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# On the validity of the stress-flow angle as a metric for ice-shelf stability

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## 1. Introduction

In their paper examining the impact of marine ice on the structural integrity of the Larsen C Ice Shelf (LCIS), Kulessa and others (2014) argue that the angle between the ice velocity vector and the direction of the first principal stress at any point in an ice shelf – the *stress-flow angle* – can be used as a ‘first-order criterion on which to judge an ice-shelf's stability’. While we have not been able to find an exact mathematical definition of the stress-flow angle in Kulessa and others (2014), or in any of the papers subsequently using this metric, it appears reasonable to conclude from their description that it can be calculated as

$$\cos\theta = \frac{\boldsymbol{u}\cdot\boldsymbol{P}_1}{\|\boldsymbol{u}\|\,\|\boldsymbol{P}_1\|} \tag{1}$$

where θ is the stress-flow angle, $\boldsymbol{u}$ is the ice velocity vector and $\boldsymbol{P}_1$ is the first principal stress vector.

Here, we demonstrate that Eqn (1) is not *frame-indifferent*, outline why this means that it is not an admissible metric for determining the calving front dynamics or stability of an ice shelf, and emphasise that the principle of frame-indifference must hold for any metric or relation which seeks to give insight into the dynamics of an ice shelf.

Before proceeding further with our short overview of this topic, we like to clarify our use of the terms *(in)stability* and *structural integrity*. We use the term *calving instability* to refer to the process of self-sustained ice-shelf collapse through calving-front retreat, and we use the term *structural integrity* to describe the ability of an ice shelf to withstand internal and external forces without failing due to fracture. Hence, in our terminology, an isolated calving event could be ascribed to a lack of structural integrity, whilst a calving instability only occurs when an initial calving event creates the conditions for further calving events, resulting in a run-away process of calving-front retreat.

The introduction of the stress-flow angle by Kulessa and others (2014) is motivated by observations that rifts on ice shelves tend to form perpendicular to the direction of ice flow. By also assuming that rifts tend to open up when orientated perpendicular to the direction of maximum tensile stress (the first principal stress direction), it follows that when the angle between the ice velocity vector, $\boldsymbol{u}$, and the first principal stress vector, $\boldsymbol{P}_1$ – the stress-flow angle – approaches zero, rift growth would be promoted. They suggest that a calving front situated in a region of low stress-flow angles could result in an unstable frontal retreat. Conversely, a stress-flow angle close to 90 degrees (when the maximum tensile stress is aligned along the rifts) would act to suppress rift growth. They used the stress-flow angle to analyse the conditions leading to the collapse of the Larsen B Ice Shelf (LBIS) in 2002, and found that stress-flow angles at the calving front of the LBIS were near-zero following the 1995 calving event, and argued that this led to its eventual large-scale demise in early 2002. For the LCIS, again using the stress-flow angle as a ‘first-order criterion’, they concluded that the calving events between 1995 and 2002 on the LBIS ‘might serve as a plausible blue-print for Larsen C's future’.

Jansen and others (2015) used the stress-flow angle approach to analyse the LCIS, in particular by examining the growing rift that would eventually result in the calving of the A68 iceberg in 2017 (Hogg and Gudmundsson, 2017). Their work suggested that the propagation of the rift – and the subsequent calving of a large, tabular iceberg – could lead the new ice shelf calving front to be in an unstable configuration. In their study of the Wilkins Ice Shelf, Rankl and others (2017) explored a number of metrics for categorising ice-shelf stability, including the stress-flow angle, and concluded that it did not provide any additional information beyond that already contained in the principal stresses and strain rates. Borstad and others (2017) also calculated the stress-flow angles for the LCIS and assessed its use as a stability criterion. They suggested that – because this metric does not consider the magnitude of the stresses – its use is limited in understating how rifts would propagate through the ice shelf. They also stated that it cannot account for regions of high stress-flow angle in which the second principal stress (by definition perpendicular to the first principal stress direction) is also tensile, meaning that rift growth would still be promoted.

## 2. Frame-indifference

One of the foundations of classical mechanics is that the laws of physics should be the same in different inertial reference frames – i.e. they must have the same form under a Galilean

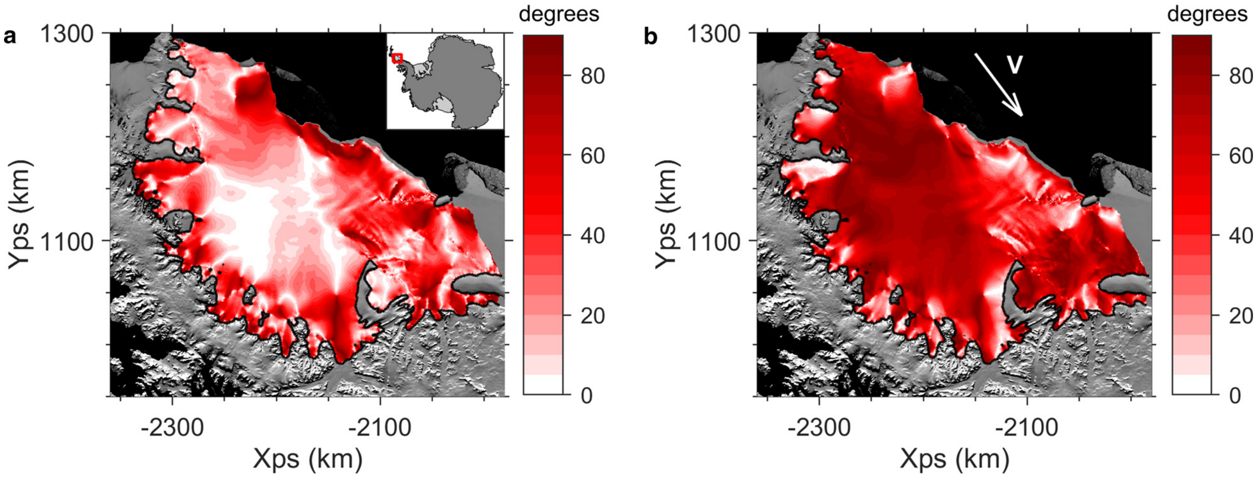

**Fig. 1.** The stress-flow angle, calculated for the LCIS in two different inertial reference frames overlain on a MODIS Mosaic of Antarctica image (Scambos and others, 2007), with the grounding line plotted in black. Panel (a) corresponds to a frame (*S*) at rest with respect to the bedrock below the ice shelf, and (b) a frame (*S*′) moving with at a constant velocity $\boldsymbol{V}$ of 3 km a$^{-1}$ in the positive *x* direction and − 4 km a$^{-1}$ in the positive *y* direction with respect to *S*, in a polar stereographic coordinate system. This relative velocity is shown by the labelled arrow in panel (b). The large differences in stress-flow angle between the two frames, particularly in the centre of the shelf, show that the two observers would reach different conclusions about the structural integrity of the shelf, violating the principle of frame-indifference.

transformation. No measurements or experiments should privilege one inertial reference frame over another. In continuum mechanics this is encompassed in the principle of *frame-indifference* (e.g. Jog, 2015, p. 204), which is typically introduced as a somewhat stricter condition, also requiring invariance with respect to time-dependent frame rotation and velocity, and including the Galilean transformation as a special case. However, any relation which is not objective under a Galilean transformation, is also not frame-indifferent.

The stress-flow angle, as previously presented and used, is constructed in such a way that it is not invariant under a Galilean transformation. This transformation takes coordinates in one inertial frame, $S$ and translates them to another inertial frame $S'$, moving at a constant velocity $\boldsymbol{V}$ relative to frame $S$. The method for transforming coordinates between inertial frames under a Galilean transformation is an elementary exercise found in most introductory textbooks on classical mechanics. In summary, the velocity vector, $\boldsymbol{u}$, is not invariant under a Galilean transformation, but the first principal stress vector, $\boldsymbol{P}_1$, is invariant. Therefore, we see from Eqn (1) that the stress-flow angle itself will be changed under a Galilean transformation, and that this metric is not frame-indifferent. Because the stress-flow angle is a metric that is calculated point-wise across an ice shelf to determine, locally, the structural integrity and calving-front stability, this lack of frame-indifference makes it inadmissible as a physical description of these processes. We now highlight this with an example on the LCIS.

### *2.1. Larsen C Ice Shelf case study*

Here, we use the vertically integrated Úa ice-flow model (Gudmundsson, 2013) to calculate the stress-flow angles on the LCIS in two inertial reference frames. The full details of the model setup and data assimilation can be found in Mitcham and others (2022). We first calculate the stress-flow angles in a frame at rest with respect to the bedrock below the ice shelf – frame $S$ – using Eqn (1). We then calculate the stress-flow angles again in a different inertial frame, $S'$, moving with a constant velocity, $\boldsymbol{V}$, of 3 km a$^{-1}$ in the positive $x$ direction and −4 km a$^{-1}$ in the positive $y$ direction with respect to $S$. The resulting fields for the stress-flow angle in $S$ and $S'$ are shown in Fig. 1.

From the two maps, we see that the observers in $S$ and $S'$ will measure very different values of the stress-flow angle. Therefore, they will also draw different conclusions about the structural integrity of the ice shelf and the stability of its calving front position from this metric, violating the principle of frame-indifference.

## 3. Concluding remarks

We have shown that the stress-flow angle metric introduced by Kulessa and others (2014), and subsequently used to asses the stability and structural integrity of ice shelves, is not frame-indifferent due to its direct dependence on the material ice-shelf velocity. For a grounded ice sheet or glacier, the ice velocity can naturally be defined relative to the solid earth with which its lower boundary is in physical contact. This relative velocity difference between the ice and the solid earth is invariant under a Galilean transformation, and can therefore be used in physically meaningful dynamic relations, such as a basal sliding law. Importantly, the equations governing the dynamics of the ice sheet are frame-indifferent by construction, and are therefore also equally valid in a frame that is not at rest with respect to the solid earth. This is not the case for the stress-flow angle metric, as has been demonstrated here.

Whilst the ice-shelf velocity can also be defined relative to the solid earth, the relative velocity between a local point on a floating ice shelf and the bedrock below it – with which it is not in contact – cannot plausibly be of any physical importance to the local calving dynamics that the stress-flow angle seeks to describe. If the stress-flow angle metric were to be updated to explicitly refer to the velocity relative to a lateral boundary velocity, this would raise further questions about what those boundary velocities should be, and the metric would no longer be local. This is not how the stress-flow angle has been presented in the existing literature.

We suggest that the principle of frame-indifference could be respected by constructing laws and metrics for ice-shelf structural integrity and calving-front stability that use the stress, strain, strain rates, etc., or any of the invariants of those tensor quantities, rather than the material ice velocity directly.

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