## Essays on the econometric analysis of treatment assignment rules and altruistic preferences

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Dissertation

# ESSAYS ON THE ECONOMETRIC ANALYSIS OF TREATMENT ASSIGNMENT RULES AND ALTRUISTIC PREFERENCES 

 by
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## Dedication

In loving memory of my mother Bayarmaa Batchuluun.

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All errors in this dissertation are my own.

# ESSAYS ON THE ECONOMETRIC ANALYSIS OF TREATMENT ASSIGNMENT RULES AND ALTRUISTIC PREFERENCES 

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#### Abstract

This dissertation has two main themes: treatment assignment rules and altruistic preferences. The first two chapters are about comparing different treatment assignment rules using observational data. The third chapter studies how altruistic preferences are affected by markets and incentives.

In Chapter 1, I develop a theoretical framework to compare different treatment assignment rules. A treatment assignment rule is a mapping from observed characteristics to binary treatment status. The welfare difference between two given treatment assignment rules is not point identified in general when data are obtained from an observational study or a randomized experiment with imperfect compliance. I characterize the sharp identified region of the welfare difference and obtain bounds under various assumptions on the unobservables with and without instrumental variables. I conduct estimation and inference of the bounds using orthogonalized moment conditions to deal with the presence of infinite-dimensional nuisance parameters.

In Chapter 2, I apply the method I proposed in Chapter 1 to examine two applications in economics. First, I study the problem of assigning individuals to job


training programs. I calculate the welfare differences between different hypothetical policies using experimental data from the National Job Training Partnership Act Study. Second, I apply the method to study public health insurance policies. Specifically, I calculate the welfare impact of Medicaid expansion using data from the Oregon Health Insurance Experiment.

Chapter 3 (joint with Ching-to Albert Ma and Daniel Wiesen) studies how altruistic preferences are changed by markets and incentives using a laboratory experiment. Subjects are asked to choose health care qualities for hypothetical patients in monopoly, duopoly, and quadropoly. Prices, costs, and patient benefits are experimental incentive parameters. We combine a theoretical model of strategic interaction with a nonparametric estimation method and find that markets tend to reduce altruism.

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## List of Abbreviations

| CARA | Constant Absolute Risk Aversion |
| :---: | :---: |
| CATE | Conditional Average Treatment Effect |
| CDF | Cumulative Distribution Function |
| CEINR | Chernozhukov, Escanciano, Ichimura, Newey, and Robins |
| GMM | Generalized Method of Moments |
| GPV | Guerre, Perrigne, and Vuong |
| KS | Kolmogorov-Smirnov |
| IV | Instrumental Variable |
| JTPA | Job Training Partnership Act |
| LATE | Local Average Treatment Effect |
| MIV | Monotone Instrumental Variable |
| MTR | Monotone Treatment Response |
| OHIE | Oregon Health Insurance Experiment |
| ORSEE | Online Recruitment System for Economic Experiments |
| PDF | Probability Density Function |
| SD | Standard Deviation |
| US | United States |

## Chapter 1

## Identification and Inference for Welfare Gains without Unconfoundedness

### 1.1 Introduction

The problem of choosing among alternative treatment assignment rules based on data is pervasive in economics and many other fields, including marketing and medicine. A treatment assignment rule is a mapping from individual characteristics to a treatment assignment. For instance, it can be a job training program eligibility criterion based on the applicants' years of education and annual earnings. Throughout the chapter, I call the treatment assignment rule a policy, and the subject who decides the treatment assignment rule a policymaker. The policymaker can be an algorithm assigning targeted ads, a doctor deciding medical treatment, or a school principal deciding which students take classes in person during a pandemic. As individuals with different characteristics might respond differently to a given policy, policymakers aim to choose a policy that generates the highest overall outcome or welfare.

Most previous work on treatment assignment in econometrics focused on estimating the optimal policy using data from a randomized experiment. I contribute to this literature by focusing on the identification and inference of the welfare gain using data from an observational study or a randomized experiment with imperfect compliance. The assumption called unconfoundedness might fail to hold for such datasets. ${ }^{1}$

[^0]By relaxing the unconfoundedness assumption, my framework accommodates many interesting and empirically relevant cases, including the use of instrumental variables to identify the effect of a treatment. The advantage of focusing on welfare gain is to provide policymakers with the ability to be more transparent when choosing among alternative policies. Policymakers may want to know how much the welfare gain or loss is in addition to the welfare ranking of competing policies when they make their decisions. They might also need to report the welfare gain.

When the unconfoundedness assumption does not hold, identification of the conditional average treatment effect (CATE) and hence identification of the welfare gain becomes a delicate matter. Without further assumptions on selection, one cannot uniquely identify the welfare gain. I take a partial identification approach whereby one obtains bounds on the parameter of interest with a minimal amount of assumptions on the unobservables and, later on, tighten these bounds by imposing additional assumptions with and without instrumental variables. The bounds, or sharp identified region, of the welfare gain can be characterized using tools from random set theory. ${ }^{2}$ The framework I use allows me to consider various assumptions that involve instrumental variables and shape restrictions on the unobservables.

I show that the lower and upper bounds of the welfare gain can, in general, be written as functions of the conditional mean treatment responses and a propensity score. Hence, estimation and inference of these bounds can be thought of as a semiparametric estimation problem in which the conditional mean treatment responses and the propensity score are infinite-dimensional nuisance parameters. Bounds that do not rely on instruments admit regular and asymptotically normal estimators. I construct orthogonalized, or locally robust, moment condition by adding an adjustment term that accounts for the first step estimation to the original moment condition,

[^1]following Chernozhukov, Escanciano, Ichimura, Newey, and Robins (2020) (CEINR, henceforth). This method leads to estimators that are first-order insensitive to estimation errors of the nuisance parameters. I calculate the adjustment term using an approach proposed by Ichimura and Newey (2017). The locally robust estimation is possible even with instrumental variables under an additional monotonicity assumption of instruments. The estimation strategy has at least two advantages. First, it allows for flexible estimation of nuisance parameters, including the possibility of using high-dimensional machine learning methods. Second, the calculation of confidence intervals for the bounds is straightforward because the asymptotic variance doesn't rely on the estimation of nuisance parameters. The results from a Monte Carlo simulation suggest that the method works well in a finite sample.

Related Literature This work is related to the literature on treatment assignment, sometimes also referred to as treatment choice, which has been growing in econometrics since the seminal work by Manski (2004). Earlier work in this literature include Dehejia (2005), Hirano and Porter (2009), Stoye (2009a, 2012), Chamberlain (2011), Bhattacharya and Dupas (2012), Tetenov (2012), Kasy (2014), and Armstrong and Shen (2015).

In a recent work, Kitagawa and Tetenov (2018) propose what they call an empirical welfare maximization method. This method selects a treatment rule that maximizes the sample analog of the average social welfare over a class of candidate treatment rules. Their method has been further studied and extended in different directions. Kitagawa and Tetenov (2019) study an alternative welfare criterion that concerns equality. Mbakop and Tabord-Meehan (2016) propose what they call a penalized welfare maximization, an alternative method to estimate optimal treatment rules. While Andrews, Kitagawa, and McCloskey (2019) consider inference for the estimated optimal rule, Rai (2018) considers inference for the optimal rule itself. These papers
and most of the earlier papers only apply to a setting in which the assumption of unconfoundedness holds.

In a dynamic setting, treatment assignment is studied by Kock and Thyrsgaard (2017), Kock, Preinerstorfer, and Veliyev (2018), Adusumilli, Geiecke, and Schilter (2020), Sakaguchi (2019), and Han (2019), among others.

This work contributes to the less explored case of using observational data to infer policy choice where the unconfoundedness assumption does not hold. Earlier work in the treatment choice literature with partial identification include Stoye (2007) and Stoye (2009b). This work is closely related to Kasy (2016), but their main object of interest is the welfare ranking of policies rather than the magnitude of welfare gain that results from switching from one policy to another policy. It is also closely related to Athey and Wager (2020) as they are concerned with choosing treatment assignment policies using observational data. However, their approach is about estimating the optimal treatment rule by point identifying the causal effect using various assumptions. Another recent work by Sasaki and Ura (2020) studies the problem using the point identification of marginal treatment effects. In a related work in statistics, Cui and Tchetgen Tchetgen (2020) propose a method to estimate optimal treatment rules using instrumental variables. More recently, Assunção, McMillan, Murphy, and Souza-Rodrigues (2019) work with a partially identified welfare criterion that also takes spillover effects into account to analyze deforestation regulations in Brazil.

The rest of the chapter is structured as follows. In Section 1.2, I set up the problem. Section 1.3 presents the identification results of the welfare gain. Section 1.4 discusses the estimation and inference of the bounds. Section 1.5 summarizes the results from a Monte Carlo simulation. Finally, Section 1.6 concludes. All proofs, some useful definitions and theorems from random set theory, and more details on
the simulation study are collected in the Appendix.

Notation Throughout this chapter, for $d \in \mathbb{N}$, let $\mathbb{R}^{d}$ denote the Euclidean space and $\|\cdot\|$ denote the Euclidean norm. Let $\langle\cdot, \cdot\rangle$ denote the inner product in $\mathbb{R}^{d}$ and $E[\cdot]$ denote the expectation operator. The notation $\xrightarrow{p}$ and $\xrightarrow{d}$ denote convergence in probability and convergence in distribution, respectively. For a sequence of numbers $x_{n}$ and $y_{n}, x_{n}=o\left(y_{n}\right)$ and $x_{n}=O\left(y_{n}\right)$ mean, respectively, that $x_{n} / y_{n} \rightarrow 0$ and $x_{n} \leq C y_{n}$ for some constant $C$ as $n \rightarrow \infty$. For a sequence of random variables $X_{n}$ and $Y_{n}$, the notation $X_{n}=o_{p}\left(Y_{n}\right)$ and $X_{n}=O_{p}\left(Y_{n}\right)$ mean, respectively, that $X_{n} / Y_{n} \xrightarrow{p} 0$ and $X_{n} / Y_{n}$ is bounded in probability. $\mathcal{N}(\mu, \Omega)$ denotes a normal distribution with mean $\mu$ and variance $\Omega$. $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal distribution.

### 1.2 Setup

Let $(\Omega, \mathfrak{A})$ be a measurable space. Let $Y: \Omega \rightarrow \mathbb{R}$ denote an outcome variable, $D$ : $\Omega \rightarrow\{0,1\}$ denote a binary treatment, and $X: \Omega \rightarrow \mathcal{X} \subset \mathbb{R}^{d_{x}}$ denote pretreatment covariates. For $d \in\{0,1\}$, let $Y_{d}: \Omega \rightarrow \mathbb{R}$ denote a potential outcome that would have been observed if the treatment status were $D=d$. For each individual, the researcher only observes either $Y_{1}$ or $Y_{0}$ depending on what treatment the individual received. Hence, the relationship between observed and potential outcomes is given by

$$
\begin{equation*}
Y=Y_{1} \cdot D+Y_{0} \cdot(1-D) \tag{1.1}
\end{equation*}
$$

Policy I consider is a treatment assignment rule based on observed characteristics of individuals. In other words, the policymaker assigns an individual with covariate $X$ to a binary treatment according to a treatment rule $\delta: \mathcal{X} \rightarrow\{0,1\} .{ }^{3}$ The welfare

[^2]criterion considered is population mean welfare. If the policymaker chooses policy $\delta$, the welfare is given by
\[

$$
\begin{align*}
u(\delta) & \equiv E\left[Y_{1} \cdot \delta(X)+Y_{0} \cdot(1-\delta(X))\right]  \tag{1.2}\\
& =E\left[E\left[Y_{1} \mid X\right] \cdot \delta(X)+E\left[Y_{0} \mid X\right] \cdot(1-\delta(X))\right]
\end{align*}
$$
\]

The object of my interest is welfare gain that results from switching from policy $\delta^{*}$ to another policy $\delta$ which is

$$
\begin{equation*}
u(\delta)-u\left(\delta^{*}\right)=E\left[\Delta(X) \cdot\left(\delta(X)-\delta^{*}(X)\right)\right], \quad \Delta(X) \equiv E\left[Y_{1}-Y_{0} \mid X\right] \tag{1.3}
\end{equation*}
$$

Remark 1.2.1. I assume that individuals comply with the assignment. This can serve as a natural baseline for choosing between policies.

The observable variables in my model are $(Y, D, X)$ and I assume that the researcher knows the joint distribution of $(Y, D, X)$ when I study identification. Later, in Section 1.4, I assume availability of data - size $n$ random sample from $(Y, D, X)$ to conduct inference on objects that depend on this joint distribution. The unobservables in my model are potential outcomes $\left(Y_{1}, Y_{0}\right)$. The conditional average treatment effect $\Delta(X)=E\left[Y_{1}-Y_{0} \mid X\right]$ and hence my object of interest welfare gain cannot be point identified in the absence of strong assumptions. One instance in which it can be point identified is when potential outcomes $\left(Y_{1}, Y_{0}\right)$ are independent of treatment $D$ conditional on $X$, i.e.,

$$
\begin{equation*}
\left(Y_{1}, Y_{0}\right) \perp D \mid X \tag{1.4}
\end{equation*}
$$

This assumption is called unconfoundedness and is a widely-used identifying assumption in causal inference. See Imbens and Rubin (2015) Chapter 12 and 21 for more discussions on this assumption. Under unconfoundedness, the conditional average
treatment effect can be identified as

$$
\begin{equation*}
E\left[Y_{1}-Y_{0} \mid X\right]=E[Y \mid D=1, X]-E[Y \mid D=0, X] . \tag{1.5}
\end{equation*}
$$

Note that the right-hand side of (1.5) is identified since the researcher knows the joint distribution of $(Y, D, X)$. If data are obtained from a randomized experiment, the assumption holds since the treatment is randomly assigned. However, if data are obtained from an observational study, the assumption is not testable and often controversial. In the next section, I relax the assumption of unconfoundedness and explore what can be learned about my parameter of interest when different assumptions are imposed on the unobservables and when there are additional instrumental variables $Z \in \mathcal{Z} \subset \mathbb{R}^{d_{z}}$ to help identify the conditional average treatment effect.

The welfare gain is related to Manski (2004)'s regret which has been used by Kitagawa and Tetenov (2018), Athey and Wager (2020), and many others in the literature to evaluate the performance of the estimated treatment rules. When $\mathcal{D}$ is the class of treatment rules to be considered, the regret from choosing treatment rule $\delta$ is $u\left(\delta^{*}\right)-u(\delta)$ where

$$
\begin{equation*}
\delta^{*}=\arg \max _{d \in \mathcal{D}} E\left[E\left[Y_{1} \mid X\right] \cdot d+E\left[Y_{0} \mid X\right] \cdot(1-d)\right] \tag{1.6}
\end{equation*}
$$

It is an expected loss in welfare that results from not reaching the maximum feasible welfare as $\delta^{*}$ is the policy that maximizes population welfare. In Kitagawa and Tetenov (2018) and others, under the assumption of unconfoundedness, the welfare criterion $u(\delta)$ in (1.2) is point-identified. Therefore, the optimal "oracle" treatment rule in (1.6) is well defined when the researcher knows the joint distribution of $(Y, D, X)$. However, when the welfare criterion in (1.2) is set-identified, one needs to specify their notion of optimality. For instance, the optimal rule could be a rule that maximizes the guaranteed or minimum welfare.

### 1.3 Identification

### 1.3.1 Sharp identified region

Partial identification approach has been proven to be a useful alternative or complement to point identification analysis with strong assumptions. See Manski (2003), Tamer (2010), and Molinari (2019) for an overview. The theory of random sets, which I use to conduct my identification analysis, is one of the tools that have been used fruitfully to address identification and inference in partially identified models. Examples include Beresteanu and Molinari (2008), Beresteanu, Molchanov, and Molinari (2011, 2012), Galichon and Henry (2011), Epstein, Kaido, and Seo (2016), Chesher and Rosen (2017), and Kaido and Zhang (2019). See Molchanov and Molinari (2018) for a textbook treatment of its use in econometrics.

My goal in this section is to characterize the sharp identified region of the welfare gain when different assumptions are imposed on the unobservables. The sharp identified region of the welfare gain is the tightest possible set that collects the values of welfare gain that results from all possible $\left(Y_{1}, Y_{0}\right)$ that are consistent with the maintained assumptions. Toward this end, I define a random set and its selections whose formal definitions can be found in Appendix 1.7.1. The random set is useful for incorporating weak assumptions in a unified framework rather than deriving bounds on a case-by-case basis.

Let $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right): \Omega \rightarrow \mathcal{F}$ be a random set where $\mathcal{F}$ is the family of closed subsets of $\mathbb{R}^{2}$. Assumptions on potential outcomes can be imposed through this random set. Then, the collection of all random vectors $\left(Y_{1}, Y_{0}\right)$ that are consistent with those assumptions equals the family of all selections of $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right)$ denoted by $\mathcal{S}\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right)$. Specific examples of a random set with more discussions on selections, namely, in the context of worst-case bounds of Manski (1990) and monotone treatment response analysis of Manski (1997), are given in Section 1.3.3. Using the random set notations

I just introduced, the sharp identified region of the welfare gain is given by

$$
\begin{equation*}
B_{I}\left(\delta, \delta^{*}\right) \equiv\left\{\beta \in \mathbb{R}: \beta=E\left[E\left[Y_{1}-Y_{0} \mid X\right] \cdot\left(\delta(X)-\delta^{*}(X)\right)\right],\left(Y_{1}, Y_{0}\right) \in \mathcal{S}\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right)\right\} \tag{1.7}
\end{equation*}
$$

The set $B_{I}\left(\delta, \delta^{*}\right)$ is a collection of all $\beta^{\prime}$ 's where $\left(Y_{1}, Y_{0}\right)$ is a selection of the random set $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right)$. Note that, from this definition, it is not yet clear how we can estimate this object when we have data on $(Y, D, X)$. In the next subsection, I characterize the identified set so that we can estimate the bounds when we have data.

### 1.3.2 Lower and upper bound

One way to achieve characterization of the sharp identified region is through a selection expectation and its support function. Their definitions can be found in Appendix 1.7.1. Let the support function of a convex set $K \subset \mathbb{R}^{d}$ be denoted by

$$
\begin{equation*}
s(v, K)=\sup _{x \in K}\langle v, x\rangle, \quad v \in \mathbb{R}^{d} . \tag{1.8}
\end{equation*}
$$

The support function appears in Beresteanu and Molinari (2008), Beresteanu, Molchanov, and Molinari (2011), Bontemps, Magnac, and Maurin (2012), Kaido and Santos (2014), Kaido (2016), and Kaido (2017), among others.

I first state a lemma that will be useful to prove my main result. It shows how expectation of a functional of potential outcomes can be bounded from below and above by expected support function of the random set $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right)$. The proof of the following lemma and all other proofs in this chapter are collected in the Appendix.

Lemma 1.3.1. Let $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right): \Omega \rightarrow \mathcal{F}$ be an integrable random set that is almost surely convex and let $\left(Y_{1}, Y_{0}\right) \in \mathcal{S}\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right)$. For any $v \in \mathbb{R}^{2}$, we have

$$
\begin{equation*}
-E\left[s\left(-v, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right] \leq v^{\prime} E\left[\left(Y_{1}, Y_{0}\right)^{\prime} \mid X\right] \leq E\left[s\left(v, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right] \quad \text { a.s. } \tag{1.9}
\end{equation*}
$$

I introduce a notation that appears in the following theorem and throughout the
chapter. Let $\theta_{10}(X) \equiv \mathbb{1}\left\{\delta(X)=1, \delta^{*}(X)=0\right\}$ be an indicator function for the sub population that are newly treated under the new policy. Similary, let $\theta_{01}(X) \equiv$ $\mathbb{1}\left\{\delta(X)=0, \delta^{*}(X)=1\right\}$ be an indicator function for the sub population that are no longer being treated because of the new policy.

Theorem 1.3.1 (General case). Suppose $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right): \Omega \rightarrow \mathcal{F}$ is an integrable random set that is almost surely convex. Let $\delta: \mathcal{X} \rightarrow\{0,1\}$ and $\delta^{*}: \mathcal{X} \rightarrow\{0,1\}$ be treatment rules. Also, let $v^{*}=(1,-1)^{\prime}$. Then, $B_{I}\left(\delta, \delta^{*}\right)$ in (1.7) is an interval $\left[\beta_{l}, \beta_{u}\right]$ where

$$
\begin{equation*}
\beta_{l}=E\left[\underline{\Delta}(X) \cdot \theta_{10}(X)-\bar{\Delta}(X) \cdot \theta_{01}(X)\right] \tag{1.10}
\end{equation*}
$$

and

$$
\begin{equation*}
\beta_{u}=E\left[\bar{\Delta}(X) \cdot \theta_{10}(X)-\underline{\Delta}(X) \cdot \theta_{01}(X)\right], \tag{1.11}
\end{equation*}
$$

where $\underline{\Delta}(X) \equiv-E\left[s\left(-v^{*}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right]$ and $\bar{\Delta}(X) \equiv E\left[s\left(v^{*}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right]$.

The lower (upper) bound on the welfare gain is achieved when the newly treated people are the ones who benefit the least (most) from the treatment and the people who are no longer being treated are the ones who benefit the most (least) from the treatment. Therefore, the lower and upper bounds of the welfare gain involve both $\underline{\Delta}(X)=-E\left[s\left(-v^{*}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right]$ and $\bar{\Delta}(X)=E\left[s\left(v^{*}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right]$, expected support functions of the random set at directions $-v^{*}=(-1,1)^{\prime}$ and $v^{*}=(1,-1)^{\prime}$. Oftentimes, these can be estimated by its sample analog estimators. I give closed form expressions of the expected support functions in Section 1.3.3 and 1.3.4 - they depend on objects such as $E[Y \mid D=1, X=x], E[Y \mid D=0, X=x]$, and $P(D=1 \mid X=x)$. To ease notation, let $\eta(d, x) \equiv E[Y \mid D=d, X=x]$ for $d \in\{0,1\}$ be the conditional mean treatment responses and $p(x) \equiv P(D=1 \mid X=x)$ be the propensity score.

While I characterize the identified region of the welfare gain directly given assumptions on the selections ( $Y_{1}, Y_{0}$ ), Kasy (2016)'s analysis is based on the identified set for CATE and their main results apply to any approach that leads to partial identification of treatment effects. The characterization I give above is related to their
characterization when no restrictions across covariate values are imposed on treatment effects (e.g., no restrictions such as $\Delta(x)$ is monotone in $x)$ and $\underline{\Delta}(x)$ and $\bar{\Delta}(x)$ are respectively lower and upper bound on the CATE $\Delta(x)$. As examples of such bounds, Kasy (2016) considers bounds that arise under instrument exogeneity as in Manski (2003) and under marginal stationarity of unobserved heterogeneity in panel data models as in Chernozhukov, Fernández-Val, Hahn, and Newey (2013). I consider bounds when there are instrumental variables that satisfy mean independence or mean monotonicity conditions as in Manski (2003) in Section 1.3.4.

In the following subsection, Section 1.3.3, I illustrate the form of the random set and show how Theorem 1.3.1 can be used to derive closed form bounds under different sets of assumptions.

### 1.3.3 Identification without Instruments




Figure 1•1: Random set $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right)$ under worst-case

Manski (1990) derived worst-case bounds on $Y_{1}$ and $Y_{0}$ when the outcome variable is bounded, i.e., $Y \in[\underline{y}, \bar{y}] \subset \mathbb{R}$ where $-\infty<\underline{y} \leq \bar{y}<\infty$. It is called worst-case bounds because no additional assumptions are imposed on their distributions. Then,
as shown in Figure 1•1, the random set $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right)$ is such that

$$
\mathcal{Y}_{1} \times \mathcal{Y}_{0}=\left\{\begin{array}{l}
\{Y\} \times[\underline{y}, \bar{y}] \text { if } D=1  \tag{1.12}\\
{[\underline{y}, \bar{y}] \times\{Y\} \text { if } D=0}
\end{array}\right.
$$

The random set in (1.12) switches its value between two sets depending on the value of $D$. If $D=1, \mathcal{Y}_{1}$ is given by a singleton $\{Y\}$ whereas $\mathcal{Y}_{0}$ is given by the entire support $[\underline{y}, \bar{y}]$. Similarly, if $D=0, \mathcal{Y}_{0}$ is given by a singleton $\{Y\}$ whereas $\mathcal{Y}_{1}$ is given by the entire support $[\underline{y}, \bar{y}]$. I plot $\mathcal{Y}_{d}$ and its selection $Y_{d}$ for $d \in\{0,1\}$ as a function of $\omega \in \Omega$ in Figure 1.2. If $D=d$, the random set $\mathcal{Y}_{d}$ is a singleton $\{Y\}$ and the family of selections consists of single random variable $\{Y\}$ as well. On the other hand, if $D=1-d$, the random set $\mathcal{Y}_{d}$ is an interval $[\underline{y}, \bar{y}]$ and the family of all selections consists of all $\mathfrak{A}$-measurable random variables that has support on $[\underline{y}, \bar{y}]$. Note that each selection $\left(Y_{1}, Y_{0}\right)$ of $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right)$ can be represented in the following way.

$$
D=d
$$

$$
D=1-d
$$




Figure 1.2: Random set $\mathcal{Y}_{d}$ and its selection $Y_{d}$ for $d \in\{0,1\}$ as a function of $\omega \in \Omega$ under worst-case

Take random variables $S_{1}: \Omega \rightarrow \mathbb{R}$ and $S_{0}: \Omega \rightarrow \mathbb{R}$ whose distributions conditional on $Y$ and $D$ are not specified and can be any probability distributions on $[\underline{y}, \bar{y}]$. Then
$\left(Y_{1}, Y_{0}\right)$ that satisfies the following is a selection of $\mathcal{Y}_{1} \times \mathcal{Y}_{0}$ :

$$
\begin{align*}
& Y_{1}=Y \cdot D+S_{1} \cdot(1-D),  \tag{1.13}\\
& Y_{0}=Y \cdot(1-D)+S_{0} \cdot D .
\end{align*}
$$

This representation makes it even clearer how I am not imposing any structure on the counterfactuals that I do not observe. $S_{1}$ and $S_{0}$ correspond to the selection mechanisms that appear in Ponomareva and Tamer (2011) and Tamer (2010).

Now, for the random set in (1.12), I can calculate its expected support function at directions $v^{*}=(1,-1)$ and $-v^{*}=(-1,1)$ to obtain the bounds of the welfare gain in closed form. As shown in Figure 1.3, the support function of random set $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right)$ in (1.12) at direction $v^{*}=(1,-1)$ is the (signed) distance (rescaled by the norm of $v^{*}$ ) between the origin and the hyperplane tangent to the random set in direction $v^{*}=(1,-1)$. Then, the bounds are given in the following Corollary to Theorem 1.3.1.


$$
v^{*}=(1,-1)
$$


$v^{*}=(1,-1)$

Figure 1.3: Support function of $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right)$ at direction $v^{*}=(1,-1)$ under worst-case

Corollary 1.3.1 (Worst-case). Let $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right)$ be a random set in (1.12). Let $\delta: \mathcal{X} \rightarrow$ $\{0,1\}$ and $\delta^{*}: \mathcal{X} \rightarrow\{0,1\}$ be treatment rules. Then, $B_{I}\left(\delta, \delta^{*}\right)$ in (1.7) is an interval
[ $\left.\beta_{l}, \beta_{u}\right]$ where

$$
\begin{align*}
\beta_{l}=E & {\left[((\eta(1, X)-\bar{y}) \cdot p(X)+(\underline{y}-\eta(0, X)) \cdot(1-p(X))) \cdot \theta_{10}(X)\right.} \\
& \left.-((\eta(1, X)-\underline{y}) \cdot p(X)+(\bar{y}-\eta(0, X)) \cdot(1-p(X))) \cdot \theta_{01}(X)\right], \tag{1.14}
\end{align*}
$$

and

$$
\begin{align*}
\beta_{u}=E & {\left[((\eta(1, X)-\underline{y}) \cdot p(X)+(\bar{y}-\eta(0, X)) \cdot(1-p(X))) \cdot \theta_{10}(X)\right.}  \tag{1.15}\\
& \left.-((\eta(1, X)-\bar{y}) \cdot p(X)+(\underline{y}-\eta(0, X)) \cdot(1-p(X))) \cdot \theta_{01}(X)\right] .
\end{align*}
$$

Worst-case analysis is a great starting point as no additional assumptions are imposed on the unobservables. However, the bounds could be too wide to be informative in some cases. In fact, the worst-case bound cover 0 all the time as $\beta_{l} \leq 0$ and $\beta_{u} \geq 0$. One could impose additional assumptions on the relationship between the unobservables and obtain tighter bounds. Towards that end, I analyze the monotone treatment response (MTR) assumption of Manski (1997).

Assumption 1.3.1 (MTR Assumption).

$$
\begin{equation*}
Y_{1} \geq Y_{0} \text { a.s. } \tag{1.16}
\end{equation*}
$$

Assumption 1.3.1 states that everyone benefits from the treatment. Suppose Assumption 1.3.1 holds. Then, the random set is such that

$$
\mathcal{Y}_{1} \times \mathcal{Y}_{0}=\left\{\begin{array}{l}
\{Y\} \times[\underline{y}, Y] \text { if } D=1  \tag{1.17}\\
{[Y, \bar{y}] \times\{Y\} \text { if } D=0}
\end{array}\right.
$$

As shown in Figure 1•4, depending on the value of $D$, the random set in (1.17) switches its value between two sets, that are smaller than those in (1.12). The bounds of the welfare gain when the random set is given by (1.17) are given in the following Corollary to Theorem 1.3.1. Notice that the lower bound on conditional average treatment effect $\underline{\Delta}(X)=-E\left[s\left(-v^{*}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right]$ equals 0 when the random set is



Figure 1.4: Random set $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right)$ under MTR Assumption
given by (1.17). It is shown geometrically in Figure $1 \cdot 5$. The expected support function of the random set in (1.17) at direction $-v^{*}=(-1,1)^{\prime}$ is always 0 as the hyperplane tangent to the random set at direction $-v^{*}=(-1,1)^{\prime}$ goes through the origin regardless of the value of $D$.



Figure 1.5: Support function of $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right)$ at direction $-v^{*}=(-1,1)^{\prime}$ under MTR Assumption

Corollary 1.3.2 (MTR). Suppose Assumption 1.3 .1 holds. Let $\delta: \mathcal{X} \rightarrow\{0,1\}$ and $\delta^{*}: \mathcal{X} \rightarrow\{0,1\}$ be treatment rules. Then, $B_{I}\left(\delta, \delta^{*}\right)$ in (1.7) is an interval $\left[\beta_{l}, \beta_{u}\right]$ where

$$
\begin{equation*}
\beta_{l}=E\left[-((\eta(1, X)-\underline{y}) \cdot p(X)+(\bar{y}-\eta(0, X)) \cdot(1-p(X))) \cdot \theta_{01}(X)\right] \tag{1.18}
\end{equation*}
$$

and

$$
\begin{equation*}
\beta_{u}=E\left[((\eta(1, X)-\underline{y}) \cdot p(X)+(\bar{y}-\eta(0, X)) \cdot(1-p(X))) \cdot \theta_{10}(X)\right] \tag{1.19}
\end{equation*}
$$

### 1.3.4 Identification with Instruments

Availability of additional variables, called instrumental variables, could help us tighten the bounds on CATE and hence the bounds on the welfare gain. In this subsection, I consider two types of assumptions: (1) mean independence (IV Assumption) and (2) mean monotonicity (MIV Assumption).

## Mean independence

Assumption 1.3.2 (IV Assumption). There exists an instrumental variable $Z \in$ $\mathcal{Z} \subset \mathbb{R}^{d_{z}}$ such that, for $d \in\{0,1\}$, the following mean independence holds:

$$
\begin{equation*}
E\left[Y_{d} \mid X, Z=z\right]=E\left[Y_{d} \mid X, Z=z^{\prime}\right] \tag{1.20}
\end{equation*}
$$

for each value of $X$ and all $z, z^{\prime} \in \mathcal{Z}$.

This assumption requires that there exists a conditionally exogeneous instrumental variable $Z$ for our treatment $D$. Specifically, the assumption means the following. Each value of $(X, Z)$ defines an observable subpopulation. Then, the assumption says that, for each value of $X$, the mean value of each potential outcome is the same across all of the subpopulations $(X, Z=z), z \in \mathcal{Z}$.

When data are obtained from a randomized experiment with imperfect compliance, the random assignment can be used as an instrumental variable to identify the effect of the treatment. When $Z \in\{0,1\}$ is random offer, for each $x \in \mathcal{X}$, we can define subpopulations $(x, 0)$, people who are described by $x$ and are in the control group, and $(x, 1)$, people who are described by $x$ and are in the treatment group. Then, the assumption says that the mean value of each potential outcome is the same in these two groups. This assumption is plausible since the offer is randomly assigned.

Suppose Assumption 1.3.2 holds. Since I am imposing an additional restriction on $\left(Y_{1}, Y_{0}\right)$, the sharp identified region of the welfare gain is given by

$$
B_{I}\left(\delta, \delta^{*}\right) \equiv\left\{\beta \in \mathbb{R}: \beta=E\left[E\left[Y_{1}-Y_{0} \mid X\right] \cdot\left(\delta(X)-\delta^{*}(X)\right)\right],\left(Y_{1}, Y_{0}\right) \in \mathcal{S}\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right)\right.
$$

$$
\begin{equation*}
\left(Y_{1}, Y_{0}\right) \text { satisfies Assumption 1.3.2\} } \tag{1.21}
\end{equation*}
$$

The following lemma corresponds to the Manski's sharp bounds for CATE under IV assumption. Manski (1990) explains it for the more general case of when there are level-set restrictions on the outcome regression. To ease notation, let $q(v, z) \equiv$ $E\left[s\left(v, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X, Z=z\right]$ be the expected support function of the random set $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right)$ at direction $v$, conditional on $(X, Z)$ when $Z$ takes value $z$.

Lemma 1.3.2 (IV). Let $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right): \Omega \rightarrow \mathcal{F}$ be an integrable random set that is almost surely convex and let $\left(Y_{1}, Y_{0}\right) \in \mathcal{S}\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right)$. Let $v_{1}=(1,0)^{\prime}$ and $v_{0}=(0,1)^{\prime}$. Suppose Assumption 1.3.2 holds. Then, we have

$$
\begin{align*}
& \sup _{z \in \mathcal{Z}}\left\{-q\left(-v_{1}, z\right)\right\}-\inf _{z \in \mathcal{Z}} q\left(v_{0}, z\right) \\
& \quad \leq E\left[Y_{1}-Y_{0} \mid X\right] \leq  \tag{1.22}\\
& \inf _{z \in \mathcal{Z}} q\left(v_{1}, z\right)-\sup _{z \in \mathcal{Z}}\left\{-q\left(-v_{0}, z\right)\right\} \quad \text { a.s. }
\end{align*}
$$

Bounds for CATE with instrumental variables involve expected support functions at directions $v_{1}=(1,0)$ and $v_{0}=(0,1)$. The support function of the random set $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right)$ at direction $v_{1}=(1,0)$ under worst-case is depicted in Figure 1•6.

Theorem 1.3.2 (IV). Suppose $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right): \Omega \rightarrow \mathcal{F}$ is an integrable random set that is almost surely convex. Let $\delta: \mathcal{X} \rightarrow\{0,1\}$ and $\delta^{*}: \mathcal{X} \rightarrow\{0,1\}$ be treatment rules. Also, let $v_{1}=(1,0)^{\prime}$ and $v_{0}=(0,1)^{\prime}$. Then, $B_{I}\left(\delta, \delta^{*}\right)$ in (1.21) is an interval $\left[\beta_{l}, \beta_{u}\right]$ where

$$
\begin{equation*}
\beta_{l}=E\left[\underline{\Delta}(X) \cdot \theta_{10}(X)-\bar{\Delta}(X) \cdot \theta_{01}(X)\right] \tag{1.23}
\end{equation*}
$$

and

$$
\begin{equation*}
\beta_{u}=E\left[\bar{\Delta}(X) \cdot \theta_{10}(X)-\underline{\Delta}(X) \cdot \theta_{01}(X)\right] \tag{1.24}
\end{equation*}
$$

where $\underline{\Delta}(X) \equiv \sup _{z \in \mathcal{Z}}\left\{-q\left(-v_{1}, z\right)\right\}-\inf _{z \in \mathcal{Z}} q\left(v_{0}, z\right)$ and $\bar{\Delta}(X) \equiv \inf _{z \in \mathcal{Z}} q\left(v_{1}, z\right)-$


Figure 1.6: Support function of $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right)$ at direction $v_{1}=(1,0)$ under worst-case
$\sup _{z \in \mathcal{Z}}\left\{-q\left(-v_{0}, z\right)\right\}$.

Identification of the welfare gain with instruments is similar to idenfication without instruments. The difference lies in the forms of lower and upper bounds on the CATE. Theorem 1.3.2 can be combined with different maintained assumptions on the potential outcomes to result in different bounds. The following corollary shows the IV bounds under worst-case assumption. To ease notation, let $\eta(d, x, z) \equiv E[Y \mid D=$ $d, X=x, Z=z]$ for $d \in\{0,1\}$ denote the conditional mean treatment responses and $p(x, z) \equiv P(D=1 \mid X=x, Z=z)$ denote the propensity score.

Corollary 1.3.3 (IV-worst-case). Let $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right)$ be a random set in (1.12). Let $\delta: \mathcal{X} \rightarrow\{0,1\}$ and $\delta^{*}: \mathcal{X} \rightarrow\{0,1\}$ be treatment rules. Then, $B_{I}\left(\delta, \delta^{*}\right)$ in (1.21) is
an interval $\left[\beta_{l}, \beta_{u}\right]$ where

$$
\begin{align*}
\beta_{l}=E & {\left[\left(\sup _{z \in \mathcal{Z}}\{\eta(1, X, z) \cdot p(X, z)+\underline{y} \cdot(1-p(X, z))\}\right.\right.} \\
& \left.-\inf _{z \in \mathcal{Z}}\{\bar{y} \cdot p(X, z)+\eta(0, X, z) \cdot(1-p(X, z))\}\right) \cdot \theta_{10}(X)  \tag{1.25}\\
& -\left(\inf _{z \in \mathcal{Z}}\{\eta(1, X, z) \cdot p(X, z)+\bar{y} \cdot(1-p(X, z))\}\right. \\
& \left.\left.-\sup _{z \in \mathcal{Z}}\{\underline{y} \cdot p(X, z)+\eta(0, X, z) \cdot(1-p(X, z))\}\right) \cdot \theta_{01}(X)\right],
\end{align*}
$$

and

$$
\begin{align*}
\beta_{u}=E & {\left[\left(\inf _{z \in \mathcal{Z}}\{\eta(1, X, z) \cdot p(X, z)+\bar{y} \cdot(1-p(X, z))\}\right.\right.} \\
& \left.-\sup _{z \in \mathcal{Z}}\{\underline{y} \cdot p(X, z)+\eta(0, X, z) \cdot(1-p(X, z))\}\right) \cdot \theta_{10}(X)  \tag{1.26}\\
& -\left(\sup _{z \in \mathcal{Z}}\{\eta(1, X, z) \cdot p(X, z)+\underline{y} \cdot(1-p(X, z))\}\right. \\
& \left.\left.-\inf _{z \in \mathcal{Z}}\{\bar{y} \cdot p(X, z)+\eta(0, X, z) \cdot(1-p(X, z))\}\right) \cdot \theta_{01}(X)\right] .
\end{align*}
$$

Next, we derive the IV bounds under MTR assumption in the following corollary.

Corollary 1.3.4 (IV-MTR). Suppose Assumption 1.3 .1 holds. Let $\delta: \mathcal{X} \rightarrow\{0,1\}$ and $\delta^{*}: \mathcal{X} \rightarrow\{0,1\}$ be treatment rules. Then, $B_{I}\left(\delta, \delta^{*}\right)$ in (1.21) is an interval $\left[\beta_{l}, \beta_{u}\right]$ where

$$
\begin{align*}
\beta_{l}=E & {\left[-\left(\inf _{z \in \mathcal{Z}}\{\eta(1, X, z) \cdot p(X, z)+\bar{y} \cdot(1-p(X, z))\}\right.\right.} \\
& \left.\left.-\sup _{z \in \mathcal{Z}}\{\underline{y} \cdot p(X, z)+\eta(0, X, z) \cdot(1-p(X, z))\}\right) \cdot \theta_{01}(X)\right], \tag{1.27}
\end{align*}
$$

and

$$
\begin{align*}
\beta_{u}=E & {\left[\left(\inf _{z \in \mathcal{Z}}\{\eta(1, X, z) \cdot p(X, z)+\bar{y} \cdot(1-p(X, z))\}\right.\right.} \\
& \left.\left.-\sup _{z \in \mathcal{Z}}\{\underline{y} \cdot p(X, z)+\eta(0, X, z) \cdot(1-p(X, z))\}\right) \cdot \theta_{10}(X)\right] . \tag{1.28}
\end{align*}
$$

Bounds obtained with instruments are functions of $\eta(1, x, z), \eta(0, x, z)$ and $p(x, z)$ and involve taking intersections across values of $Z$. If $Z$ is continuous, this would
amount to infinitely many intersections. However, bounds can be simplified in some empirically relevant cases such as the following.

Assumption 1.3.3 (Binary IV with monotonic first step). Suppose $Z \in\{0,1\}$ is a binary instrumental variable that satisfies Assumption 1.3.2. Suppose further that for all $x \in \mathcal{X}$,

$$
\begin{equation*}
p(x, 1)=P(D=1 \mid X=x, Z=1) \geq P(D=1 \mid X=x, Z=0)=p(x, 0) \tag{1.29}
\end{equation*}
$$

When $Z \in\{0,1\}$ is random offer and $D \in\{0,1\}$ is program participation, this means that someone who received an offer to participate in the program is more likely to participate in the program than someone who didn't receive an offer.

Lemma 1.3.3. Suppose Assumption 1.3.3 holds. Then,

$$
\begin{align*}
& 1=\arg \max _{z \in\{0,1\}}\{\eta(1, X, z) \cdot p(X, z)+\underline{y} \cdot(1-p(X, z))\},  \tag{1.30}\\
& 0=\arg \min _{z \in\{0,1\}}\{\bar{y} \cdot p(X, z)+\eta(0, X, z) \cdot(1-p(X, z))\},  \tag{1.31}\\
& 1=\arg \min _{z \in\{0,1\}}\{\eta(1, X, z) \cdot p(X, z)+\bar{y} \cdot(1-p(X, z))\},  \tag{1.32}\\
& 0=\arg \max _{z \in\{0,1\}}\{\underline{y} \cdot p(X, z)+\eta(0, X, z) \cdot(1-p(X, z))\} . \tag{1.33}
\end{align*}
$$

Under Assumption 1.3.3, using Lemma 1.3.3, bounds in (1.25) and (1.26) are simplified as

$$
\begin{align*}
\beta_{l}=E & {[((\eta(1, X, 1) \cdot p(X, 1)+\underline{y} \cdot(1-p(X, 1))} \\
& -(\bar{y} \cdot p(X, 0)+\eta(0, X, 0) \cdot(1-p(X, 0)))) \cdot \theta_{10}(X)  \tag{1.34}\\
& -((\eta(1, X, 1) \cdot p(X, 1)+\bar{y} \cdot(1-p(X, 1))) \\
& \left.-(\underline{y} \cdot p(X, 0)+\eta(0, X, 0) \cdot(1-p(X, 0)))) \cdot \theta_{01}(X)\right]
\end{align*}
$$

and

$$
\begin{align*}
\beta_{u}=E & {[((\eta(1, X, 1) \cdot p(X, 1)+\bar{y} \cdot(1-p(X, 1)))} \\
& -(\underline{y} \cdot p(X, 0)+\eta(0, X, 0) \cdot(1-p(X, 0)))) \cdot \theta_{10}(X)  \tag{1.35}\\
& -((\eta(1, X, 1) \cdot p(X, 1)+\underline{y} \cdot(1-p(X, 1))) \\
& \left.-(\bar{y} \cdot p(X, 0)+\eta(0, X, 0) \cdot(1-p(X, 0)))) \cdot \theta_{01}(X)\right] .
\end{align*}
$$

Bounds in (1.27) and (1.28) can also be simplified similarly.

## Mean monotonicity

Next, I consider monotone instrumental variable (MIV) assumption introduced by Manski and Pepper (2000) which weakens Assumption 1.3.2 by replacing the equality in (1.20) by an inequality. An instrumental variable which satisfies this assumption could also help us obtain tighter bounds.

Assumption 1.3.4 (MIV Assumption). There exists an instrumental variable $Z \in$ $\mathcal{Z} \subset \mathbb{R}^{d_{z}}$ such that, for $d \in\{0,1\}$, the following mean monotonicity holds:

$$
\begin{equation*}
E\left[Y_{d} \mid X, Z=z\right] \geq E\left[Y_{d} \mid X, Z=z^{\prime}\right] \tag{1.36}
\end{equation*}
$$

for each value of $X$ and all $z, z^{\prime} \in \mathcal{Z}$ such that $z \geq z^{\prime}$.

In the job training program example, the pre-program earnings can be used as a monotone instrumental variable when the outcome variable is post-program earnings. If we were to use the pre-program earnings as an instrumental variable that holds in mean independent sense, then we would have to assume that people with different prior earnings have the same mean post-program earnings. That might not be a reasonable assumption to make. On the other hand, if we use the prior earnings as a monotone instrumental variable, we assume that people with higher prior earnings have weakly higher mean post-program earnings compared to those with lower prior earnings. That is much more reasonable assumption to make in this context.

Suppose Assumption 1.3.4 holds. Then, the sharp identified region of the welfare gain is given by

$$
B_{I}\left(\delta, \delta^{*}\right) \equiv\left\{\beta \in \mathbb{R}: \beta=E\left[E\left[Y_{1}-Y_{0} \mid X\right] \cdot\left(\delta(X)-\delta^{*}(X)\right)\right],\left(Y_{1}, Y_{0}\right) \in \mathcal{S}\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right)\right.
$$

$$
\begin{equation*}
\left(Y_{1}, Y_{0}\right) \text { satisfies Assumption 1.3.4\}. } \tag{1.37}
\end{equation*}
$$

Similarly to the IV analysis, the following lemma illustrates the bounds for CATE under MIV assumption.

Lemma 1.3.4 (MIV). Let $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right): \Omega \rightarrow \mathcal{F}$ be an integrable random set that is almost surely convex and let $\left(Y_{1}, Y_{0}\right) \in \mathcal{S}\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right)$. Let $v_{1}=(1,0)^{\prime}$ and $v_{0}=(0,1)^{\prime}$. Suppose Assumption 1.3.4 holds. Then, we have

$$
\begin{align*}
& \sum_{z \in \mathcal{Z}} P(Z=z) \cdot\left(\sup _{z_{1} \leq z}\left\{-q\left(-v_{1}, z_{1}\right)\right\}-\inf _{z_{2} \geq z} q\left(v_{0}, z_{2}\right)\right) \\
& \quad \leq E\left[Y_{1}-Y_{0} \mid X\right] \leq  \tag{1.38}\\
& \sum_{z \in \mathcal{Z}} P(Z=z) \cdot\left(\inf _{z_{2} \geq z} q\left(v_{1}, z_{2}\right)-\sup _{z_{1} \leq z}\left\{-q\left(-v_{0}, z_{1}\right)\right\}\right) \quad \text { a.s. }
\end{align*}
$$

Recall that under IV assumption, the intersections are taken over all of values of $Z$. On the other hand, as for MIV bounds, for each value of $Z$, the intersections are taken for values above or below that value. The bounds involve integrating them over values of $Z$. Note that when $\mathcal{Z}$ is not finite, the summation notation in (1.38) can be replaced by a Lebesgue integral.

Theorem 1.3.3 (MIV). Suppose $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right): \Omega \rightarrow \mathcal{F}$ is an integrable random set that is almost surely convex. Let $\delta: \mathcal{X} \rightarrow\{0,1\}$ and $\delta^{*}: \mathcal{X} \rightarrow\{0,1\}$ be treatment rules. Also, let $v_{1}=(1,0)^{\prime}$ and $v_{0}=(0,1)^{\prime}$. Then, $B_{I}\left(\delta, \delta^{*}\right)$ in (1.37) is an interval $\left[\beta_{l}, \beta_{u}\right]$ where

$$
\begin{equation*}
\beta_{l}=E\left[\underline{\Delta}(X) \cdot \theta_{10}(X)-\bar{\Delta}(X) \cdot \theta_{01}(X)\right] \tag{1.39}
\end{equation*}
$$

and

$$
\begin{equation*}
\beta_{u}=E\left[\bar{\Delta}(X) \cdot \theta_{10}(X)-\underline{\Delta}(X) \cdot \theta_{01}(X)\right] \tag{1.40}
\end{equation*}
$$

where

$$
\underline{\Delta}(X) \equiv \sum_{z \in \mathcal{Z}} P(Z=z) \cdot\left(\sup _{z_{1} \leq z}\left\{-q\left(-v_{1}, z_{1}\right)\right\}-\inf _{z_{2} \geq z} q\left(v_{0}, z_{2}\right)\right)
$$

and

$$
\bar{\Delta}(X) \equiv \sum_{z \in \mathcal{Z}} P(Z=z) \cdot\left(\inf _{z_{2} \geq z} q\left(v_{1}, z_{2}\right)-\sup _{z_{1} \leq z}\left\{-q\left(-v_{0}, z_{1}\right)\right\}\right)
$$

This identification result under MIV assumption is similar to the identification result under IV assumption. Again, the difference lies in the forms of lower and upper bounds on the CATE. As before, Theorem 1.3.3 can be combined with different maintained assumptions on the potential outcomes to result in different bounds. The following corollary shows the MIV bounds under worst-case assumption.

Corollary 1.3.5 (MIV-worst-case). Let $\delta: \mathcal{X} \rightarrow\{0,1\}$ and $\delta^{*}: \mathcal{X} \rightarrow\{0,1\}$ be treatment rules. Then, $B_{I}\left(\delta, \delta^{*}\right)$ in (1.37) is an interval $\left[\beta_{l}, \beta_{u}\right]$ where

$$
\begin{align*}
\beta_{l}=E & {\left[\sum _ { z \in \mathcal { Z } } P ( Z = z ) \cdot \left(\sup _{z_{1} \leq z}\left\{\eta\left(1, X, z_{1}\right) \cdot p\left(X, z_{1}\right)+\underline{y} \cdot\left(1-p\left(X, z_{1}\right)\right)\right\}\right.\right.} \\
& \left.-\inf _{z_{2} \geq z}\left\{\bar{y} \cdot p\left(X, z_{2}\right)+\eta\left(0, X, z_{2}\right) \cdot\left(1-p\left(X, z_{2}\right)\right)\right\}\right) \cdot \theta_{10}(X) \\
& -\sum_{z \in \mathcal{Z}} P(Z=z) \cdot\left(\inf _{z_{2} \geq z}\left\{\eta\left(1, X, z_{2}\right) \cdot p\left(X, z_{2}\right)+\bar{y} \cdot\left(1-p\left(X, z_{2}\right)\right)\right\}\right.  \tag{1.41}\\
& \left.\left.-\sup _{z_{1} \leq z}\left\{\underline{y} \cdot p\left(X, z_{1}\right)+\eta\left(0, X, z_{1}\right) \cdot\left(1-p\left(X, z_{1}\right)\right)\right\}\right) \cdot \theta_{01}(X)\right]
\end{align*}
$$

and

$$
\begin{align*}
\beta_{u}=E & {\left[\sum _ { z \in \mathcal { Z } } P ( Z = z ) \cdot \left(\inf _{z_{2} \geq z}\left\{\eta\left(1, X, z_{2}\right) \cdot p\left(X, z_{2}\right)+\bar{y} \cdot\left(1-p\left(X, z_{2}\right)\right)\right\}\right.\right.} \\
& \left.-\sup _{z_{1} \leq z}\left\{\underline{y} \cdot p\left(X, z_{1}\right)+\eta\left(0, X, z_{1}\right) \cdot\left(1-p\left(X, z_{1}\right)\right)\right\}\right) \cdot \theta_{10}(X)  \tag{1.42}\\
& -\sum_{z \in \mathcal{Z}} P(Z=z) \cdot\left(\sup _{z_{1} \leq z}\left\{\eta\left(1, X, z_{1}\right) \cdot p\left(X, z_{1}\right)+\underline{y} \cdot\left(1-p\left(X, z_{1}\right)\right)\right\}\right. \\
& \left.\left.-\inf _{z_{2} \geq z}\left\{\bar{y} \cdot p\left(X, z_{2}\right)+\eta\left(0, X, z_{2}\right) \cdot\left(1-p\left(X, z_{2}\right)\right)\right\}\right) \cdot \theta_{01}(X)\right] .
\end{align*}
$$

Next, we derive the MIV bounds under MTR assumption in the following corollary.

Corollary 1.3.6 (MIV-MTR). Suppose Assumption 1.3.1 holds. Let $\delta: \mathcal{X} \rightarrow\{0,1\}$ and $\delta^{*}: \mathcal{X} \rightarrow\{0,1\}$ be treatment rules. Then, $B_{I}\left(\delta, \delta^{*}\right)$ in (1.37) is an interval $\left[\beta_{l}, \beta_{u}\right]$ where

$$
\begin{align*}
\beta_{l}=E & {\left[\sum _ { z \in \mathcal { Z } } P ( Z = z ) \cdot \left(\sup _{z_{1} \leq z}\left\{E\left[Y \mid X, Z=z_{1}\right]\right\}\right.\right.} \\
& \left.-\inf _{z_{2} \geq z}\left\{E\left[Y \mid X, Z=z_{2}\right]\right\}\right) \cdot \theta_{10}(X) \\
& -\sum_{z \in \mathcal{Z}} P(Z=z) \cdot\left(\inf _{z_{2} \geq z}\left\{\eta\left(1, X, z_{2}\right) \cdot p\left(X, z_{2}\right)+\bar{y} \cdot\left(1-p\left(X, z_{2}\right)\right)\right\}\right.  \tag{1.43}\\
& \left.\left.-\sup _{z_{1} \leq z}\left\{\underline{y} \cdot p\left(X, z_{1}\right)+\eta\left(0, X, z_{1}\right) \cdot\left(1-p\left(X, z_{1}\right)\right)\right\}\right) \cdot \theta_{01}(X)\right]
\end{align*}
$$

and

$$
\begin{align*}
\beta_{u}=E & {\left[\sum _ { z \in \mathcal { Z } } P ( Z = z ) \cdot \left(\inf _{z_{2} \geq z}\left\{\eta\left(1, X, z_{2}\right) \cdot p\left(X, z_{2}\right)+\bar{y} \cdot\left(1-p\left(X, z_{2}\right)\right)\right\}\right.\right.} \\
& \left.-\sup _{z_{1} \leq z}\left\{\underline{y} \cdot p\left(X, z_{1}\right)+\eta\left(0, X, z_{1}\right) \cdot\left(1-p\left(X, z_{1}\right)\right)\right\}\right) \cdot \theta_{10}(X) \\
& -\sum_{z \in \mathcal{Z}} P(Z=z) \cdot\left(\sup _{z_{1} \leq z}\left\{E\left[Y \mid X, Z=z_{1}\right]\right\}\right.  \tag{1.44}\\
& \left.\left.-\inf _{z_{2} \geq z}\left\{E\left[Y \mid X, Z=z_{2}\right]\right\}\right) \cdot \theta_{01}(X)\right] .
\end{align*}
$$

Notice that the MIV-MTR bounds involve $E[Y \mid X=x, Z=z]$ other than $\eta(d, x, z)=$ $E[Y \mid D=d, X=x, Z=z]$ and $p(x, z)=P(D=1 \mid X=x, Z=z)$. For ease of exposition, Table 1.1 summarizes the forms of lower and upper bounds on CATE under different sets of assumptions.

### 1.4 Estimation and Inference

The bounds developed in Section 1.3 are functions of conditional mean treatment responses $\eta(1, x)$ and $\eta(0, x)$, and propensity score $p(x)$ in the absence of instruments. The bounds with instruments are functions of conditional mean treatment responses $\eta(1, x, z)$ and $\eta(0, x, z)$, and propensity score $p(x, z)$. Let $F$ be the joint distribution of $W=(Y, D, X, Z)$ and suppose we have a size $n$ random sample $\left\{w_{i}\right\}_{i=1}^{n}$ from $W$.

Table 1.1: Lower and upper bounds on CATE

| Assumptions | $\Delta(X)$ | $\bar{\Delta}(X)$ |
| :---: | :---: | :---: |
| worst-case | $\begin{aligned} & (\eta(1, X)-\bar{y}) \cdot p(X) \\ & \quad+(\underline{y}-\eta(0, X)) \cdot(1-p(X)) \end{aligned}$ | $\begin{aligned} & (\eta(1, X)-y) \cdot p(X) \\ & \quad+(\bar{y}-\eta \overline{(0, X)}) \cdot(1-p(X)) \end{aligned}$ |
| MTR | 0 | same as worst-case |
| IV-worst-case | $\begin{aligned} & \sup _{z \in \mathcal{Z}}\{\eta(1, X, z) \cdot p(X, z) \\ & \quad+y \cdot(1-p(X, z))\} \\ & \inf _{z \in \mathcal{Z}}\{\bar{y} \cdot \bar{p}(X, z) \\ & \quad+\eta(0, X, z) \cdot(1-p(X, z))\}) \end{aligned}$ | $\begin{aligned} & \inf _{z \in \mathcal{Z}}\{\eta(1, X, z) \cdot p(X, z) \\ &+\bar{y} \cdot(1-p(X, z))\} \\ &-\sup _{z \in \mathcal{Z}}\{\underline{y} \cdot p(X, z) \\ &+\eta(0, X, z) \cdot(1-p(X, z))\}) \end{aligned}$ |
| IV-MTR | 0 | same as IV-worst-case |
| MIV-worst-case | $\begin{aligned} \sum_{z \in \mathcal{Z}} P(Z=z) \cdot\left(\operatorname { s u p } _ { z _ { 1 } \leq z } \left\{\eta\left(1, X, z_{1}\right) \cdot p\left(X, z_{1}\right)\right.\right. \\ \left.+\underline{y} \cdot\left(1-p\left(X, z_{1}\right)\right)\right\} \\ -\inf _{z_{2} \geq z}\left\{\bar{y} \cdot p\left(X, z_{2}\right)\right. \\ \left.\left.+\eta\left(0, X, z_{2}\right) \cdot\left(1-p\left(X, z_{2}\right)\right)\right\}\right) \end{aligned}$ | $\begin{array}{r} \sum_{z \in \mathcal{Z}} P(Z=z) \cdot\left(\operatorname { i n f } _ { z _ { 2 } \geq z } \left\{\eta\left(1, X, z_{2}\right) \cdot p\left(X, z_{2}\right)\right.\right. \\ \left.+\bar{y} \cdot\left(1-p\left(X, z_{2}\right)\right)\right\} \\ -\sup _{z_{1} \leq z}\left\{\underline{y} \cdot p\left(X, z_{1}\right)\right. \\ \left.\left.+\eta\left(0, X, z_{1}\right) \cdot\left(1-p\left(X, z_{1}\right)\right)\right\}\right) \end{array}$ |
| MIV-MTR | $\begin{array}{r} \sum_{z \in \mathcal{Z}} P(Z=z) \cdot\left(\sup _{z_{1} \leq z}\left\{E\left[Y \mid X, Z=z_{1}\right]\right\}\right. \\ \left.-\inf _{z_{2} \geq z}\left\{E\left[Y \mid X, Z=z_{2}\right]\right\}\right) \end{array}$ | same as MIV-worst-case |

This table reports the form of $\underline{\Delta}(X)$ and $\bar{\Delta}(X)$ under different assumptions.

If the conditioning variables $X$ and $Z$ are discrete and take finitely many values, conditional mean treatment responses and propensity scores can be estimated by the corresponding empirical means. If there is a continuous component, conditional mean treatment responses and propensity scores can be estimated using nonparametric regression methods. I start with bounds that do not rely on instruments. Let $\hat{\eta}(1, x)$, $\hat{\eta}(0, x)$, and $\hat{p}(x)$ be those estimated values. A natural sample analog estimator for the lower bound under the worst-case in (1.14) can be constructed by first plugging these estimated values into (1.14) and then by taking average over $i$ as follows:

$$
\begin{align*}
\hat{\beta}_{l}=\frac{1}{n} & \sum_{i=1}^{n}\left[\left(\left(\hat{\eta}\left(1, x_{i}\right)-\bar{y}\right) \cdot \hat{p}\left(x_{i}\right)+\left(\underline{y}-\hat{\eta}\left(0, x_{i}\right)\right) \cdot\left(1-\hat{p}\left(x_{i}\right)\right)\right) \cdot \theta_{10}\left(x_{i}\right)\right.  \tag{1.45}\\
& \left.-\left(\left(\hat{\eta}\left(1, x_{i}\right)-\underline{y}\right) \cdot \hat{p}\left(x_{i}\right)+\left(\bar{y}-\hat{\eta}\left(0, x_{i}\right)\right) \cdot\left(1-\hat{p}\left(x_{i}\right)\right)\right) \cdot \theta_{01}\left(x_{i}\right)\right] .
\end{align*}
$$

In this estimation problem, $\eta(1, x), \eta(0, x)$, and $p(x)$ are nuisance parameters that need to be estimated nonparametrically. In what follows, I collect these possibly
infinite-dimensional nuisance parameters and denote it as follows: ${ }^{4}$

$$
\begin{equation*}
\gamma=(\eta(1, \cdot), \eta(0, \cdot), p(\cdot)) \tag{1.46}
\end{equation*}
$$

Estimation of these parameters can affect the sampling distribution of $\hat{\beta}_{l}$ in a complicated manner. To mitigate the effect of this first step nonparametric estimation, one could use an orthogonalized moment condition, which I describe below, to estimate $\beta_{l}$.

Let $\beta_{*}$ denote either the lower bound or the upper bound, i.e., $\beta_{*} \in\left\{\beta_{l}, \beta_{u}\right\}$. I write my estimator as a generalized method of moments (GMM) estimator in which the true value $\beta_{*, 0}$ of $\beta_{*}$ satisfies a single moment restriction

$$
\begin{equation*}
E\left[m\left(w_{i}, \beta_{*, 0}, \gamma_{0}\right)\right]=0 \tag{1.47}
\end{equation*}
$$

where

$$
\begin{equation*}
m\left(w, \beta_{l}, \gamma\right)=\underline{\Delta}(\gamma) \cdot \theta_{10}(x)-\bar{\Delta}(\gamma) \cdot \theta_{01}(x)-\beta_{l} \tag{1.48}
\end{equation*}
$$

and

$$
\begin{equation*}
m\left(w, \beta_{u}, \gamma\right)=\bar{\Delta}(\gamma) \cdot \theta_{10}(x)-\underline{\Delta}(\gamma) \cdot \theta_{01}(x)-\beta_{u} \tag{1.49}
\end{equation*}
$$

$\underline{\Delta}(\gamma)$ and $\bar{\Delta}(\gamma)$ denote the lower and upper bound on CATE respectively and are functions of the nuisance parameters $\gamma$.

We would like our moment function to have an orthogonality property so that the estimation of parameter of interest would be first-order insensitive to nonparametric estimation errors in the nuisance parameter. This allows for the use of various nonparametric estimators of these parameters including high-dimensional machine learning estimators. I construct such moment function by adding influence function adjustment term for first step estimation $\phi\left(w, \beta_{*}, \gamma\right)$ to the original moment function

[^3]$m\left(w, \beta_{*}, \gamma\right)$ as in CEINR. Let the orthogonalized moment function be denoted by
\[

$$
\begin{equation*}
\psi\left(w, \beta_{*}, \gamma\right)=m\left(w, \beta_{*}, \gamma\right)+\phi\left(w, \beta_{*}, \gamma\right) \tag{1.50}
\end{equation*}
$$

\]

Let $F_{\tau}=(1-\tau) F_{0}+\tau G$ for $\tau \in[0,1]$, where $F_{0}$ is the true distribution of $W$ and $G$ is some alternative distribution. Then, we say that the moment condition satisfies the Neyman orthogonality condition or is locally robust if

$$
\begin{equation*}
\left.\frac{d}{d \tau} E\left[\psi\left(w_{i}, \beta_{*, 0}, \gamma\left(F_{\tau}\right)\right)\right]\right|_{\tau=0}=0 \tag{1.51}
\end{equation*}
$$

The orthogonality has been used in semiparametric problems by Newey (1990, 1994), Andrews (1994), Robins and Rotnitzky (1995), among others. More recently, in a high-dimensional setting, it has been used by Belloni, Chen, Chernozhukov, and Hansen (2012), Belloni, Chernozhukov, and Hansen (2014), Farrell (2015), Belloni, Chernozhukov, Fernández-Val, and Hansen (2017), Athey, Imbens, and Wager (2018), and Chernozhukov, Chetverikov, Demirer, Duflo, Hansen, Newey, and Robins (2018), among others. Recently, Sasaki and Ura (2018) proposed using orthogonalized moments for the estimation and inference of a parameter called policy relevant treatment effect whose explanation can be found in Heckman and Vytlacil (2007). Much like our problem, the estimation of the policy relevant treatment effect involves estimation of multiple nuisance parameters.

### 1.4.1 Influence function calculation

In this subsection, I show how I derive the adjustment term $\phi\left(w, \beta_{l}, \gamma\right)$ for the lower bound under the worst-case assumption. This illustrates how I derive the adjustment term for the cases in which $\underline{\Delta}(\gamma)$ and $\bar{\Delta}(\gamma)$ are differentiable with respect to $\gamma$, i.e., cases in which we do not have instrumental variables. Additional assumptions need to be imposed for the cases where $\underline{\Delta}(\gamma)$ and $\bar{\Delta}(\gamma)$ are non-differentiable with respect
to $\gamma$.
Under the worst-case assumption, the original moment function for lower bound takes the following form:

$$
\begin{align*}
m\left(w, \beta_{l}, \gamma\right)= & ((\eta(1, x)-\bar{y}) \cdot p(x)+(\underline{y}-\eta(0, x)) \cdot(1-p(x))) \cdot \theta_{10}(x)  \tag{1.52}\\
& -((\eta(1, x)-\underline{y}) \cdot p(x)+(\bar{y}-\eta(0, x)) \cdot(1-p(x))) \cdot \theta_{01}(x)-\beta_{l} .
\end{align*}
$$

Assumption 1.4.1. $\eta(1, x), \eta(0, x)$, and $p(x)$ are continuous at every $x$.
Lemma 1.4.1. If Assumption 1.4.1 is satisfied then the influence function of $E\left[m\left(w, \beta_{l, 0}, \gamma(F)\right)\right]$ is $\phi\left(w, \beta_{l, 0}, \gamma_{0}\right)$ which is given by

$$
\begin{equation*}
\phi\left(w, \beta_{l, 0}, \gamma_{0}\right)=\phi_{1}+\phi_{2} \tag{1.53}
\end{equation*}
$$

where

$$
\begin{align*}
& \phi_{1}=\left(\theta_{10}(x)-\theta_{01}(x)\right) \cdot\left(\eta_{0}(1, x)+\eta_{0}(0, x)-(\underline{y}+\bar{y})\right) \cdot\left(d-p_{0}(x)\right),  \tag{1.54}\\
& \phi_{2}=\left(\theta_{10}(x)-\theta_{01}(x)\right) \cdot\left[y-\eta_{0}(1, x)\right]^{d} \cdot\left[-\left(y-\eta_{0}(0, x)\right)\right]^{1-d} .
\end{align*}
$$

Note that we have $E\left[\phi\left(w, \beta_{l, 0}, \gamma_{0}\right)\right]=0$ so that the orthogonalized moment condition $\psi\left(w, \beta_{l}, \gamma\right)$ still identifies our parameter of interest with $E\left[\psi\left(w, \beta_{l, 0}, \gamma_{0}\right)\right]=0$. The adjustment term consists of two terms. While term $\phi_{1}$ represents the effect of local perturbations of the distribution of $D \mid X$ on the moment, term $\phi_{2}$ represents the effect of local perturbations of the distribution of $Y \mid D, X$ on the moment.

### 1.4.2 GMM estimator and its asymptotic variance

Following CEINR, I use cross-fitting, a version of sample splitting, in the construction of sample moments. Cross-fitting works as follows. Let $K>1$ be a number of folds. Partitioning the set of observation indices $\{1,2, \ldots, n\}$ into $K$ groups $\mathcal{I}_{k}, k=1, \ldots, K$, let $\hat{\gamma}_{k}$ be the first step estimates constructed from all observations not in $\mathcal{I}_{k}$. Then,
$\hat{\beta}_{*}$ can be obtained as a solution to

$$
\begin{equation*}
\frac{1}{n} \sum_{k=1}^{K} \sum_{i \in \mathcal{I}_{k}} \psi\left(w_{i}, \hat{\beta}_{*}, \hat{\gamma}_{k}\right)=0 \tag{1.55}
\end{equation*}
$$

Following CEINR, I give the conditions under which the estimator is root- $n$ asymptotically normal. $\|\cdot\|$ denotes a mean-square norm, also known as $L_{2}$-norm in the following.

Assumption 1.4.2. For each $k=1, \ldots, K$,
$(i) \int\left\|\psi\left(w, \beta_{*, 0}, \hat{\gamma}_{k}\right)-\psi\left(w, \beta_{*, 0}, \gamma_{0}\right)\right\|^{2} F_{0}(d w) \xrightarrow{p} 0$, (ii) $\left\|\int \psi\left(w, \beta_{*, 0}, \hat{\gamma}_{k}\right) F_{0}(d w)\right\| \leq C\left\|\hat{\gamma}_{k}-\gamma_{0}\right\|^{2}$ for $C>0$, (iii) $\left\|\hat{\gamma}_{k}-\gamma_{0}\right\|=o_{p}\left(n^{-1 / 4}\right)$
(iv) there is $\zeta>0$ and $d\left(w_{i}\right)$ with $E\left[d\left(w_{i}\right)^{2}\right]<\infty$ such that for $\left\|\beta_{*}-\beta_{*, 0}\right\|$ and $\left\|\gamma-\gamma_{0}\right\|$ small enough

$$
\left\|\psi\left(w_{i}, \beta_{*}, \gamma\right)-\psi\left(w_{i}, \beta_{*, 0}, \gamma_{0}\right)\right\| \leq d\left(w_{i}\right)\left(\left\|\beta_{*}-\beta_{*, 0}\right\|^{\zeta}+\left\|\gamma-\gamma_{0}\right\|^{\zeta}\right) .
$$

Assumption 1.4.2 $(i)$ is a mean square consistency condition for $\hat{\gamma}_{k}$. It is a much weaker condition compared to other stochastic equicontinuity results in the literature, which generally involve boundedness of some derivatives of $\hat{\gamma}_{k}$. This is made possible because of cross-fitting.

Assumption 1.4.2 (ii) holds if $\int \psi\left(w, \beta_{*, 0}, \gamma\right) F_{0}(d w)$ is twice continuously Frechet differentiable. Assumption 1.4.2 (iii) is the familiar condition from semiparametric estimation literature which requires that $\hat{\gamma}_{k}$ converges at $n^{-1 / 4}$ rate.

Assumption 1.4.2 (iv) is needed to insure the consistency of a variance estimator to be introduced below. This condition takes into account the presence of $\hat{\beta}_{*}$ in the variance estimator. Now, I state the formal asymptotic result in the following theorem.

Theorem 1.4.1. Suppose that $\left\{w_{i}\right\}_{i=1}^{n}$ are i.i.d., Assumption 1.4.2 (i), (ii), and (iii)
are satisfied, $\hat{\beta}_{*} \xrightarrow{p} \beta_{*, 0}$, and $\Omega_{*} \equiv E\left[\psi\left(w_{i}, \beta_{*, 0}, \gamma_{0}\right)^{2}\right]<\infty$. Then

$$
\begin{equation*}
\sqrt{n}\left(\hat{\beta}_{*}-\beta_{*, 0}\right) \xrightarrow{d} \mathcal{N}\left(0, \Omega_{*}\right) . \tag{1.56}
\end{equation*}
$$

Moreover, if Assumption 1.4.2 (iv) is also satisfied, a consistent estimator for the asymptotic variance can be constructed as

$$
\begin{equation*}
\hat{\Omega}_{*}=\frac{1}{n} \sum_{k=1}^{K} \sum_{i \in \mathcal{I}_{k}} \psi\left(w_{i}, \hat{\beta}_{*}, \hat{\gamma}_{k}\right)^{2} . \tag{1.57}
\end{equation*}
$$

I show the forms of a locally robust estimator of the worst-case lower bound and a consistent estimator of its asymptotic variance in the following corollary.

Corollary 1.4.1 (Locally robust estimator of the lower bound under the worst-case and a consistent estimator of its asymptotic variance). A locally robust estimator $\hat{\beta}_{l}$ of the lower bound under the worst-case takes the form

$$
\begin{align*}
\hat{\beta}_{l}=\frac{1}{n} & \sum_{k=1}^{K} \sum_{i \in \mathcal{I}_{k}}\left[\left(\left(\hat{\eta}_{k}\left(1, x_{i}\right)-\bar{y}\right) \cdot \hat{p}_{k}\left(x_{i}\right)\right.\right. \\
& \left.+\left(\underline{y}-\hat{\eta}_{k}\left(0, x_{i}\right)\right) \cdot\left(1-\hat{p}_{k}\left(x_{i}\right)\right)\right) \cdot \theta_{10}\left(x_{i}\right) \\
& -\left(\left(\hat{\eta}_{k}\left(1, x_{i}\right)-\underline{y}\right) \cdot \hat{p}_{k}\left(x_{i}\right)\right.  \tag{1.58}\\
& \left.+\left(\bar{y}-\hat{\eta}_{k}\left(0, x_{i}\right)\right) \cdot\left(1-\hat{p}_{k}\left(x_{i}\right)\right)\right) \cdot \theta_{01}\left(x_{i}\right) \\
& +\left(\theta_{10}\left(x_{i}\right)-\theta_{01}\left(x_{i}\right)\right) \cdot\left(\hat{\eta}_{k}\left(1, x_{i}\right)+\hat{\eta}_{k}\left(0, x_{i}\right)-(\underline{y}+\bar{y})\right) \cdot\left(d_{i}-\hat{p}_{k}\left(x_{i}\right)\right) \\
& \left.+\left(\theta_{10}\left(x_{i}\right)-\theta_{01}\left(x_{i}\right)\right) \cdot\left[y_{i}-\hat{\eta}_{k}\left(1, x_{i}\right)\right]^{d_{i}} \cdot\left[-\left(y_{i}-\hat{\eta}_{k}\left(0, x_{i}\right)\right)\right]^{1-d_{i}}\right] .
\end{align*}
$$

Moreover, a consistent estimator of its asymptotic variance takes the form

$$
\begin{align*}
\hat{\Omega}_{l}= & \frac{1}{n} \sum_{k=1}^{K} \sum_{i \in \mathcal{I}_{k}} \psi\left(w_{i}, \hat{\beta}_{l}, \hat{\gamma}_{k}\right)^{2} \\
=\frac{1}{n} & \sum_{k=1}^{K} \sum_{i \in \mathcal{I}_{k}}\left[\left(\left(\hat{\eta}_{k}\left(1, x_{i}\right)-\bar{y}\right) \cdot \hat{p}_{k}\left(x_{i}\right)\right.\right. \\
& \left.+\left(\underline{y}-\hat{\eta}_{k}\left(0, x_{i}\right)\right) \cdot\left(1-\hat{p}_{k}\left(x_{i}\right)\right)\right) \cdot \theta_{10}\left(x_{i}\right)  \tag{1.59}\\
& -\left(\left(\hat{\eta}_{k}\left(1, x_{i}\right)-\underline{y}\right) \cdot \hat{p}_{k}\left(x_{i}\right)\right. \\
& \left.+\left(\bar{y}-\hat{\eta}_{k}\left(0, x_{i}\right)\right) \cdot\left(1-\hat{p}_{k}\left(x_{i}\right)\right)\right) \cdot \theta_{01}\left(x_{i}\right)-\hat{\beta}_{l} \\
& +\left(\theta_{10}\left(x_{i}\right)-\theta_{01}\left(x_{i}\right)\right) \cdot\left(\hat{\eta}_{k}\left(1, x_{i}\right)+\hat{\eta}_{k}\left(0, x_{i}\right)-(\underline{y}+\bar{y})\right) \cdot\left(d_{i}-\hat{p}_{k}\left(x_{i}\right)\right) \\
& \left.+\left(\theta_{10}\left(x_{i}\right)-\theta_{01}\left(x_{i}\right)\right) \cdot\left[y_{i}-\hat{\eta}_{k}\left(1, x_{i}\right)\right]^{d_{i}} \cdot\left[-\left(y_{i}-\hat{\eta}_{k}\left(0, x_{i}\right)\right)\right]^{1-d_{i}}\right]^{2} .
\end{align*}
$$

Given locally robust estimators $\hat{\beta}_{l}$ and $\hat{\beta}_{u}$ of the lower and upper bound $\beta_{l}$ and $\beta_{u}$, and consistent estimators $\hat{\Omega}_{l}$ and $\hat{\Omega}_{u}$ of their asymptotic variance $\Omega_{l}$ and $\Omega_{u}$, we can construct the $100 \cdot \alpha \%$ confidence interval for the lower bound $\beta_{l}$ and upper bound $\beta_{u}$ as

$$
\begin{equation*}
C I_{\alpha}^{\beta_{l}}=\left[\hat{\beta}_{l}-C_{\alpha} \cdot\left(\hat{\Omega}_{l} / n\right)^{1 / 2}, \hat{\beta}_{l}+C_{\alpha} \cdot\left(\hat{\Omega}_{l} / n\right)^{1 / 2}\right] \tag{1.60}
\end{equation*}
$$

and

$$
\begin{equation*}
C I_{\alpha}^{\beta_{u}}=\left[\hat{\beta}_{u}-C_{\alpha} \cdot\left(\hat{\Omega}_{u} / n\right)^{1 / 2}, \hat{\beta}_{u}+C_{\alpha} \cdot\left(\hat{\Omega}_{u} / n\right)^{1 / 2}\right] \tag{1.61}
\end{equation*}
$$

where $C_{\alpha}$ satisfies

$$
\begin{equation*}
\Phi\left(C_{\alpha}\right)-\Phi\left(-C_{\alpha}\right)=\alpha \tag{1.62}
\end{equation*}
$$

In other words, $C_{\alpha}$ is the value that satisfies $\Phi\left(C_{\alpha}\right)=(\alpha+1) / 2$, i.e, the $(\alpha+1) / 2$ quantile of the standard normal distribution. For example, when $\alpha=0.95, C_{\alpha}$ is 1.96.

### 1.4.3 Bounds with instruments

When there are additional instrumental variables, $\underline{\Delta}(\gamma)$ and $\bar{\Delta}(\gamma)$ in (1.48) and (1.49) are non-differentiable with respect to $\gamma$ as they involve sup and inf operators. However, under additional monotonicity assumption, the bounds can be simplified. In this section, I derive the influence function for the IV-worst-case lower bound under the monotonicity assumption. Under monotonicity, the moment condition for the IV-worst-case lower bound is

$$
\begin{align*}
m\left(w, \beta_{l}, \gamma\right)= & (\eta(1, x, 1) \cdot p(x, 1)+\underline{y} \cdot(1-p(x, 1)) \\
& -\bar{y} \cdot p(x, 0)-\eta(0, x, 0) \cdot(1-p(x, 0))) \cdot \theta_{10}(x)  \tag{1.63}\\
& -(\eta(1, x, 1) \cdot p(x, 1)+\bar{y} \cdot(1-p(x, 1)) \\
& -\underline{y} \cdot p(x, 0)-\eta(0, x, 0) \cdot(1-p(x, 0))) \cdot \theta_{01}(x)-\beta_{l} .
\end{align*}
$$

Lemma 1.4.2. If Assumption 1.4.1 is satisfied then the influence function of $E\left[m\left(w, \beta_{l, 0}, \gamma(F)\right)\right]$ is $\phi\left(w, \beta_{l, 0}, \gamma_{0}\right)$ which is given by

$$
\begin{equation*}
\phi\left(w, \beta_{l, 0}, \gamma_{0}\right)=\phi_{1}+\phi_{2} \tag{1.64}
\end{equation*}
$$

where

$$
\begin{align*}
\phi_{1}= & {\left[\left(\left(\eta_{0}(1, x, 1)-\underline{y}\right) \cdot \theta_{10}(x)-\left(\eta_{0}(1, x, 1)-\bar{y}\right) \cdot \theta_{01}(x)\right) \cdot\left(d-p_{0}(x, 1)\right)\right]^{z} } \\
& \cdot\left[\left(\left(\eta_{0}(0, x, 0)-\bar{y}\right) \cdot \theta_{10}(x)-\left(\eta_{0}(0, x, 0)-\underline{y}\right) \cdot \theta_{01}(x)\right) \cdot\left(d-p_{0}(x, 0)\right)\right]^{1-z}  \tag{1.65}\\
\phi_{2}= & \left(\theta_{10}(x)-\theta_{01}(x)\right) \cdot\left(\mathbb{1}\{d=1, z=1\} \cdot\left(y-\eta_{0}(1, x, 1)\right)\right. \\
& \left.+\mathbb{1}\{d=0, z=0\} \cdot\left(-\left(y-\eta_{0}(0, x, 0)\right)\right)\right) .
\end{align*}
$$

Notice again that we have $E\left[\phi\left(w, \beta_{l, 0}, \gamma_{0}\right)\right]=0$ so that the orthogonalized moment condition $\psi\left(w, \beta_{l}, \gamma\right)$ still identifies our parameter of interest with $E\left[\psi\left(w, \beta_{l, 0}, \gamma_{0}\right)\right]=0$. The adjustment term again consists of two terms. In this case, while term $\phi_{1}$ represents the effect of local perturbations of the distribution of $D \mid X, Z$ on the moment, term $\phi_{2}$ represents the effect of local perturbations of the distribution of $Y \mid D, X, Z$
on the moment.

### 1.5 Monte Carlo Simulation

Mimicking the JTPA experiment studied in Kitagawa and Tetenov (2018), I consider the following data generating process. Let $X$ be a discrete random variable whose probability mass function is given in Figure 1.7.

Figure 1.7: Probability mass function of $X$


Conditional on $X=x$, let

$$
\begin{gather*}
Z \mid X=x \sim \operatorname{Bernoulli}(2 / 3)  \tag{1.66}\\
U \mid X=x, Z=z \sim \operatorname{Unif}[0,1] \text { for } z \in\{0,1\},  \tag{1.67}\\
D=\mathbb{1}\{p(X, Z) \geq U\} \tag{1.68}
\end{gather*}
$$

$$
Y_{1} \mid X=x, Z=z, U=u \sim \operatorname{Lognormal}\left(\log \frac{m_{1}^{2}(x, u)}{\sqrt{\sigma_{1}^{2}+m_{1}^{2}(x, u)}}, \sqrt{\log \left(\frac{\sigma_{1}^{2}}{m_{1}^{2}(x, u)}+1\right)}\right),
$$

$$
\begin{equation*}
Y_{0} \mid X=x, Z=z, U=u \sim \text { Lognormal }\left(\log \frac{m_{0}^{2}(x, u)}{\sqrt{\sigma_{0}^{2}+m_{0}^{2}(x, u)}}, \sqrt{\log \left(\frac{\sigma_{0}^{2}}{m_{0}^{2}(x, u)}+1\right)}\right), \tag{1.69}
\end{equation*}
$$

where

$$
\begin{align*}
p(x, z) & =\frac{1}{1+e^{-(-4.89+0.05 \cdot x+5 \cdot z)}},  \tag{1.71}\\
m_{1}(x, u) & =E\left[Y_{1} \mid X=x, Z=z, U=u\right]=5591+1027 \cdot x+2000 \cdot u,  \tag{1.72}\\
m_{0}(x, u) & =E\left[Y_{0} \mid X=x, Z=z, U=u\right]=-1127+1389 \cdot x+1000 \cdot u,  \tag{1.73}\\
\sigma_{1}^{2} & =\operatorname{Var}\left[Y_{1} \mid X=x, Z=z, U=u\right]=11000^{2},  \tag{1.74}\\
\sigma_{0}^{2} & =\operatorname{Var}\left[Y_{0} \mid X=x, Z=z, U=u\right]=11000^{2} . \tag{1.75}
\end{align*}
$$

$X$ corresponds to years of education and takes values from 7 to 18. $Z$ corresponds to random offer and follows Bernoulli $(2 / 3)$ to reflect the fact that probability of being randomly assigned to the treatment group is $2 / 3$ irrespective of applicants' years of education. $D$ corresponds to program participation and equals 1 whenever $p(x, z)$ exceeds the value of $U$ which is uniformly distributed on $[0,1] . Y_{1}$ and $Y_{0}$ are potential outcomes and observed outcome $Y=Y_{1} \cdot D+Y_{0} \cdot(1-D)$ corresponds to 30 -month post-program earnings. For $d \in\{0,1\}, Y_{d}$ conditional on $X, Z$, and $U$ follows a lognormal distribution whose mean is $m_{d}(x, u)$ and variance is $\sigma_{d}^{2}$. Under this structure, we have

$$
\begin{equation*}
E\left[Y_{d} \mid X, Z\right]=E\left[Y_{d} \mid X\right] \text { for } d \in\{0,1\} \tag{1.76}
\end{equation*}
$$

Consider the following pair of policies:

$$
\begin{equation*}
\delta^{*}(x)=1\{x \leq 11\} \text { and } \delta(x)=1\{x \leq 12\} \tag{1.77}
\end{equation*}
$$

Policy $\delta^{*}$ corresponds to treating everyone who has less than or equal to 11 years of education, and policy $\delta$ corresponds to treating everyone who has less than or equal to 12 years of education. Then, the population welfare gain is 1,236 . The population worst-case bounds are $(-31,191,37,608)$ and IV-worst-case bounds are (-
$2,380,21,227)$. I set $\underline{y}=0$ and $\bar{y}=160,000$ to calculate the bounds. More details on the calculation of these population quantities can be found in Appendix 1.7.4.

I focus on worst-case lower bound and report coverage probabilities and average lengths of $95 \%$ confidence intervals, for samples sizes $n \in\{100,1000,5000,10000\}$, out of 1000 Monte Carlo replications in Table 1.2. I use empirical means in the first step estimation of conditional mean treatment responses and propensity scores. I construct the confidence intervals using original and debiased moment conditions with and without cross-fitting.

Panel A shows the results without cross-fitting. Panel B shows the results with cross-fitting when the number of fold is 2 . There is not much difference between these two cases and in both cases, confidence intervals constructed using original moments are invalid, and as expected, show undercoverage. However, confidence intervals obtained using debiased moment conditions show good coverage even with small sample size. Lastly, in Panel C, I report the results when true values of nuisance parameters are used to construct the confidence intervals. In that case, the coverage probability is around 0.95 for both original and debiased moments, as expected. In all cases, the average length of the confidence intervals is wider when debiased moments are used.

### 1.6 Conclusion

In this chapter, I consider identification and inference of the welfare gain that results from switching from one policy to another policy. Understanding how much the welfare gain is under different assumptions on the unobservables allows policymakers to make informed decisions about how to choose between alternative treatment assignment policies. I use tools from theory of random sets to obtain the identified set of this parameter. I then employ orthogonalized moment conditions for the estimation

Table 1.2: Monte Carlo results

| Original moment <br> Sample size |  |  |  | Debiased moment <br> Coverage |  | Average length | Coverage | Average length |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel A: without cross-fitting |  |  |  |  |  |  |  |  |
| 100 | 0.80 | 13976 | 0.94 | 21316 |  |  |  |  |
| 1000 | 0.79 | 4454 | 0.95 | 6797 |  |  |  |  |
| 5000 | 0.78 | 1995 | 0.94 | 3045 |  |  |  |  |
| 10000 | 0.80 | 1412 | 0.96 | 2154 |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| Panel B: with | cross-fitting | $($ K=2) |  |  |  |  |  |  |
| 100 | 0.79 | 14180 | 0.94 | 21316 |  |  |  |  |
| 1000 | 0.78 | 4462 | 0.95 | 6797 |  |  |  |  |
| 5000 | 0.78 | 1996 | 0.94 | 3045 |  |  |  |  |
| 10000 | 0.80 | 1412 | 0.96 | 2154 |  |  |  |  |
| Panel C: when true values of |  |  |  |  |  |  |  |  |
| 100 | 0.95 | 14008 | 0.94 | 21316 |  |  |  |  |
| 1000 | 0.94 | 4449 | 0.95 | 6797 |  |  |  |  |
| 5000 | 0.95 | 1991 | 0.94 | 3045 |  |  |  |  |
| 10000 | 0.95 | 1408 | 0.96 | 2154 |  |  |  |  |

Note: $95 \%$ confidence interval for worst-case lower bound. Number of Monte Carlo replications is 1000 .
and inference of these bounds. I conduct Monte Carlo simulations to assess the finite sample performance of the estimators.

### 1.7 Appendix

### 1.7.1 Random Set Theory

In this appendix, I introduce some definitions and theorems from random set theory that are used throughout the chapter. See Molchanov (2017) and Molchanov and Molinari (2018) for more detailed treatment of random set theory. Let $(\Omega, \mathfrak{A}, P)$ be a complete probability space and $\mathcal{F}$ be the family of closed subsets of $\mathbb{R}^{d}$.

Definition 1.7.1 (Random closed set). A map $X: \Omega \rightarrow \mathcal{F}$ is called a random closed set if, for every compact set $K$ in $\mathbb{R}^{d}$,

$$
\begin{equation*}
\{\omega \in \Omega: X(\omega) \cap K \neq \emptyset\} \in \mathfrak{A} . \tag{1.78}
\end{equation*}
$$

Definition 1.7.2 (Selection). A random vector $\xi$ with values in $\mathbb{R}^{d}$ is called a (measurable) selection of $X$ if $\xi(\omega) \in X(\omega)$ for almost all $\omega \in \Omega$. The family of all selections of $X$ is denoted by $\mathcal{S}(X)$.

Definition 1.7.3 (Integrable selection). Let $L^{1}=L^{1}\left(\Omega ; \mathbb{R}^{d}\right)$ denote the space of $\mathfrak{A}$ measurable random vectors with values in $\mathbb{R}^{d}$ such that the $L^{1}$-norm $\|\xi\|_{1}=E[\|\xi\|]$ is finite. If $X$ is a random closed set in $\mathbb{R}^{d}$, then the family of all integrable selections of $X$ is given by

$$
\begin{equation*}
\mathcal{S}^{1}(X)=\mathcal{S}(X) \cap L^{1} \tag{1.79}
\end{equation*}
$$

Definition 1.7.4 (Integrable random sets). A random closed set $X$ is called integrable if $\mathcal{S}^{1}(X) \neq \emptyset$.

Definition 1.7.5 (Selection (or Aumann) expectation). The selection (or Aumann) expectation of $X$ is the closure of the set of all expectations of integrable selections, i.e.

$$
\begin{equation*}
\mathbb{E}[X]=c l\left\{\int_{\Omega} \xi d P: \xi \in \mathcal{S}^{1}(X)\right\} \tag{1.80}
\end{equation*}
$$

Note that I use $\mathbb{E}[\cdot]$ for the Aumann expectation and reserve $E[\cdot]$ for the expectation of random variables and random vectors.

Definition 1.7.6 (Support function). Let $K \subset \mathbb{R}^{d}$ be a convex set. The support
function of a set $K$ is given by

$$
\begin{equation*}
s(v, K)=\sup _{x \in K}\langle v, x\rangle, \quad v \in \mathbb{R}^{d} \tag{1.81}
\end{equation*}
$$

Theorem 1.7.1 (Theorem 3.11 in Molchanov and Molinari (2018)). If an integrable random set $X$ is defined on a nonatomic probability space, or if $X$ is almost surely convex, then

$$
\begin{equation*}
E[s(v, X)]=s(v, \mathbb{E}[X]), \quad v \in \mathbb{R}^{d} . \tag{1.82}
\end{equation*}
$$

### 1.7.2 Proofs and Useful Lemmas

## Proof of Lemma 1.3.1

By the definition of selection expectation, we have $E\left[\left(Y_{1}, Y_{0}\right)^{\prime} \mid X\right] \in \mathbb{E}\left[\mathcal{Y}_{1} \times \mathcal{Y}_{0} \mid X\right]$. Then by the definition of support function and Theorem 1.7.1, for any $v \in \mathbb{R}^{2}$, we have

$$
\begin{align*}
v^{\prime} E\left[\left(Y_{1}, Y_{0}\right)^{\prime} \mid X\right] & \leq s\left(v, \mathbb{E}\left[\mathcal{Y}_{1} \times \mathcal{Y}_{0} \mid X\right]\right)  \tag{1.83}\\
& =E\left[s\left(v, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right] .
\end{align*}
$$

For any $v \in \mathbb{R}^{2}$, we can write

$$
\begin{align*}
-v^{\prime} E\left[\left(Y_{1}, Y_{0}\right)^{\prime} \mid X\right] & \leq s\left(-v, \mathbb{E}\left[\mathcal{Y}_{1} \times \mathcal{Y}_{0} \mid X\right]\right)  \tag{1.84}\\
& =E\left[s\left(-v, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right] .
\end{align*}
$$

Thus, we also have

$$
\begin{equation*}
v^{\prime} E\left[\left(Y_{1}, Y_{0}\right)^{\prime} \mid X\right] \geq-E\left[s\left(-v, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right] \tag{1.85}
\end{equation*}
$$

## Proof of Theorem 1.3.1

We write $\Delta(X) \equiv E\left[Y_{1}-Y_{0} \mid X\right]=v^{* \prime} E\left[\left(Y_{1}, Y_{0}\right)^{\prime} \mid X\right]$ for $v^{*}=(1,-1)^{\prime}$. By Lemma 1.3.1, we have

$$
\begin{equation*}
\underline{\Delta}(X)=-E\left[s\left(-v^{*}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right] \leq \Delta(X) \leq E\left[s\left(v^{*}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right]=\bar{\Delta}(X) \quad \text { a.s. } \tag{1.86}
\end{equation*}
$$

Since $\delta(X)-\delta^{*}(X)$ can take values in $\{-1,0,1\}$, we consider two cases: (i) $\delta(X)-$ $\delta^{*}(X)=1$ and (ii) $\delta(X)-\delta^{*}(X)=-1$. When (i) $\delta(X)-\delta^{*}(X)=1$, the upper bound on $\Delta(X) \cdot\left(\delta(X)-\delta^{*}(X)\right)$ is $\bar{\Delta}(X)$. When (ii) $\delta(X)-\delta^{*}(X)=-1$, the upper bound on $\Delta(X) \cdot\left(\delta(X)-\delta^{*}(X)\right)$ is $-\underline{\Delta}(X)$. Hence, the upper bound on $E\left[\Delta(X) \cdot\left(\delta(X)-\delta^{*}(X)\right)\right]$ should be

$$
\begin{equation*}
\beta_{u}=E\left[\bar{\Delta}(X) \cdot \theta_{10}(X)-\underline{\Delta}(X) \cdot \theta_{01}(X)\right] . \tag{1.87}
\end{equation*}
$$

Similarly, the lower bound on $E\left[\Delta(X) \cdot\left(\delta(X)-\delta^{*}(X)\right)\right]$ should be

$$
\begin{equation*}
\beta_{l}=E\left[\underline{\Delta}(X) \cdot \theta_{10}(X)-\bar{\Delta}(X) \cdot \theta_{01}(X)\right] . \tag{1.88}
\end{equation*}
$$

Lemma 1.7.1. Suppose $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right): \Omega \rightarrow \mathcal{F}$ is of the following form:

$$
\mathcal{Y}_{1} \times \mathcal{Y}_{0}=\left\{\begin{array}{l}
\{Y\} \times\left[Y_{L, 0}, Y_{U, 0}\right] \text { if } D=1  \tag{1.89}\\
{\left[Y_{L, 1}, Y_{U, 1}\right] \times\{Y\} \text { if } D=0}
\end{array}\right.
$$

where $Y$ is a random variable and each of $Y_{L, 0}, Y_{U, 0}, Y_{L, 1}$, and $Y_{U, 1}$ can be a constant
or a random variable. Let $v^{*}=(1,-1)^{\prime}$, $v_{1}=(1,0)^{\prime}$, and $v_{0}=(0,1)^{\prime}$. Then, we have

$$
\begin{aligned}
E\left[s\left(v_{1}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right]= & E[Y \mid D=1, X] \cdot P(D=1 \mid X) \\
& +E\left[Y_{U, 1} \mid D=0, X\right] \cdot P(D=0 \mid X) \\
-E\left[s\left(-v_{1}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right]= & E[Y \mid D=1, X] \cdot P(D=1 \mid X) \\
& +E\left[Y_{L, 1} \mid D=0, X\right] \cdot P(D=0 \mid X) \\
E\left[s\left(v_{0}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right]= & E\left[Y_{U, 0} \mid D=1, X\right] \cdot P(D=1 \mid X) \\
& +E[Y \mid D=0, X] \cdot P(D=0 \mid X), \\
-E\left[s\left(-v_{0}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right]= & E\left[Y_{L, 0} \mid D=1, X\right] \cdot P(D=1 \mid X) \\
& +E[Y \mid D=0, X] \cdot P(D=0 \mid X), \\
E\left[s\left(v^{*}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right]= & \left(E[Y \mid D=1, X]-E\left[Y_{L, 0} \mid D=1, X\right]\right) \cdot P(D=1 \mid X) \\
& +\left(E\left[Y_{U, 1} \mid D=0, X\right]-E[Y \mid D=0, X]\right) \cdot P(D=0 \mid X) \\
-E\left[s\left(-v^{*}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right]= & \left(E[Y \mid D=1, X]-E\left[Y_{U, 0} \mid D=1, X\right]\right) \cdot P(D=1 \mid X) \\
& +\left(E\left[Y_{L, 1} \mid D=0, X\right]-E[Y \mid D=0, X]\right) \cdot P(D=0 \mid X) .
\end{aligned}
$$

Proof. We have

$$
\begin{aligned}
E\left[s\left(v_{1}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right]= & E\left[\sup _{\left(y_{1}, y_{0}\right) \in \mathcal{Y}_{1} \times \mathcal{Y}_{0}} y_{1} \mid X\right] \\
= & E[Y \mid D=1, X] \cdot P(D=1 \mid X) \\
& +E\left[Y_{U, 1} \mid D=0, X\right] \cdot P(D=0 \mid X), \\
-E\left[s\left(-v_{1}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right]= & -E\left[\sup _{\left(y_{1}, y_{0}\right) \in \mathcal{Y}_{1} \times \mathcal{Y}_{0}}-y_{1} \mid X\right] \\
= & E\left[\inf _{\left(y_{1}, y_{0}\right) \in \mathcal{Y}_{1} \times \mathcal{Y}_{0}} y_{1} \mid X\right] \\
= & E[Y \mid D=1, X] \cdot P(D=1 \mid X) \\
& +E\left[Y_{L, 1} \mid D=0, X\right] \cdot P(D=0 \mid X)
\end{aligned}
$$

$$
\begin{aligned}
& E\left[s\left(v_{0}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right]=E\left[\sup _{\left(y_{1}, y_{0}\right) \in \mathcal{Y}_{1} \times \mathcal{Y}_{0}} y_{0} \mid X\right] \\
& =E\left[Y_{U, 0} \mid D=1, X\right] \cdot P(D=1 \mid X) \\
& +E[Y \mid D=0, X] \cdot P(D=0 \mid X), \\
& -E\left[s\left(-v_{0}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right]=-E\left[\sup _{\left(y_{1}, y_{0}\right) \in \mathcal{Y}_{1} \times \mathcal{Y}_{0}}-y_{0} \mid X\right] \\
& =E\left[\inf _{\left(y_{1}, y_{0}\right) \in \mathcal{Y}_{1} \times \mathcal{Y}_{0}} y_{0} \mid X\right] \\
& =E\left[Y_{L, 0} \mid D=1, X\right] \cdot P(D=1 \mid X) \\
& +E[Y \mid D=0, X] \cdot P(D=0 \mid X), \\
& E\left[s\left(v^{*}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right]=E\left[\sup _{\left(y_{1}, y_{0}\right) \in \mathcal{Y}_{1} \times \mathcal{Y}_{0}} y_{1}-y_{0} \mid X\right] \\
& =\left(E[Y \mid D=1, X]-E\left[Y_{L, 0} \mid D=1, X\right]\right) \cdot P(D=1 \mid X) \\
& +\left(E\left[Y_{U, 1} \mid D=0, X\right]-E[Y \mid D=0, X]\right) \cdot P(D=0 \mid X), \\
& -E\left[s\left(-v^{*}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right]=-E\left[\sup _{\left(y_{1}, y_{0}\right) \in \mathcal{Y}_{1} \times \mathcal{Y}_{0}}-y_{1}+y_{0} \mid X\right] \\
& =E\left[\inf _{\left(y_{1}, y_{0}\right) \in \mathcal{Y}_{1} \times \mathcal{Y}_{0}} y_{1}-y_{0} \mid X\right] \\
& =\left(E[Y \mid D=1, X]-E\left[Y_{U, 0} \mid D=1, X\right]\right) \cdot P(D=1 \mid X) \\
& +\left(E\left[Y_{L, 1} \mid D=0, X\right]-E[Y \mid D=0, X]\right) \cdot P(D=0 \mid X) .
\end{aligned}
$$

## Proof of Corollary 1.3.1

By setting $Y_{L, 1}=Y_{L, 0}=\underline{y}$ and $Y_{U, 1}=Y_{U, 0}=\bar{y}$ in Lemma 1.7.1, we have

$$
\begin{align*}
E\left[s\left(v^{*}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right] & =(\eta(1, X)-\underline{y}) \cdot p(X)+(\bar{y}-\eta(0, X)) \cdot(1-p(X)),  \tag{1.90}\\
-E\left[s\left(-v^{*}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right] & =(\eta(1, X)-\bar{y}) \cdot p(X)+(\underline{y}-\eta(0, X)) \cdot(1-p(X)) . \tag{1.91}
\end{align*}
$$

Plugging these in, the result follows from Theorem 1.3.1.

## Proof of Corollary 1.3.2

By setting $Y_{L, 0}=\underline{y}, Y_{U, 0}=Y_{L, 1}=Y$, and $Y_{U, 1}=\bar{y}$ in Lemma 1.7.1, we have

$$
\begin{align*}
E\left[s\left(v^{*}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right] & =(\eta(1, X)-\underline{y}) \cdot p(X)+(\bar{y}-\eta(0, X)) \cdot(1-p(X)),  \tag{1.92}\\
-E\left[s\left(-v^{*}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right] & =0 \tag{1.93}
\end{align*}
$$

Plugging these in, the result follows from Theorem 1.3.1.

## Proof of Lemma 1.3.2

By the definition of selection expectation, we have $E\left[\left(Y_{1}, Y_{0}\right)^{\prime} \mid X, Z\right] \in \mathbb{E}\left[\mathcal{Y}_{1} \times \mathcal{Y}_{0} \mid X, Z\right]$. By arguments that appear in Lemma 1.3.1, for any $v \in \mathbb{R}^{2}$ and for all $z \in \mathcal{Z}$, we have

$$
\begin{equation*}
-q(-v, z) \leq v^{\prime} E\left[\left(Y_{1}, Y_{0}\right)^{\prime} \mid X, Z=z\right] \leq q(v, z) \tag{1.94}
\end{equation*}
$$

Assumption 1.3.2 implies that

$$
\begin{equation*}
E\left[Y_{d} \mid X, Z\right]=E\left[Y_{d} \mid X\right], \quad d \in\{0,1\} . \tag{1.95}
\end{equation*}
$$

Hence, for all $z \in \mathcal{Z}$, the following holds:

$$
\begin{equation*}
-q(-v, z) \leq v^{\prime} E\left[\left(Y_{1}, Y_{0}\right)^{\prime} \mid X\right] \leq q(v, z) \tag{1.96}
\end{equation*}
$$

By replacing $v$ with $v_{1}=(1,0)^{\prime}$ and $v_{0}=(0,1)^{\prime}$, we obtain the following:

$$
\begin{gather*}
E\left[Y_{1} \mid X\right] \geq \sup _{z \in \mathcal{Z}}\left\{-q\left(-v_{1}, z\right)\right\}  \tag{1.97}\\
E\left[Y_{1} \mid X\right] \leq \inf _{z \in \mathcal{Z}} q\left(v_{1}, z\right)  \tag{1.98}\\
E\left[Y_{0} \mid X\right] \geq \sup _{z \in \mathcal{Z}}\left\{-q\left(-v_{0}, z\right)\right\}  \tag{1.99}\\
E\left[Y_{0} \mid X\right] \leq \inf _{z \in \mathcal{Z}} q\left(v_{0}, z\right) \tag{1.100}
\end{gather*}
$$

Then, the upper bound in (1.22) can be obtained by subtracting the lower bound on $E\left[Y_{0} \mid X\right]$ (1.99) from the upper bound on $E\left[Y_{1} \mid X\right]$ (1.98). Similarly, the lower bound in (1.22) can be obtained by subtracting the upper bound on $E\left[Y_{0} \mid X\right]$ (1.100) from the lower bound on $E\left[Y_{1} \mid X\right]$ (1.97).

## Proof of Theorem 1.3.2

Bounds on $\Delta(X)$ is derived in Lemma 1.3.2. The remaining part of the proof is the same as that of Theorem 1.3.1.

## Proof of Corollary 1.3.3

The statements in Lemma 1.7.1 still hold when we condition on an additional variable $Z$. Hence, by setting $Y_{L, 1}=Y_{L, 0}=\underline{y}$ and $Y_{U, 1}=Y_{U, 0}=\bar{y}$ in Lemma 1.7.1, we have

$$
\begin{align*}
q\left(v_{1}, z\right) & =\eta(1, X, z) \cdot p(X, z)+\bar{y} \cdot(1-p(X, z)),  \tag{1.101}\\
-q\left(-v_{1}, z\right) & =\eta(1, X, z) \cdot p(X, z)+\underline{y} \cdot(1-p(X, z)),  \tag{1.102}\\
q\left(v_{0}, z\right) & =\bar{y} \cdot p(X, z)+\eta(0, X, z) \cdot(1-p(X, z)),  \tag{1.103}\\
-q\left(-v_{0}, z\right) & =\underline{y} \cdot p(X, z)+\eta(0, X, z) \cdot(1-p(X, z)) . \tag{1.104}
\end{align*}
$$

for all $z \in \mathcal{Z}$. Plugging these in, the result follows from Theorem 1.3.2.

## Proof of Corollary 1.3.4

The statements in Lemma 1.7.1 still hold when we condition on an additional variable Z. Hence, by setting $Y_{L, 0}=\underline{y}, Y_{U, 0}=Y_{L, 1}=Y$, and $Y_{U, 1}=\bar{y}$ in Lemma 1.7.1, we
have

$$
\begin{align*}
q\left(v_{1}, z\right) & =\eta(1, X, z) \cdot p(X, z)+\bar{y} \cdot(1-p(X, z)),  \tag{1.105}\\
-q\left(-v_{1}, z\right) & =E[Y \mid X, Z=z],  \tag{1.106}\\
q\left(v_{0}, z\right) & =E[Y \mid X, Z=z],  \tag{1.107}\\
-q\left(-v_{0}, z\right) & =\underline{y} \cdot p(X, z)+\eta(0, X, z) \cdot(1-p(X, z)), \tag{1.108}
\end{align*}
$$

for all $z \in \mathcal{Z}$. Plugging these in, the result follows from Theorem 1.3.2.

## Proof of Lemma 1.3.4

By the definition of selection expectation, we have $E\left[\left(Y_{1}, Y_{0}\right)^{\prime} \mid X, Z\right] \in \mathbb{E}\left[\mathcal{Y}_{1} \times \mathcal{Y}_{0} \mid X, Z\right]$. By arguments that appear in Lemma 1.3.1, for any $v \in \mathbb{R}_{+}^{2}$ and for all $z \in \mathcal{Z}$, we have

$$
\begin{equation*}
-q(-v, z) \leq v^{\prime} E\left[\left(Y_{1}, Y_{0}\right)^{\prime} \mid X, Z=z\right] \leq q(v, z) \tag{1.109}
\end{equation*}
$$

By Assumption 1.3.4, the following holds for all $z \in \mathcal{Z}$ :

$$
\begin{equation*}
\sup _{z_{1} \leq z}\left\{-q\left(-v, z_{1}\right)\right\} \leq v^{\prime} E\left[\left(Y_{1}, Y_{0}\right)^{\prime} \mid X, Z=z\right] \leq \inf _{z_{2} \geq z} q\left(v, z_{2}\right) \tag{1.110}
\end{equation*}
$$

By replacing $v$ with $v_{1}=(1,0)^{\prime}$ and $v_{0}=(0,1)^{\prime}$ and integrating everything with respect to $Z$, we obtain the following:

$$
\begin{gather*}
E\left[Y_{1} \mid X\right] \geq \sum_{z \in \mathcal{Z}} P(Z=z) \cdot\left(\sup _{z_{1} \leq z}\left\{-q\left(-v_{1}, z_{1}\right)\right\}\right)  \tag{1.111}\\
E\left[Y_{1} \mid X\right] \leq \sum_{z \in \mathcal{Z}} P(Z=z) \cdot\left(\inf _{z_{2} \geq z} q\left(v_{1}, z_{2}\right)\right)  \tag{1.112}\\
E\left[Y_{0} \mid X\right] \geq \sum_{z \in \mathcal{Z}} P(Z=z) \cdot\left(\sup _{z_{1} \leq z}\left\{-q\left(-v_{0}, z_{1}\right)\right\}\right)  \tag{1.113}\\
E\left[Y_{0} \mid X\right] \leq \sum_{z \in \mathcal{Z}} P(Z=z) \cdot\left(\inf _{z_{2} \geq z} q\left(v_{0}, z_{2}\right)\right) \tag{1.114}
\end{gather*}
$$

Then, the upper bound in (1.38) can be obtained by subtracting the lower bound on $E\left[Y_{0} \mid X\right]$ (1.113) from the upper bound on $E\left[Y_{1} \mid X\right]$ (1.112). Similarly, the lower bound in (1.38) can be obtained by subtracting the upper bound on $E\left[Y_{0} \mid X\right]$ (1.114) from the lower bound on $E\left[Y_{1} \mid X\right]$ (1.111).

## Proof of Theorem 1.3.3

Bounds on $\Delta(X)$ is derived in Lemma 1.3.4. The remaining part of the proof is the same as that of Theorem 1.3.1.

## Proof of Corollary 1.3.5

The statements in Lemma 1.7.1 still hold when we condition on an additional variable $Z$. Hence, by setting $Y_{L, 1}=Y_{L, 0}=\underline{y}$ and $Y_{U, 1}=Y_{U, 0}=\bar{y}$ in Lemma 1.7.1, for all $z \in \mathcal{Z}$, we have

$$
\begin{align*}
q\left(v_{1}, z\right) & =\eta(1, X, z) \cdot p(X, z)+\bar{y} \cdot(1-p(X, z)),  \tag{1.115}\\
-q\left(-v_{1}, z\right) & =\eta(1, X, z) \cdot p(X, z)+\underline{y} \cdot(1-p(X, z)),  \tag{1.116}\\
q\left(v_{0}, z\right) & =\bar{y} \cdot p(X, z)+\eta(0, X, z) \cdot(1-p(X, z)),  \tag{1.117}\\
-q\left(-v_{0}, z\right) & =\underline{y} \cdot p(X, z)+\eta(0, X, z) \cdot(1-p(X, z)) . \tag{1.118}
\end{align*}
$$

Plugging these in, the result follows from Theorem 1.3.3.

## Proof of Corollary 1.3.6

The statements in Lemma 1.7.1 still hold when we condition on an additional variable $Z$. Hence, by setting $Y_{L, 0}=\underline{y}, Y_{U, 0}=Y_{L, 1}=Y$, and $Y_{U, 1}=\bar{y}$ in Lemma 1.7.1, for all
$z \in \mathcal{Z}$, we also have

$$
\begin{align*}
q\left(v_{1}, z\right) & =\eta(1, X, z) \cdot p(X, z)+\bar{y} \cdot(1-p(X, z)),  \tag{1.119}\\
-q\left(-v_{1}, z\right) & =E[Y \mid X, Z=z]  \tag{1.120}\\
q\left(v_{0}, z\right) & =E[Y \mid X, Z=z]  \tag{1.121}\\
-q\left(-v_{0}, z\right) & =\underline{y} \cdot p(X, z)+\eta(0, X, z) \cdot(1-p(X, z)), \tag{1.122}
\end{align*}
$$

Plugging these in, the result follows from Theorem 1.3.3.

## Proof of Lemma 1.4.1

For $0 \leq \tau \leq 1$, let

$$
\begin{equation*}
F_{\tau}=(1-\tau) F_{0}+\tau G_{w}^{j} \tag{1.123}
\end{equation*}
$$

where $F_{0}$ is the true distribution of $F$ and $G_{w}^{j}$ is a family of distributions approaching the CDF of a constant $w$ as $j \rightarrow \infty$. Let $F_{0}$ be absolutely continuous with pdf $f_{0}(w)=$ $f_{0}(y, d, x)$. Let the marginal, conditional, and joint distributions and densities under $F_{0}$ be denoted by $F_{0}(x), F_{0}(d \mid x), F_{0}(y \mid d, x), F_{0}(d, x)$ and $f_{0}(x), f_{0}(d \mid x), f_{0}(y \mid d, x), f_{0}(d, x)$, etc. and the expectations under $F_{0}$ be denoted by $E_{0}$. As in Ichimura and Newey (2017), let

$$
\begin{equation*}
G_{w}^{j}(\tilde{w})=E\left[1\left\{w_{i} \leq \tilde{w}\right\} \varphi\left(w_{i}\right)\right] \tag{1.124}
\end{equation*}
$$

where $\varphi\left(w_{i}\right)$ is a bounded function with $E\left[\varphi\left(w_{i}\right)\right]=1$. This $G_{w}^{j}(\tilde{w})$ will approach the cdf of the constant $\tilde{w}$ as $\varphi(w) f_{0}(w)$ approaches a spike at $\tilde{w}$. For small enough $\tau, F_{\tau}$ will be a cdf with pdf $f_{\tau}$ that is given by

$$
\begin{equation*}
f_{\tau}(\tilde{w})=f_{0}(\tilde{w})[1-\tau+\tau \varphi(w)]=f_{0}(\tilde{w})(1+\tau S(w)), S(w)=\varphi(w)-1 \tag{1.125}
\end{equation*}
$$

Let the marginal, conditional, and joint distributions and densities under $F_{\tau}$ be similarly denoted by $F_{\tau}(x), F_{\tau}(d \mid x), F_{\tau}(y \mid d, x), F_{\tau}(d, x)$ and $f_{\tau}(x), f_{\tau}(d \mid x), f_{\tau}(y \mid d, x), f_{\tau}(d, x)$, etc. and the expectations under $F_{\tau}$ be denoted by $E_{\tau}$. By Ichimura and Newey (2017)'s Lemma A1, we have

$$
\begin{equation*}
\frac{d}{d \tau} E_{\tau}[Y \mid D=d, X=x]=E_{0}\left[\left\{Y-E_{0}[Y \mid D=d, X=x]\right\} \varphi(W) \mid D=d, X=x\right] \tag{1.126}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{d}{d \tau} E_{\tau}[1\{D=d\} \mid X=x]=E_{0}\left[\left\{1\{D=d\}-E_{0}[1\{D=d\} \mid X=x]\right\} \varphi(W) \mid X=x\right] \tag{1.127}
\end{equation*}
$$

The influence function can be calculated as

$$
\begin{equation*}
\phi(w, \beta, \gamma)=\lim _{j \rightarrow \infty}\left[\left.\frac{d}{d \tau} E_{\tau}\left[m\left(w_{i}, \beta, \gamma\left(F_{\tau}\right)\right)\right]\right|_{\tau=0}\right] \tag{1.128}
\end{equation*}
$$

We first denote the conditional mean treatment response and the propensity score under $F_{\tau}$ by

$$
\begin{equation*}
\eta_{\tau}(d, x) \equiv \int y d F_{\tau}(y \mid d, x) \tag{1.129}
\end{equation*}
$$

and

$$
\begin{equation*}
p_{\tau}(x) \equiv \int 1\{d=1\} d F_{\tau}(d \mid x) \tag{1.130}
\end{equation*}
$$

Then, by the chain rule, we have

$$
\begin{aligned}
& \frac{d}{d \tau} E_{\tau}\left[m\left(w_{i}, \beta, \gamma\left(F_{\tau}\right)\right)\right] \\
& =\frac{d}{d \tau} E_{\tau}\left[m\left(w_{i}, \beta, \gamma\left(F_{0}\right)\right)\right]+\frac{d}{d \tau} E_{0}\left[m\left(w_{i}, \beta, \gamma\left(F_{\tau}\right)\right)\right] \\
& =\frac{d}{d \tau}\left[\int \left(\left(\left(\eta_{0}(1, x)-\bar{y}\right) p_{0}(x)+\left(\underline{y}-\eta_{0}(0, x)\right)\left(1-p_{0}(x)\right)\right) \theta_{10}(x)\right.\right. \\
& \left.\left.\left.\quad-\left(\left(\eta_{0}(1, x)-\underline{y}\right) p_{0}(x)+\left(\bar{y}-\eta_{0}(0, x)\right)\left(1-p_{0}(x)\right)\right) \theta_{01}(x)\right)-\beta\right) d F_{\tau}(x)\right] \\
& \quad+\frac{d}{d \tau}\left[\int \left(\left(\left(\eta_{0}(1, x)-\bar{y}\right) p_{\tau}(x)+\left(\underline{y}-\eta_{0}(0, x)\right)\left(1-p_{\tau}(x)\right)\right) \theta_{10}(x)\right.\right. \\
& \left.\left.\left.\quad-\left(\left(\eta_{0}(1, x)-\underline{y}\right) p_{\tau}(x)+\left(\bar{y}-\eta_{0}(0, x)\right)\left(1-p_{\tau}(x)\right)\right) \theta_{01}(x)\right)-\beta\right) d F_{0}(x)\right] \\
& \quad+\frac{d}{d \tau}\left[\int \left(\left(\left(\eta_{\tau}(1, x)-\bar{y}\right) p_{0}(x)+\left(\underline{y}-\eta_{\tau}(0, x)\right)\left(1-p_{0}(x)\right)\right) \theta_{10}(x)\right.\right. \\
& \left.\left.\left.\quad-\left(\left(\eta_{\tau}(1, x)-\underline{y}\right) p_{0}(x)+\left(\bar{y}-\eta_{\tau}(0, x)\right)\left(1-p_{0}(x)\right)\right) \theta_{01}(x)\right)-\beta\right) d F_{0}(x)\right] .
\end{aligned}
$$

First, we have

$$
\begin{aligned}
& \frac{d}{d \tau} E_{\tau}\left[m\left(w_{i}, \beta, \gamma\left(F_{0}\right)\right)\right] \\
& =\int\left(\left(\eta_{0}(1, x)-\bar{y}\right) p_{0}(x)+\left(\underline{y}-\eta_{0}(0, x)\right)\left(1-p_{0}(x)\right)\right) \theta_{10}(x) \\
& \left.\quad-\left(\left(\eta_{0}(1, x)-\underline{y}\right) p_{0}(x)+\left(\bar{y}-\eta_{0}(0, x)\right)\left(1-p_{0}(x)\right)\right) \theta_{01}(x)\right) d G(x)-\beta
\end{aligned}
$$

Next, we want to find $\frac{d}{d \tau} E_{0}\left[m\left(w_{i}, \beta, \gamma\left(F_{\tau}\right)\right)\right]$. In order to do that, first note that we have

$$
\begin{aligned}
& \frac{d}{d \tau} \int \theta(x) \eta_{\tau}(d, x) f_{0}(d \mid x) f_{0}(x) d x \\
& =\int \theta(x) \frac{d}{d \tau}\left[\eta_{\tau}(d, x)\right] f_{0}(d \mid x) f_{0}(x) d x \\
& =\int \theta(x) E_{0}\left[\left\{Y-\eta_{0}(d, x)\right\} \varphi(W) \mid D=d, X=x\right] f_{0}(d \mid x) f_{0}(x) d x \\
& =\int \theta(x)\left[\int\left\{y-\eta_{0}(d, x)\right\} \frac{g(y, d, x)}{f_{0}(y, d, x)} f_{0}(y \mid d, x) d y\right] f_{0}(d \mid x) f_{0}(x) d x \\
& =\int \theta(x)\left[\int\left\{y-\eta_{0}(d, x)\right\} \frac{g(y, d, x)}{f_{0}(y, d, x)} \frac{f_{0}(y, d, x)}{f_{0}(d, x)} d y\right] \frac{f_{0}(d, x)}{f_{0}(x)} f_{0}(x) d x \\
& =\int \theta(x)\left[\int\left\{y-\eta_{0}(d, x)\right\} g(y, d, x) d y\right] d x \\
& =\int \theta(x)\left\{y-\eta_{0}(d, x)\right\} g(y, d, x) d y d x
\end{aligned}
$$

The second equality follows from equation (1.126). The third equality follows from choosing $\varphi(w)$ to be a ratio of a sharply peaked pdf to the true density:

$$
\begin{equation*}
\varphi(\tilde{w})=\frac{g(\tilde{w}) 1\left(f_{0}(\tilde{w}) \geq 1 / j\right)}{f_{0}(\tilde{w})} \tag{1.131}
\end{equation*}
$$

where as in Ichimura and Newey (2017), $g(w)$ is specified as follows. Letting $K(u)$ be a pdf that is symmertic around zero, has bounded support, and is continuously differentiable of all orders with bounded derivatives, we let

$$
\begin{equation*}
g(\tilde{w})=\prod_{l=1}^{r} \kappa_{l}^{j}\left(\tilde{w}_{l}\right), \kappa_{l}^{j}\left(\tilde{w}_{l}\right)=\frac{j K\left(\left(w_{l}-\tilde{w}_{l}\right) j\right)}{j \int K\left(\left(w_{l}-\tilde{w}_{l}\right) j\right) d \mu_{l}\left(\tilde{w}_{l}\right)} . \tag{1.132}
\end{equation*}
$$

Hence, we obtain

$$
\begin{aligned}
& \frac{d}{d \tau}\left[\int \left(\left(\left(\eta_{\tau}(1, x)-\bar{y}\right) p_{0}(x)+\left(\underline{y}-\eta_{\tau}(0, x)\right)\left(1-p_{0}(x)\right)\right) \theta_{10}(x)\right.\right. \\
& \left.\left.\left.-\left(\left(\eta_{\tau}(1, x)-\underline{y}\right) p_{0}(x)+\left(\bar{y}-\eta_{\tau}(0, x)\right)\left(1-p_{0}(x)\right)\right) \theta_{01}(x)\right)-\beta\right) d F_{0}(x)\right] \\
& =\int\left(\theta_{10}(x)-\theta_{01}(x)\right)\left\{y-\eta_{0}(1, x)\right\} g(y, 1, x) d y d x \\
& -\int\left(\theta_{10}(x)-\theta_{01}(x)\right)\left\{y-\eta_{0}(0, x)\right\} g(y, 0, x) d y d x
\end{aligned}
$$

With the similar argument, but using equation (1.127), we also have

$$
\begin{aligned}
& \frac{d}{d \tau} \int \theta(x) p_{\tau}(x) f_{0}(x) d x \\
& =\int \theta(x) \frac{d}{d \tau}\left[p_{\tau}(x)\right] f_{0}(x) d x \\
& =\int \theta(x) E_{0}\left[\left\{1\{D=1\}-p_{0}(x)\right\} \varphi(W) \mid X=x\right] f_{0}(x) d x \\
& =\int \theta(x)\left[\int\left\{1\{d=1\}-p_{0}(x)\right\} \frac{g(y, d, x)}{f_{0}(y, d, x)} f_{0}(y, d \mid x) d y d d\right] f_{0}(x) d x \\
& =\int \theta(x)\left[\int\left\{1\{d=1\}-p_{0}(x)\right\} \frac{g(y, d, x)}{f_{0}(y, d, x)} \frac{f_{0}(y, d, x)}{f_{0}(x)} d y d d\right] f_{0}(x) d x \\
& =\int \theta(x)\left\{1\{d=1\}-p_{0}(x)\right\} g(y, d, x) d y d d d x
\end{aligned}
$$

Hence,

$$
\begin{aligned}
& \frac{d}{d \tau}\left[\int \left(\left(\left(\eta_{0}(1, x)-\bar{y}\right) p_{\tau}(x)+\left(\underline{y}-\eta_{0}(0, x)\right)\left(1-p_{\tau}(x)\right)\right) \theta_{10}(x)\right.\right. \\
& \left.\left.\left.-\left(\left(\eta_{0}(1, x)-\underline{y}\right) p_{\tau}(x)+\left(\bar{y}-\eta_{0}(0, x)\right)\left(1-p_{\tau}(x)\right)\right) \theta_{01}(x)\right)-\beta\right) d F_{0}(x)\right] \\
& =\int\left(\theta_{10}(x)-\theta_{01}(x)\right)\left(\eta_{0}(1, x)+\eta_{0}(0, x)-(\underline{y}+\bar{y})\right)\left\{1\{d=1\}-p_{0}(x)\right\} g(y, d, x) d y d d d x
\end{aligned}
$$

Therefore, as $j \rightarrow \infty$, since $\eta(1, x), \eta(0, x)$, and $p(x)$ are continuous at $x$, we obtain

$$
\begin{aligned}
\phi(w, \beta, \gamma) & =\left(\left(\eta_{0}(1, x)-\bar{y}\right) p_{0}(x)+\left(\underline{y}-\eta_{0}(0, x)\right)\left(1-p_{0}(x)\right)\right) \theta_{10}(x) \\
& \left.-\left(\left(\eta_{0}(1, x)-\underline{y}\right) p_{0}(x)+\left(\bar{y}-\eta_{0}(0, x)\right)\left(1-p_{0}(x)\right)\right) \theta_{01}(x)\right)-\beta \\
& +\left(\theta_{10}(x)-\theta_{01}(x)\right)\left(\eta_{0}(1, x)+\eta_{0}(0, x)-(\underline{y}+\bar{y})\right)\left\{1\{d=1\}-p_{0}(x)\right\} \\
& +\left(\theta_{10}(x)-\theta_{01}(x)\right)\left\{y-\eta_{0}(1, x)\right\}^{d}\left\{-\left(y-\eta_{0}(0, x)\right)\right\}^{1-d} .
\end{aligned}
$$

## Proof of Theorem 1.4.1

Let

$$
\begin{equation*}
\hat{\psi}\left(\beta_{l}\right)=\frac{1}{n} \sum_{k=1}^{L} \sum_{i \in \mathcal{I}_{k}} \psi\left(w_{i}, \beta_{l}, \hat{\gamma}_{k}\right) . \tag{1.133}
\end{equation*}
$$

First we show that

$$
\begin{equation*}
\sqrt{n} \hat{\psi}\left(\beta_{0}\right)=\frac{1}{\sqrt{n}} \sum_{i=1}^{n} \psi\left(w_{i}, \beta_{0}, \gamma_{0}\right)+o_{p}(1) \tag{1.134}
\end{equation*}
$$

holds. Under Assumption 1.4.2 (i), (ii), and (iii), the result follows. Following CEINR, we provide a sketch of the argument. Let

$$
\begin{equation*}
\hat{\Delta}_{i k} \equiv \psi\left(w_{i}, \beta_{0}, \hat{\gamma}_{k}\right)-\bar{\psi}\left(\hat{\gamma}_{k}\right)-\psi\left(w_{i}, \beta_{0}, \gamma_{0}\right) \tag{1.135}
\end{equation*}
$$

and

$$
\begin{equation*}
\bar{\Delta}_{k} \equiv \frac{1}{n} \sum_{i \in \mathcal{I}_{k}} \bar{\Delta}_{i k} . \tag{1.136}
\end{equation*}
$$

Let $n_{k}$ be the number of observations with $i \in \mathcal{I}_{k}$ and $W_{k}$ denote a vector of all observations $w_{i}$ for $i \notin \mathcal{I}_{k}$. Note that for any $i, j \in \mathcal{I}_{k}, i \neq j$, we have $E\left[\hat{\Delta}_{i k} \hat{\Delta}_{j k} \mid W_{k}\right]=$ $E\left[\hat{\Delta}_{i k} \mid W_{k}\right] E\left[\hat{\Delta}_{j k} \mid W_{k}\right]=0$ since by construction $E\left[\hat{\Delta}_{i k} \mid W_{k}\right]=0$. By Assumption 1.4.2
(i),

$$
\begin{equation*}
E\left[\bar{\Delta}_{k}^{2} \mid W_{k}\right]=\frac{1}{n^{2}} \sum_{i \in \mathcal{I}_{k}} E\left[\hat{\Delta}_{i k}^{2} \mid W_{k}\right] \leq \frac{n_{k}}{n^{2}} \int\left\{\psi\left(w, \beta_{0}, \hat{\gamma}_{k}\right)-\psi\left(w, \beta_{0}, \gamma_{0}\right)\right\}^{2} F_{0}(d w)=o_{p}\left(n_{k} / n^{2}\right) \tag{1.137}
\end{equation*}
$$

This implies that, for each $k$, we have $\bar{\Delta}_{k}=o_{p}\left(\sqrt{n_{k}} / n\right)$. Then it follows that

$$
\begin{equation*}
\sqrt{n}\left[\hat{\psi}\left(\beta_{0}\right)-\frac{1}{n} \sum_{i=1}^{n} \psi\left(w_{i}, \beta_{0}, \gamma_{0}\right)-\frac{1}{n} \sum_{k=1}^{L} n_{k} \bar{\psi}\left(\hat{\gamma}_{k}\right)\right]=\sqrt{n} \sum_{k=1}^{L} \bar{\Delta}_{k}=o_{p}\left(\sqrt{n_{k} / n}\right) \xrightarrow{p} 0 \tag{1.138}
\end{equation*}
$$

By Assumption 1.4.2 (ii) and (iii), we have

$$
\begin{equation*}
\sqrt{n}\left|\bar{\psi}\left(\hat{\gamma}_{k}\right)\right| \leq \sqrt{n} C\left\|\hat{\gamma}_{k}-\gamma_{0}\right\|^{2} \xrightarrow{p} 0 \tag{1.139}
\end{equation*}
$$

Then (1.134) follows by the triangle inequality. Since (1.134) holds and $\left\{w_{i}\right\}_{i=1}^{n}$ are i.i.d., by central limit theorem

$$
\begin{equation*}
\sqrt{n} \hat{\psi}\left(\beta_{0}\right) \xrightarrow{d} N(0, \Omega), \tag{1.140}
\end{equation*}
$$

where $\Omega=E\left[\psi\left(w_{i}, \beta_{0}, \gamma_{0}\right)^{2}\right]$. The rest of the proof is standard as in Newey and McFadden (1994) and we provide a sketch of the argument. Let $M=E\left[\left.\frac{\partial \psi\left(w, \beta, \gamma_{0}\right)}{\partial \beta}\right|_{\beta=\beta_{0}}\right]$ and $\hat{M}=\frac{\partial \hat{\psi}(\hat{\beta})}{\partial \beta}$. The first order condition is

$$
\begin{equation*}
0=\hat{M} \hat{\psi}(\hat{\beta}) \tag{1.141}
\end{equation*}
$$

We expand $\hat{\psi}(\hat{\beta})$ around $\beta_{0}$ to obtain

$$
\begin{equation*}
\hat{\psi}(\hat{\beta})=\hat{\psi}\left(\beta_{0}\right)+\bar{M}\left(\hat{\beta}-\beta_{0}\right) \tag{1.142}
\end{equation*}
$$

where $\bar{M}=\frac{\partial \hat{\psi}\left(\beta_{u}\right)}{\partial \beta}$ and $\bar{\beta}$ is the mean value. Substituting this back into the first order condition, we get

$$
\begin{equation*}
0=\hat{M} \hat{\psi}\left(\beta_{0}\right)+\hat{M} \bar{M}\left(\hat{\beta}-\beta_{0}\right) \tag{1.143}
\end{equation*}
$$

Solving this for $\hat{\beta}-\beta_{0}$ and multiplying by $\sqrt{n}$, we obtain

$$
\begin{equation*}
\sqrt{n}\left(\hat{\beta}-\beta_{0}\right)=-(\hat{M} \bar{M})^{-1} \hat{M} \sqrt{n} \hat{\psi}\left(\beta_{0}\right) . \tag{1.144}
\end{equation*}
$$

We also have $\hat{M} \xrightarrow{p} M$ and $\bar{M} \xrightarrow{p} M$ and by the continuous mapping theorem,

$$
\begin{equation*}
-(\hat{M} \bar{M})^{-1} \hat{M} \xrightarrow{p}-M^{-1} . \tag{1.145}
\end{equation*}
$$

Then, by the Slutzky theorem,

$$
\begin{equation*}
\sqrt{n}\left(\hat{\beta}-\beta_{0}\right) \xrightarrow{d}-M^{-1} N(0, \Omega)=N\left(0, M^{-2} \Omega\right) . \tag{1.146}
\end{equation*}
$$

In our case, $M=E\left[\left.\frac{\partial \psi\left(w, \beta, \gamma_{0}\right)}{\partial \beta}\right|_{\beta=\beta_{0}}\right]=1$ and so the asymptotic variance is $\Omega$. Finally, CEINR showed that Assumption 1.4.2 (iii) insures that $\hat{V} \xrightarrow{p} V$.

## Proof of Lemma 1.4.2

The proof is similar to that of Lemma 1.4.1. Since we have an additional variable $Z \in\{0,1\}$, we make slight adjustments. By the chain rule, we have

$$
\begin{aligned}
& \frac{d}{d \tau} E_{\tau}\left[m\left(w_{i}, \beta, \gamma\left(F_{\tau}\right)\right)\right] \\
&= \frac{d}{d \tau} E_{\tau}\left[m\left(w_{i}, \beta, \gamma\left(F_{0}\right)\right)\right]+\frac{d}{d \tau} E_{0}\left[m\left(w_{i}, \beta, \gamma\left(F_{\tau}\right)\right)\right] \\
&= \frac{d}{d \tau}\left[\int \left(\left(\eta_{0}(1, x, 1) \cdot p_{0}(x, 1)+\underline{y} \cdot\left(1-p_{0}(x, 1)\right)\right.\right.\right. \\
&\left.\quad-\bar{y} \cdot p_{0}(x, 0)-\eta_{0}(0, x, 0) \cdot\left(1-p_{0}(x, 0)\right)\right) \cdot \theta_{10}(x) \\
& \quad-\left(\eta_{0}(1, x, 1) \cdot p_{0}(x, 1)+\bar{y} \cdot\left(1-p_{0}(x, 1)\right)\right. \\
&\left.\left.\left.\quad-\underline{y} \cdot p_{0}(x, 0)-\eta_{0}(0, x, 0) \cdot\left(1-p_{0}(x, 0)\right)\right) \cdot \theta_{01}(x)-\beta\right) d F_{\tau}(x)\right] \\
& \quad+\frac{d}{d \tau}\left[\int \left(\left(\eta_{0}(1, x, 1) \cdot p_{\tau}(x, 1)+\underline{y} \cdot\left(1-p_{\tau}(x, 1)\right)\right.\right.\right. \\
&\left.\quad-\bar{y} \cdot p_{\tau}(x, 0)-\eta_{0}(0, x, 0) \cdot\left(1-p_{\tau}(x, 0)\right)\right) \cdot \theta_{10}(x) \\
& \quad-\left(\eta_{0}(1, x, 1) \cdot p_{\tau}(x, 1)+\bar{y} \cdot\left(1-p_{\tau}(x, 1)\right)\right. \\
&\left.\left.\left.\quad-\underline{y} \cdot p_{\tau}(x, 0)-\eta_{0}(0, x, 0) \cdot\left(1-p_{\tau}(x, 0)\right)\right) \cdot \theta_{01}(x)-\beta\right) d F_{0}(x)\right] \\
& \quad+\frac{d}{d \tau}\left[\int \left(\left(\eta_{\tau}(1, x, 1) \cdot p_{0}(x, 1)+\underline{y} \cdot\left(1-p_{0}(x, 1)\right)\right.\right.\right. \\
&\left.\quad-\bar{y} \cdot p_{0}(x, 0)-\eta_{\tau}(0, x, 0) \cdot\left(1-p_{0}(x, 0)\right)\right) \cdot \theta_{10}(x) \\
& \quad-\left(\eta_{\tau}(1, x, 1) \cdot p_{0}(x, 1)+\bar{y} \cdot\left(1-p_{0}(x, 1)\right)\right. \\
&\left.\left.\left.\quad-\underline{y} \cdot p_{0}(x, 0)-\eta_{\tau}(0, x, 0) \cdot\left(1-p_{0}(x, 0)\right)\right) \cdot \theta_{01}(x)-\beta\right) d F_{0}(x)\right] .
\end{aligned}
$$

By the arguments in the proof of Lemma 1.4.1, we have

$$
\begin{equation*}
\frac{d}{d \tau} \int \theta(x) \eta_{\tau}(d, x, z) f_{0}(x) d x=\int \theta(x)\left\{y-\eta_{0}(d, x, z)\right\} g(y, d, x) d y d x \tag{1.147}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{d}{d \tau} \int \theta(x) p_{\tau}(x, z) f_{0}(x) d x=\int \theta(x)\left\{1\{d=1\}-p_{0}(x, z)\right\} g(y, d, x) d y d d d x \tag{1.148}
\end{equation*}
$$

Hence, we obtain

$$
\begin{aligned}
& \frac{d}{d \tau}\left[\int \left(\left(\eta_{0}(1, x, 1) \cdot p_{\tau}(x, 1)+\underline{y} \cdot\left(1-p_{\tau}(x, 1)\right)\right.\right.\right. \\
& \left.\quad-\bar{y} \cdot p_{\tau}(x, 0)-\eta_{0}(0, x, 0) \cdot\left(1-p_{\tau}(x, 0)\right)\right) \cdot \theta_{10}(x) \\
& \quad-\left(\eta_{0}(1, x, 1) \cdot p_{\tau}(x, 1)+\bar{y} \cdot\left(1-p_{\tau}(x, 1)\right)\right. \\
& \left.\left.\left.\quad-\underline{y} \cdot p_{\tau}(x, 0)-\eta_{0}(0, x, 0) \cdot\left(1-p_{\tau}(x, 0)\right)\right) \cdot \theta_{01}(x)-\beta\right) d F_{0}(x)\right] \\
& =\int\left(\left(\eta_{0}(1, x, 1)-\underline{y}\right) \cdot \theta_{10}(x)\right. \\
& \left.\quad-\left(\eta_{0}(1, x, 1)-\bar{y}\right) \cdot \theta_{01}(x)\right) \cdot\left(d-p_{0}(x, 1)\right) \\
& \quad+\left(\left(\eta_{0}(0, x, 0)-\bar{y}\right) \cdot \theta_{10}(x)\right. \\
& \left.\quad-\left(\eta_{0}(0, x, 0)-\underline{y}\right) \cdot \theta_{01}(x)\right) \cdot\left(d-p_{0}(x, 0)\right) g(y, d, z, x) d y d d d z d x
\end{aligned}
$$

Also,

$$
\begin{aligned}
\frac{d}{d \tau} & {\left[\int \left(\left(\eta_{\tau}(1, x, 1) \cdot p_{0}(x, 1)+\underline{y} \cdot\left(1-p_{0}(x, 1)\right)\right.\right.\right.} \\
& \left.-\bar{y} \cdot p_{0}(x, 0)-\eta_{\tau}(0, x, 0) \cdot\left(1-p_{0}(x, 0)\right)\right) \cdot \theta_{10}(x) \\
& -\left(\eta_{\tau}(1, x, 1) \cdot p_{0}(x, 1)+\bar{y} \cdot\left(1-p_{0}(x, 1)\right)\right. \\
& \left.\left.\left.-\underline{y} \cdot p_{0}(x, 0)-\eta_{\tau}(0, x, 0) \cdot\left(1-p_{0}(x, 0)\right)\right) \cdot \theta_{01}(x)-\beta\right) d F_{0}(x)\right] \\
= & \int\left(\theta_{10}(x)-\theta_{01}(x)\right) \cdot p_{0}(x, 1) \cdot\left(y-\eta_{0}(1, x, 1)\right) g(y, d, z, x) d y d d d z d x \\
& -\int\left(\theta_{10}(x)-\theta_{01}(x)\right) \cdot\left(1-p_{0}(x, 0)\right) \cdot\left(y-\eta_{0}(0, x, 0)\right) g(y, d, z, x) d y d d d z d x
\end{aligned}
$$

### 1.7.3 More General Case

I show how my result can be extended to a more general setting. Let $\delta: \mathcal{X} \rightarrow[0,1]$ so that the treatment rules can be randomized treatment rules. Also, let $w: \mathcal{X} \rightarrow \mathbb{R}_{+}$ be some weighting function so that the policymaker cares about the weighted average welfare $E\left[w(X) \cdot\left(E\left[Y_{1} \mid X\right] \cdot \delta(X)+E\left[Y_{0} \mid X\right] \cdot(1-\delta(X))\right)\right]$ rather than the mean welfare. Then, by letting $\psi(X) \equiv w(X) \cdot\left(\delta(X)-\delta^{*}(X)\right)$, my object of interest becomes

$$
\begin{equation*}
E\left[\Delta(X) \cdot w(X) \cdot\left(\delta(X)-\delta^{*}(X)\right)\right]=E[\Delta(X) \cdot \psi(X)] \tag{1.149}
\end{equation*}
$$

I derive the identification of this parameter in the following theorem.
Theorem 1.7.2 (More general case). Suppose $\left(\mathcal{Y}_{1} \times \mathcal{Y}_{0}\right): \Omega \rightarrow \mathcal{F}$ is an integrable random set. Let $\delta: \mathcal{X} \rightarrow[0,1]$ and $\delta^{*}: \mathcal{X} \rightarrow[0,1]$ be treatment rules and $w: \mathcal{X} \rightarrow \mathbb{R}_{+}$ be a weighting function. Also, let $\psi(X) \equiv w(X) \cdot\left(\delta(X)-\delta^{*}(X)\right)$ and $p=(1,-1)^{\prime}$. Then, $B_{I}\left(\delta, \delta^{*}\right)$ in (1.7) is an interval $\left[\beta_{l}, \beta_{u}\right]$ where

$$
\begin{equation*}
\beta_{l}=E[\underline{\Delta}(X) \cdot|\psi(X)| \cdot \mathbb{1}\{\psi(X) \geq 0\}-\bar{\Delta}(X) \cdot|\psi(X)| \cdot \mathbb{1}\{\psi(X)<0\}], \tag{1.150}
\end{equation*}
$$

and

$$
\begin{equation*}
\beta_{u}=E[\bar{\Delta}(X) \cdot|\psi(X)| \cdot \mathbb{1}\{\psi(X) \geq 0\}-\underline{\Delta}(X) \cdot|\psi(X)| \cdot \mathbb{1}\{\psi(X)<0\}], \tag{1.151}
\end{equation*}
$$

where $\bar{\Delta}(X) \equiv E\left[s\left(v^{*}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right]$ and $\underline{\Delta}(X) \equiv-E\left[s\left(-v^{*}, \mathcal{Y}_{1} \times \mathcal{Y}_{0}\right) \mid X\right]$.

Proof. The proof is similar to that of Theorem 1.3.1. I still have (1.86) to bound $\Delta(X)$. Since $\psi(X) \in \mathbb{R}$, I consider two cases: (i) $\psi(X) \geq 0$ and (ii) $\psi(X)<0$. When (i) $\psi(X) \geq 0$, the upper bound on $\Delta(X) \cdot \psi(X)$ is $\bar{\Delta}(X) \cdot|\psi(X)|$. When (ii) $\psi(X)<0$, the upper bound on $\Delta(X) \cdot \psi(X)$ is $-\underline{\Delta}(X) \cdot|\psi(X)|$. Hence, the lower bound should be

$$
\begin{equation*}
\beta_{l}=E[\underline{\Delta}(X) \cdot|\psi(X)| \cdot \mathbb{1}\{\psi(X) \geq 0\}-\bar{\Delta}(X) \cdot|\psi(X)| \cdot \mathbb{1}\{\psi(X)<0\}] . \tag{1.152}
\end{equation*}
$$

Similarly, the upper bound on $E[\Delta(X) \cdot \psi(X)]$ should be

$$
\begin{equation*}
\beta_{u}=E[\bar{\Delta}(X) \cdot|\psi(X)| \cdot \mathbb{1}\{\psi(X) \geq 0\}-\underline{\Delta}(X) \cdot|\psi(X)| \cdot \mathbb{1}\{\psi(X)<0\}] . \tag{1.153}
\end{equation*}
$$

### 1.7.4 More Details on the Simulation Study

The population welfare gain that results from switching from $\delta^{*}(x)=1\{x \leq 11\}$ to $\delta(x)=1\{x \leq 12\}$ is

$$
\begin{align*}
\beta & =E\left[E\left[Y_{1}-Y_{0} \mid X\right] \cdot\left(\delta(X)-\delta^{*}(X)\right)\right] \\
& =P(X=12) \cdot E\left[Y_{1}-Y_{0} \mid X=12\right] \\
& =0.43 \cdot \int_{0}^{1}\left\{m_{1}(12, u)-m_{0}(12, u)\right\} d u  \tag{1.154}\\
& =1236
\end{align*}
$$

The integration is done using integrate() function on R. Given the structure in Section 1.5, we have

$$
\left.\begin{array}{rl}
E[Y \mid D=0, X, Z] & =E\left[Y_{0} \mid U>p(X, Z), X, Z\right] \\
& =\frac{1}{1-p(X, Z)} \int_{p(X, Z)}^{1} m_{0}(X, u) d u \\
E[Y \mid D=1, X, Z] & =E\left[Y_{1} \mid U \leq p(X, Z), X, Z\right] \\
& =\frac{1}{p(X, Z)} \int_{0}^{p(X, Z)} m_{1}(X, u) d u
\end{array}\right] \begin{aligned}
& P(D=1 \mid X)=2 / 3 \cdot p(X, 1)+1 / 3 \cdot p(X, 0)
\end{aligned}
$$

$$
\begin{array}{r}
P(Z=1 \mid D=0, X)=\frac{P(D=0 \mid Z=1, X) P(Z=1 \mid X)}{P(D=0 \mid X)} \\
=\frac{2 / 3 \cdot(1-p(X, 1))}{1-2 / 3 \cdot p(X, 1)-1 / 3 \cdot p(X, 0)}, \\
P(Z=0 \mid D=0, X)=\frac{1 / 3 \cdot(1-p(X, 0))}{1-2 / 3 \cdot p(X, 1)-1 / 3 \cdot p(X, 0)}, \\
P(Z=1 \mid D=1, X)=\frac{P(D=1 \mid Z=1, X) P(Z=1 \mid X)}{P(D=1 \mid X)} \\
=\frac{2 / 3 \cdot p(X, 1)}{2 / 3 \cdot p(X, 1)+1 / 3 \cdot p(X, 0)}, \\
P(Z=0 \mid D=1, X)=\frac{1 / 3 \cdot p(X, 0)}{2 / 3 \cdot p(X, 1)+1 / 3 \cdot p(X, 0)}, \\
E[Y \mid D=0, X]=E[Y \mid D=0, X, Z=1] \cdot P(Z=1 \mid D=0, X) \\
+E[Y \mid D=0, X, Z=0] \cdot P(Z=0 \mid D=0, X), \\
E[Y \mid D=1, X]=E[Y \mid D=1, X, Z=1] \cdot P(Z=1 \mid D=1, X)  \tag{1.163}\\
+E[Y \mid D=1, X, Z=0] \cdot P(Z=0 \mid D=1, X)
\end{array}
$$

Given these quantities, worst-case and IV-worst-case bounds can be calculated similarly.

## Chapter 2

## Two Applications of Treatment Assignment Rules

### 2.1 Introduction

In this chapter, I apply the method I proposed in the previous chapter to examine two applications in economics. First, I study the problem of assigning individuals to job training programs. I calculate the welfare differences between different hypothetical policies using experimental data from the National Job Training Partnership Act (JTPA) Study. Second, I apply the method to study public health insurance policies. Specifically, I calculate the welfare impact of Medicaid expansion using data from the Oregon Health Insurance Experiment.

### 2.2 National Job Training Partnership Act Study

In this section, I use experimental data from the National JTPA Study which was commissioned by the United States Department of Labor in 1986. The goal of this randomized experiment was to measure the benefits and costs of training programs funded under the JTPA of 1982. Applicants who were randomly assigned to a treatment group were allowed access to the program for 18 months while the ones assigned to a control group were excluded from receiving JTPA services in that period. The original evaluation of the program is based on data of 15,981 applicants. More detailed information about the experiment and program impact estimates can be found
in Bloom, Orr, Bell, Cave, Doolittle, Lin, and Bos (1997).

### 2.2.1 Data

I follow Kitagawa and Tetenov (2018) and focus on adult applicants with available data on 30-month earnings after the random assignment, years of education, and pre-program earnings. I downloaded the dataset that Kitagawa and Tetenov (2018) used in their analysis from Econometrica's supplementary material repository and supplemented this dataset with that of Abadie, Angrist, and Imbens (2002) to obtain a variable that indicates program participation. ${ }^{1}$ Table 2.1 shows the summary statistics of this sample. The sample consists of 9223 observations, of which 6133 (roughly $2 / 3$ ) were assigned to the treatment group, and 3090 (roughly $1 / 3$ ) were assigned to the control group. The means and standard deviations of program participation, 30-month earnings, years of education, and pre-program earnings are given for the entire sample, the treatment group subsample, and the control group subsample.

Treatment variable is the job training program participation and equals 1 for individuals who actually participated in the program. Only $65 \%$ of those who got assigned to the treatment group actually participated in the training program. I look at the joint distribution of assigned and realized treatment status in Table 2.2 to further investigate the compliance issue. Outcome variable is 30 -month earnings and is on average $\$ 16,093$ and ranges from $\$ 0$ to $\$ 155,760$ with median earnings $\$ 11,187$. Note that this is 30 -month earnings and is not annual earnings. In the analysis below, based on this range, I set $\underline{y}=\$ 0$ and $\bar{y}=\$ 160,000$. Treatment group assignees earned $\$ 16,487$ on average while control group assignees earned $\$ 15,311$. The $\$ 1,176$ difference between these two group averages is an estimate of the JTPA impact on 30-month earnings from an intention-to-treat perspective. Pretreatment

[^4]covariates I consider are years of education and pre-program annual earnings. Years of education are on average 11.61 years and range from 7 to 18 years with median 12 years. Pre-program annual earnings are on average $\$ 3,232$ and range from $\$ 0$ to $\$ 63,000$ with median earnings $\$ 1,600 .^{2}$ Both variables are roughly balanced by assignment status due to random assignment and large samples involved.

Table 2.1: Summary statistics

|  | Entire sample | Assigned to treatment | Assigned to control |
| :---: | :---: | :---: | :---: |
| Treatment |  |  |  |
| Job training | $\begin{gathered} 0.44 \\ (0.50) \end{gathered}$ | $\begin{gathered} 0.65 \\ (0.48) \end{gathered}$ | $\begin{gathered} 0.01 \\ (0.12) \end{gathered}$ |
| Outcome variable |  |  |  |
| 30-month earnings | $\begin{gathered} 16,093 \\ (17,071) \end{gathered}$ | $\begin{gathered} 16,487 \\ (17,391) \end{gathered}$ | $\begin{gathered} 15,311 \\ (16,392) \end{gathered}$ |
| Pretreatment covariates |  |  |  |
| Years of education | $\begin{gathered} 11.61 \\ (1.87) \end{gathered}$ | $\begin{aligned} & 11.63 \\ & (1.87) \end{aligned}$ | $\begin{aligned} & 11.58 \\ & (1.88) \end{aligned}$ |
| Pre-program earnings | $\begin{gathered} 3,232 \\ (4,264) \end{gathered}$ | $\begin{gathered} 3,205 \\ (4,279) \end{gathered}$ | $\begin{gathered} 3,287 \\ (4,234) \end{gathered}$ |
| Number of observations | 9223 | 6133 | 3090 |
| This table reports the means and standard deviations (in brackets) of variables in our sample. Treatment variable is job training program participation and equals 1 for individuals who actually participated in the program. The outcome variable is 30 -month earnings after the random assignment. Pretreatment covariates are years of education and pre-program annual earnings. The earnings are in US Dollars. |  |  |  |

Compliance Although the offer of treatment was randomly assigned, the compliance was not perfect. Table 2.2 shows the joint distribution of assigned and realized

[^5]treatment. Assigned treatment equals 1 for individuals who got offered the training program and realized treatment equals 1 for individuals who actually participated in the training. As can be seen from this table, the realized treatment is not equal to assigned treatment for roughly $23 \%$ of the applicants. Therefore, the program participation is self-selected and likely to be correlated with potential outcomes. Since the assumption of unconfoundedness fails to hold in this case, the conditional treatment effects are not point identified. Although the random offer can be used as a treatment variable to point identify the intention-to-treat effect as in Kitagawa and Tetenov (2018), the actual program participation should be used to identify the treatment effect itself.

Table 2.2: The joint distribution of assigned and realized treatment

|  | Assigned treatment |  |  |
| :---: | :---: | :---: | :---: |
| Realized treatment | 1 | 0 | Total |
| 1 | 4015 | 43 | 4058 |
| 0 | 2118 | 3047 | 5165 |
| Total | 6133 | 3090 | 9223 |

This table reports the joint distribution of assigned and realized treatment in our sample. Assigned treatment equals 1 for individuals who got offered job training and realized treatment equals 1 for individuals who actually participated in the training. It shows the compliance issue in our sample.

### 2.2.2 Example 1

Applicants were eligible for training if they faced a certain barriers to employment. This included being a high school dropout. Suppose the benchmark policy is to treat everyone with less than high school education, i.e., people who have less than or equal to 11 years of education. Now, consider implementing a new policy in which
we include people with high school degree. In other words, let

$$
\begin{align*}
\delta^{*} & =\mathbb{1}\{\text { education } \leq 11\},  \tag{2.1}\\
\delta & =\mathbb{1}\{\text { education } \leq 12\} . \tag{2.2}
\end{align*}
$$

These are the policies I considered in the Monte Carlo experiment in Chapter 1. The estimates of lower and upper bounds on the welfare gain from this new policy under various assumptions and different instrumental variables are summarized in Table 2.4. In this example, a random offer is used as an instrumental variable and pre-program earnings is used as a monotone instrumental variable. Since the eligibility was randomly assigned, the IV assumption (1.20) holds. As for the MIV assumption (1.36), it requires that conditional on years of education, the average potential earnings that would have been observed in each state (participating versus not participating in the training program) is higher for those who were earning more compared to those who were earning less.

For the first step estimation, I use cross-fitting with $K=2$ and estimate $\hat{\eta}(1, x)$, $\hat{\eta}(0, x)$ and $\hat{p}(x)$ by empirical means. Those empirical means out of whole sample are depicted in Figure 2•1, $2 \cdot 2$ and $2 \cdot 3$. Empirical means and distributions when years of education is used as $X$ and random offer is used as $Z$ are summarized in Table 2.3.

Table 2.3: Empirical means and distributions when $X$ is years of education

| $X$ | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| sample size | 34 | 616 | 642 | 984 | 1167 | 3940 | 660 | 602 | 197 | 260 | 111 | 10 |
| $P(X)$ | 0.004 | 0.067 | 0.07 | 0.107 | 0.127 | 0.427 | 0.072 | 0.065 | 0.021 | 0.028 | 0.012 | 0.001 |
| $E[Y \mid X]$ | 7998 | 12252 | 12509 | 14095 | 13492 | 16982 | 18210 | 20204 | 20837 | 20875 | 20032 | 11606 |
| $E[Y \mid D=1, X]$ | 7747 | 14916 | 14860 | 14706 | 15622 | 18391 | 18713 | 21093 | 21369 | 20678 | 22082 | 9207 |
| $E[Y \mid D=0, X]$ | 8102 | 10469 | 10908 | 13662 | 12014 | 15786 | 17745 | 19520 | 20322 | 21033 | 18411 | 14005 |
| $P(D=1 \mid X)$ | 0.294 | 0.401 | 0.405 | 0.415 | 0.41 | 0.459 | 0.48 | 0.435 | 0.492 | 0.446 | 0.441 | 0.5 |
| $E[Y \mid D=1, Z=1, X]$ | 7747 | 14932 | 14860 | 14687 | 15639 | 18448 | 18833 | 21272 | 21369 | 20678 | 22082 | 9207 |
| $E[Y \mid D=1, Z=0, X]$ | 0 | 11011 | 14886 | 17263 | 14000 | 13530 | 6089 | 13449 | 0 | 0 | 0 | 0 |
| $E[Y \mid D=0, Z=1, X]$ | 6194 | 9778 | 10114 | 13944 | 11752 | 15345 | 16299 | 18776 | 21010 | 16767 | 16904 | 12636 |
| $E[Y \mid D=0, Z=0, X]$ | 8888 | 10980 | 11546 | 13451 | 12196 | 16084 | 18661 | 20028 | 19781 | 23669 | 19429 | 14918 |
| $P(D=1 \mid Z=1, X)$ | 0.588 | 0.61 | 0.601 | 0.622 | 0.626 | 0.676 | 0.702 | 0.65 | 0.688 | 0.678 | 0.662 | 0.714 |
| $P(D=1 \mid Z=0, X)$ | 0 | 0.005 | 0.019 | 0.009 | 0.012 | 0.016 | 0.014 | 0.029 | 0 | 0 | 0 | 0 |

This table reports the empirical means and distributions when $X$ is years of education. Y denotes the outcome variable which is 30 -month earnings in US Dollars. $D$ denotes the program participation and equals 1 for individuals who participated in the program. $Z$ denotes the random assignment to treatment and equals 1 for individuals who got offered job training.

Figure 2•1: Estimated mean outcome for the treated group when X is years of education


Figure 2.2: Estimated mean outcome for the untreated group when X is years of education


Figure 2.3: Estimated propensity score when X is years of education


As can be seen from Table 2.4, the worst-case bounds cover 0, as I explained earlier. Although we cannot rank which policy is better, we quantify the no-assumption scenario as a welfare loss of $\$ 31,423$ and a welfare gain of $\$ 36,928$. Under the MTR assumption, the lower bound is 0 . That is because the MTR assumption states that everyone benefits from the treatment, and under the new policy, we are expanding the treated population. The upper bound under MTR is the same as the upper bound under the worst-case. When we use a random offer as an instrumental variable, the bounds are tighter than the worst-case bounds and still cover 0. However, when we use pre-program earnings as a monotone instrumental variable, the bounds do not cover 0 , and it is even tighter if we impose an additional MTR assumption. Therefore, if the researcher is comfortable with the validity of the MIV assumption, she can conclude that implementing the new policy is guaranteed to improve welfare and that improvement is between $\$ 3,569$ and $\$ 36,616$.

Table 2.4: Welfare gains in Example 1

| Assumptions | lower bound | upper bound |
| :--- | :---: | :---: |
| worst-case | $-31,423$ | 36,928 |
|  | $(-32,564,-30,282)$ | $(35,699,38,158)$ |
| MTR | 0 | same as worst-case |
| IV-worst-case | $-2,486$ | 20,787 |
|  | $(-2,774,-2,198)$ | $(19,881,21694)$ |
| IV-MTR | 0 | same as IV-worst-case |
| MIV-worst-case | 3,569 | 36,616 |
| MIV-MTR | 7,167 | same as MIV-worst-case |

This table reports the estimated welfare gains and their $95 \%$ confidence intervals (in brackets) in Example 1 under various assumptions. The welfare is in terms of 30 -month earnings in US Dollars.

### 2.2.3 Example 2

One class of treatment rules that Kitagawa and Tetenov (2018) considered is a class of quadrant treatment rules:

$$
\begin{align*}
& \mathscr{G}=\left\{\left\{x: s_{1}\left(\text { education }-t_{1}\right)>0 \text { and } s_{2}\left(\text { pre-program earnings }-t_{2}\right)>0\right\},\right.  \tag{2.3}\\
& \left.\left.s_{1}, s_{2} \in\{-1,1\}, t_{1}, t_{2} \in \mathbb{R}\right\}\right\} .
\end{align*}
$$

One's education level and pre-program earnings have to be above or below some specific thresholds to be assigned to treatment according to this treatment rule. Within this class of treatment rules, the empirical welfare maximizing treatment rule that Kitagawa and Tetenov (2018) calculates is $\mathbb{1}$ \{education $\leq 15$, prior earnings $\leq$ $\$ 19,670\}$. Let this policy be the benchmark policy and consider implementing another policy that lowers the education threshold to be 12. In fact, that policy is another empirical welfare maximizing policy that takes into account the treatment assignment cost which is $\$ 774$ per assignee. I calculate the welfare difference between these two policies. In other words, let

$$
\begin{align*}
\delta^{*} & =\mathbb{1}\{\text { education } \leq 15, \text { pre-program earnings } \leq \$ 19,670\},  \tag{2.4}\\
\delta & =\mathbb{1}\{\text { education } \leq 12, \text { pre-program earnings } \leq \$ 19,670\} . \tag{2.5}
\end{align*}
$$

I illustrate the policies in Figure $2 \cdot 4$. The estimation results are summarized in Table 2.5. In this example, a random offer is used as an instrumental variable. For the first step estimation, I use cross-fitting with $K=2$ and estimate $\hat{\eta}(1, x)$ and $\hat{\eta}(0, x)$ by polynomial regression of degree 2 and $\hat{p}(x)$ by logistic regression with polynomial of degree 2. Those estimated conditional mean treatment responses and propensity score out of whole sample are depicted in Figures 2•5, $2 \cdot 6$ and $2 \cdot 7$.

Figure 2.4: Hypothetical policies in Example 2


Policy 1 is $\mathbb{1}\{$ education $\leq 15$, pre-program earnings $\leq \$ 19,670\}$, and Policy 2 is $\mathbb{1}\{$ education $\leq 12$, pre-program earnings $\leq \$ 19,670\}$.

Figure 2.5: Estimated mean outcome for the treated group when X is years of education and pre-program annual earnings


Figure 2.6: Estimated mean outcome for the untreated group when X is years of education and pre-program annual earnings


Figure 2.7: Estimated propensity score when X is years of education and pre-program annual earnings


As can be seen from Table 2.5, again, the worst-case bounds cover 0. However, we quantify the no-assumption scenario as a welfare loss of $\$ 13,435$ and a welfare gain of $\$ 11,633$. Under the MTR assumption, the upper bound is 0 . That is because the MTR assumption states that everyone benefits from the treatment, and under the new policy, we are shrinking the treated population. The lower bound under MTR is the same as the lower bound under the worst-case. When we use a random offer as an instrumental variable, the bounds are tighter and still cover 0 as well. Using IV assumption alone, which is a credible assumption since the offer was randomly assigned in the experiment, we quantify the difference as a welfare loss of $\$ 7,336$ and a welfare gain of $\$ 1,035$. In this case, the researcher cannot be sure whether implementing the new policy is guaranteed to worsen or improve welfare. However, if she decides that the welfare gain being at most $\$ 1,035$ is not high enough, she can go ahead with the first policy.

Table 2.5: Welfare gains in Example 2

| Assumptions | lower bound | upper bound |
| :--- | :---: | :---: |
| worst-case | $-13,435$ | 11,633 |
|  | $(-14,361,-12,510)$ | $(10,871,12,394)$ |
| MTR | same as worst-case | 0 |
| IV-worst-case | $-7,336$ | 1,035 |
| IV-MTR | $(-7,911,-6,763)$ | $(862,1,208)$ |
|  | same as IV-worst-case | 0 |

This table reports the estimated welfare gains and their $95 \%$ confidence intervals (in brackets) in Example 2 under various assumptions. The welfare is in terms of 30 -month earnings in US Dollars.

### 2.3 Oregon Health Insurance Experiment

In this section, I use experimental data from the Oregon Health Insurance Experiment (OHIE) to study the impact of different health insurance policies. Medicaid is a public insurance program that provides health coverage to low-income families and individuals in the United States. In 2008, the state of Oregon randomly selected a group of uninsured adults by lottery, out of a waiting list of approximately 90,000 people, and provided them with an opportunity to apply for Medicaid. This lottery allowed researchers to help understand the causal impact of having access to health insurance and having actual insurance coverage on various outcomes. See Finkelstein, Taubman, Wright, Bernstein, Gruber, Newhouse, Allen, Baicker, and Group (2012) for more detailed information about the experiment and survey findings in the year after random assignment. ${ }^{3}$

### 2.3.1 Data

I use publicly available data that consist of both administrative and survey data. ${ }^{4}$ The original full sample consists of 74,922 individuals (some individuals, out of the initial approximately 90,000 individuals, were excluded from the analysis for various reasons). I use a subsample of this dataset, with 26,423 individuals, who responded to the initial survey.

Table 2.6 summarizes the summary statistics of the variables I use in the analysis. It reports the means and the standard deviations for the entire sample, the treatment group subsample, and the control group subsample. The treatment group consists of those who were randomly selected to apply for Medicaid. The control group consists of those who were not given that opportunity. After dropping observations with a

[^6]missing value in any of the variables I consider, the final sample consists of 21,743 observations.

The treatment variable $(D)$ is a binary variable that equals 1 for individuals who actually enrolled in Medicaid and equals 0 otherwise. Discussion on the compliance issue to follow later. I use household income as a percent of the federal poverty line as a pretreatment covariate $(X)$. I consider two distinct outcome variables $(Y)$ : one for health outcome and one for health care utilization. For health outcome, following Finkelstein, Taubman, Wright, Bernstein, Gruber, Newhouse, Allen, Baicker, and Group (2012), I use a binary variable that equals 1 if the self-reported health is "excellent" /"very good" /"good" and equals 0 if the self-reported health is "fair" / "poor." For health care utilization, I use a variable that indicates whether the respondent had any primary care visits in the last six months.

Table 2.6: Summary statistics

|  | Entire sample | Assigned to <br> treatment | Assigned to <br> control |
| :--- | :---: | :---: | :---: |
| Treatment |  |  |  |
| Ever on Medicaid | 0.29 | 0.44 | 0.13 |
|  | $(0.45)$ | $(0.50)$ | $(0.34)$ |
| Pretreatment covariate |  |  |  |
| Income (percent of federal poverty line) | 71.65 | 72.97 | 70.30 |
|  | $(65.13)$ | $(65.20)$ | $(65.04)$ |
| Outcome variables |  |  |  |
| Self-reported health | 0.60 | 0.62 | 0.59 |
|  | $(0.49)$ | $(0.49)$ | $(0.49)$ |
| Primary care visits | 0.57 | 0.58 | 0.57 |
|  | $(0.49)$ | $(0.49)$ | $(0.49)$ |
| Number of observations | 21743 | 10968 | 10775 |

This table reports the means and standard deviations (in brackets) of variables in our sample.

Compliance Although the lottery winners were randomly chosen, the compliance was not perfect. Similar to the JTPA analysis, I look at the joint distribution of assigned and realized treatment for this sample in Table 2.7. Assigned treatment equals 1 for those who won the lottery to apply for Medicaid and equals 0 for those who did not win. Realized treatment equals 1 for those who actually enrolled in Medicaid and equals 0 for those who did not enroll. As shown in the table, the realized treatment is not equal to the assigned treatment for roughly $35 \%$ of the applicants. Therefore, the enrollment is likely to be self-selected and to be correlated with potential outcomes.

Table 2.7: The joint distribution of assigned and realized treatment

|  | Assigned treatment |  |  |
| :---: | :---: | :---: | :---: |
| Realized treatment | 1 | 0 | Total |
| 1 | 0.22 | 0.07 | 0.29 |
| 0 | 0.28 | 0.43 | 0.71 |
| Total | 0.50 | 0.50 | 1 |

This table reports the joint distribution of assigned and realized treatment in our sample. Assigned treatment equals 1 for individuals who won the lottery for Medicaid and realized treatment equals 1 for individuals who actually enrolled in Medicaid. It shows the compliance issue in our sample.

### 2.3.2 Medicaid expansion

The 2011 Affordable Care Act called for the expansion of Medicaid to cover people with incomes below $133 \%$ of the federal poverty level starting from 2014, compared to the $100 \%$ that was previously in place. While some states have adopted the Medicaid expansion, some states have yet to adopt the expansion. This allowed researchers to use difference-in-differences type of methods to evaluate the impact of the expansion in various contexts.

I take a different approach and use the method I proposed in Chapter 1 to quantify the impact of this expansion in terms of two welfare measures. In other words, I calculate the welfare difference between the following two policies:

$$
\begin{align*}
\delta^{*} & =\mathbb{1}\{\text { income } \leq 100 \%\},  \tag{2.6}\\
\delta & =\mathbb{1}\{\text { income } \leq 133 \%\} . \tag{2.7}
\end{align*}
$$

The first policy is to target people whose income level is up to $100 \%$ of the federal poverty level. The second policy is to target people whose income is up to $133 \%$ of the federal poverty level.

Health outcome Table 2.8 reports the estimated welfare gains in terms of health outcome. The random assignment is used as a treatment variable to compute the welfare gain from an intent-to-treat perspective. In other cases, the random assignment is used as an IV. Since the health outcome is a binary variable, I set $\underline{y}=0$ and $\bar{y}=1$.

Table 2.8: Welfare gains in terms of overall health

| Assumptions | lower bound | upper bound |
| :--- | :---: | :---: |
| intent-to-treat |  | 0.003 |
| worst-case | -0.089 | 0.057 |
|  | $(-0.093,-0.086)$ | $(0.054,0.061)$ |
| MTR | 0 | same as worst-case |
| IV-worst-case | -0.070 | 0.046 |
| IV-MTR | $(-0.073,-0.067)$ | $(0.044,0.049)$ |

This table reports the estimated welfare gains under various assumptions. The welfare is in terms of self-reported overall health.

From an intent-to-treat perspective, the welfare gain of Medicaid expansion in
terms of self-reported health is 0.003 . In other words, it is an increase in the probability of reporting one's health as "good", "very good", or "excellent." However, the welfare gain of enrolling in Medicaid is not point-identified. The worst-case bounds are (-0.089, 0.057). When we use the random assignment as an IV, we get tighter bounds of $(-0.070,0.046)$. As discussed before, the worst-case bounds always cover 0 . In this example, even with IV, the bounds cover 0 as well. However, if we are willing to assume that MTR holds, i.e., the self-reported health is always improved by having an insurance, the lower bound is 0 as we are expanding the treated population under this policy.

Finkelstein, Taubman, Wright, Bernstein, Gruber, Newhouse, Allen, Baicker, and Group (2012) find that insurance is associated with statistically significant improvements in self-reported health. They compute it from the intent-to-treat perspective and also compute the local average treatment effect (LATE) using enrollment as a treatment variable. Although not perfect, this finding favors the MTR assumption.

Health care utilization Table 2.9 reports the estimated welfare gains in terms of health care utilization. Again, the random assignment is used as a treatment variable to compute the welfare gain from an intent-to-treat perspective. In other cases, the random assignment is used as an IV. Since the health care utilization is a binary variable, I again set $\underline{y}=0$ and $\bar{y}=1$.

From an intent-to-treat perspective, the calculated welfare gain in terms of health care utilization is -0.0004 . However, the welfare gain of enrolling in Medicaid is not point-identified. The worst-case bounds are ( $-0.079,0.068$ ). When we use the random assignment as an IV, we get tighter bounds of (-0.063, 0.053). Again, even with IV, we get bounds that cover 0 in this case.

As before, the lower bound is 0 if we are willing to assume that MTR holds, i.e., the probability of having a primary care visit is increased by having an insurance.

Table 2.9: Welfare gains in terms of health care utilization

| Assumptions | lower bound | upper bound |
| :--- | :---: | :---: |
| intent-to-treat |  | -0.0004 |
| worst-case | -0.079 | 0.068 |
|  | $(-0.082,-0.075)$ | $(0.065,0.071)$ |
| MTR | 0 | same as worst-case |
| IV-worst-case | -0.063 | 0.053 |
|  | $(-0.066,-0.060)$ | $(0.051,0.056)$ |
| IV-MTR | 0 | same as IV-worst-case |

This table reports the estimated welfare gains under various assumptions. The outcome is whether the respondent had any primary care visits in the last 6 months.

This assumption is even more plausible here since we expect the health insurance to increase health care utilization by lowering the price of health care. It is also consistent with the findings in Finkelstein, Taubman, Wright, Bernstein, Gruber, Newhouse, Allen, Baicker, and Group (2012) - the LATE estimates suggest that insurance coverage is associated with an increase in the number of outpatient visits.

### 2.4 Conclusion

In this chapter, I use the method I propose in Chapter 1 and examine two applications in economics. First, I consider the problem of assigning individuals to a job training program. Second, I study the welfare impact of Medicaid expansion. In both cases, I use data from randomized experiments with imperfect compliance and calculate the welfare differences between various different policies of interest.

## Chapter 3

## Changing Preferences: An Experiment and Estimation of Market-Incentive Effects on Altruism (with Ching-to Albert Ma and Daniel Wiesen)

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Experimental Research (CLER) of the University of Cologne.

### 3.1 Introduction

Recent economic research has questioned the monolithic power of incentives and markets. Economists now legitimately question if more competition or high-powered incentives must result in more outputs or worker efforts. A multifaceted approach has been advanced. Economic agents' preferences may include more than utility from financial reward and disutility from cost or effort. In fact, economic agents may be fair minded, altruistic, and socially responsible, but may also be spiteful.

Clearly, social and individual preferences determine behaviors and market outcomes. The usual research methodology says that given multi-dimensional preferences, economists can write analytical and empirical models to study incentives and markets. A deeper question, of course, is what determines social preferences. There, economists often concede that anthropologists, sociologists, psychologists, and neuroscientists may have identified plausible factors such as climate, cultural-historical events, physiology, and genetics to explain preferences. But when these factors remain exogenous, the usual methodology remains valid.

In this chapter, we assess whether social preferences can be changed by markets and incentives, the key social institutions that economists study. Our focus is on altruism, market competition, and incentives. ${ }^{1}$ This chapter presents experimental evidence that altruistic preferences can be diminished by competition and altered by incentives. In other words, economic models that analyze altruism's effect on market and incentives must confront the possibility that markets and incentives themselves

[^7]may change altruism.
Our research proceeds in three steps. First, we use a structural model to decompose behavioral changes into preference effects and market-incentive effects. This is the key conceptual step. Behavioral outcomes are interactions between preferences and market-incentive institutions. ${ }^{2}$ Must altruistic preferences remain immutable when markets and incentives change? Our structural model allows altruistic preferences to be influenced by markets and incentives. Behavioral changes are then results of markets and incentives changing preferences as well as equilibria.

Second, we use a laboratory experiment in which incentives and market competition are exogenously varied for subjects. Identifying preference and market-incentive effects by real data is too daunting a task. A controlled environment offers a better chance. Our experimental framing is health care provision, and subjects are primed to be altruistic. They choose health care qualities which affect their own payoffs and which benefit patients through a transfer to a charity for the treatment of ophthalmic patients outside the laboratory. We also have taken care to isolate subjects, so such confounding factors as fairness, collusion, and spitefulness could be minimized. Each subject experiences different markets and incentive configurations. Our withinsubject design is appropriate because we claim that preferences change, not just that preferences are heterogenous (which could be identified by a between-subject design).

Third, we adapt the nonparametric econometric method by Guerre, Perrigne, and Vuong (2000) to estimate preference distributions. We estimate subjects' altruism distributions separately as subjects experience different markets and different incentive configurations. The nonparametric method does not restrict us to prespecified distribution classes, and is straightforward to implement.

We show that subjects become less altruistic when they have to compete against

[^8]others in a duopoly or in a quadropoly, compared to when they are monopolists. The flip side is that when subjects become monopolists, they become more altruistic. Our contribution can be likened to the classic Lucas critique in policy evaluations. Given preferences, equilibrium outcomes result from market-incentive institutions. This common view is inadequate: preferences are not given. The structure of an economic model consists of equilibrium decisions based on economic agents' preferences. When preferences vary systematically with changes in the market-incentive environment, rather than remaining exogenous, a change in the market-incentive environment will systematically alter the structure of the economic model.

For the theoretical model, we specify that a subject's preferences are given by a weighted average of patients' benefits from health care quality, and profits. By choosing a higher quality, the subject reduces profit, but raises patient benefits. A more altruistic subject puts a higher weight on patients' benefits. The tradeoff between benefits and profits depend on three experimental parameters: a subject's price (revenue) per patient, quality cost, and patient benefit.

A subject makes decisions in three markets: monopoly, duopoly, and quadropoly. Under monopoly a subject chooses the quality for the entire patient population. Under duopoly and quadropoly, subjects move simultaneously and each subject's market share depends on the entire profile of subjects' quality choices, according to a logistic demand function. A total of 361 subjects participated in experimental sessions in October 2017 and April 2018 at the University of Cologne. Within each of the three markets, we systematically vary the incentives using a $2 \times 2 \times 2$ factorial design. Price, cost, and patient benefit assume binary values for a total of eight incentive configurations. In total, each subject played 24 games.

Each basic game is one of incomplete information: a player's altruism is his own private information, but each is uncertain about another player's altruism. We assume
that the uncertainty on altruism is described by a distribution. It is this distribution that we would estimate, for each market and for each incentive configuration. The estimation is by means of symmetric Bayes-Nash equilibria.

Nonparametric estimation yield very different altruism distributions for the 24 games. The striking pattern is that for each incentive configuration, estimated altruism distributions exhibit lower means in duopoly relative to monopoly, and yet even lower means in quadropoly. Subjects have become less altruistic and value profits more when the market becomes more competitive. What is more striking, however, is that the observed equilibrium qualities are much higher in duopoly and quadropoly than monopoly. Although subjects have become less altruistic, the competition disciplinary force is stronger.

These results offer a deeper interpretation than the usual, reduced-form approach. If only behavioral results are considered, then markets and incentives are shown to raise qualities, so one would conclude that there is no crowding out. We reject the simplistic conclusion. In fact, quality changes result from two effects: preference changes, and market-incentive changes. The effects go in opposite directions. Markets reduce altruism, but also discipline subjects. In our experiment, the market-incentive effect is stronger than the preference-change effect. Together they produce the observed behavioral results. Our structural approach permits counter factual calculations. It also allows straightforward robustness checks.

It has not escaped our notice that the ultimate questions are: why has competition, according to our evidence, diminished altruism, and why has the competitive disciplinary effect turned out to be stronger? These questions, perhaps, strike a counterpoint to the usual exogenous assumptions for analysis of economic models. Recent advances in neuroscience have adopted a reductionist principle that all behaviors can be traced to electrochemical activities in the brain. We are neither in any position
to render an opinion nor did we manage to use brain scans to assess if competition triggered specific neural activities. However, we can speculate. When subjects play monopoly, they only have to consider a tradeoff between profits and patient benefits. When subjects play duopoly, they are presented with an additional concern: the competitor's quality choice. The tradeoff between profits and patient benefits now depends on what the rival subject would choose. Complexity has increased, and perhaps the higher cognitive demand has diluted the concern for patient benefits.

The plan of the chapter is as follows. The next subsection is a literature review. The model is set up in Section 3.2. The experimental design and sessions are described in Section 3.3. In Section 3.4, we present quality choice descriptive statistics, the nonparametric estimator, and then estimation results on altruism. We also perform nonparametric tests on the equality of the estimated altruism distributions. We end the section with some counterfactual quality estimations. Section 3.5 presents the reduced-form analysis. The last section draws some conclusion. Appendix A contains experiment materials. Appendix B contains robustness checks. We consider an alternate utility function, and a between-subject subsample.

### 3.1.1 Literature review

Our work is related to three strands in the literature. First, we relate to the growing body of work on the impact of markets on moral and prosocial behaviors. Evidence from laboratory experiments in such framing and contexts as altruistic motives, freeriding, and social responsibility indicates that competition reduces moral behavior.

Falk and Szech (2013) show that repeated interactions in bilateral and multilateral free-offer markets reduce morals compared to individual decisions. Subjects are more willing to accept a negative externality imposed on a third-party (a mouse getting killed) in markets. Using a consumer and firms in a laboratory experiment, Bartling, Weber, and Yao (2015) analyze socially responsible behavior in posted-price markets.

They find evidence for socially responsible behaviors in markets, but such behaviors are stronger in non-market contexts. In a follow-up study, Bartling, Valero, and Weber (2019) report similar patterns for different externalities. Kirchler, Huber, Stefan, and Sutter (2016) analyze how trading anonymity, involvement with the traded good, and punishment influence moral behavior in a double auction with negative externalities imposed on third parties (voiding measles vaccine donations). Building on Kirchler, Huber, Stefan, and Sutter (2016), Sutter, Huber, Kirchler, Stefan, and Walzl (2020) report that moral behaviors are consistent with lower trading volume in markets with negative externalities, but externalities do not affect market prices.

The literature aims to show that social preferences may be altered by the market structure. Shifts in preferences are typically inferred from observed behaviors. In single-person decision environments, this seems a natural inference. However, we consider a multi-person strategic interaction environment. Therefore, behaviors are equilibrium outcomes, which in turn depend on preferences and the market structure. Hence, our use of a structural model to identify changes in altruistic preferences is necessary to decompose behavioral changes into those due to preferences and strategic changes. Bartling, Weber, and Yao (2015) also employ a structural modelling approach. They estimate consumers' preferences by a conditional logit choice model. Whereas they show that the average buyer cares for a third-party's earnings, preferences are assumed to remain unchanged in different market treatments. However, in their setup, consumers simply make purchase decisions after firms have chosen products and prices, so do not engage in a strategic game against firms.

Further, the above studies use a between-subject design. Instead, we use a withinsubject design to identify preferences changes. Also, we examine a (regulated) market in which economic agents on the supply-side compete for market share by benefiting third parties. We do not let subjects do harm to third parties. ${ }^{3}$ Neither do we let

[^9]subjects receive feedbacks about their decisions, so learning, reputation, and peer effects are well controlled; for further discussion, see Bartling, Weber, and Yao (2015) and Breyer and Weimann (2015). Furthermore, we change competition and incentives one at a time, so the confounding effects of multiple changes between manipulations can be avoided.

Our study relates to the literature on prosocial behavior, incentives, and crowdingout. Ultimatum, dictator, public good, trust, and gift-exchange game experiments analyze behavioral changes due to incentives; see Bowles and Polania-Reyes (2012) for a summary. Economic incentives are often found to reduce pro-social behaviors. Experimental evidence tends to confirm crowding out; see, for example, Falk and Kosfeld (2006), Gneezy and Rustichini (2000a,b), and Mellström and Johannesson (2008). Our work points to the inadequacy of identifying crowding out only in terms of outcomes. Incentive schemes are disciplinary, even when they may erode social motives. The missing link is that market-incentive mechanisms and social motives pull in different directions, and it is an empirical matter which is stronger.

Finally, our nonparametric estimation relates to the literature on structurally estimating preferences. This literature has been on measuring inequity aversion and reciprocity (e.g., Charness and Rabin (2002); Bellemare, Kröger, and Van Soest (2008)), and altruism (e.g., Andreoni (1989); Andreoni and Miller (2002); Fisman, Kariv, and Markovits (2007)); see DellaVigna (2018) for a summary. Using data from field experiments, a few chapters structurally infer social preferences to identify differences between charitable giving and worker effort; see DellaVigna, List, and Malmendier (2012) and DellaVigna, List, Malmendier, and Rao (2016). We, however, use a nonparametric estimator developed by Guerre, Perrigne, and Vuong (2000) that has originated from estimating Bayes-Nash equilibria in first-price auctions. We have used behavior is markets for public services (e.g., Besley and Ghatak (2005)) and credence goods (e.g., Dulleck and Kerschbamer (2006); Dulleck, Kerschbamer, and Sutter (2011)), and health care (e.g., Arrow (1963)).
the monotonicity of qualities in altruism for identification; this is similar to identification by the monotonicity of auction bids in valuations in Guerre, Perrigne, and Vuong (2000).

### 3.2 A model of altruism and competition

The experiment frames subjects to provide medical services at some quality to a set of patients. ${ }^{4}$ We study three market games: monopoly, duopoly, and quadropoly. The monopoly game is a single-person decision problem, and the simultaneous-move duopoly and quadropoly games are strategic problems.

### 3.2.1 Demand

In each market, there are 100 patients. Under monopoly, each subject simply makes the quality decision, $q$ between 0 and 10 , for all the 100 patients. In duopoly and quadropoly, subjects choose qualities simultaneously. Then the subjects' quality profile determines each subject's market share according to a logistic demand system. For duopoly, let $q_{1}$ and $q_{2}$ be qualities chosen by subject 1 and subject 2 . The numbers of patients for subjects 1 and 2 are, respectively,

$$
\begin{equation*}
\frac{100 \exp \left(b q_{1}\right)}{\exp \left(b q_{1}\right)+\exp \left(b q_{2}\right)} \quad \text { and } \quad \frac{100 \exp \left(b q_{2}\right)}{\exp \left(b q_{1}\right)+\exp \left(b q_{2}\right)} \tag{3.1}
\end{equation*}
$$

where $b>0$ is a patient-benefit parameter. For quadropoly, let $q_{1}, q_{2}, q_{3}$, and $q_{4}$ denote the four subjects' quality choices. Subject $i$ who chooses quality $q_{i}$ will have

$$
\begin{equation*}
\frac{100 \exp \left(b q_{i}\right)}{\exp \left(b q_{1}\right)+\exp \left(b q_{2}\right)+\exp \left(b q_{3}\right)+\exp \left(b q_{4}\right)} \tag{3.2}
\end{equation*}
$$

[^10]patients. The logistic demand guarantees that each subject gets some patients under any quality profile, and is commonly used for discrete-choice situations when consumers' utilities may be subject to Type I Extreme Value disturbances.

### 3.2.2 Quality choices and preferences

A subject receives a fixed payment $p>0$ for each patient that he treats. For the theoretical model, a subject's quality choice is a continuous variable between 0 and 10 (although in the experiment we set the possible qualities to be integers between 0 and 10, a total of 11 choices). The subject bears the per-patient quality cost at $c q^{2}$ when he provides medical service at quality $q$, where $c>0$ is a cost parameter. Medical service at quality $q$ gives a benefit $b q$ to a patient. We call the environment defined by the three parameters, payment $p$, $\operatorname{cost} c$, and patient benefit $b$, an incentive configuration.

Our health care framing primes an experiment subject for an altruistic motive when qualities are chosen. We assume that a subject's preferences are represented by $\alpha b q+U\left(p-c q^{2}\right)$, for some parameter $\alpha$ and an increasing and concave function $U$, so preferences are linear combinations of the patient benefit $b q$, and the utility of his own profit $U\left(p-c q^{2}\right)$. We maintain the assumption that a subject earns altruistic utility from his own patients. (We will discuss "global" altruism, in which a subject values all patient benefits, in the next Subsection.) Framing and priming affect subjects differently, so we assume that the preference weight on patient benefit, $\alpha$, is a random variable on an interval $[\underline{\alpha}, \bar{\alpha}] \subset \mathbb{R}$ with some distribution.

### 3.2.3 Monopoly, duopoly and quadropoly

In monopoly, each subject simply chooses a quality for his 100 patients. If a subject's altruism parameter is $\alpha$ and he chooses quality $q$, the subject's per-patient payoff is $\alpha b q+U\left(p-c q^{2}\right)$. A profit-maximizing subject (whose $\alpha$ is set at 0 ) chooses $q=0$,
whereas a subject who only cares about patient benefit chooses the maximum quality, which is 10 . Otherwise, a subject's "interior" optimal quality is given by the firstorder condition:

$$
\begin{equation*}
\alpha b-U^{\prime}\left(p-c q^{2}\right) \times 2 c q=0 \tag{3.3}
\end{equation*}
$$

In monopoly, altruism is the only reason behind a subject choosing a strictly positive quality. In fact, the first-order condition (3.3) defines a monotone relationship between $\alpha$ and the optimal quality:

$$
\begin{equation*}
\alpha=\frac{2 c q}{b} U^{\prime}\left(p-c q^{2}\right) . \tag{3.4}
\end{equation*}
$$

A more altruistic subject is willing to forgo more profit for a higher quality for patients. Given a utility function $U$, equation (3.4) allows us to infer the value of $\alpha$ from subjects' quality choices.

The experiment subjects also play the duopoly and quadropoly games. We will lay out all the details in duopoly, but we will be rather succinct in quadropoly. In duopoly, two subjects are randomly paired. They simultaneously choose qualities, say $q_{1}$ and $q_{2}$, which result in market shares in (3.1). The subjects' payoffs are

$$
\begin{aligned}
& {\left[\alpha_{1} b q_{1}+U\left(p-c q_{1}^{2}\right)\right] \times \frac{100 \exp \left(b q_{1}\right)}{\exp \left(b q_{1}\right)+\exp \left(b q_{2}\right)}} \\
& \quad \text { and } \\
& {\left[\alpha_{2} b q_{2}+U\left(p-c q_{2}^{2}\right)\right] \times \frac{100 \exp \left(b q_{2}\right)}{\exp \left(b q_{1}\right)+\exp \left(b q_{2}\right)},}
\end{aligned}
$$

where $\alpha_{1}$ and $\alpha_{2}$ are the subjects' altruism parameters.
We model duopoly as a Bayesian game. We assume that each subject' altruism parameter, $\alpha$, is drawn independently from a random variable with distribution $F$ and density $f$ on support $[\underline{\alpha}, \bar{\alpha}]$. Each subject observes his own altruism parameter, but not an opponent's altruism parameter. The uncertainty on the altruism parameter
$\alpha$ is the basis for the Bayesian perspective, and stems from framing having different effects on different subjects.

A subject's strategy in the duopoly game is a function that maps the altruism parameter $\alpha$ to a quality, say, $q:[\underline{\alpha}, \bar{\alpha}] \rightarrow[0,10]$. If subject 1 has altruism parameter $\alpha_{1}$ and chooses $q_{1}$ when the rival subject 2 follows a strategy $q^{\prime}:[\underline{\alpha}, \bar{\alpha}] \rightarrow[0,10]$, subject 1's expected utility is

$$
\begin{align*}
E U\left(q_{1} ; q^{\prime}\right) & =\int_{\underline{\alpha}}^{\bar{\alpha}}\left\{\left[\alpha_{1} b q_{1}+U\left(p-c q_{1}^{2}\right)\right]\left[\frac{100 \exp \left(b q_{1}\right)}{\exp \left(b q_{1}\right)+\exp \left(b q^{\prime}(x)\right)}\right]\right\} \mathrm{d} F(x) \\
& =\left[\alpha_{1} b q_{1}+U\left(p-c q_{1}^{2}\right)\right] \times \int_{\underline{\alpha}}^{\bar{\alpha}}\left[\frac{100 \exp \left(b q_{1}\right)}{\exp \left(b q_{1}\right)+\exp \left(b q^{\prime}(x)\right)}\right] \mathrm{d} F(x) \tag{3.5}
\end{align*}
$$

We assume that a subject does not earn any utility from patient benefits provided by a rival subject. The expression in (3.5) only concerns those patients the subject serves. An alternate form of "global" altruism, which includes patient benefits the rival subject provides is outside our current consideration. A first reason is tractability; Bayes-Nash equilibria will be more complicated, and the estimation of the equilibria would become difficult. Second, global altruism will inevitably involve some concern about free riding, and some notion about peer effect of quality provision, which we have chosen to suppress. The suppression of concern other than private altruism will be discussed in the experimental design section.

The market share is defined by

$$
S\left(q_{1} ; q^{\prime}\right) \equiv \frac{\exp \left(b q_{1}\right)}{\exp \left(b q_{1}\right)+\exp \left(b q^{\prime}\right)},
$$

so we can rewrite the expected utility in (3.5) as

$$
\begin{equation*}
E U\left(q_{1} ; q^{\prime}\right)=\left[\alpha_{1} b q_{1}+U\left(p-c q_{1}^{2}\right)\right] \times \int_{\underline{\alpha}}^{\bar{\alpha}} 100 S\left(q_{1} ; q^{\prime}(x)\right) \mathrm{d} F(x) . \tag{3.6}
\end{equation*}
$$

The market share term is the difference between monopoly and duopoly. A subject choosing a higher quality earns a higher market share:

$$
\frac{\mathrm{d} S\left(q_{1} ; q^{\prime}\right)}{\mathrm{d} q_{1}}=b S\left(q_{1} ; q^{\prime}\right)\left[1-S\left(q_{1} ; q^{\prime}\right)\right]>0
$$

In duopoly, even a purely profit-maximizing subject $(\alpha=0)$ has an incentive to offer quality because a higher quality gains market share which generates profits.

For each value of $\alpha_{1} \in[\underline{\alpha}, \bar{\alpha}]$, we let

$$
\begin{equation*}
q\left(\alpha_{1} ; q^{\prime}\right)=\arg \max _{q_{1}}\left[\alpha_{1} b q_{1}+U\left(p-c q_{1}^{2}\right)\right] \times \int_{\underline{\alpha}}^{\bar{\alpha}} 100 S\left(q_{1} ; q^{\prime}(x)\right) \mathrm{d} F(x) \tag{3.7}
\end{equation*}
$$

be subject 1's best response against the rival's strategy $q^{\prime}(\alpha):[\underline{\alpha}, \bar{\alpha}] \rightarrow[0,10]$. A subject's optimal quality choice is still a tradeoff between profit and patient benefit. However, a subject's payoff depends on what he believes about his rival subject's qualities, which are chosen according to the strategy $q^{\prime}$. A symmetric Bayes-Nash equilibrium strategy specifies a subject's quality choice for each value of the altruism parameter that maximizes the subject's expected utility, given that the rival subject uses the same strategy.

Definition 3.2.1 (Duopoly Bayes-Nash Equilibrium). The strategy $q^{*}:[\underline{\alpha}, \bar{\alpha}] \rightarrow$ $[0,10]$ is a symmetric Bayes-Nash equilibrium, if, at each $\alpha \in[\underline{\alpha}, \bar{\alpha}]$,

$$
\begin{equation*}
q^{*}(\alpha)=\arg \max _{q}\left[\alpha b q+U\left(p-c q^{2}\right)\right] \times \int_{\underline{\alpha}}^{\bar{\alpha}} 100 S\left(q ; q^{*}(x)\right) d F(x) \tag{3.8}
\end{equation*}
$$

The usual characterization of an equilibrium is by means of the first-order condition for the maximization of (3.6) or the best response in (3.7). Given a rival's strategy $q^{\prime}$, for the maximization of expected utility in (3.6), we obtain the first-order
derivative with respect to $q_{1}$ :

$$
\begin{align*}
& \frac{\partial E U\left(q_{1} ; q^{\prime}\right)}{\partial q_{1}}=\left[\alpha_{1} b-2 c q_{1} U^{\prime}\left(p-c q_{1}^{2}\right)\right] \times \int_{\underline{\alpha}}^{\bar{\alpha}} 100 S\left(q_{1} ; q^{\prime}(x)\right) \mathrm{d} F(x) \\
+ & {\left[\alpha_{1} b q_{1}+U\left(p-c q_{1}^{2}\right)\right] \times \int_{\underline{\alpha}}^{\bar{\alpha}} 100 b S\left(q_{1} ; q^{\prime}(x)\right)\left[1-S\left(q_{1} ; q^{\prime}(x)\right)\right] \mathrm{d} F(x) . } \tag{3.9}
\end{align*}
$$

We assume that the expected utility in (3.6) is quasi-concave for the incentive configurations under consideration. Hence, by setting the first-order derivative to zero, we obtain the implicit function that defines the best response at $\alpha$.

To characterize the symmetric Bayes-Nash equilibrium $q^{*}:[\underline{\alpha}, \bar{\alpha}] \rightarrow[0,10]$, we note that at the equilibrium, each subject has the same first-order condition. The equilibrium $q^{*}$ therefore is defined by the equation from setting (3.9) to 0 at each $\alpha \in[\underline{\alpha}, \bar{\alpha}]$ with $q^{\prime}$ set to $q^{*}:$

$$
\begin{gather*}
{\left[\alpha b-2 c q^{*}(\alpha) U^{\prime}\left(p-c q^{*}(\alpha)^{2}\right)\right] \times \int_{\underline{\alpha}}^{\bar{\alpha}} 100 S\left(q^{*}(\alpha) ; q^{*}(x)\right) \mathrm{d} F(x)}  \tag{3.10}\\
+\left[\alpha b q^{*}(\alpha)+U\left(p-c q^{*}(\alpha)^{2}\right)\right] \times \int_{\underline{\alpha}}^{\bar{\alpha}} 100 b S\left(q^{*}(\alpha) ; q^{*}(x)\right)\left[1-S\left(q^{*}(\alpha) ; q^{*}(x)\right)\right] \mathrm{d} F(x)=0 .
\end{gather*}
$$

Being the solution of an integral equation, a symmetric Bayes-Nash equilibrium is difficult to compute, even for simple functional forms of the utility $U$ and distribution $F$. Fortunately, we do not have to rely on this computation. In fact, what makes our model operational is the following.

Lemma 3.2.1. Equilibrium strategy $q^{*}:[\underline{\alpha}, \bar{\alpha}] \rightarrow[0,10]$ is monotone increasing in $\alpha$.

Proof of Lemma 3.2.1: Using the first-order derivative of $E U$ with respect to
$q_{1}$ in (3.9), we further differentiate this with respect to $\alpha_{1}$ to obtain

$$
\begin{aligned}
\frac{\partial^{2} E U\left(q_{1} ; q^{\prime}\right)}{\partial \alpha_{1} \partial q_{1}}= & b \int_{\underline{\alpha}}^{\bar{\alpha}} 100 S\left(q_{1} ; q^{\prime}(x)\right) \mathrm{d} F(x) \\
& +b q_{1} \int_{\underline{\alpha}}^{\bar{\alpha}} 100 b S\left(q_{1} ; q^{\prime}(x)\right)\left[1-S\left(q_{1} ; q^{\prime}(x)\right)\right] \mathrm{d} F(x)>0
\end{aligned}
$$

By assumption $E U$ is quasi-concave in $q_{1}$, so as $\alpha_{1}$ increases, the optimal quality increases. This is true for any given strategy $q^{\prime}$, so remains valid at the equilibrium $q^{*}$.

Because $\alpha$ is a random variable, the equilibrium strategy $q^{*}(\alpha)$ is also a random variable. An equilibrium duopoly is the pair of qualities specified by the equilibrium strategy, $\left(q^{*}\left(\alpha_{1}\right), q^{*}\left(\alpha_{2}\right)\right)$, for two independent realizations of $\alpha$, namely $\alpha_{1}$ for the first subject, and $\alpha_{2}$ for the second subject.

Remark 3.2.1 (Duopoly Equilibrium Quality Distribution). The Bayes-Nash equilibrium $q^{*}$ induces a joint distribution of the two subjects' equilibrium qualities on $[0,10] \times[0,10]$. By symmetry and independence, the marginal density is the one induced by the equilibrium strategy $q^{*}$. Denoting this marginal distribution by $G^{*}$ : $[0,10] \rightarrow[0,1]$, we conclude that for $\widetilde{q} \in[0,10], G^{*}(\widetilde{q})=F(\widetilde{\alpha})$, where $q^{*}(\widetilde{\alpha})=\widetilde{q}$.

The actual play of the duopoly are realizations of $G^{*}$. By the monotonicity of the equilibrium $q^{*}$, the distribution $F$ of $\alpha$ and the equilibrium quality distribution $G^{*}$ are isomorphic. Whereas we have no data on $F$, we do have data on qualities from equilibrium play. This is the key to the estimation of the altruism distribution $F$ under duopoly, and Subsection 3.4.2 will present the estimation of $G^{*}$ by the empirical quality distribution.

Next, we discuss quadropoly. There are now four subjects, and the demands are in (3.2). Otherwise, there is not much conceptual difference between duopoly and quadropoly. The definition of a symmetric Bayes-Nash equilibrium has exactly the
same form. If subject $i$ chooses quality $q_{i}$, his market share now is

$$
S\left(q_{i} ; q_{-i}\right)=\frac{\exp \left(b q_{i}\right)}{\sum_{j=1}^{4} \exp \left(b q_{j}\right)}
$$

where we use $q_{-i}$ to denote the quality vector $\left(q_{1}, q_{2}, q_{3}, q_{4}\right)$ with the $i^{\text {th }}$ element omitted. Given strategies $q_{j}, j=1,2,3,4, j \neq i$, if subject $i$ chooses quality $q_{i}$ his expected utility is

$$
\left[\alpha_{i} b q_{i}+U\left(p-c q_{i}^{2}\right)\right] \times \iiint 100 S\left(q_{i} ; q_{-i}\left(\alpha_{-i}\right)\right) \prod_{j=1, j \neq i}^{4} \mathrm{~d} K\left(\alpha_{j}\right)
$$

where the notation $q_{-i}\left(\alpha_{-i}\right)$ is a short hand for $\left(q_{j}\left(\alpha_{j}\right), j=1,2,3,4, j \neq i\right)$, and $K$ is the distribution of $\alpha$ in quadropoly.

Definition 3.2.2 (Quadropoly Bayes-Nash Equilibrium). The strategy $q^{* *}(\alpha)$ is a symmetric Bayes-Nash equilibrium, if, at each $\alpha \in[\underline{\alpha}, \bar{\alpha}]$,

$$
\begin{align*}
q^{* *}(\alpha)= & \arg \max _{q}\left[\alpha b q+U\left(p-c q^{2}\right)\right] \\
& \times \iiint\left\{100 S\left(q ; q_{-i}^{* *}\left(\alpha_{-i}\right)\right)\right\} \prod_{j=1, j \neq i}^{4} d K\left(\alpha_{j}\right) . \tag{3.11}
\end{align*}
$$

We can use the first-order condition to characterize the equilibrium strategy $q^{* *}$. It is straightforward to verify the same monotonicity property.

Lemma 3.2.2. Equilibrium strategy $q^{* *}:[\underline{\alpha}, \bar{\alpha}] \rightarrow[0,10]$ is monotone increasing in $\alpha$.

Remark 3.2.2 (Quadropoly Equilibrium Quality Distribution). The Bayes-Nash equilibrium $q^{* *}$ induces a joint distribution of the four subjects' equilibrium qualities on $[0,10]^{4}$. By symmetry and independence, the marginal density is the one induced by the equilibrium strategy $q^{* *}$. We denote this marginal distribution by $L^{* *}:[0,10] \rightarrow[0,1]$.

Notice that we have used the notation $F$ to denote the altruism distribution in duopoly, but we have used a different notation $K$ for that in quadropoly. Although we
have the same set of subjects in 3 markets and 8 incentive configurations, we do allow altruism distributions to vary according to markets and incentive configurations. We now turn to the experiment.

### 3.3 The experiment

### 3.3.1 Design

The experimental design implements the theoretical model just described. Role playing as physicians, subjects decide on the quality of health care for a set of hypothetical patients. ${ }^{5}$ Each subject chooses a medical-service quality $q$ from a finite set $\{0,1,2, \ldots, 10\} .^{6}$ Three parameters determine payoffs. These are the capitation payment to the physician $p$, the quality cost parameter $c$, and the patient benefit parameter, $b$. The subject bears the quality cost, so if he chooses a quality $q$, his profit becomes $p-c q^{2}$, whereas the patient benefit is $b q$, exactly the same as in the theoretical model.

We use a $2 \times 2 \times 2$ factorial design to vary each of the $p, c$, and $b$ parameters systematically. The capitation payment $p$ may be low or high, set at 10 and 15 , respectively. The cost parameter $c$ can be either 0.075 or 0.1 , whereas the benefit parameter $b$ can be either 0.5 or 1 . All monetary amounts were in terms of the experimental currency, Taler, which was later converted to Euro at the rate of 100:1. A full set of parameters can be found in Table 3.12 in Appendix B. We call a game with a profile of price-cost-benefit parameters an incentive configuration. The $2 \times 2 \times 2$ variations set up a total of 8 incentive configurations. There are 3 markets: monopoly,

[^11]duopoly, and quadropoly. Each subject plays 24 games in the entire experiment: 8 incentive configurations by 3 markets.

The experiment uses a within-subject design. Subjects experience different markets and incentive configurations, and we aim to investigate how subjects' quality choices and preferences change according to their experiences. In the actual implementation, subjects played all 8 incentive-configuration games in one market, and then moved onto the next market. Subjects were not informed of the market up until they were to play the 8 incentive-configuration games in that market. ${ }^{7}$

There are 6 different ways to order the three markets, displayed in Table 3.1. For example, in "3 (D-Q-M)" a subject plays the duopoly game first, followed by quadropoly, and finally monopoly. We roughly assigned about $1 / 6$ of the subject population to each of the 6 orders. The last column in Table 3.1 lists the number of subjects who participated in each order. We randomize the order in which the 8 incentive configurations are presented to subjects. In each market, each subject plays the 8 games in the following order: 1 st, $(p=10, c=0.1, b=1) ; 2 \mathrm{nd},(p=10$, $c=0.075, b=1) ; 3 \mathrm{rd},(p=15, c=0.1, b=0.5) ; 4 \mathrm{th},(p=15, c=0.1, b=1) ; 5$ th, $(p=10, c=0.1, b=0.5) ; 6$ th, $(p=10, c=0.075, b=0.5) ; 7$ th $(p=15, c=0.075$, $b=1)$ and 8 th, $(p=15, c=0.075, b=0.5)$.

We used the common "random-choice" payment method to determine profits and patient benefits. One of the 8 incentive-configuration games in each market would be chosen randomly for determining the subject's profit and the patient benefits. The random-choice payment method was implemented for each subject independently.

A subject never learns others' decisions for any of the 8 incentive-configuration games in a market. However, at the end of one market session, each subject is given a summary information of actual demands, profits, and patient benefits, aggre-

[^12]Table 3.1: Market orders in the experiment

|  |  | Order of markets |
| :---: | :---: | :---: | | Number |
| :---: |
| of subjects |

gated over the 8 games. In duopoly and quadropoly, subjects are randomly paired or grouped. When subjects are done with one market, say duopoly, the match will be dissolved. Then subjects will be randomly matched for the next market, say quadropoly. Subjects do play a normal form game against others randomly drawn from a population.

Our design rules out repeated plays, learning, and reputation. We have thought about the design tradeoff. On the one hand, we would like to keep altruism as the main frame, and would like to avoid issues about norms and collusions. On the other hand, we would have to face the possibility that subjects having to learn to play a Bayes-Nash equilibrium. In the end, we have come down with a design that would rely on subjects playing a Bayes-Nash equilibrium with preferences governed by altruism. This explains our suppressing information of subjects' play and outcomes. Our approach also gives supports about the rejection of global altruism. Subjects do not have information about patient benefits other than those patients he has chosen benefits for. We have maintained the altruism frame throughout. It is inappropriate to introduce a control that eliminates the patient benefits, or to make the benefits independent of subjects' quality choices. ${ }^{8}$

[^13]We do want to find out if subjects' preferences change according to markets and incentive configurations. Randomly assigning subjects to play different market and incentive-configuration games would identify differences, not changes. However, we can use a subsample for a between-subject design. We construct this subsample by taking data from a subject's experiences in the market he or she first participates. Given that we have 361 subjects, a between-subject design would put only about 120 subjects in one market, and each subject would then play only 8 games. The betweensubject subsample serves as a comparison with the main within-subject design. The analysis is in Appendix B. Broadly, the results are consistent with the complete sample for the within-subject design.

Although there are no real patients, the health benefits accrued in the laboratory are converted into monetary transfers to a charity dedicated to providing surgeries for ophthalmic patients. The patient benefit is thus made salient. A subject's consideration of patients' benefit from costly quality choices have real empirical and health-related consequences.

### 3.3.2 Experimental sessions

Experimental sessions were carried out in October 2017 and in April 2018, at the Cologne Laboratory for Experimental Research of the University of Cologne. Subjects in the experiment were mostly students from the University of Cologne, Germany. Participants were invited via the ORSEE platform (Greiner (2015)). In total, 361 subjects participated in the experiment. ${ }^{9}$ Subjects on average were about 24 years old, and $55 \%$ of them were female. Among the subjects who were students, 131
control or variant. Besides, we would not be able to control what subjects would think about what qualities were doing.
${ }^{9}$ We dropped three subjects who did not complete their last, monopoly sessions due to technical problems (one subject in condition 3 (D-Q-M), and two in condition 5 (Q-D-M)). However, these three subjects did interact with other subjects before they played their last monopoly session. We have kept data of others who played against these three subjects in duopoly and quadropoly.
were in law and social sciences, 22 in medicine, 42 in arts and humanities, 49 in mathematics and natural sciences, 35 in theology. There were 21 in other disciplines or non-students; 61 subjects did not provide their faculty information.

The experiment was programmed in zTree (Fischbacher (2007)). Upon arrival, subjects were randomly assigned to cubicles. Initial instructions informed subjects that the experiment consisted of three parts. Detailed instructions of each part would only be given at the start of that part. Each part corresponded to one of the three markets (monopoly, duopoly, and quadropoly). Participants had adequate time to read the instructions. The instructions can be found in Appendix A.1. Participants were allowed to ask clarifying questions, which were answered in private. For each market, subjects needed to answer several control questions. Subjects should understand the price, cost, and benefit parameters, and how quality choices might affect demands. Each subject must answer all control questions correctly to ensure an adequate understanding before the start of each part of the experiment. The control questions can be found in Appendix A.2.

When making a decision, each subject was informed of the incentive-configuration parameters, as well as profits and the patient benefits as functions of the quality that can be one in $\{0,1,2 \ldots, 10\}$. In monopoly, each subject had 100 patients. In duopoly and quadropoly, a subject had a logistic demand which depended on the quality profile of matched subjects. The zTree program provided a calculator, which allowed subjects to practice inputting own and other players' qualities to calculate the resultant demands (number of patients), profits, and patient benefits for all players. A screen shot of the calculator is in Appendix A.3. After subjects played the 8 incentive-configuration games in a market, they were informed of their and their paired subject's or subjects' total demands (number of patients), and total patient benefits in the 8 games. Data about individual games in each incentive configuration
were not given. Our design gets each of 361 subjects to play 24 games. We have taken steps to guard against "experimenter demand effects" (see, for example, Charness, Gneezy, and Kuhn (2012) by not telling subjects all three markets in advance.

One subject was randomly chosen to be a monitor. After the experiment, the monitor verified that a money order equal to the total patient benefit was issued by the Finance Department of the University of Cologne. The money order was payable to an organization, Christoffel Blindenmission, which supports ophthalmologists performing cataract surgeries in a hospital in Masvingo, Zimbabwe. The money order was sealed in an envelope, and the monitor and an experiment assistant then deposited the envelope in the nearest mailbox. The monitor was paid an additional $€ 5$. Subjects were told in advance that the experimental patient benefits would be for real patients, but not for those in a developing country to avoid motives of compassion and rather to remain in a health context. A similar procedure for making patient benefits meaningful to subjects has been applied by, for example, Hennig-Schmidt, Selten, and Wiesen (2011), Kesternich, Schumacher, and Winter (2015), and Brosig-Koch, Hennig-Schmidt, Kairies-Schwarz, and Wiesen (2017).

Sessions lasted, on average, for about 90 minutes, and subjects earned, on average, about $€ 14.20$ ( $€ 18.20$ including show-up fee). The average benefit per patient was about €8.10. In total, €2,923.60 were transferred to the Christoffel Blindenmission. Average costs for a cataract operation for adults are about $€ 30$, so our experiment supported about 100 surgeries. ${ }^{10}$

[^14]
### 3.4 Estimation of altruism distributions from experimental data

We first present data of subjects' quality choices. Then we describe how we estimate structurally the $\alpha$ altruism distribution for each market and in each incentive configuration.

### 3.4.1 Descriptive statistics on subjects' quality choices

Table 3.2 presents some summary statistics of the 361 subjects' quality choices in the 8 incentive-configuration games in the 3 markets. Clearly, subjects chose higher qualities in duopoly and quadropoly than in monopoly, and the standard deviations of subjects' quality choices were also much smaller. Raising the intensity of competition from duopoly to quadropoly increases qualities only slightly more. Within a market, quality variations between the 8 incentive-configuration games seem quite modest.

Table 3.2: Means and standard deviations of subjects' quality choices

| Incentive configurations | Monopoly |  | Duopoly |  | Quadropoly |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | mean | SD | mean | SD |  | SD |
| ( $p=10, c=0.075, b=0.5)$ | 4.17 | 2.99 | 7.75 | 1.58 | 8.26 | 1.40 |
| ( $p=10, c=0.075, b=1$ ) | 4.15 | 2.99 | 7.98 | 1.59 | 8.31 | 1.56 |
| $(p=10, c=0.1, b=0.5)$ | 3.79 | 2.79 | 6.94 | 1.35 | 7.34 | 1.34 |
| $(p=10, c=0.1, b=1)$ | 3.73 | 2.80 | 7.09 | 1.52 | 7.46 | 1.34 |
| $(p=15, c=0.075, b=0.5)$ | 4.82 | 3.43 | 8.82 | 1.53 | 9.09 | 1.32 |
| $(p=15, c=0.075, b=1)$ | 4.83 | 3.41 | 8.98 | 1.60 | 9.15 | 1.43 |
| $(p=15, c=0.1, b=0.5)$ | 4.51 | 3.27 | 8.19 | 1.63 | 8.55 | 1.47 |
| $(p=15, c=0.1, b=1)$ | 4.44 | 3.19 | 8.40 | 1.62 | 8.65 | 1.61 |
| Total | 4.31 | 3.14 | 8.02 | 1.70 | 8.35 | 1.57 |

For each of the 24 games, we draw the quality histograms; they are in Figures $3 \cdot 1$ to $3 \cdot 3$, and the actual frequency of each quality between 0 and 10 is written at the top of each vertical bar. These frequencies will be used for estimating subjects' altruism parameter.

Figure 3•1: Quality histograms in monopoly


Figure 3.2: Quality histograms in duopoly


Figure 3•3: Quality histograms in quadropoly


Clearly, the 24 histograms show higher qualities in duopoly and quadropoly than monopoly. Nevertheless, the difference between duopoly and quadropoly does not appear to be very significant. Quality frequencies are needed for the estimation of altruism parameters, to which we now turn.

### 3.4.2 Nonparametric estimation of altruism distribution by Bayes-Nash equilibria

We adapt a nonparametric estimation method developed by Guerre, Perrigne, and Vuong (2000) (abbreviated to GPV) for first-price auctions. We use duopoly to illustrate the adaptation. First, from the equilibrium strategy $q^{*}$ in (3.10), we invert and obtain $\alpha$ in terms of the equilibrium quality $q^{*}(\alpha)$, the utility function $U$, and incentive parameters:

$$
\alpha=\frac{\left\{\begin{array}{c}
2 c q^{*}(\alpha) U^{\prime}\left(p-c q^{*}(\alpha)^{2}\right) \int_{\underline{\alpha}}^{\bar{\alpha}} S\left(q^{*}(\alpha) ; q^{*}(x)\right) \mathrm{d} F(x)  \tag{3.12}\\
-U\left(p-c q^{*}(\alpha)^{2}\right) \times \int_{\underline{\alpha}}^{\bar{\alpha}} b S\left(q^{*}(\alpha) ; q^{*}(x)\right)\left[1-S\left(q^{*}(\alpha) ; q^{*}(x)\right)\right] \mathrm{d} F(x)
\end{array}\right\}}{\left\{\begin{array}{c}
b \int_{\underline{\alpha}}^{\bar{\alpha}} S\left(q^{*}(\alpha) ; q^{*}(x)\right) \mathrm{d} F(x) \\
+b q^{*}(\alpha) \int_{\underline{\alpha}}^{\bar{\alpha}} b S\left(q^{*}(\alpha) ; q^{*}(x)\right)\left[1-S\left(q^{*}(\alpha) ; q^{*}(x)\right)\right] \mathrm{d} F(x)
\end{array}\right\}} .
$$

Given an equilibrium $q^{*}$, the uncertainty concerning a rival subject's altruism is equivalent to the uncertainty of the rival's quality choices. From Remark 3.2.1, we can replace the altruism distribution $F$ by the equilibrium quality distribution $G^{*}$. Then, using $q$ to denote the equilibrium quality chosen by the subject with altruism parameter $\alpha$, we rewrite (3.12) as

$$
\begin{align*}
\alpha= & \left(2 c q U^{\prime}\left(p-c q^{2}\right) \int_{0}^{10} S(q ; x) \mathrm{d} G^{*}(x)-U\left(p-c q^{2}\right) \times \int_{0}^{10} b S(q ; x)[1-S(q ; x)] \mathrm{d} G^{*}(x)\right) / \\
& \left(b \int_{0}^{10} S(q ; x) \mathrm{d} G^{*}(x)+b q \int_{0}^{10} b S(q ; x)[1-S(q ; x)] \mathrm{d} G^{*}(x)\right) \tag{3.13}
\end{align*}
$$

This says that given an equilibrium $q^{*}$, we can use the equilibrium quality distri-
bution $G^{*}$ to express a subject's altruism parameter $\alpha$ in terms of his quality choice $q$. We estimate the $\alpha$ distribution by recovering their values from subjects' quality choices. The estimated $\alpha$ is a nonlinear map of the chosen quality $q$, and the equilibrium quality distribution $G^{*}$, given the game's parameters.

The argument suggests that we adapt the GPV two-step method as follows. In Step 1, the densities of equilibrium quality distribution $G^{*}$ are estimated by the empirical quality densities in each market-incentive-configuration constellation. Let $\widehat{g}(x)$ denote the empirical quality densities; it is the fraction of subjects (out of the total of 361 ) who have chosen quality $x=0,1, \ldots, 10$. We use $\widehat{g}(x)$ to estimate the $G^{*}$ 's densities. The empirical densities of the 24 games are those in Figures $3 \cdot 1$ to $3 \cdot 3$.

The term $\int_{0}^{10} S(q ; x) \mathrm{d} G^{*}(x)$ in (3.13) is now estimated by $\sum_{x=0}^{10} S(q ; x) \widehat{g}(x) ;$ similarly, the term $\int_{0}^{10} b S(q ; x)[1-S(q ; x)] \mathrm{d} G^{*}(x)$ in (3.13) is estimated by $\sum_{x=0}^{10} b S(q ; x)[1-$ $S(q ; x)] \widehat{g}(x)$. For each subject $i=1, \ldots, 361$, we use (3.13) to calculate:

$$
\begin{align*}
\hat{\alpha}_{i}= & \left(2 c q_{i} U^{\prime}\left(p-c q_{i}^{2}\right) \sum_{x=0}^{10} S\left(q_{i} ; x\right) \widehat{g}(x)-U\left(p-c q_{i}^{2}\right) \sum_{x=0}^{10} b S\left(q_{i} ; x\right)\left[1-S\left(q_{i} ; x\right)\right] \widehat{g}(x)\right) / \\
& \left(b \sum_{x=0}^{10} S\left(q_{i} ; x\right) \widehat{g}(x)+b q_{i} \sum_{x=0}^{10} b S\left(q_{i} ; x\right)\left[1-S\left(q_{i} ; x\right)\right] \widehat{g}(x)\right) \tag{3.14}
\end{align*}
$$

which is an estimate of subject $i$ 's $\alpha$. In Step 2, we use the sample of estimated $\alpha$ 's to estimate nonparametrically the altruism distribution:

$$
\begin{equation*}
\widehat{F}(a)=\frac{1}{361} \sum_{i=1}^{361} I\left\{\hat{\alpha_{i}} \leq a\right\} \tag{3.15}
\end{equation*}
$$

where $I$ is the indicator function that takes the value 1 when the condition inside the curly brackets is satisfied, and 0 otherwise.

The estimation procedures are similar for monopoly and quadropoly. In monopoly, we use the first-order condition (3.4) to recover a subject's $\alpha$ value from his quality
choice in any given incentive-market configuration. In other words, in the first step, for each $i=1, \ldots, 361$, we compute

$$
\hat{\alpha}_{i}=\frac{2 c q_{i} U^{\prime}\left(p-c q_{i}^{2}\right)}{b}
$$

Then these estimated $\alpha$ 's are used to estimate the distribution of altruism in the second step.

For quadropoly, in the first step, we compute the following

$$
\begin{align*}
\hat{\alpha}_{i}= & \left(2 c q_{i} U^{\prime}\left(p-c q_{i}^{2}\right) \sum_{x, y, z=0}^{10} S\left(q_{i} ; x, y, z\right) \widehat{l}(x) \widehat{l}(y) \widehat{l}(z)\right. \\
& \left.-U\left(p-c q_{i}^{2}\right) \sum_{x, y, z=0}^{10} b S\left(q_{i} ; x, y, z\right)\left[1-S\left(q_{i} ; x, y, z\right)\right] \widehat{l}(x) \widehat{l}(y) \widehat{l}(z)\right) /  \tag{3.16}\\
& \left(b \sum_{x, y, z=0}^{10} S\left(q_{i} ; x, y, z\right) \widehat{l}(x) \widehat{l}(y) \widehat{l}(z)\right. \\
& \left.+b q_{i} \sum_{x, y, z=0}^{10} b S\left(q_{i} ; x, y, z\right)\left[1-S\left(q_{i} ; x, y, z\right)\right] \widehat{l}(x) \widehat{l}(y) \widehat{l}(z)\right)
\end{align*}
$$

where $\widehat{l}(x), x=0,1, \ldots, 10$ is the empirical density function of quality in quadropoly. In the second step, these estimated $\alpha$ 's are used to estimate the altruism distribution $K$.

Given preferences and a symmetric equilibrium, our Bayesian game with independent values is identified by the equilibrium quality being monotone in altruism. The basic games and identification are the same as in GPV, whose two-step estimator for bidders' valuation distribution in first-price auctions is consistent and achieves optimal convergence rate with a properly chosen bandwidth. These results depend on the assumption that the unknown valuation distribution is smooth. However, subjects in our game choose from only 11 possible qualities. We therefore can only estimate the unknown altruism distribution by histograms with 11 possible values.

Even with more subjects, we would be unable to approximate a smooth distribution by histograms with a limited number of values.

### 3.4.3 Estimates of altruism distributions

We assume that the utility function $U$ is linear: $U(x)=x$. In this case, $\alpha$ is the marginal rate of substitution between patient benefit $b q$ and profit $p-c q^{2}$. Our main results will be based on this structural assumption. When $U$ is linear, for monopoly we have

$$
\begin{equation*}
\alpha=\frac{2 c q}{b} \tag{3.17}
\end{equation*}
$$

for duopoly, we have

$$
\begin{equation*}
\alpha=\frac{2 c q \int_{0}^{10} S(q ; x) \mathrm{d} G(x)-\left(p-c q^{2}\right) \times \int_{0}^{10} b S(q ; x)[1-S(q ; x)] \mathrm{d} G(x)}{b \int_{0}^{10} S(q ; x) \mathrm{d} G(x)+b q \int_{0}^{10} b S(q ; x)[1-S(q ; x)] \mathrm{d} G(x)} . \tag{3.18}
\end{equation*}
$$

For brevity, we do not write down the corresponding expression for $\alpha$ under quadropoly.
We have also used the alternate assumption of the utility function exhibiting a constant coefficient of absolute risk aversion. ${ }^{11}$ The estimation results for $U(x) \equiv$ $1-\exp (-r x)$ are in Appendix B. There we set the coefficient of absolute risk aversion $r$ at 0.10. (We have also obtained results for $r$ set at 0.05 and 0.15 . Results turn out to be similar and are available from the authors.) The drawback is that the marginal rate of substitution between patient benefit and profit varies with the quality level, so the estimated value of $\alpha$ is not so easy to interpret.

We first present summary statistics of the estimated altruism distributions. Table 3.3 lists the means of the estimated $\alpha$ distributions in monopoly. We use these

[^15]Table 3.3: Estimated means of $\alpha$ in monopoly

| Incentive configurations | mean |
| :--- | :--- |
| $(p=10, c=0.075, b=0.5)$ | 1.252 |
| $(p=10, c=0.075, b=1)$ | 0.622 |
| $(p=10, c=0.1, b=0.5)$ | 1.515 |
| $(p=10, c=0.1, b=1)$ | 0.746 |
| $(p=15, c=0.075, b=0.5)$ | 1.446 |
| $(p=15, c=0.075, b=1)$ | 0.725 |
| $(p=15, c=0.1, b=0.5)$ | 1.805 |
| $(p=15, c=0.1, b=1)$ | 0.889 |

estimated monopoly means as normalization. In duopoly and quadropoly, for each incentive configuration, we subtract the estimated monopoly mean from each estimated $\alpha$. This normalization uses the estimated monopoly mean as the origin. In Table 3.4, we present the normalized means and standard deviations of the 24 altruism distributions. Due to the normalization, each reported monopoly $\alpha$ distribution in Table 3.4 has a zero mean. Across a row in Table 3.4, for example, the magnitude -1.335 for the duopoly $\alpha$ mean in incentive configuration $(p=10, c=0.075, b=0.5)$ says that when the market changes from monopoly to duopoly, the average altruism parameter has decreased by 1.335 .

Table 3.4: Normalized means and standard deviations of $\alpha$ distributions

| Incentive configurations | Monopoly |  | Duopoly |  | Quadropoly |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | mean | SD | mean | SD | mean |  | SD |
| $(p=10, c=0.075, b=0.5)$ | 0 | 0.898 | -1.335 | 0.939 | -1.579 | 0.766 |  |
| $(p=10, c=0.075, b=1)$ | 0 | 0.448 | -0.812 | 0.612 | -0.985 | 0.657 |  |
| $(p=10, c=0.1, b=0.5)$ | 0 | 1.117 | -1.378 | 0.903 | -2.233 | 1.710 |  |
| $(p=10, c=0.1, b=1)$ | 0 | 0.559 | -0.882 | 0.725 | -1.069 | 0.822 |  |
| $(p=15, c=0.075, b=0.5)$ | 0 | 1.028 | -1.980 | 0.928 | -2.382 | 0.980 |  |
| $(p=15, c=0.075, b=1)$ | 0 | 0.512 | -1.244 | 0.767 | -1.471 | 1.138 |  |
| $(p=15, c=0.1, b=0.5)$ | 0 | 1.308 | -2.001 | 1.327 | -2.428 | 1.147 |  |
| $(p=15, c=0.1, b=1)$ | 0 | 0.638 | -1.207 | 0.827 | -1.485 | 1.016 |  |

The striking observation is that across each row, the average altruism has decreased from monopoly to duopoly, and then decreased further more from duopoly to quadropoly! This is clear evidence that competition reduces altruism on average. Standard deviations also tend to be different, but the pattern is not so uniform.

We now present the estimated $\alpha$ 's and their frequencies. Each of the $\alpha$ estimate is a nonlinear transformation of the chosen quality and the empirical quality distribution, and market and incentive-configuration parameters. The frequency for each $\alpha$ estimate is the same as the quality frequency, which are in Figures $3 \cdot 1$ to $3 \cdot 3$, so we do not write the frequencies again. We maintain the normalization by measuring $\alpha$ estimates from the means, which are in Table 3.3. In Table 3.5, we list the normalized estimated $\alpha$ 's corresponding to each quality between 0 and 10 .

Table 3.5: Estimated monopoly $\alpha$ values, normalized at mean

| $q=0 \quad q=1 \quad q=2 \quad q=3$ | $q=4$ | $q=5$ | $q=6$ | $q=7$ | $q=8$ | $q=9$ | $q=10$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & (p=10, c=0.075, b=0.5) \\ & -1.252-0.952-0.652-0.352 \end{aligned}$ | -0.052 | 0.248 | 0.548 | 0.848 | 1.148 | 1.448 | 1.748 |
| $\begin{aligned} & (p=10, c=0.075, b=1) \\ & -0.622 \quad-0.472 \\ & -0.322 \end{aligned}$ | -0.022 | 0.128 | 0.278 | 0.428 | 0.578 | 0.728 | 0.878 |
| $\begin{aligned} & (p=10, c=0.1, b=0.5) \\ & -1.515^{-1.115}-0.715-0.315 \end{aligned}$ | 0.085 | 0.485 | 0.885 | 1.285 | 1.685 | 2.085 | 2.485 |
| $\begin{array}{lll} (p=10, c=0.1, b=1) \\ -0.746 & -0.546 & -0.346 \end{array}-0.146$ | 0.054 | 0.254 | 0.454 | 0.654 | 0.854 | 1.054 | 1.254 |
| $\begin{aligned} & (p=15, c=0.075, b=0.5) \\ & -1.446 \quad-1.146 \\ & -0.846 \end{aligned}$ | -0.246 | 0.054 | 0.354 | 0.654 | 0.954 | 1.254 | 1.554 |
|  | -0.125 | 0.025 | 0.175 | 0.325 | 0.475 | 0.625 | 0.775 |
| $\begin{aligned} & (p=15, c=0.1, b=0.5) \\ & -1.805-1.405-1.005-0.605 \end{aligned}$ | -0.205 | 0.195 | 0.595 | 0.995 | 1.395 | 1.795 | 2.195 |
| $\begin{array}{lll} (p=15, c=0.1, b=1) \\ -0.889 & -0.689 & -0.489 \end{array}-0.289$ | -0.089 | 0.111 | 0.311 | 0.511 | 0.711 | 0.911 | 1.111 |

The frequencies of these normalized estimated $\alpha$ 's are in the following histograms in Figure $3 \cdot 4$. In these histograms, and later ones to be presented, we do not use
identical scales on the horizontal axis. The 8 histograms exhibit various spreads. Due to the nonlinear transformation from the observed qualities to the estimated $\alpha$, the actual values differ considerably across different incentive configurations. However, these histograms show that altruism distributions are diverse.

Next, we turn to estimated duopoly $\alpha$ (again normalized by the corresponding monopoly mean) in Table 3.6; we do not report those $\alpha$ when the corresponding quality was chosen by none of the subjects. The corresponding histograms are in Figure 3.5. The frequency for each $\alpha$ estimate is the same as the corresponding quality frequency, which is in Figure 3•2.

Table 3.6: Estimated duopoly $\alpha$ values, normalized at monopoly mean

| $q=0 \quad q=1 \quad q=2 \quad q=3$ | $q=4$ | $q=5$ | $q=6$ | $q=7$ | $q=8$ | $q=9$ | $q=10$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $(p=10, c=0.075, b=0.5)$ $-10.486 \quad-\quad-5.422-4.272$ | -3.430 | -2.758 | -2.186 | -1.668 | -1.177 | -0.689 | -0.187 |
| $\begin{aligned} & (p=10, c=0.075, b=1) \\ & -8.148 \end{aligned}$ | -2.079 | -1.682 | -1.359 | -1.071 | -0.792 | -0.506 | -0.187 |
| $(p=10, c=0.1, b=0.5)-4.289$ | -3.364 | -2.608 | -1.942 | -1.321 | -0.710 | -0.088 | 0.559 |
| $\begin{aligned} & (p=10, c=0.1, b=1) \\ & -8.824-4.912 \end{aligned}$ | -2.038 | -1.607 | -1.244 | -0.900 | -0.542 | -0.141 | 0.332 |
| $\begin{gathered} (p=15, c=0.075, b=0.5)_{-6.430} \end{gathered}$ | -5.252 | -4.349 | -3.613 | -2.979 | -2.403 | -1.851 | -1.296 |
| $\begin{aligned} & (p=15, c=0.075, b=1) \\ & -11.376-6.272-4.496 \end{aligned}$ | -2.923 | -2.443 | -2.079 | -1.772 | -1.489 | -1.213 | -0.900 |
| $\begin{aligned} & (p=15, c=0.1, b=0.5) \\ & -15.714-6.486 \end{aligned}$ | -5.255 | -4.284 | -3.468 | -2.744 | -2.071 | -1.412 | -0.741 |
| $\begin{aligned} & (p=15, c=0.1, b=1) \\ & -11.589 \end{aligned} \quad-3.956$ | -3.156 | $-2.551$ | -2.082 | -1.688 | -1.326 | -0.967 | -0.568 |

Figure 3.4: Histograms of estimated $\alpha$ in each incentive configuration in monopoly


The estimated values of $\alpha$ are very different from those in monopoly. The range has become much wider. From the histograms, we see that the higher values of estimated $\alpha$ 's have higher densities, but all of these higher values are below the corresponding monopoly mean. Subjects have become much less altruistic. Besides the stronger concentration, the $\alpha$ distributions appear to be strongly left-skewed in duopoly.

Table 3.7 presents the (normalized) $\alpha$ estimates for quadropoly, and Figure 3.6 presents the histogram. The frequency for each $\alpha$ estimate is the same as the corresponding quality frequency, which is in Figure $3 \cdot 3$. Similar to duopoly, quadropoly $\alpha$ distributions show a stronger concentration below the normalized monopoly mean and are left-skewed, as in duopoly.

Table 3.7: Estimated quadropoly $\alpha$ values, normalized at monopoly mean

| $q=0 \quad q=1 \quad q=2$ | $q=3$ | $q=4$ | $q=5$ | $q=6$ | $q=7$ | $q=8$ | $q=9$ | $q=10$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $(p=10, c=0.075, b=0.5)$ -11.194 | ) | -3.733 | -3.079 | $-2.540$ | -2.073 | -1.648 | -1.245 | -0.845 |
| $(p=10, c=0.075, b=1)$ -10.619 | -2.838 | -2.258 | -1.843 | -1.521 | -1.253 | -1.015 | -0.788 | -0.550 |
| $\begin{aligned} & (p=10, c=0.1, b=0.5) \\ & -21.505-11.209 \end{aligned}$ | - | -4.539 | -3.651 | -2.941 | -2.322 | -1.730 | -1.095 | -0.331 |
| $\begin{aligned} & (p=10, c=0.1, b=1) \\ & -10.742 \end{aligned}$ | $-2.866$ | -2.258 | -1.815 | -1.460 | -1.154 | -0.864 | -0.560 | -0.197 |
| $\begin{aligned} & (p=15, c=0.075, b=0.5) \\ & -16.391 \end{aligned}$ | ) | -5.598 | -4.707 | -3.992 | -3.390 | -2.860 | -2.374 | -1.908 |
| $(p=15, c=0.075, b=1)$ -15.717 | -4.191 | -3.362 | -2.783 | $-2.346$ | -1.995 | -1.698 | -1.429 | -1.163 |
| $\begin{aligned} & (p=15, c=0.1, b=0.5) \\ & -16.729 \end{aligned}$ | -6.908 | -5.671 | -4.721 | -3.944 | -3.277 | -2.678 | -2.117 | -1.566 |
| $\begin{aligned} & (p=15, c=0.1, b=1) \\ & -15.883 \end{aligned}$ | -4.259 | -3.403 | -2.796 | -2.329 | -1.947 | -1.614 | -1.305 | -0.987 |

Figure 3.5: Histograms of estimated $\alpha$ in each incentive configuration in duopoly


Estimations show striking differences between monopoly $\alpha$ distributions and the duopoly and quadropoly $\alpha$ distributions. Whereas preferences tend to exhibit diversity in monopoly, they are less diverse in duopoly, and becoming less so in quadropoly. Densities of estimated $\alpha$ 's tend to vary quite a lot in monopoly, but a lot less so in duopoly and quadropoly. Moreover, estimated $\alpha$ distributions tend to be left-skewed and being more concentrated at the high end of the distribution.

### 3.4.4 Statistical tests on altruism distributions

We can perform standard two-sample Kolmogorov-Smirnov (KS) tests on the (null) hypothese that two estimated altruisms are drawn from the same continuous distribution. ${ }^{12}$ The test statistic, KS distance, is the largest absolute difference between two empirical distribution functions; see, for example, Conover (1998). For two estimated $\alpha$ distributions, say $\widehat{F}_{1}$ and $\widehat{F}_{2}$, their KS distance is defined by $K S_{1,2} \equiv$ $\sup _{a}\left|\widehat{F}_{1}(a)-\widehat{F}_{2}(a)\right|$. We have plotted the 24 estimated $\alpha$ distributions in Figure 3•7. Note that these are plots of actual estimated $\alpha$ 's, not normalized at the mean of the monopoly $\alpha$.

In each of the 8 incentive configurations, we compare $3 \alpha$-distribution pairs: i) monopoly versus duopoly (M-D), ii) monopoly versus quadropoly (M-Q), and iii) duopoly versus quadropoly (D-Q). Table 3.8 presents the KS distances for all 24 pairs; all the $p$-values are very small (reported to be less than $2.2 \times 10^{-16}$ by the software $R$, so omitted in the table). Except in one incentive configuration ( $p=10, c=0.1, b=0.5$ ), the KS distances are highest for M-Q, followed by M-D, and then D-Q. For incentive configuration ( $p=10, c=0.1, b=0.5$ ), the only difference is that D-Q distance is higher than M-D distance. The intepretation is that competition has an increasing effect on the reduction of altruim distribution. Because the $p$-values are so small, we

[^16]Figure 3.6: Histograms of estimated $\alpha$ in each incentive configuration in quadropoly


Figure 3.7: Distributions of estimated $\alpha$ in each market and in each incentive configuration




$p=15, c=0.075, b=0.5$


$\mathrm{p}=15, \mathrm{c}=0.1, \mathrm{~b}=0.5$

$p=15, c=0.1, b=1$

reject the equality of the estimated $\alpha$ distributions in all comparisons.
Table 3.8: KS distances of $\alpha$ distributions between two markets for each incentive configuration

| pair KS distance | pair KS distance |
| :---: | :---: |
| $p=10, c=0.1, b=1$ | $p=10, c=0.1, b=0.5$ |
| M-D 0.587 | M-D 0.521 |
| M-Q 0.781 | M-Q 0.873 |
| D-Q 0.399 | D-Q 0.554 |
| $p=10, c=0.075, b=1$ | $p=10, c=0.075, b=0.5$ |
| M-D 0.654 | M-D 0.595 |
| M-Q 0.825 | M-Q 0.742 |
| D-Q 0.388 | D-Q 0.399 |
| $p=15, c=0.1, b=0.5$ | $p=15, c=0.075, b=1$ |
| M-D 0.662 | M-D 1 |
| M-Q 0.828 | M-Q 1 |
| D-Q 0.504 | D-Q 0.559 |
| $p=15, c=0.1, b=1$ | $p=15, c=0.075, b=0.5$ |
| M-D 0.737 | M-D 0.831 |
| M-Q 1 | M-Q 1 |
| D-Q 0.532 | D-Q 0.679 |

Next, for each of the 3 markets, we consider $\alpha$ distributions from the 8 different incentive configurations. There are 28 pairs for comparisons in each market. Table 3.9 presents the KS distances for these distributions. There, pairs are labeled by the order in which they were presented in Section 3.3.1, on page 95; for instance, the label 1-2 denotes the incentive-configuration pair ( $p=10, c=0.1, b=1$ ) and $(p=10, c=0.075, b=1)$. The KS distances vary across different pairs. All $p$ values are much smaller than 0.01 (and have been omitted in the table); we reject the hypothesis that any pair of the estimated $\alpha$ distributions are identical.

Table 3.9: KS distances of $\alpha$ distributions between two markets for each incentive configuration

| pair | KS distances |  | Quadropoly |
| :---: | :---: | :---: | :---: |
|  | Monoply | Duopoly |  |
| 1-2 | 0.227 | 0.327 | 0.260 |
| 1-3 | 0.512 | 0.329 | 0.382 |
| 1-4 | 0.169 | 0.183 | 0.399 |
| 1-5 | 0.432 | 0.313 | 0.440 |
| 1-6 | 0.382 | 0.199 | 0.244 |
| 1-7 | 0.152 | 0.715 | 0.803 |
| 1-8 | 0.449 | 0.413 | 0.803 |
| 2-3 | 0.582 | 0.305 | 0.343 |
| 2-4 | 0.327 | 0.199 | 0.343 |
| 2-5 | 0.540 | 0.482 | 0.393 |
| 2-6 | 0.440 | 0.307 | 0.285 |
| 2-7 | 0.183 | 0.648 | 0.748 |
| 2-8 | 0.507 | 0.282 | 0.748 |
| 3-4 | 0.465 | 0.504 | 0.343 |
| 3-5 | 0.144 | 0.177 | 0.255 |
| 3-6 | 0.365 | 0.216 | 0.341 |
| 3-7 | 0.582 | 0.504 | 0.557 |
| 3-8 | 0.271 | 0.504 | 0.557 |
| 4-5 | 0.379 | 0.346 | 0.335 |
| 4-6 | 0.288 | 0.307 | 0.463 |
| 4-7 | 0.252 | 0.532 | 0.607 |
| 4-8 | 0.404 | 0.335 | 0.607 |
| 5-6 | 0.252 | 0.374 | 0.335 |
| 5-7 | 0.526 | 0.681 | 0.499 |
| 5-8 | 0.155 | 0.681 | 0.498 |
| 6-7 | 0.376 | 0.612 | 0.739 |
| 6-8 | 0.149 | 0.307 | 0.739 |
| 7-8 | 0.471 | 0.482 | 0.595 |

### 3.4.5 Counterfactual monopoly qualities from estimated duopoly and quadropoly altruism

Whereas Table 3.2 and Figures $3 \cdot 1$ to $3 \cdot 3$ report the outcomes, our structural estimation of $\alpha$ distributions in 3.4.2 can separately identify the effects (i) due to preferences change and (ii) due to market-incentive changes. However, results in Subsections 3.4.2 and 3.4.3 are obtained without explicit derivations of Bayes-Nash equilibria. One could not easily compute duopoly or quadropoly equilibrium quality distributions under the counterfactual that preference distributions remained unchanged at the monopoly configuration.

Instead, we perform counterfactual of the following sort. We use the estimated altruism distributions in an incentive configuration in duopoly or quadropoly to calculate the optimal qualities under monopoly. That is, we take $\alpha$ values and their frequencies from Tables 3.6 and 3.7 and feed them into the monopoly first-order condition (3.4) to calculate optimal qualities. The next two figures show the counterfactual histograms of monopoly qualities when $\alpha$ 's are those identified in duopoly and quadropoly. In each counterfactual computation, the optimal qualities need not be integers, and we have limited the optimal qualities to be nonnegative. (Those estimated $\alpha$ in duopoly and quadropoly that are negative have been replaced by 0 to ensure a nonnegative optimal monopoly quality.)

Differences between empirical monopoly qualities and counterfactual qualities are striking. Histograms in Figures 3.8 and 3.9 have no resemblance to those in the empirical quality distributions in Figure $3 \cdot 1$. This indeed indicates that markets and incentives do change preferences.

### 3.5 Reduced-form analysis of experimental data

We now present reduced-form analysis of subjects' quality choices. Table 3.2 already describes the 24 quality means and standard deviations for the 3 markets and 8 incentive configurations, and Figures $3 \cdot 1$ to $3 \cdot 3$ show the quality histograms. Here, we first present some aggregated descriptive statistics, and then regression results.

A subject makes 8 quality choices in each market. Of these 8 , four of them are made with one fixed incentive-configuration parameter. For example, under monopoly at $p=10$, a subject chooses 4 qualities, while cost and patient-benefit parameters vary between low and high. We record the average of these 4 qualities for each subject, and then we find the average of all 361 subjects (the average of a total of 1,444 quality choices). In Table 3.10, the first entry 3.959 records the mean of

Figure 3•8: Counterfactual monopoly quality histogram from duopoly altruism $\alpha$


Figure 3.9: Counterfactual monopoly quality histogram from quadropoly altruism $\alpha$

subjects' average quality choices at $p=10$, and 2.900 is the corresponding standard deviation. Across that row, when the price is set at 15, the higher level, the mean becomes 4.652 , and the standard deviation becomes 3.327 . The relative difference, 0.175 , equals $(4.652-3.959) / 3.959$. The rest of Table 3.10 presents the quality-choice averages for each parameter in each market. ${ }^{13}$

From the first three rows with data entries in Table 3.10, average quality is higher in each market when the price is set at the higher level, but the relative difference declines as the market becomes more competitive. From the second set of data entries, average quality becomes lower when cost is set at the higher level, although the relative difference remains almost the same across markets. For patient benefits, quality averages exhibit a different pattern. For monopoly, a higher patient benefit results in a slightly lower average quality, whereas for duopoly and quadropoly, a high patient benefit results in slightly higher quality averages. But in all three markets, the relative difference seems very small.

Table 3.10: Descriptives on the variations in price, costs, and patient benefit

| Parameter | Low parameter level ( $\mathrm{N}=1,444$, per market) |  | High parameter level ( $\mathrm{N}=1,444$, per market) |  | Relative difference |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Price ( $p=10 ; p=15$ ) |  |  |  |  |  |
| Monopoly | 3.959 | 2.900 | 4.652 | 3.327 | 0.175 |
| Duopoly | 7.442 | 1.573 | 8.595 | 1.625 | 0.155 |
| Quadropoly | 7.841 | 1.479 | 8.862 | 1.484 | 0.130 |
| Cost ( $c=0.075 ; c=0.1$ ) |  |  |  |  |  |
| Monopoly | 4.493 | 3.227 | 4.118 | 3.038 | -0.083 |
| Duopoly | 8.380 | 1.660 | 7.657 | 1.662 | -0.086 |
| Quadropoly | 8.704 | 1.489 | 8.000 | 1.564 | -0.081 |
| Patient benefit ( $b=0.5 ; b=1$ ) |  |  |  |  |  |
| Monopoly | 4.323 | 3.150 | 4.287 | 3.128 | -0.008 |
| Duopoly | 7.925 | 1.668 | 8.112 | 1.726 | 0.024 |
| Quadropoly | 8.310 | 1.523 | 8.393 | 1.608 | 0.010 |

[^17]We next use ordinary least square regressions to study the effect of market competition and incentive-configurations:

$$
\begin{equation*}
q_{i}=\beta_{0}+\beta_{1} D+\beta_{2} Q+\gamma_{1} \text { Price }+\gamma_{2} \text { Cost }+\gamma_{3} \text { Benefit }+\psi \mathbf{X}_{i}+\varepsilon_{i} \tag{3.19}
\end{equation*}
$$

where $q_{i}$, the dependent variable, is subject $i$ 's quality choice, and $\beta_{0}$ is the intercept. Experimental manipulations are defined by a set of dummies. Regarding monopoly as the reference market, we use the dummy variables $D$ and $Q$ to represent duopoly and quadropoly, respectively; a dummy is set to 1 when the quality on the left-hand side has been chosen under the corresponding market condition. The Price, Cost, and Benefit variables are also dummies. The variable Price takes the value of 1 when price $p$ is equal to the high level of 15 ; it takes the value at 0 otherwise. Similarly, Cost takes the value of 1 when $c=0.1$, and Benefit takes the value of 1 when patient benefit $b=1$; otherwise, they are 0 . Equation (3.19) includes a vector of additional control $\mathbf{X}_{i}$ of market orders (see Table 3.1) and session dummies, and finally $\varepsilon_{i}$ is an error term. Model (1) in Table 3.11 presents the estimation results. In Model (2), we add market and incentive-configuration interaction terms.

From Table 3.11, quality is significantly higher in duopoly and quadropoly compared to monopoly, and the magnitudes are similar in both models. Wald tests indicate a highly significant difference between Duopoly and Quadropoly ( $p<0.001$ ). For incentive configurations with a high price, a low cost, and a high patient benefits, qualities are significantly higher; see Model (1). With interaction terms in Model (2), the effects of price and cost remain qualitatively similar but the magnitudes have declined. The average benefit effect becomes insignificant; this suggests that the patient-benefit effect may be market specific. Using Wald tests, we find that market effects are significantly larger than market-configuration effects (at $p<0.001$ ).

Table 3.11: Quality regressions

| Model | (1) | (2) |
| :---: | :---: | :---: |
| Duopoly ( $D$ ) | $3.713^{* * *}$ | $3.545 * * *$ |
|  | (0.158) | (0.157) |
| Quadropoly (Q) | $4.046^{* * *}$ | $3.987^{* * *}$ |
|  | (0.157) | (0.156) |
| High price $(=1$ if $p=15)$ | $0.955^{* * *}$ | $0.693^{* * *}$ |
|  | (0.029) | (0.050) |
| High cost ( $=1$ if $c=0.1)$ | $-0.601^{* * *}$ | $-0.375^{* * *}$ |
|  | (0.024) | (0.046) |
| High benefit ( $=1$ if $b=1$ ) | $0.078^{* * *}$ | $-0.036$ |
|  | $(0.024)$ | $(0.043)$ |
| Duopoly $\times$ High price |  | $0.461^{* * *}$ |
|  |  | (0.066) |
| Quadropoly $\times$ High price |  | $0.328^{* * *}$ |
|  |  | (0.061) |
| Duopoly $\times$ High cost |  | $-0.348^{* * *}$ |
|  |  | (0.056) |
| Quadropoly $\times$ High cost |  | $-0.328^{* * *}$ |
|  |  | $(0.055)$ |
| Duopoly $\times$ High benefit |  |  |
|  |  | $(0.056)$ |
| Quadropoly $\times$ High benefit |  | $0.119 * *$ |
|  |  | (0.055) |
| Market order and session dummies | Yes | Yes |
| Constant | 3.971 *** | 4.047 *** |
|  | (0.400) | (0.399) |
|  | 8,664 | 8,664 |
| Subjects | 361 | 361 |
| $R^{2}$ | 0.445 | 0.447 |

[^18]From Models (1) and (2) results, more intense market competition has implemented higher equilibrium qualities. An interpretation of an unqualified success of competition (under regulated prices) on implementing higher qualities, however, is misguided. Bayes-Nash equilibrium qualities depend on preferences, markets, and incentive configurations. Our structural estimation supports reduction in altruism, which generally reduces subjects' qualities in equilibrium. The scenario is more appropriately described as a tug of war-between altruism reduction and competitionincentive disciplinary powers. In our setting, competition-incentive powers have won over altruism reduction.

### 3.6 Concluding remarks

Using behavioral data from an experiment in a health frame, we show that the altruistic preferences are affected by markets and incentives. We model subjects' preferences through a linear utility function whose marginal rate of substitution is interpreted as the degree of altruism. Subjects play a simultaneous-move incomplete information game when they compete with each other. Using the experimental data, we estimate the altruism distribution in each market-incentive environment. The estimation results show that subjects are less altruistic when they have to compete against each other.

Although our conclusion is that altruism has changed, we have maintained certain assumptions, both in the theoretical model and in the experiment. The structural model does require some consistency in preferences between different markets and incentive configurations. So to speak, we can estimate changing preferences only if those changes are not so drastic. We narrow down our study to one altruism parameter.

The assumption that individuals are interested only in profits and patient benefits
is maintained throughout. We would not be in a position to test if subjects would become spiteful, winning oriented, or fair-minded when they participate in duopoly or quadropoly. Our design does minimize these contaminations, however. We have only told subjects very sparse outcome information. Subjects never have learned that they have been "disadvantaged" by the rival, that their qualities have been higher or lower than rivals', or that their choices turn out to be similar or very different from the population averages. We have limited subjects' ability to learn about each other by implementing a simultaneous-move game. Interaction between subjects and learning about the population are both impossible in our design. Every attempt has been made to ensure that a subject is playing against another randomly drawn subject, and only once.

We make the point that economic institutions may affect preferences in nontrivial ways. Economic institutions may shape preferences just as climate, cultural-historical events, physiology, and genetics.

## Appendix A Materials for the experiment

## A. 1 Instructions

You are taking part in an economic decision-making experiment. Please carefully read the instructions. It is very important that you do not speak with other participants for the duration of the experiment. If you break these rules, you could be excluded from the experiment and not receive any payment. If you do not understand something, please take another look at the instructions. If you still have questions, please raise your hand. We will come to you at your cubicle and answer your questions in private.

You can earn money in the course of the experiment. The amount of your earnings depends on your decisions and decisions made by other participants. At no time will you be told the names of the other participants. They will also not at any time be informed about your identity.

For showing up you will receive a fee of EUR 2.50.
All monetary amounts in this experiment are expressed in Taler, whereby the following applies: Taler $100=$ EUR 1.

At the end of the experiment, the amount of money you earned will be paid to you in cash. Your decisions are made on the computer screen present in your cubicle. All data and answers will be evaluated anonymously. You were asked to draw your own personal cubicle number in order to maintain anonymity.

The experiment will last around 60 minutes and consists of three parts. Before each of the three parts you will receive detailed instructions and be asked to answer control questions pertaining to these instructions. Please note: Neither your decisions in the first part nor in the second part of the experiment have an influence on the other parts of the experiment.

We will ask you to answer a few questions at the end of the experiment. You will receive an additional payment for answering this questionnaire.

First part of the Experiment. In the first part of experiment, you will take on the role of a physician and make decisions about the treatment of various patients. In total, you will determine the quality of care that you would like provide for eight different types of patients. For each of these patients you can choose quality of 0,1 , $2,3,4,5,6,7,8,9$ or 10 .

The demand for medical care by the various patient types is determined only after you have made your decisions about the quality of care for all eight types.
[Duopoly: You are randomly matched with another participant. This participant also decides in the role of a physician. Also this physician determines the quality for the same eight types of patients. The matching with this participants remains throughout the entire second part of the experiment. You and the other physician chose the quality simultaneously and independently from each other.]
[Quadropoly: You are randomly matched with three other participants. These participants also decide in the role of a physicians. Also these physicians determine the quality for the same eight types of patients. The matching with these participants remains throughout the entire third part of the experiment. You and the other physicians chose the quality simultaneously and independently from each other.]

In total, 100 patients of each type demand medical care. It will only be determined after you have made your decisions about the quality of care for all eight types how many of the 100 patients of each type wish to seek treatment from you.
[Duopoly: Only after you and the other physician, you are matched with, decided upon the quality of medical treatment for the eight patients, it is determined how many of the 100 patients seek treatment from you and the other physician.]
[Quadrupoly: Only after you and the others physicians, you are matched with, decided upon the quality of medical treatment for the eight patients, it is determined how many of the 100 patients seek treatment from you and the other physicians.]

Earnings. For each patient who seeks medical care from you, you receive a lump sum that is independent of the quality of care you have selected. You incur costs with your selection of the quality of care. These costs depend on the quality level you choose and can vary between the different patient types. Your earnings for each patient type are as follows:

Earnings $=($ Lumpsum-Costs $) \times$ Number of patients who seek medical care from you
(when read: your earnings are equal to the difference between the lump sum and the costs that arise from the quality of care you have chosen, multiplied by the number of patients who seek treatment from you.)

With the quality of care you choose, you determine not only your own earnings, but also the utility enjoyed by the patient. The amount of the lump sum, your costs, your earnings, and the patient's utility will be displayed on your screen (as illustrated in Subsection A.3) for each patient type.

Before you choose the quality of care for each patient type, you have the opportunity to click on the "calculator" button and thereby calculate patients' potential demand for treatment (as illustrated in Subsection A.3). You can enter the quality you would like to provide as many times as you want. Clicking on the "calculate" button provides you with information about the number of patients who would seek care given the quality level you entered. In addition, you receive information about the resulting earnings and patient utility. You define the quality of care that you wish to provide by entering that quality in the field "your decision" and confirming this entry with "OK."

Payment. After the conclusion of the experiment, one of the 8 decisions will be randomly chosen to function as the relevant round for determining your payment for this part of the experiment. The earnings from this randomly-chosen round will be
converted into Euro at the end of the experiment and paid out to you in cash. There are no participants present in the lab who take on the role of patients. An actual patient will benefit from the patient utility resulting from the quality of care you selected in the randomly-chosen round: A monetary value equaling the patient utility derived from your decision, multiplied by the number of patients who seek treatment from you, will be transferred to Christoffel Blindenmission Deutschland e.V., 64625 Bensheim. This organization will use the funds to enable the treatment of patients suffering from cataracts, a serious eye condition.

Control questions. Before proceeding to the decisions in the experiment, we would like to ask you to answer several control questions. These control questions should make it easier for you become acquainted with the decision-making situation. If you have questions about this, please raise your hand. The first part of the experiment will begin after all participants have correctly answered the control questions.

Payment Procedure. In order to ensure that payments to the participants and the transfer of the monetary donation to Christoffel Blindenmission Deutschland e.V. are carried out correctly, an overseer will be randomly chosen after the third part of the experiment. The overseer receives a fee of Euro 5 in addition to his or her regular payment from the experiment. The overseer will affirm that the transfer to Christoffel Blindenmission is correctly carried out by the financial administration of the University of Cologne. For the transfer to Christoffel Blindenmission, the overseer will fill out a payment order to Christoffel Blindenmission with the amount, in Euro, that corresponds to the patient utility realized in the randomly-selected round. The financial administration of the University of Cologne will then execute payment of the donation to Christoffel Blindenmission using funds allocated for this experiment. The form will be placed in a stamped envelope addressed to the financial administration of the University of Cologne. The overseer and the experimenter will jointly deposit
this envelope in the nearest mailbox.
The overseer will confirm by signing a form that he or she properly carried out the assigned tasks, as described above. A copy of this form, as well as a copy of the confirmation from Christoffel Blindenmission that the donation was received, can be requested by all participants from the office of the Seminar of Personnel Economics and Human Resource Management. The copies will be sent by e-mail.

## A. 2 Control questions of the experiment

## Comprehension questions

[The comprehension questions are presented for the market order Monopoly-Duopoly-Quadropoly. Question that are the same irrespective of the market setting are marked with an asterisk $\left({ }^{*}\right)$. ]

## Monopoly

1. In the first part of the experiment, you decide in the role of a $\qquad$ about the treatment of $\qquad$ .(*)
2. For how many different patient types, do you decide on quality of treatment
3. How many patients of each type demand medical services in total? $\qquad$
4. How many physicians decide on the quality of medical services beside you in a market? $\qquad$
5. Is the following statement true or false? "Your quality choice for a patient does not only determine your profit but also the patient's benefit." (*)
$\square$ TrueFalse
To answer the following two questions, please consider the examples on your computer screen.
6. Please consider Example A on your computer screen. Please assume that you would choose a quality of $\mathbf{1}$ for patients of this type. For one patient, what is
a. your capitation? $\qquad$
b. your costs? $\qquad$
c. your profit? $\qquad$
d. the patient's benefit? $\qquad$
7. Again, please consider Example A on your computer screen. Please assume, that you would choose a quality of 4 for the patients of this type (Hint: To answer the questions below, please use the calculator on your computer screen.).
a. What is the patient demand for your treatment quality? $\qquad$
b. What is your profit? $\qquad$
c. What is the patient's benefit? $\qquad$
8. Now, please consider Example B on your computer screen. Please assume, that you would choose a quality of $\mathbf{7}$ for patients of this type. For one patient, what is
a. your capitation? $\qquad$
b. your costs? $\qquad$
c. your profit? $\qquad$
d. the patient's benefit? $\qquad$
9. Again, please consider Example B on your computer screen. Please assume, that you would choose a quality of 5 for the patients of this type (Hint: To answer the questions below, please use the calculator on your computer screen.).
a. What is the patient demand for your treatment quality? $\qquad$
b. What is your profit? $\qquad$
c. What is the patient's benefit? $\qquad$
10. Which of the following statements is true? (*)
$\square$ Your quality choice for a patient type determines the number of patients of this type who demand your treatment quality. For those patients, who demand your treatment, the quality choice determines the patient benefit. In addition, your quality choice determines your profit for the patient type.
$\square$ Your quality choice for a patient type determines the number of patients of that type who demand your treatment quality. While your quality choice has no influence on the patient benefit it determines your profit.
$\square$ Your quality choice for a patient type does not determine the number of patients of that type who demand your treatment quality. Your quality choice has no influence on the patient benefit and only determines your profit.
$\downarrow$ None.
11. Please complete the following sentence!

After the completion of the experiment, it will be determined $\qquad$ which of your $\qquad$ decisions from this part of the experiment is relevant for determining your payment and the patient's benefit. (*)

## Duopoly

1. For how many different patient types, do you decide on quality of treatment
2. How many patients of each type demand medical services in total? $\qquad$
3. How many physicians decide on the quality of medical services beside you in a market? $\qquad$
To answer the following two questions, please consider the examples on your computer screen.
4. Please consider Example A on your computer screen. Please assume that you would choose a quality of $\mathbf{1}$ for patients of this type. For one patient, what is
a. your capitation? $\qquad$
b. your costs? $\qquad$
c. your profit? $\qquad$
d. the patient's benefit? $\qquad$
5. Again, please consider Example A on your computer screen. Please assume, that you would choose a quality of 4 for the patients of this type. The other physician would choose a quality of $\mathbf{3}$ (Hint: To answer the questions below, please use the calculator on your computer screen.).
a. What is the patient demand for your treatment quality? $\qquad$
b. What is the patient demand for the other physician's treatment quality?
$\qquad$
c. What is your profit? $\qquad$
d. What is the other physician's profit? $\qquad$
e. What is the patient's benefit resulting from your quality decision? $\qquad$
e. What is the patient's benefit resulting from the other physician's quality decision? $\qquad$
6. Now, please consider Example B on your computer screen. Please assume, that you would choose a quality of 7 for patients of this type. For one patient, what is
a. your capitation? $\qquad$
b. your costs? $\qquad$
c. your profit? $\qquad$
d. the patient's benefit? $\qquad$
7. Again, please consider Example B on your computer screen. Please assume, that you would choose a quality of 5 for the patients of this type. The other physician would choose a quality of 6 (Hint: To answer the questions below, please use the calculator on your computer screen.).
a. What is the patient demand for your treatment quality? $\qquad$
b. What is the patient demand for the other physician's treatment quality?
c. What is your profit? $\qquad$
d. What is the other physician's profit? $\qquad$
e. What is the patient's benefit resulting from your quality decision? $\qquad$
e. What is the patient's benefit resulting from the other physician's quality decision? $\qquad$

## Quadropoly

1. For how many different patient types, do you decide on quality of treatment
2. How many patients of each type demand medical services in total?
3. How many physicians decide on the quality of medical services beside you in a market? $\qquad$
To answer the following two questions, please consider the examples on your computer screen.
4. Please consider Example A on your computer screen. Please assume that you would choose a quality of $\mathbf{1}$ for patients of this type. For one patient, what is
a. your capitation? $\qquad$
b. your costs? $\qquad$
c. your profit? $\qquad$
d. the patient's benefit? $\qquad$
5. Again, please consider Example A on your computer screen. Please assume, that you would choose a quality of 4 for the patients of this type. The other physicians would choose a quality of $\mathbf{3}$ (Hint: To answer the questions below, please use the calculator on your computer screen.).
a. What is the patient demand for your treatment quality? $\qquad$
b. What is the patient demand for the second physician's treatment quality?
$\qquad$
c. What is the patient demand for the third physician's treatment quality?
$\qquad$
d. What is the patient demand for the fourth physician's treatment quality?
$\qquad$
e. What is your profit? $\qquad$
f. What is the second physician's profit? $\qquad$
g. What is the third physician's profit? $\qquad$
h. What is the fourth physician's profit? $\qquad$
i. What is the patient's benefit resulting from your quality decision? $\qquad$
j. What is the patient's benefit resulting from the second physician's quality decision? $\qquad$
k. What is the patient's benefit resulting from the third physician's quality decision? $\qquad$
6. What is the patient's benefit resulting from the fourth physician's quality decision? $\qquad$
7. Now, please consider Example B on your computer screen. Please assume, that you would choose a quality of $\mathbf{7}$ for patients of this type. For one patient, what is
a. your capitation? $\qquad$
b. your costs? $\qquad$
c. your profit? $\qquad$
d. the patient's benefit? $\qquad$
8. Again, please consider Example B on your computer screen. Please assume, that you would choose a quality of 5 for the patients of this type. The second and the third physician would choose a quality of 6 . The fourth physician would choose a quality of 4. (Hint: To answer the questions below, please use the calculator on your computer screen.).
a. What is the patient demand for your treatment quality? $\qquad$
b. What is the patient demand for the second physician's treatment quality?
c. What is the patient demand for the third physician's treatment quality?
$\qquad$
d. What is the patient demand for the fourth physician's treatment quality?
e. What is your profit? $\qquad$
f. What is the second physician's profit? $\qquad$
g. What is the third physician's profit? $\qquad$
h. What is the fourth physician's profit? $\qquad$
i. What is the patient's benefit resulting from your quality decision? $\qquad$
j. What is the patient's benefit resulting from the second physician's quality decision? $\qquad$
k. What is the patient's benefit resulting from the third physician's quality decision? $\qquad$
l. What is the patient's benefit resulting from the fourth physician's quality decision? $\qquad$

## A. 3 Screen shots and experiment parameters

Figure 3•10: Decision screenshot


Figure 3•11: Duopoly calculator screenshot


Figure 3•12: Duopoly calculator screenshot with qualities inputted


Figure 3.13: Quadropoly calculator screenshot


Table 3.12: Experiment parameters

|  | Quality, $q$ |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Incentive configuration $1(p=10, c=0.1, b=1)$ |  |  |  |  |  |  |  |  |  |  |  |
| Capitation, $p$ | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| Cost, $c(q)$ | 0 | 0.1 | 0.4 | 0.9 | 1.6 | 2.5 | 3.6 | 4.9 | 6.4 | 8.1 | 10 |
| Profit, $p-c(q)$ | 10 | 9.9 | 9.6 | 9.1 | 8.4 | 7.5 | 6.4 | 5.1 | 3.6 | 1.9 | 0 |
| Patient benefit, $q$ | 0 | 1 | 2 | 3 |  | 5 | 6 | 7 | 8 | 9 | 10 |
| Incentive configuration $2(p=10, c=0.075, b=1)$ |  |  |  |  |  |  |  |  |  |  |  |
| Capitation, $p$ | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| Cost, $c(q)$ | 0 | 0.075 | 0.3 | 0.675 | 1.2 | 1.875 | 2.7 | 3.675 | 4.8 | 6.075 | 7.5 |
| Profit, $p-c(q)$ |  | 9.925 | 9.7 | 9.325 | 8.8 | 8.125 | 7.3 | 6.325 | 5.2 | 3.925 | 2.5 |
| Patient benefit, $q$ | 0 | 1 | 2 |  | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Incentive configuration $3(p=15, c=0.1, b=0.5)$ |  |  |  |  |  |  |  |  |  |  |  |
| Capitation, $p$ | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 |
| Cost, $c(q)$ | 0 | 0.1 | 0.4 | 0.9 | 1.6 | 2.5 | 3.6 | 4.9 | 6.4 | 8.1 | 10 |
| Profit, $p-c(q)$ | 15 | 14.9 | 14.6 | 14.1 | 13.4 | 12.5 | 11.4 | 10.1 | 8.6 | 6.9 | 5 |
| Patient benefit, $q$ | 0 | 0.5 |  | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 |
| Incentive configuration $4(p=15, c=0.1, b=1)$ |  |  |  |  |  |  |  |  |  |  |  |
| Capitation, $p$ | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 |
| Cost, $c(q)$ | 0 | 0.1 | 0.4 | 0.9 | 1.6 | 2.5 | 3.6 | 4.9 | 6.4 | 8.1 | 10 |
| Profit, $p-c(q)$ | 15 | 14.9 | 14.6 | 14.1 | 13.4 | 12.5 | 11.4 | 10.1 | 8.6 | 6.9 | 5 |
| Patient benefit, $q$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Incentive configuration $5(p=10, c=0.1, b=0.5)$ |  |  |  |  |  |  |  |  |  |  |  |
| Capitation, $p$ | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| Cost, $c(q)$ | 0 | 0.1 | 0.4 | 0.9 | 1.6 | 2.5 | 3.6 | 4.9 | 6.4 | 8.1 | 10 |
| Profit, $p-c(q)$ | 10 | 9.9 | 9.6 | 9.1 | 8.4 | 7.5 | 6.4 | 5.1 | 3.6 | 1.9 | 0 |
| Patient benefit, $q$ | 0 | 0.5 |  | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 |
| Incentive configuration 6 ( $p=10, c=0.075, b=0.5)$ |  |  |  |  |  |  |  |  |  |  |  |
| Capitation, $p$ | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| Cost, $c(q)$ | 0 | 0.075 | 0.3 | 0.675 | 1.2 | 1.875 | 2.7 | 3.675 | 4.8 | 6.075 | 7.5 |
| Profit, $p-c(q)$ |  | 9.925 |  | 9.325 | 8.8 | 8.125 | 7.3 | 6.325 | 5.2 | 3.925 | 2.5 |
| Patient benefit, $q$ | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 |
| Incentive configuration $7(p=15, c=0.075, b=1)$ |  |  |  |  |  |  |  |  |  |  |  |
| Capitation, $p$ | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 |
| Cost, $c(q)$ |  | 0.075 | 0.3 | 0.675 | 1.2 | 1.875 | 2.7 | 3.675 | 4.8 | 6.075 | 7.5 |
| Profit, $p-c(q)$ | 15 | 14.925 | 14.7 | 14.325 | 13.8 | 13.125 | 12.3 | 11.325 | 10.2 | 8.925 | 7.5 |
| Patient benefit, $q$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Incentive configuration $8(p=15, c=0.075, b=0.5)$ |  |  |  |  |  |  |  |  |  |  |  |
| Capitation, $p$ | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 |
| Cost, $c(q)$ | 0 | 0.075 | 0.3 | 0.675 | 1.2 | 1.875 | 2.7 | 3.675 | 4.8 | 6.075 | 7.5 |
| Profit, $p-c(q)$ | 15 | 14.925 | 14.7 | 14.325 | 13.8 | 13.125 | 12.3 | 11.325 | 10.2 | 8.925 | 7.5 |
| Patient benefit, $q$ | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 |

## Appendix B Robustness

## B. 1 Constant absolute risk aversion utility

Instead of the linear utility function, we now assume that utility takes the form $U(x)=1-\exp (-0.1 x)$, where the coefficient of absolute risk aversion is set at 0.1. Table 3.13 reports the means of estimated $\alpha$ 's

Table 3.13: Estimated means of $\alpha$ in monopoly under CARA

| Incentive configurations | mean |
| :--- | :--- |
| $(p=10, c=0.075, b=0.5)$ | 0.066 |
| $(p=10, c=0.075, b=1)$ | 0.033 |
| $(p=10, c=0.1, b=0.5)$ | 0.084 |
| $(p=10, c=0.1, b=1)$ | 0.041 |
| $(p=15, c=0.075, b=0.5)$ | 0.051 |
| $(p=15, c=0.075, b=1)$ | 0.026 |
| $p=15, c=0.1, b=0.5)$ | 0.071 |
| $(p=15, c=0.1, b=1)$ | 0.034 |

The relative magnitudes between these means are quite close to those for the linear utility function in Table 3.3. For example, the mean $\alpha$ in incentive configuration ( $p=10, c=0.075, b=0.5$ ) is two times of that in configuration $(p=10, c=0.075$, $b=1$ ). The same is true for the linear utility model; see the first two rows in Table 3.3. Using the same normalization (subtracting the monopoly mean), we report the means and standard deviations of estimated $\alpha$ 's in Duopoly and Quadropoly in Table 3.14.

Table 3.14: Normalized means and standard deviations of $\alpha$ distributions under CARA

| Incentive configurations | Monopoly |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Duopoly <br> mean |  | Quadropoly <br> mean |  | SD |  |
| mean |  |  |  |  |  |  |  | SD

Again, the means have all become lower when the market becomes more competitive. The differences between the normalized duopoly and quadropoly means also point in the same direction as those in the linear utility model although the magnitudes have now become smaller (see Table 3.4).

For brevity, we do not present the estimated $\alpha$ values. Figures 3•14, 3•15 and $3 \cdot 16$ are the histograms of estimated normalized altruism distributions for the three markets. The comparisons between these with those under linear utility (histograms in Figures $3 \cdot 4,3 \cdot 5$, and $3 \cdot 6$ ) just show the differences in estimated values.

## B. 2 Between-subject subsample

We use subjects' first experiences for a between-subject experiment. From Table 3.1, roughly a third of the 361 subjects played each of the three markets in their first round, so we only can use about $1 / 3$ of the entire data. In the experiments, 124 subjects played the monopoly game first, 119 played the duopoly first, and 118 played the quadropoly first. The 8 decisions of these first games constitute the subsample.

Table 3.15 presents the first-round summary statistics of the 8 incentive-configuration games in the 3 markets. There are some small differences in the means and standard
deviations between the smaller, between-subject subsample and the full sample. Nevertheless, the means and standard deviations follow the same pattern in Table 3.2. Figures $3 \cdot 17$ to $3 \cdot 19$ present the quality choice distributions by incentive configurations for the three markets.

Table 3.15: Between-subject subsample summary statistics

| Incentive configurations | $\begin{array}{c}\text { Monopoly } \\ \text { mean }\end{array}$ |  | $(n=124)$ | $\begin{array}{l}\text { D. } \\ \text { mean }\end{array}$ | $\begin{array}{l}\text { SD }\end{array}$ | $\begin{array}{l}\text { Quadropoly } \\ \text { mean }\end{array}$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | $(n=118)$ |  |  |  |  |
| SD |  |  |  |  |  |  |$)$

Figure 3•14: Histograms of estimated $\alpha$ for CARA in each incentive configuration in monopoly


Figure 3•15: Histograms of estimated $\alpha$ for CARA in each incentive configuration in duopoly


Figure 3•16: Histograms of estimated $\alpha$ for CARA in each incentive configuration in quadropoly


Table 3.16 presents the means of estimated $\alpha$ 's in Monopoly, and they are similar to those in the full sample in Table 3.3.

Table 3.16: Estimated means of $\alpha$ in monopoly

| Incentive configurations | mean |
| :--- | :--- |
| $(p=10, c=0.075, b=0.5)$ | 1.321 |
| $(p=10, c=0.075, b=1)$ | 0.669 |
| $(p=10, c=0.1, b=0.5)$ | 1.626 |
| $(p=10, c=0.1, b=1)$ | 0.774 |
| $(p=15, c=0.075, b=0.5)$ | 1.534 |
| $(p=15, c=0.075, b=1)$ | 0.790 |
| $(p=15, c=0.1, b=0.5)$ | 1.929 |
| $(p=15, c=0.1, b=1)$ | 0.947 |

In Table 3.17, we present the means and standard deviations of estimated $\alpha$ 's in Duopoly and Quadropoly (under the same normalization as before). There are some differences from Table 3.4. In particular, the means tend to be higher in magnitude than those in the full sample. The standard deviations are also bigger, but that can be accounted for by the smaller sample size.

Table 3.17: Normalized means and standard deviations of $\alpha$ distributions

| Incentive configurations | Monopoly |  | Duopoly <br> mean <br> SD |  | Quadropoly |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | SD | mean | SD |
| $(p=10, c=0.075, b=0.5)$ | 0 | 0.798 | -1.532 | 1.170 | -1.789 | 1.076 |  |
| $(p=10, c=0.075, b=1)$ | 0 | 0.403 | -0.893 | 0.531 | -1.141 | 1.005 |  |
| $(p=10, c=0.1, b=0.5)$ | 0 | 1.027 | -1.639 | 1.053 | -2.762 | 2.782 |  |
| $(p=10, c=0.1, b=1)$ | 0 | 0.504 | -1.011 | 0.588 | -1.315 | 1.322 |  |
| $(p=15, c=0.075, b=0.5)$ | 0 | 0.902 | -2.188 | 1.045 | -2.665 | 1.511 |  |
| $(p=15, c=0.075, b=1)$ | 0 | 0.453 | -1.345 | 0.733 | -1.743 | 1.903 |  |
| $(p=15, c=0.1, b=0.5)$ | 0 | 1.156 | -2.377 | 1.708 | -2.832 | 1.641 |  |
| $(p=15, c=0.1, b=1)$ | 0 | 0.586 | -1.323 | 0.706 | -1.743 | 1.585 |  |

We next present the histograms of the actual qualities in the subsample in Figures $3 \cdot 17,3 \cdot 18$, and $3 \cdot 19$, with the frequencies written on top of each quality level. Qual-
ities in monopoly in the full and between-subject subsample show more variations. However, the duopoly and quadropoly quality distributions are remarkably similar.

Figures $3 \cdot 20,3 \cdot 21$, and $3 \cdot 22$ plot the histograms of estimated $\alpha$ distributions. (Again for brevity, we have omitted the actual estimated values.) As with the case of qualities, the estimated $\alpha$ distributions in monopoly show more differences between the full sample and the between-subject sample, but the estimated $\alpha$ distributions in duopoly and quadropoly are remarkably similar. Overall, we think that our results are robust with respect to between-subject and within-subject designs.

## B.2.1 Reduced-form analysis for between-subject subsample

Table 3.18 reports descriptive statistics on subjects' first-experience average qualities for low and high parameter levels of price, cost, and patient benefits. The entries are written with the same convention as in Table 3.10. The average qualities in Table 3.18 exhibit the same pattern as those in Table 3.10. The average quality is higher in each market at the higher price, but the relative difference declines as the market becomes more competitive. Average qualities are lower at higher cost, but the relative difference hardly varies with competition. Patient benefit does not seem to affect average qualities much. We conclude that the reduced-form analysis is robust with respect to the between-subject and within-subject designs.

Regression results for the between-subject analysis are reported in Table ??. The notation here is the same as in Table 3.11, except of course that there are no marketorder dummies. Because of the smaller sample, the $R^{2}$ 's are uniformly smaller than regressions in Table 3.11. Most estimates happen to be a little smaller in their magnitudes than in Table 3.11, but their significance remains the same.

Table 3.18: Descriptives on the variations in price, costs, and patient benefit in the subjects' first market session

|  | Low parameter level <br> Mean <br> Parameter |  | High parameter level <br> Mean |  | Relative <br> difference | N |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Price $(p=10 ; p=15)$ |  |  |  |  |  |  |
| Monopoly | 4.200 | 2.614 | 4.984 | 2.962 | 0.187 | 496 |
| Duopoly | 7.105 | 1.736 | 8.189 | 1.623 | 0.153 | 476 |
| Quadropoly | 7.466 | 1.720 | 8.509 | 1.832 | 0.140 | 472 |
| Cost $(c=0.075 ; c=0.1)$ |  |  |  |  |  |  |
| Monopoly | 4.811 | 2.866 | 4.373 | 2.757 | -0.091 | 496 |
| Duopoly | 8.074 | 1.734 | 7.221 | 1.693 | -0.106 | 476 |
| Quadropoly | 8.413 | 1.778 | 7.561 | 1.826 | -0.101 | 472 |
| Patient benefit $\left(b=0.5 ; b_{H}=1\right)$ |  |  |  |  |  |  |
| Monopoly | 4.601 | 2.807 | 4.583 | 2.834 | -0.004 | 496 |
| Duopoly | 7.529 | 1.781 | 7.765 | 1.743 | 0.031 | 476 |
| Quadropoly | 7.943 | 1.781 | 8.032 | 1.919 | 0.011 | 472 |

Figure 3•17: Between-subject quality histograms in monopoly


Figure 3•18: Between-subject quality histograms in duopoly


Figure 3•19: Between-subject quality histograms in quadropoly


Figure 3.20: Between-subject histograms of estimated monopoly $\alpha$




$p=15, c=0.075, b=0.5$
$p=15, c=0.075, b=1$


$p=15, c=0.1, b=0.5$
$p=15, c=0.1, b=1$



Figure 3•21: Between-subject histograms of estimated duopoly $\alpha$


Figure 3.22: Between-subject histograms of estimated quadropoly $\alpha$


Table 3.19: Between-subject quality regressions

| Model | (1) | (2) |
| :---: | :---: | :---: |
| Duopoly | 3.194*** | $3.125^{* * *}$ |
|  | (0.373) | (0.371) |
| Quadropoly | 3.809*** | $3.834^{* * *}$ |
|  | $\underset{0.967 * * *}{(0.39)}$ | $(0.387)$ $0.784^{* * *}$ |
| High price ( $=1$ if $p=15$ ) | (0.0459) | (0.0761) |
| High cost ( $=1$ if $c=0.1$ ) | -0.710*** | -0.437*** |
|  | (0.0437) | (0.0811) |
| High benefit ( $=1$ if $b=1$ ) | 0.100** | -0.0181 |
|  | (0.0423) | (0.0660) |
| Duopoly $\times$ High price |  | $0.300 * * *$ |
|  |  | (0.107) |
| Quadropoly $\times$ High price |  | $\begin{gathered} 0.258^{* *} \\ (0.114) \end{gathered}$ |
| Duopoly $\times$ High cost |  | -0.415*** |
|  |  | (0.111) |
| Quadropoly $\times$ High cost |  | $-0.414^{* * *}$ |
|  |  | (0.102) |
| Duopoly $\times$ High benefit |  | $0.253^{* *}$ |
| Quadropoly $\times$ High benefit |  | 0.107 |
|  |  | (0.101) |
| Session dummies | Yes | Yes |
| Constant | 4.051*** | $4.066^{* * *}$ |
|  | (0.334) | (0.331) |
| Observations | 2,888 | 2,888 |
| Subjects | 361 | 361 |
| $R^{2}$ | 0.386 | 0.388 |

Notes: OLS; robust standard errors clustered for subjects in brackets; ${ }^{* * *}$ for $p<0.01 ;{ }^{* *}$ for $p<0.05$

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## CURRICULUM VITAE


[^0]:    ${ }^{1}$ The assumption of unconfoundedness is also known as selection on observables and assumes that treatment is independent of potential outcomes conditional on observable characteristics.

[^1]:    ${ }^{2}$ The terms identified region, identified set, and bounds are used interchangeably throughout the chapter.

[^2]:    ${ }^{3}$ I consider deterministic treatment rules in my framework. See Appendix 1.7.3 for discussions on randomized treatment rules.

[^3]:    ${ }^{4}$ I use $\eta(1, \cdot), \eta(0, \cdot)$, and $p(\cdot)$ instead of $\eta(1, x), \eta(0, x)$, and $p(x)$ to highlight the fact that they are functions.

[^4]:    ${ }^{1}$ The links to download the datasets are: https://www.econometricsociety.org/content/supplement-who-should-be-treated-empirical-welfare-maximization-methods-treatment-choice and https://economics.mit.edu/faculty/angrist/data1/data/abangim02.

[^5]:    ${ }^{2}$ For reference, median earnings in the US in 1986 was $\$ 25,260$ for men and $\$ 16,230$ for women. This information is based on the March 1987 Current Population Survey conducted by the Bureau of the Census. The report can be found at the following link: https://www.census.gov/library/publications/1987/demo/p60-157.html.

[^6]:    ${ }^{3}$ More details about the OHIE and further study results can be found at http://www.nber.org/oregon.
    ${ }^{4}$ Data are available at http://www.nber.org/oregon/data.html.

[^7]:    ${ }^{1}$ We use the term altruism to mean a decision maker's enjoyment when his action in a game benefits others who are outside of the game. This has been called pure altruism in the literature (see, for example, Andreoni $(1989,1990)$ ). By contrast, a broader notion of altruism would allow a decision maker also to gain enjoyment from actions performed by his rivals in the same game. This notion of global altruism, which involves social norm or peer effect, is not our focus.

[^8]:    ${ }^{2}$ A few recent papers have made claims that markets do erode morality or social responsibility, but our view is simply that the decomposition is the key. The recent papers fall short of this; more details are in the literature review subsection.

[^9]:    ${ }^{3}$ A prominent example in which social preferences are an important concern for supply side's

[^10]:    ${ }^{4}$ There were no real patients in the laboratory, and the subjects were not medical doctors. We operationalized the quality of medical services by converting it to actual cash payments that benefited real patients outside of the laboratory. See footnote 5 and the end of Subsection 3.3.1.

[^11]:    ${ }^{5}$ Hypothetical patient profiles, characterizing patients through different benefits from medical treatment decisions, have been employed in several behavioral experiments in health with medical and non-medical students (e.g., Hennig-Schmidt, Selten, and Wiesen (2011); Kesternich, Schumacher, and Winter (2015); and Brosig-Koch, Hennig-Schmidt, Kairies-Schwarz, and Wiesen (2017)) and practicing physicians (e.g., Brosig-Koch, Hennig-Schmidt, Kairies-Schwarz, and Wiesen (2016); Brosig-Koch, Hennig-Schmidt, Kairies-Schwarz, Kokot, and Wiesen (2019)).
    ${ }^{6}$ This is the only difference from the continuous quality choice assumption in the theoretical model.

[^12]:    ${ }^{7}$ It was impractical to get subjects to play the 24 games in a random order. Too much back-and-forth between markets and incentive configurations could be confusing to subjects. Random rematching for 16 times for each subject also would be too time consuming.

[^13]:    ${ }^{8}$ To eliminate patient benefit, we would have to write a new set of instructions, and let subjects see different screens in the experiments. It is questionable how such a setup could be argued as any

[^14]:    ${ }^{10}$ For more on activities of the Christoffel Blindenmission related to cataract, see www.cbm.de/spendenCBM_Spenden_Sie_fuer_Operationen_am_Grauen_Star-494570.html.

[^15]:    ${ }^{11}$ CARA is a common functional form for risk preferences in the literature. See, for example, Barseghyan, Molinari, O'Donoghue, and Teitelbaum (2018). It has been used for estimating risk preferences from individual-level data in contexts such as property insurance (Cohen and Einav (2007); Barseghyan, Molinari, and Teitelbaum (2016)), game shows (Beetsma and Schotman (2001); Andersen, Harrison, Lau, and Rutström (2008)), and health insurance (Einav, Finkelstein, Ryan, Schrimpf, and Cullen (2013); Handel and Kolstad (2015)). In experiments, the CARA specification also has been used for estimating risk preferences (Harrison and Rutström (2008)).

[^16]:    ${ }^{12}$ Whereas the KS test is on drawn samples, our $\alpha$ 's are estimates. We did not manage to obtain the $\alpha$ 's sampling distributions, so our KS tests would not take sampling errors into account. However, as we show below, the rejections are very strong, so it is unlikely that KS tests performed poorly.

[^17]:    ${ }^{13}$ Table 3.10 aggregates the information in Table 3.2 , which contains quality-choice means and standard deviations in each incentive-configuration-market constellation.

[^18]:    Notes: OLS; robust standard errors clustered for subjects in brackets;
    *** for $p<0.01 ;^{* *}$ for $p<0.05$

