Preservice teachers' beliefs about mathematical digital competency – a "hidden variable" in teaching mathematics with digital technology?

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Recently the construct of mathematical digital competency (MDC) was put forward in which mathematical competency and digital competency are seen as a connected whole. This entails that student understanding of mathematical concepts may be almost inseparable from digital tools. We report on a quantitative study with n=238 preservice teachers (PSTs) from Germany that investigates PSTs' beliefs about such a "connected position" of MDC. Results show that a large group of PSTs believe in the potential of digital technology but at the same time opposes the notion of MDC and rather believe that mathematical and digital competency should be separated. Furthermore, PSTs' beliefs about MDC are largely independent from epistemological beliefs. We hypothesize, that beliefs about MDC may be an overlooked variable which may influence how teacher think about and use digital technology in the mathematics classroom.

Keywords: Beliefs, digital technology, teacher education, graphing calculators, self-efficacy

Introduction

Teacher beliefs are an important factor for implementing digital technology (DT) in the mathematics classroom (Thurm & Barzel, 2021). The question arises which facets of teacher beliefs are relevant for teaching mathematics with (DT). In this respect, previous research has shown that teachers' beliefs about the potentials of DT use, teacher epistemological beliefs (i.e., beliefs about the nature of mathematics and teaching and learning mathematics) and self-efficacy are important dimensions of teacher beliefs (Thurm & Barzel, 2020). In this paper we investigate a somewhat novel dimension of teacher beliefs in the context of teaching mathematics with DT. Starting point for this research study was the work of Geraniou and Jankvist (2019) who argue that mathematical competencies and digital competencies are rarely seen as a connected whole even though students will have to simultaneously activate and use these competencies. Therefore, they conceptualize the construct of "mathematical digital competency" (MDC), which describes an amalgam of mathematical and digital competencies. In particular, they use the theories of conceptual fields (Vergnaud, 2009) and instrumental genesis (Guin & Trouche, 1999) to show that such an amalgam entails that a student's understanding of a mathematical concept may almost inseparably be connected to digital tools and the student's instrumented techniques. In this paper we investigate how PSTs think about such an amalgam and how their beliefs are related to other belief facets like epistemological beliefs and beliefs about affordances and risk of DT use. We start by elaborating in more detail on the two main theoretical frameworks for our study, namely MDC and teacher beliefs. Throughout the paper the term "digital technology" (DT) refers to mathematic-specific digital technologies like function plotters, dynamic geometry systems, computer algebra systems and multi-representational tools.

Theoretical background

The notion of competency has gradually gained momentum and is nowadays a key construct in the educational paradigm (Geraniou & Jankvist, 2019). Mathematical competency can be defined as "someone's insightful readiness to act appropriately in response to a specific sort of mathematical challenge in given situations" (Niss & Højgaard, 2019, p.14) while digital competency has been conceptualized as "the set of knowledge, skills and attitudes […] that are required when using ICT and digital media to perform tasks; solve problems; communicate; manage information; collaborate; create and share content; and build knowledge" (Ferrari, 2012, p.43). Geraniou and Jankvist (2019) linked mathematical and digital competencies by using the theory of instrumental genesis (TIG) and the theory of conceptual fields (TCF). TIG (Guin & Trouche, 1999) describes the process of transforming a digital tool (an artefact) into a mathematical instrument which is a psychological construct that combines (parts of) the artefact and cognitive schemes in which technical knowledge about the artefact and the domain-specific mathematical knowledge are intertwined. TCF (Vergnaud, 2009) takes, similar to TIG, a developmental point of view. TCF highlights that a concept is not only referring to explicit objects of thought, but comprises a set of schemes, a set of situations and a set of linguistic and symbolic tools of representation. In particular, Vergnaud (2009) stresses that different concepts and situations are interconnected forming conceptual fields, which he defines as "a set of situations and a set of concepts tied together" (Vergnaud, 2009, p.86). The set of situations gives meaning to the concept and acts as a point of reference.

Geraniou and Jankvist (2019) show that both TIG and TCF can serve as a lens to investigate the simultaneous activation and development of mathematical and digital competency which they call "mathematical digital competency" (MDC). In particular they highlight, that the situations that make up students' conceptual fields "*may be embedded so deeply in a techno-mathematical discourse that, potentially, also their understanding of the mathematical concepts involved is almost inseparable from the digital tools and the students' instrumented techniques*" (p. 42). This could for example mean that the set of situations that students use as points of reference to give meaning to a concept will largely comprise situations involving DT. Moreover, a student may only be able to do (some) mathematically activities within a digital environment. Hence students might only be able to think about, explain and do mathematics with reference to a digital tool. Starting from this notion of a close amalgam of student's mathematical competencies and digital competencies, the question arises what teachers believe about such a potentially close interwovenness.

According to the broadly accepted definition proposed by Philipp (2007), teacher beliefs can be defined as "psychologically held understandings, premises, or propositions about the world that are thought to be true" (p.259). Beliefs are part of teachers' conception - a general notion or mental structure encompassing beliefs, meanings, concepts, propositions, rules, mental images, and preferences (Philipp, 2007). Teacher beliefs are an important factor for teaching mathematics with DT since they act as a bridge between knowledge and action (Thurm & Barzel 2020; 2021). However, teacher beliefs are not an unidimensional construct but can be differentiated into various dimensions (clusters) and sub-dimensions which form a differentiated belief system in which (clusters of) beliefs can be logically connected and some (clusters of) beliefs are more important than others (Philipp, 2007; Leder et al. 2002; Thurm & Barzel, 2020; 2021). With respect to teaching with DT, three dimensions of teacher beliefs have so far been identified as particularly important: (i) beliefs about teaching and learning with DT, (ii) self-efficacy beliefs and (iii) epistemological beliefs (Thurm & Barzel, 2020). These dimensions of teacher beliefs can be further differentiated into various sub-dimensions (Thurm & Barzel, 2020; see figure 1).

However, teacher *beliefs about MDC* have not yet been investigated, which is not surprising given that the notion of MDC has just recently emerged with the work of Geraniou and Jankvist in 2019. Clearly, a teacher can have different belief positions with respect to the relation of mathematical and digital competencies. On one extreme one can fully embrace the "*connected position*" of MDC (e.g., that students understanding of the mathematical concepts involved may be almost inseparable from DT). On the other extreme, someone could strongly favor a "*independent position*" believing that mathematical and digital competencies should not be closely interwoven but rather clearly separated (i.e., a student should be able to think about, explain and do mathematics without a DT). Clearly if teachers oppose a close interwovenness ("*connected position*") as conceptualized by MDC this will be problematic if the goal is to develop students MDC or if students learn mathematical concepts with the support of DT. In the following, we will refer to beliefs about the interwovenness of mathematical competencies and digital competencies simply as "beliefs about MDC".

Research questions and methodology

In our exploratory research study, we investigate two distinct but interconnected research questions:

RQ1: What are preservice teachers' beliefs about MDC? (Belief position)

RQ2: How are preservice teachers' beliefs about MDC related to epistemological beliefs, beliefs about teaching and learning with DT and self-efficacy beliefs? (Belief system)

To answer the research questions, we used quantitative instruments to measure PSTs beliefs about MDC, about the nature of mathematics and mathematical learning, about teaching and learning with DT and PSTs self-efficacy beliefs. The reason why we use questionnaires to catch teacher beliefs is that questionnaires allow to simultaneously measure different belief dimensions and relate them to each other by statistical analysis. Furthermore, questionnaires also provide opportunity to investigate the discriminant validity of the dimensions (Fives & Gill, 2015). In the following we briefly elaborate how the different dimensions were assessed through multi-item-scales (figure 1 gives an overview of all scales).

Since there were no items/scales available to measure PSTs beliefs about MDC, we started to construct a scale following the recommendations for scale and item construction of Simms (2008). The item design was guided by the goal to write items that capture PSTs believe whether or not it is acceptable if a student's understanding of a mathematical concept is almost inseparable from digital tools. If it is inseparable, this would mean for example that a student is only able to think about, explain and do mathematics with reference to a digital tool. We set up an initial pool of eight items (sample items are given in table 1) which were further refined with PSTs and experts. Response format for all items was a 6-point Likert scale ranging from "1=strongly disagree" to "6=strongly agree" (hence higher values indicate a more "independent position"). After administering the scale to the PSTs, we conducted an exploratory factor analysis (EFA). Based on the results four of the eight items were dropped and the final MDC-scale was constructed using the mean of the items MDC1-MDC4 displayed in table 1. Hence high values on the MDC-scale reflect that PSTs oppose the notion of a close link between mathematical competencies and digital competencies (which we call “independent position”) while low values of the MDC-score reflect a more “connected position” in line with the MDC concept. Reliability (Cronbach’s alpha) of the scale was 0.75 indicating an acceptable reliability score.

MDC1	A student should be able to explain a mathematical concept or relationship without referring to a digital mathematical tool.
MDC2	A student should be able to solve a mathematical problem without a digital mathematical tool.
MDC3	A student’s understanding of a mathematical concept should be independent from digital mathematical tools.
MDC4	A student should be able to give examples of a mathematical concept or relationship without referring to a digital mathematical tool.

Table 1: Sample items for the scale to measure PSTs beliefs about MDC

- To measure PSTs’ epistemological beliefs about the nature of mathematics we used three shortened multi-item-scales from the international TEDS-M-Study (Blömeke & Kaiser, 2014). One scale captured to what extend PSTs believe that mathematics is a static collection of rules and procedures (Scale: “Rules and Procedures”, E1). The second scale captured to what extend teachers view mathematics as a dynamic science which consists of problem-solving processes and the discovery of mathematical structures and regularities (Scale: “Inquiry”, E2). The third scale captured to what extend PSTs believe that mathematics is discovered rather than invented (Scale: “platonic conception”, E3).
  Epistemological beliefs about the learning of mathematics were captured with two scales. A shortened scale from the COACTIV-Study (Kunter & Baumert, 2013) measured to what extent students believe that learning mathematics is best achieved by receptive learning (Scale: “Instructivist”, E4). We also asked PSTs to rank the four conceptions of learning mathematics put forward by Kuhs and Ball (1986) by preference (classroom-focused; content-focused with an emphasis on performance; content-focused with an emphasis on conceptual understanding and learner-focused). By taking the mean of the ranks of the two instructivist conceptions (classroom-focus, content-focused with an emphasis on performance) we derived a scale (E5) where higher values indicate a more constructivist approach to teaching.
- To measure PSTs’ beliefs about teaching and learning with DT we used five established multi-item-scales (all items can be found in Thurm (2020)). Beliefs about the potentials of teaching with DT were captured by the scales “Supports discovery learning”" (T1) and “Support of multiple representation” (T2), whereas negative beliefs were captured by the scales “loss of computational / by-hand-skills” (T3) and “Mindless working” (T4). The scale “Prior mastery of mathematics by hand” (T5) captured whether students believe that DT should only be used when the mathematics is thoroughly understood without DT. Response format for all items of the scales was a 6-point Likert scale ranging from “1=strongly disagree” to “6=strongly agree”.
- Self-efficacy was measured using two established multi-item-scales (all items can be found in Thurm (2020) capturing PSTs’ self-efficacy for task design and selection (S1) and lesson design

and implementation (S2). Following Bandura (2006) response was given on a scale ranging from 0 to 100, where higher values indicate higher self-efficacy.

The online-questionnaire which comprised all previous described scales was administered in 2021 to n=238 PSTs from two German universities in the state of North Rhine-Westphalia. Participation was voluntarily and anonymous. To ensure a certain level of basic expertise in mathematics education and a basic exposure to digital mathematical tools during university studies, the questionnaire was only administered to PSTs who had completed at least four semesters of teacher education. 35% of the participants were male while 65% were female. The PSTs indicated that they had used digital mathematical tools for their own learning approximately once a week during their university studies.

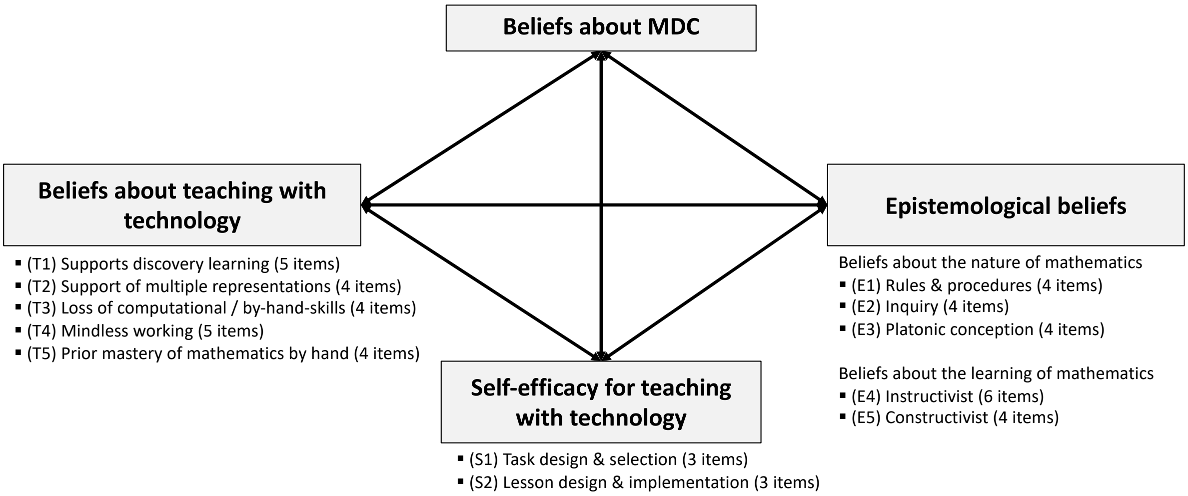

Figure 1: Overview of the different dimensions of teacher beliefs that were investigated

Results

RQ1: What are preservice teachers' beliefs about MDC? (*Belief position*)

Figure 2 shows the distribution of the MDC-scale (average of the items MDC1-MDC4). The mean of the MDC-scale is 4.51 indicating that PSTs on average clearly agreed more with an "*independent position*". In particular, 55.5% of the PSTs show an MDC-score ≥ 4.5 and thus strongly identified with the "*independent position*". A group of 40.3% of the PSTs was somewhat undecided/moderate (MDC-score between 2.5 and 4.5), while only 4.2% identified strongly with a *"connected position"* (MDC-score ≤2.5).

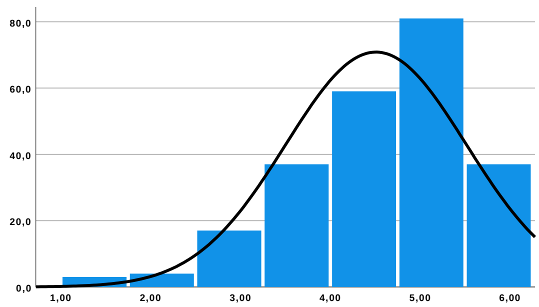

Figure 2: Frequency histogram of the scale measuring PSTs beliefs about MDC (mean of the items MDC1-MDC4)

RQ2: How are preservice teachers' beliefs about MDC related to epistemological beliefs, beliefs about teaching and learning with DT and self-efficacy beliefs? (*Belief system*)

Table 2 shows the means, standard deviation and reliability of all scales used in this study. Table 3 gives the correlation among the different constructs. First, none of the correlations are extremely high which indicates that the MDC-scale is indeed measuring a distinct construct different from the other constructs. The correlations in table 3 also show that a stronger "*independent position*" (higher values on the MDC-scale) is significantly associated with more negative beliefs about teaching and learning with DT (T1-discovery learning: ρ=-0.31**, T3-loss of skills: ρ=0.34**, T4-mindless working: ρ=0.39***). However, a closer look at the subgroup of PSTs that holds a particularly strong "*independent position*" (MDC-score ≥ 4.5, see table 2) reveals that this subgroup has still very positive beliefs about the potentials of DT for teaching and learning (DT supports discovery learning=4.65 on a 6-point scale, DT supports multiple representations=5.36 on a 6-point-scale). Remarkably, there were almost no significant correlations between MDC and epistemological beliefs (E1-E4) or self-efficacy beliefs (S1, S2). Only scales E2 ("Inquiry") and E5 ("Constructivist") were slightly negatively correlated with a more "*independent position*".

Scale	***α***	**Mean total (SD)**	**Mean MDC≥ 4.5**	**Scale**	***α***	**Mean total (SD)**	**Mean MDC ≥4.5**
MDC	.75	4.51 (1.00)	5.23	(S1) Task design & selection	.84	64.63 (20.12)	64.64
(T1) Supports discovery learning	.86	4.76 (0.86)	4.65	(S2) Lesson design & implementation	.87	62.47 (21.11)	62.64
(T2) Support of multiple representations	.70	5.44 (0.58)	5.36	(E1) Rules & procedures	.65	3.33 (0.94)	3.34
(T3) Loss of comp. / by-hand-skills	.87	4.03 (1.12)	4.28	(E2) Inquiry	.70	5.21 (0.66)	5.15
(T4) Mindless working	.90	4.23 (1.12)	4.44	(E3) Platonic conception	.77	4.25 (1.08)	4.38
(T5) Prior mastery of mathematics by hand	.94	4.29 (1.36)	4.69	(E4) Instructivist	.80	3.54 (0.91)	3.55
				(E5) Constructivist		3.03 (0.63)	3.05

Table 2: Mean (M), standard deviation (SD) and reliability (Cronbach's alpha) for the total sample (Mean total) and the subgroup with strong "independent position" (Mean MDC≥4.5)

	T1	**T2**	**T3**	**T4**	**T5**	**E1**	**E2**	**E3**	**E4**	**E5**	**S1**	**S2**
MDC	-0.31 **	-0.12	0.34 ***	0.39 ***	0.58 ***	0.12	-0.2 *	-0.04	0.07	-0.14 *	0.04	0.01

Table 3: Correlation between the MDC-scale and scales measuring teacher beliefs about teaching and learning with DT (T1-T5), epistemological beliefs (E1-E5) and self-efficacy beliefs (S1-S2). (* < .05, **< .01, *< .001)**

Discussion and conclusion

In this study, we took a first step to investigate PSTs' beliefs about the relation between mathematical and digital competency. We found that many PSTs in the sample strongly agreed with an *"independent position"* meaning that they believe that a student should be able to think about, explain and do mathematics independently from digital tools. This clearly opposes a more "*connected position*" of mathematical and digital competencies as conceptualized in the concept of MDC by Geraniou and Jankvist (2019).

Remarkably, beliefs about MDC were only barely associated with epistemological beliefs by the PSTs. Moreover, we found, that even the PSTs who were very strongly favoring an *"independent position"* at the same time strongly believed in the potentials of teaching and learning with DT (e.g., for discovery and investigation). These are interesting findings since they may point to some contradiction in the PSTs' belief system. If a PST highly values the potential of DT (e.g., to support discovery learning) but at the same time believes that mathematical understanding should be independent from DT, it will not be easy to balance these two views. In fact, if DT is used in a constructivist way for discovery learning of a new concept, the conceptual fields and the set of situations that students use as points of reference to give meaning to a concept will not be independent of the DT. Rather students' conceptual fields will be constructed around and therefore interwoven with DT. Hence, if PSTs hold an *"independent position"* this may limit the use of DT to understand and learn mathematics, or even reduces the use of? DT to "do mathematics". We hypothesize that PSTs may not be aware of this contradiction in their beliefs system. The observation that most PSTs agreed strongly that DT should only be used if the mathematics is thoroughly understood, may be a consequence of managing the tension of using DT (acting in line with their positive beliefs about DT) and at the same time maintaining an independence between mathematical understanding and DT (acting in line with their "*independent position*").

In total, the results of this study indicate that beliefs about MDC might be an overlooked variable—especially if the goal is to support students to develop MDC. In particular, the results of the study point to a discrepancy: If students learn with DT, they may develop some form of MDC (Geraniou & Jankvist, 2019). Yet, teachers expect students' understanding to be independent of DT and to be able to think about, explain and to do mathematics without DT. This discrepancy will be problematic and likely impact how teachers use DT. Consequently, it might be fruitful to engage PSTs in reflection about their beliefs on MDC and how these influence / conflict other beliefs in their beliefs system (e.g., beliefs about the potentials of DT for learning). Finally, we would like to mention one main limitation of the study – namely, that the belief position measured by the MDC-scale was not triangulated with qualitative data. Currently we are conducting qualitative interviews with PSTs after they have answered the MDC-scale to validate whether the MDC-scale is indeed measuring teachers' beliefs about MDC, and to uncover the PSTs' belief argumentation for their position.

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