Design considerations for facilitating mathematical learning online

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This paper presents a graduate student's reflections on the design of learning opportunities using the Desmos tool for carrying out mathematical activities and the online Zoom platform for facilitating mathematical learning. Using the theory of instrumental orchestration as our interpretative framework, we discuss the different types of orchestrations when a digital tool is used to support mathematical learning not in the familiar face-to-face classroom-based environment, but online instead. The contribution of this paper lies in the discussion of the design considerations and orchestrations to overcome the challenges of online learning and at the same time to capitalise on the opportunities it offers for mathematical learning.

Keywords: Online mathematical learning, distance learning environment, instrumental orchestration.

Introduction

There is a vast range of digital technologies (DTs), designed for and used in mathematics education, serving different purposes. Such technologies offer affordances for and constraints on students' learning, but also on teaching practices. DTs refer to digital tools and applications used "(a) as a support for the organisation of the teacher's work (producing worksheets, keeping grades) and (b) as a support for new ways of doing and representing mathematics" (Sinclair & Robutti, 2014, p. 598).

Using DTs for teaching mathematics at a distance has also been progressively researched, with many authors discussing the great opportunities of such a mode of teaching and learning, as well as the challenges teachers and students face (e.g. Silverman & Hoyos, 2018; Drijvers et al., 2021). Classroom-based strategies and activities, such as think-pair-share, role playing, group discussion, gesturing, modelling, assessing, etc., that are known to be engaging and effective for students' learning in the face-to-face classroom environment, and therefore widely used, present new challenges for teachers and students in online learning environments (Collison, Elbaum, Haavind, & Tinker, 2000). During online synchronous teaching, the teacher-students interactions are certainly limited and the opportunities to observe individual students' work and intervene when necessary decrease (e.g. Silverman & Hoyos, 2018). In such online settings, assessing individual students' learning is challenged and the tools for doing mathematics need to be carefully chosen. In face-to-face classroom-based lessons, learning takes place in a social context, in which students and teachers use various strategies to communicate with one another, including verbal and non-verbal strategies. In online remote learning, certain aspects of such communications are more challenging to achieve, such as writing and sharing each other's written mathematical work, pointing at notations and symbols written on the board, and other gesturing used by students to support their explanations. In fact, while verbal communications are possible online (via video/audio/chat features), supporting such conversation with written notes is more challenging online, since such online spaces allow limited mathematical learning input (Leventhall, 2004; Aldon et al., 2021; Drijvers et al., 2021).

In this paper, we share how a graduate student on our MA in Mathematics Education master's course, Tania, capitalised on the opportunities DTs offer for remotely supporting the learning of mathematics. She designed learning opportunities to be used via the online Zoom platform that replaced the usual face-to-face classroom environment. Aware of the constraints of the online Zoom platform with regards to mathematical input and notation, she also planned for using Desmos, a digital tool that would 'make up' for constraints with respect to writing mathematics and sharing mathematical work in real time. The data presented in this paper is based on Tania's written assignment, which was submitted in September 2020 as part of the master's module on 'Digital Technologies for Mathematical Learning' (DTML). In her assignment, Tania shared her reflections on how best to support learners make links between different representations of quadratic functions using Desmos, and how face-to-face classroom-based activities were adapted to online learning via Zoom. We start by introducing the Theory of Instrumental Orchestration (TIO), which we used as our interpretative framework to present and analyse Tania's pedagogical considerations (or in other words orchestrations) when planning for mathematical learning with Desmos, and where the face-to-face classroom environment was replaced by the Zoom online learning space. We then give an overview of Tania's design considerations for online learning, followed by an analysis of Tania's reflections on how best to support online mathematical learning applying the TIO framework. We conclude with some reflections on broadening the TIO framework's orchestrations, as a result of overcoming the constraints, while capitalising on the opportunities when facilitating online mathematical learning.

Theoretical Background

The integration of DT into mathematics education has been an ongoing and non-trivial issue, mainly due to the complexity of the use of DTs. In order to describe how a teacher manages the use of DTs, steers students' instrumental genesis and orchestrates mathematical situations, the TIO was developed by Trouche (2004). Trouche (2004) introduced TIO by arguing that an instrumental orchestration describes the teacher's organisation and use of the different artefacts within a learning environment in a mathematical situation, so as to guide students' instrumental genesis (as cited in Drijvers et al., 2010, pp. 214–215). Trouche (2004) argued that an instrumental orchestration is defined by "didactical configurations (i.e. the layout of the artifact available in the environment) and by "exploitation modes of these configurations" (p. 296). A *didactical configuration* involves the teaching set-up and the artefacts available in the teaching environment and set-up. An *exploitation mode* involves the approach a teacher decides to take when exploiting a didactical configuration, aiming at supporting their didactical intentions (ibid). For example, how tasks are introduced to students and how they are solved, what roles the artefacts might play, or what schemes and techniques students may develop, are a teacher's decisions (ibid). Drijvers et al. (2010) added to Trouche's (2004) two elements: 'didactical configuration' and 'exploitation mode', a third element that of 'didactical performance'. *Didactical performance* involves the decisions a teacher takes while teaching, considering their chosen 'didactical configuration' and 'exploitation mode' (Drijvers et al., 2010). As Drijvers et al. (2014) add "what question to pose now, how to do justice to […] any particular student input, how to deal with an unexpected aspect of the mathematical task or the technological tool, or other emerging goals" (p. 191). Six orchestration types have been identified in the literature regarding whole class teaching and the seventh one involves students working on their own or in pairs with technology (Drijvers, Tacoma, Besamusca, van den Heuvel, Doorman, & Boon, 2014). These are:

> Technical-demo orchestration concerns the demonstration of tool techniques by the teacher […]. Explain-the-screen orchestration concerns whole-class explanation by the teacher, guided by what happens on the computer screen […]. In the Link-screen-board orchestration, the teacher stresses the relationship between what happens in the technological environment and how this is represented in conventional mathematics of paper, book, and blackboard […]. The Discuss-the-screen orchestration concerns a whole-class discussion about what happens on the computer screen […]. In the Spot-and-show orchestration, student reasoning is brought to the fore through the identification of interesting DME student work during preparation of the lesson, and its deliberate use in a classroom discussion […]. In the Sherpa-at-work orchestration, a so-called Sherpa-student uses the technology to present his or her work, or to carry out actions the teacher requests. (Trouche & Drijvers, 2010, pp. 219–220).
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> In the Work-and-walk-by orchestration, the didactical configuration and the corresponding resources consist of the students sitting at their technological devices, and the teacher walking by in the classroom (Drijvers et al., 2014, p. 192).

Design Considerations for Online Learning

In this section, we present data from one graduate student's (Tania) reflections on designing activities to support eighteen 15-16 years old students' making links between different representations of quadratics functions using Desmos via Zoom. Tania was a student on the DTML masters' module and therefore a *learner* herself, who gained knowledge about technology-enriched practices in mathematics education. In this module, students learn about the affordances of various digital tools and critically discuss their value for mathematical learning. For their assignment they are expected to trial a digital tool with learners and critically reflect on the mathematical learning. Tania's assignment stood out, as she offered a particularly detailed account of and reflections on not just the mathematical learning with a DT, but also of doing so online. For this reason, we selected Tania's case study to illuminate how TIO can be applied to analyse orchestrations for online mathematical learning and how her design decisions showcase good practice in technology-rich learning.

The mathematics topic and choice of a tool

In her reflective writing, Tania justified her choice of a digital tool (Desmos – the technological artefact) for supporting eighteen Year 11 (15-16 years old) students' learning about 'quadratics'. In preparation for their mathematics examination (iGCSE), the students learned how to plot and recognise quadratic graphs; how to factorise, expand, complete the square of quadratic expressions; how to solve quadratic equations in various formats; and how to apply the 'rules' for graph transformations. By engaging with relevant mathematics education research (e.g. van der Meij & de Jong, 2006) as part of her the DTML module, Tania was aware that students master all of the above as separate knowledge and skills about quadratics, but may not necessarily develop a fuller, more holistic understanding of 'quadratics'. Inspired by her recent experience with graphware digital tools and knowledge of their potential for mathematics learning she gained in DTML, Tania thus planned for activities where Desmos would be used to support students to engage with the multiple representations of quadratics.

Planning for Online Learning

Tania trialed these activities with students via Zoom. In her written reflections, Tania was explicit about how much she learned from studying on the DTML and how it influenced her design decisions for mathematical activities which she ended up carrying out online. She became aware of the affordances and constraints of Zoom and Desmos, and there is evidence in her written reflections that such awareness affected how she orchestrated the learning of the students, as will be discussed next.

Tania's Instrumental Orchestrations

In this section we apply the TIO to describe and analyse Tania's own re-count of and reflection on her own practices to support online learning, where Desmos was used with the learning objective of supporting students make connections between different representations of quadratics. More precisely, we will be using Drijvers et al.'s (2014) interpretation of an instrumental orchestration consisting of three elements: a didactical configuration, an exploitation mode and a didactical performance. We will be using quotes from Tania's assignment, which we will indicate using single quotation marks '_'.

Tania wanted to find out about students' prior knowledge about the mathematics topic under scrutiny. She set a pre-task for the students to carry out on paper. Students then emailed her their scanned work in advance of trialing the online activities.

The *didactical configuration* for carrying out the online activity included the online platform (Zoom) as the online learning space which replaced the usual face-to-face classroom-based learning environment. Tania had to quickly become familiar with Zoom's functionalities, i.e. video camera (to make herself visible to the students), audio and chat features (to 'see' and talk with the students), breakout rooms for group activities, sharing screens and files (to share the resources she prepared for the online activities), Whiteboard, and certain functionalities that Zoom offered and which she referred to as the 'Pace', 'Pause' and 'Response' features; the Desmos tool; the mouse; and her computer. She referred to a 'Pace' feature of the Zoom platform to restrict students' access to specific screens. Tania also mentioned how she used the 'Pause' feature to remove students' ability to interact with the screen to capture everybody's attention and focus on the next activity; asking a student to provide an oral explanation; discuss a screen, etc. According to Tania, the 'Response' feature (how she referred to the chat box) allowed students to offer an answer for any questions or tasks that were posed.

Tania carefully thought about her *exploitation mode* regarding the online activity. For example, aware of the need for students to have plenty of time to explore the mathematics with the DT, Tania facilitated many such opportunities by organising students to work in pairs in breakout rooms. To better capture Tania's design considerations for the online activity, she created for the 18 students, we present these in Figure 1 below. In this figure, we exemplify the elements of mathematical learning Tania planned for (left column), the challenges she faced due to carrying out the activities online (middle column), and how Tania decided to exploit the artefacts mentioned above in her *didactical configuration orchestration* for the online activities (right column). Afterwards, we present her reflections on the elements involved with the *exploitation mode* regarding online learning and how she was able to overcome the challenges she was faced with, based on the affordances of the DTs she used.

Planning for…	Challenges for online teaching and learning	Tania's *Exploitation Mode* for online teaching and learning
Assessing prior knowledge	Writing mathematics in real time	Setting work for students to carry out on paper and then email their scanned work
Sharing work in lesson	Hand writing equations Hand sketching graphs	Using new symbolic representation x^2; Using Desmos for writing equations and sketching graphs; Treating computer screen like paper Sharing screens
Sketching graphs	Hand drawing graphs	Using Desmos to write equations and draw graphs, in parallel with verbal and written explanations
Managing the lesson	Presence and gesturing	Using the 'Pause class' Zoom feature
Monitoring and assessing students' progress	Walking through the classroom and checking on students' work	Joining break out rooms; Monitoring chats of break out rooms; Asking students to type explanations in the chat
Supporting collaboration - pair work; whole class	Students sharing working out in their books or on the board	Use of breakout groups; Students sharing screens
Gesturing	Using hand gestures to point at written work, to explain mathematics, to show own work on paper	Mouse used like a finger/pen to point to parts of the screen, while offering verbal explanation; 'Pause Class' for focus and joint attention
Sharing oral explanations	Facilitating rich conversations between teacher&students and among students	Teacher monitoring students' work by visiting breakout rooms, then asking the students to share their screens and explain their solutions.

Figure 1: Tania's design considerations for online learning

Tania's *didactical performance* involved the use of the 'Pause' and 'Pace' Zoom features in order to allow all students to see or work on the same screen together or have whole-group conversations and be provided with further support or introductions to new tasks, without many distractions. In her assignment, she referred to Godwin and Beswetherick (2003) by stating that 'teachers can be reluctant to use technology in the classroom if they are concerned that their students will lose focus and misbehave', and while she did not provide evidence that this did not happen online, she certainly hinted that she spent little time on managing students' transition to new activities. By asking the students to type their reasoning in the chat, Tania was pleased to be able to assess students' understanding, as she was able to quickly see their answers in the chat box. *Work-and-walk-by orchestration* and *Sherpa-at-work orchestration* were carried out with much ease, moving from one breakout room to another, checking on students' work, and hence monitoring students' work in less time than in a face-to-face classroom-based learning environment for example.

At the start of the online activities, Tania ensured that the students themselves had access to the Desmos application on their own devices, and they knew how to share screens. Her *technical-demo orchestration* included modeling to students how to input mathematics superscripts so that Desmos would recognise quadratic functions by using the '^' symbol for powers. Tania commented on how using Desmos to test out and modify their answers without judgement encouraged students to hypothesise about what would happen if variables changed. Tania noted that that was the case for all students, as she was able to quicky monitor their responses in the chat boxes, therefore gaining a window to every student's mathematical thinking. After sending the students to work in pairs in breakout rooms for the set activities, Tania would bring them back together and would always seek explanations from randomly selected students about how they (in pairs) found the solution to the mathematics question posed. *Spot-and-show orchestration* was used often in these online activities, due to the ease of sharing screens (hence students' work) and inviting students to explain their reasoning, followed by the *discuss-the-screen orchestration*, where their peers were invited to reflect and discuss the volunteer-student's contribution. Tania wrote explicitly about how she was able to check on students' progress through the 'Response' mode, being able to offer individual support, or lead a group discussion by *explaining-the-screen*, all of which were also highlighted by Tania as benefits of online learning. Tania's *didactical performance* was visible in her actions: 'In the 'Response' mode I could see students were happy to try out different equations. Only one pair were not inputting anything so I joined their breakout room' and while this aspect was clearly an advantage of online learning, Tania appreciated that it could be a limitation, as the rich mathematical conversations she had with pairs of students in breakout rooms were missed out by the rest of the students. She admitted '[All] students would also benefit from being able to listen in to each other's conversations' in a face-to-face learning situation.

For one of the activities, she designed (presented in Figure 2), Tania sent the students in breakout rooms for another opportunity to work with and consolidate their understanding of links between different representations. She particularly liked this activity as it asked the students to 'explain their thinking'. The justification box completed by students in the breakout rooms would instantly be visible to her, meaning that she was able to monitor students' understanding and support them when and if needed. At one point in the online activity, she noticed that one pair of students provided the answer as a quadratic expression in the expanded form, and not in the form that would evidence their understanding of the links between graphs as horizontal translations. She joined their breakout room to praise the students for finding the answer and asked them if their equation could be written in any other format. One student proceeded to re-write the equation of the parabola passing through the purple points on the graph as $y = (x + 4)^2$, while the other student immediately then said 'oh, because it's moved to the left four' having made the link to their knowledge of transformations of graphs. Tania brought everyone back together and asked this pair of students to explain what they had discovered by sharing screenshots and pictures of their graphs, their written work, and by actively interacting with Desmos while explaining their work. Using Drijvers et al.'s (2014) framework, there is evidence in Tania's reflections that the *link-screen-board orchestration* happened often in this online activity. In fact, throughout the activity, Desmos screens were shared via Zoom, which together with the written and verbal justifications, and pointing-at-screen by students and Tania, facilitated the *link-screenS* (Desmos and Zoom)-*board* orchestrations, supporting thus transitioning and making connections between the two DTs and conventional mathematical work.

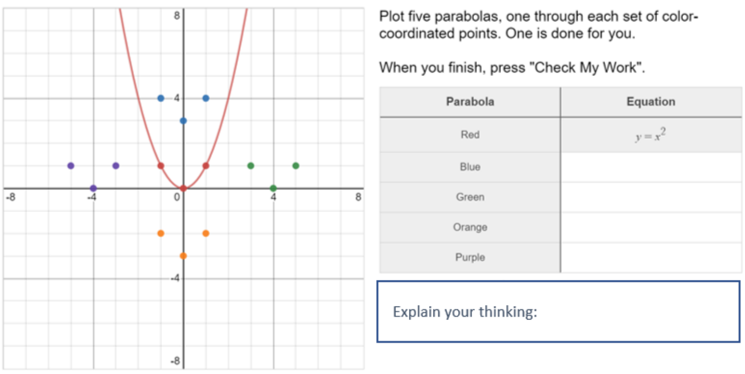

Figure 2: Mathematical Investigation with Desmos

Conclusion

The application of TIO to the data (graduate student's written reflections on her design considerations for online mathematical learning with a DT) we presented above indicates that the three elements of the TIO framework, namely *didactical configuration, exploitation mode* and *didactical performance*, which were developed for face-to-face classroom teaching and learning, provided us with a useful framework for describing the orchestrations necessary for designing and carrying out online learning activities. We investigated the reflections of a graduate student on practices involved when technology-rich learning activities take place online instead of a face-to-face learning situation. The nature of social interaction normally observed in face-to-face classroom-based mathematical learning activities, such as pair-work, students' and teachers' non-verbal gesturing, instant assessment of and feedback to students' learning, needed to be orchestrated for carrying out a mathematical activity online to ensure students' learning did take place. Breakout rooms, contributions to chat, freezing screens, pacing the learning, ease of use of mathematics specific DTs, were used to promote the same learning outcomes for students.

In many respects, such orchestrations facilitated learning in more productive ways according to Tania, as the ease of instant or timely access to each student's work and their contributions in the chat tool at the click of a button were pointed out as advantages of online learning. The ease of monitoring the students' work led to offering instant and timely individual support, and Tania reflected on these aspects as being more productive in an online environment. In other cases, Tania's orchestrations hindered certain pedagogical practices. For example, Tania had rich mathematical conversations with some pairs of students in the breakout rooms, but then the rest of the students missed these conversations as they were not in the 'same room'. This of course would not have necessarily been the case if Tania and the students were all in the same 'physical' room, where she could have easily initiated a whole group discussion.

Online input of mathematical writing is a well-known challenge (Leventhall, 2004), and Tania found a way to 'make up' for this limitation by designing the online activities around students' mathematical investigations using Desmos. Her pedagogical decision for this DT was taken in order to address both the learning objective of the mathematical activity (linking different representations of quadratics), but also to support students to do and share mathematical work through Desmos (graph drawings), and chat boxes (typing explanations). We argue that there are ways around such issues (for example Tania would take notes of the key messages in the breakout conversations she had and mention these to all students when they are back in the same virtual room), but this strategy, and the many others that Tania shared with us in her reflections, clearly indicate the need for careful considerations and investment of time in designing activities for online mathematical learning.

This paper's contribution lies in exemplifying how the TIO framework's orchestrations can be broadened as a result of overcoming the challenges and tapping into the opportunities of online learning, as supported by careful design considerations for technology enriched practices based on certain affordances of DTs. We discussed how a graduate student addressed the challenges of online learning and capitalized on the opportunities offered by DTs, while developing her own knowledge of and expertise with technology enriched mathematical learning. Tania had to quickly broaden her field of pedagogical expertise, by identifying how best learners interact online with their peers (in breakout rooms and the main Zoom room), and with technology (Desmos) to do mathematics and therefore accommodating their needs. These considerations should be central to any design decisions when orchestrating students' mathematical learning online.

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