Venus mountain waves in the upper atmosphere simulated by a time-invariant linear full-wave spectral model

Supplementary Information

Momentum Fluxes at 20 km Altitude

The wave momentum fluxes at 20 km altitude calculated using the full range of parameters considered in this study are shown in Tables S1a and S1b for LT = 11 h and 16 h, respectively. The smallest momentum fluxes occur for large near-surface eddy diffusivities (profiles A and D), the nominal lower atmosphere stability, and a westward surface wind speed of 1 m s^{-1}. The values are ~ 10^{-8} and 10^{-7} kg m^{-1} s^{-2} (LT = 11 h) and ~ 10^{-11} and 10^{-9} kg m^{-1} s^{-2} (LT = 16 h) for eddy diffusivity profiles A and D, respectively. The largest momentum flux of ~ 1 kg m^{-1} s^{-2} (for LT = 11 h and 16 h) occurs for the smallest near-surface eddy diffusivity (profile C), nominal lower atmosphere stability, and a westward surface wind speed of 3 m s^{-1} (the 5 m s^{-1} wind leads to a similar value of momentum flux).

For the smallest values of eddy diffusivity (profile C) and for the low stability lower atmosphere the momentum flux for LT = 16 increases by about an order of magnitude as the surface wind speed increases from 1 to 5 m/s. In this case the effect of the near-surface wind is to increase the vertical wavelength and thereby reduce the wave dissipation. This is not seen in the LT = 11 h case because the dissipation is less sensitive to the value of the near-surface wind for small N^2. For eddy profile C, the nominal lower atmosphere stability and for both LT = 11 h and 16 h, there is an overall increase of momentum flux with increasing surface wind speed up to 3 m/s, after which the momentum flux declines. For larger values of eddy diffusivity, the momentum flux can increase by several orders of magnitude as the surface wind speed increases from 1 m/s to 5 m/s. This dramatic increase in the momentum flux is due to the significant decrease of wave dissipation as the wind speed (and the vertical wavelength) increases, as discussed previously with respect to Figure 9.

The impact of the lower atmosphere stability on momentum fluxes is most noticeable for large eddy diffusivities and smaller surface wind speeds. For eddy diffusivity profiles A and D, and for a given value of the surface wind speed, the momentum fluxes are always greater in the low stability lower atmosphere. This is because the vertical wavelength is greater in the low stability lower atmosphere, as discussed previously, which reduces the dissipation rate. When the eddy diffusivities are extremely small (profile C), the momentum fluxes are greater in the nominal stability lower atmosphere at any surface wind speed.

For eddy diffusivity profile B, the momentum fluxes are greater in the low stability lower atmosphere only for the smallest surface wind speed considered (1 m/s).

For LT = 11 h, surface winds speeds of 2, 3 and 5 m/s, and for the low stability lower atmosphere, the momentum fluxes appear to be fairly insensitive to the low-level value of eddy diffusivity. This is due to the extremely small values of N^2 at the lowest altitudes, leading to large vertical wavelengths near the lower boundary. This lack of sensitivity does not occur for the nominal stability lower atmosphere because the near-surface values of N^2 are larger. Furthermore, for the low stability lower atmosphere the near-surface values of N^2 are larger at LT = 16 h than for LT = 11h, and so the momentum fluxes are more sensitive to the low-level eddy diffusivities for LT = 16 h.

$\bar{U}_{sfc}$ \ η_{eddy}	C		B		A		D	
	Low	Nom	Low	Nom	Low	Nom	Low	Nom
-1	1.6E-2	5.8E-2	1.7E-2	6.8E-4	8.5E-3	8.0E-9	2.0E-3	7.8E-8
-2	3.3E-3	4.5E-1	3.3E-3	2.0E-1	3.3E-3	8.3E-6	3.2E-3	2.3E-6
-3	4.6E-3	1.5E0	4.6E-3	9.8E-1	4.7E-3	7.4E-4	5.1E-3	3.0E-5
-5	1.2E-2	7.1E-1	1.2E-2	6.9E-1	1.2E-2	8.3E-2	1.3E-2	1.3E-3

Table S1a. Wave momentum fluxes (kg m^{-1} s^{-2}) at z = 20 km for LT = 11 h, and for different values of surface wind ($\bar{U}_{sfc}$, m/s), eddy diffusivity profiles η_{eddy} (A, B, C and D), and lower atmosphere stability (low or nominal). The surface values of η_{eddy} are 10^{-3}, 10^{-2}, 1 and 10 m^2 s^{-1} for profiles C, B, A and D, respectively.

$\bar{U}_{sfc}$ \ η_{eddy}	C		B		A		D	
	Low	Nom	Low	Nom	Low	Nom	Low	Nom
-1	1.7E-2	2.9E-2	2.4E-2	3.2E-5	1.9E-3	4.4E-11	3.5E-5	1.6E-9
-2	3.4E-2	3.2E-1	6.2E-2	1.2E-1	2.8E-2	1.0E-7	2.4E-3	8.8E-8
-3	5.9E-2	1.1E0	6.6E-2	5.8E-1	5.9E-2	3.4E-5	1.7E-2	1.7E-6
-5	1.9E-1	5.1E-1	1.5E-1	5.9E-1	1.6E-1	2.5E-2	6.6E-2	1.7E-4

Table S1b. The same as for Table S1a, except for LT = 16 h.

Brightness-Weighted Temperatures

Values of T'_W based on Eq. (8) are presented for different combinations of surface wind (-1, -2, -3 and -5 m/s), eddy diffusivity profile (A, B, C and D), lower atmospheric stability (low and nominal), and local time (LT = 11 h and 16 h). Results are shown for LT = 11 h and 16 h in Tables S2a and S2b, respectively. Not surprisingly, these values tend to correlate with the corresponding values of momentum flux presented previously in Tables S1a and S1b. For both nominal and low values of lower atmosphere stability, values of T'_W generally increase with increasing surface wind speed and with decreasing surface eddy diffusivity. For all eddy diffusion profiles T'_W increases as the wind speed increases, but in the case of eddy diffusion profiles A and D this increase is by several orders of magnitude while for eddy diffusion profiles B and C it is far more modest. For the largest values of surface eddy diffusivity (profiles A and D), T'_W is significantly smaller for the larger (that is, nominal) stability of the lower atmosphere. However, for surface wind speeds of -2 m/s or more and for smaller values of surface eddy diffusivity (profiles B and C) T'_W is larger when the stability of the lower atmosphere is larger. The reason for this behavior follows the explanation of the results presented in section 5.1 The eddy diffusivity preferentially damps waves of shorter vertical wavelengths (large Scorer parameter), and shorter vertical wavelengths are favored by smaller wind speeds ($\bar{U}$) and greater atmospheric stability (N^2). Therefore, the eddy diffusivity is particularly important for the nominal lower atmosphere stability and small surface winds. However, when the eddy diffusivity is small, the waves propagate easier when the lower atmosphere has greater stability (that is, in the nominal stability case). Waves are less sensitive to the eddy diffusivity when the stability of the lower atmosphere is low.

The modeled values of T'_W shown in Tables S2a and S2b are similar to the ~ 2 K observed by *Fukuhara et al*. (2017) only for a limited range of conditions. For LT = 11 h (Table S2a) values of T'_W tend to be in more reasonable agreement with observed values for the low stability lower atmosphere conditions and for surface winds of 2 – 3 m/s and all eddy diffusivity profiles, and 1 m/s surface winds only for the larger surface eddy diffusivities (cases A and D). For the nominal stability lower atmosphere, a reasonable match to the observations is more problematic for LT = 11 h. However, *Navarro et al*. (2018) have noted that the morning atmosphere tends to have smaller winds and low stability near the surface, suggesting that our low stability lower atmosphere results shown in Table S2a are most appropriate.

η_{eddy} / $\bar{U}_{sfc}$	C		B		A		D	
	Low	Nom	Low	Nom	Low	Nom	Low	Nom
-1	4.9	5.3	4.9	0.51	3.0	2×10^{-3}	1.2	5×10^{-3}
-2	2.3	16	2.3	9.1	2.3	5×10^{-2}	1.8	2×10^{-2}
-3	2.7	29	2.7	23	2.7	0.56	2.4	9×10^{-2}
-5	4.0	24	4.0	23	4.1	6.4	3.9	0.61

Table S2a. Maximum brightness-weighted temperature perturbation T_W' (K) for LT = 11 h, and different values of surface wind ($\bar{U}_{sfc}$), eddy diffusivity profiles η_{eddy} (A, B, C and D) and lower atmosphere stability (low or nominal). The surface values of η_{eddy} are 10^{-3}, 10^{-2}, 1 and 10 $m^2 s^{-1}$ for profiles C, B, A and D, respectively.

For LT = 16 h (Table S2b) and the low stability lower atmosphere, surface winds of 2 to 3 m/s in combination with large eddy diffusivity (profile D) results in T_W' values of ~1 K to 3 K, while a surface wind of 1 m/s in combination with modest eddy diffusivity (profile A) results in T_W' ~ 1.4 K. A surface wind of 1 m/s and eddy diffusivity profile C results in a value of T_W' ~ 4.5 K, while a surface wind of 5 m/s in combination with modest eddy diffusivity (profile A) results in T_W' ~ 3.3 K. For the nominal stability lower atmosphere, only the surface wind value of 5 m/s provides a match to the observations for eddy diffusion profile A. For the nominal stability lower atmosphere, eddy diffusion profiles B, C and D do not provide a match with the observations for any value of surface wind speed considered.

η_{eddy} / $\bar{U}_{sfc}$	C		B		A		D	
	Low	Nom	Low	Nom	Low	Nom	Low	Nom
-1	5.7	4.5	6.1	0.1	1.4	9×10^{-5}	0.1	8×10^{-4}
-2	6.1	14	6.4	6.9	5.2	4×10^{-3}	1.0	4×10^{-3}
-3	11	30	10	19	8.7	0.1	3.2	2×10^{-2}
-5	15	23	14	22	13	3.3	5.7	0.2

Table S2b. The same as for Table S2a except for LT = 16 h.

Mean State Accelerations

The maximum values of the accelerations that occur near 100 km altitude for LT = 11 h are shown in Table S3a. In those cases where the waves were convectively unstable (i.e., when *S* < -1), wave-induced diffusion was included to maintain stable amplitudes, and those results are shown with an asterisk. Note that for the nominal lower atmosphere stability, the values of wave-induced diffusion needed to bring some of the wave amplitudes down to stable values would be extremely large. In those cases (shaded in the table) we have not included any wave-induced diffusion.

η_{eddy} / $\bar{U}_{sfc}$	C		B		A		D	
	Low	Nom	Low	Nom	Low	Nom	Low	Nom
-1	154(47*)	341(106*)	108(46*)	3.5^s	67(30*)	$5x10^{-5}$	7.1^S	$1x10^{-4}$
-2	30(20*)	$2x10^3$	21(19*)	801	23^s	$4x10^{-2}$	9.7^S	$3x10^{-3}$
-3	41(27*)	$7x10^3$	29(26*)	$4x10^3$	32(28*)	3.8^S	15^S	$4x10^{-2}$
-5	105(50*)	$3x10^3$	74(53*)	$2x10^3$	81(42*)	392	39^S	2.5^s

Table S3a. Maximum accelerations (m/s/h) for LT = 11 h, for different values of surface wind ($\bar{U}_{sfc}$) eddy diffusivity profiles η_{eddy} (A, B, C and D) and lower atmosphere stability (low or nominal). The surface values of η_{eddy} are 10^{-3}, 10^{-2}, 1 and 10 $m^2 s^{-1}$ for profiles C, B, A and D, respectively. Superscript "S" signifies stable; superscript "*" denotes with wave-induced diffusion; shaded signifies unstable amplitudes and no wave-induced diffusion applied.

For LT = 16 h, the maximum mean-state accelerations (Table S3b) in the thermosphere associated with the viscous dissipation of the waves always maximized near 175 km altitude, irrespective of the surface wind speeds, the eddy diffusivity profile, and the lower atmosphere stability. However, the peak values of the accelerations strongly depended on the lower atmosphere parameters. These peak values provided in table S3b, below, do not include wave-induced diffusion.

η_{eddy} / $\bar{U}_{sfc}$	C		B		A		D	
	Low	Nom	Low	Nom	Low	Nom	Low	Nom
-1	$>10^4$	$>10^5$	$>10^4$	140	10^4	$2.2x10^{-5}$	17.4	10^{-4}
-2	$>10^5$	$>10^5$	$>10^4$	$>10^5$	$>10^4$	0.25	$1.5x10^3$	$8x10^{-3}$
-3	$>10^5$	$>10^6$	$>10^5$	$>10^5$	$>10^5$	169	$8x10^3$	0.25
-5	$>10^6$	$>10^6$	$>10^5$	$>10^5$	$>10^5$	$>10^5$	$3x10^4$	50

Table S3b. The same as for Table S3a except for LT = 16 h.

For a given eddy diffusivity profile, the value of the peak acceleration scales approximately with the low-level momentum fluxes shown in Tables S1a and S1b.