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Abstract

Supply chain contracting is known to suffer from inefficiency in the presence of asymmetric information. Full vertical integration would eliminate the informational inefficiency but can be strategically undesirable. Yet today's supply chain partnerships exhibit a certain degree of partial vertical integration via equity ties between the firms. Such governance forms received limited attention in supply chain research. Management literature suggests that partial vertical integration may help the firms to ease contracting problems by aligning their incentives, and thus improve the total surplus. We address this proposition by studying a model of a partially integrated supply chain in which the buyer holds an equity stake in the supplier. We adopt an operational perspective and investigate contracting between the firms within the joint economic lot size framework. We demonstrate that in this classical setting, partial integration can in fact be sufficient for eliminating informational inefficiency and achieving coordination. However, contrary to what one may expect, a tighter integration may harm supply chain performance and defeat coordination. We explain the underlying mechanism and investigate it analytically and numerically. Our results characterize sensitivity of supply chain performance to the degree of integration and stress the importance of the operational planning perspective for strategic decision making.

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1 Introduction

Most supply chain coordination models assume either independent firms engaging in a supply chain relationship or a vertically integrated supply chain structure with a common ownership (Cachon, 2003; Choi & Cheng, 2011). Nonetheless, today's economy provides a large number of examples where firms are neither fully integrated nor entirely independent, being, to a certain degree, integrated partially—by owning equity in one another (Fee et al., 2006; Hunold & Stahl, 2016), cross-shareholding, or having joint ownership of another firm. The prominent examples are the Japanese *keiretsu* (Lincoln et al., 1992; Ahmadjian & Lincoln, 2001), international joint ventures (Desai et al., 2004), or, more generally, business groups and alliances (Granovetter, 1995; Gulati & Singh, 1998).

Such hybrid forms of governance in the supplier–buyer relationships emerge for a variety of reasons. Globalization of production and trade and competition on the global scale are often cited as the major driving forces behind partnerships with shared ownership (Al-Saadon & Das, 1996; Buckley & Ghauri, 2004; Lee, 2004). Firms entering local markets may, in fact, be restricted by the regulation in choosing the size of the foreign ownership; however, even in the absence of such legal requirements, they may still prefer a partial equity stake as entry mode—e.g., for the purpose of retaining control over intangible assets (Gulati & Singh, 1998; Nakamura, 2005), monitoring and access to information (Mjoen & Tallman, 1997; Nakamura & Xie, 1998), overcoming institutional complications (Gatignon & Anderson, 1988; Yamin & Golesorkhi, 2010; Bowe et al., 2014), or retaining flexibility in continuing or leaving the business (Powell, 1990; Buckley & Ghauri, 2004). It happens, therefore, that firms are negotiating and contracting "with companies that they partly own" (Powell, 1990; Fee et al., 2006; Krishnaswami et al., 2013; Sasaki et al., 2010).

While partial vertical integration has been extensively studied in the economics literature (see Sect. 2), it received limited attention in supply-chain research. The recent work by Li et al. (2020b) and Chen et al. (2017) are notable exceptions. In these studies, the authors investigated the impact of partial integration on supply-chain coordination and competition while assuming complete information. They have shown that partial integration benefits supply chains and represents an equilibrium outcome under certain conditions. Our work extends this line of research by studying a setting with asymmetric information, which takes into account the uncertainty pertinent to supply-chain contracting. It is well known that private information, while being "a key feature of real supply chain relationships" (Liu & Cetinkaya, 2009), makes contracting between the parties problematic and can lead to inefficiency and coordination failure (Cachon, 2003). At the same time, extant literature suggests that partial integration may help the partners to ease contracting problems by aligning their incentives, and so improve the total surplus (Pisano, 1989; Gulati, 1995; Allen & Phillips, 2000; Fee et al., 2006). The purpose of this paper is to investigate this proposition in a supply chain study setting. To this end, we adopt the joint economic lot size framework as a conventional tool for studying supply chain coordination (Corbett & de Groote, 2000; Voigt & Inderfurth, 2011; Zissis et al., 2015; Kerkkamp et al., 2018; Sucky, 2006; Pishchulov & Richter, 2016).

In an earlier pilot study, Pishchulov et al. (2016) have demonstrated that dependence between the level of partial integration and supply-chain performance may be counterintuitive. Specifically, partial integration can indeed remove informational inefficiency and enable coordination, yet a tighter integration may defeat it. However, Pishchulov et al. (2016) did not provide further insights into the drivers and properties of this dependence. This study closes this gap, first, by explaining the mechanism giving rise to such a counterintuitive dependence between the level of integration and supply-chain performance. Second, we obtain conditions,

under which (i) supply-chain performance consistently improves with the level of integration, and (ii) supply-chain coordination, once achieved, persists with tighter integration. Third, our analysis establishes favourable properties of partial integration. Specifically, we show that a minor variation in the level of integration is likely to yield a substantial performance improvement in the absence of coordination, but it would not significantly detract from performance in the presence of coordination. Overall, the present work stresses the importance of taking operational-level planning into account when making strategic business decisions, and offers an analytical framework to this end.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the analytical model of coordination under partial vertical integration. Section 4 provides illustrative examples of dependence between the level of integration and supply-chain performance. Section 5 offers interpretation of that dependence. It further obtains conditions for achieving supply-chain coordination and its persistence with tighter integration. Section 6 establishes favourable properties of partial vertical integration via sensitivity analysis. Section 7 wraps up with a discussion and an outlook for future research.

2 Literature review

To date, partial vertical integration has been subject of substantial formal analysis in the economics literature and, to a smaller extent, in the supply chain literature.

Regarding the economics research, we divide the literature into cooperative and non-cooperative settings. Much of the work on non-cooperative settings has analyzed partial vertical integration as partial mergers without control rights. Flath (1989) studied a vertical channel with two tiers and Cournot competition between multiple identical firms in each tier. His analysis demonstrates that shareholdings by the suppliers in the buyers reduce the consumer price, while shareholdings in the opposite direction would countervail this effect. Greenlee and Raskovich (2006) considered a Cournot setting with a single supplier and multiple buyers, who can acquire non-controlling partial ownership in the supplier. Their results show that the consumer surplus does not depend on ownership, whereas the total surplus does. However, in the Bertrand setting with differentiated substitute products, the consumer surplus depends on the ownership structure. Hunold and Stahl (2016) found that when competition is sufficiently intense, the downstream buyers opt for non-controlling partial ownership in the most efficient supplier, which increases the consumer prices. Reiffen (1998) and Gilo and Spiegel (2011) discussed the effect of partial vertical integration on the economic mechanisms of foreclosure and analysed examples from industry practice.

Research on cooperative settings has a particular focus on incomplete contracting. Riordan (1991) studied a principal–agent setting in which a downstream firm (the principal) can *ex ante* acquire non-controlling partial ownership in an upstream supplier (the agent). His analysis reveals that partial integration effectively leads the principal to cost sharing with the agent and, therefore, reduces the principal's incentive to exercise his monopsony power, thus increasing the agent's incentive to expend an effort. On the other hand, with more integration, the agent's incentive is diminishing as she receives a smaller share of the residual profit while still bearing the full costs of the unobservable effort. The interplay of these two effects ultimately determines an optimal degree of vertical integration. In the model by Aghion and Tirole (1994), an upstream R&D firm is to deliver an innovation whose value for the downstream buyer is non-contractible. Their analysis determines the optimal allocation of property rights on the innovation and shows that the buyer's upstream ownership does not

help to mitigate the effort and allocation inefficiencies. Dasgupta and Tao (2000) showed, however, that, in the presence of other potential downstream buyers, equity stake in the upstream partner does matter, as it decreases the attractiveness of the outside option to her and increases the likelihood of her relationship-specific investment. The analysis by Harbaugh (2001) further demonstrated that with *ex post* bargaining, a partial equity stake can be effective at overcoming the classical holdup problem. Van den Steen (2002) extended these insights to bilateral dependency, cross-shareholding and vertical joint ventures. Güth et al. (2007) studied a bilateral trade with one-sided asymmetric information and showed that non-controlling cross-shareholding with minority stakes allows the parties to achieve the first-best solution despite information asymmetry, whereas one-sided shareholding may not allow that.

In the supply chain literature, Chen et al. (2017) have been the first to study partial integration via cross-shareholding in a non-cooperative newsvendor setting with Stackelberg power structure. They found that supply-chain performance does not depend on the equity share held by the follower in the leader, while it improves with an equity share held in the opposite direction. Li et al. (2020b) have recently studied a setting with two competing supply chains that sell substitutable products. They found that partial vertical integration represents an equilibrium outcome as long as the degree of product substitutability is below a threshold. Nevertheless, Chen et al. (2017) and Li et al. (2020b) assume symmetric information, while pointing to information asymmetry as a further research direction—which we aim to address in this study.

It is known that asymmetric information can potentially worsen supply-chain performance, squeezing the pie that the parties share. A large body of supply-chain research studied coordination under asymmetric information in a variety of different settings, e.g. with the supplier being privately informed of capacity cost (Bolandifar et al., 2018), manufacturing cost (Cachon & Zhang, 2006; Liu & Cetinkaya, 2009; Çakanyıldırım et al., 2012; Shao et al., 2020; Davis & Hyndman, 2021), cost and quality (Chen & Hu, 2015), and supply reliability (Yang et al., 2009; Li et al., 2020a); or the buyer being privately informed of holding cost (Corbett & de Groote, 2000; Voigt & Inderfurth, 2011; Zissis et al., 2015; Kerkkamp et al., 2018), ordering and holding costs (Sucky, 2006; Pishchulov & Richter, 2016), manufacturing cost (Ha, 2001; Liu & Cetinkaya, 2009), demand (Burnetas et al., 2007), or shortage cost and service level (Lutze & Özer, 2008). Notably, study settings in Zissis et al. (2020) and Schoenmeyr and Graves (2021) involve bilateral information asymmetry between a supplier and a buyer. Recent reviews (Shen et al., 2019; Vosooghidizaji et al., 2020) offer a taxonomy of this body of work based on the underlying supply-chain structure, nature of demand, contract type and other features. However, this literature does not consider the possibility of partial vertical integration via equity participation. Our work differs in this regard. We study a setting involving both, information asymmetry and partial vertical integration. The latter potentially allows supply-chain partners to better align their interests by increasing the size of the pie they share. In contrast to Chen et al. (2017), we find that equity holding by the less powerful party (the buyer) in the more powerful party (the supplier) can help to improve supply-chain performance and achieve coordination. Furthermore, we find that supply-chain performance in our setting can deteriorate with tighter integration—due to changing incentives of the privately informed party.

For the purposes of this analysis, we employ the conventional joint economic lot size model under asymmetric information following related work (Corbett & de Groote, 2000; Sucky, 2006; Voigt & Inderfurth, 2011; Zissis et al., 2015; Pishchulov & Richter, 2016; Pishchulov et al., 2016; Kerkkamp et al., 2018; Zissis et al., 2020), as it allows us to study the problem on hand using a simple yet meaningful model of supply-chain coordination (Kohli & Park, 1989;

Reyniers, 2001; Pibernik et al., 2011) and allows for closed-form solutions, which facilitates the subsequent analysis. To this end, we shall use the framework of Pishchulov et al. (2016) which extends the conventional model to the case of partial vertical integration. We adopt the operational planning perspective for two reasons. First, it is known that supply chains require coordination at the operational level (Banerjee, 1986; Sucky, 2006; Kerkkamp et al., 2018). Second, the analysis by Reyniers (2001) demonstrates that disregarding the operational-level aspects, such as lot-sizing and inventory-related costs, when making a strategic decision about vertical integration in a supply chain, may lead to outcomes not predicted by classical economics models. In this way, the operational setting can support strategic decision making.

3 The model

3.1 Basic assumptions and notation

Consider a supply chain consisting of a single supplier (*she*) and a single buyer (*he*), as depicted in Fig. 1 (in the following, we explain the notation used in the figure). The supplier and the buyer are labelled with literals P and A for *principal* and *agent*, respectively. The buyer serves a deterministic constant demand for a particular product and satisfies the demand from stock. Since the demand is deterministic, stock shortages are not permitted, which is a standard assumption in the literature (Banerjee, 1986; Corbett & de Groote, 2000; Sucky, 2006; Kerkkamp et al., 2018). The buyer replenishes his stock by re-ordering the product from the supplier. The supplier follows the *lot-for-lot* policy: upon receiving the buyer's order, she sets up a production batch, manufactures the order, and ships it to the buyer. The lot-for-lot policy is commonly assumed in the literature (Corbett & de Groote, 2000; Sucky, 2006; Voigt & Inderfurth, 2011; Zissis et al., 2015; Kerkkamp et al., 2018), and we maintain this assumption too. Note that while manufacturing, the supplier accumulates the finished product until the order is completed and shipped, which leads to stock holding at the supplier side. Figure 2 illustrates evolution of the inventory levels at both firms. Note also that the buyer has to place the order ahead of the lead time, which is assumed constant. Problem data is comprised at the supplier side of:

> p production rate, in units per time unit,
> R fixed setup cost per production batch,
> h_P unit inventory holding cost per time unit.

At the buyer side, the problem data is comprised of

> d deterministic constant demand for the product, in units per time unit ($d < p$),

which is known and verifiable to the supplier, and inventory-related cost parameters whose values represent the buyer's private information and are not known to the supplier with certainty. We assume a discrete set of possible values for these parameters, as this assumption is realistic in practical applications (Zissis et al., 2020). Specifically, we assume that there are two possible cost structures that the buyer can possess, each respectively comprising the following two parameters (Sucky, 2006):

> B_i fixed ordering cost assumed for the buyer's cost structure no. i,
> $h_{A,i}$ unit holding cost per time unit assumed for his cost structure no. i ($i = 1, 2$).

For brevity, we shall refer to the buyer with cost structure no. i as *buyer type i* (Harsanyi, 1967; Myerson, 1979; Burnetas et al., 2007; Kerkkamp et al., 2018). We assume that the supplier
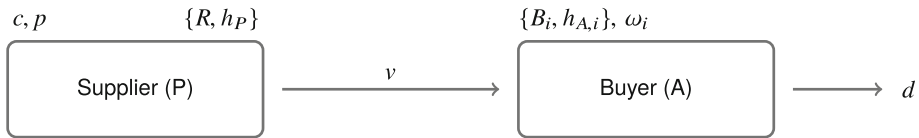
Fig. 1 The single-supplier–single-buyer supply chain under consideration (Pishchulov & Richter, 2016; Pishchulov et al., 2016). *Note* see Table 1 for description of notation
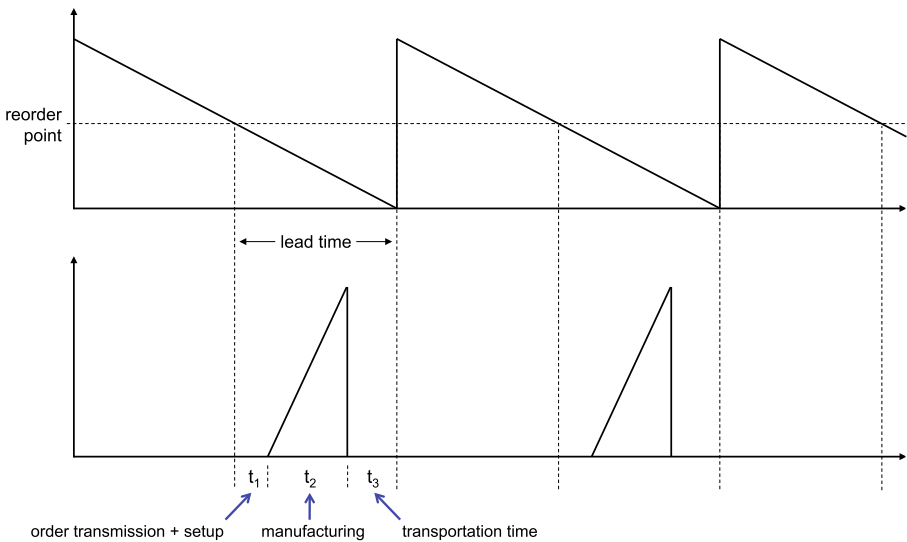

Fig. 2 Evolution of the inventory level over time at the buyer (above) and the supplier (below) (Banerjee, 1986)

knows the probability distribution over the buyer types—e.g., as the relative frequency of their occurrence in the entire population of buyers (Harsanyi, 1967; Myerson, 1979). This probability distribution is represented by

ω_i probability for the buyer to be of type i ($\omega_i > 0$, $i = 1, 2$, and $\omega_1 + \omega_2 = 1$).

We express the inventory-related costs of the supplier and the buyer per time unit as the following functions of the order size x (Banerjee, 1986; Sucky, 2006), where the superscript indicates the supply-chain member, and the subscript—the buyer type:

$$K^P(x) = R \cdot \frac{d}{x} + \frac{x}{2} \cdot \frac{d}{p} h_P, \tag{1}$$

$$K_i^A(x) = B_i \cdot \frac{d}{x} + \frac{x}{2} \cdot h_{A,i} \qquad (i = 1, 2). \tag{2}$$

The above cost functions are known to be strictly convex and unimodal—provided that all cost parameters are positive, which we assume to hold. Note that transportation time t_3 in Fig. 2 has been assumed negligibly small and excluded from consideration to simplify the exposition. For tractability, and in line with the literature (Corbett & de Groote, 2000; Sucky, 2006; Zissis et al., 2015; Kerkkamp et al., 2018), we assume that (1) and (2) are the only decision-relevant costs.

3.2 Partial vertical integration

We shall address a setting in which the supplier and the buyer are partially integrated via a direct equity stake. To examine how the supply-chain contracting becomes affected by this, we shall assume that the buyer holds an ownership share in the supplier. The magnitude of this share will be denoted by α, where $0 < \alpha < 1$. This entitles the buyer to appropriate the fraction α of the supplier's profit. Note that the extreme case of $\alpha = 0$ corresponds to the setting with fully independent firms (Sucky, 2006; Pishchulov & Richter, 2016), while the extreme case of $\alpha = 1$ represents a vertically integrated supply chain. As in most of the literature reviewed in Sect. 2, we will focus our analysis on the pure financial interest of the investing firm and exclude the control rights aspect from consideration. Thus, we assume that the supplier acts in her own interest—which is the maximization of her own profit. To proceed, we additionally introduce the following notation:

c supplier's unit production cost,
v supplier's wholesale price.

As the parties' optimal lot sizes in (1) and (2) may differ, the supplier may find it beneficial to offer a side payment y to the buyer for adopting a particular order size x. The supplier's net profit per time unit can accordingly be expressed as a function of the order size x and the side payment y as follows:

$$N^P(x, y) = (v - c)d - K^P(x) - y. \tag{3}$$

Note that such a contract between the parties translates into a unit price $v - y/d$ charged to the buyer, which gives him a discount off the regular price v (Banerjee, 1986). The total relevant costs of the buyer are reduced by the side payment y and the fraction α of the supplier's net profit (3), and equal for the buyer type i to

$$K_i^A(x) - y - \alpha N^P(x, y) = K_i^A(x) + \alpha K^P(x) - (1 - \alpha)y - \alpha(v - c)d. \tag{4}$$

Let

$$K_i^\alpha(x) := K_i^A(x) + \alpha K^P(x) \tag{5}$$

respectively represent the buyer's own inventory-related costs plus those of the supplier which become partially internalized by the buyer (as indicated by the superscript α). The order size that minimizes (5) is

$$x_{\alpha,i}^* = \sqrt{2d \cdot \frac{B_i + \alpha R}{h_{A,i} + \alpha \cdot \frac{d}{p} h_P}}. \tag{6}$$

Note that with $\alpha = 0$, it represents the classical economic order quantity (EOQ) of the buyer type i (Silver et al., 1998), and with $\alpha = 1$, it represents the joint economic lot size (JELS) of the supplier and that buyer type (Banerjee, 1986). In the latter case, we will denote the order size $x_{\alpha,i}^*$ by

$$\text{JELS}_i \equiv x_{1,i}^* \qquad (i = 1, 2). \tag{7}$$

For $\alpha \in (0, 1)$, we will refer to $x_{\alpha,i}^*$ as the *partially joint economic lot size* (PJELS) because in this case, the buyer holds a partial stake in the supplier. Since $x_{\alpha,i}^*$ minimizes the cost expression in (5) and, ultimately, in (4), this order size will be preferred by the respective buyer type i. On the other hand, the supplier's preferred order size is the one that minimizes

her cost function $K^P(x)$ defined in (1), being thus represented by her economic lot size (ELS):

$$x_P^* = \sqrt{\frac{2pR}{h_P}}. \tag{8}$$

We assume the supplier to have the initiative of offering contracts to the buyer, which is a common assumption in the related literature (Corbett & de Groote, 2000; Sucky, 2006; Voigt & Inderfurth, 2011; Zissis et al., 2015; Kerkkamp et al., 2018); we maintain this assumption for all $\alpha \in [0, 1)$ in line with the studies reviewed in Sect. 2 (Dasgupta & Tao, 2000; Harbaugh, 2001; Van den Steen, 2002). Knowing the two possible buyer types $i \in \{1, 2\}$, the supplier can anticipate the respective order sizes $x_{\alpha,i}^*$. When an order size $x_{\alpha,i}^*$ for some $i \in \{1, 2\}$ happens to be unfavorable to her, she can offer the buyer a side payment for adopting a different order size instead—preferably, the joint economic lot size $x_{1,i}^*$ (Banerjee, 1986). Since there are two possible buyer types, the supplier generally needs to prepare two such offers, each tailored to the specific cost structure of the buyer and comprising the respective JELS and the corresponding side payment. However, there is an opportunity for the buyer to pick an offer intended for the cost structure other than his true one if it is beneficial for him to do so (Sucky, 2006). This leads to opportunistic behavior of the buyer, raising the supplier's costs and negatively impacting her net profit. Thus, the supplier's contracting problem represents a principal–agent problem with *adverse selection* (Laffont & Martimort, 2002). Its solution can accordingly be approached as follows.

In view of the uncertainty about the buyer's cost structure, the supplier needs to design the offers in a way that would maximize her expected net profit. According to the *revelation principle*, this can be achieved by means of a truth-inducing *direct revelation* mechanism (Myerson, 1979, 1982) that requests the buyer to reveal his type while ensuring that he does so truthfully. Such a mechanism can be implemented by designing a menu of *take-it-or-leave-it* offers:

$$(x_1, y_1) \qquad \text{and} \qquad (x_2, y_2),$$

each consisting of an order size x_i and a side payment y_i, such that the buyer type i would prefer the i-th offer. This procedure is known as *screening*, which leads to *self-selection* by the buyer (Kreps, 1990). By virtue of (3)–(6), the supplier solves the following non-linear optimization problem:

$$\min \quad \omega_1 \cdot (K^P(x_1) + y_1) + \omega_2 \cdot (K^P(x_2) + y_2) \tag{9}$$
$$\text{s.t.} \quad K_1^\alpha(x_1) - (1-\alpha)y_1 \ \leq \ K_1^\alpha(x_{\alpha,1}^*) \tag{10}$$
$$K_2^\alpha(x_2) - (1-\alpha)y_2 \ \leq \ K_2^\alpha(x_{\alpha,2}^*) \tag{11}$$
$$K_1^\alpha(x_1) - (1-\alpha)y_1 \ \leq \ K_1^\alpha(x_2) - (1-\alpha)y_2 \tag{12}$$
$$K_2^\alpha(x_2) - (1-\alpha)y_2 \ \leq \ K_2^\alpha(x_1) - (1-\alpha)y_1 \tag{13}$$

where x_1, x_2, y_1, y_2 are decision variables and $x_1, x_2 > 0$ (note that $y_1, y_2 \geq 0$ by virtue of (10) and (11)). Objective (9) minimizes the supplier's expected costs, which is equivalent to maximizing her expected net profit as per (3).[1] Constraints (10)–(11) represent the *individual rationality* constraints. They require that accepting the respective offer does not increase the buyer's costs (5) beyond the level guaranteed by his optimal lot-sizing (6). These constraints

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[1] In shared ownership the supplier minimizes the fraction $1 - \alpha$ of her expected costs, which is equivalent to (9) (Fiocco, 2016).

hence ensure that the buyer of either type accepts the respective offer. Constraints (12)–(13) represent the *incentive-compatibility* constraints, which require the offers to be designed in a way that the buyer type 1 weakly prefers the 1st offer to the 2nd, and vice versa—which ensures truthful revelation of the buyer type. We collate the model notation in Table 1. Table 2 summarizes the model assumptions. To facilitate the model solution, we apply the following transformation to problem (9)–(13):

$$
\begin{aligned}
z_i &:= (1 - \alpha) \cdot y_i & (i = 1, 2), & \quad (14) \\
\beta &:= 1/(1 - \alpha) & (0 \leq \alpha < 1). & \quad (15)
\end{aligned}
$$

Note that z_i represents the *effective* side payment received by the buyer type i, due to the buyer's ownership share α in the supplier. By virtue of (14)–(15), we can rewrite problem (9)–(13) as follows:

$$
\begin{aligned}
\min \quad & \omega_1 \cdot (K^P(x_1) + \beta z_1) + \omega_2 \cdot (K^P(x_2) + \beta z_2) & (16) \\
\text{s.t.} \quad & K_1^\alpha(x_1) - z_1 \leq K_1^\alpha(x_{\alpha,1}^*) & (17) \\
& K_2^\alpha(x_2) - z_2 \leq K_2^\alpha(x_{\alpha,2}^*) & (18) \\
& K_1^\alpha(x_1) - z_1 \leq K_1^\alpha(x_2) - z_2 & (19) \\
& K_2^\alpha(x_2) - z_2 \leq K_2^\alpha(x_1) - z_1 & (20)
\end{aligned}
$$

We solve problem (16)–(20) in closed form, which facilitates the analysis of optimal contracting in dependence from the ownership share α. A closed-form solution to (16)–(20) is derived using Karush–Kuhn–Tucker (KKT) conditions (Bazaraa et al., 2006, Chapter 4). The solution approach consists of (i) determining all *KKT solutions* of problem (16)–(20), and (ii) choosing from the KKT solutions one with the smallest objective value.

With regard to (i), we collate the results in Table 3 (detailed derivations are available from the authors). Specifically, Table 3 comprises 15 candidate solutions, each representing a menu of offers. The menus are divided into eight generic *menu types*, each corresponding to one possible combination of non-binding constraints in the problem (16)–(20) (Sucky, 2006; Pishchulov et al., 2016). Menu types 1–3 each comprise a single menu, called *menus* 1–3. Menu types 4, 5 and 7, 8 each comprise two menus, to which we shall refer as 4a–b, 5a–b, 7a–b and 8a–b, respectively. Menu type 6 comprises four menus, to which we shall refer as 6a–d. Sub-indexing of menus of the types 4 to 6 with literals a, b, c, d follows the lexicographic order of lot sizes $x_{i_{1,2}}$ in the respective menu type in Table 3 ($i = 1, 2$): e.g., in the menu 4a, the order size x_2 is set equal to x_{2_1}, while in the menu 4b, to x_{2_2}. Sub-indexing of menus of types 7 and 8 follows the order of their appearance in Table 3 (Pishchulov & Richter, 2016). If a candidate menu of offers satisfies the respective feasibility and necessary optimality conditions (Table 3), it qualifies as a KKT solution. There may exist multiple KKT solutions, hence in step (ii) we choose a KKT solution with the smallest objective value. Finally, by substituting $y_i = \beta z_i$ ($i = 1, 2$), we obtain a menu of offers that solves the original problem (9)–(13). In the following, we shall elaborate properties of optimal solutions that will allow us to simplify steps (i) and (ii). For ease of exposition, we postpone this analysis to Sect. 6.

Table 1 Notation summary

Model data	
α	Buyer's ownership share in the supplier
B_i	Fixed ordering cost of the buyer with cost structure no. i (buyer type i)
c	Supplier's unit production cost
d	Buyer's demand for the product, in units per time unit
$h_{A,i}$	Unit holding cost per time unit of the buyer with cost structure no. i (buyer type i)
h_P	Supplier's unit inventory holding cost per time unit
ω_i	Probability for the buyer to have cost structure $\{B_i, h_{A,i}\}$
p	Supplier's production rate, in units per time unit ($p > d$)
R	Supplier's fixed setup cost per production lot
v	Supplier's wholesale price
Decision variables	
x	Order and production lot size
y	Supplier's side payment to the buyer
(x_i, y_i)	Supplier's offer to the buyer type i
z_i	Supplier's effective side payment to the buyer type i
Functions	
$K^P(x)$	Supplier's relevant costs
$K_i^A(x)$	Own relevant costs of the buyer type i
$K_i^\alpha(x)$	Effective (own and internalized) relevant costs of the buyer type i given ownership share α
$K_i(x)$	Total relevant supply-chain costs when the buyer is of type i
$\overline{K}(x)$	Difference between the total relevant supply-chain costs given buyer types 1 and 2
$\widetilde{K}(\alpha)$	Difference between the effective relevant costs of buyer types 1 and 2 acting individually
$N^P(x, y)$	Supplier's net profit
Order sizes and menus of offers	
M	$= \{1, 2, 3, 4a\text{–}b, 5a\text{–}b, 6a\text{–}d, 7a\text{–}b, 8a\text{–}b\}$ — index set of candidate menus of offers
M^*	$= \{1, 2, 3, 4a\text{–}b, 5a\text{–}b, 7a, 8a\}$ — index set of potentially optimal candidate menus of offers
$\overline{M}$	$= \{1, 2, 3, 7a, 8a\}$ — index set of menus of offers that yield a constant supply-chain performance
$\widetilde{M}$	$= \{4a\text{–}b, 5a\text{–}b\}$ — index set of menus of offers that yield a variable supply-chain performance
$v_m(\alpha)$	Candidate menu of offers with index $m \in M$ given ownership share α
$x_{\alpha,i}^*$	Partially joint economic lot size given ownership share α and buyer type i (PJELS$_i$)
$x_{0,i}^*$	Economic order quantity of the buyer type i (EOQ$_i$)
$x_{1,i}^*$	Joint economic lot size when the buyer is of type i (JELS$_i$)
x_P^*	Supplier's economic lot size (ELS)
$(\hat{x}_{\alpha,i}, \hat{y}_{\alpha,i})$	Offer for the buyer type i in the optimal menus of offers given ownership share α
Supply-chain performance	
K^*	Optimal (first-best) supply-chain performance
$\hat{K}(\alpha)$	Supply-chain performance resulting from optimal contracting given ownership share α
Aggregate and derived parameters	
$\check{\alpha}, \hat{\alpha}$	Potential local extrema of function $\widetilde{K}(\alpha)$
$\widetilde{\alpha}_0$	Extremum of $\widetilde{K}(\alpha)$ on the interval $(\alpha_0, 1)$, if any

Table 1 continued

β	$= 1/(1-\alpha)$
$\overline{B}$	$= B_1 - B_2$
$\overline{h}_A$	$= h_{A,1} - h_{A,2}$
H_i	$= \frac{d}{p}h_P + h_{A,i}$
H_P	$= \frac{d}{p}h_P$
S_i	$= R + B_i$

Table 2 Summary of model assumptions

Assumption 1. The supply chain comprises a single supplier and a single buyer

Assumption 2. The buyer serves a deterministic constant demand for a single product

Assumption 3. Lot-for-lot policy: upon receiving the buyer's order, the supplier manufactures the quantity ordered and ships it to the buyer

Assumption 4. Stock shortages are not permitted

Assumption 5. The supplier's cost structure comprises a fixed setup cost and a unit holding cost per time unit

Assumption 6. The buyer's cost structure comprises a fixed ordering cost and a unit holding cost per time unit

Assumption 7. The buyer's cost structure represents his private information and defines the buyer type

Assumption 8. There are two possible buyer types with a probability distribution over them that is known to the supplier

Assumption 9. The supplier has the initiative of offering contracts to the buyer

Assumption 10. The buyer may own a partial equity stake in the supplier

Assumption 11. The parties are acting in their own interest and seek to maximise their own profit

4 Effects of partial vertical integration on supply-chain coordination and performance

The optimal contracting solution presented in the previous section allows us to study the effect that the degree of partial integration between the supplier and the buyer takes on the level of coordination achieved in the supply chain. A supply chain is said to be *coordinated* if the actions taken by its members in their own interest happen to be optimal for the supply chain as a whole. If a particular form of the contract between supply-chain members is capable of inducing such behavior—by providing proper incentives to each party—then such contract form is said to coordinate the supply chain (Cachon, 2003).

Several studies have, however, demonstrated that coordinating contracts may lose their coordinating capability in the presence of asymmetric information (see e.g. Cachon, 2003; Corbett & de Groote, 2000; Corbett et al., 2004; Ha, 2001). This holds also true in our setting. To see this, consider first the case of fully independent firms—by assuming $\alpha = 0$. We know from Sect. 3 that order size JELS$_i$ minimizes the supply-chain costs when the buyer is of type i. Hence, the supply chain is coordinated if the order size adopted by the supply-chain members happens to be JELS$_i$, given that the buyer's true cost structure is $\{B_i, h_{A,i}\}$. In the case of symmetric information—i.e., when the buyer's true type $\hat{i}$ is known to both parties,

Table 3 Eight menu types comprising 15 menus of offers $((x_1, y_1), (x_2, y_2))$ (Pishchulov et al., [2016])

Menu type	Order sizes	Side payments: $y_i = \beta z_i$	Feasibility conditions[a]	Necessary optimality conditions[a]
1.	$x_1 = \text{JELS}_1$ $x_2 = \text{JELS}_2$	$z_1 = K_1^\alpha(x_1) - K_1^\alpha(x_{\alpha,1}^*)$ $z_2 = K_2^\alpha(x_2) - K_2^\alpha(x_{\alpha,2}^*)$	$K_2^\alpha(x_1) - z_1 \geq K_2^\alpha(x_{\alpha,2}^*)$ $K_1^\alpha(x_2) - z_2 \geq K_1^\alpha(x_{\alpha,1}^*)$	
2.	$x_1 = \sqrt{2d \cdot \dfrac{\omega_1 S_1 + \omega_2 \overline{B}}{\omega_1 H_1 + \omega_2 h_A}}$ $x_2 = \text{JELS}_2$	$z_1 = K_1^\alpha(x_1) - K_1^\alpha(x_{\alpha,1}^*)$ $z_2 = K_2^\alpha(x_2) - K_2^\alpha(x_1) + z_1$	$K_2^\alpha(x_1) - z_1 \leq K_2^\alpha(x_{\alpha,2}^*)$ $K_1^\alpha(x_2) - z_2 \geq K_1^\alpha(x_{alpha,1}^*)$	
3.	$x_1 = \text{JELS}_1$ $x_2 = \sqrt{2d \cdot \dfrac{\omega_2 S_2 - \omega_1 \overline{B}}{\omega_2 H_2 - \omega_1 h_A}}$	$z_1 = K_1^\alpha(x_1) - K_1^\alpha(x_2) + z_2$ $z_2 = K_2^\alpha(x_2) - K_2^\alpha(x_{\alpha,2}^*)$	$K_2^\alpha(x_1) - z_1 \geq K_2^\alpha(x_{\alpha,2}^*)$ $K_1^\alpha(x_2) - z_2 \leq K_1^\alpha(x_{\alpha,1}^*)$	
4.	$x_1 = \text{JELS}_1$ $x_{2_{1,2}} = \sqrt{2d} \cdot \dfrac{\sqrt{s_1} \pm \sqrt{s_2}}{\sqrt{h_1} \pm \sqrt{h_2}}$	$z_1 = K_1^\alpha(x_1) - K_1^\alpha(x_{\alpha,1}^*)$ $z_2 = K_2^\alpha(x_2) - K_2^\alpha(x_{\alpha,2}^*)$	$K_2^\alpha(x_1) - z_1 \geq K_2^\alpha(x_{\alpha,2}^*)$	$x_{\alpha,1}^* = x_{\alpha,2}^*,\ x_2 = \text{JELS}_2$ $0 \leq \mathcal{M}_1(x_2) \leq \omega_1/\omega_2$
5.	$x_{1_{1,2}} = \sqrt{2d} \cdot \dfrac{\sqrt{s_1} \pm \sqrt{s_2}}{\sqrt{h_1} \pm \sqrt{h_2}}$ $x_2 = \text{JELS}_2$	$z_1 = K_1^\alpha(x_1) - K_1^\alpha(x_{\alpha,1}^*)$ $z_2 = K_2^\alpha(x_2) - K_2^\alpha(x_{\alpha,2}^*)$	$K_1^\alpha(x_2) - z_2 \geq K_1^\alpha(x_{\alpha,1}^*)$	$x_{\alpha,1}^* = x_{\alpha,2}^*,\ x_1 = \text{JELS}_1$ $0 \leq \mathcal{M}_2(x_1) \leq \omega_2/\omega_1$
6.	$x_{1,2_{1,2}} = \sqrt{2d} \cdot \dfrac{\sqrt{s_1} \pm \sqrt{s_2}}{\sqrt{h_1} \pm \sqrt{h_2}}$	$z_1 = K_1^\alpha(x_1) - K_1^\alpha(x_{\alpha,1}^*)$ $z_2 = K_2^\alpha(x_2) - K_2^\alpha(x_{\alpha,2}^*)$		$x_{\alpha,1}^* = x_{\alpha,2}^*,\ x_1 = \text{JELS}_1,\ x_2 = \text{JELS}_2$ $\mathcal{M}_{1,2}(x_{2,1}) \geq 0,\ -\omega_2 \leq \omega_2 \mathcal{M}_1(x_2) - \omega_1 \mathcal{M}_2(x_1) \leq \omega_1$

Table 3 continued

Menu type	Order sizes	Side payments: $y_i = \beta z_i$	Feasibility conditions[a]	Necessary optimality conditions[a]
7.	$x_1 = x_2 = \sqrt{A}$ $x_1 = \sqrt{2d \cdot \dfrac{\omega_1 S_2 + \omega_2 H_2 \cdot \overline{B}/\overline{h}_A}{\omega_1 H_2 + \omega_2 S_2 \cdot \overline{h}_A/\overline{B}}}, \quad x_2 = \dfrac{A}{x_1}$	$z_1 = K_2^\alpha(x_1) - K_2^\alpha(x_{\alpha,2}^*)$ $z_2 = K_2^\alpha(x_2) - K_2^\alpha(x_{\alpha,2}^*)$	$K_1^\alpha(x_2) - z_2 \leq K_1^\alpha(x_{\alpha,1}^*)$	$x_1 = x_2 = \text{JELS}_1 = \text{JELS}_2$ $\mathcal{M}_2(x_2) \leq -1/\omega_2$
8.	$x_1 = x_2 = \sqrt{A}$ $x_1 = \sqrt{2d \cdot \dfrac{\omega_1 S_1 + \omega_2 H_1 \cdot \overline{B}/\overline{h}_A}{\omega_1 H_1 + \omega_2 S_1 \cdot \overline{h}_A/\overline{B}}}, \quad x_2 = \dfrac{A}{x_1}$	$z_1 = K_1^\alpha(x_1) - K_1^\alpha(x_{\alpha,1}^*)$ $z_2 = K_1^\alpha(x_2) - K_1^\alpha(x_{\alpha,1}^*)$	$K_2^\alpha(x_1) - z_1 \leq K_2^\alpha(x_{\alpha,2}^*)$	$x_1 = x_2 = \text{JELS}_1 = \text{JELS}_2$ $\mathcal{M}_1(x_1) \leq -1/\omega_1$

Each menu represents a candidate solution to problem (9)–(13) (Pishchulov et al., 2016)

[a] In addition, $x_1, x_2 > 0$ must hold for feasibility. A tuple (x_1, x_2, z_1, z_2) satisfying the respective feasibility and necessary optimality conditions represents a KKT solution of problem (16)–(20)

After the substitution $y_i = \beta z_i$, $i = 1, 2$, it becomes a KKT solution of problem (9)–(13). A horizontal separator line in rows 4–8 stands for the logical 'or'

Notation: ($i = 1, 2$)

$S_i = R + B_i \qquad s_i = \alpha R + B_i \qquad \overline{B} = B_1 - B_2 \qquad x_{\alpha,i}^* = \sqrt{2d\, s_i/\overline{h}_i} \qquad \mathcal{M}_1(x) = \sqrt{2d\, s_i/h_i} \qquad \mathcal{M}_1(x) = \dfrac{H_2 x^2 - 2d S_2}{\overline{h}_A x^2 - 2d B} \qquad \mathcal{A} = 2d \cdot \dfrac{\overline{B}}{\overline{h}_A}$

$H_i = \dfrac{d}{p} h_P + h_{A,i} \quad h_i = \alpha \cdot \dfrac{d}{p} h_P + h_{A,i} \quad \overline{h}_A = h_{A,1} - h_{A,2} \quad \text{JELS}_i = \sqrt{2d S_i/H_i} \quad \mathcal{M}_2(x) = -\dfrac{H_1 x^2 - 2d S_1}{\overline{h}_A x^2 - 2d B} \quad \beta = \dfrac{1}{1-\alpha}$

they can contract on the order size and size payment (x, y) defined as

$$x = \text{JELS}_i \qquad \text{and} \qquad y = K_i^A(x) - K_i^A(x_{0,i}^*)$$

where $x_{0,i}^*$ represents the buyer's economic order quantity as per equation (6). This uniquely determined contract minimizes the supplier's costs without making the buyer worse off, and, at the same time, it coordinates the supply chain by leading the parties to adopt the respective joint economic lot size (Banerjee, 1986; Sucky, 2006). The situation is, however, different in the presence of asymmetric information. The opportunity for the buyer to abuse his private information leads the supplier to solving the contracting problem (9)–(13). Obviously, the supply chain will become coordinated—regardless of the buyer's true cost structure— whenever the optimal solution is represented by a menu of offers with order sizes $x_1 = \text{JELS}_1$ and $x_2 = \text{JELS}_2$. We can infer from Table 3 that these are, in particular: (i) the menu of offers of type 1 (*menu 1*), (ii) the first of the two menus of type 7 (*menu 7a*), and (iii) the first of the two menus of type 8 (*menu 8a*). We elaborate in Propositions 1 and 2 below the conditions under which these menus of offers represent an optimal solution to the supplier's problem. This can indeed be the case for certain problem instances (see Example 4 in Table 4 below). In that event, the resulting supply-chain costs will, with certainty, be at their minimum due to coordination. When, however, optimal contracting leads to a menu of offers other than 1, 7a, 8a, supply-chain coordination will not be achieved with some non-zero probability. The loss of efficiency (in the expectation) occurs in such situations particularly due to the ability of the buyer to extract information rent from his private information, which is in line with the existing studies (Laffont & Martimort, 2002; Zissis et al., 2015; Kerkkamp et al., 2018).

A proposition made in the literature is that partial vertical integration via equity partici-pation may facilitate contracting between supply-chain partners by aligning their incentives, thus improving the total surplus (Pisano, 1989; Gulati, 1995; Allen & Phillips, 2000; Fee et al., 2006). 'Common sense' may suggest that the firms' incentives become progressively aligned with the increase of partial integration (manifested in parameter α), so that the total surplus increases in α, approaching its maximum as α approaches unity. We below demon-strate that this intuition does not, in general, hold true. We first introduce additional definitions and notation based on Pishchulov et al. (2016).

Given the constancy of demand d met by the supply chain under study, as well as the constancy of unit costs and revenues assumed for our setting, a reduction of total supply-chain costs represents accordingly an improvement of total surplus, which is thus maximized when the supply chain becomes coordinated. Therefore, we will use the minimum total costs of the vertically integrated supply chain as a benchmark for the performance of that supply chain when it lacks full vertical integration and is controlled by the participating firms in a decentralized fashion. We denote the total relevant supply-chain costs when the buyer is of type i and x is the order size adopted by the parties by

$$K_i(x) = K^P(x) + K_i^A(x). \tag{21}$$

From both the supplier's and the supply-chain perspective, there is uncertainty about the buyer's true cost structure—embodied in probabilities ω_i—which leads us to considering the expected value of (21) as the performance indicator of the supply chain (see also Corbett et al., 2004). We will be interested in the supply-chain performance under optimal contracting as determined by an optimal solution of problem (9)–(13). Let such solution be represented, for the given α, by the menu of offers

$$(\hat{x}_{\alpha,1}, \hat{y}_{\alpha,1}) \qquad \text{and} \qquad (\hat{x}_{\alpha,2}, \hat{y}_{\alpha,2}). \tag{22}$$

Table 4 Data for problem instances in Examples 1–7

No.	Supplier			Buyer	(Cost structure no. 1)			(Cost structure no. 2)		
	p	R	h_P	d	B_1	$h_{A,1}$	ω_1	B_2	$h_{A,2}$	ω_2
1.	100,000	700	390	14,500	300	200	0.55	25	785	0.45
2.	31,851	2175	3	10,617	1305	9	0.5	145	11.5	0.5
3.	15,000	100	20	10,000	60	100	0.5	20	60	0.5
4.	100,000	1000	40	60,000	400	840	0.33	10	280	0.67
5.	15,000	100	10	5000	100	30	0.75	40	20	0.25
6.	100,000	230	760	47,000	250	630	0.5	16	12	0.5
7.	100,000	900	950	75,000	40	160	0.5	940	140	0.5

Data in Example 2 is based on empirical estimates from Lal and Staelin (1984)

We then define the following expected costs under optimal contracting:

$$\hat{K}(\alpha) = \omega_1 K_1(\hat{x}_{\alpha,1}) + \omega_2 K_2(\hat{x}_{\alpha,2}) \tag{23}$$

as the performance level realized by the supply chain, with α representing the degree of partial integration. We assume that, if there are several menus of offers (22) that solve (9)–(13) and, thus, happen to be optimal to the supplier, then she chooses from them one for which the expected supply-chain costs in (23) are minimal. The resulting performance level $\hat{K}(\alpha)$ is then to be compared with the benchmark

$$K^* = \omega_1 K_1(x^*_{1,1}) + \omega_2 K_2(x^*_{1,2}) \tag{24}$$

which represents the minimum expected supply-chain costs under full integration, and where $x^*_{1,i} \equiv \text{JELS}_i$, as defined in (6). The difference $\hat{K}(\alpha) - K^*$ is called *coordination deficit*, which is obviously a non-negative value that captures the informational inefficiency of contracting. In accordance with the definition of coordination given at the beginning of this section, the supply chain is *coordinated* when the coordination deficit equals zero. Indeed, in this case, actions taken by the supply-chain members in their own interest yield a supply-chain performance matching the optimal level K^*. Otherwise, when the coordination deficit is positive, supply-chain performance is suboptimal, and hence the supply chain lacks coordination.

We next consider a number of examples that illustrate possible effects of partial integration on supply-chain coordination and performance. Table 4 presents seven problem instances, of which we refer here to *Examples 1 to 5*. Figure 3a–e depict supply-chain performance $\hat{K}(\alpha)$ in the respective examples.

In each graph, the degree α of partial integration varies from 0 to 1. The respective performance benchmark K^* is indicated in each figure as well. Each plot also highlights the menu type (cf. Table 3) that proves to be optimal in the respective range of α; the numbers in circles indicate that menu type. Note that we employ closed-form solutions derived in Sect. 3 to produce Fig. 3a–e. Specifically, $\hat{K}(\alpha)$ and K^* are defined by Eqs. (23)–(24). Equation (23), in turn, uses an optimal menu of contracts which we obtain in Sect. 3 as a solution to contracting problem (16)–(20) and present in closed form in Table 3. Equation (24) uses optimal lot sizes presented in closed form in equation (7). Figure 3a–e confirm the observations below (Pischulov et al., 2016).

Observation 1 *In certain problem instances, a partial vertical integration via a minority equity holding can be sufficient for achieving supply-chain coordination under optimal contracting, thus obviating the need for the full vertical integration (Fig. 3a, c).*
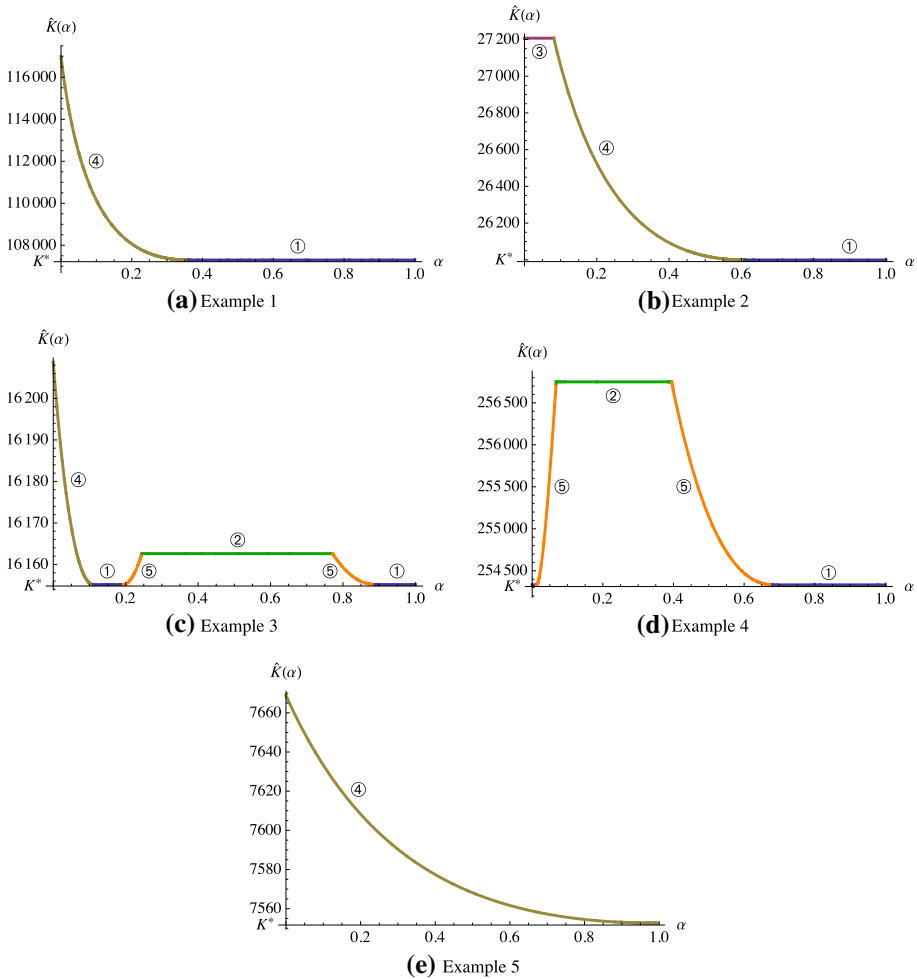
Fig. 3 Supply-chain performance realized at various degrees of partial vertical integration in Examples 1–5. K^* represents the performance benchmark in each example. Graph colouring highlights the ranges of α over which a specific menu type is optimal (see Table 3). The numbers in circles indicate menu type

Observation 2 *Supply-chain performance needs not to be strictly monotonic in the degree of partial vertical integration (Fig. 3b), nor does it need to be weakly monotonic (Fig. 3c, d).*

Observation 3 *Coordination can be lost with increasing degree of partial integration (Fig. 3c), even if it persisted from the outset (Fig. 3d)—whereas in other cases, coordination may not be achievable at any degree of partial vertical integration (Fig. 3e).*

Observation 1 highlights the cases in which partial vertical integration takes a positive effect on supply-chain performance and coordination. It comes about because equity participation creates a bond between the partners, helping them to align their incentives and overcome contractual complications, as discussed in Sect. 1. In such situations, tighter integration strengthens this bond by making the investing firm "partially internalize the effect of its actions on a trading partner" (Fee et al., 2006). Observation 2 indicates, however,

that supply-chain performance does not need to always improve with tightening integration because the partners' incentives may remain insensitive to equity participation within certain bounds or may even become distorted so that supply-chain performance deteriorates. Indeed, Observation 3 stresses that increasing integration beyond a certain point may become detrimental for supply-chain coordination when this creates incentives for the investing firm to act opportunistically. In such situations, there are limits to which firms can usefully internalize the effect of their actions. In practice, limits to partial integration may also exist due to reluctance of the investee to be exposed to too much influence by the investing partner (Allen & Phillips, 2000; Fee et al., 2006).

Observation 1 further indicates that coordination can be achieved in our setting with a minority stake of the less powerful party (the buyer) in the more powerful party (the supplier), which is in contrast to findings by Chen et al. (2017) for their non-cooperative setting. Furthermore, Observation 2 is in marked contrast with the monotonic dependence of supply-chain performance on the degree of integration in their setting. Observations similar to ours have been made by Illing (1992) in a bargaining setting with shared property rights. It becomes of interest to derive more general conclusions about the relationship between the degree of partial integration and supply-chain performance and coordination. In the following section, we establish its key properties and make them subject to interpretation.

5 Emergence and persistence of coordination

In this section we investigate conditions, under which supply-chain coordination emerges and persists with varying degrees of partial integration. We divide this analysis into two cases: when the system-optimal JELS does not depend on the buyer's cost structure, and when it does. To simplify the subsequent exposition, we employ the following notation:

$$ S_i = R + B_i \qquad \text{and} \qquad H_i = \frac{d}{p} h_P + h_{A,i} \qquad (i = 1, 2). \qquad (25) $$

Furthermore, let $\text{ELS} \equiv x_P^*$ represent the supplier's economic lot size and $\text{EOQ}_i \equiv x_{0,i}^*$—the economic order quantity of the buyer type i as per (6), (8).

5.1 When JELS does not depend on the buyer's cost structure

Consider first the class of problem instances in which the buyer's specific cost structure does not affect the system-optimal lot sizing—so that $\text{JELS}_1 = \text{JELS}_2$. We obtain the following result, which is a generalisation of Pishchulov and Richter (2016, Proposition 3).

Proposition 1 *Let $JELS_1 = JELS_2$ hold. Then the buyer's possible cost structures either coincide or strictly dominate one another in Pareto sense. Without loss of generality, assume that $B_1 \geq B_2$ and $h_{A,1} \geq h_{A,2}$. Then the following is true:*

i) *If $EOQ_1 = EOQ_2 = ELS$ then the supply chain happens to be coordinated at every degree of partial vertical integration $\alpha \in [0, 1)$ without contracting on the lot size.*

ii) *Otherwise, if $\omega_1 \leq H_2/H_1$ holds, then optimal contracting leads to supply-chain coordination at every degree of partial vertical integration $\alpha \in [0, 1)$.*

iii) *Otherwise, optimal contracting does not lead to supply-chain coordination at any $\alpha \in [0, 1)$.*

Proof The proof follows that of Pishchulov and Richter (2016, Prop. 3) and is available on request. □

Assertion *i)* of the proposition concerns cases when contracting on the lot size is unnecessary, and is in accordance with intuition. For all other cases, assertions *ii)–iii)* state that optimal contracting either yields coordination irrespective of the degree of partial vertical integration, or is otherwise unable to do so at any degree $\alpha \in [0, 1)$—despite the identity $\mathrm{JELS}_1 = \mathrm{JELS}_2$ (see Fig. 3e). Specifically, the proof of Proposition 1 shows that in *ii)*, it is optimal for the supplier to offer a menu 1, 7a, or 8a to the buyer, which yields coordination, whereas in *iii)*, the supplier will offer a menu of type 4 or 5, so that coordination fails. Pishchulov and Richter (2016, p. 717) provide interpretation of this result for the case of $\alpha = 0$, which carries over to $\alpha > 0$ in a straightforward way (we therefore omit this exposition here for reasons of space).

To summarize, by applying Proposition 1 to a problem instance with $\mathrm{JELS}_1 = \mathrm{JELS}_2$, we can easily determine once and for all $\alpha \in [0, 1)$ whether coordination is achievable. The implication of the proposition is that managerial decision-making with regard to the choice of α becomes significantly simplified when the system-optimal policy is insensitive to the buyer's cost structure. Especially when assertions *i)* or *ii)* of the proposition hold, the decision-makers are assured that the supply chain can be coordinated under any specific choice of α. Otherwise, when assertion *iii)* holds, the supply chain is expected to operate inefficiently at any degree of partial integration. In the latter case, the expected amount of inefficiency still depends on α. We will investigate this dependence in Sect. 6 below.

5.2 When JELS depends on the buyer's cost structure

We next refer to the class of instances with $\mathrm{JELS}_1 \neq \mathrm{JELS}_2$—i.e., when the buyer's cost structure affects the system-optimal lot sizing. For brevity, let

$$H_P = \frac{d}{p} h_P \qquad \overline{B} = B_1 - B_2 \qquad \overline{h}_A = h_{A,1} - h_{A,2}. \qquad (26)$$

Further, for an order size x, let

$$\overline{K}(x) = K_1(x) - K_2(x) \qquad (27)$$

represent the difference in the supply-chain costs under the buyer's cost structures no. 1 and 2. Let further

$$\widetilde{K}(\alpha) = K_1^\alpha(x_{\alpha,1}^*) - K_2^\alpha(x_{\alpha,2}^*)$$

represent the difference in the *effective* inventory-related costs incurred by the buyer types 1 and 2 when adopting the respectively optimal partially joint economic lot sizes. We then obtain the following result.

Proposition 2 *Let $\mathrm{JELS}_1 \neq \mathrm{JELS}_2$ hold, and let $\alpha \in [0, 1)$ be given. Optimal contracting yields supply-chain coordination if and only if the following holds:*

$$\overline{K}(\mathrm{JELS}_1) \leq \widetilde{K}(\alpha) \leq \overline{K}(\mathrm{JELS}_2). \qquad (28)$$

Proof The proof is straightforward and based on the feasibility conditions of the menu of offers of type 1 (see Table 3). We present the proof in Sect. A.1 of Appendix A in full detail. □
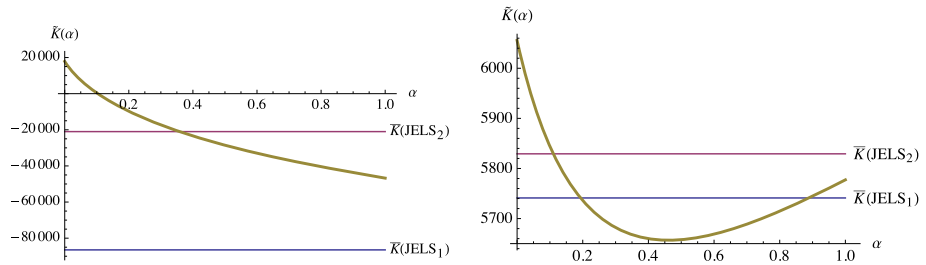
Fig. 4 Achieving supply-chain coordination by virtue of condition (28) in Examples 1 (left) and 3 (right)

Thus, given $\mathrm{JELS}_1 \neq \mathrm{JELS}_2$, optimal contracting yields coordination whenever $\widetilde{K}(\alpha)$ is found within the bounds $[\overline{K}(\mathrm{JELS}_1), \overline{K}(\mathrm{JELS}_2)]$. Figure 4 illustrates this in Examples 1 and 3 of Sect. 4 (Fig. 3a, c). Note that given $\mathrm{JELS}_1 \neq \mathrm{JELS}_2$, coordination can only be achieved with the menu of offers of type 1, whereas menus 7a and 8a cannot be coordinating. The reason of this is that the latter menus are either undefined (when the buyer's possible cost structures are weakly Pareto-efficient) or fail to offer the order sizes $x_i = \mathrm{JELS}_i$ to the buyer types $i = 1, 2$ (see Table 3).

Using condition (28), we can easily determine whether optimal contracting leads to supply-chain coordination without any vertical integration. If not, then it becomes of interest to know the minimum degree of (partial) integration at which coordination would emerge. This question is answered by the following corollary from Proposition 2 and continuity of $\widetilde{K}(\alpha)$.

Corollary 1 *Let $\mathrm{JELS}_1 \neq \mathrm{JELS}_2$ hold. If optimal contracting does not lead to supply-chain coordination in the absence of partial integration then the minimum degree of integration at which it does is the smallest α, for which any of the two inequalities in (28) holds as equality.*

Figure 4 illustrates this. Note that exactly one of the two inequalities in condition (28) is violated when there is no coordination. When the graph of $\widetilde{K}(\alpha)$ enters the corridor of values $[\overline{K}(\mathrm{JELS}_1), \overline{K}(\mathrm{JELS}_2)]$ for the first time, condition (28) becomes satisfied as equality, which enables coordination. We have not been able to derive a closed-form expression of such a minimum 'coordinating' degree of partial integration, but it can be easily obtained for a given problem instance by solving the respective equation numerically. Below we gain insight into the mechanism underlying the 'coordinating' capability of partial integration.

Interpretation 1 (**Achieving coordination via partial integration**) We use Example 1 for this purpose, see Fig. 3a. Figure 5a depicts the cost curves and the menu of offers of type 1 when $\alpha = 0$. There is obviously a lack of coordination because the menu of type 1 is not incentive-compatible, as buyer type 1 prefers to pick the 2nd offer. In turn, Fig. 5b represents the situation with $\alpha = 0.36$, where the menu of type 1 happens to be incentive-compatible and yields coordination (note that for ease of exposition, we represent the side payments in terms of z_1, z_2). Thus, increasing α from 0 to 0.36 is just sufficient to make the incentives of the system-optimal menu of offers compatible with the buyer types.

To see the reason of this, observe that at each value of α, the (in)compatibility of the said incentives results from interaction between three curves: $K^P(x)$, $K_1^A(x)$ and $K_2^A(x)$. Specifically, a degree α of partial vertical integration makes the buyer type i act in terms of the cost function $K_i^\alpha(x) = K_i^A(x) + \alpha K^P(x)$. Thus, with growing α, his effective cost function becomes determined to an ever bigger extent by the supplier's cost function $K^P(x)$. This changes the shape and position of both buyer types' curves $K_i^\alpha(x)$, as illustrated by
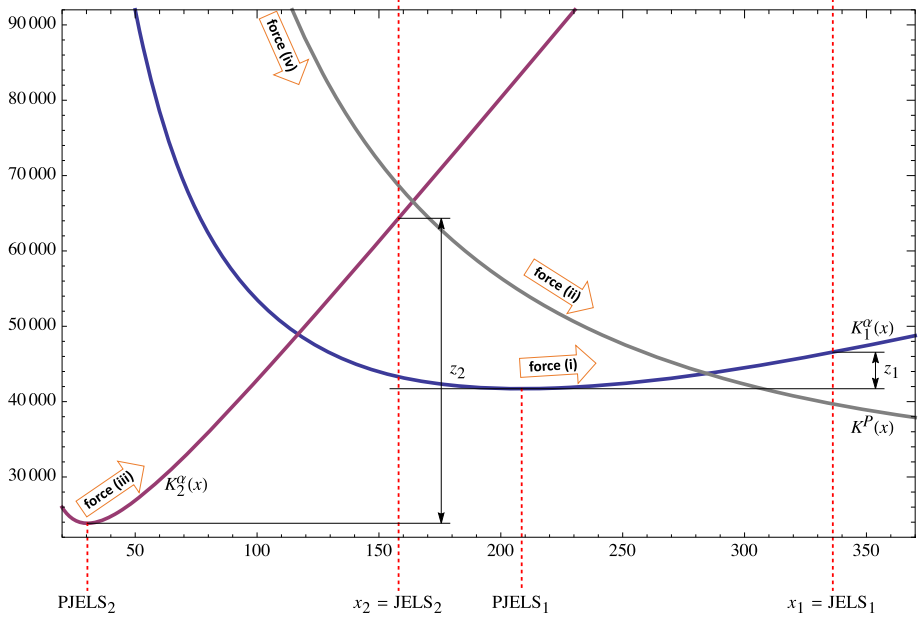
(a) $\alpha = 0$, incentive-incompatibility
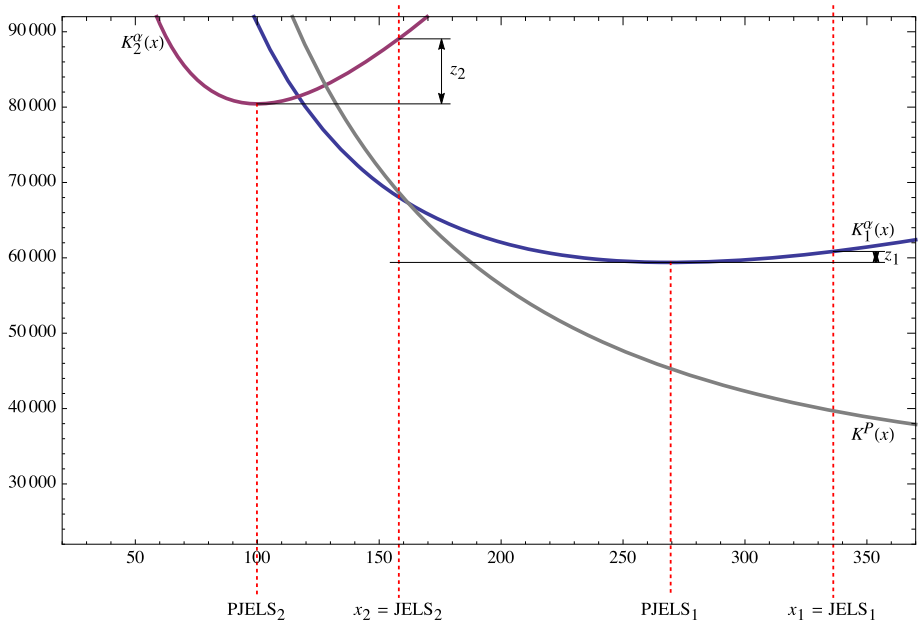
(b) $\alpha = 0.36$, incentive-compatibility

Fig. 5 Menu of offers of type 1 in Example 1 with $\alpha = 0$ (above) and $\alpha = 0.36$ (below)

Table 5 Forces (i)–(iv) driving a buyer type's cost and compensation under varying α when he sticks to another type's offer

	Buyer type's own	Buyer's internalized
Cost	(i)	(ii)
Compensation	(iii)	(iv)

Fig. 5a, b. This, in turn, has two consequences: first, the order size PJELS_1 moves away from $x_2 = \text{JELS}_2$, which increases the cost of adopting x_2 instead of PJELS_1 for the buyer type 1; second, the order size PJELS_2 moves closer to $x_2 = \text{JELS}_2$, which decreases the side payment z_2 intended as compensation for adopting the order size x_2. Both these effects—increasing cost and decreasing compensation—jointly reduce the attractiveness of choosing (x_2, z_2) for the buyer type 1, until any benefit of doing so disappears at $\alpha \approx 0.36$, as it is presented in Fig. 5b.

We can gain a deeper insight into the magnitude of the above two effects by considering the components $K_i^A(x)$ and $\alpha K^P(x)$ of the i-th buyer type cost function separately. Note that in Fig. 5a, function $K_i^\alpha(x)$ represents just $K_i^A(x)$, $i = 1, 2$. Let $\alpha = 0$, and refer to buyer type 1 first. We know that with increasing α, his individually optimal order size PJELS_1 shifts to the right. Looking at Fig. 5a, we can see that two forces then come into play: (i) the component $K_1^A(\text{PJELS}_1)$ of his optimum costs will increase, so that the contribution $K_1^A(\text{JELS}_2) - K_1^A(\text{PJELS}_1)$ to his cost of adopting JELS_2 instead of PJELS_1 will decrease; and (ii) the supplier's cost $K^P(\text{PJELS}_1)$ will decrease, so that the contribution $\alpha \cdot [K^P(\text{JELS}_2) - K^P(\text{PJELS}_1)]$ to his cost of adopting JELS_2 instead of PJELS_1 will increase. Note that force (i) drives this buyer type's *own* cost, whereas force (ii)—his *internalized* cost due to shared ownership. The second force is dominating, so that, in effect, the cost of adopting JELS_2 instead of PJELS_1 by the buyer type 1 will increase.

We proceed in a similar way with the buyer type 2. We know that, with increasing α, his individually optimal order size PJELS_2 shifts to the right. We can see in Fig. 5a that the following two forces then emerge: (iii) the cost component $K_2^A(\text{PJELS}_2)$ will increase, so that the respective contribution $K_2^A(\text{JELS}_2) - K_2^A(\text{PJELS}_2)$ to the compensation z_2 will decrease; and (iv) the supplier's cost $K^P(\text{PJELS}_2)$ will decrease, but the negative contribution $\alpha \cdot [K^P(\text{JELS}_2) - K^P(\text{PJELS}_2)]$ to the compensation z_2 will decrease with α either (i.e., its absolute value will increase)—which happens due to smallness of the multiplier α.[2] Thus, both forces jointly lessen the amount of compensation z_2.

Table 5 collates the meaning of forces (i)–(iv). Summarizing, forces (ii)–(iv) act to restore incentive-compatibility, while force (i) counteracts. The former are dominating to the extent that incentive-compatibility becomes restored when $\alpha \approx 0.36$. In general, the strength and direction of these forces depend on the shape and position of the cost curves $K^P(x)$, $K_1^A(x)$, $K_2^A(x)$, and so does their net effect. □

Figure 4 suggests that tightening integration may, however, defeat coordination, forcing $\widetilde{K}(\alpha)$ to leave the corridor of values $[\overline{K}(\text{JELS}_1), \overline{K}(\text{JELS}_2)]$. Figure 3c, d in Sect. 4 illustrate this by depicting supply-chain performance $\hat{K}(\alpha)$. We below explain this mechanism in greater detail.

Interpretation 2 (**Coordination loss due to partial integration**) We use Example 4 for this purpose. Recall that in this example, supply-chain coordination is achieved without any

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[2] The effect of small α can be explained as follows: consider a negative-valued, increasing function $f(\alpha)$. Then the derivative $(\alpha f(\alpha))' = f(\alpha) + \alpha f'(\alpha)$ is obviously negative for sufficiently small values of α, making $\alpha f(\alpha)$ decrease.

vertical integration (see Fig. 3d). Figure 6a shows that the system-optimal menu of offers of type 1 is indeed incentive-compatible when $\alpha = 0$: none of the buyer types would benefit from choosing the offer intended for the other type. A slight increase of α to ≈ 0.012 leads, however, to coordination failure as it becomes more attractive for the buyer type 2 to choose the 1st offer. For illustrative purposes, we depict this situation in Fig. 6b with $\alpha = 0.12$. Note that, as α increases, both PJELS_1 and PJELS_2 move to the right.

The above Interpretation 1 has identified forces (i)–(iv) that govern the development of incentives in the system-optimal menu of offers with varying α (Table 5). We can similarly observe action of these forces here by referring to Fig. 6a: (i) the contribution $K_2^A(\text{JELS}_1) - K_2^A(\text{PJELS}_2)$ to the cost of adopting JELS_1 instead of PJELS_2 by the buyer type 2 is decreasing; (ii) the contribution $\alpha \cdot [K^P(\text{JELS}_1) - K^P(\text{PJELS}_2)]$ to that cost is decreasing either (i.e., its absolute value is increasing)—due to smallness of the multiplier α; (iii) the contribution $K_1^A(\text{JELS}_1) - K_1^A(\text{PJELS}_1)$ to the compensation z_1 is decreasing; and (iv) the contribution $\alpha \cdot [K^P(\text{JELS}_1) - K^P(\text{PJELS}_1)]$ to the compensation z_1 is also decreasing. Thus, both the costs of adopting JELS_1 by the buyer type 2 and the compensation for that are decreasing. The former effect is, however, more pronounced—so that the compensation exceeds the costs. This is determined by the position of the curves $K_{1,2}^A(x)$: first, observe that the order size $\text{EOQ}_2 \equiv \text{PJELS}_2|_{\alpha=0}$ deviates from JELS_2 more so than $\text{EOQ}_1 \equiv \text{PJELS}_1|_{\alpha=0}$ deviates from JELS_1 (Fig. 6a). As a result, PJELS_2 moves with increasing α faster to the right than PJELS_1 does, as Fig. 6b illustrates. This leads to stronger growth along the curve $K_2^A(x)$ in Fig. 6a than along the curve $K_1^A(x)$, despite the latter growing steeper than the former. As a result, force (i) is stronger than force (iii). Secondly, although the curve $K^P(x)$ exhibits a bigger reduction in the magnitude of decline over the interval $[\text{PJELS}_2, \text{JELS}_1]$ than it does over $[\text{PJELS}_1, \text{JELS}_1]$, force (ii) is still stronger than force (iv) due to smallness of α. In effect, the cost of adopting JELS_1 by the buyer type 2 declines faster than the respective compensation, which distorts the incentives and leads to the loss of coordination despite a higher degree of partial integration. □

It thus becomes of interest to derive the conditions under which supply-chain coordination, once attained at a particular α_0, persists also for $\alpha > \alpha_0$. To this end, let $H_P, \overline{B}, \overline{h}_A$ be as defined in (26). Consider the following two *critical numbers*:

$$\check{\alpha} = \frac{B_1 h_{A,2} - B_2 h_{A,1}}{R\overline{h}_A - H_P \overline{B}} \qquad \hat{\alpha} = \frac{R^2 h_{A,1} h_{A,2} - H_P^2 B_1 B_2}{H_P(B_1 + B_2) - R(h_{A,1} + h_{A,2})} \cdot \frac{1}{R H_P} \qquad (29)$$

Note that a critical number is undefined if its expression in (29) has a zero denominator. We will below prove that critical numbers $\hat{\alpha}$ and $\check{\alpha}$, when defined, represent stationary points of $\widetilde{K}(\alpha)$. The following intermediate result is helpful in characterizing the persistence of coordination.

Lemma 1 *At most one of the two critical numbers $\hat{\alpha}$ and $\check{\alpha}$ is defined and positive.*

Proof See Sect. A.2 of Appendix A. □

Hence, by virtue of Lemma 1, critical numbers $\hat{\alpha}$ and $\check{\alpha}$, when both are defined, cannot be both positive. Let now some $\alpha_0 \in [0, 1)$ be given. If there is a critical number that lies within the open interval $(\alpha_0, 1)$ then this number is uniquely determined by virtue of Lemma 1. We will denote this number by $\widetilde{\alpha}_0$, and otherwise consider $\widetilde{\alpha}_0$ as undefined. Thus, in formal
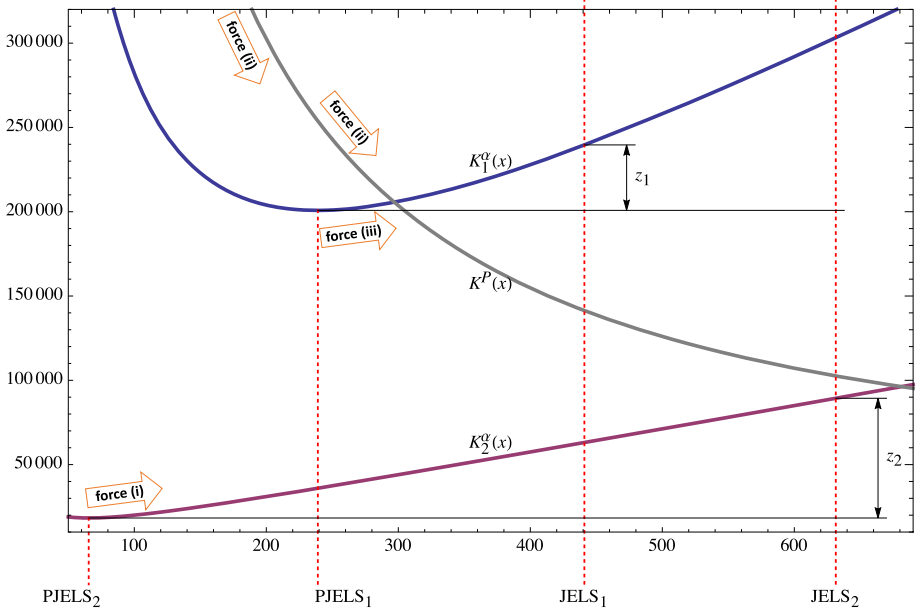
(a) $\alpha = 0$, incentive-compatibility
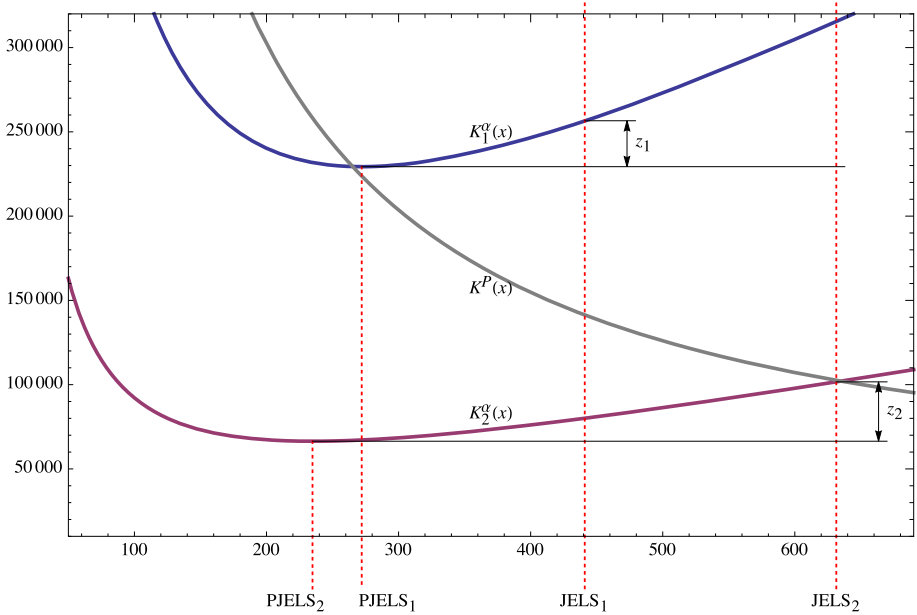
(b) $\alpha = 0.12$, incentive-incompatibility

Fig. 6 Menu of offers of type 1 in Example 4 with $\alpha = 0$ (above) and $\alpha = 0.12$ (below)

terms:

$$\widetilde{\alpha}_0 = \begin{cases} \check{\alpha}, & \text{if } \check{\alpha} \text{ is defined and } \alpha_0 < \check{\alpha} < 1, \\ \hat{\alpha}, & \text{if } \hat{\alpha} \text{ is defined and } \alpha_0 < \hat{\alpha} < 1, \\ \text{undefined}, & \text{otherwise}. \end{cases} \tag{30}$$

Then, we obtain the following

Proposition 3 *Let $JELS_1 \neq JELS_2$ hold, and let $\alpha_0 \in [0, 1)$ be given. Optimal contracting yields supply-chain coordination at every $\alpha \in [\alpha_0, 1)$ if and only if condition (28) holds for $\alpha = \alpha_0$ and for $\alpha = \widetilde{\alpha}_0$ when the latter is defined.*

Proof See Sect. A.3 of Appendix A. □

Using Proposition 3, we can easily establish whether supply-chain coordination persists with an increasing degree of partial vertical integration in a problem instance when $JELS_1 \neq JELS_2$. If coordination is achieved with some $\alpha_0 \in [0, 1)$, it requires then to verify condition (28) for at most one additional value of α. Indeed, when condition (28) holds at $\alpha = \alpha_0$, and $\widetilde{K}(\alpha)$ has no local extrema on $(\alpha_0, 1)$, then (28) will automatically hold for all $\alpha \in [\alpha_0, 1)$—as it can be shown that (28) is guaranteed to hold for $\alpha = 1$. This is illustrated by the left-hand graph in Fig. 4. The proof of Proposition 3 shows that, in such cases, $\widetilde{\alpha}_0$ is undefined. However, when $\widetilde{K}(\alpha)$ has a local extremum on $(\alpha_0, 1)$, then it must occur at $\widetilde{\alpha}_0$; for this reason, it suffices to check condition (28) at $\alpha = \widetilde{\alpha}_0$ in order to establish the persistence of coordination for all $\alpha \in [\alpha_0, 1)$. For example, the right-hand graph in Fig. 4 exhibits an extremum beyond the corridor $[\overline{K}(JELS_1), \overline{K}(JELS_2)]$. As a result, there exists a range of values around $\widetilde{\alpha}_0 \approx 0.46$ at which optimal contracting does not lead to supply-chain coordination (cf. Fig. 3c). Propositions 1 and 3 further imply:

Corollary 2 *If optimal contracting yields coordination at some α_0 then there exists at most one interval $(\underline{\alpha}, \bar{\alpha}) \subseteq (\alpha_0, 1)$, maximal by inclusion, where optimal contracting will not yield coordination.*

Figures 3a–d demonstrate this for problem instances when $JELS_1 \neq JELS_2$: once supply-chain performance $\hat{K}(\alpha)$ reaches the benchmark level K^* at some α_0, it leaves that level with increasing α at most once. This is because function $\widetilde{K}(\alpha)$ has at most one extremum on interval $(0, 1)$, as explained by the following

Interpretation 3 (**Corollary** 2) We refer to Example 3 for this purpose. Figure 3c depicts supply-chain performance $\hat{K}(\alpha)$, whereas the right-hand graph in Fig. 4 depicts the corresponding function $\widetilde{K}(\alpha)$. At $\alpha = 0$, $\widetilde{K}(\alpha)$ is found beyond the corridor $[\overline{K}(JELS_1), \overline{K}(JELS_2)]$, which implies incentive-incompatibility of the menu of offers of type 1 and lack of coordination, as per Proposition 2. However, increasing α allows to gradually remove incentive-incompatibility, as per Interpretation 1, so that $\widetilde{K}(\alpha)$ enters the said corridor. Increasing α even further distorts the incentives again, as per Interpretation 2. The maximum distortion obviously occurs at the extremum of $\widetilde{K}(\alpha)$, after which incentives begin to restore towards compatibility, ultimately leading to coordination. This turnaround in the development of incentives can be explained in terms of forces (i)–(iv) (see Table 5). In fact, these forces come to balancing each other at the functions's extremum because the latter occurs at $\alpha = \check{\alpha}$, which is the degree of partial integration at which $PJELS_1$ and $PJELS_2$ coincide. By inspecting the contributions to the cost and compensation driven by forces (i)–(iv), as expressed in Interpretation 2, it is easy to verify that such α for which $PJELS_1 = PJELS_2$ holds indeed makes the marginal cost (a combined effect of forces (i)–(ii)) equal to the

marginal compensation (a combined effect of forces (iii)–(iv)). Thus the process of incentives distortion, manifested in the stronger increase of compensation received by the buyer type 2 when picking the 1st offer, as compared to his cost of doing so, stops at this point and begins to move in the opposite direction towards restoration of incentive-compatibility. By inspecting the motion of PJELS$_1$ and PJELS$_2$ in Fig. 6a, b, it is easy to see that they may cross only once. Furthermore, the proof of Proposition 3 reveals that the extremum of $\widetilde{K}(\alpha)$ may also occur at $\alpha = \hat{\alpha}$, which is the degree at which the supplier's ELS is equal to the geometric mean of PJELS$_1$ and PJELS$_2$. Again, by inspecting Fig. 6a, b, it easy to see that this is not possible to happen in the given example. This explains the uniqueness of extremum of $\widetilde{K}(\alpha)$ and the assertion of Corollary 2 that coordination can be lost 'at most once'. □

The key outcomes and implications of the analysis in this section are three-fold. First, when the system-optimal policy does not depend on the buyer's cost structure (so that JELS$_1$ = JELS$_2$), Proposition 1 states that coordination will either persist at all degrees of partial integration or cannot be achieved at any. In contrast, when the system-optimal policy depends on the buyer's cost structure (so that JELS$_1 \neq$ JELS$_2$), dependence of supply-chain coordination on the degree of partial integration is more intricate: an ownership share of the buyer in the supplier can help the supply chain to remove informational inefficiency from the operational stage and thus achieve coordination. This share can be determined by means of Proposition 2 and Corollary 1. Yet increasing this share must not necessarily do better, as it may distort the buyer's incentives to the extent that inefficiency reappears and coordination fails. Proposition 3 derives conditions for this to happen, which needs to be accordingly checked for when choosing the level of integration.

6 Dependence of supply-chain performance on the degree of partial integration

We now turn to investigating properties of dependence between the degree of partial vertical integration and supply-chain performance. From Sect. 3, we know that candidate solutions to contracting problem (9)–(13) are determined by 15 expressions of menus of offers, which are divided into eight generic types according to the feasibility and necessary optimality conditions. Table 3 specifies expressions of these 15 menus of offers. We can treat these expressions as vector-valued functions

$$v_m : \alpha \mapsto (x_1, y_1, x_2, y_2), \qquad m \in M := \{1, 2, 3, 4a\text{–}b, 5a\text{–}b, 6a\text{–}d, 7a\text{–}b, 8a\text{–}b\},$$

where the menus of the same type are sub-indexed with literals (see Sect. 3). For the given value of α, an optimal menu of offers is selected from $\{v_m(\alpha)\}_{m \in M}$ as described in Sect. 4: see eqs. (22) and (23). The following result stems directly from Pishchulov and Richter (2016, Proposition 2) and indicates that we can restrict attention to certain menus only.

Proposition 4 *In an optimal menu of offers, $x_i =$ JELS$_i$ holds for at least one $i \in \{1, 2\}$.*

Proof The proof follows that of Pishchulov and Richter (2016, Prop. 2) and is available on request. □

Hence, for the given α, we may take only those menus into consideration that offer $x_i =$ JELS$_i$ to at least one buyer type i. Further, consider the following subset of M:

$$M^* = \{1, 2, 3, 4a\text{–}b, 5a\text{–}b, 7a, 8a\} \subset M.$$

Then, the proof of Proposition 4 implies the following

Corollary 3 *For the given* α, *an optimal menu of offers* $v_o(\alpha)$ *with* $o \in \{6a–d, 7b, 8b\}$ *must coincide with some other menu* $v_m(\alpha)$ *such that* $m \in M^*$.

We can therefore disregard menus 6a–d, 7b, 8b and evaluate only those contained in M^*. Further, based on the derivation of the 15 candidate menus of offers in Sect. 3 and their specification in Table 3, it is straightforward to verify the following properties [see also Pishchulov and Richter (2016, Section 4)]: (i) if $JELS_1 = JELS_2$ then menus 7a and 8a either coincide with or replace menus 3 and 2, respectively; (ii) if $JELS_1 \neq JELS_2$ then menus 7a and 8a cannot be optimal. Hence in either case, we can restrict the analysis to just seven candidate menus of five types (1, 4a–b, 5a–b, 7a, 8a, and 1, 2, 3, 4a–b, 5a–b, respectively). Further to this, we can establish that (iii) menus 7a and 8a are the only candidate menus in M^* that *pool* buyer types—that is, contain an identical offer for them both; thus, (iv) with $JELS_1 \neq JELS_2$, pooling never occurs in an optimal menu of offers, which means that in such a case, it is always optimal for the supplier to discriminate between the buyer types.

Consider further the partition of M^* into two subsets:

$$\overline{M} = \{1, \ 2, \ 3, \ 7a, \ 8a\} \qquad \text{and} \qquad \widetilde{M} = \{4a–b, \ 5a–b\}.$$

Assume that for some $m \in \overline{M}$, the menu of offers $v_m(\alpha)$ is respectively optimal for each α in an open interval $(\underline{\alpha}, \bar{\alpha})$. Then supply-chain performance $\hat{K}(\alpha)$ is constant on $(\underline{\alpha}, \bar{\alpha})$. Indeed, Table 3 reveals that the order sizes in the menus of types 1–3 and 7–8 do not depend on α. As a result, optimality of a menu of any of these types for $\alpha \in (\underline{\alpha}, \bar{\alpha})$ implies that varying α within these limits does not affect supply-chain performance $\hat{K}(\alpha)$. From the preceding discussion, we know that optimality of a menu $v_m(\alpha)$ with $m \in \{1, 7a, 8a\}$ leads to supply-chain coordination and, hence, to the best possible—and constant—supply-chain performance on $(\underline{\alpha}, \bar{\alpha})$. Given optimality of a menu of offers of type 2 or 3, supply-chain performance will remain at a constant level too, which, however, does not need to be the best possible one. This is illustrated by Fig. 3b–d in which menus of offers of type 2 and 3 happen to be optimal over certain ranges of α, so that varying the degree of partial integration within these ranges does not help to improve supply-chain performance. The reason for this invariance of supply-chain performance is that one of the individual rationality constraints (10)–(11) is not binding, so that the optimal solution of the contracting problem (9)–(13) responds to variation of α within $(\underline{\alpha}, \bar{\alpha})$ by properly adjusting side payments $\hat{y}_{\alpha,1}$, $\hat{y}_{\alpha,2}$ while keeping the order sizes $\hat{x}_{\alpha,1}$, $\hat{x}_{\alpha,2}$ unchanged (see also Illing, 1992). This is illustrated on Example 2 in Fig. 7: offering $JELS_2$ with the effective side payment $z_2^{\min}$ to buyer type 2 is not feasible because it would lead to buyer type 1 picking this offer. Instead, $(\hat{x}_{\alpha,2}, \hat{z}_{\alpha,2})$ is offered to buyer type 2. To prevent buyer type 1 from picking that offer, an effective side payment $\hat{z}_{\alpha,1} = \hat{z}_{\alpha,2} + K_1^{\alpha}(\hat{x}_{\alpha,1}) - K_1^{\alpha}(\hat{x}_{\alpha,2})$ must be offered to buyer type 1.

We therefore conclude that supply-chain performance can vary only within those ranges of α where a menu of offers of type 4 or 5 holds optimal. Indeed, individual rationality constraints (10)–(11) happen to be binding at these menus of offers, so that one of the order sizes in the optimal solution becomes subject to variation with varying α, which makes supply-chain performance $\hat{K}(\alpha)$ vary as well (cf. Illing, 1992). To further characterize the latter dependence, let

$$EOQ_{\min} = \min\{EOQ_1, EOQ_2\} \qquad \text{and} \qquad EOQ_{\max} = \max\{EOQ_1, EOQ_2\}.$$

Proposition 5 *Assume that, for some* $m \in \widetilde{M}$, *the menu of offers* $v_m(\alpha)$ *is respectively optimal for each* α *in an open interval* $(\underline{\alpha}, \bar{\alpha})$. *If supply-chain performance* $\hat{K}(\alpha)$ *is varying on* $(\underline{\alpha}, \bar{\alpha})$,
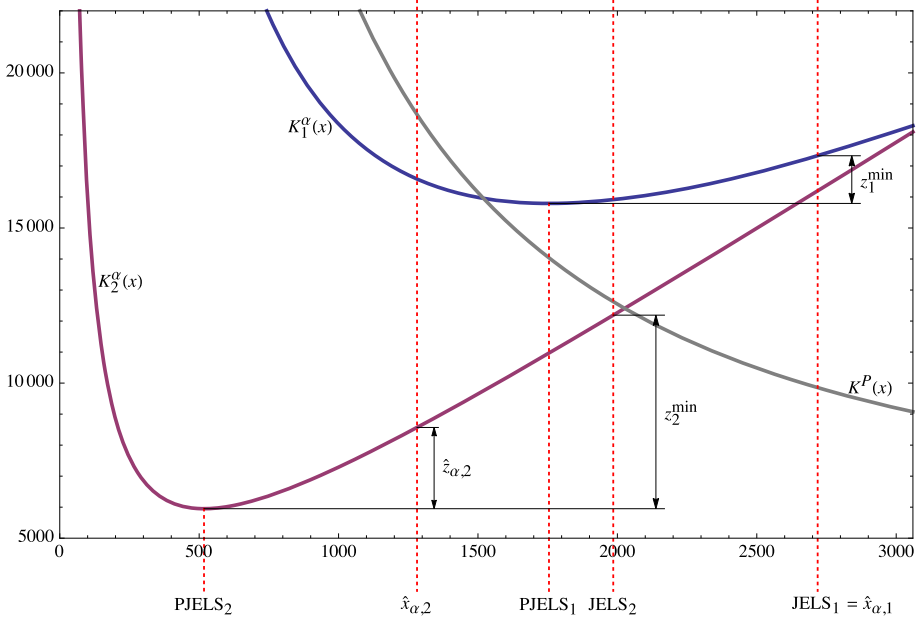
(a) $\alpha = 0$
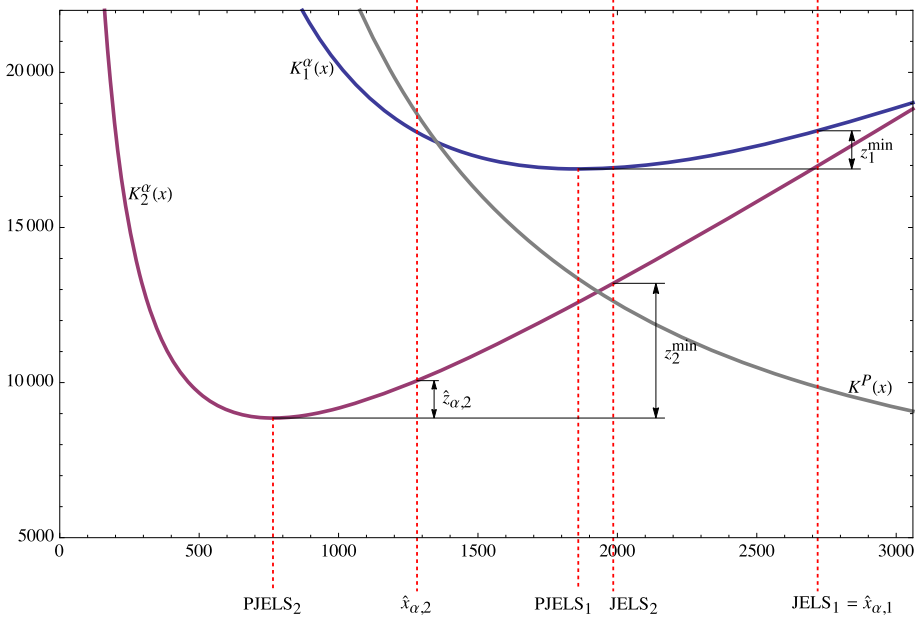
(b) $\alpha = 0.08$

Fig. 7 Emergence of the menu of type 3 in Example 2 with $\alpha = 0$ (above) and $\alpha = 0.08$ (below)
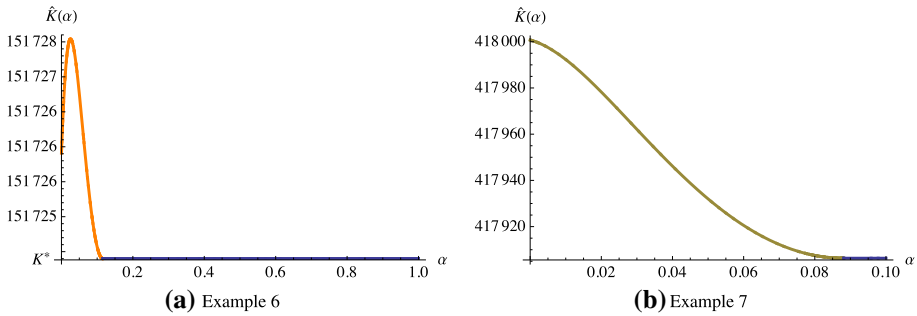
Fig. 8 Supply-chain performance realized at various degrees of partial vertical integration in Examples 6 and 7

then it does so in a strictly monotone way provided that the following condition holds:

$$ELS \leq EOQ_{\min} \qquad or \qquad ELS \geq EOQ_{\max} \qquad or \qquad \hat{\alpha} \notin (\underline{\alpha}, \bar{\alpha}). \qquad (31)$$

Proof See Sect. A.4 of Appendix A. □

Figure 3a–e in Sect. 4 illustrate a strictly monotone dependence of supply-chain performance $\hat{K}(\alpha)$ on α within those ranges where menus of offers of type 4 and 5 happen to be optimal. Note that when the supplier's economic lot size ELS is found between the buyer's two possible economic order sizes EOQ_1 and EOQ_2, condition (31) requires that the critical number $\hat{\alpha}$, as defined in (29), does not lie within the interval $(\underline{\alpha}, \bar{\alpha})$ to ensure strict monotonicity of $\hat{K}(\alpha)$ on that interval. This condition cannot be relaxed, as the following *Example 6* demonstrates. Problem data for this example are given in Table 4. It holds that $EOQ_1 < ELS < EOQ_2$, and $\hat{\alpha} \approx 0.026$. A menu of offers of type 5 happens to be optimal within the range of $\alpha \in [0, 0.11]$. Figure 8a shows the dependence of supply-chain performance on α. Since $\hat{\alpha} \in [0, 0.11]$, we observe that $\hat{K}(\alpha)$ is changing its behaviour at this point, from increasing to decreasing.

In order to gain insight into the likelihood of the emergence of this effect, we have conducted an extensive numerical study encompassing 100,000 randomly generated problem instances (Pishchulov et al., 2016). Specifically, we have set $p = 100\,000$ for all problem instances, and sampled the values of the remaining parameters independently from a uniform distribution. Table 6 indicates sampling ranges. Results reveal that 2,932 problem instances (ca. 3% of their total number) satisfy conditions $EOQ_{\min} < ELS < EOQ_{\max}$ and $\hat{\alpha} \in [0, 1]$. At the same time, only 16 of these instances (ca. 0.02% of the total number) exhibit a non-monotonic behavior of supply-chain performance $\hat{K}(\alpha)$ within an optimality interval of a menu of type 4 or 5 (Pishchulov et al., 2016) —for the reason that only in that few instances, $\hat{\alpha}$ falls within such an interval.[3]

To summarize, increasing or decreasing the degree of partial integration within certain ranges may not have any impact on supply-chain performance. Otherwise, supply-chain performance is likely to change monotonically with the varying degree of integration. An important consequence of the proof of Proposition 5 is the following

Corollary 4 *If supply-chain coordination is attained at some $\bar{\alpha} \in [0, 1)$ then $\hat{K}'(\bar{\alpha}) = 0$.*

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[3] We note that the figures reported are specific to the sample used and may differ depending on the sampling procedure.

Table 6 Sampling ranges in the numerical study

Supplier		Buyer	(Cost structure no. 1)			(Cost structure no. 2)	
R	h_P	d	B_1	$h_{A,1}$	ω_1	B_2	$h_{A,2}$
[10, 1000]	[10, 1000]	$[10^4, 10^5]$	[10, 1000]	[10, 1000]	[0.01, 0.99]	[10, 1000]	[10, 1000]

Values of all parameters, except for ω_1, have been rounded to the nearest integer after sampling. Values of ω_1 have been rounded to the fourth decimal place (Pishchulov et al., 2016). It holds that $p = 100\,000$ and $\omega_2 = 1 - \omega_1$

Figures 3 and 8 illustrate this: at every point where the graph of supply-chain performance $\hat{K}(\alpha)$ attains the benchmark K^*, the gradient of $\hat{K}(\alpha)$ vanishes. Therefore, slight variations of the degree of integration in a supply chain that happens to be coordinated should not have a substantial adverse impact on its performance.

Figure 3a–e in Sect. 4 illustrated dependence of supply-chain performance on the degree of partial vertical integration under different parametrizations of the model. These figures may further suggest that when supply-chain performance is monotonic in α, then it exhibits convex behaviour. The following *Example 7* demonstrates that this does not need to be the case. Problem data for this example is given in Table 4. Figure 8b shows that, despite a monotonic decrease of $\hat{K}(\alpha)$ over interval [0, 0.088], its behavior on this interval is non-convex—in contrast to the previously considered examples. We have not been able to obtain a closed-form characterization of convexity similar to monotonicity results of Proposition 5. However, we found in Pishchulov et al. (2016) that only $\approx 0.03\%$ of the problem instances in the above numerical study exhibit monotonic but not strictly convex behavior of $\hat{K}(\alpha)$.[3] This suggests that, whenever supply-chain performance can be improved by increasing or decreasing the degree of integration, this is likely to occur in a super-linear way; thus, a slight initial increase of α in Fig. 3a–c is capable of quickly absorbing a substantial portion of coordination deficit.

7 Conclusion and outlook

This paper studied the effects of partial vertical integration on supply-chain coordination and performance—a topic that has been previously understudied in the supply-chain literature. We have employed a conventional model of supply-chain coordination and studied it in a setting with asymmetric information, which may lead to coordination failure and underperformance. Our objective was to investigate the extent to which the proposition made in the literature with regard to the ability of partial integration to improve the total surplus, holds true for the setting under study. From the managerial perspective, 'common sense' suggesting that supply-chain performance will always improve with tighter integration, proves wrong in our setting. Specifically, a tighter integration may actually harm supply-chain performance, or not have any effect.

Further, our results demonstrate that supply-chain coordination under asymmetric information may be achieved with just a minority equity stake—which is able to absorb coordination deficit and yield the efficiency of a fully integrated supply chain. We provided an economic interpretation of this capability. A 'coordinating' degree of partial integration turns out to be an outcome of the interplay of four forces that drive the buyer's cost and compensation, both own and internalized. Specifically, these forces act so that cost and compensation come to balance each other at a 'coordinating' degree of integration—to the extent that the

buyer does not benefit from choosing the offer intended for another buyer type. Furthermore, we have analytically characterized the persistence of efficiency with tighter integration and established conditions under which supply-chain performance would consistently improve with varying degree of partial integration. We have observed such consistent behaviour in most simulated instances. Our analysis also revealed other favourable properties of partial integration: its minor variation is likely to yield a substantial performance improvement in an uncoordinated supply chain, but it would not significantly hurt the performance of a coordinated supply chain.

It must be noted that coordination may be attained in some problem instances at a degree of partial integration of over 50%—so that the assumption of absence of the buyer's control rights may not hold in practice, unless the buyer obtains his majority ownership by acquiring non-voting shares. In fact, offering non-voting stock can be an attractive option for the supplier when it helps to improve overall efficiency while avoiding dilution of control (Van den Steen, 2002); furthermore, using non-voting stock can help to reduce anticompetitive effects of partial integration (Gilo & Spiegel, 2011; Gilo, 2000).

Our study refers to supply-chain coordination and performance at the operational level, following a strategic decision about partial integration. We acknowledge that negotiating the level of integration requires its own analysis which goes beyond the scope of this study and represents an interesting avenue for future research. The closed-form solution derived in our study facilitates such analysis and can be easily implemented in software. Future work should further investigate coordination effects of partial integration in settings with more general production and shipment policies, settings involving more than two possible cost structures of the buyer, more than two supply-chain members, and closed-loop supply chain settings. Furthermore, it would be of interest to study contracting between the parties when their bargaining power depends on the level of integration.

Declarations

Appendix A. Proofs of propositions

A.1 Proof of Proposition 2

The following lemma is helpful in proving the proposition.

Lemma 2 *Assume that for the given $\alpha \in [0, 1)$, the menu of offers of type 1 satisfies the respective feasibility conditions as per Table 3. Then this menu of offers represents a unique optimal solution of the contracting problem (9)–(13) and yields supply-chain coordination.*

Proof Suppose that the supply-chain members adopt an order size x and the supplier makes a side payment y to the buyer. Then the costs effectively borne by the supplier due to the ownership share $1 - \alpha$ in its own company can be expressed as

$$(1 - \alpha) \cdot \left[K^P(x) + y \right] \equiv (1 - \alpha)K^P(x) + (1 - \alpha)y, \tag{A.1}$$

while the total costs effectively borne by the buyer type $i \in \{1, 2\}$ due to ownership share α in the supplier's company amount to

$$K_i^\alpha(x) - (1 - \alpha)y. \tag{A.2}$$

Their joint total costs express accordingly as

$$(1 - \alpha)K^P(x) + K_i^\alpha(x) \equiv K^P(x) + K_i^A(x). \tag{A.3}$$

Referring to the right-hand side above, we can immediately see that the joint economic lot size

$$x_i = \text{JELS}_i \tag{A.4}$$

uniquely minimizes (A.3) by its definition in (7). Hence order size (A.4) and the side payment y_i such that

$$(1 - \alpha)y_i = K_i^\alpha(x_i) - K_i^\alpha(x_{\alpha,i}^*), \tag{A.5}$$

uniquely minimize the supplier's costs (A.1) [see also Sucky (2006, Section 4.2)]. It is then easy to see that the tuple (x_1, y_1, x_2, y_2) defined by (A.4)–(A.5) with $i = 1, 2$ is the unique global minimum of the objective function (9). At the same time, this tuple obviously represents the menu of offers of type 1 as per Table 3. Under the hypothesis of the lemma, a menu of type 1 happens to be a KKT solution of the contracting problem (9)–(13) (cf. Sect. 3). Taking into account that it uniquely delivers the smallest possible value to the objective function, we conclude that it indeed represents the unique optimal solution of the contracting problem. It obviously yields coordination because it leads the buyer types 1 and 2 to adopting the order sizes $x_1 = \text{JELS}_1$ and $x_2 = \text{JELS}_2$ respectively. □

To prove now the proposition, refer to the menu of offers of type 1, see Table 3. Note that the feasibility conditions for that menu can be rewritten in an obvious way as follows:

$$K_1^\alpha(x_1) - K_2^\alpha(x_1) \ \leq \ K_1^\alpha(x_{\alpha,1}^*) - K_2^\alpha(x_{\alpha,2}^*) \ \leq \ K_1^\alpha(x_2) - K_2^\alpha(x_2). \tag{A.6}$$

By definition of the functions involved, the above is obviously equivalent to

$$K_1(x_1) - K_2(x_1) \ \leq \ K_1^\alpha(x_{\alpha,1}^*) - K_2^\alpha(x_{\alpha,2}^*) \ \leq \ K_1(x_2) - K_2(x_2). \tag{A.7}$$

Noting that $x_1 = \text{JELS}_1$ and $x_2 = \text{JELS}_2$ for the menu of offers in question, and taking into account the definition of function $\overline{K}(x)$ in (27), we can thus rewrite (A.7) as

$$\overline{K}(\text{JELS}_1) \quad \leq \quad K_1^{\alpha}(x_{\alpha,1}^*) - K_2^{\alpha}(x_{\alpha,2}^*) \quad \leq \quad \overline{K}(\text{JELS}_2), \tag{A.8}$$

what represents condition (28) of the proposition. Thus when that condition holds, the menu of type 1 happens to be feasible and represents a KKT solution of contracting problem (9)–(13) as per Table 3. Then, by Lemma 2, this menu is a unique optimal solution of the contracting problem which yields supply-chain coordination. This proves the sufficient condition of the proposition.

We now prove the necessary condition. Assume that optimal contracting leads to supply-chain coordination. This, in turn, means that the optimal menu of offers contains

$$x_1 = \text{JELS}_1 \qquad \text{and} \qquad x_2 = \text{JELS}_2 \tag{A.9}$$

as the respective order sizes. Following Pishchulov and Richter (2016, proof of Proposition 1), it is straightforward to show that an optimal solution of the transformed problem (16)–(20) must necessarily represent its KKT solution. Therefore, we conclude that an optimal solution of the original problem (9)–(13) must coincide with some menu of offers from Table 3. Obviously, the menu of type 1 satisfies (A.9). Further, it is straightforward to verify that under the hypothesis $\text{JELS}_1 \neq \text{JELS}_2$ of the proposition, no other menu in Table 3, if defined, satisfies (A.9), unless it coincides with the menu of type 1. Hence the menu of type 1 must be an optimal solution of the contracting problem (9)–(13) and must necessarily represent its KKT solution. This, in turn, means that the menu of offers of type 1 satisfies the respective feasibility conditions as per Table 3, which implies that condition (A.8) holds by construction. This completes the proof.

A.2 Proof of Lemma 1

Lemma 1 asserts that of the two critical numbers $\check{\alpha}$, $\hat{\alpha}$, whichever are defined, at most one is positive. Assume that $\check{\alpha}$, $\hat{\alpha}$ are both defined, since otherwise the assertion of the proposition is obviously true. Suppose that, contrary to the assertion of the proposition, both critical numbers happen to be positive. By definition of $\check{\alpha}$ in (29) and its positiveness we then have

$$\frac{B_1 h_{A,2} - B_2 h_{A,1}}{R(h_{A,1} - h_{A,2}) - H_P(B_1 - B_2)} \; > \; 0. \tag{A.10}$$

Hence the numerator and the denominator in (A.10) have the same sign. We can obviously assume without loss of generality that both are positive. By the positiveness of the numerator we have:

$$\frac{B_1}{h_{A,1}} \; > \; \frac{B_2}{h_{A,2}}, \tag{A.11}$$

while by the positiveness of the denominator in (A.10), there holds:

$$R(h_{A,1} - h_{A,2}) \; > \; H_P(B_1 - B_2). \tag{A.12}$$

Further, by definition of $\hat{\alpha}$ in (29) and its positiveness, there holds:

$$\frac{R^2 h_{A,1} h_{A,2} - H_P^2 B_1 B_2}{H_P(B_1 + B_2) - R(h_{A,1} + h_{A,2})} \; > \; 0. \tag{A.13}$$

Assume first that the denominator in (A.13) is positive. From this and (A.12) we then obtain:

$$\frac{B_2}{h_{A,2}} > \frac{R}{H_P}.$$

But the above inequality and (A.11) together imply that the numerator in (A.13) is negative, and thus (A.13) does not hold true—a contradiction.

Assume now that the denominator in (A.13) is negative. From this and (A.12) we obtain:

$$\frac{R}{H_P} > \frac{B_1}{h_{A,1}}.$$

The above inequality and (A.11) imply, however, that the numerator in (A.13) is positive, and thus (A.13) does not hold true—a contradiction. This proves that $\check{\alpha}$ and $\hat{\alpha}$ cannot be both positive.

A.3 Proof of Proposition 3

Let the hypothesis $\text{JELS}_1 \neq \text{JELS}_2$ of the proposition hold. We first prove its sufficient condition. Assume that (28) holds for $\alpha = \alpha_0$ as well as for $\alpha = \tilde{\alpha}_0$ if the latter is defined. By definition of the functions involved, it is easy to verify that (28) holds for $\alpha = 1$ as well. We can thus make

Observation A.1 *At the endpoints of interval* $[\alpha_0, 1]$, *function* $\widetilde{K}(\alpha)$ *is found within the bounds defined by the left and right-hand sides of* (28).

Obviously, $\widetilde{K}(\alpha)$ is continuous and differentiable in α everywhere on $[0, 1]$. We next determine stationary points of $\widetilde{K}(\alpha)$ on the open interval $(\alpha_0, 1)$, if there are any. To this end, let

$$s_i = \alpha R + B_i \qquad\qquad h_i = \alpha H_P + h_{A,i}, \qquad (A.14)$$

where $i = 1, 2$. By definition of functions $K_i^\alpha(x)$ and due to the expression of the partially joint economic lot sizes $x_{\alpha,i}^*$ in (6), it is then straightforward to obtain:

$$\widetilde{K}(\alpha) = \sqrt{2ds_1h_1} - \sqrt{2ds_2h_2},$$

so that:

$$\widetilde{K}'(\alpha) = \sqrt{2d} \cdot \left[\frac{Rh_1 + s_1 H_P}{2\sqrt{s_1h_1}} - \frac{Rh_2 + s_2 H_P}{2\sqrt{s_2h_2}} \right].$$

Setting the above equal to zero and rearranging the terms successively yields:

$$\frac{Rh_1 + s_1 H_P}{\sqrt{s_1h_1}} = \frac{Rh_2 + s_2 H_P}{\sqrt{s_2h_2}} \qquad \Leftrightarrow$$

$$Rh_1\sqrt{s_2h_2} + s_1 H_P\sqrt{s_2h_2} = Rh_2\sqrt{s_1h_1} + s_2 H_P\sqrt{s_1h_1} \qquad \Leftrightarrow$$

$$R\sqrt{h_1h_2}\left(\sqrt{s_2h_1} - \sqrt{s_1h_2}\right) = H_P\sqrt{s_1s_2}\left(\sqrt{s_2h_1} - \sqrt{s_1h_2}\right) \qquad \Leftrightarrow$$

$$\underbrace{\left(\sqrt{s_2h_1} - \sqrt{s_1h_2}\right)}_{=\check{E}} \cdot \underbrace{\left(R\sqrt{h_1h_2} - H_P\sqrt{s_1s_2}\right)}_{=\hat{E}} = 0. \qquad (A.15)$$

Denote the two parenthetical expressions in (A.15) by $\check{E}$ and $\hat{E}$ respectively. (A.15) obviously holds whenever $\check{E} = 0$ or $\hat{E} = 0$, or both. We accordingly distinguish between the following two cases.

Case 1 $\check{E} = 0$. This obviously holds true if and only if

$$\frac{s_1}{h_1} = \frac{s_2}{h_2}, \tag{A.16}$$

or, equivalently, $x_{\alpha,1}^* = x_{\alpha,2}^*$. By virtue of definitions in (A.14), equation (A.16) can be rewritten after an obvious simplification in the following form:

$$\alpha \cdot \left[R(h_{A,1} - h_{A,2}) - H_P(B_1 - B_2) \right] = B_1 h_{A,2} - B_2 h_{A,1}. \tag{A.17}$$

We can distinguish between the following three sub-cases.

a) If the bracketed expression and the right-hand side in (A.17) are both equal to zero then (A.17) holds obviously true for all α. We can however exclude this sub-case from consideration because it implies that $JELS_1 = JELS_2$, which contradicts the hypothesis of the proposition.
b) If the bracketed expression in (A.17) is zero while the right-hand side is not, then (A.17) does not have a solution.
c) If the right-hand side and the bracketed expression in (A.17) are both non-zero then the solution of (A.17) is uniquely determined as

$$\check{\alpha} = \frac{B_1 h_{A,2} - B_2 h_{A,1}}{R(h_{A,1} - h_{A,2}) - H_P(B_1 - B_2)}.$$

This is exactly the critical number $\check{\alpha}$ as defined by (29).

Case 2 $\hat{E} = 0$. This obviously holds true if and only if

$$\frac{R^2}{H_P^2} = \frac{s_1 s_2}{h_1 h_2},$$

what can be rewritten after an obvious simplification in the following form:

$$\alpha \cdot \left[RH_P^2(B_1 + B_2) - R^2 H_P(h_{A,1} + h_{A,2}) \right] = R^2 h_{A,1} h_{A,2} - H_P^2 B_1 B_2. \tag{A.18}$$

We can then distinguish between the three possible sub-cases as in the above Case 1—which in a similar way imply that (A.18) holds uniquely for the value of α determined as

$$\hat{\alpha} = \frac{R^2 h_{A,1} h_{A,2} - H_P^2 B_1 B_2}{H_P(B_1 + B_2) - R(h_{A,1} + h_{A,2})} \cdot \frac{1}{RH_P},$$

if the latter is defined. This value represents then the critical number $\hat{\alpha}$ as per (29).

The above analysis of Cases 1 and 2 reveals that equation (A.15) holds for $\alpha = \hat{\alpha}$ and $\alpha = \check{\alpha}$ only, whichever are defined. By Lemma 1, at most one of the critical numbers $\hat{\alpha}$, $\check{\alpha}$ happens to be defined and positive. If such critical number exists and belongs to interval $(\alpha_0, 1)$ then we denote it by $\widetilde{\alpha}_0$ as per (30), otherwise $\widetilde{\alpha}_0$ is considered undefined. Thus when $\widetilde{\alpha}_0$ is defined then it represents the unique stationary point of $\widetilde{K}(\alpha)$ on $(\alpha_0, 1)$, and therefore, if $\widetilde{K}(\alpha)$ has a local extremum on $(\alpha_0, 1)$, it must occur at $\widetilde{\alpha}_0$. If, on the other hand, $\widetilde{\alpha}_0$ is undefined then $\widetilde{K}(\alpha)$ has no extrema on $(\alpha_0, 1)$. By assumption, condition (28) holds at $\alpha = \widetilde{\alpha}_0$ if the latter is defined, and we thus make

Observation A.2 *A strict local extremum of function $\widetilde{K}(\alpha)$ on $(\alpha_0, 1)$, if there is any, is found within the bounds defined by the left and right-hand sides of* (28).

We therefore conclude from Observations A.1 and A.2 that $\widetilde{K}(\alpha)$ remains within the bounds defined by the left and right-hand sides of (28) over the entire interval $[\alpha_0, 1]$. As a result, condition (28) holds for all $\alpha \in [\alpha_0, 1)$. Hence, by Proposition 2, optimal contracting leads to supply-chain coordination at every $\alpha \in [\alpha_0, 1)$. This proves the sufficient condition of the proposition.

We now prove the necessary condition. Assume that optimal contracting leads to supply-chain coordination at every $\alpha \in [\alpha_0, 1)$. Then, by Proposition 2, condition (28) holds for every $\alpha \in [\alpha_0, 1)$, and in particular for $\alpha = \alpha_0$ and $\alpha = \widetilde{\alpha}_0$ when the latter is defined. This completes the proof.

A.4 Proof of Proposition 5

We will prove that for $v_m(\alpha)$ representing a menu of type 4, supply-chain performance $\hat{K}(\alpha)$ is either constant or, otherwise, strictly monotone on $(\underline{\alpha}, \bar{\alpha})$ provided that condition (31) holds; it is straightforward to extend the proof to menus of type 5.

Let H_P, S_i, H_i be as defined in (25), (26) and let further, in accordance with Table 3,

$$s_i(\alpha) = \alpha R + B_i \qquad\qquad h_i(\alpha) = \alpha H_P + h_{A,i},$$

where $i = 1, 2$. As Table 3 indicates, a menu of offers of type 4 is characterized by the order sizes

$$x_1 = \text{JELS}_1 \qquad x_{2_{1,2}}(\alpha) = \sqrt{2d} \cdot \frac{\sqrt{s_1(\alpha)} \pm \sqrt{s_2(\alpha)}}{\sqrt{h_1(\alpha)} \pm \sqrt{h_2(\alpha)}} \qquad (A.19)$$

where the same sign (plus or minus) applies in the numerator and the denominator of the expression defining the order size x_2. We below assume that the former is the case, while the latter case can be treated in the same way as follows below. Hence the optimal menu of offers is featuring the order sizes

$$(\hat{x}_{\alpha,1}, \ \hat{x}_{\alpha,2}) = (x_{1,1}^*, \ x_{2_1}(\alpha))$$

for each $\alpha \in (\underline{\alpha}, \bar{\alpha})$. Then the supply-chain performance represents, by its definition in (23), the following function of α on the interval $(\underline{\alpha}, \bar{\alpha})$:

$$\hat{K}(\alpha) = \omega_1 K_1(x_{1,1}^*) + \omega_2 K_2(x_{2_1}(\alpha)). \qquad (A.20)$$

Note that (i) the 1$^{\text{st}}$ term in (A.20) does not depend on α, (ii) the 2$^{\text{nd}}$ term is twice differentiable in α, and (iii) $\omega_2 > 0$. Thus $\hat{K}(\alpha)$ happens to be either constant or strictly monotone on $(\underline{\alpha}, \bar{\alpha})$ if and only if the derivative of the function

$$\hat{K}_2(\alpha) := K_2(x_{2_1}(\alpha))$$

remains either zero, positive, or negative on the entire interval $(\underline{\alpha}, \bar{\alpha})$. Consider any $\alpha \in (\underline{\alpha}, \bar{\alpha})$. By definition of $K_2(x)$ in (21) we obviously have

$$\hat{K}_2(\alpha) = \frac{S_2 d}{x_{2_1}(\alpha)} + \frac{H_2}{2} \cdot x_{2_1}(\alpha)$$

and

$$\hat{K}_2'(\alpha) = \left(-\frac{S_2 d}{x_{2_1}^2(\alpha)} + \frac{H_2}{2} \right) \cdot x_{2_1}'(\alpha). \qquad (A.21)$$

Assuming $h_{A,1} \neq h_{A,2}$, we obtain from (A.19):

$$x_{2_1}(\alpha) = \sqrt{2d} \cdot \frac{\sqrt{s_1(\alpha)} + \sqrt{s_2(\alpha)}}{\sqrt{h_1(\alpha)} + \sqrt{h_2(\alpha)}} = \sqrt{2d} \cdot \frac{\left(\sqrt{s_1(\alpha)} + \sqrt{s_2(\alpha)}\right)\left(\sqrt{h_1(\alpha)} - \sqrt{h_2(\alpha)}\right)}{h_1(\alpha) - h_2(\alpha)} =$$

$$= \frac{\sqrt{2d}}{h_{A,1} - h_{A,2}} \cdot \left(\sqrt{s_1(\alpha)} + \sqrt{s_2(\alpha)}\right)\left(\sqrt{h_1(\alpha)} - \sqrt{h_2(\alpha)}\right),$$

so that

$$x'_{2_1}(\alpha) = \frac{\sqrt{2d}}{h_{A,1} - h_{A,2}} \cdot \left[\left(\frac{R}{2\sqrt{s_1(\alpha)}} + \frac{R}{2\sqrt{s_2(\alpha)}}\right)\left(\sqrt{h_1(\alpha)} - \sqrt{h_2(\alpha)}\right) + \right.$$

$$\left. + \left(\sqrt{s_1(\alpha)} + \sqrt{s_2(\alpha)}\right)\left(\frac{H_P}{2\sqrt{h_1(\alpha)}} - \frac{H_P}{2\sqrt{h_2(\alpha)}}\right)\right] =$$

$$= x_{2_1}(\alpha) \cdot \frac{1}{2}\left(\frac{R}{\sqrt{s_1(\alpha)s_2(\alpha)}} - \frac{H_P}{\sqrt{h_1(\alpha)h_2(\alpha)}}\right). \tag{A.22}$$

It is easy to verify that the same result holds also in the case when $h_{A,1} = h_{A,2}$. We can thus rewrite equation (A.21) as follows:

$$\hat{K}'_2(\alpha) = \underbrace{\left(-\frac{S_2 d}{x_{2_1}(\alpha)} + \frac{H_2}{2} \cdot x_{2_1}(\alpha)\right)}_{=I(\alpha)} \cdot \frac{1}{2} \cdot \underbrace{\left(\frac{R}{\sqrt{s_1(\alpha)s_2(\alpha)}} - \frac{H_P}{\sqrt{h_1(\alpha)h_2(\alpha)}}\right)}_{=J(\alpha)} \tag{A.23}$$

To proceed, denote the expression in the first parentheses in (A.23) by $I(\alpha)$, and that in the second parentheses—by $J(\alpha)$. Note that the menu of offers in question is optimal by assumption and therefore represents a KKT solution of problem (9)–(13) (cf. Sect. 3). It hence satisfies the feasibility and necessary optimality conditions for menus of offers of type 4, as specified in Table 3. The necessary optimality conditions require, in particular, that either

$$x_{2_1}(\alpha) = \text{JELS}_2 \quad \text{and} \quad x^*_{\alpha,1} = x^*_{\alpha,2} \tag{A.24}$$

or

$$\frac{H_2 x^2_{2_1}(\alpha) - 2d S_2}{(h_{A,1} - h_{A,2})x^2_{2_1}(\alpha) - 2d(B_1 - B_2)} \geq 0 \tag{A.25}$$

holds. We will below distinguish between the following two cases.

Case 1 Let (A.24) hold at a particular $\alpha \in (\underline{\alpha}, \bar{\alpha})$. We will show that (A.24) holds then at each $\alpha \in (\underline{\alpha}, \bar{\alpha})$. Indeed, the second equality in (A.24) obviously yields:

$$\frac{s_1(\alpha)}{h_1(\alpha)} = \frac{s_2(\alpha)}{h_2(\alpha)}. \tag{A.26}$$

Thus, by definition of $x_{2_1}(\alpha)$ in (A.19), the following holds for some $c > 0$:

$$x_{2_1}(\alpha) = \sqrt{2d} \cdot \frac{\sqrt{s_1(\alpha)} + \sqrt{s_2(\alpha)}}{\sqrt{h_1(\alpha)} + \sqrt{h_2(\alpha)}} = \sqrt{2d} \cdot \frac{c\sqrt{s_2(\alpha)} + \sqrt{s_2(\alpha)}}{c\sqrt{h_2(\alpha)} + \sqrt{h_2(\alpha)}} = \sqrt{2d} \cdot \frac{\sqrt{s_2(\alpha)}}{\sqrt{h_2(\alpha)}} \tag{A.27}$$

Taking into account the first equality in (A.24) and definition of order size JELS_2, we thus obtain:

$$\frac{S_2}{H_2} = \frac{s_2(\alpha)}{h_2(\alpha)}.$$

By definition of parameters involved, we can rewrite the above equality as follows:

$$\frac{R + B_2}{H_P + h_{A,2}} = \frac{\alpha R + B_2}{H_P + h_{A,2}},$$

It is then straightforward to derive from the above that

$$\frac{R}{H_P} = \frac{B_2}{h_{A,2}}. \tag{A.28}$$

This equality implies however that the expression on the right in (A.27) yields a constant value for all $\alpha \in [0, 1]$. Therefore, the first equality in (A.24) must equally hold for all $\alpha \in [0, 1]$.

Further, we obviously have from (A.28) and (A.26):

$$\frac{R}{H_P} = \frac{B_2}{h_{A,2}} = \frac{s_2(\alpha)}{h_2(\alpha)} = \frac{s_1(\alpha)}{h_1(\alpha)}.$$

From the above equality of the ratios on the left and on the right we then easily derive:

$$\frac{R}{H_P} = \frac{B_1}{h_{A,1}}. \tag{A.29}$$

Eqs. (A.28) and (A.29) imply then that the second equality in (A.24) holds for all $\alpha \in [0, 1]$, too.

Hence (A.24) indeed holds for all $\alpha \in (\underline{\alpha}, \bar{\alpha})$. This in turn means—by the definition of the menu of offers in question—that it coincides with the menu of offers of type 1 on the entire interval $(\underline{\alpha}, \bar{\alpha})$. Furthermore, $I(\alpha) = J(\alpha) = 0$ holds for all $\alpha \in (\underline{\alpha}, \bar{\alpha})$, which yields $\hat{K}'_2(\alpha) \equiv 0$ and proves supply-chain performance $\hat{K}(\alpha)$ to be constant on that interval.

Case 2 Let condition (A.25) hold at a particular $\alpha \in (\underline{\alpha}, \bar{\alpha})$. We first prove that (A.25) then holds at each $\alpha \in (\underline{\alpha}, \bar{\alpha})$. Indeed, assume the opposite: that (A.25) does not hold at some $\mathring{\alpha} \in (\underline{\alpha}, \bar{\alpha})$. Then condition (A.24) must necessarily hold at $\mathring{\alpha}$—and therefore at each $\alpha \in (\underline{\alpha}, \bar{\alpha})$, as the analysis of the above Case 1 reveals. However, following the approach of Pishchulov and Richter (2016, Appendix D.3), it can be shown that the second equality in (A.24) is equivalent to having the denominator in the left-hand side of (A.25) equal to zero. Therefore, condition (A.25) cannot hold at any $\alpha \in (\underline{\alpha}, \bar{\alpha})$ because it requires the said denominator to be non-zero. This leads to a contradiction.

We therefore conclude that the denominator in the left-hand side of (A.25) never turns to zero for $\alpha \in (\underline{\alpha}, \bar{\alpha})$. Since the expression in the said denominator obviously represents a continuous function of α, we make the following

Observation A.3 *The denominator in the left-hand side of condition* (A.25) *remains either positive or negative on* $(\underline{\alpha}, \bar{\alpha})$.

We will next prove that under condition (31) of the proposition, expression $J(\alpha)$ as defined in (A.23) remains either positive or negative on $(\underline{\alpha}, \bar{\alpha})$. To proceed, let ELS $\equiv x_P^*$ represent the supplier's economic lot size and EOQ$_i \equiv x_{0,i}^*$—the economic order quantity of the buyer type i, as defined by (6), (8). Without loss of generality, let EOQ$_1 \leq$ EOQ$_2$. We can distinguish between three sub-cases:

1) ELS $\leq$ EOQ$_1 \leq$ EOQ$_2$. Then, by definition of these order sizes, the following obviously holds:

$$\frac{R}{H_P} \leq \frac{B_1}{h_{A,1}} \leq \frac{B_2}{h_{A,2}},$$

where $H_P = \frac{d}{p} h_P$, as defined in (26). Note that the above two inequalities cannot simultaneously hold as equalities because otherwise $x^*_{\alpha,1} = x^*_{\alpha,2}$ must hold for any $\alpha \geq 0$; as explained above, this is equivalent to having the denominator in the left-hand side of (A.25) equal to zero for any α, which contradicts Observation A.3. Thus we conclude that the following holds true:

$$\frac{R}{H_P} \leq \frac{B_1}{h_{A,1}} \qquad \text{and} \qquad \frac{R}{H_P} < \frac{B_2}{h_{A,2}}.$$

By referring to the definition of parameters $s_1(\alpha)$ and $h_1(\alpha)$, it is then straightforward to verify that the ratio $s_1(\alpha)/h_1(\alpha)$ is either constant or strictly monotone decreasing in α, with $\lim_{\alpha \to \infty} s_1(\alpha)/h_1(\alpha) = R/H_P$. Similarly, it can be shown that the ratio $s_2(\alpha)/h_2(\alpha)$ is strictly monotone decreasing in α, with $\lim_{\alpha \to \infty} s_2(\alpha)/h_2(\alpha) = R/H_P$. This allows us to conclude that the inequalities

$$\frac{R}{H_P} \leq \frac{s_1(\alpha)}{h_1(\alpha)} \qquad \text{and} \qquad \frac{R}{H_P} < \frac{s_2(\alpha)}{h_2(\alpha)}$$

hold true for any $\alpha \geq 0$. This in turn implies that the inequality

$$\frac{R}{H_P} < \sqrt{\frac{s_1(\alpha)s_2(\alpha)}{h_1(\alpha)h_2(\alpha)}}$$

is satisfied at every $\alpha \geq 0$. From this and the definition of $J(\alpha)$ in (A.23) we obviously have that $J(\alpha) < 0$, for all $\alpha \geq 0$.

2) $\text{EOQ}_1 \leq \text{EOQ}_2 \leq \text{ELS}$. Taking the same approach as above, we can show that this sub-case implies $J(\alpha) > 0$, for all $\alpha \geq 0$.

3) $\text{EOQ}_1 < \text{ELS} < \text{EOQ}_2$. By referring to the expression of $J(\alpha)$ in (A.23), it is easy to see that $J(\alpha) = 0$ holds for any such α that solves the equation

$$\frac{R}{H_P} = \sqrt{\frac{s_1(\alpha)s_2(\alpha)}{h_1(\alpha)h_2(\alpha)}}. \tag{A.30}$$

It can be shown that such α can satisfy the condition $J'(\alpha) = 0$ only when Case 1 analysed above applies. Therefore $J'(\alpha) \neq 0$ must hold for any α that satisfies (A.30) in the case under consideration. Thus $J(\alpha)$ changes its sign at any such α, and does not do so anywhere else. We hence conclude that if equation (A.30) does not have a solution within the interval $(\underline{\alpha}, \bar{\alpha})$ then $J(\alpha)$ does not change sign on this interval, and thus remains either positive or negative on it. It is straightforward to verify that an only solution of equation (A.30) happens to be the critical number $\hat{\alpha}$ as defined in (29) (see also the analysis of Case 2 in the proof of Proposition 3).

The analysis of the above three sub-cases thus allows us to conclude that under condition (31), $J(\alpha)$ remains either positive or negative on $(\underline{\alpha}, \bar{\alpha})$. Furthermore, the expression of $x'_{2_1}(\alpha)$ in (A.22) allows us to conclude that the order size $x_{2_1}(\alpha)$ is strictly monotone in α on $(\underline{\alpha}, \bar{\alpha})$.

From Observation A.3 we know that the denominator in the left-hand side of (A.25) remains either positive or negative on $(\underline{\alpha}, \bar{\alpha})$. Assume the former; the opposite case can be treated similarly. We then have from condition (A.25) that the numerator in its left-hand side must be non-negative:

$$H_2 x^2_{2_1}(\alpha) - 2dS_2 \geq 0,$$

on the entire interval $(\underline{\alpha}, \bar{\alpha})$. This is obviously equivalent to having $I(\alpha) \geq 0$ hold on the entire interval $(\underline{\alpha}, \bar{\alpha})$, see the expression of $I(\alpha)$ in (A.23). Taking into account that $J(\alpha)$ remains either positive or negative on $(\underline{\alpha}, \bar{\alpha})$, and that

$$\hat{K}_2'(\alpha) = \frac{1}{2} \cdot I(\alpha) J(\alpha) \tag{A.31}$$

holds by virtue of (A.23), we conclude that $\hat{K}_2(\alpha)$ is weakly monotone on $(\underline{\alpha}, \bar{\alpha})$. We will next prove its strict monotonicity. Indeed, observe that $I(\alpha) = 0$ holds if and only if $x_{2_1}(\alpha) =$ JELS$_2$. Assume that the latter equality holds at some $\mathring{\alpha} \in (\underline{\alpha}, \bar{\alpha})$. But then the menu of offers in question coincides with the menu of offers of type 1, what yields supply-chain coordination (cf. Lemma 2) and implies that supply-chain costs $\hat{K}_2(\mathring{\alpha})$ are lowest possible and equal to $K_2(x_{1,2}^*)$, where $x_{1,2}^* \equiv$ JELS$_2$, as defined in (7). By the weak monotonicity of $\hat{K}_2(\alpha)$ we further have that $\hat{K}_2(\alpha) \equiv K_2(x_{1,2}^*)$ must hold either on $(\underline{\alpha}, \mathring{\alpha}]$ or on $[\mathring{\alpha}, \bar{\alpha})$, and so must the equality $x_{2_1}(\alpha) =$ JELS$_2$. This is however not possible due to the strict monotonicity of $x_{2_1}(\alpha)$ on the interval $(\underline{\alpha}, \bar{\alpha})$ established above. We thus have to conclude that $x_{2_1}(\alpha) \neq$ JELS$_2$ holds for all $\alpha \in (\underline{\alpha}, \bar{\alpha})$. Hence the inequality $I(\alpha) > 0$ holds for all $\alpha \in (\underline{\alpha}, \bar{\alpha})$ as well. Taking into account that $J(\alpha)$ remains either positive or negative on $(\underline{\alpha}, \bar{\alpha})$, we then have by virtue of (A.31) that $K_2(\alpha)$ is strictly monotone on $(\underline{\alpha}, \bar{\alpha})$. This completes the proof.

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