes data. The parameters required were the wave frequency, magnetic field, and plasma density. In the previously published paper, a robust 3-step procedure to identify the exact values of the wave perpendicular and parallel wavelengths. This procedure was demonstrated with application to some representative observational event to show that the 3-step procedure can be used for any event if it is within the mathematical limitations of the model.

The second study I will be discussing is a statistical analysis of both narrow and wide HDD events that is in the submission process to be published. The study focuses on observations by the NASA Van Allen Probe's satellites in the equatorial magnetosphere. This is due to a great number of ducting events related to narrow and wide ducts occurring in the equatorial magnetosphere $[28,33,34,26,7,30]$. Observations show that these ducting events have many similar features. This paper contains the results from the statistical study of VLF waves in the high-density ducts. Our goals were to identify primary locations where these events are observed, and quantify parameters of the high density ducts and the waves trapped in these ducts.

## CHAPTER 2 OBSERVATIONS

Observations analyzed in this study consist of the wave frequency, electron density, and background magnetic field measured by the Van Allen Probe A and B (also known as RBSP-A and RBSP-B) satellites in the equatorial magnetosphere. The data is obtained by the Electric Field and Wave (EFW) instrument [35], and the Electric and Magnetic Field Instrument Suite and Integrated Science instrument (EMFISIS) [36]. The electric field is measured in the spinning satellite coordinate system ( $u, v, w$ ), where the $w$ axis is coaligned with the spin axis of the satellite that points towards the Sun. While whistlers can be seen in the $u, v, w$ components, the events considered will only be observed in the $w$ component.

### 2.1 Example of Narrow-Duct Event

Provided below is the example event that was used for simulations [1]. Figure 2.1A shows the PSD of the $E_{w}$ component of the electric field with the values displayed by the color bar. The PSD was retrieved from the EMFISIS instrument in the frequency range 100-1000 Hz . Only the $w$-component is being examined as the equatorial plane it corresponds to is the perpendicular component of the electric field. The density of the event is ploted as the white line over the PSD in Figure 2.1A. Below this is the background magnetic field of the event displayed in Figure 2.1B to demonstrate that the background magnetic field changes insignificantly compared to the change in density. The background magnetic field was observed to be 170.5 nT at the time of 11:17:01 UT which corresponds to the peak PSD. Over the time interval of the entire event, the magnetic field changes from 173 nT to 164 nT or by only $\approx 5.5 \%$. This change is tiny compared to the variation in density of $\approx 70$ $\mathrm{cm}^{-3}$ to more than $150 \mathrm{~cm}^{-3}$ or by more than $100 \%$. Figure 2.1 C is a slice of the PSD at the peak value of PSD in the duct that was simulated. The four frequencies in which the procedure was tested are marked with the dotted lines and were chosen as they represent peaks in the PSD region. The exact frequencies used were 215.0, 285.0, 337.5, and 425.0 Hz where 285.0 Hz was the primary frequency focused on. The exact location of RBSP-B is shown in Figures 2.1D, 2.1E, and 2.1F. The exact position of RBSP-B during the event was on the magnetic shell line $L=5.592$, at MLT $=8.294$, and $\mathrm{MLat}=1.238^{\circ}$. The orbit of RBSP-B is displayed with the red line and the location of the satellite is displayed as the red dot on the line. These panels were created using the orbital plot tool from the Van Allen Probes Science Gateway website at https://rbspgway.jhuapl.edu.

Figure 2.2 is an event that was not run through the simulation as it was used for the statistical study that is in the submission process. Figure 2.2 is an example of a typical narrow HDD that was ducts observed by the RBSP-A satellite in the equatorial magnetosphere on 08/26/2015 from 11:15:30 to 11:20:30 UT. Figure 2.2A shows the PSD of the $E_{w}$ component of the electric field with the values displayed by the color bar. The PSD was retrieved from the EMFISIS instrument in the frequency range $200-1000 \mathrm{~Hz}$. The white
line displays the calculated ratio of $n / n_{1}$ of the observed density with the important value of $n / n_{1}=1$ marked as the magenta dotted line. This ratio is calculated for $\omega=2.07 \times 10^{3}$ $\mathrm{rad} / \mathrm{s}(f=330 \mathrm{~Hz})$ or the frequency that corresponds to the peak value of PSD. The importance of this is discussed in detail in the theory section. Below in Figure 2.2B, the background magnetic field and the density are displayed of the observed event. Similiarly to Figure 2.1, the background magneic field variation and the variation in density must be compared to show that this event is in fact a narrow HDD event. It was found during the event that the magnetic field changes from 202 to 214 nT or by $6 \%$ while the variation in density was from $45 \mathrm{~cm}^{-3}$ to $81 \mathrm{~cm}^{-3}$ or by $80 \%$. This event was observed 3400 km interior to the plasmapause and was found to be $\approx 400 \mathrm{~km}$ in size.


Figure 2.1: Displayed is the example event that was modeled in the simulation.A) The Power spectral density (PSD) of $E_{w}$ component of the electric field is shown above the graph with a color pallet. The electron density is displayed with a white line and was measured by the RBSP-B satellite from 10:37 to 11:47 UT on 02/27/2016. B) Displayed is the magnitude of the background magnetic field that was observed by the satellite. C) The PSD of $E_{w}$ versus frequency at the time of peak PSD of 11:17:01 UT during the time interval from 11:09:00 to 11:22:00 UT. D), E), and F) displays the orbit of the satellite in GSE coordinates in the time interval of 06:47:00 to 15:47:00 UT on 02/27/2016.


Figure 2.2: Displayed is an example of a narrow high-density duct observed by the RBSPA satellite from 11:15:30 to 11:20:30 UT on 10/26/2015. A) The Power spectral density (PSD) of $E_{w}$ component of the electric field is shown above the graph with a color pallet. The electron density is displayed with a white line and was measured by the RBSP-B satellite. B) The magnitude of the background magnetic field measured by the satellite is plotted in the left axis while the observed electron density of the event is displayed on the right hand axis. The peak frequency and the range of parallel wavelengths used for the ratio calculation is shown in the top right.

### 2.2 Example of Wide-Duct Event

Figure 2.3 displays an example of a wide HDD event that occurred on 08/21/2015 from 08:23:00 UT to 09:54:00 UT. Figure 2.3A shows the PSD of the $E_{w}$ component of the electric field with the values displayed by the color bar. The PSD was retrieved from the EMFISIS instrument in the frequency range $200-1000 \mathrm{~Hz}$. The white line displays the calculated ratio of $n / n_{1}$ of the observed density with the important value of $n / n_{1}=1$ marked as the magenta dotted line. Unlike for narrow events, this ratio must also take into account the variation in the background magnetic field as it changes a significant amount during the time interval of the event. To prove this, Figure 2.3B shows the background magnetic field, displayed as the dotted blue line, changes from 500 nT to 4500 nT or $800 \%$ and the plasma density, displayed as the red line, changes from 300 to $2800 \mathrm{~cm}^{-3}$ or $833 \%$. The ratio $n / n_{1}$ was calculated for $\omega=2.51 \times 10^{3} \mathrm{rad} / \mathrm{s}(f=400 \mathrm{~Hz})$ which corresponds to the max observed value of the PSD in this region. The parallel wavelength of the trapped waves are assumed to change linearly from 10.5 km to 16 km during the event. Details of the analysis of VLF waves inside wide ducts are given in [30].


Figure 2.3: Displayed is an example of a narrow high-density duct observed by the RBSPA satellite from 8:23 to 9:54 UT on 8/21/2015. A) Power spectral density (PSD) of $E_{w}$ component of the electric field displayed above the graph with a color pallet. The electron density ratio of $n / n_{1}$ is shown with a white line and was calculated based on the observed values by the RBSP-A satellite B ) The magnitude of the background magnetic field measured by the satellite is plotted in the left axis while the observed electron density of the event is displayed on the right hand axis. The peak frequency and the range of parallel wavelengths used for the ratio calculation is shown in the top right.

## CHAPTER 3

## THEORY

Mathematical model for VLF whistler-mode waves in the magnetosphere consists of the momentum equation for electrons and Maxwell's equation for the electric and magnetic fields [12, 37].

$$
\begin{align*}
\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v} \cdot \nabla \mathbf{v} & =-\frac{e}{m_{e}}(\mathbf{E}+\mathbf{v} \times \mathbf{B})  \tag{3.1}\\
\nabla \times \mathbf{B} & =\mu_{0} \mathbf{j}  \tag{3.2}\\
\nabla \times \mathbf{E} & =-\frac{\partial \mathbf{B}}{\partial t}  \tag{3.3}\\
\nabla \cdot \mathbf{B} & =0 \tag{3.4}
\end{align*}
$$

This model assumes that the plasma is cold $\left(T_{e, i}=0\right)$, ions are stationary, and the current is carried by the electrons only $\mathbf{j}=-e n \mathbf{v}$; where $\mathbf{v}$ is the electron velocity. The plasma is considered to be quasi-neutral, therefore the plasma density is constant in time and the model does not include the electron density continuity equation. The model is further simplified by the 'quasi-longitudinal" approximation of the Electron Magnetohydrodynamics (EMHD). The 'quasi-longitudinal" approximation simplifies the equations by excluding the displacement current in Ampere's Law (3.2). The 'quasi-longitudinal" approximation is valid for waves that satisfy the condition $\omega<\omega_{c e} \ll \omega_{p e}$ [19] where $\omega$ is the wave angular frequency, $\omega_{c e}$ is the electron gyrofrequency, and $\omega_{p e}$ is the electron plasma frequency.

The majority of whistler waves satisfy the condition allowing for the 'quasi-longitudinal" approximation to be valid. Figure 2.1 is an example of this as the wave with the frequency $f=285.0 \mathrm{~Hz}, \omega=1.79 \times 10^{3} \mathrm{rad} / \mathrm{s}, \omega_{c e}=3.00 \times 10^{4} \mathrm{rad} / \mathrm{s}\left(B_{0}=170.5 \mathrm{nT}\right)$, and $\omega_{p e}=6.909 \times 10^{5} \mathrm{rad} / \mathrm{s}\left(n_{0}=150 \mathrm{~cm}^{-3}\right)$. These values are found for every observed event as a way to confirm them as whistler events and to also allow for any observed event to be an excellent event to model. This model assumes that the plasma density is constant in the direction of the field lines and varies in the perpendicular direction, or across the ambient magnetic field. It is assumed that the ambient magnetic field is constant in all directions. Looking back to Figure 2.1 and Figure 2.2, it can be seen over the course of the event that the background magnetic field only changed by $6 \%$ and $7 \%$ respectively, which allows for the assumption that the magnetic field is homogenous across the duct. Assuming that the magnetic field and the density are homogeneous in the parallel direction due to the relative small size of the modeling region in the equatorial magnetosphere (Mlat $\approx 1.3^{\circ}$ ).

Assuming the background magnetic field is uniform, it is implied that $\mathbf{v}_{0}=0$ and $\mathbf{B}=$ $B_{0} \bar{z}$ allows Equations 3.1-3.4 to be linearized to

$$
\begin{gather*}
m_{e} \frac{\partial \mathbf{v}}{\partial t}=q\left(\overline{\mathbf{E}}+\overline{\mathbf{v}} \times B_{0} \bar{z}\right)  \tag{3.5}\\
\nabla \times \overline{\mathbf{B}}=\mu_{0} n_{0} q \mathbf{v} \tag{3.6}
\end{gather*}
$$

$$
\begin{equation*}
\nabla \times \overline{\mathbf{E}}=-\frac{\partial \overline{\mathbf{B}}}{\partial t} \tag{3.7}
\end{equation*}
$$

The curl of equation 3.5 is taken twice, after the first curl substitute in Faraday's Law and after the second curl substitute in Ampere's law turning equation 3.5 into

$$
\begin{equation*}
m_{e} \frac{\partial \nabla \times \nabla \times \mathbf{v}}{\partial t}=q\left(-\frac{\partial \mu_{0} n_{0} q \overline{\mathbf{v}}}{\partial t}+\nabla \times \nabla \times \overline{\mathbf{v}} \times B_{0} \hat{z}\right) \tag{3.8}
\end{equation*}
$$

Then multiply by $m_{e}^{-1} \omega_{c e}^{-1} c^{2} / \omega_{p_{e}}{ }^{2}$

$$
\begin{equation*}
\frac{\partial \bar{\nabla} \times \bar{\nabla} \times \mathbf{v}}{\partial \omega_{c e} t}=-\frac{\mu_{0} n_{0} q^{2}}{m_{e}} \frac{c^{2}}{\omega_{p e}} \frac{\partial \overline{\mathbf{v}}}{\partial \omega_{c} t}+\frac{q B_{0}}{m_{e} \omega_{c e}} \bar{\nabla} \times \bar{\nabla} \times(\overline{\mathbf{v}} \times \hat{z}) \tag{3.9}
\end{equation*}
$$

Note that since $q=-e$ makes the coefficients in front of the second term on the righthand side equal to -1 and the coefficients in front of the first term on the right-hand side equal to 1 , equation 3.9 can be rewritten to

$$
\begin{equation*}
\frac{\partial}{\partial \tau}(\overline{\mathbf{v}}+\bar{\nabla} \times \bar{\nabla} \times(\overline{\mathbf{v}})=-\bar{\nabla} \times \bar{\nabla} \times(\overline{\mathbf{v}} \times \hat{z}) \tag{3.10}
\end{equation*}
$$

where $\tau=\omega_{c e} t$ is the normalized time. Assuming $v(\overline{\mathbf{x}}, \tau)=v_{o} e^{i \overline{\mathbf{k}} \cdot \overline{\mathrm{x}}+i \tau \frac{\omega}{\omega_{c e}}}$ time dependence so

$$
\begin{equation*}
i \frac{\omega}{\omega_{c e}}(\overline{\mathbf{v}}+i \overline{\mathbf{k}} \times(i \overline{\mathbf{k}} \times \overline{\mathbf{v}}))=i \overline{\mathbf{k}} \times(i \overline{\mathbf{k}} \times(\overline{\mathbf{v}} \times \hat{z})) \tag{3.11}
\end{equation*}
$$

Further simplified using $i \overline{\mathbf{k}} \cdot \overline{\mathbf{v}}=0$ and $\overline{\mathbf{v}}=\xi+\overline{\mathbf{k}} \times \xi$ where $\overline{\mathbf{k}} \cdot \xi=0$ becomes

$$
\begin{equation*}
\frac{\omega}{\omega_{c e}}(\overline{\mathbf{v}}-\overline{\mathbf{k}} \times(\overline{\mathbf{k}} \times \overline{\mathbf{v}}))=i \overline{\mathbf{k}} \times(\overline{\mathbf{k}} \times(\overline{\mathbf{v}} \times \hat{z})) \tag{3.12}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathbf{k} \times(\mathbf{k} \times \overline{\mathbf{v}})=\mathbf{k} \mathbf{k} \cdot \overline{\mathbf{v}}-k^{2} \overline{\mathbf{v}}=-k^{2} \overline{\mathbf{v}} \tag{3.13}
\end{equation*}
$$

and

$$
\begin{equation*}
\overline{\mathbf{k}} \times(\overline{\mathbf{k}} \times(\overline{\mathbf{v}} \times \hat{z}))=k_{z} \mathbf{k} \times \overline{\mathbf{v}} \tag{3.14}
\end{equation*}
$$

so then equation 3.12 bceomes

$$
\begin{equation*}
\frac{\omega}{\omega_{c e}} \overline{\mathbf{v}}\left(1+k^{2}\right)=i k_{z} \mathbf{k} \times \overline{\mathbf{v}} \tag{3.15}
\end{equation*}
$$

and crossing this equation with k creates the second equation need

$$
\begin{equation*}
\frac{\omega}{\omega_{c e}} \overline{\mathbf{k}} \times \overline{\mathbf{v}}\left(1+k^{2}\right)=-i k_{z} k^{2} \overline{\mathbf{v}} \tag{3.16}
\end{equation*}
$$

Equations 3.15 and 3.16 are then combined into a matrix equation. The determinant is then set equal to zero

$$
\left[\begin{array}{cc}
\frac{\omega}{\omega_{c e}}\left(1+k^{2}\right) & -i k_{z}  \tag{3.17}\\
i k_{z} k^{2} & \frac{\omega}{\omega_{c e}}\left(1+k^{2}\right)
\end{array}\right] \cdot\left[\begin{array}{c}
\overline{\mathbf{v}} \\
\overline{\mathbf{k}} \times \overline{\mathbf{v}}
\end{array}\right]=0
$$

becoming

$$
\begin{equation*}
{\frac{\omega}{\omega_{c e}}}^{2}\left(1+k^{2}\right)^{2}-k_{z}^{2} k^{2}=0 \tag{3.18}
\end{equation*}
$$

which after simplified provides the wave dispersion relation

$$
\begin{equation*}
k^{2}-\frac{\omega_{c e}}{\omega} k_{\|} k+\frac{\omega_{p e}^{2}}{c^{2}}=0 . \tag{3.19}
\end{equation*}
$$

where $k_{z}$ is replaced with $k_{\|}$as $z$ is set to the direction in which the wave propagates. Here, $k_{\|}$and $k_{\perp}$ are parallel and perpendicular to the $\mathbf{B}_{0}$ component of the wave vector and

$$
\begin{equation*}
k^{2}=k_{\|}^{2}+k_{\perp}^{2} \tag{3.20}
\end{equation*}
$$

To explain the physical meaning of ducting, we use (3.19) to express $k_{\perp}$ as a function of $\omega, k_{\|}, \omega_{c e}$ and $\omega_{p e}$ :

$$
\begin{equation*}
k_{\perp_{1,2}}=k_{\|}\left[\frac{\omega_{c e}^{2}}{4 \omega^{2}}\left(1 \mp \sqrt{1-\frac{n}{n_{2}}}\right)^{2}-1\right]^{1 / 2} \tag{3.21}
\end{equation*}
$$

where

$$
\begin{equation*}
n_{2}=k_{\|}^{2} \frac{m_{e}}{\mu_{0} e^{2}}\left(\frac{\omega_{c e}}{2 \omega}\right)^{2} . \tag{3.22}
\end{equation*}
$$

Expression (3.20) shows that the wave with some particular $\omega$ and $k_{\|}$can propagate in the media with some particular $B_{0}\left(\right.$ or $\left.\omega_{c e}\right)$ and $n$ (or $\omega_{p e}$ ) if,

$$
\begin{equation*}
n / n_{2}<1, \tag{3.23}
\end{equation*}
$$

and,

$$
\begin{equation*}
\frac{\omega_{c e}}{2 \omega}\left(1+\sqrt{1-\frac{n}{n_{2}}}\right)>1 . \tag{3.24}
\end{equation*}
$$

Therefore, if the conditions $(3.2,3.24)$ are satisfied there must be a minimum of one whistler wave trapped in the duct with a specified $\omega$ and real $k_{\|}$and $k_{\perp}$. These conditions are simplified as it has been demonstrated that high-density ducts only trap waves with the $k_{\perp}$ corresponding to the "-" sign in the expression $(3.21)[38,23]$. This allows for the conditions for ducting to be greatly simplified as

$$
\begin{equation*}
\frac{\omega_{c e}}{2 \omega}\left(1-\sqrt{1-\frac{n}{n_{2}}}\right)>1, \text { or } n / n_{1}>1, \tag{3.25}
\end{equation*}
$$

where

$$
\begin{equation*}
n_{1}=k_{\|}^{2} \frac{m_{e}}{\mu_{0} e^{2}}\left(\frac{\omega_{c e}}{\omega}-1\right) \tag{3.26}
\end{equation*}
$$

However, for the condition (3.27) to be satisfied,

$$
\begin{equation*}
\omega<\omega_{c e} / 2 \tag{3.27}
\end{equation*}
$$

Meaning that only low-frequency whistlers can satisfy these conditions and are the only waves considered in this paper. This condition (3.26) is the most important condition as if condition (3.26) is satisfied then condition (3.25) is also satisfied.

Figure 3.1 displays $n$ as a function of $k_{\perp}$ calculated from (3.19) using the retrieved parameters from the event shown in Figure 2.1. These values specifically are $\omega=1.79 \times 10^{3}$ $\mathrm{rad} / \mathrm{s}(f=285 \mathrm{~Hz}), \omega_{c e}=3.00 \times 10^{4} \mathrm{rad} / \mathrm{s}\left(B_{0}=170.5 \mathrm{nT}\right)$, and $k_{\|}=2 \pi / \lambda_{\|}$, with $\lambda_{\|}=11$, $12,13,14$, and 15 km . Every set of these parameters will be calculated to find the values of $n_{1}$ and $n_{2}$ to allow for the range of successful ducting parameters to be found for testing. It can be seen from Figure 3.1, when $k_{\perp}=0$ that $n_{1}=n$ and that $n_{2}$ is the maximum value of $n$ when

$$
\begin{equation*}
k_{\perp_{1}}=k_{\perp_{2}}=k_{\|}\left(\omega_{c e}^{2} / 4 \omega^{2}-1\right)^{1 / 2} \tag{3.28}
\end{equation*}
$$

Figure 3.2A displays that no whistler can be trapped when $n>n_{2}$ and Figure 3.2B displays how there is only one wave if $n<n_{1}$ but two different waves with real $k_{\perp_{1}}$ and $k_{\perp_{2}}$ when $n_{1}<n<n_{2}$. Figure 3.2 shows how a HDD is a localized density enhanced region where the density outside of the duct is $n<n_{1}$ and the density in the duct is in the range of $n_{1}<n<n_{2}$. A whistler with $k_{\perp_{1}}$ can be trapped inside a HDD with some degree of leakage [38,23]. These plots were also made for the other selected frequencies based on Figure 2.1 to find the range of parameters that allow for some degree of ducting to occur inside the observed duct.

### 3.1 Optimal Ducting Parameter

A three step procedure has been created to easily determine the optimal parallel and perpendicular wave numbers of the whistler waves observed inside the high-density ducts[1]. First is to find an observed event to retrieve the required parameters from the observed duct and wave. Second is to use these retrieved parameters to calculate the range of wavelengths (or wave numbers) that would allow a whistler wave to be confined in the observed duct. Third is to run the simulation to find which parameters allow for the least energy to be leaked from the the duct, or the "most trapped" wave [1].

The Van Allen Probes allow for all the necessary parameters to be retrieve as seen in Figure 2.1. These necessary parameters are wave frequency, background magnetic field, and the electron density. To check if an observed event is a real event, $\omega$, $\omega_{c e}$, and $\omega_{c e}$ must be calculated to confirm the event satisfies the condition that $\omega<\omega_{c e} \ll \omega_{p e}$.

The second step is to find the ranges of trapped $\lambda_{\|}$and $\lambda_{\perp}$ for the observed events parameters. The observed parameters are then used to calculate the range of $l_{\|}$(or $k_{\|}$) so that $n / n_{2}<1$ during the event and that $n / n_{1}>1$ inside the duct and that $n / n_{1}<1$ outside the duct as can be seen in Figure 3.2. Figure 3.2A is the calculated $n / n_{2}$ ratio for $f=285$

Hz and the observed electron density and the magnetic field. This figure displays that real $k_{\perp}$ exists only when the parallel wavelength less than 20.5 km . Figure 3.2 B displays the ratio of $n / n_{1}$, it shows that waves having parallel wavelengths within the range of 11 km to 15.5 km will be trapped inside the HDD.

The final step is to compare the simulated propagation results to select the optimal or the "most trapped" waves from these ranges. This step is achieved by running twodimensional, time dependent simulations of the waves with varying parameters inside the duct, as discussed in Chapter 4. There are two rationales for this step: the first one is that all criteria used in this paper are obtained from the dispersion relation (3.8), derived for the case of homogeneous media. To make sure that these criteria can be applied to ducts, which by definition are very inhomogeneous structures, the predictions about wave trapping made by these criteria must be verified with simulations of the entire EMHD model. The second rationale is that high-density ducts confine VLF waves with some degree of leakage, which depends on the parameters of the wave as well as the duct [38, 32]. The satellite may cross the duct at any location close or far from the region where the waves are generated. Therefore, it is reasonable to expect that the observed waves are the ones that have minimal leakage of energy from the duct (or have a maximum amplitude in the duct). The parameters chosen to be optimal are the parameters that lose the least amplitude over the course of 200 wave periods.

### 3.2 Wide Ducts

For an event to be considered a wide duct, the ambient magnetic field must change significantly along side the change in density of the duct. When this occurs, the ambient magnetic field must be taken into account for the calculations of $n_{1}$ and $n_{2}$. The combination of the magnetic field and the density gradient creates a pseudo duct in which whistler waves can be trapped. The condition required for this pseudo duct to be formed is that both of the gradients are in the same direction, or simply stated, the magnetic field and density must both either increase or decrease in the same perpendicular direction. These ducts have been discussed by Woodroffe and Streltsov [28]. In the recent paper by Streltsov [30], it was shown that these ducts can be observed and some of the events were successfully recreated by simulations. Wide events were then observed and cataloged to greater understand these mostly ignored phenomena, compared to narrow events.


Figure 3.1: Plasma density as a function of the perpendicular wave number when $f=285$ $\mathrm{Hz}, B_{0}=170.5 \mathrm{nT}$, and $\lambda_{\|}=11,12,13,14$, and 15 km .


Figure 3.2: (A) The ratio $n / n_{2}$ for the waves with $f=285 \mathrm{~Hz}$ and $\lambda_{\|}=20.5 \mathrm{~km}$ and 23.0 km . (B) The ratio $n / n_{1}$ for the waves with $f=285 \mathrm{~Hz}$ and $\lambda_{\|}=11.0$ and 15.5 km .

## CHAPTER 4 SIMULATION MODEL

The discussion of the simulation model must begin with an understanding of the assumptions made and the understanding of the axis. All work in the model is done in a 2 D rectangular computational domain, described with orthogonal coordinate system $(x, z)$ where the $z$ direction and the $x$ direction are perpendicular. The constant background magnetic field is in the $z$ direction while the inhomogeneity in the plasma is in the $x$ direction. The plasma density is uniform in the $x$ direction to create the duct. The boundary conditions are periodic in the $z$ direction, and are of a simple Dirichlet type (all quantities are set equal to 0 ) in the $x$ direction. The wave is unable to propagate through the $x$ boundaries to avoid reflections of the wave that would bounce back and interact with the duct. This is done through dissipation layers near the boundaries. The fields of $\boldsymbol{E}, \boldsymbol{B}$, and $\boldsymbol{v}$ are still considered to be three-dimensional vector quantities [23].

The initial conditions for the simulation are to specify waves with prescribed $\omega, \lambda_{\|}$, and $\lambda_{\perp}$ standing in the $x$ direction. The wave propagates in the $z$ direction which has its size set equal to $1 \lambda_{\|}\left(l_{z}=\lambda_{\|}\right)$. The $z$ domain is discretized with 20 uniformly spaced grid points. The size of the $x$ domain is defined by the size of the observed duct. The density profile is stretched from the lowest point on each edge of the duct to create an isolated duct as seen in Figures 3.2 and 5.1. This extended region contains the dissipation layers. From testing, it was discovered that the entire transverse size of the domain equal to three widths of the duct is a reasonable size for this domain. The testing also showed that allocating $1 / 6$ of the transverse size of the domain to the size of the each dissipation layer near the boundaries in the $x$-direction is adequate to dampen the waves. This dampening ensures no waves bounce back into the system disturbing the ducting. As the simulation is only in two dimensions, the model does not require any input about the normal angle and the planarity of the event as it would be unnecessary information about the observed waves [1].

The duct that was modeled previously was observed by RBSP-B on 02/27/2016 from 11:09:00 and 11:22:00 UT [1]. It occurred in between $L=5.525$ and $L=5.621$ which corresponds to $\approx 612 \mathrm{~km}$ distance in the equatorial region. As previously stated, the total transverse size of the domain for the simulation was defined to be $3 \times$ duct size or 1836 km . For a transverse domain of this size, looking at Table 5.1, there are $2661 \lambda_{\perp_{2}}$ for the waves with $f=285 \mathrm{~Hz}$ and $\lambda_{\|}=11 \mathrm{~km}$. To be able to resolve these waves with at least 5 grid points, the domain is discretized with 13,305 points. While this could be achieved, it was not necessary for the goal of providing a robust procedure and thus the domain was scaled down 12 times, so the size of the duct in the simulations is 50 km , and the total size of the domain is 150 km or $217 \lambda_{\perp_{2}}$. Note that even though the domain was scaled down, all the parameters in the simulations and observations are the same and therefore the simulated waves are the same as the observed waves. The size of the $x$ domain is still 217 times larger then the perpendicular wavelength allowing for the same wave behavior to be expected as if the $x$ domain was 2661 times larger. Another argument for bringing down the domain size to save on computation is the fact that all of the tested parameters are being tested in
a domain scaled down the same amount. Therefore the comparative study between these waves is expected to still give the same outcome as the waves behaviors remain relative to each other. Scaling down our computational domain allowed us to achieve our goal with significantly less simulations.

## CHAPTER 5

## RESULTS

The results presented are first from the simulation results in the Propagation Analysis section [1]. The results from the statistical analysis paper, which is in the submission process, are displayed in the Statistics of Narrow-Duct Events and Statistics of Wide-Duct Events sections below.

### 5.1 Propagation Analysis

Based on the peaks in frequency as seen in Figure 2.1, the simulations were performed on the four specified frequencies. For each frequency, the parallel and perpendicular wavelengths were varied based on the range that was calculated based on the ratios of density. The results from the simulations for the waves with $f=285 \mathrm{~Hz}$ and $\lambda_{\|}=11,12,13,14,15$ km are illustrated in Figures 5.1 and 5.2. Figure 5.1 show dynamics of the $E_{y}$ component of the electric field across the $x$-cut of the domain $E_{y}\left(x, z=l_{z} / 2, t\right)$ to allow for the dynamics of the wave to be visualized. The duct that the simulation was representing is shown at the top of Figure 5.1 along with the density ratio where $1=n / n_{1}$ as the magenta line. The vertical dashed red lines show the predicted boundary of the duct and continues down to the visualization to allow for each boundary of the duct to be clear seen. This boundry is determined based on the dispersion relation (3.5), where $n / n_{1}>1$. As the value of $n_{1}$ increases as the $\lambda_{\|}$decreases, the duct confining waves with $\lambda_{\|}=11 \mathrm{~km}$ should be less than the width of the duct for the waves with $\lambda_{\|}=15 \mathrm{~km}$. The simulation succeeds in confirming this prediction.

The center of the $E_{y}$ domain, $E_{y}\left(x=l_{x} / 2, z=l_{z} / 2, t\right)$, with $f=285 \mathrm{~Hz}$ and $\lambda_{\|}=11$, $12,13,14,15 \mathrm{~km}$ is displayed in Figure 5.2 to allow for the amplitudes of the waves to be easily compared to find which wave loses the least amplitude and therefore is the optimal $\lambda_{\|}$for the duct at this frequency. The simulation ran for $\approx 200$ wave periods for significant energy leakage to occur. The simulation confirms that HDD are able to effectively trap whistler in a range of parameters such as parallel (and perpendicular) wavelengths and frequency. The waves with $\lambda_{\|}=13$ and 14 km show that these waves are less confined in the duct than the waves with $\lambda_{\|}=11$ and 15 km . It was concluded that the wave with $\lambda_{\|}$ $=11 \mathrm{~km}$ retains the largest amplitude in the duct and therefore is the optimal $\lambda_{\|}$for $f=$ 285 Hz in the observed duct. Analysis of these five $\lambda_{\|}$allow for the behaviors of different parameters to be observed confirming the statement that HDD function as a filter as The waves with $\lambda_{\|}$of 11 km and 15 km show a better ability to be trapped compared to waves with $\lambda_{\|}$of $12 \mathrm{~km}, 13 \mathrm{~km}$ and 14 km .

The simulation was ran for all four of the frequencies previously discussed but figures were not created for them as they would be redundant. Instead, Table (make table) is used to display the final results from all the simulations. Over the $\approx 200$ wave periods, a percentage of retained normalized amplitude was found for every run. It was found that the most optimal situation had $90.9 \%$ of the initial wave amplitude while the least optimal
situation had $10.2 \%$ of the initial wave amplitude. This result leads to the conclusion that the most probable wave that will be observed in this duct far away from the source region, is a wave with $f=285 \mathrm{~Hz}$ and $\lambda_{\|}=11 \mathrm{~km}$. The perpendicular and parallel wavelengths of the "most trapped" wave are marked with "*" in Table 5.1. In short, the waves with frequencies $215.0,337.5$, and 390.0 Hz show that $\lambda_{\|}$of the most trapped waves with these frequencies are $13.00,11.00$, and 12.00 km respectfully.

Table 5.2 displays the percent efficiency of the best and worst parallel wavelength for each frequency tested based off the observed event. Table 2 allows for analysis of determining optimal parameters for ducting events based on the percentage of the wave amplitude that was retained over approximately 200 wave periods. The amplitudes of each are normalized to allow for better viewing of the results. The fact that some of the tested parameters only retained $10.2 \%$ of the initial wave amplitude compared to retaining $90.9 \%$ of the initial wave amplitude shows significant differences in the optimal parameters ducting ability compared to non-optimal parameters. This result leads to the conclusion that it is the most probable wave with $f=285 \mathrm{~Hz}$, which can be detected in this duct far away from the source region. The perpendicular and parallel wavelengths of the "most trapped" wave are marked with "*" in Table 1. The source for error would be if the parallel plasma density is inhomogeneous. This does not affect the model in the equatorial region as the parallel density profile is nearly homogeneous, but would affect the results nearer to the poles and the atmosphere as the parallel density profile would change significantly. Simulations showed that the lowest frequency consistently had the highest trapping efficiency which was not expected.

From this study, the simulations were able to show consistent and robust results. the simulation was successful in showing that the dispersion relation (2.5) in the case of homogeneous plasma and constant magnetic field, is able to accurately predict the boundaries of the density inhomogeneity duct for a whistler event. The results also show that whistler ducting events can be observed by the RBSP-B satellite in the equatorial magnetosphere region. The results most importantly show that the method discussed for finding the optimal ducting parameters based on an observed duct is an accurate and valid approach which can be applied to future studies of whistler ducting events.

| $f[\mathrm{~Hz}]$ | $\lambda_{\\|}[\mathrm{km}]$ | $\lambda_{\perp_{1}}[\mathrm{~km}]$ | $\lambda_{\perp_{2}}[\mathrm{~km}]$ |
| :---: | :---: | :---: | :---: |
|  | $\lambda_{\\|}^{*}[\mathrm{~km}]$ | $\lambda_{\perp_{1}}^{*}[\mathrm{~km}]$ | $\lambda_{\perp_{2}}^{*}[\mathrm{~km}]$ |
| 215.0 | $12.75-18.00$ | $49.40-9.31$ | $0.60-0.90$ |
|  | 13.00 | 32.76 | 0.62 |
| 285.0 | $11.00-15.50$ | $40.50-8.04$ | $0.69-1.04$ |
|  | 11.00 | 40.50 | 0.69 |
| 337.5 | $10.00-14.25$ | $44.20-6.85$ | $0.77-1.21$ |
|  | 11.00 | 14.24 | 0.86 |
| 390.0 | $9.25-13.50$ | $40.40-5.83$ | $0.83-1.40$ |
|  | 12.00 | 7.72 | 1.16 |

Table 5.1: Ranges of the calculated $\lambda_{\|}, \lambda_{\perp_{1}}$, and $\lambda_{\perp_{2}}$ that allow for trapping of whistlermode waves to occur with the frequencies $215.0,285.0,337.5$, and 390.0 Hz as selected from Figure 2.1(C). The optimal parameters for each set of parameters are displayed as $\lambda_{\|}^{*}$, $\lambda_{\perp_{1}}^{*}$, and $\lambda_{\perp_{2}}^{*}$.

| $f[\mathrm{~Hz}]$ | $\lambda_{\\|}[\mathrm{km}]$ <br> Efficiency Percent | $\lambda_{\\|}[\mathrm{km}]$ <br> Efficiency Percent |
| :---: | :---: | :---: |
| 215.0 | 13 | 16 |
|  | 90.9 | 83.3 |
| 285.0 | 11.00 | 13 |
|  | 72.7 | 49.7 |
| 337.5 | 11 | 13 |
|  | 54.5 | 26.4 |
| 390.0 | 12 | 10 |
|  | 42.1 | 10.2 |

Table 5.2: The most optimal and least optimal ducting $\lambda_{\|}$found from the frequencies 215.0, $285.0,337.5$, and 390.0 Hz in the high-density duct are displayed along side their ducting efficiency to allow for comparison. These percentages are found by comparing the starting and ending wave amplitude.


Figure 5.1: Displayed are the dynamics of $E_{y}$ in the simulations visualized for the waves with $f=285 \mathrm{~Hz}$ and $\lambda_{\|}=11,12,13,14,15 \mathrm{~km}$.


Figure 5.2: The center line of the dynamics of $E_{y}$ in the simulation domain for the waves with $f=285 \mathrm{~Hz}$ and $\lambda_{\|}=11,12,13,14,15 \mathrm{~km}$ as shown in Figure 5.1.

### 5.2 Statistics of Narrow-Duct Events

From the work for the paper in the publication process, two separate time intervals were examined for whistler ducting events. The first period from 4/1/2013 to 5/1/2013 (Period 1) and the second period from $7 / 24 / 2015$ to $9 / 1 / 2015$ (Period 2). These periods were selected as they were from two different regions of the magnetosphere. They were also selected because they had varying amounts of geomagnetic activity so that both periods could have calm and active days to see if the events behave differently based on geomagnetic activity. During Period 1 , the orbit was in the night-side magnetosphere. Period 2 had an orbit in the dusk side of the magnetosphere, or the transitional region between the day and night side. Figure 5.5 displays the orbits of the RBSP-A satellite during the two intervals. The red dots display all the events during Period 1 while the blue dots display all observed events during Period 2. Combined there were a total of 164 HDD events observed.

To find narrow HDD events that were observed by RBSP-A, a simple procedure is followed. First is to find a peak in the density. Peaks are determined by an at least $50 \%$ increase in the density. Next, this increase is checked to see if it correlates with an increase in the PSD of the electric field. If that is the case, the potential event is checked by verifying that the wave angular frequency $\omega$ is less then the electron gyrofrequency $\omega_{c e}$. If the event is confirmed, the parameters observed by the RBSP-A are retrieved for the event. These parameters are the size of the duct, the wave frequency, and location of the duct relative to the plasmapause. These parameters are then used to create distribution plot for analysis.

During both time periods, $78.18 \%$ of the narrow-duct events magnetic field value all occurred in the range of $100-300 \mathrm{nT}$, and $88.28 \%$ of these events had a peak PSD corresponding to a frequency of less that 500 Hz . Period 1 was found to have 2.03 events per day and Period 2 was found to be more active having an average of 3.7 events per day. Period 2 was also found to have larger ducts on average. Period 1 had an average duct width of 253.69 km while Period 2 had an average duct width of 427.76 km . Both of these average sizes are shifted larger due to a few outlier events. This can be seen in Figure 5.6, how approximately $70 \%$ of observed events occur with duct widths less than 250 km but have a few 1000 km or more. It can also be seen from the figure that Period 1 had $50.84 \%$ of events occur with duct width less than 100 km while Period 2 had $42.37 \%$ of events in this same range reinforcing the statement that Period 2 had larger ducts on average. Figure 5.6 shows that the majority of the observed events were in the ducts with a radial size less than 50 km . The graph is displayed with the events cataloged into bins where the first bar represents all events from $0-50 \mathrm{~km}$, the next shows $50-100 \mathrm{~km}$, and the next $100-150 \mathrm{~km}$. Period 2 was also found to have a higher peak frequency on average as seen in Figure 5.7. Period 2 was found to have an average peak frequency of 320.90 Hz while Period 1 had a lower average of 240.49 Hz . It can be seen that the majority of these waves reach their maximum at frequency between $f=200 \mathrm{~Hz}$ and $f=400 \mathrm{~Hz}$. The graph is displayed with the events cataloged into bins where the first bar represents all events from $0-100 \mathrm{~Hz}$, the next shows $100-200 \mathrm{~Hz}$, and the next $300-400 \mathrm{~Hz}$.

From looking at the active and inactive geomagnetic days based on the DST measured on these days, no significant differences are found in the events due to the activity. Comparing the active and inactive time intervals, it was found that the active time-frame was 11.9 \% higher frequency peaks than the low activity time and the duct width during the active
time was $15.3 \%$ wider than during the low activity time. This variation could be caused by a single large outlier event and no other differences were seen in the observed events.

Figure 5.8 displays the distrubution of the varitations of the density amplitudes in the narrow HDD events. It was found that the variation of events on average increases by $161 \%$. The majority of events see an increase of $50 \mathrm{~cm}^{-3}$ to $150 \mathrm{~cm}^{-3}$. The observed narrow events commonly occur in areas of low density allowing for increases as small as $50 \mathrm{~cm}^{-3}$ to become percent increases of $100 \%$ or more. The graph is displayed with the events cataloged into bins where the first bar represents all events from $0-50 \mathrm{~cm}^{-3}$, the next shows $50-100 \mathrm{~cm}^{-3}$, and the next $100-150 \mathrm{~cm}^{-3}$. As seen in Figure 5.9, the distance between the observed events and the observed plasmapause is displayed. The events plotted with negative distances shows the events that occurred inside of the observed plasmapause while positive distances shows events outside or past the observed plasmapause. The graph is displayed with the events cataloged into bins where the first bar represents all events from $0-0.15 R_{E}$, the next shows $0.15-0.30 R_{E}$, and the next $0.30-0.45 R_{E}$. It was found that more events occurred inside the plasmapause in Period 1 and Period 2 was almost evenly split between inside and outside the plasmapause. Period 1 was found to have $36.7 \%$ of the events occurred outside of the plasmapause while $63.3 \%$ of the events occurred inside the plasmapause. Period 2 was found to have $54.8 \%$ of the events occurred outside of the plasmapause while $45.2 \%$ of the events occurred inside the plasmapause. It was seen that the events most commonly occur at the radial distance less than $0.15 R_{E}$ from the plasmapause.


Figure 5.3: The location of every observed event is graphed on the orbital path of RBSP-A. The day side is shown by the white side of the gradient circle in the center. The red set of points are all the events from April of 2013 and the blue set of points are all the events from August of 2015.


Figure 5.4: The widths of the narrow ducts are displayed. The events are divided into 50 km intervals with the displayed number showing the upper limit of the interval. A) All events observed during Period 1. B) All events observed during period 2. C) All narrowduct events observed over both periods.


Figure 5.5: The frequency peaks that correlate to the region of maximum PSD of the narrow ducts are displayed. The events are divided into 100 Hz intervals with the displayed number showing the upper limit of the interval. A) All events observed during Period 1. B) All events observed during period 2. C) All narrow-duct events observed over both periods.


Figure 5.6: The variation of electron density of the narrow ducts are displayed. The events are divided into $50 \mathrm{~cm}^{-3}$ intervals with the displayed number showing the upper limit of the interval. A) All events observed during Period 1. B) All events observed during period 2. C) All narrow-duct events observed over both periods.


Figure 5.7: The distance to the plasmapause of the narrow ducts are displayed. The negative distances display events that occurred inside of the plasmapause. The events are divided into $0.15 \mathrm{R}_{E}$ intervals with the displayed number showing the upper limit of the interval. A) All events observed during Period 1. B) All events observed during period 2. C) All narrow-duct events observed over both periods.

### 5.3 Statistics of Wide-Duct Events

To identify a wide event, a modified procedure must be done compared to the previously discussed procedure. This is due to the fact that for wide events, the variation in the background magnetic field and the variation of the density both must be taken into account when finding the ratio of $n / n_{1}$ for determining if ducting is in fact occurring in a possible observed event. This relation is discussed in great detail in the recent paper by Streltsov [30]. Similarly to narrow events, a region where an unusual increase in density is first found and then check to see if it correlates to an increase in the electric field PSD. The difference is that the magnetic field variations must be taken into account when finding the ratio of $n / n_{1}$. The data for both the density and the background magnetic fields must be retrieved for the calculation to occur. When finding this ration the parallel wavelength is a free parameter in the model, so it is adjusted to find where ducting may be occurring for the potential event [30].

For wide events, only Period 2 was used as a sufficient amount of events were observed and wide duct events are easier to observe then the narrow events. This is due to the fact that the region they can occur is restricted to small regions in the orbit of the RSBP satellites due to the required increase in magnetic field. Figure 5.10 displays the orbit of RBSP-A during Period 2 and the location of all the events recorded. A clarification must be made as at first glance the wide events seem to occur in a greater region of space but as the satellite is moving much faster during through these regions, these regions of data make up only a small portion of the observed data in one orbit. During the time interval 49 events were identified, these events are displayed as blue dots in Figure 5.10.

Figure 5.11 shows the distribution of wide event widths during Period 2. The average size of the ducts was found to be 4402 km with most events occurring having ducts smaller then 2000 km . Figure 5.12 shows the distribution of the peak frequency of the observed events. The average value and the most common frequency range line up much closer here then we have seen in the other figures. The average frequency peak is 435.5 Hz and the most common range of frequency was $300-400 \mathrm{~Hz}$ with $400-500 \mathrm{~Hz}$ as the next most common range. Figure 5.13 displays all the recorded distances from the plasmapause. Unlike narrow events, all events occurred inside the plasmapause. As all events occurred inside the plasmapause, they are all displayed with positive distance as the distinction was not necessary like for the narrow events. The wide events most commonly occured within the distance of $0.5 \mathrm{R}_{E}$ to the plasmapause which is much farther then was common for the narrow events. Figure 5.14 shows the distribution of the variation in density over the width of a wide-duct event. This is the most unique finding of the analysis as the largest and smallest variation are just as common as each other. The average change in density is 1546 $\mathrm{cm}^{-3}$.

It was found that $67 \%$ of the observed wide events had both a decrease in the density and background magnetic field. Another interesting finding is that multiple events were commonly observed on the same day (for example, see Figure 1 in [30]). Over the course of 37 days, 15 of them had multiple wide events be observed. There were also days where no events were observed. Looking at the two parameters that make an event a wide event, it was found that the average change in density is $1546 \mathrm{~cm}^{-3}$, while the average change in the background magnetic field is 1825 nT . These average change values are only $\approx 18 \%$
different showing how the variation in both values are important for ducting to occurr in a wide HDD.


Figure 5.8: The location of every observed wide event is graphed on the orbital path of RBSP-A. The day side is shown by the white side of the gradient circle in the center.


Figure 5.9: The widths of the wide ducts are displayed. The events are divided into 1000 km intervals with the displayed number showing the upper limit of the interval.


Figure 5.10: The frequency peaks that correlate to the region of maximum PSD of the wide ducts are displayed. The events are divided into 100 Hz intervals with the displayed number showing the upper limit of the interval.


Figure 5.11: The distance to the plasmapause of the wide ducts are displayed. All the events occurred inside of the plasmapause. The events are divided into $0.5 \mathrm{R}_{E}$ intervals with the displayed number showing the upper limit of the interval.


Figure 5.12: The variation of electron density of the wide ducts are displayed. The events are divided into $500 \mathrm{~cm}^{-3}$ intervals with the displayed number showing the upper limit of the interval.

## CHAPTER 6 CONCLUSION

This paper presents the results found from the statistical study and the wave ducting propagation of whistler waves observed in the equatorial magnetosphere by the Van Allen Probes. These combined results have added to the understanding and knowledge we have on whistler wave ducting events. The findings also show how common these events are and allow for future research to greater understand one of the common phenomena that affect the dynamics of the equatorial magnetosphere.

### 6.1 Conclusions from the Propagation Study

The event simulated was used as a test to provide evidence that the procedure we have developed works for a real world, observed event. The event was observed in the equatorial magnetosphere by the RBSP-B satellite on 02/27/2016. Packages of VLF waves with frequencies in the range from 100 to 1000 Hz localized inside field-aligned density enhancements, or high-density ducts, were observed. Previous studies have suggested that high-density duct functions as a filter due to how high-density ducts are able to retain waves with specific parameters while leaking away waves rapidly if their parameters do not match the specific parameters for ducting. The main goal of this study was to create and display a novel, robust 3-step procedure to identify the exact values of the wave perpendicular and parallel wavelengths and then illustrate this procedure through application to a representative observational event [1].

This goal was achieved by developing a simple three-step procedure based on the values that can be retrieved from the Van Allen Probes to calculate the parallel and perpendicular wave numbers of the whistler-mode wave trapped in the observed duct. These known parameters are the frequency propagating in the high-density duct, the background magnetic field and the density values. This three-step procedure is:

1. From $n, B$, and $\omega$ identify the range of $\lambda_{\|}$providing wave trapping in the duct
2. From $n, B, \omega$, and $\lambda_{\|}$identify the range of corresponding $\lambda_{\perp_{1}}$.
3. From 2D EMHD simulations identify $\lambda_{\|}$and $\lambda_{\perp}$ of the "most trapped" wave.

This process was used on the event displayed in Figure 2.1. The results display a reliable and consistent way of determining wavevectors of the VLF waves for multiple frequencies selected based on the regions of peak PSD in the electric field.

The model is applicable for events that occur over relatively short distances along the magnetic field lines in the equatorial magnetosphere. As the limitation to the model is that it assumes that the background magnetic field and the plasma density are homogeneous in field aligned direction, which is a reasonable assumption over relatively short distances in the equatorial magnetosphere. Therefore, the model breaks down at low altitudes where
the magnetic field and the plasma density change rapidly along the field lines. This study focuses on the region in which the model is adequate for our purposes.

### 6.2 Conclusions from Statistical Analysis

From analysing a large amount of data from a long and varied time interval of data from the Van Allen Probes, major conclusions were drawn. The most important conclusion is that ducting or trapping of VLF waves in regions of enhanced plasma density is a common phenomena in the Earth's magnetosphere.

Narrow high-density ducts are formed by the enhancements of the plasma density where the background magnetic field varies a relatively insignificant amount compared to the variation of plasma density over the course of the duct. The ducts are normally small in size and the background magnetic field normally changes less than $6 \%-10 \%$ over the width of the duct. Therefore the duct only occurs due to the variation in density. Over the two periods observed, a total of 169 events were analyzed leading to the major findings that the narrow HDD events occur inside and outside of the plasmasphere mostly inside the radial distance $0.15 R_{E}$ from the plamapause, have a duct width of less than 50 km , the density variation most commonly increases $50 \mathrm{~cm}^{-3}$ to $150 \mathrm{~cm}^{-3}$, and lastly that the frequency peaks are most often in the range of $200-400 \mathrm{~Hz}$.

Wide ducts are formed by the enhancements of the plasma density where the background magnetic field varies a comparable or significant amount compared to the variation of plasma density over the course of the duct and normally require that the parallel wavelength of the trapped waves changes inside the duct. Over the two periods observed, a total of 49 events were analyzed leading to the major findings that the wide events occur inside of the plasmasphere at the radial distance $0.5 R_{E}$ from the plamapause, have a duct width between $0-1000 \mathrm{~km}$, the density variation on average is $1546 \mathrm{~cm}^{-3}$, the magnetic field varies an average of 1825 nT , and lastly that the frequency peaks are most often in the range of $400-500 \mathrm{~Hz}$.

## Appendices

## APPENDIX A

 OBSERVATIONS

Figure A.1: Displayed is the example event that was modeled in the simulation.A) The Power spectral density (PSD) of $E_{w}$ component of the electric field is shown above the graph with a color pallet. The electron density is displayed with a white line and was measured by the RBSP-B satellite from 10:37 to 11:47 UT on 02/27/2016. B) Displayed is the magnitude of the background magnetic field that was observed by the satellite. C) The PSD of $E_{w}$ versus frequency at the time of peak PSD of 11:17:01 UT during the time interval from 11:09:00 to 11:22:00 UT. D), E), and F) displays the orbit of the satellite in GSE coordinates in the time interval of 06:47:00 to 15:47:00 UT on 02/27/2016.


Figure A.2: Displayed is an example of a narrow high-density duct observed by the RBSPA satellite from 11:15:30 to 11:20:30 UT on 10/26/2015. A) The Power spectral density (PSD) of $E_{w}$ component of the electric field is shown above the graph with a color pallet. The electron density is displayed with a white line and was measured by the RBSP-B satellite. B) The magnitude of the background magnetic field measured by the satellite is plotted in the left axis while the observed electron density of the event is displayed on the right hand axis. The peak frequency and the range of parallel wavelengths used for the ratio calculation is shown in the top right.


Figure A.3: Displayed is an example of a narrow high-density duct observed by the RBSPA satellite from 8:23 to 9:54 UT on 8/21/2015. A) Power spectral density (PSD) of $E_{w}$ component of the electric field displayed above the graph with a color pallet. The electron density ratio of $n / n_{1}$ is shown with a white line and was calculated based on the observed values by the RBSP-A satellite B ) The magnitude of the background magnetic field measured by the satellite is plotted in the left axis while the observed electron density of the event is displayed on the right hand axis. The peak frequency and the range of parallel wavelengths used for the ratio calculation is shown in the top right.

## APPENDIX B

RESULTS

| $f[\mathrm{~Hz}]$ | $\lambda_{\\|}[\mathrm{km}]$ | $\lambda_{\perp_{1}}[\mathrm{~km}]$ | $\lambda_{\perp_{2}}[\mathrm{~km}]$ |
| :---: | :---: | :---: | :---: |
|  | $\lambda_{\\|}^{*}[\mathrm{~km}]$ | $\lambda_{\perp_{1}}^{*}[\mathrm{~km}]$ | $\lambda_{\perp_{2}}^{*}[\mathrm{~km}]$ |
| 215.0 | $12.75-18.00$ | $49.40-9.31$ | $0.60-0.90$ |
|  | 13.00 | 32.76 | 0.62 |
| 285.0 | $11.00-15.50$ | $40.50-8.04$ | $0.69-1.04$ |
|  | 11.00 | 40.50 | 0.69 |
| 337.5 | $10.00-14.25$ | $44.20-6.85$ | $0.77-1.21$ |
|  | 11.00 | 14.24 | 0.86 |
| 390.0 | $9.25-13.50$ | $40.40-5.83$ | $0.83-1.40$ |
|  | 12.00 | 7.72 | 1.16 |

Table B.1: Ranges of the calculated $\lambda_{\|}, \lambda_{\perp_{1}}$, and $\lambda_{\perp_{2}}$ that allow for trapping of whistlermode waves to occur with the frequencies $215.0,285.0,337.5$, and 390.0 Hz as selected from Figure 2.1(C). The optimal parameters for each set of parameters are displayed as $\lambda_{\|}^{*}$, $\lambda_{\perp_{1}}^{*}$, and $\lambda_{\perp_{2}}^{*}$.

| $f[\mathrm{~Hz}]$ | $\lambda_{\\|}[\mathrm{km}]$ <br> Efficiency Percent | $\lambda_{\\|}[\mathrm{km}]$ <br> Efficiency Percent |
| :---: | :---: | :---: |
| 215.0 | 13 | 16 |
|  | 90.9 | 83.3 |
| 285.0 | 11.00 | 13 |
|  | 72.7 | 49.7 |
| 337.5 | 11 | 13 |
|  | 54.5 | 26.4 |
| 390.0 | 12 | 10 |
|  | 42.1 | 10.2 |

Table B.2: The most optimal and least optimal ducting $\lambda_{\|}$found from the frequencies $215.0,285.0,337.5$, and 390.0 Hz in the high-density duct are displayed along side their ducting efficiency to allow for comparison. These percentages are found by comparing the starting and ending wave amplitude.


Figure B.1: Displayed are the dynamics of $E_{y}$ in the simulations visualized for the waves with $f=285 \mathrm{~Hz}$ and $\lambda_{\|}=11,12,13,14,15 \mathrm{~km}$.


Figure B.2: The center line of the dynamics of $E_{y}$ in the simulation domain for the waves with $f=285 \mathrm{~Hz}$ and $\lambda_{\|}=11,12,13,14,15 \mathrm{~km}$ as shown in Figure 5.1.


Figure B.3: The location of every observed event is graphed on the orbital path of RBSP-A. The day side is shown by the white side of the gradient circle in the center. The red set of points are all the events from April of 2013 and the blue set of points are all the events from August of 2015.


Figure B.4: The widths of the narrow ducts are displayed. The events are divided into 50 km intervals with the displayed number showing the upper limit of the interval. A) All events observed during Period 1. B) All events observed during period 2. C) All narrowduct events observed over both periods.


Figure B.5: The frequency peaks that correlate to the region of maximum PSD of the narrow ducts are displayed. The events are divided into 100 Hz intervals with the displayed number showing the upper limit of the interval. A) All events observed during Period 1. B) All events observed during period 2. C) All narrow-duct events observed over both periods.


Figure B.6: The variation of electron density of the narrow ducts are displayed. The events are divided into $50 \mathrm{~cm}^{-3}$ intervals with the displayed number showing the upper limit of the interval. A) All events observed during Period 1. B) All events observed during period 2. C) All narrow-duct events observed over both periods.


Figure B.7: The distance to the plasmapause of the narrow ducts are displayed. The negative distances display events that occurred inside of the plasmapause. The events are divided into $0.15 \mathrm{R}_{E}$ intervals with the displayed number showing the upper limit of the interval. A) All events observed during Period 1. B) All events observed during period 2. C) All narrow-duct events observed over both periods.


Figure B.8: The location of every observed wide event is graphed on the orbital path of RBSP-A. The day side is shown by the white side of the gradient circle in the center.


Figure B.9: The widths of the wide ducts are displayed. The events are divided into 1000 km intervals with the displayed number showing the upper limit of the interval.


Figure B.10: The frequency peaks that correlate to the region of maximum PSD of the wide ducts are displayed. The events are divided into 100 Hz intervals with the displayed number showing the upper limit of the interval.


Figure B.11: The distance to the plasmapause of the wide ducts are displayed. All the events occurred inside of the plasmapause. The events are divided into $0.5 \mathrm{R}_{E}$ intervals with the displayed number showing the upper limit of the interval.


Figure B.12: The variation of electron density of the wide ducts are displayed. The events are divided into $500 \mathrm{~cm}^{-3}$ intervals with the displayed number showing the upper limit of the interval.

## APPENDIX C <br> SIMULATION CODE

## C. 1 Convert File

```
Program Convert
Integer, Parameter :: nx=2001, nz=20000
Integer, Parameter :: nx1=1001, nz1=2000
Real*8 :: Ex (nx , nz ), Ey (nx , nz ), Ez (nx , nz )
Real*8 :: Ex1(nx1, nz1), Ey1(nx1, nz1), Ez1(nx1, nz1)
open (10,file='Exfield.dat',status='old')
open (20,file='Eyfield.dat',status='old')
open (30,file='Ezfield.dat',status='old')
read (10,*) n1, n2
read (10,*) Ex
read (20,*) n1, n2
read (20,*) Ey
read (30,*) n1, n2
read (30,*) Ez
close (10); close (20); close (30)
    do k = 1 , nz1
    k1 = k*10
        do i = 1 , nx1
            i1 = (i-1)*2 + 1
            i1 = i
    Ex1(i,k) = Ex(i1,k1)
    Ey1(i,k) = Ey(i1,k1)
    Ez1(i,k) = Ez(i1,k1)
        end do
    end do
open (10,file='ExfieldS.dat',status='unknown')
open (20,file='EyfieldS.dat',status='unknown')
open (30,file='EzfieldS.dat',status='unknown')
write (10,11) nx1, nz1
write (10,12) Ex1
write (20,11) nx1, nz1
write (20,12) Ey1
write (30,11) nx1, nz1
write (30,12) Ez1
close (10); close (20); close (30)
Format (2(i5,1x))
Format (10(e12.5,1x))
```

End Program Convert

## C. 2 Input File

| $7.02 \mathrm{e}-06$ | ! time step in s |
| :---: | :---: |
| 101000011 | ! times |
| 150000.0 | ! X size of the domain (in m) |
| 11000.0 | ! Z size of the domain (in m) |
| 285.0 | ! Wave frequency (in Hz ) |
| 11000.0 | ! Parallel wavelength (in m) |
| 150.0 | ! Density for dispersion relation |
| 1 | ! number of the perp. wavelength (1 or 2) |
| 0.001 | ! amplitude of the E field in the antenna |
| 0.333 | ! X-size of the Antenna |
| 6.0 | ! Antenna exponnent (2, 4, or 6) |
| 0.17 | ! Size of the dissipation layer |
| 10.0 | ! Ampl. for dissipation |
| 1.55 | ! Weight for SOR |
| 0 | ! Key (1/0) for convection |

## C. 3 Main

MODULE Global_Data
SAVE
Integer, Parameter ::
$n x=1501, n x S=n x+1$, $n \times 1=n x-1, n \times 2=n x-2, n \times 3=n x-3, n \times 4=n x-4$,
$n z=21, \quad n z S=n z+1$,
$n z 1=n z-1, n z 2=n z-2, n z 3=n z-3, n z 4=n z-4$,
$k b=1, k b 1=k b+1, k b 2=k b+2$,
$k t=n z, k t 1=k t-1, k t 2=k t-2$
Real*8 : :
StepX, StepZ, StepX12, StepZ12, StepXX12, StepZZ12, StepXZ12, D1X , D2X , D3X , D4X , D1Z , D2Z , D3Z , D4Z , DD1X, DD2X, DD3X, DD4X, DD0X, DD1Z, DD2Z, DD3Z, DD4Z, DD0Z,

X_WORK(1:nx,1:nz), Y_WORK(1:nx,1:nz), Z_WORK(1:nx,1:nz),
\& $\operatorname{DeN}(1: n x, 1: n z), \operatorname{DenV}(1: n x, 1: n z), B B(1: n x, 1: n z)$
Real*8 : :
SOR0X(1:nx,1:nz), SOR1X(1:nx,1:nz),
$\operatorname{SOROY}(1: n x, 1: n z), \operatorname{SOR1Y}(1: n x, 1: n z)$,
SOROZ(1:nx,1:nz), SOR1Z(1:nx,1:nz),
$\operatorname{RRX}(1: n x, 1: n z), \operatorname{RRY}(1: n x, 1: n z), \operatorname{RRZ}(1: n x, 1: n z)$, $\operatorname{EECX}(n x, n z), \operatorname{EECY}(n x, n z), \operatorname{EECZ}(n x, n z)$, C1_SOR, C0_SOR, Eps_SOR

Integer : :
I_SOR, Imax_SOR
Real*8, Parameter ::
$\mathrm{ccc} 0=299790.0 \mathrm{E}+3$, pi=3.14159265D0, Pi2 = 2.d0*pi, mu_0=3.14159265D0*4.0E-7, e_0=1.6022E-19, m_e=9.10940E-31, eps_0=8.8542E-12, extr1=-3.d0, extr2=3.d0, two=2.d0, D8=8.D0, D16=16.D0, D30=30.D0, D05=0.5D0

End Module Global_Data

```
This is a version of whist10pd.f with a higher (4) order of
    the derivatives
    Use global_data
    IMPLICIT DOUBLE PRECISION (A-H,O-Z)
    Real*8 ::
    EX(1:nx,1:nz), EY(1:nx,1:nz), EZ(1:nx,1:nz),
    BX(1:nx,1:nz), BY(1:nx,1:nz), BZ(1:nx,1:nz),
    VX(1:nx,1:nz), VY(1:nx,1:nz), VZ(1:nx,1:nz),
    BX_P(1:nx,1:nz), BY_P(1:nx,1:nz), BZ_P(1:nx,1:nz),
    VX_P(1:nx,1:nz), VY_P(1:nx,1:nz), VZ_P(1:nx,1:nz),
    Dis(1:nx,1:nz)
    Real*8, TARGET ::
        rpvx1(1:nx,1:nz), rpvx2(1:nx,1:nz), rpvx3(1:nx,1:nz),
        rpvx4(1:nx,1:nz),
        rpvy1(1:nx,1:nz), rpvy2(1:nx,1:nz), rpvy3(1:nx,1:nz),
        rpvy4(1:nx,1:nz),
        rpvz1(1:nx,1:nz), rpvz2(1:nx,1:nz), rpvz3(1:nx,1:nz),
        rpvz4(1:nx,1:nz),
    rpbx1(1:nx,1:nz), rpbx2(1:nx,1:nz), rpbx3(1:nx,1:nz),
    rpbx4(1:nx,1:nz),
    rpby1(1:nx,1:nz), rpby2(1:nx,1:nz), rpby3(1:nx,1:nz),
    rpby4(1:nx,1:nz),
    rpbz1(1:nx,1:nz), rpbz2(1:nx,1:nz), rpbz3(1:nx,1:nz),
    rpbz4(1:nx,1:nz)
```

    Real*8, POINTER : : ! Pointers
        p_vx1(:,:), p_vx2(:,:), p_vx3(:,:), p_vx4(:,:),
        p_vy1(:,:), p_vy2(:,:), p_vy3(:,:), p_vy4(:,:),
        p_vz1(:,:), p_vz2(:,:), p_vz3(:,:), p_vz4(:,:),
        p_bx1(:,:), p_bx2(:,:), p_bx3(:,:), p_bx4(:,:),
        p_by1(:,:), p_by2(:,:), p_by3(:,:), p_by4(:,:),
        p_bz1(:,:), p_bz2(:,:), p_bz3(:,:), p_bz4(:,:),
        p_work(:, :)
    Real*8, Allocatable : :
        DuctX(:), B_in(:),
        \(\operatorname{ExA}(:), \operatorname{EyA} \overline{( }:), \operatorname{EzA}(:), \operatorname{BxA}(:), \operatorname{ByA}(:), \operatorname{BzA}(:)\),
    Num=(/'0','1','2','3','4','5','6','7','8','9'/)
    NULLIFY (p_vx1, p_vx2, p_vx3, p_vx4,
            p_vy1, p_vy2, p_vy3, p_vy4,
        p_vz1, p_vz2, p_vz3, p_vz4,
            p_bx1, p_bx2, p_bx3, p_bx4,
            p_by1, p_by2, p_by3, p_by4,
                p_bz1, p_bz2, p_bz3, p_bz4,
                        p_work)
    * Input Parameters m
* Input Parameters m
* 

```
```

open (20,file='rbsp_dat_new3.dat',status='old')

```
open (20,file='rbsp_dat_new3.dat',status='old')
    read (20,*) ht
    read (20,*) ht
    read (20,*) ntime1, ntime2, ntime3
    read (20,*) ntime1, ntime2, ntime3
    read (20,*) SizeX ! Sizes of the BOX in m
    read (20,*) SizeX ! Sizes of the BOX in m
    read (20,*) SizeZ ! Sizes of the BOX in m
    read (20,*) SizeZ ! Sizes of the BOX in m
    read (20,*) Omega ! Wave frequency (in Hz)
    read (20,*) Omega ! Wave frequency (in Hz)
    read (20,*) wLz ! Wavelength in X
    read (20,*) wLz ! Wavelength in X
    read (20,*) D000 ! Density for the Disp. Relation
    read (20,*) D000 ! Density for the Disp. Relation
    read (20,*) Key_Kx ! Number of the perpendicular wavelength
    read (20,*) Key_Kx ! Number of the perpendicular wavelength
    read (20,*) AmpA ! Ampl of the Antenna
    read (20,*) AmpA ! Ampl of the Antenna
    read (20,*) SizeA ! X-size of the Antenna
    read (20,*) SizeA ! X-size of the Antenna
    read (20,*) GradA ! Gradient of the Antenna (4 or 6)
    read (20,*) GradA ! Gradient of the Antenna (4 or 6)
    read (20,*) SizeDis ! size of the dissipation layer (1/6-1/8)
    read (20,*) SizeDis ! size of the dissipation layer (1/6-1/8)
    read (20,*) AmpC ! ampl. of dissipation
    read (20,*) AmpC ! ampl. of dissipation
    read (20,*) C1_SOR ! Weight function for SOR
    read (20,*) C1_SOR ! Weight function for SOR
    read (20,*) Key_Conv ! Key to activate convective term
    read (20,*) Key_Conv ! Key to activate convective term
close (20)
```

```
*
*********************************************************
* Allocation of Arrays *
*********************************************************
*
            icen = nx/2.0 + 1
            kcen = nz/2.0 + 1
        icount1 = 0
        icount2 = 0
            iten = 5
            ht5 = ht * iten
                nw = ntime1 * ntime2 / iten + 2
    ALLOCATE (DuctX(nx), ExA(nw), EyA(nw), EzA(nw),
&
&
        ExM(nx,nw), EyM(nx,nw), EzM(nx,nw))
*
********************************************************
* Association of Pointers *
*********************************************************
*
```

```
rpvx1 = 0.0; rpvx2 = 0.0; rpvx3 = 0.0; rpvx4 = 0.0
```

rpvx1 = 0.0; rpvx2 = 0.0; rpvx3 = 0.0; rpvx4 = 0.0
rpvy1 = 0.0; rpvy2 = 0.0; rpvy3 = 0.0; rpvy4 = 0.0
rpvy1 = 0.0; rpvy2 = 0.0; rpvy3 = 0.0; rpvy4 = 0.0
rpvz1 = 0.0; rpvz2 = 0.0; rpvz3 = 0.0; rpvz4 = 0.0
rpvz1 = 0.0; rpvz2 = 0.0; rpvz3 = 0.0; rpvz4 = 0.0
p_vx1 => rpvx1
p_vx1 => rpvx1
p_vx2 => rpvx2
p_vx2 => rpvx2
p_vx3 => rpvx3
p_vx3 => rpvx3
p_vx4 => rpvx4
p_vx4 => rpvx4
p_vy1 => rpvy1
p_vy1 => rpvy1
p_vy2 => rpvy2
p_vy2 => rpvy2
p_vy3 => rpvy3
p_vy3 => rpvy3
p_vy4 => rpvy4
p_vy4 => rpvy4
p_vz1 => rpvz1
p_vz1 => rpvz1
p_vz2 => rpvz2
p_vz2 => rpvz2
p_vz3 => rpvz3
p_vz3 => rpvz3
p_vz4 => rpvz4
p_vz4 => rpvz4
rpbx1 = 0.0; rpbx2 = 0.0; rpbx3 = 0.0; rpbx4 = 0.0
rpbx1 = 0.0; rpbx2 = 0.0; rpbx3 = 0.0; rpbx4 = 0.0
rpby1 = 0.0; rpby2 = 0.0; rpby3 = 0.0; rpby4 = 0.0
rpby1 = 0.0; rpby2 = 0.0; rpby3 = 0.0; rpby4 = 0.0
rpbz1 = 0.0; rpbz2 = 0.0; rpbz3 = 0.0; rpbz4 = 0.0
rpbz1 = 0.0; rpbz2 = 0.0; rpbz3 = 0.0; rpbz4 = 0.0
p_bx1 => rpbx1
p_bx1 => rpbx1
p_bx2 => rpbx2
p_bx2 => rpbx2
p_bx3 => rpbx3

```
p_bx3 => rpbx3
```

```
        p_bx4 => rpbx4
        p_by1 => rpby1
        p_by2 => rpby2
        p_by3 => rpby3
        p_by4 => rpby4
        p_bz1 => rpbz1
        p_bz2 => rpbz2
        p_bz3 => rpbz3
        p_bz4 => rpbz4
***
* }\quad*********************** Space
***
StepX = SizeX/float(nx-1)
StepZ = SizeZ/float(nz)
StepXX = 1.D0/StepX/StepX
StepZZ = 1.D0/StepZ/StepZ
StepXX12 = 1.D0/(12.D0*StepX*StepX)
StepZZ12 = 1.D0/(12.D0*StepZ*StepZ)
StepX12 = 1.D0/(12.d0*StepX)
StepZ12 = 1.D0/(12.d0*StepZ)
StepXZ12 = StepX12*StepZ12
Work = 280.0D0*StepX
    D1X = 224.0D0/Work
    D2X = -56.0D0/Work
    D3X = 32.0D0/Work/3.0D0
    D4X = - 1.0D0/Work
Work = 560.0D0 * StepX * StepX
DD1X = 896.D0/Work
DD2X =-112.D0/Work
DD3X = 128.D0/Work/9.D0
DD4X =- 1.D0/Work
DD0X =-14350.D0/Work/9.D0
Work = 280.0D0*StepZ
    D1Z = 224.0D0/Work
    D2Z = -56.0D0/Work
    D3Z = 32.0D0/Work/3.0D0
    D4Z = - 1.0D0/Work
Work = 560.0D0*StepZ*StepZ
DD1Z = 896.D0/Work
DD2Z =-112.D0/Work
```

> DD3Z = 128.D0/Work/9.D0

DD4Z $=-1$. D0/Work
DD0Z =-14350.D0/Work/9.D0

```
**
****************************** Time
**
    ht24 = ht/24.0
        ab_1 = 55.*ht24
        ab_2 = -59.*ht24
        ab_3 = 37.*ht24
        ab_4 = -9.*ht24
        am_1 = 9.*ht24
        am_2 = 19.*ht24
        am_3 = -5.*ht24
        am_4 = ht24
**
*********************************************************
** *
** First step: T = 0 **
** ******************************************************************
**
```

```
    ExM = 0.d0
```

    ExM = 0.d0
    EyM = 0.d0
    EyM = 0.d0
    EzM = 0.d0
    EzM = 0.d0
    X_WORK = 0.d0
    X_WORK = 0.d0
    Y_WORK = 0.d0
    Y_WORK = 0.d0
    Z_WORK = 0.d0
    Z_WORK = 0.d0
    !
    !
    ! Density inside the domain
    ! Density inside the domain
    !
    !
    open (10,file='Dens_Bfield_In.dat',status='old')
    open (10,file='Dens_Bfield_In.dat',status='old')
    read (10,*) n_dens
    read (10,*) n_dens
    Allocate (Dens(n_dens), B_in(n_dens), Dist(n_dens))
    Allocate (Dens(n_dens), B_in(n_dens), Dist(n_dens))
    read (10,*) Dist
    read (10,*) Dist
    read (10,*) Dens ! in cm-3
    read (10,*) Dens ! in cm-3
    read (10,*) B_in ! in nT
    read (10,*) B_in ! in nT
    close (10)
    close (10)
    x = - StepX
    x = - StepX
    do i = 1, nx
    ```
    do i = 1, nx
```

```
        x = x + StepX
    do j = 1 , n_dens-1
    if (x .ge. Dist(j) .and. x .le. Dist(j+1)) then
        arg = (x-Dist(j))/(Dist(j+1)-Dist(j))
Den(i,1) = Dens(j) + (Dens(j+1)-Dens(j)) * arg
BB (i,1) = B_in(j) + (B_in(j+1)-B_in(j)) * arg
                    Goto 700
                                    end if
    end do
700
    continue
    end do
```

```
        do k = 2 , nz
```

        do k = 2 , nz
        Den(:,k) = Den(:,1)
        Den(:,k) = Den(:,1)
        BB (:,k) = BB (:,1)
        BB (:,k) = BB (:,1)
        end do
        end do
        Den = Den*1.0E+6
        Den = Den*1.0E+6
        BB = BB *1.0E-9
        BB = BB *1.0E-9
        D000 = D000 * 1.0E+6
        D000 = D000 * 1.0E+6
    open (10,file='Density_Out.dat',status='unknown')
open (10,file='Density_Out.dat',status='unknown')
write (10,52) nx,nz
write (10,52) nx,nz
write (10,50) Den
write (10,50) Den
close (10)
close (10)
open (10,file='Bfield_Out.dat',status='unknown')
open (10,file='Bfield_Out.dat',status='unknown')
write (10,52) nx,nz
write (10,52) nx,nz
write (10,50) BB
write (10,50) BB
close (10)
close (10)
DenV = 1.d0/(mu_0*e_0*Den) ! Parameter to calculate the RHP
DenV = 1.d0/(mu_0*e_0*Den) ! Parameter to calculate the RHP
! for Velocity
! for Velocity
**
*******************************************************************

```
**
**
****************************************************************
**
    SizeA = SizeA*SizeX*0.5d0
        x = - StepX - SizeX/2.0D0
        do i = 1, nx
            x = x + StepX
            arg = x / SizeA
    DuctX(i) = 1.0/exp(arg**GradA)
    end do
**
*******************************************************************
** Wave's Parameters *
*******************************************************************
**
```

```
            B000 = BB(nx/2+1,kb)
```

            B000 = BB(nx/2+1,kb)
            EmE = e_0 / m_e
            EmE = e_0 / m_e
    Omega_PE = sqrt(D000*e_0*EmE/eps_0)
Omega_PE = sqrt(D000*e_0*EmE/eps_0)
Omega_CE = EmE * B000
Omega_CE = EmE * B000
ALPHA = Omega_PE / ccc0
ALPHA = Omega_PE / ccc0
ALPHA = ALPHA * ALPHA
ALPHA = ALPHA * ALPHA
Omega = Pi2 * Omega
Omega = Pi2 * Omega
wKz = Pi2 / wLz
wKz = Pi2 / wLz
work1 = 0.5d0 * Omega_CE / Omega
work1 = 0.5d0 * Omega_CE / Omega
work1 = work1 * work1
work1 = work1 * work1
work2 = (wKz*Omega_CE/Omega)**2.d0
work2 = (wKz*Omega_CE/Omega)**2.d0
work2 = work2*m_e/e_0/e_0/4.0d0/mu_0
work2 = work2*m_e/e_0/e_0/4.0d0/mu_0
work2 = D000/work2
work2 = D000/work2
work2 = sqrt(1.d0 - work2)
work2 = sqrt(1.d0 - work2)
wKx1 = wKz*sqrt(work1*(1-work2)**2.0 - 1.d0)
wKx1 = wKz*sqrt(work1*(1-work2)**2.0 - 1.d0)
wKx2 = wKz*sqrt(work1*(1+work2)**2.0 - 1.d0)
wKx2 = wKz*sqrt(work1*(1+work2)**2.0 - 1.d0)
wKx22 = wKx2 ! For output only
wKx22 = wKx2 ! For output only
wLx1 = Pi2 / wKx1
wLx1 = Pi2 / wKx1
wLx2 = Pi2 / wKx2
wLx2 = Pi2 / wKx2
wKx = wKx1

```
    wKx = wKx1
```

```
        if (Key_Kx .eq. 2) wKx = wKx2
        wKx2 = wKx*wKx
        wKz2 = wKz*wKz
        wK2 = wKx2 + wKz2
        wK = sqrt(wK2)
**
***********************************************************
**
Dissipation
*********************************************************
**
            AmpC = AmpC * Omega
            X11 = SizeDis*SizeX
            X22 = SizeX - X11
            Dis = 0.0
                    x = -StepX
            Do i = 1 , nx
                x = x + StepX
            if (x < X11)
            & Dis(i,kb) = AmpC*((x-X11)/X11)**2.0
                if (x > X22)
            & Dis(i,kb) = AmpC*((x-X22)/X11)**2.0
                End Do
            Do k = 2 , nz
            Dis(:,k) = Dis(:,kb)
            End Do
**
*********************************************************
** Parameters SOR
************************************************************
**
```

```
Imax_SOR = 1e+4
```

Imax_SOR = 1e+4
Eps_SOR = 1.0e-8
Eps_SOR = 1.0e-8
do i = 1 , nx
do i = 1 , nx
do k = 1 , nz
do k = 1 , nz
EECX(i,k) = EmE - DenV(i,k)* DD0Z
EECX(i,k) = EmE - DenV(i,k)* DD0Z
EECY(i,k) = EmE - DenV(i,k)*(DD0X + DD0Z)
EECY(i,k) = EmE - DenV(i,k)*(DD0X + DD0Z)
EECZ(i,k) = EmE - DenV(i,k)* DD0X
EECZ(i,k) = EmE - DenV(i,k)* DD0X
end do

```
end do
```

* 

*****
***************************** SOR's training
*****
*
IF (C1_SOR == 0.0) then
print *,' Start SOR training'
print *,' SOR training is complete '
else
print *,' SOR is not trained '
C0_SOR = 1.D0 - C1_SOR
end if
**
********************************************************************
** Initial Conditions for E, B, V \& RHPs
********************************************************************
**

```
```

    AmpEy = AmpA
    ```
    AmpEy = AmpA
    AmpEx = AmpEy * Omega * (((Omega_CE/Omega)**2.0 - 1.0)
    AmpEx = AmpEy * Omega * (((Omega_CE/Omega)**2.0 - 1.0)
& * wK2/ALPHA - 1.0)/Omega_CE
& * wK2/ALPHA - 1.0)/Omega_CE
    AmpEz = AmpEx * wKz * wKx / (ALPHA + wKx2)
    AmpEz = AmpEx * wKz * wKx / (ALPHA + wKx2)
    AmpBx = - AmpEy * wKz / Omega
    AmpBx = - AmpEy * wKz / Omega
    AmpBy = -(AmpEz * wKx - AmpEx * wKz) / Omega
    AmpBy = -(AmpEz * wKx - AmpEx * wKz) / Omega
    AmpBz = - AmpEy * wKx / Omega
    AmpBz = - AmpEy * wKx / Omega
    AmpVx = wKz * (AmpEx * wKz - AmpEz * wKx) / Omega
    AmpVx = wKz * (AmpEx * wKz - AmpEz * wKx) / Omega
    AmpVy = - AmpEy * (wKx2 + wKz2) / Omega
    AmpVy = - AmpEy * (wKx2 + wKz2) / Omega
    AmpVz = wKx * (AmpEx * wKz - AmpEz * wKx) / Omega
    AmpVz = wKx * (AmpEx * wKz - AmpEz * wKx) / Omega
    SizeZ2 = SizeZ/2.00
    SizeZ2 = SizeZ/2.00
    SizeX2 = SizeX/2.00
    SizeX2 = SizeX/2.00
        time = -ht
        time = -ht
        do J = 1 , 4
```

        do J = 1 , 4
    ```
```

        time = time + ht
            t1 = time * Omega
        x = - StepX - SizeX2
    do i = 1 , nx
        x = x + StepX
    X_Cos = Cos(x * wKx) * DuctX(i)
X_Sin = Sin(x * wKx) * DuctX(i)
C X Cos = Cos(x * wKx) ! TEST
C X_Sin = Sin(x * wKx) ! TEST
z = - StepZ - SizeZ2
do k = 1, nz
z = z + StepZ
z1 = z * wKz - t1
Z_Cos = Cos(z1)
Z_Sin = Sin(z1)
Ex(i,k) = AmpEx * X_Cos * Z_Sin
Ey(i,k) = AmpEy * X_Cos * Z_Cos
Ez(i,k) = AmpEz * X_Sin * Z_Cos
Bx(i,k) = AmpBx * X_Cos * Z_Cos
By(i,k) = AmpBy * X_Cos * Z_Sin
Bz(i,k) = AmpBz * X_Sin * Z_Sin
Vx(i,k) = AmpVx * X_Cos * Z_Cos * DenV(i,k)
Vy(i,k) = AmpVy * X_Cos * Z_Sin * DenV(i,k)
Vz(i,k) = AmpVz * X_Sin * Z_Sin * DenV(i,k)
end do
end do
IF (Key_Conv == 1) CALL Convect (Vx, Vy, Vz)
RRX = -EmE * (Vy*(Bz+BB)-Vz*By) - X_WORK
RRY = EmE * (Vx*(Bz+BB)-Vz*Bx) - Y_WORK
RRZ = -EmE * (Vx*By - Vy*Bx) - Z_WORK
SOR1X = Ex
SOR1Y = Ey

```
```

SOR1Z = Ez

```

CALL SolverE
```

SELECT CASE (J)

```
CASE (1)
                    CALL RHP_V
        P_vx4 = RRX
        P_vy4 = RRY
        P_VZ4 = RRZ
            CALL RHP_B
        P_bx4 = RRX
        P_by4 = RRY
        P_bz4 = RRZ
CASE (2)
            CALL RHP_V
        P_vx3 = RRX
        P_vy3 = RRY
        P_vz3 = RRZ
            CALL RHP_B
    P_bx3 = RRX
    P_by3 = RRY
    P_bz3 = RRZ
CASE (3)
            CALL RHP_V
    P_vx2 = RRX
    P_vy2 = RRY
    P_vz2 = RRZ
            CALL RHP_B
    P_bx2 = RRX
    P_by2 = RRY
    P_bz2 = RRZ
CASE (4)
CASE DEFAULT
END SELECT
    end do
print *,' Initialization is complete '
DEALLOCATE (DuctX)
```

***
******************************* PRINT INFO
***
open (30,file='INFO',status='unknown')
write (30,900)
write (30,901) B000*1.0e+9, D000*1.0e-6
write (30,902) SizeX, SizeZ
write (30,903) wKz, wLz
write (30,904) wKx1, wLx1
write (30,905) wKx22, wLx2
write (30,906) Omega, Omega/Pi2
write (30,907) Omega_PE, Omega_CE
format(1x,' PARAMETERS:')
format(1x,' Bo (nT) ',e12.4,3x,'N (cm-3) ',e12.4)
format(1x,' Size X (m) ',e12.4,3x,'Size Z (m) ',e12.4)
format(1x,' WNum Z (rad/m) ',e12.4,3x,'Wlength Z (m) ',e12.4)
format(1x,' WNum1 X (rad/m) ',e12.4,3x,'Wlength1 X (m) ',e12.4)
format(1x,' WNum2 X (rad/m) ',e12.4,3x,'Wlength2 X (m) ',e12.4)
format(1x,' Omeg (r/s) ',e12.4,3x,'Freq. (Hz) ',e12.4)
format(1x,' Omeg_P (r/s) ',e12.4,3x,'Omeg_CE (r/s) ',e12.4)
close (30)
*
********************************************************************

* Multisteps *
*******************************************************************
* 

```


CALL RHP_V
P_VX1 = RRX
P_vy1 = RRY
P_vz1 = RRZ
CALL RHP_B
P_bx1 = RRX
P_by1 = RRY
P_bz1 = RRZ
\(!\)
\(!\)
\(V x \_P=V x+a b \_1^{*} p \_v x 1+a b \_2 * p \_v x 2\)
\&
\[
+a b \_3 * p \_v x 3+a b \_4 * p \_v x 4
\]
\(B x \_P=B x+a b \_1^{*} p \_b x 1+a b \_2 * p \_b x 2\)
\&
\(+a b \_3 * p \_b x 3+a b \_4 * p \_b x 4\)
\(!\quad Y\) componnents
\(V y \_P=V y+a b \_1 * p \_v y 1+a b \_2 * p \_v y 2\)
\[
+a b \_3 * p \_v y 3+a b \_4 * p \_v y 4
\]
\(B y \_P=B y+a b \_1 * p \_b y 1+a b \_2 * p \_b y 2\) + ab_3*p_by3 + ab_4*p_by4
Z componnents
Vz_P = Vz + ab_1*p_vz1 + ab_2*p_vz2 \(+a b \_3 * p \_v z 3+a b \_4 * p \_v z 4\)
\(B z \_P=B z+a b \_1 * p \_b z 1+a b \_2 * p \_b z 2\)
\[
+a b \_3 * p \_b z 3+a b \_4 * p \_b z 4
\]
**
\(* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *\)
\(* * \quad \mathrm{E}\) field I
\(* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *\) **

IF (Key_Conv == 1) CALL Convect (Vx_P, Vy_P, Vz_P)
```

    RRX = -EmE * (Vy_P*(Bz_P+BB)-Vz_P*By_P) - X_WORK
    & -Dis * Vx_P
    RRY = EmE * (Vx_P*(Bz_P+BB)-Vz_P*Bx_P) - Y_WORK
                -Dis * Vy_P
    RRZ = -EmE * (Vx_P*By_P - Vy_P*Bx_P) - Z_WORK
        -Dis * Vz_P
    CALL SolverE
    if (I_SOR > Ima_SOR) Ima_SOR = I_SOR
    * 

*********************************************************

*     * 
* Re-organization of POINTERS *
*     * 
* p_work => p_vx1
p_vx1 => p_vx4
p_vx4 => p_vx3
p_vx3 => p_vx2
p_vx2 => p_work
p_work => p_vy1
p_vy1 => p_vy4
p_vy4 => p_vy3
p_vy3 => p_vy2
p_vy2 => p_work
p_work => p_vz1
p_vz1 => p_vz4
p_vz4 => p_vz3
p_vz3 => p_vz2
p_vz2 => p_work
p_work => p_bx1
p_bx1 => p_bx4
p_bx4 => p_bx3
p_bx3 => p_bx2
p_bx2 => p_work
p_work => p_by1
p_by1 => p_by4
p_by4 => p_by3
p_by3 => p_by2
p_by2 => p_work

```
```

    p_work => p_bz1
    p_bz1 => p_bz4
    p_bz4 => p_bz3
    p_bz3 => p_bz2
    p_bz2 => p_work
    **
*****************************************************************
**
Corrector
*****************************************************************
**
**
****************************************************************
** Right hand Parts
*****************************************************************
**

```

CALL RHP_V
    P_Vx1 = RRX
    P_vy1 = RRY
    P_vz1 = RRZ

CALL RHP_B

P_bx1 = RRX
P_by1 = RRY
P_bz1 = RRZ
!
! X componnents
!
\(V x=V x+a m \_1^{*} p \_v x 1+a m \_2 * p \_v x 2\)
\&
\[
+a m \_3 * p \_v x 3+a m \_4 * p \_v x 4
\]
\(B x=B x+a m \_1 * p \_b x 1+a m \_2 * p \_b x 2\)
\&
!
\(!\quad Y\) componnents
!
\(V y=V y+a m \_1 * p \_v y 1+a m \_2 * p \_v y 2\)
\& + am_3*p_vy3 + am_4*p_vy4

By \(=\) By + am_1*p_by1 + am_2*p_by2
\[
+ \text { am_3*p_by3 + am_4*p_by4 }
\]
```

            ! Z componnents
    Vz = Vz + am_1*p_vz1 + am_2*p_vz2
    &
    Bz = Bz + am_1*p_bz1 + am_2*p_bz2
    & + am_3*p_bz3 + am_4*p_bz4
    **
*******************************************************************
** E field II
********************************************************************
**
IF (Key_Conv == 1) CALL Convect (Vx, Vy, Vz)
RRX = -EmE * (Vy*(Bz+BB)-Vz*By) - X_WORK
-Dis * Vx
RRY = EmE * (Vx*(Bz+BB)-Vz*Bx) - Y_WORK
-Dis * Vy
RRZ = -EmE * (Vx*By - Vy*Bx) - Z_WORK
-Dis * Vz
CALL SolverE
if (I_SOR > Ima_SOR) Ima_SOR = I_SOR
icount1 = icount1 + 1
if (icount1 == iten) then

```
```

            icount1 = 0
    ```
            icount1 = 0
            icount2 = icount2 + 1
            icount2 = icount2 + 1
        ExA(icount2) = SOR1X(icen,kcen)
        ExA(icount2) = SOR1X(icen,kcen)
        EyA(icount2) = SOR1Y(icen,kcen)
        EyA(icount2) = SOR1Y(icen,kcen)
        EzA(icount2) = SOR1Z(icen,kcen)
        EzA(icount2) = SOR1Z(icen,kcen)
ExM(:,icount2) = SOR1X(:,kcen)
EyM(:,icount2) = SOR1Y(:,kcen)
EzM(:,icount2) = SOR1Z(:,kcen)
        BxA(icount2) = Bx(icen,kcen)
        ByA(icount2) = By(icen,kcen)
        BzA(icount2) = Bz(icen,kcen)
            end if
end do
                                ! End of The Main JJJ Loop
```

```
*
****
*********
****
*
print *,' Step # ',iii,' Max # of itter. ',Ima_SOR
if (iii >= ntime3) then
name1 = 'vx000'
name2 = 'vy000'
name3 = 'vz000'
name4 = 'ex000'
name5 = 'ey000'
name6 = 'ez000'
name7 = 'bx000'
name8 = 'by000'
name9 = 'bz000'
iw = iii/1000
    name1(2:2) = Num (iw + 1)
    name2(2:2) = Num (iw + 1)
    name3(2:2) = Num (iw + 1)
    name4(2:2) = Num (iw + 1)
    name5(2:2) = Num (iw + 1)
    name6(2:2) = Num (iw + 1)
    name7(2:2) = Num (iw + 1)
    name8(2:2) = Num (iw + 1)
    name9(2:2) = Num (iw + 1)
iw1 = (iii - iw*1000)/100
name1(3:3) = Num (iw1 + 1)
name2(3:3) = Num (iw1 + 1)
name3(3:3) = Num (iw1 + 1)
name4(3:3) = Num (iw1 + 1)
name5(3:3) = Num (iw1 + 1)
name6(3:3) = Num (iw1 + 1)
name7(3:3) = Num (iw1 + 1)
name8(3:3) = Num (iw1 + 1)
name9(3:3) = Num (iw1 + 1)
iw2 = (iii - iw*1000 - iw1*100)/10
name1(4:4) = Num (iw2 + 1)
name2(4:4) = Num (iw2 + 1)
name3(4:4) = Num (iw2 + 1)
name4(4:4) = Num (iw2 + 1)
name5(4:4) = Num (iw2 + 1)
name6(4:4) = Num (iw2 + 1)
```

```
name7(4:4) = Num (iw2 + 1)
name8(4:4) = Num (iw2 + 1)
name9(4:4) = Num (iw2 + 1)
iw3 = iii - iw*1000 - iw1*100 - iw2*10
name1(5:5) = Num (iw3 + 1)
name2(5:5) = Num (iw3 + 1)
name3(5:5) = Num (iw3 + 1)
name4(5:5) = Num (iw3 + 1)
name5(5:5) = Num (iw3 + 1)
name6(5:5) = Num (iw3 + 1)
name7(5:5) = Num (iw3 + 1)
name8(5:5) = Num (iw3 + 1)
name9(5:5) = Num (iw3 + 1)
open (20, file=name1, status='unknown')
write (20,52) nx,nz
write (20,50) Vx
close (20)
open (20, file=name2, status='unknown')
write (20,52) nx,nz
write (20,50) Vy
close (20)
open (20, file=name3, status='unknown')
write (20,52) nx,nz
write (20,50) Vz
close (20)
open (20, file=name4, status='unknown')
write (20,52) nx,nz
write (20,50) Ex
close (20)
open (20, file=name5, status='unknown')
write (20,52) nx,nz
write (20,50) Ey
close (20)
open (20, file=name6, status='unknown')
write (20,52) nx,nz
write (20,50) Ez
close (20)
open (20, file=name7, status='unknown')
write (20,52) nx,nz
write (20,50) Bx
close (20)
open (20, file=name8, status='unknown')
```

```
    write (20,52) nx,nz
    write (20,50) By
    close (20)
    open (20, file=name9, status='unknown')
    write (20,52) nx,nz
    write (20,50) Bz
    close (20)
        end if
    End Do ! End of The Main III Loop
    open (20, file='EBfields.dat', status='unknown')
    write (20,52) icount2
    time = 0
    do i = 1, icount2
    time = time + ht5
    write (20,50) time, ExA(i), EyA(i), EzA(i),
&
end do
close (20)
open (20, file='Exfield.dat', status='unknown')
write (20,52) nx,icount2
write (20,50) ExM
close (20)
open (20, file='Eyfield.dat', status='unknown')
write (20,52) nx,icount2
write (20,50) EyM
close (20)
open (20, file='Ezfield.dat', status='unknown')
write (20,52) nx,icount2
write (20,50) EzM
close (20)
format (10(1x,e11.5))
format (1x, i6)
format (1x,2i6)
format (1x,5i6)
format (1x,5F16.7)
END
```

```
**
***
**
*
Subroutine SolverE
```



Use global_data
IMPLICIT DOUBLE PRECISION (A-H,O-Z)

```
I_SOR = 0
I_SOR = I_SOR + 1
if (i_sor > Imax_SOR) then
                    print *,' Emergency: Iterations diverge !!!'
                stop
                    end if
```

760
Sorex = Sor1X
SoreY = Sor1Y
SoreZ = Sor1Z
do $\mathrm{i}=1$, nx

```
ip1 = i + 1 ; if (ip1 > nx) ip1 = ip1 - nx
ip2 = i + 2 ; if (ip2 > nx) ip2 = ip2 - nx
ip3 = i + 3 ; if (ip3 > nx) ip3 = ip3 - nx
ip4 = i + 4 ; if (ip4 > nx) ip4 = ip4 - nx
im1 = i - 1 ; if (im1 < 1) im1 = im1 + nx
im2 = i - 2 ; if (im2 < 1) im2 = im2 + nx
im3 = i - 3 ; if (im3 < 1) im3 = im3 + nx
im4 = i - 4 ; if (im4 < 1) im4 = im4 + nx
do k = 1 , nz
        kp1 = k + 1 ; if (kp1 > nz) kp1 = kp1 - nz
        kp2 = k + 2 ; if (kp2 > nz) kp2 = kp2 - nz
        kp3 = k + 3 ; if (kp3 > nz) kp3 = kp3 - nz
        kp4 = k + 4 ; if (kp4 > nz) kp4 = kp4 - nz
        km1 = k - 1 ; if (km1 < 1) km1 = km1 + nz
```

```
km2 = k - 2 ; if (km2 < 1) km2 = km2 + nz
km3 = k - 3 ; if (km3 < 1) km3 = km3 + nz
km4 = k - 4 ; if (km4 < 1) km4 = km4 + nz
W1U = D1X*(Sor1Z(ip1,kp1)-Sor1Z(im1,kp1))
    + D2X*(Sor1Z(ip2,kp1)-Sor1Z(im2,kp1))
    + D3X*(Sor1Z(ip3,kp1)-Sor1Z(im3,kp1))
    + D4X*(Sor1Z(ip4,kp1)-Sor1Z(im4,kp1))
W2U = D1X*(Sor1Z(ip1,kp2)-Sor1Z(im1,kp2))
    + D2X*(Sor1Z(ip2,kp2)-Sor1Z(im2,kp2))
    + D3X*(Sor1Z(ip3,kp2)-Sor1Z(im3,kp2))
    + D4X*(Sor1Z(ip4,kp2)-Sor1Z(im4,kp2))
W3U = D1X*(Sor1Z(ip1,kp3)-Sor1Z(im1,kp3))
    + D2X*(Sor1Z(ip2,kp3)-Sor1Z(im2,kp3))
    + D3X*(Sor1Z(ip3,kp3)-Sor1Z(im3,kp3))
    + D4X*(Sor1Z(ip4,kp3)-Sor1Z(im4,kp3))
W4U = D1X*(Sor1Z(ip1,kp4)-Sor1Z(im1,kp4))
    + D2X*(Sor1Z(ip2,kp4)-Sor1Z(im2,kp4))
    + D3X*(Sor1Z(ip3,kp4)-Sor1Z(im3,kp4))
    + D4X*(Sor1Z(ip4,kp4)-Sor1Z(im4,kp4))
W1D = D1X*(Sor1Z(ip1,km1)-Sor1Z(im1,km1))
    + D2X*(Sor1Z(ip2,km1)-Sor1Z(im2,km1))
    + D3X*(Sor1Z(ip3,km1)-Sor1Z(im3,km1))
    + D4X*(Sor1Z(ip4,km1)-Sor1Z(im4,km1))
W2D = D1X*(Sor1Z(ip1,km2)-Sor1Z(im1,km2))
    + D2X*(Sor1Z(ip2,km2)-Sor1Z(im2,km2))
    + D3X*(Sor1Z(ip3,km2)-Sor1Z(im3,km2))
    + D4X*(Sor1Z(ip4,km2)-Sor1Z(im4,km2))
W3D = D1X*(Sor1Z(ip1,km3)-Sor1Z(im1,km3))
    + D2X*(Sor1Z(ip2,km3)-Sor1Z(im2,km3))
    + D3X*(Sor1Z(ip3,km3)-Sor1Z(im3,km3))
    + D4X*(Sor1Z(ip4,km3)-Sor1Z(im4,km3))
W4D = D1X*(Sor1Z(ip1,km4)-Sor1Z(im1,km4))
    + D2X*(Sor1Z(ip2,km4)-Sor1Z(im2,km4))
    + D3X*(Sor1Z(ip3,km4)-Sor1Z(im3,km4))
    + D4X*(Sor1Z(ip4,km4)-Sor1Z(im4,km4))
Work1 = D1Z*(W1U - W1D) + D2Z*(W2U - W2D)
    + D3Z*(W3U - W3D) + D4Z*(W4U - W4D)
Work2 = DD1Z*(Sor1X(i,km1) + Sor1X(i,kp1))
    + DD2Z*(Sor1X(i,km2) + Sor1X(i,kp2))
    + DD3Z*(Sor1X(i,km3) + Sor1X(i,kp3))
```

\&
+ DD4Z*(Sor1X(i,km4) + Sor1X(i,kp4))

```
Sor1X(i,k) = (RRX(i,k) - DenV(i,k)
    * (Work1 - Work2))/EECX(i,k)
Sor1Y(i,k) = (RRY(i,k) + DenV(i,k) *
    (DD1X*(Sor1Y(im1,k) + Sor1Y(ip1,k))
    + DD2X*(Sor1Y(im2,k) + Sor1Y(ip2,k))
    + DD3X*(Sor1Y(im3,k) + Sor1Y(ip3,k))
    + DD4X*(Sor1Y(im4,k) + Sor1Y(ip4,k))
    + DD1Z*(Sor1Y(i,kp1) + Sor1Y(i,km1))
    + DD2Z*(Sor1Y(i,kp2) + Sor1Y(i,km2))
    + DD3Z*(Sor1Y(i,kp3) + Sor1Y(i,km3))
    + DD4Z*(Sor1Y(i,kp4) + Sor1Y(i,km4))))
    / EECY(i,k)
W1U = D1X*(Sor1X(ip1,kp1)-Sor1X(im1,kp1))
    + D2X*(Sor1X(ip2,kp1)-Sor1X(im2,kp1))
    + D3X*(Sor1X(ip3,kp1)-Sor1X(im3,kp1))
    + D4X*(Sor1X(ip4,kp1)-Sor1X(im4,kp1))
```

```
W2U = D1X*(Sor1X(ip1,kp2)-Sor1X(im1,kp2))
    + D2X*(Sor1X(ip2,kp2)-Sor1X(im2,kp2))
    + D3X*(Sor1X(ip3,kp2)-Sor1X(im3,kp2))
    + D4X*(Sor1X(ip4,kp2)-Sor1X(im4,kp2))
```

W3U = D1X*(Sor1X(ip1,kp3)-Sor1X(im1,kp3))
+ D2X*(Sor1X(ip2,kp3)-Sor1X(im2,kp3))
+ D3X*(Sor1X(ip3,kp3)-Sor1X(im3,kp3))
+ D4X*(Sor1X(ip4,kp3)-Sor1X(im4,kp3))
W4U = D1X*(Sor1X(ip1,kp4)-Sor1X(im1,kp4))
+ D2X*(Sor1X(ip2,kp4)-Sor1X(im2,kp4))
+ D3X*(Sor1X(ip3,kp4)-Sor1X(im3,kp4))
+ D4X*(Sor1X(ip4,kp4)-Sor1X(im4,kp4))

```
W1D = D1X*(Sor1X(ip1,km1)-Sor1X(im1,km1))
    + D2X*(Sor1X(ip2,km1)-Sor1X(im2,km1))
    + D3X*(Sor1X(ip3,km1)-Sor1X(im3,km1))
    + D4X*(Sor1X(ip4,km1)-Sor1X(im4,km1))
```

W2D = D1X*(Sor1X(ip1,km2)-Sor1X(im1,km2))
+ D2X*(Sor1X(ip2,km2)-Sor1X(im2,km2))
+ D3X*(Sor1X(ip3,km2)-Sor1X(im3,km2))
+ D4X*(Sor1X(ip4,km2)-Sor1X(im4,km2))
W3D $=$ D1X*(Sor1X(ip1,km3)-Sor1X(im1,km3))
+ D2X*(Sor1X(ip2,km3)-Sor1X(im2,km3))
+ D3X*(Sor1X(ip3,km3)-Sor1X(im3,km3))
+ D4X*(Sor1X(ip4,km3)-Sor1X(im4,km3))

```
        W4D = D1X*(Sor1X(ip1,km4)-Sor1X(im1,km4))
    + D2X*(Sor1X(ip2,km4)-Sor1X(im2,km4))
    + D3X*(Sor1X(ip3,km4)-Sor1X(im3,km4))
    + D4X*(Sor1X(ip4,km4)-Sor1X(im4,km4))
Work1 = D1Z*(W1U - W1D) + D2Z*(W2U - W2D)
    + D3Z*(W3U - W3D) + D4Z*(W4U - W4D)
Work2 = DD1X*(Sor1Z(im1,k) + Sor1Z(ip1,k))
    + DD2X*(Sor1Z(im2,k) + Sor1Z(ip2,k))
    + DD3X*(Sor1Z(im3,k) + Sor1Z(ip3,k))
    + DD4X*(Sor1Z(im4,k) + Sor1Z(ip4,k))
    Sor1Z(i,k) = (RRZ(i,k) - DenV(i,k) *
        (Work1 - Work2)) / EECZ(i,k)
    Sor1X(i,k) = C1_SOR*SOr1X(i,k) + C0_SOR*Sor0X(i,k)
    Sor1Y(i,k) = C1_SOR*Sor1Y(i,k) + C0_SOR*Sor0Y(i,k)
    Sor1Z(i,k) = C1_SOR*Sor1Z(i,k) + C0_SOR*Sor0Z(i,k)
        end do
    end do
    work1 = MAXVAL(ABS(sor1X - sor0X))
    work0 = MAXVAL(ABS(sor1X
    if (work1 > Eps_SOR*work0) GOTO 760
    work1 = MAXVAL(ABS(sor1Y - sor0Y))
    work0 = MAXVAL(ABS(sor1Y ))
    if (work1 > Eps_SOR*work0) GOTO 760
    work1 = MAXVAL(ABS(sor1Z - sor0Z))
work0 = MAXVAL(ABS(sor1Z
    if (work1 > Eps_SOR*work0) GOTO 760
End Subroutine SolverE
*
**
***
**
*
Subroutine RHP_V
The suroutine calculates Laplace of E
```

```
Use global_data
do i = 1 , nx
ip1 = i + 1 ; if (ip1 > nx) ip1 = ip1 - nx
ip2 = i + 2 ; if (ip2 > nx) ip2 = ip2 - nx
ip3 = i + 3 ; if (ip3 > nx) ip3 = ip3 - nx
ip4 = i + 4 ; if (ip4 > nx) ip4 = ip4 - nx
im1 = i - 1 ; if (im1 < 1) im1 = im1 + nx
im2 = i - 2 ; if (im2 < 1) im2 = im2 + nx
im3 = i - 3 ; if (im3 < 1) im3 = im3 + nx
im4 = i - 4 ; if (im4< 1) im4 = im4 + nx
do k = 1 , nz
    kp1 = k + 1 ; if (kp1 > nz) kp1 = kp1 - nz
    kp2 = k + 2 ; if (kp2 > nz) kp2 = kp2 - nz
    kp3 = k + 3 ; if (kp3 > nz) kp3 = kp3 - nz
    kp4 = k + 4 ; if (kp4 > nz) kp4 = kp4 - nz
    km1 = k - 1 ; if (km1 < 1) km1 = km1 + nz
    km2 = k - 2 ; if (km2 < 1) km2 = km2 + nz
    km3 = k - 3 ; if (km3 < 1) km3 = km3 + nz
    km4 = k - 4 ; if (km4 < 1) km4 = km4 + nz
    W1U = D1X*(Sor1Z(ip1,kp1)-Sor1Z(im1,kp1))
        + D2X*(Sor1Z(ip2,kp1)-Sor1Z(im2,kp1))
        + D3X*(Sor1Z(ip3,kp1)-Sor1Z(im3,kp1))
        + D4X*(Sor1Z(ip4,kp1)-Sor1Z(im4,kp1))
    W2U = D1X*(Sor1Z(ip1,kp2)-Sor1Z(im1,kp2))
        + D2X*(Sor1Z(ip2,kp2)-Sor1Z(im2,kp2))
        + D3X*(Sor1Z(ip3,kp2)-Sor1Z(im3,kp2))
        + D4X*(Sor1Z(ip4,kp2)-Sor1Z(im4,kp2))
W3U = D1X*(Sor1Z(ip1,kp3)-Sor1Z(im1,kp3))
    + D2X*(Sor1Z(ip2,kp3)-Sor1Z(im2,kp3))
    + D3X*(Sor1Z(ip3,kp3)-Sor1Z(im3,kp3))
    + D4X*(Sor1Z(ip4,kp3)-Sor1Z(im4,kp3))
W4U = D1X*(Sor1Z(ip1,kp4)-Sor1Z(im1,kp4))
    + D2X*(Sor1Z(ip2,kp4)-Sor1Z(im2,kp4))
    + D3X*(Sor1Z(ip3,kp4)-Sor1Z(im3,kp4))
    + D4X*(Sor1Z(ip4,kp4)-Sor1Z(im4,kp4))
W1D = D1X*(Sor1Z(ip1,km1)-Sor1Z(im1,km1))
    + D2X*(Sor1Z(ip2,km1)-Sor1Z(im2,km1))
    + D3X*(Sor1Z(ip3,km1)-Sor1Z(im3,km1))
```

    + D4X*(Sor1Z(ip4,km1)-Sor1Z(im4,km1))
    ```
    W2D = D1X*(Sor1Z(ip1,km2)-Sor1Z(im1,km2))
    + D2X*(Sor1Z(ip2,km2)-Sor1Z(im2,km2))
    + D3X*(Sor1Z(ip3,km2)-Sor1Z(im3,km2))
    + D4X*(Sor1Z(ip4,km2)-Sor1Z(im4,km2))
```

```
W3D = D1X*(Sor1Z(ip1,km3)-Sor1Z(im1,km3))
    + D2X*(Sor1Z(ip2,km3)-Sor1Z(im2,km3))
    + D3X*(Sor1Z(ip3,km3)-Sor1Z(im3,km3))
    + D4X*(Sor1Z(ip4,km3)-Sor1Z(im4,km3))
```

    W4D \(=\) D1X* (Sor1Z(ip1,km4)-Sor1Z(im1,km4))
    + D2X*(Sor1Z(ip2,km4)-Sor1Z(im2,km4))
    + D3X*(Sor1Z(ip3,km4)-Sor1Z(im3,km4))
    + D4X*(Sor1Z(ip4,km4)-Sor1Z(im4,km4))
    $R R X(i, k)=D 1 Z *(W 1 U-W 1 D)+D 2 Z^{*}(W 2 U-W 2 D)$
+ D3Z* (W3U - W3D) + D4Z* (W4U - W4D)
- DD1Z*(Sor1X(i,kp1) + Sor1X(i,km1))
- DD2Z*(Sor1X(i,kp2) + Sor1X(i,km2))
- DD3Z*(Sor1X(i,kp3) + Sor1X(i,km3))
- DD4Z*(Sor1X(i,kp4) + Sor1X(i,km4))
- DD0Z* Sor1X(i ,k)
$\operatorname{RRY}(i, k)=\operatorname{DD1X*}(\operatorname{Sor} 1 Y(i p 1, k)+\operatorname{Sor} 1 Y(i m 1, k))$
+ DD2X*(Sor1Y(ip2,k) + Sor1Y(im2,k))
+ DD3X*(Sor1Y(ip3,k) + Sor1Y(im3,k))
+ DD4X*(Sor1Y(ip4,k) + Sor1Y(im4,k))
$+\operatorname{Sor} 1 Y(i, k) *(D D 0 X+D D 0 Z)$
+ DD1Z* (Sor1Y(i,kp1) + Sor1Y(i,km1))
+ DD2Z*(Sor1Y(i,kp2) + Sor1Y(i,km2))
+ DD3Z*(Sor1Y(i,kp3) + Sor1Y(i,km3))
$+\operatorname{DD4Z}$ (Sor1Y(i,kp4) + Sor1Y(i,km4))

```
W1U = D1X*(Sor1X(ip1,kp1)-Sor1X(im1,kp1))
    + D2X*(Sor1X(ip2,kp1)-Sor1X(im2,kp1))
    + D3X*(Sor1X(ip3,kp1)-Sor1X(im3,kp1))
    + D4X*(Sor1X(ip4,kp1)-Sor1X(im4,kp1))
```

```
W2U = D1X*(Sor1X(ip1,kp2)-Sor1X(im1,kp2))
    + D2X*(Sor1X(ip2,kp2)-Sor1X(im2,kp2))
    + D3X*(Sor1X(ip3,kp2)-Sor1X(im3,kp2))
    + D4X*(Sor1X(ip4,kp2)-Sor1X(im4,kp2))
```

W3U = D1X*(Sor1X(ip1,kp3)-Sor1X(im1,kp3))
+ D2X*(Sor1X(ip2,kp3)-Sor1X(im2,kp3))
+ D3X*(Sor1X(ip3,kp3)-Sor1X(im3,kp3))
+ D4X*(Sor1X(ip4,kp3)-Sor1X(im4,kp3))

```
    W4U = D1X*(Sor1X(ip1,kp4)-Sor1X(im1,kp4))
    + D2X*(Sor1X(ip2,kp4)-Sor1X(im2,kp4))
    + D3X*(Sor1X(ip3,kp4)-Sor1X(im3,kp4))
    + D4X*(Sor1X(ip4,kp4)-Sor1X(im4,kp4))
    W1D = D1X*(Sor1X(ip1,km1)-Sor1X(im1,km1))
    + D2X*(Sor1X(ip2,km1)-Sor1X(im2,km1))
    + D3X*(Sor1X(ip3,km1)-Sor1X(im3,km1))
    + D4X*(Sor1X(ip4,km1)-Sor1X(im4,km1))
    W2D = D1X*(Sor1X(ip1,km2)-Sor1X(im1,km2))
    + D2X*(Sor1X(ip2,km2)-Sor1X(im2,km2))
    + D3X*(Sor1X(ip3,km2)-Sor1X(im3,km2))
    + D4X*(Sor1X(ip4,km2)-Sor1X(im4,km2))
    W3D = D1X*(Sor1X(ip1,km3)-Sor1X(im1,km3))
    + D2X*(Sor1X(ip2,km3)-Sor1X(im2,km3))
    + D3X*(Sor1X(ip3,km3)-Sor1X(im3,km3))
    + D4X*(Sor1X(ip4,km3)-Sor1X(im4,km3))
    W4D = D1X*(Sor1X(ip1,km4)-Sor1X(im1,km4))
    + D2X*(Sor1X(ip2,km4)-Sor1X(im2,km4))
    + D3X*(Sor1X(ip3,km4)-Sor1X(im3,km4))
    + D4X*(Sor1X(ip4,km4)-Sor1X(im4,km4))
RRZ(i,k) = D1Z*(W1U - W1D) + D2Z*(W2U - W2D)
        + D3Z*(W3U - W3D) + D4Z*(W4U - W4D)
        - DD1X*(Sor1Z(ip1,k) + Sor1Z(im1,k))
        - DD2X*(Sor1Z(ip2,k) + Sor1Z(im2,k))
        - DD3X*(Sor1Z(ip3,k) + Sor1Z(im3,k))
        - DD4X*(Sor1Z(ip4,k) + Sor1Z(im4,k))
        - DD0X* Sor1Z(i ,k)
        end do
end do
RRX = RRX * DenV
RRY = -RRY * DenV
RRZ = RRZ * DenV
End Subroutine RHP_V
Subroutine RHP_B
```

The suroutine calculates right-hand-part of the Faraday's law

```
Use global_data
do i = 1 , nx
ip1 = i + 1 ; if (ip1 > nx) ip1 = ip1 - nx
ip2 = i + 2 ; if (ip2 > nx) ip2 = ip2 - nx
ip3 = i + 3 ; if (ip3 > nx) ip3 = ip3 - nx
ip4 = i + 4 ; if (ip4 > nx) ip4 = ip4 - nx
im1 = i - 1 ; if (im1 < 1) im1 = im1 + nx
im2 = i - 2 ; if (im2 < 1) im2 = im2 + nx
im3 = i - 3 ; if (im3< 1) im3 = im3 + nx
im4 = i - 4 ; if (im4 < 1) im4 = im4 + nx
do k = 1 , nz
        kp1 = k + 1 ; if (kp1 > nz) kp1 = kp1 - nz
        kp2 = k + 2 ; if (kp2 > nz) kp2 = kp2 - nz
        kp3 = k + 3 ; if (kp3 > nz) kp3 = kp3 - nz
        kp4 = k + 4 ; if (kp4 > nz) kp4 = kp4 - nz
        km1 = k - 1 ; if (km1 < 1) km1 = km1 + nz
        km2 = k - 2 ; if (km2 < 1) km2 = km2 + nz
        km3 = k - 3 ; if (km3 < 1) km3 = km3 + nz
        km4 = k - 4 ; if (km4 < 1) km4 = km4 + nz
RRX(i,k) = D1Z*(Sor1Y(i,kp1) - Sor1Y(i,km1))
            + D2Z*(Sor1Y(i,kp2) - Sor1Y(i,km2))
            + D3Z*(Sor1Y(i,kp3) - Sor1Y(i,km3))
            + D4Z*(Sor1Y(i,kp4) - Sor1Y(i,km4))
RRY(i,k) = D1X*(Sor1Z(ip1,k) - Sor1Z(im1,k))
    + D2X*(Sor1Z(ip2,k) - Sor1Z(im2,k))
    + D3X*(Sor1Z(ip3,k) - Sor1Z(im3,k))
    + D4X*(Sor1Z(ip4,k) - Sor1Z(im4,k))
    - D1Z*(Sor1X(i,kp1) - Sor1X(i,km1))
    - D2Z*(Sor1X(i,kp2) - Sor1X(i,km2))
    - D3Z*(Sor1X(i,kp3) - Sor1X(i,km3))
    - D4Z*(Sor1X(i,kp4) - Sor1X(i,km4))
RRZ(i,k) = D1X*(Sor1Y(ip1,k) - Sor1Y(im1,k))
    + D2X*(Sor1Y(ip2,k) - Sor1Y(im2,k))
    + D3X*(Sor1Y(ip3,k) - Sor1Y(im3,k))
    + D4X*(Sor1Y(ip4,k) - Sor1Y(im4,k))
```

end do
end do

```
RRZ = - RRZ
End Subroutine RHP_B
Subroutine Convect (Vx, Vy, Vz)
The suroutine calculates \(X\) convective term in the momentum equation
```

Use global_data
IMPLICIT DOUBLE PRECISION (A-H,O-Z)

Real*8 : :
\& $\quad V x(n x, n z), V y(n x, n z), V z(n x, n z)$
do $i=1$, $n x$
i1 = i + 1 ; if (i1 > nx) i1 = 1
$i 2=i-1$; if (i2 < 1 ) i2 = nx
do $k=1$, $n z$
$\mathrm{k} 1=\mathrm{k}+1$; if (k1 >nz) k1 = 1 $k 2=k-1$; if $(k 2<1) k 2=n z$
$V x \_L=V x(i 2, k)+V x(i, k)$
$V x \_R=V x(i 1, k)+V x(i, k)$
$V z \_D=V z(i, k 2)+V z(i, k)$
$V z \_U=V z(i, k 1)+V z(i, k)$
if (Vx_L > 0) then
i_L = i2
else
i_L = i
end if
if (Vx_R > 0) then
$i \_R=i$
else
i $R=i 1$
end if
if (Vz_D > 0) then

```
            k_D \(=\) k2
                            else
                k_D = k
                                    end if
            if (Vz_U > 0) then
                k_U = k
                else
            k_U = k1
                    end if
\(X \_\operatorname{WORK}(i, k)=\left(V x \_R * \operatorname{Den}\left(i \_R, k\right) * V x\left(i \_R, k\right)\right.\)
    - Vx_L*Den(i_L,k)*Vx(i_L,k))/StepX
    \(+\left(V z \_U^{*} \operatorname{Den}\left(i, k \_U\right) * V x\left(i, k \_U\right)\right.\)
    - \(\left.\quad V z_{-}^{-} D * \operatorname{Den}\left(i, k_{-} D\right) * V x\left(i, k \_D\right)\right) /\) StepZ
Y_WORK \((i, k)=\left(V x \_R * \operatorname{Den}\left(i \_R, k\right) * V y\left(i \_R, k\right)\right.\)
    - \(V x\) _L*Den(i_L,k)*Vy(i_L,k))/StepX
    \(+\left(V z \_U^{*} \operatorname{Den}\left(i, k \_U\right) * V y\left(i, k \_U\right)\right.\)
    - Vz_D*Den(i,k_D)*Vy(i,k_D))/StepZ
Z_WORK (i,k) = (Vx_R*Den(i_R,k)*Vz(i_R,k)
    - \(V x\) _L*Den(i_L,k)*Vz(i_L,k))/StepX
    \(+\left(V z \_U^{*} \operatorname{Den}\left(i, k \_U\right) * V z\left(i, k \_U\right)\right.\)
    - Vz_D*Den(i,k_D)*Vz(i,k_D))/StepZ
end do
end do
X_WORK = D05 * X_WORK / Den
Y_WORK = D05 * Y_WORK / Den
Z_WORK = D05 * Z_WORK / Den
End Subroutine Convect
```


## APPENDIX D

WIDE DUCT CODE

```
clc
clear
me = 9.1094e-31;
qe = 1.6022e-19;
mu0 = pi*4.0e-7;
const = me/(mu0*qe*qe);
j = 888;
b = load('wideb.mat', 'data');
n = load('widen.mat', 'data');
b = cell2mat(struct2cell(b));
n = cell2mat(struct2cell(n));
figure
plot(n)
figure
plot(b)
q_par1 = 10500; %parrallel wavelength in m
q_par2 = 16000; %parrallel wavelength in m
omega = 400; %freq in Hz
step_size = (q_par2 - q_par1)/(j-1);
omega = 2*pi*omega;
qqq1 = const*q_par2;
for i = 1: j
q_par(i) = q_par1 + step_size * (i-1);
q_par(i) = 2 * pi / q_par(i);
q_par2(i) = q_par(i) * q_par(i) * 1e-6;
qqq1(i)= const * q_par2(i);
omega_ce(i) = 176*b(i);
ratio(i) = omega_ce(i)/omega;
n1(i) = qqq1(i) * (ratio(i)-1);
end
n1 = n1.';
```

```
for i = 1: j
```

non1 (i) $=n(i) / n 1(i) ;$
end
figure
plot(non1);




## APPENDIX E

## EVENT CATALOG

## E. 1 Narrow Catalog

|  |  | Start | End |  | Start |  | End |  | Start | End | MLT | Lshell | mlat |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Date | Time | B [ nT ] |  | n peak [ $\mathrm{cm}^{-3}$ ] | f range [ Hz ] |  | $\mathrm{f}[\mathrm{Hz}]$ |  | change in $n$ |  |  |  |  |
| 4/2/2013 | 2013-04-02 6:19:02.594780 to 6:31:35.958179 | 176 | 166 | 135 | 230 |  | 90 | 1300 | 112 | 136 | 23.11908 | 5.436467 | 7.700768 |
|  |  |  |  |  |  |  |  |  |  |  | 23.25608 | 5.569585 | 8.057347 |
|  | 2013-04-02 6:57:00.323487 to 6:59:47.301862 | 153 | 150 | 77 | 140 |  | 70 | 470 | 63 | 84 | 23.52188 | 5.782365 | 8.707446 |
|  |  |  |  |  |  |  |  |  |  |  | 23.54902 | 5.800148 | 8.770689 |
|  | 2013-04-02 7:16:01.099319 to 7:24:08.320010 | 147 | 144 | 57 | 170 |  | 70 | 770 | 44 | 63 | 23.71112 | 5.889441 | 9.137064 |
|  |  |  |  |  |  |  |  |  |  |  | 23.79014 | 5.921903 | 9.30885 |
|  | 2013-04-02 9:48:43.740698 to 9:55:15.496275 | 245 | 262 | 102 | 190 |  | 60 | 780 | 63 | 124 | 1.390978 | 5.192628 | 12.4532 |
|  |  |  |  |  |  |  |  |  |  |  | 1.489191 | 5.098059 | 12.65575 |
| 4/3/2013 | 2013-04-03 7:30:33.186333 to 7:33:03.152260 | 745 | 695 | 630 | 210 |  | 90 | 1050 | 430 | 760 | 21.28404 | 3.439253 | -4.058747 |
|  |  |  |  |  |  |  |  |  |  |  | 21.35317 | 3.500801 | -3.801936 |
|  | 2013-04-03 16:49:51.697184 to 16:52:02.190930 | 503 | 484 | 350 | 200 |  | 110 | 480 | 292 | 392 | 21.79444 | 4.103601 | -11.97625 |
|  |  |  |  |  |  |  |  |  |  |  | 21.83686 | 4.148676 | -11.88338 |
| 4/4/2013 | 2013-04-04 21:13:46.173954 to 21:22:46.060156 | 164 | 154 | 136 | 270 |  | 80 | 630 | 102 | 136 | 23.04597 | 5.382083 | -1.879642 |
|  |  |  |  |  |  |  |  |  |  |  | 23.14249 | 5.462838 | -1.503069 |
| 4/6/2013 | 2013-04-06 18:49:02.267441 to 18:53:46.373612 | 142 | 139 | 165 | 310 |  | 50 | 1650 | 123 | 200 | 23.4028 | 5.744885 | -6.241293 |
|  |  |  |  |  |  |  |  |  |  |  | 23.44766 | 5.763239 | -6.073248 |
| 4/7/2013 | 2013-04-07 19:51:59.205918 to 19:57:49.803343 | 345 | 317 | 470 | 240 |  | 110 | 900 | 355 | 520 | 21.93876 | 4.33869 | -4.627309 |
|  |  |  |  |  |  |  |  |  |  |  | 22.03807 | 4.450406 | -4.386001 |
|  | 2013-04-07 20:08:27.514169 to 20:13:26.442861 | 273 | 257 | 220 | 210 |  | 100 | 700 | 163 | 267 | 22.21045 | 4.645332 | -3.929089 |
|  |  |  |  |  |  |  |  |  |  |  | 22.28303 | 4.727119 | -3.722232 |
|  | 2013-04-07 23:24:51.745315 to 23:30:39.807296 | 143.5 | 147 | 80 | 310 |  | 90 | 700 | 22 | 84 | 0.356595 | 5.636743 | 4.27293 |
|  |  |  |  |  |  |  |  |  |  |  | 0.419716 | 5.60041 | 4.496279 |
| 4/8/2013 | 2013-04-08 0:43:50.371715 to 0:45:36.405518 | 259 | 263 | 200 | 190 |  | 90 | 380 | 150 | 220 | 1.399393 | 4.769014 | 7.071793 |
|  |  |  |  |  |  |  |  |  |  |  | 1.43004 | 4.739051 | 7.124513 |
|  | 2013-04-08 15:36:06.386685 to 15:44:23.795370 | 159 | 153 | 190 | 600 |  | 280 | 1100 | 165 | 200 | 23.25628 | 5.750517 | -7.801772 |
|  |  |  |  |  |  |  |  |  |  |  | 23.33775 | 5.788013 | -7.575145 |
|  | 2013-04-08 16:59:20.243053 to 17:14:53.589463 | 147 | 152 | 183 | 570 |  | 280 | 1350 | 150 | 200 | 0.083918 | 5.784707 | -5.576484 |
|  |  |  |  |  |  |  |  |  |  |  | 0.245501 | 5.705807 | -5.165359 |
| 4/10/2013 | 2013-04-10 2:55:58.776521 to 3:13:30.153469 | 189 | 176 | 170 | 1200 |  | 90 | 1300 | 84 | 200 | 22.82202 | 5.550618 | 10.45878 |
|  |  |  |  |  |  |  |  |  |  |  | 23.00879 | 5.734965 | 11.06933 |
|  | 2013-04-10 3:38:45.397643 to 3:46:22.133539 | 162 | 160 | 105 | 600 |  | 70 | 950 | 70 | 112 | 23.27073 | 5.942176 | 11.91896 |
|  |  |  |  |  |  |  |  |  |  |  | 23.34434 | 5.987707 | 12.15405 |
|  | 2013-04-10 4:25:19.628842 to 4:32:55.162297 | 158 | 159.5 | 35 | 240 |  | 60 | 600 | 18 | 39 | 23.73106 | 6.123343 | 13.34274 |
|  |  |  |  |  |  |  |  |  |  |  | 23.80427 | 6.128727 | 13.55684 |
|  | 2013-04-10 4:34:59.451803 to 4:42:37.466709 | 159 | 161.5 | 38 | 230 |  | 40 | 700 | 22 | 43 | 23.8276 | 6.129101 | 13.62422 |
|  |  |  |  |  |  |  |  |  |  |  | 23.90161 | 6.125982 | 13.83531 |
|  | 2013-04-10 6:11:53.632004 to 6:18:24.878324 | 248 | 262 | 170 | 100 |  | 60 | 500 | 85 | 183 | 0.938516 | 5.539638 | 16.34458 |
|  |  |  |  |  |  |  |  |  |  |  | 1.032267 | 5.454174 | 16.53228 |
| 4/11/2013 | 2013-04-11 8:07:23.348492 to 8:41:32.885832 | 170 | 197 | 110 | 160 |  | 80 | 260 | 55 | 124 | 0.155819 | 5.933601 | 12.32747 |
|  |  |  |  |  |  |  |  |  |  |  | 0.544503 | 5.687824 | 12.98805 |
| 4/12/2013 | 2013-04-12 20:02:29.039618 to 20:28:27.592069 | 155 | 170 | 180 | 200 |  | 100 | 650 | 92 | 200 | 0.096338 | 5.662261 | -2.207404 |
|  |  |  |  |  |  |  |  |  |  |  | 0.389183 | 5.471456 | -1.336224 |
| 4/13/2013 | 2013-04-13 19:46:48.213128 to 20:02:07.390300 | 290 | 245 | 340 | 170 |  | 50 | 1100 | 150 | 430 | 21.9357 | 4.615856 | -3.35558 |
|  |  |  |  |  |  |  |  |  |  |  | 22.15415 | 4.86239 | -2.70611 |
|  | 2013-04-13 20:09:45.147892 to 20:25:18.915089 | 228 | 201 | 270 | 100 |  | 60 | 1100 | 200 | 283 | 22.25749 | 4.975825 | -2.374475 |
|  |  |  |  |  |  |  |  |  |  |  | 22.45094 | 5.178316 | -1.714022 |
|  | 2013-04-13 21:44:06.677094 to 21:54:48.735026 | 142.5 | 141 | 115 | 170 |  | 70 | 900 | 84 | 124 | 23.29301 | 5.760352 | 1.587477 |
|  |  |  |  |  |  |  |  |  |  |  | 23.39644 | 5.784719 | 2.011251 |


| 4/14/2013 | 2013-04-14 0:36:29.697263 to 1:03:47.025720 | 315 | 480 | 500 | 250 | 60 | 1350 | 180 | 837 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2013-04-14 1:09:09.897747 to 1:15:50.883873 | 529 | 605 | 700 | 230 | 140 | 1100 | 356 | 770 |
| 4/16/2013 | 2013-04-16 11:56:18.679896 to 12:03:00.101894 | 170.5 | 166 | 220 | 200 | 80 | 900 | 135 | 240 |
|  | 2013-04-16 12:29:02.942586 to 12:35:00.247913 | 154 | 152.5 | 150 | 230 | 80 | 800 | 112 | 166 |
|  | 2013-04-16 12:48:46.819176 to 12:52:49.879481 | 150 | 149.5 | 125 | 200 | 70 | 500 | 52 | 138 |
|  | 2013-04-16 12:56:43.298727 to 13:02:59.392307 | 149 | 149 | 123 | 180 | 70 | 450 | 77 | 137 |
|  | 2013-04-16 13:19:46.943710 to 13:24:42.992053 | 151.3 | 152.5 | 160 | 200 | 70 | 400 | 113 | 181 |
| 4/17/2013 | 2013-04-17 13:00:22.818142 to 13:03:32.332842 | 550 | 517 | 760 | 210 | 70 | 500 | 298 | 850 |
|  | 2013-04-17 13:20:18.959076 to 13:26:49.996528 | 385 | 352 | 700 | 170 | 40 | 900 | 430 | 765 |
|  | 2013-04-17 23:24:44.588044 to 23:43:53.101753 | 180 | 164 | 200 | 130 | 60 | 920 | 165 | 223 |
| 4/18/2013 | 2013-04-18 3:08:47.305018 to 3:15:52.402162 | 282 | 305 | 360 | 175 | 40 | 1100 | 200 | 390 |
| 4/19/2013 | 2013-04-19 2:11:58.500197 to 2:20:18.918694 | 204 | 194 | 220 | 150 | 80 | 500 | 165 | 242 |
|  | 2013-04-19 2:32:45.235847 to 2:35:40.960924 | 182 | 179 | 135 | 130 | 90 | 300 | 103 | 148 |
|  | 2013-04-19 2:43:17.789971 to 2:49:39.375991 | 174 | 170 | 105 | 100 | 80 | 300 | 62 | 128 |
|  | 2013-04-19 3:45:32.527503 to 3:53:16.134978 | 155 | 156 | 52 | 190 | 70 | 400 | 35 | 57 |
| 4/20/2013 | 2013-04-20 4:13:23.438778 to 4:18:25.494333 | 351 | 329 | 430 | 230 | 110 | 600 | 265 | 471 |
|  | 2013-04-20 4:46:30.282881 to 4:49:31.642468 | 247 | 240 | 170 | 220 | 120 | 600 | 122 | 181 |
|  | 2013-04-20 4:51:00.260339 to 4:53:28.674564 | 236.8 | 232 | 165 | 300 | 160 | 580 | 124 | 165 |
|  | 2013-04-20 5:15:03.907007 to 5:21:12.553604 | 198 | 190 | 103 | 280 | 100 | 630 | 57 | 113 |
| 4/22/2013 | 2013-04-22 1:49:03.667100 to 2:08:01.330090 | 223 | 197 | 280 | 250 | 90 | 480 | 175 | 322 |
|  | 2013-04-22 21:02:50.905022 to 21:07:55.352866 | 149.2 | 147.6 | 170 | 130 | 60 | 340 | 124 | 182 |
|  | 2013-04-22 21:55:31.764971 to 22:05:03.908385 | 143 | 144.7 | 137 | 140 | 60 | 360 | 86 | 150 |
|  | 2013-04-22 22:26:47.995572 to 22:46:43.761494 | 153 | 165 | 220 | 100 | 50 | 250 | 137 | 240 |
| 4/25/2013 | 2013-04-25 10:23:26.050947 to 10:51:47.819388 | 206 | 143 | 160 | 380 | 200 | 1270 | 43 | 182 |
| 4/26/2013 | 2013-04-26 21:36:37.184423 to 21:45:00.971140 | 359 | 311 | 400 | 220 | 70 | 1360 | 290 | 478 |


| 1.418473 | 4.543941 | 7.675598 |
| ---: | ---: | ---: |
| 2.040842 | 3.967871 | 8.09639 |
| 2.198376 | 3.831712 | 8.122143 |
| 2.399147 | 3.665233 | 8.113683 |
| 22.78544 | 5.580639 | -2.447977 |
| 22.85483 | 5.625414 | -2.312175 |
| 23.12155 | 5.752345 | -1.833581 |
| 23.17909 | 5.769721 | -1.737913 |
| 23.31894 | 5.796513 | -1.513985 |
| 23.35661 | 5.799965 | -1.455426 |
| 23.39793 | 5.801924 | -1.391907 |
| 23.45919 | 5.801321 | -1.298936 |
| 23.63046 | 5.777905 | -1.044692 |
| 23.68039 | 5.765251 | -0.971613 |
| 21.16432 | 4.046022 | -11.45722 |
| 21.22949 | 4.115151 | -11.24809 |
| 21.55002 | 4.466775 | -10.21837 |
| 21.65858 | 4.588526 | -9.872516 |
| 22.43377 | 5.367368 | 6.711624 |
| 22.64386 | 5.564049 | 7.489326 |
| 1.02058 | 4.981374 | 14.5726 |
| 1.147017 | 4.858521 | 14.73917 |
| 22.28212 | 5.422294 | 11.22693 |
| 22.37808 | 5.530915 | 11.53365 |
| 22.51867 | 5.679705 | 11.98341 |
| 22.54807 | 5.709031 | 12.0773 |
| 22.63115 | 5.788163 | 12.34198 |
| 22.69763 | 5.847212 | 12.55269 |
| 23.26689 | 6.16479 | 14.27515 |
| 23.34297 | 6.178974 | 14.48921 |
| 21.47427 | 4.475992 | 8.354903 |
| 21.55452 | 4.579053 | 8.616611 |
| 21.96437 | 5.103698 | 9.88745 |
| 22.00195 | 5.15035 | 9.997655 |
| 22.02283 | 5.17608 | 10.05838 |
| 22.05264 | 5.212533 | 10.14443 |
| 22.31453 | 5.514982 | 10.86624 |
| 22.38396 | 5.588264 | 11.04674 |
| 21.99966 | 5.251215 | 11.14467 |
| 22.22638 | 5.518822 | 11.86038 |
| 22.81049 | 5.72643 | 1.974272 |
| 22.85979 | 5.745349 | 2.16437 |
| 23.34646 | 5.794241 | 3.986628 |
| 23.44428 | 5.774516 | 4.330122 |
| 23.68158 | 5.689646 | 5.114597 |
| 23.90906 | 5.565228 | 5.79006 |
| 21.67376 | 4.880979 | -2.44787 |
| 22.02984 | 5.253688 | -1.519531 |
| 20.94126 | 4.087099 | 4.366391 |
| 21.09448 | 4.267834 | 4.732718 |
|  |  |  |


|  | 2013-04-26 21:53:24.967501 to 21:54:59.139282 | 275 | 268 | 245 | 190 | 120 | 720 | 122 | 294 | 21.23839 | 4.440902 | 5.101644 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  | 21.26181 | 4.469252 | 5.163946 |
| 4/27/2013 | 2013-04-27 3:38:23.700782 to 3:50:01.224320 | 722 | 1000 | 900 | 360 | 180 | 870 | 270 | 1050 | 2.267663 | 3.572634 | 15.08566 |
|  |  |  |  |  |  |  |  |  |  | 2.762311 | 3.201992 | 14.62542 |
| 4/28/2013 | 2013-04-28 2:53:45.741703 to 3:07:01.480427 | 157.4 | 156.6 | 40 | 170 | 70 | 390 | 20 | 43 | 22.6882 | 6.144279 | 14.77188 |
|  |  |  |  |  |  |  |  |  |  | 22.8225 | 6.198459 | 15.13948 |
|  | 2013-04-28 3:10:57.173661 to 3:21:33.029005 | 156.7 | 157.7 | 43 | 150 | 50 | 530 | 18 | 47 | 22.86487 | 6.211223 | 15.25248 |
|  |  |  |  |  |  |  |  |  |  | 22.97212 | 6.234203 | 15.53177 |
|  | 2013-04-28 3:29:18.022137 to 3:35:33.860099 | 158.8 | 160.3 | 33 | 170 | 50 | 420 | 13 | 36 | 23.05412 | 6.242808 | 15.73848 |
|  |  |  |  |  |  |  |  |  |  | 23.11802 | 6.244191 | 15.8953 |
|  | 2013-04-28 4:44:26.560498 to 4:47:34.476653 | 202.5 | 206.8 | 65 | 185 | 60 | 470 | 35 | 69 | 23.91305 | 5.924647 | 17.52344 |
|  |  |  |  |  |  |  |  |  |  | 23.95076 | 5.897236 | 17.5862 |
| 4/29/2013 | 2013-04-29 5:26:39.124447 to 5:30:03.697011 | 166.7 | 165.5 | 77 | 280 | 120 | 450 | 43 | 84 | 23.91305 | 5.924647 | 17.52344 |
|  |  |  |  |  |  |  |  |  |  | 23.95076 | 5.897236 | 17.5862 |
|  | 2013-04-29 5:42:06.341894 to 5:46:25.500630 | 161.8 | 161 | 76 | 189 | 80 | 600 | 34 | 85 | 22.56194 | 6.090415 | 13.77257 |
|  |  |  |  |  |  |  |  |  |  | 22.60451 | 6.110341 | 13.8499 |
|  | 2013-04-29 7:35:17.755748 to 8:16:10.075329 | 196 | 270 | 93 | 180 | 100 | 950 | 52 | 102 | 23.80346 | 5.86813 | 15.50381 |
|  |  |  |  |  |  |  |  |  |  | 0.385861 | 5.382064 | 16.15536 |
|  | 2013-04-29 8:33:50.426950 to 8:42:04.622330 | 312 | 345 | 93 | 310 | 70 | 890 | 69 | 101 | 0.695451 | 5.093963 | 16.4965 |
|  |  |  |  |  |  |  |  |  |  | 0.852759 | 4.946274 | 16.6682 |
| 7/24/2015 | 2015-07-24 8:46:00.946546 to 9:03:03.855101 | 295 | 237 | 150 | 330 | 200 | 980 | 48 | 160 | 15.47339 | 4.594625 | -5.938303 |
|  |  |  |  |  |  |  |  |  |  | 15.72135 | 4.870134 | -5.257051 |
|  | 2015-07-24 14:12:26.036040 to 14:14:42.057369 | 557 | 586 | 630 | 190 | 0 | 300 | 133 | 630 | 20.07262 | 3.711624 | 0.470637 |
|  |  |  |  |  |  |  |  |  |  | 20.1435 | 3.656605 | 0.440773 |
|  | 2015-07-24 14:14:43.015316 to 14:16:57.834352 | 587 | 619 | 700 | 160 | 110 | 300 | 320 | 700 | 20.1495 | 3.651985 | 0.438145 |
|  |  |  |  |  |  |  |  |  |  | 20.2168 | 3.600623 | 0.407058 |
|  | 2015-07-24 14:16:57.767784 to 14:20:53.671239 | 619 | 685 | 850 | 200 | 90 | 320 | 470 | 850 | 20.22298 | 3.595928 | 0.404126 |
|  |  |  |  |  |  |  |  |  |  | 20.35803 | 3.495413 | 0.334258 |
|  | 2015-07-24 14:20:48.530597 to 14:24:50.144260 | 680 | 763 | 1100 | 190 | 90 | 400 | 350 | 1200 | 20.36464 | 3.490565 | 0.330656 |
|  |  |  |  |  |  |  |  |  |  | 20.50901 | 3.386816 | 0.245807 |
|  | 2015-07-24 17:13:31.084146 to 17:19:20.385412 | 447 | 405 | 510 | 300 | 170 | 1200 | 100 | 510 | 14.89247 | 3.9899 | 9.810192 |
|  |  |  |  |  |  |  |  |  |  | 15.00839 | 4.127857 | 10.08399 |
|  | 2015-07-24 17:20:40.613601 to 17:26:37.927515 | 395 | 358 | 520 | 400 | 90 | 2700 | 240 | 520 | 15.03708 | 4.162563 | 10.15271 |
|  |  |  |  |  |  |  |  |  |  | 15.14738 | 4.297804 | 10.42052 |
|  | 2015-07-24 17:26:34.611047 to 17:30:36.977717 | 358 | 337 | 360 | 500 | 30 | 3000 | 200 | 390 | 15.15072 | 4.30194 | 10.42873 |
|  |  |  |  |  |  |  |  |  |  | 15.22266 | 4.391553 | 10.60666 |
|  | 2015-07-24 17:42:50.971822 to 17:48:56.502211 | 287 | 270 | 168 | 700 | 10 | 2800 | 78 | 166 | 15.42891 | 4.652452 | 11.13084 |
|  |  |  |  |  |  |  |  |  |  | 15.52161 | 4.77062 | 11.37319 |
| 7/25/2015 | 2015-07-25 3:06:27.378687 to 3:10:30.655514 | 220 | 211 | 111 | 390 | 110 | 800 | 50 | 111 | 15.77871 | 4.943936 | 3.617546 |
|  |  |  |  |  |  |  |  |  |  | 15.83151 | 5.002289 | 3.743551 |
|  | 2015-07-25 3:12:22.926053 to 3:16:49.152494 | 206.5 | 197 | 47 | 350 | 210 | 540 | 29 | 47 | 15.8575 | 5.03064 | 3.803671 |
|  |  |  |  |  |  |  |  |  |  | 15.91329 | 5.090599 | 3.92854 |
|  | 2015-07-25 3:29:55.764998 to 3:43:12.075453 | 182 | 170 | 69 | 300 | 60 | 600 | 18 | 69 | 16.07847 | 5.259444 | 4.264714 |
|  |  |  |  |  |  |  |  |  |  | 16.23111 | 5.401154 | 4.531734 |
|  | 2015-07-25 12:36:45.724940 to 12:43:41.798983 | 173 | 167 | 113 | 340 | 40 | 1500 | 50 | 113 | 16.16126 | 5.332767 | -0.260642 |
|  |  |  |  |  |  |  |  |  |  | 16.2384 | 5.399216 | -0.034585 |
|  | 2015-07-25 12:49:34.712924 to 12:56:28.308113 | 162.7 | 158 | 84 | 300 | 40 | 1300 | 57 | 85 | 16.30794 | 5.455511 | 0.170373 |
|  |  |  |  |  |  |  |  |  |  | 16.38235 | 5.511654 | 0.390584 |
|  | 2015-07-25 13:00:17.167786 to 13:03:28.834163 | 155.5 | 153 | 76 | 310 | 30 | 1080 | 45 | 76 | 16.42604 | 5.542543 | 0.520163 |
|  |  |  |  |  |  |  |  |  |  | 16.45956 | 5.565148 | 0.619635 |


|  | 2015-07-25 13:21:29.366579 to 13:27:16.433787 | 144 | 141.5 | 43 | 220 | 20 | 1100 | 26.5 | 43 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2015-07-25 20:24:50.689552 to 20:58:33.106264 | 378 | 260 | 360 | 710 | 500 | 2350 | 135 | 430 |
|  | 2015-07-25 22:13:55.306561 to 22:21:27.294689 | 174.3 | 171.7 | 32 | 110 | 80 | 300 | 10 | 33 |
| 7/26/2015 | 2015-07-26 16:07:52.183977 to 16:13:00.492559 | 153.5 | 152 | 125 | 330 | 70 | 1050 | 70 | 124 |
|  | 2015-07-26 16:13:07.314887 to 16:15:35.586815 | 152 | 151.5 | 93 | 320 | 80 | 910 | 70 | 93 |
|  | 2015-07-26 16:15:51.616364 to 16:24:21.165838 | 151.3 | 149 | 125 | 340 | 60 | 1080 | 64 | 125 |
|  | 2015-07-26 16:26:53.975594 to 16:39:47.204267 | 148.3 | 146.5 | 120 | 220 | 60 | 1100 | 40 | 125 |
|  | 2015-07-26 16:45:59.000760 to 16:51:38.753958 | 146.3 | 145.9 | 104 | 190 | 20 | 1050 | 57 | 104 |
|  | 2015-07-26 16:52:10.494269 to 16:55:05.178195 | 145.9 | 146 | 103 | 130 | 20 | 860 | 57 | 103 |
|  | 2015-07-26 17:37:42.582900 to 17:40:59.269361 | 156 | 158.3 | 104 | 200 | 30 | 1350 | 30 | 104 |
|  | 2015-07-26 17:40:50.139148 to 17:43:58.159644 | 158.2 | 159.5 | 104 | 250 | 30 | 1480 | 50 | 104 |
|  | 2015-07-26 17:45:17.580866 to 17:47:05.866471 | 159.9 | 160.8 | 105 | 300 | 30 | 1490 | 40 | 105 |
|  | 2015-07-26 17:49:23.001302 to 17:50:46.201664 | 162.1 | 163.1 | 86 | 310 | 20 | 1330 | 36 | 86 |
| 7/27/2015 | 2015-07-27 4:20:38.710002 to 4:39:11.255199 | 320 | 409 | 265 | 430 | 330 | 650 | 110 | 265 |
|  | 2015-07-27 4:46:41.117490 to 4:59:59.894148 | 460 | 588 | 390 | 520 | 360 | 840 | 265 | 400 |
|  | 2015-07-27 4:59:57.572331 to 5:06:34.377080 | 589 | 680 | 430 | 490 | 370 | 680 | 150 | 460 |
|  | 2015-07-27 5:06:33.559815 to 5:09:37.830936 | 673 | 721 | 390 | 500 | 410 | 780 | 240 | 390 |
|  | 2015-07-27 13:07:57.298278 to 13:12:54.144158 | 231 | 244 | 199 | 520 | 60 | 1500 | 123 | 199 |
| 7/28/2015 | 2015-07-28 13:07:00.611833 to 13:08:31.954894 | 136.4 | 138.8 | 113 | 140 | 60 | 245 | 84 | 113 |
|  | 2015-07-28 19:52:46.825705 to 19:56:38.344557 | 547 | 510 | 135 | 580 | 30 | 1300 | 32 | 135 |
| 7/29/2015 | 2015-07-29 9:55:38.091934 to 10:23:58.086089 | 270 | 351 | 210 | 480 | 180 | 1050 | 50 | 220 |
|  | 2015-07-29 15:12:30.280694 to 15:20:01.105198 | 198 | 188 | 150 | 340 | 120 | 1690 | 102 | 150 |
|  | 2015-07-29 15:32:53.983040 to 15:45:47.435773 | 176.8 | 168 | 124 | 390 | 110 | 1660 | 70 | 124 |
|  | 2015-07-29 16:11:15.171293 to 16:14:50.345066 | 156.3 | 155.2 | 57 | 320 | 60 | 1000 | 43 | 57 |
|  | 2015-07-29 16:19:41.422069 to 16:28:56.413883 | 153.7 | 151.9 | 92 | 320 | 40 | 1500 | 92 |  |


| 16.65517 | 5.677089 | 1.199178 |
| ---: | ---: | ---: |
| 16.71432 | 5.703876 | 1.373272 |
| 15.152 | 4.487131 | 12.73356 |
| 15.67824 | 5.185457 | 14.20815 |
| 16.5749 | 6.144936 | 16.28544 |
| 16.65364 | 6.197198 | 16.43428 |
| 16.50991 | 5.69791 | 8.078702 |
| 16.56409 | 5.73161 | 8.246988 |
| 16.56601 | 5.732756 | 8.252971 |
| 16.59104 | 5.747339 | 8.330601 |
| 16.59681 | 5.750634 | 8.348482 |
| 16.68483 | 5.796713 | 8.620395 |
| 16.71341 | 5.810062 | 8.708219 |
| 16.84629 | 5.861236 | 9.112103 |
| 16.91454 | 5.880517 | 9.315872 |
| 16.97147 | 5.892897 | 9.4835 |
| 16.97907 | 5.894302 | 9.505704 |
| 17.0076 | 5.899018 | 9.588617 |
| 17.47251 | 5.860232 | 10.8247 |
| 17.50716 | 5.849102 | 10.90637 |
| 17.50716 | 5.849102 | 10.90637 |
| 17.54209 | 5.836804 | 10.987 |
| 17.55863 | 5.830608 | 11.02458 |
| 17.57731 | 5.823362 | 11.06658 |
| 17.60656 | 5.811395 | 11.13126 |
| 17.61916 | 5.806026 | 11.15875 |
| 18.94905 | 4.859946 | 13.81251 |
| 19.34732 | 4.494221 | 14.03611 |
| 19.54079 | 4.322363 | 14.14931 |
| 19.9107 | 4.009053 | 14.36599 |
| 19.91629 | 4.004478 | 14.36922 |
| 20.12906 | 3.834735 | 14.49002 |
| 20.13527 | 3.829896 | 14.49348 |
| 20.23723 | 3.751439 | 14.54946 |
| 18.72641 | 4.791027 | 0.458526 |
| 18.81216 | 4.714241 | 0.491518 |
| 16.50282 | 5.660607 | 1.25851 |
| 16.51828 | 5.668384 | 1.302126 |
| 14.60883 | 3.972174 | 11.94053 |
| 14.6894 | 4.070665 | 12.18568 |
| 18.5724 | 4.872251 | 0.990139 |
| 19.12281 | 4.382769 | 1.184779 |
| 15.89118 | 5.281431 | 6.724099 |
| 15.97978 | 5.368498 | 6.979367 |
| 16.13029 | 5.505039 | 7.420877 |
| 16.27372 | 5.619807 | 7.84777 |
| 16.55104 | 5.791805 | 8.675159 |
| 16.58756 | 5.809085 | 8.783027 |
| 5.832107 | 8.94071 |  |
| 16.73688 | 5.865977 | 9.218278 |
|  |  |  |


|  | 2015-07-29 16:30:31.123228 to 16:42:00.313457 | 151.7 | 152 | 64 | 500 | 50 | 1900 | 27 | 64 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2015-07-29 16:57:17.467747 to 17:01:36.727446 | 151.7 | 152.5 | 48 | 1000 | 90 | 1750 | 29 | 48 |
| 7/30/2015 | 2015-07-30 8:42:01.513579 to 8:50:52.278379 | 273 | 253 | 242 | 270 | 160 | 1300 | 163 | 240 |
|  | 2015-07-30 9:39:41.761257 to 9:42:20.392098 | 186.5 | 184.3 | 113 | 280 | 160 | 1700 | 69 | 113 |
|  | 2015-07-30 9:56:48.250938 to 10:01:08.865233 | 174 | 171.7 | 92.5 | 230 | 170 | 1500 | 63 | 92.5 |
|  | 2015-07-30 11:02:40.710098 to 11:18:23.396483 | 159 | 160 | 80 | 610 | 180 | 1510 | 47 | 84 |
| 7/31/2015 | 2015-07-31 5:43:18.940205 to 5:45:40.087516 | 158 | 176 | 70 | 210 | 100 | 1650 | 40 | 70 |
|  | 2015-07-31 5:45:54.511068 to 5:47:48.654047 | 161 | 162.5 | 70 | 380 | 90 | 1400 | 47 | 70 |
|  | 2015-07-31 5:49:16.796053 to 5:52:42.880154 | 163.5 | 166 | 75 | 290 | 90 | 1330 | 47 | 77 |
|  | 2015-07-31 5:55:03.813958 to 6:04:50.448637 | 167 | 175 | 76 | 210 | 100 | 2000 | 30 | 76 |
|  | 2015-07-31 6:10:05.680895 to 6:17:12.647594 | 182.3 | 192 | 79 | 290 | 30 | 2350 | 40 | 79 |
|  | 2015-07-31 7:49:51.474995 to 7:59:50.306578 | 614 | 785 | 630 | 660 | 60 | 3000 | 350 | 630 |
|  | 2015-07-31 10:56:06.007017 to 11:01:31.667984 | 428 | 390 | 630 | 600 | 80 | 2350 | 355 | 630 |
|  | 2015-07-31 11:13:32.641951 to 11:26:37.155304 | 330 | 290 | 113 | 520 | 110 | 1200 | 32 | 113 |
| 8/1/2015 | 2015-08-01 19:13:06.836611 to 19:19:16.178359 | 350 | 383 | 220 | 260 | 40 | 1050 | 110 | 240 |
| 8/3/2015 | 2015-08-03 3:31:16.533667 to 4:02:44.358540 | 170 | 153 | 123 | 270 | 70 | 1500 | 95 | 135 |
| 8/5/2015 | 2015-08-05 1:20:19.710546 to 1:37:16.862935 | 168 | 169 | 85 | 90 | 50 | 960 | 57 | 86 |
|  | 2015-08-05 19:26:26.074515 to 19:50:41.567082 | 166 | 172 | 84 | 220 | 130 | 550 | 52 | 84 |
| 8/6/2015 | 2015-08-06 11:55:53.684092 to 12:02:42.863377 | 179 | 172 | 150 | 90 | 20 | 830 | 102 | 150 |
|  | 2015-08-06 12:36:45.557210 to 12:47:13.726159 | 150 | 146.5 | 77 | 110 | 70 | 710 | 52 | 77 |
|  | 2015-08-06 12:57:36.664956 to 13:17:32.946799 | 142.5 | 136 | 150 | 140 | 30 | 880 | 50 | 165 |
|  | 2015-08-06 13:21:32.993218 to 14:10:27.840403 | 136 | 136 | 150 | 130 | 90 | 1200 | 92 | 150 |
|  | 2015-08-06 18:54:01.899889 to 19:00:31.236988 | 970 | 820 | 470 | 320 | 90 | 1500 | 180 | 470 |
|  | 2015-08-06 21:07:00.549789 to 21:16:03.120514 | 198 | 192 | 102 | 90 | 40 | 1180 | 28 | 102 |
| 8/7/2015 | 2015-08-07 16:35:59.132521 to 16:47:03.071605 | 150 | 155 | 64 | 110 | 40 | 530 | 47 | 77 |


| 16.75599 | 5.871627 | 9.273154 |
| ---: | ---: | ---: |
| 16.87468 | 5.89842 | 9.608417 |
| 17.04066 | 5.911791 | 10.05813 |
| 17.0855 | 5.91065 | 10.17511 |
| 15.50364 | 4.892455 | -4.417536 |
| 15.62094 | 5.016762 | -4.122452 |
| 16.20979 | 5.536002 | -2.74954 |
| 16.23582 | 5.553393 | -2.693185 |
| 16.39563 | 5.647265 | -2.355621 |
| 16.44234 | 5.670331 | -2.259665 |
| 17.10244 | 5.769782 | -1.042772 |
| 17.27437 | 5.727892 | -0.773298 |
| 17.55661 | 5.64522 | 6.467391 |
| 17.85275 | 5.456819 | 6.515882 |
| 17.60745 | 5.616461 | 6.475094 |
| 17.62756 | 5.604634 | 6.478184 |
| 17.64778 | 5.592498 | 6.481326 |
| 17.68861 | 5.567263 | 6.487783 |
| 17.72073 | 5.546758 | 6.49299 |
| 17.84794 | 5.460249 | 6.514999 |
| 17.9211 | 5.407063 | 6.528867 |
| 18.01665 | 5.334345 | 6.548539 |
| 19.97581 | 3.671417 | 7.335337 |
| 20.34283 | 3.403709 | 7.521828 |
| 14.77763 | 4.103047 | -2.692801 |
| 14.87678 | 4.214532 | -2.472865 |
| 15.09047 | 4.457876 | -1.97898 |
| 15.29407 | 4.68941 | -1.485019 |
| 19.06652 | 4.475237 | 12.27805 |
| 19.20761 | 4.345326 | 12.19742 |
| 16.12651 | 5.623097 | 6.435934 |
| 16.46725 | 5.792243 | 6.667898 |
| 16.66495 | 6.147704 | 14.2053 |
| 16.84803 | 6.142367 | 14.26086 |
| 16.73472 | 6.223544 | 16.41218 |
| 17.00375 | 6.208982 | 16.89615 |
| 15.73693 | 5.382745 | 0.631022 |
| 15.81265 | 5.444108 | 0.817412 |
| 16.1977 | 5.686171 | 1.757282 |
| 16.30941 | 5.73195 | 2.023117 |
| 16.42452 | 5.766798 | 2.291401 |
| 16.63764 | 5.797876 | 2.768024 |
| 16.68491 | 5.798931 | 2.869461 |
| 17.24257 | 5.663781 | 3.9069 |
| 13.5575 | 3.260631 | 11.22519 |
| 13.7602 | 3.46672 | 11.86323 |
| 15.86721 | 6.034953 | 17.17302 |
| 15.9671 | 6.121801 | 17.34717 |
| 16.85065 | 5.905179 | 10.41456 |
| 16.976 | 5.879712 | 10.66768 |
|  |  |  |


|  | 2015-08-07 17:47:46.645435 to 17:51:07.574363 | 202 | 206 | 77 | 240 | 90 | 550 | 47 | 77 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2015-08-07 18:03:11.063271 to 18:07:37.927146 | 221 | 226 | 81 | 300 | 80 | 600 | 39 | 92 |
|  | 2015-08-07 18:08:13.563608 to 18:11:06.025796 | 227 | 231 | 90 | 260 | 120 | 600 | 57 | 93 |
|  | 2015-08-07 18:28:20.948710 to 18:32:22.371050 | 271 | 284 | 124 | 310 | 130 | 1200 | 83 | 137 |
| 8/8/2015 | 2015-08-08 8:44:52.051252 to 8:53:19.615611 | 193 | 184 | 124 | 310 | 80 | 1100 | 93 | 124 |
|  | 2015-08-08 8:54:40.241900 to 9:10:27.588159 | 182.5 | 167 | 123 | 290 | 130 | 1500 | 93 | 136 |
|  | 2015-08-08 18:38:23.408099 to 18:42:32.067865 | 164.5 | 163.3 | 77 | 180 | 70 | 930 | 47 | 77 |
|  | 2015-08-08 18:45:03.021604 to 18:51:41.037428 | 162.5 | 162.5 | 80 | 220 | 60 | 850 | 53 | 84 |
|  | 2015-08-08 18:56:38.637359 to 18:58:33.624984 | 162.8 | 162 | 84 | 160 | 90 | 700 | 47 | 84 |
|  | 2015-08-08 19:03:38.692166 to 19:10:02.548488 | 163.2 | 163.2 | 84 | 260 | 80 | 930 | 43 | 84 |
|  | 2015-08-08 19:10:14.689884 to 19:30:07.274312 | 163.5 | 168 | 80 | 180 | 70 | 810 | 35 | 90 |
| 8/9/2015 | 2015-08-09 7:20:39.791817 to 7:23:11.610357 | 688 | 731 | 470 | 135 | 0 | 490 | 175 | 475 |
|  | 2015-08-09 13:05:05.741280 to 13:10:17.042176 | 148 | 147.5 | 29 | 120 | 70 | 370 | 19 | 29 |
|  | 2015-08-09 13:52:02.390043 to 14:00:39.802092 | 154.6 | 155.4 | 43 | 190 | 60 | 450 | 22 | 43 |
| 8/10/2015 | 2015-08-10 1:36:16.395346 to 1:46:09.570360 | 1220 | 1680 | 1100 | 260 | 80 | 900 | 80 | 1110 |
|  | 2015-08-10 7:28:18.031874 to 7:37:32.488155 | 139 | 142 | 60 | 180 | 50 | 700 | 35 | 70 |
|  | 2015-08-10 7:38:47.385532 to 7:41:37.790320 | 142.3 | 143.5 | 70 | 170 | 0 | 700 | 18 | 70 |
|  | 2015-08-10 7:53:56.329063 to 7:55:47.567680 | 147 | 147.5 | 60 | 170 | 60 | 600 | 16 | 70 |
|  | 2015-08-10 8:04:52.659151 to 8:11:36.308971 | 151.3 | 155 | 60 | 130 | 70 | 550 | 24 | 63 |
|  | 2015-08-10 8:41:25.200428 to 8:50:26.584345 | 180 | 192 | 80 | 190 | 130 | 610 | 42 | 92 |
|  | 2015-08-10 15:27 to 16:51 | 158 | 162.5 | 100 | 140 | 30 | 1350 | 53 | 102 |
| 8/11/2015 | 2015-08-11 3:55:48.164650 to 4:03:12.929859 | 508 | 584 | 180 | 740 | 100 | 180 | 50 | 240 |
| 8/12/2015 | 2015-08-12 2:58:09.781744 to 3:14:22.378404 | 172 | 164 | 30 | 250 | 200 | 500 | 12 | 35 |
| 8/13/2015 | 2015-08-13 6:31:58.610514 to 6:33:47.641298 | 138.2 | 138.4 | 70 | 210 | 50 | 600 | 36 | 76 |
|  | 2015-08-13 14:08:00.371107 to 14:13:38.170876 | 200 | 192 | 120 | 220 | 80 | 2200 | 76 | 125 |


|  |  |  |
| ---: | ---: | ---: |
| 17.75431 | 5.460492 | 11.76466 |
| 17.79894 | 5.426199 | 11.80079 |
| 17.98543 | 5.275179 | 11.91981 |
| 18.05465 | 5.216464 | 11.95102 |
| 18.06638 | 5.206401 | 11.95563 |
| 18.11084 | 5.167943 | 11.97133 |
| 18.42036 | 4.890657 | 12.0033 |
| 18.49313 | 4.823902 | 11.99188 |
| 15.61627 | 5.348012 | -2.333947 |
| 15.71431 | 5.427908 | -2.147526 |
| 15.73113 | 5.440947 | -2.116187 |
| 15.91049 | 5.566545 | -1.793915 |
| 16.22367 | 6.109351 | 15.6812 |
| 16.26692 | 6.129897 | 15.77358 |
| 16.29639 | 6.142807 | 15.83578 |
| 16.36707 | 6.170076 | 15.98246 |
| 16.42208 | 6.187693 | 16.09397 |
| 16.44174 | 6.19322 | 16.13324 |
| 16.49887 | 6.206961 | 16.24545 |
| 16.56807 | 6.219041 | 16.37755 |
| 16.57203 | 6.219566 | 16.38497 |
| 16.79068 | 6.224154 | 16.77114 |
| 19.89147 | 3.540639 | 7.620644 |
| 19.98001 | 3.476479 | 7.640745 |
| 16.51871 | 5.797984 | 2.792067 |
| 16.57438 | 5.799409 | 2.903281 |
| 17.05469 | 5.699522 | 3.746958 |
| 17.15737 | 5.654702 | 3.895614 |
| 20.67197 | 3.289512 | 17.89164 |
| 21.22634 | 2.911896 | 17.16619 |
| 16.78418 | 5.757138 | 1.438176 |
| 16.88813 | 5.729385 | 1.461608 |
| 16.90495 | 5.724093 | 1.465039 |
| 16.93664 | 5.713503 | 1.471238 |
| 17.08532 | 5.653982 | 1.496227 |
| 17.10511 | 5.644879 | 1.499092 |
| 17.22144 | 5.586325 | 1.513996 |
| 17.30424 | 5.539656 | 1.522759 |
| 17.71962 | 5.255677 | 1.547977 |
| 17.85405 | 5.150416 | 1.550425 |
| 16.10474 | 5.808328 | 8.945737 |
| 17.05166 | 5.817357 | 10.92277 |
| 19.25405 | 4.209363 | 14.78619 |
| 19.47199 | 4.026161 | 14.79082 |
| 15.78922 | 5.661701 | 7.865834 |
| 15.96906 | 5.760923 | 7.945665 |
| 16.17203 | 5.759317 | 1.585178 |
| 16.18994 | 5.763244 | 1.592279 |
| 15.19679 | 5.20031 | 7.102015 |
| 15.26568 | 5.269801 | 7.25861 |
|  |  |  |


|  | 2015-08-13 14:20:16.802991 to 14:26:26.290538 | 184 | 176.5 | 77 | 220 | 60 | 1700 | 58 | 85 | 15.35099 | 5.352417 | 7.454998 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  | 15.42348 | 5.419292 | 7.623662 |
|  | 2015-08-13 14:32:26.350870 to 14:34:10.615424 | 168.2 | 165.7 | 70 | 400 | 30 | 1500 | 52 | 70 | 15.49901 | 5.485439 | 7.800822 |
|  |  |  |  |  |  |  |  |  |  | 15.51823 | 5.501667 | 7.846081 |
| 8/14/2015 | 2015-08-14 11:40:13.132387 to 11:42:53.732172 | 220 | 225 | 130 | 420 | 230 | 600 | 21 | 150 | 17.81125 | 5.075531 | 0.307717 |
|  |  |  |  |  |  |  |  |  |  | 17.85286 | 5.041602 | 0.307414 |
|  | 2015-08-14 11:50:43.622132 to 11:53:15.707132 | 242 | 248 | 120 | 390 | 240 | 600 | 63 | 125 | 17.98831 | 4.929166 | 0.297802 |
|  |  |  |  |  |  |  |  |  |  | 18.02978 | 4.894247 | 0.292269 |
|  | 2015-08-14 18:19:12.366210 to 18:31:27.212612 | 167 | 167 | 55 | 290 | 100 | 1070 | 15 | 57 | 16.03647 | 6.131753 | 15.90866 |
|  |  |  |  |  |  |  |  |  |  | 16.17045 | 6.181599 | 16.15656 |
| 8/15/2015 | 2015-08-15 15:45:57.658520 to 16:03:33.677455 | 445 | 740 | 500 | 820 | 400 | 2700 | 65 |  | 19.34225 | 3.777685 | 3.088808 |
|  |  |  |  |  |  |  |  |  |  | 19.97678 | 3.308256 | 1.982978 |
|  | 2015-08-15 19:21:43.807755 to 19:27:15.740321 | 323 | 305 | 140 | 350 | 100 | 1900 | 20 | 165 | 14.5709 | 4.973718 | 16.23226 |
|  |  |  |  |  |  |  |  |  |  | 14.65262 | 5.082088 | 16.4081 |
| 8/16/2015 | 2015-08-16 1:04:37.998480 to 1:08:38.863845 | 894 | 1001 | 300 | 720 | 200 | 1500 | 65 | 385 | 20.0789 | 3.579312 | 17.89162 |
|  |  |  |  |  |  |  |  |  |  | 20.24891 | 3.446731 | 17.69308 |
|  | 2015-08-16 9:43:03.097069 to 9:44:21.644246 | 497 | 512 | 45 | 310 | 0 | 800 | 30 | 50 | 19.31778 | 3.769509 | 1.327874 |
|  |  |  |  |  |  |  |  |  |  | 19.35246 | 3.742656 | 1.315873 |
|  | 2015-08-16 23:33:30.222832 to 23:45:42.736661 | 193 | 185 | 38 | 220 | 100 | 1350 | 15 | 40 | 19.31778 | 3.769509 | 1.327874 |
|  |  |  |  |  |  |  |  |  |  | 19.35246 | 3.742656 | 1.315873 |
| 8/17/2015 | 2015-08-17 3:21:02.913420 to 3:36:40.941182 | 400 | 525 | 100 | 550 | 300 | 1650 | 42 | 135 | 18.74899 | 4.501029 | 14.90776 |
|  |  |  |  |  |  |  |  |  |  | 19.17623 | 4.12698 | 14.82712 |
|  | 2015-08-17 10:43:52.257021 to 10:50:48.256904 | 134 | 134 | 35 | 110 | 40 | 400 | 18 | 35 | 17.05552 | 5.542446 | 0.174378 |
|  |  |  |  |  |  |  |  |  |  | 17.14689 | 5.488097 | 0.203607 |
| 8/18/2015 | 2015-08-18 5:42:42.723827 to 6:12:02.890861 | 233 | 333 | 120 | 900 | 100 | 1500 | 20 | 135 | 17.98204 | 4.904326 | 7.890586 |
|  |  |  |  |  |  |  |  |  |  | 18.59399 | 4.370723 | 7.746493 |
| 8/19/2015 | 2015-08-19 7:41:10.526539 to 7:47:06.879467 | 121 | 111 | 57 | 280 | 0 | 700 | 29 | 63 | 17.05233 | 5.5122 | 2.058166 |
|  |  |  |  |  |  |  |  |  |  | 17.13087 | 5.463521 | 2.035376 |
|  | 2015-08-19 14:44:16.685371 to 14:53:43.254209 | 138 | 137 | 45 | 210 | 40 | 1350 | 16 | 52 | 15.67092 | 5.765465 | 8.72439 |
|  |  |  |  |  |  |  |  |  |  | 15.77701 | 5.81566 | 8.942756 |
|  | 2015-08-19 16:25:10.562364 to 16:28:16.761973 | 156 | 158 | 37 | 130 | 40 | 850 | 18 | 29 | 16.84851 | 5.762008 | 10.69845 |
|  |  |  |  |  |  |  |  |  |  | 16.88554 | 5.743749 | 10.7373 |
| 8/22/2015 | 2015-08-22 14:09:08.526530 to 14:19:25.770569 | 162 | 157 | 112 | 300 | 20 | 1200 | 93 | 112 | 15.28574 | 5.593722 | 8.151457 |
|  |  |  |  |  |  |  |  |  |  | 15.40373 | 5.674872 | 8.383571 |
| 8/23/2015 | 2015-08-23 11:35:33.819591 to 11:43:07.352231 | 195 | 243 | 190 | 110 | 80 | 1200 | 114 | 200 | 18.07206 | 4.623642 | -0.178482 |
|  |  |  |  |  |  |  |  |  |  | 18.23072 | 4.488609 | $-0.289122$ |
|  | 2015-08-23 16:36:48.145087 to 16:44:40.419882 | 199 | 190.5 | 112 | 320 | 0 | 1350 | 77 | 124 | 14.89646 | 5.516586 | 14.56892 |
|  |  |  |  |  |  |  |  |  |  | 14.99311 | 5.614822 | 14.74012 |
| 8/25/2015 2015-08-25 17:34:08.199443 to 17:43:11.424159 |  | 294 | 333 | 95 | 480 | 20 | 900 | 63 | 101 | 18.1577 | 4.587755 | 10.13024 |
|  |  | 18.3629 |  |  |  |  |  |  |  | 4.403708 | 9.888437 |
| 8/26/2015 | 2015-08-26 7:51 to 8:57 |  | 185 | 126 | 95 | 250 | 110 | 1500 | 39 | 110 | 15.09595 | 5.478105 | $-0.580177$ |
|  |  | 15.86446 |  |  |  |  |  |  |  |  | 5.783175 | -0.03785 |
|  | 2015-08-26 11:16:30.353043 to 11:21:10.510296 | 202 | 214 | 190 | 90 | 0 | 1200 | 136 | $200$ | 17.82697 | 4.753787 | $-0.232436$ |
|  |  |  |  |  |  |  |  |  |  | 17.91926 | 4.675949 | $-0.296037$ |
| 8/27/2015 2015-08-27 1:26:51.760683 to 1:48:23.615203 |  | 170 | $150$ | $90$ | 220 | $40$ | $1100$ | 16 | $105$ | 14.80408 | 5.40012 | 9.747294 |
|  |  | 15.07021 |  |  |  |  |  |  |  | 5.619099 | 9.869081 |
| 8/28/2015 | 2015-08-28 3:52:46.016480 to 4:11:09.415629 |  | 225 | 189 | 65 | 250 | 90 | 2000 | 29 | 75 | 14.35321 | 4.88091 | 2.400523 |
|  |  |  |  |  |  |  |  |  |  | 14.60801 | 5.137269 | 2.712995 |
|  | 2015-08-28 5:15:55.354244 to 5:21:19.887394 | 114.5 | 116 | 70 | 280 | 90 | 900 | 41 | 77 | 15.40746 | 5.700184 | 3.052799 |
|  |  |  |  |  |  |  |  |  |  | 15.46788 | 5.723049 | 3.046849 |


| 2015-08-28 8:02:44.428254 to 8:06:29.867340 | 147 | 149 | 93 | 80 | 20 | 1100 | 47 | 102 | 17.64338 | 4.86123 | 1.843303 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  | 17.70999 | 4.805646 | 1.799105 |
| 2015-08-28 13:24:27.230084 to 13:32:15.880013 | 173 | 163.5 | 47 | 280 | 110 | 800 | 14 | 52 | 14.78853 | 5.374757 | 7.633758 |
|  |  |  |  |  |  |  |  |  | 14.88468 | 5.457665 | 7.794899 |
| 2015-08-28 18:06:15.921728 to 18:11:17.705944 | 511 | 588 | 270 | 720 | 220 | 1600 | 120 | 320 | 19.2927 | 3.539139 | 7.567462 |
|  |  |  |  |  |  |  |  |  | 19.47535 | 3.400821 | 7.117967 |

## E. 2 Wide Catalog

|  |  | Start | E B [ nT$]$ | n peak [ $\mathrm{cm}^{-3}$ ] | Start | End | Start |  | End |  | Lshell | mlat |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Date | Time |  |  |  | f range [ Hz ] |  | $\mathrm{f}[\mathrm{Hz}]$ | change in n |  | MLT |  |  |
| 24-Jul | 2015-07-24 5:31:53.336482 to 5:47:18.494205 | 925 | 1500 | 620 | - 77 | 1500 | 400 | 135 | 600 | 20.76074 | 3.43162 | 14.70607 |
| 2015-07-24 7:28:36.175525 to 8:10:06.111539 |  |  |  |  |  |  |  |  |  | 21.54963 | 2.943187 | 15.07163 |
|  |  | 1900 | 545 | 1230 | - 172 | 1768 | 500 | 1100 | 580 | 13.37941 | 2.672893 | -11.2689 |
|  |  |  |  |  |  |  |  |  |  | 14.79672 | 3.848469 | -7.83358 |
| 2015-07-24 16:31:17.900718 to 17:08:00.734850 |  | 1300 | 500 | 1100 | - 200 | 1500 | 550 | 760 | 160 | 13.57292 | 2.746361 | 7.015373 |
|  |  |  |  |  |  |  |  |  |  | 14.76393 | 3.841574 | 9.513436 |
| $25-\mathrm{Jul}$ | 2015-07-25 1:24:06.940735 to 1:57:20.704320 | 2000 | 680 | 1250 | - 200 | 1200 | 330 | 800 | 390 | 13.26764 | 2.580174 | -6.23389 |
|  |  |  |  |  |  |  |  |  |  | 14.51981 | 3.537203 | -0.71004 |
|  | 2015-07-25 10:08:16.096570 to 10:53:08.875502 | 3600 | 677 | 1600 | - 120 | 2000 | 300 | 2500 | 560 | 12.18744 | 2.079519 | -6.46915 |
|  |  |  |  |  |  |  |  |  |  | 14.43004 | 3.483542 | -4.45034 |
| $26-\mathrm{Jul}$ | 2015-07-26 2:32:49.343238 to 2:51:25.415319 | 1250 | 2380 | 1000 | - 130 | 2700 | 820 | 390 | 190 | 21.0956 | 3.344045 | 18.80054 |
|  |  |  |  |  |  |  |  |  |  | 22.27769 | 2.614706 | 17.74618 |
| 7/27/2015 | 2015-07-27 22:53:42.305801 to 23:39:00.664013 | 535 | 1750 | 900 | - 90 | 2350 | 620 | 220 | 1210 | 19.78914 | 4.207414 | 18.1184 |
|  |  |  |  |  |  |  |  |  |  | 21.8272 | 2.65501 | 15.23411 |
| 7/28/2015 | 2015-07-28 18:55:34.896877 to 19:39:34.263515 | 3530 | 695 | 2000 | - 120 | 1480 | 390 | 2850 | 280 | 12.0711 | 2.028206 | 3.464753 |
|  |  |  |  |  |  |  |  |  |  | 14.26654 | 3.578934 | 10.87074 |
| 7/29/2015 | 2015-07-29 2:22:22.861066 to 2:47:51.930135 | 1260 | 3370 | 1790 | - 120 | 1550 | 280 | 145 | 1930 | 21.05304 | 3.306867 | 18.63825 |
|  |  |  |  |  |  |  |  |  |  | 22.86755 | 2.254597 | 16.29639 |
|  | 2015-07-29 22:03:16.221022 to 22:25:28.414409 | 2000 | 920 | 2000 | - 200 | 1200 | 520 | 2900 | 490 | 12.94258 | 2.45271 | 1.533294 |
|  |  |  |  |  |  |  |  |  |  | 13.92101 | 3.188437 | 5.848187 |
| 7/30/2015 | 2015-07-30 5:26:51.035439 to 5:40:49.394631 | 1687 | 3000 | 2200 | - 240 | 830 | 400 | 600 | 2900 | 21.53227 | 2.857043 | 15.01393 |
|  |  |  |  |  |  |  |  |  |  | 22.59315 | 2.34361 | 14.80242 |
|  | 2015-07-30 7:16 to 8:36 | 1321 | 280 | 800 | 200 | 1350 | 380 | 1000 | 200 | 13.6146 | 2.986238 | -9.45798 |
|  |  |  |  |  |  |  |  |  |  | 15.41432 | 4.795151 | -4.6466 |
|  | 2015-07-30 16:23:37.973649 to 17:04:55.448892 | 870 | 410 | 800 | 110 | 1680 | 500 | 850 | 309 | 13.9326 | 3.246145 | 8.935134 |
|  |  |  |  |  |  |  |  |  |  | 14.92092 | 4.338557 | 11.07823 |
| 7/31/2015 | 2015-07-31 0:59:05.537579 to 1:33:10.364456 | 2194 | 693 | 1500 | 140 | 1670 | 350 | 1900 | 330 | 12.8999 | 2.492543 | -5.59273 |
|  |  |  |  |  |  |  |  |  |  | 14.23358 | 3.496275 | 0.397599 |
|  | 2015-07-31 9:54:55.254080 to 10:53:50.287758 | 2280 | 430 | 1700 | 250 | 1680 | 500 | 2380 | 380 | 12.71747 | 2.400115 | -5.70456 |
|  |  |  |  |  |  |  |  |  |  | 14.73134 | 4.051497 | -2.79338 |
| 8/1/2015 | 2015-08-01 21:47:32.027443 to 22:20:23.926454 | 2500 | 800 | 1500 | 210 | 1600 | 490 | 2600 | 460 | 12.54611 | 2.298397 | 0.888672 |
|  |  |  |  |  |  |  |  |  |  | 13.99984 | 3.39472 | 7.295979 |
| 8/2/2015 | 2015-08-02 6:39:03.357570 to 7:36:15.977948 | 3990 | 540 | 1100 | 250 | 1490 | 400 | 2500 | 525 | 11.97631 | 2.127847 | -11.9437 |
|  |  |  |  |  |  |  |  |  |  | 14.43218 | 3.839041 | -6.56637 |
| 8/3/2015 | 2015-08-03 0:44:07.351193 to 1:32:00.246493 | 2600 | 540 | 1600 | 210 | 1100 | 380 | 2350 | 630 | 11.97631 | 2.127847 | -11.9437 |
|  |  |  |  |  |  |  |  |  |  | 14.43218 | 3.839041 | -6.56637 |
|  | 2015-08-03 9:49:24.776174 to 10:26:28.697126 | 1795 | 595 | 1600 | 160 | 1500 | 410 | 2190 | 290 | 12.95469 | 2.597821 | -5.14453 |
|  |  |  |  |  |  |  |  |  |  | 14.257 | 3.672824 | -3.23671 |
| 8/6/2015 | 2015-08-06 18:22:45.303293 to 18:51:57.477744 | 3302 | 1035 | 1850 | 180 | 630 | 450 | 2860 | 311 | 11.87962 | 2.097295 | 5.494687 |
|  |  |  |  |  |  |  |  |  |  | 13.48193 | 3.187817 | 10.98292 |
| 8/7/2015 | 2015-08-07 12:30:50.386563 to 12:45:17.931254 | 1840 | 1078 | 1350 | 200 | 2200 | 400 | 1450 | 830 | $12.62755$ | 2.46732 | 2.421188 |
|  |  |  |  |  |  |  |  |  |  | $13.30157$ | $2.932881$ | $3.087568$ |
| 8/8/2015 | 2015-08-08 6:09:54.106910 to 6:22:51.557171 | 5900 | 2800 | 2000 | 330 | 630 | 500 | 2400 | 1200 | 11.07254 | 1.881529 | -12.0866 |
|  |  |  |  |  |  |  |  |  |  | 12.2531 | 2.338281 | -10.4624 |
| 8/10/2015 | 2015-08-10 1:48:50.590194 to 1:57:36.975814 | 1856 | 2660 | 2200 | 100 | 1100 | 350 | 790 | 2500 | $21.41472$ | $2.793137$ | 16.87138 |
|  |  |  |  |  |  |  |  |  |  | $22.06551$ | $2.417491$ | 15.643 |
|  | 2015-08-10 3:17:26.071292 to 3:35:05.733341 | 2700 | 1288 | 2000 | 160 | 1500 | 510 | 2600 | 880 | 12.30736 | 2.412199 | -9.76769 |
|  |  |  |  |  |  |  |  |  |  | 13.18275 | 2.968389 | -6.44406 |
| 8/11/2015 | 2015-08-11 5:56:44.722842 to 6:38:50.589465 | 6530 | 930 | 1750 | 180 | 1300 | 400 | 2500 | 490 | $10.74501$ | 1.815032 | -11.9945 |
|  |  |  |  |  |  |  |  |  |  | 13.42612 | 3.208437 | -7.03963 |
| 8/13/2015 | 2015-08-13 9:27:44.025597 to 10:18:50.738694 | 334 | 1016 | 700 | 40 | 1500 | 250 | 300 | 1500 | 19.66657 | 3.340976 | 2.544842 |
|  |  |  |  |  |  |  |  |  |  | 23.54437 | 1.622954 | 2.422873 |
|  | 2015-08-13 12:12 to 13:39 | 1500 | 258 | 600 | 40 | 2400 | 530 | 1200 | 200 |  | 3.844413 | 2.325904 |
|  |  |  |  |  |  |  |  |  |  | 15.12239 | 5.35144 | 5.396094 |
| 8/16/2015 | 2015-08-16 1:08:26.768157 to 1:41:40.940207 | 990 | 3560 | 750 | 170 | 1650 | 720 | 120 | 1570 | 21.14961 | 2.650606 | 16.80005 |
|  |  |  |  |  |  |  |  |  |  | 1.29947 | 1.312807 | 1.50897 |
|  | 2015-08-16 2:55:28.799820 to 3:15:40.814132 | 2540 | 1100 | 1700 | 180 | 2120 | 550 | 2400 | 240 | 12.8728 | 3.110479 | -6.10429 |
|  |  |  |  |  |  |  |  |  |  | 13.46464 | 3.650661 | -3.81406 |
|  | 2015-08-16 10:10:00.733461 to 10:36:31.293648 | 1070 | 3270 | 1300 | 70 | 2150 | 120 | 360 | 2650 | 21.1332 | 2.441669 | 1.626622 |
|  |  |  |  |  |  |  |  |  |  | 15.35671 | 1.485978 | 0.987431 |
| 8/17/2015 | 2015-08-17 13:09:57.059945 to 13:31:24.080451 | 1260 | 3310 | 920 | 170 | 2660 | 1450 | 373 | 1700 | 20.72582 | 2.626156 | -1.92959 |
|  |  |  |  |  |  |  |  |  |  | 22.57194 | 1.868774 | -4.43144 |
|  | 2015-08-17 14:31:56.086644 to 14:50:17.291397 | 5350 | 1980 | 1300 | 200 | 1200 | 440 | 1962 | 444 | 11.09993 | 2.045954 | 6.123859 |
|  |  |  |  |  |  |  |  |  |  | 12.29893 | 2.70247 | 8.350626 |
| 8/18/2015 | 2015-08-18 7:20:08.652283 to 7:35:10.088026 | 2052 | 4348 | 1700 | 160 | 1100 | 430 | 780 | 2250 | 21.17586 | 2.4258 | 8.056177 |
|  |  |  |  |  |  |  |  |  |  | 22.57551 | 1.875051 | 7.93757 |
|  | 2015-08-18 8:32:21.699493 to 8:45:33.650284 | 4760 | 2390 | 1800 | 20 | 600 | 290 | 2700 | 1067 | 10.78209 | 1.970807 | -5.62582 |
|  |  |  |  |  |  |  |  |  |  | 11.82316 | 2.422226 | -5.03592 |
| 8/19/2015 | 2015-08-19 2:31:16.652681 to 2:46:15.633176 | 5290 | 2280 | 1900 | 200 | 1350 | 430 | 2705 | 1072 | 10.21851 | 1.856405 | -14.4499 |
|  |  |  |  |  |  |  |  |  |  | 11.61416 | 2.371651 | -10.4131 |
|  | 2015-08-19 20:30:03.307457 to 20:43:55.330567 | 4039 | 2004 | 1900 | 150 | 950 | 380 | 2806 | 596 | 9.481398 | 1.613638 | -7.07535 |
|  |  |  |  |  |  |  |  |  |  | 11.14094 | 2.083289 | -0.06279 |
| 8/20/2015 | 2015-08-20 4:05:06.902548 to 4:30:01.891474 | 1510 | 4810 | 1100 | 170 | 1200 | 500 | 270 | 2236 | 19.72499 | 3.360586 | 14.29399 |
|  |  |  |  |  |  |  |  |  |  | 21.19852 | 2.505584 | 13.91659 |
|  | 2015-08-20 5:23:25.601622 to 5:28:56.350147 | 6238 | 4438 | 2100 | 270 | 600 | 340 | 2593 | 1621 | 7.610326 | 1.409944 | -11.2049 |
|  |  |  |  |  |  |  |  |  |  | 8.892296 | 1.558074 | -11.961 |
|  | 2015-08-20 5:47:55.167281 to 6:12:35.173746 | 1774 | 786 | 950 | 70 | 2405 | 500 | 1100 | 423 | 11.32936 | 2.236353 | -10.3419 |
|  |  |  |  |  |  |  |  |  |  | 12.62291 | 3.041739 | -7.63468 |
| 8/21/2015 | 2015-08-21 8:27:53.198148 to 8:58:09.383781 | 3271 | 984 | 1400 | 180 | 1050 | 320 | 2118 | 756 | 7.393255 | 1.372269 | -3.4349 |
|  |  |  |  |  |  |  |  |  |  | 11.62599 | 2.372232 | -3.84145 |
| 8/22/2015 | 2015-08-22 2:25:53.928142 to 2:54:01.253733 | 3640 | 1090 | 1600 | 200 | 960 | 320 | 2562 | 620 | 5.688779 | 1.266966 | -15.1306 |
|  |  |  |  |  |  |  |  |  |  | 11.02609 | 2.152243 | -11.7168 |
| 8/23/2015 | 2015-08-23 3:55:51.098495 to 4:08:44.957967 | 1646 | 2851 | 1800 | 58 | 1200 | 120 | 716 | 2861 | 18.76216 | 3.984859 | 13.34372 |
|  |  |  |  |  |  |  |  |  |  | 19.19754 | 3.642481 | 13.35106 |
|  | 2015-08-23 21:43:22.520038 to 22:03:01.971349 | 1061 | 2210 | 1600 | 80 | 2150 | 275 | 570 | 1780 | 18.28351 | 4.569508 | 18.28919 |
|  |  |  |  |  |  |  |  |  |  | 18.85373 | 4.027617 | 17.70958 |


| 8/25/2015 | 2015-08-25 2:19:58.393568 to 2:47:40.170049 | 2720 | 930 | 1600 | 220 | 650 | 350 | 2881 | 470 | 0.290365 | 1.364237 | 5.370131 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  | 7.8729 | 1.451518 | -15.902 |
| 8/26/2015 | 2015-08-26 5:15:32.068436 to 6:03:13.545030 | 2700 | 583 | 1100 | 200 | 2100 | 590 | 2370 | 390 | 22.78164 | 1.759804 | 10.27634 |
|  |  |  |  |  |  |  |  |  |  | 9.784819 | 1.762774 | -9.62318 |
|  | 2015-08-26 11:46:05.674807 to 12:32:26.893990 | 355 | 1115 | 600 | 0 | 1500 | 250 | 129 | 1222 | 16.8672 | 5.285959 | 1.148424 |
|  |  |  |  |  |  |  |  |  |  | 17.66723 | 4.620872 | 1.038898 |
|  | 2015-08-26 14:13:01.922324 to 14:27:13.656327 | 2325 | 1312 | 1900 | 0 | 1300 | 300 | 2905 | 199 | 22.23962 | 1.876133 | -6.41983 |
|  |  |  |  |  |  |  |  |  |  | 0.237394 | 1.404345 | -9.99883 |
| 8/29/2015 2015-08-29 3:27:13.376999 to 3:45:59.051061 |  | 1307 | 2807 | 1050 | 160 | 600 | 320 | 200 | 1400 | 17.25566 | 5.080919 | 11.6714 |
|  |  |  |  |  |  |  |  |  |  | 17.61707 | 4.75514 | 11.55763 |

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