School mathematics education and digital technologies

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A discussion paper for JMC, May 2021

Introduction

It is generally agreed that mathematicians operate with objects that do not physically exist but are mentally constructed (Davies et al., 2011). Mathematicians strive for objectivity (Gowers, 2002) where results are invariant to the methods, contexts or technologies used or the people who derived them, although mathematical objects and associated meanings are constructed and negotiated by humans (Khait, 2005). Mathematicians commonly strive for abstract universal approaches irrespective of contexts and generally aim for a minimum of equipment to solve complex problems, using logic and abstraction as a main vehicle (Lyakhova et al., 2019). Thus, the use of technology in mathematics is secondary to mathematical thought (Gowers, 2002) [1].

However, in the diverse range of everyday life, employment and scholarly applications of mathematics, digital technologies are often used for what might be termed the *outsourcing* of mathematical processes. In learning mathematics, as well as when applying mathematics in a workplace or further study, it is important to be able to source, choose, and apply appropriate mathematical results and procedures established elsewhere without necessarily deriving them, interpreting and critiquing the outcomes. In schools, mathematics learners' use of computation tools is typically complemented by experience with hands-on processes and representations in order to develop mathematical concepts and connections. In teaching and learning of mathematics, technology is also used for visualising and communicating mathematical concepts and ideas. The use of technology in school mathematics should therefore reflect the needs of the discipline, of school mathematics and of pedagogies.

The development of digital technologies has to date had few implications for the 'what' of the school mathematics curriculum (with learning how to use technology for outsourcing mathematical processes being one exception), while it has somewhat influenced the 'how'. Technology has traditionally largely been used for teaching a pre-digital syllabus in traditional face-to-face classrooms, as well as by learners for learning mathematics outside the classroom in preparation for learning in class (such as flipped learning), in the digital (synchronous or asynchronous) classroom, by learners outside the classroom for enriching their experience of mathematics, and for developing teacher knowledge. This expanding range is already disrupting traditional student/teacher roles and power relationships, and blurring distinctions between in-school and beyond-school for many learners. Additionally, as the current pandemic has shown, technology plays an important role for (mathematics) education in times of crisis. Globally, there are now moves to re-visit and re-think the uses of digital technologies for mathematics education. This paper presents one short overview of the current state of the field.

Overview

What follows should be interpreted differentially according to the age/stage of the learners, for example, 'independent' learning can be boosted through the use of digital technologies, but in different ways and to different extents as children mature. However, from early years, children can experience digital enrichment of their mathematics learning, experiences, and developing applications of those. A broad body of work (e.g. Hunt et al., 2011) suggests students need to experience concrete, pictorial and manually-generated embodiments

[1] This, however, may be challenged in the near future, with digital technologies playing an increasingly important role for discovering new mathematics or verification of new mathematics results which humans may not be able to do 'by hand' (see, e.g., the Kepler Conjecture as one example https://en.wikipedia.org/wiki/Kepler_conjecture).

and exploration of mathematical concepts, as well as any digital representations, if concepts and processes are to be used appropriately and their affordances and limitations understood. Traditional and digital approaches are not mutually exclusive: they may be interchangeable or complementary and both need to be considered, including for inclusion purposes. Connection-making, both within and beyond mathematics, is key to mathematics knowledge formation and critical for informed use of mathematics. Within and beyond classrooms, digital technologies can support a range of connection-making in mathematics. In the workplace and in wider life it is important to be able to make connections between the mathematical model and representation, its interpretations, limitations and the situation being worked with, but the conceptual grasp and connections required by the general user of that model are more limited than those required by the person selecting and populating the model/representation – as we have seen, for example, in coming to understand the progress of the coronavirus pandemic.

We first set out some arguments for the harnessing of digital technologies for mathematics education purposes; we give an overview of a range of current such uses and then point to some key related issues, concluding with a high level overview of the current 'state of play'. The appendix gives a summary of the findings of the 2011 JMC report 'Digital technologies and mathematics education'.

Why use digital tools in mathematics education?

- **Conceptual development** in school mathematics can be supported by digital tools: 'relationships that are key for mathematical understanding are highlighted, made more tangible and manipulable. The computer screen affords the opportunity for teachers and learners to make explicit that which is implicit, and draw attention to that which is often left unnoticed' (Hoyles 2018, p12). They can support conceptual exploration and conjecture, including for example with geometry, functions or sets of data, and offer immediate feedback. However, the effect sizes in terms of enhancing conceptual engagement reported in experimental studies in the field are significant but only small to moderate (Drijvers, 2018a).
- Digital tools can **outsource algorithmic functions**, enabling greater focus on other aspects of a task (with a concomitant potential challenge to what makes an appropriate mathematics curriculum in schools or HEIs).
- For **practising mathematical skills**/building up experience, digital tools have much to offer, such as variation and randomisation of tasks, automated and intelligent feedback, and a personal environment in which one can safely make mistakes and learn from them (Drijvers, 2018b). They can enable insights into evidence of student thinking and practice, since those are captured digitally, supporting formative (as well as summative) assessment. Through digital technologies, combining traditional face-to-face with online teaching and learning could offer the best of both worlds, bringing the collaborative/3-D responsive element together with extended opportunities for reflection, both of which are valuable for mathematics learning. Such developing forms of education may assume a greater learner maturity but can also nurture independent learning skills in students (Golding et al., 2021).
- More broadly, digital technologies can **widen access** to mathematics education among poorly represented groups, as in e.g. AMSP, FSMPW work in Wales. Remote teaching has been traditionally valued for the realisation of social justice and widening access in education and, at school level in particular, for efficient use of resources.
- Students' **wider employment, further study and personal needs** require them to employ digital tools in using and applying mathematics. Even in 2011, the JMC digital technologies report (p3) argued young people should therefore have opportunity to experience some of those wider mathematical purposes within the school curriculum: 'What is needed in schools and colleges is student-led mathematical modelling, problem solving and computer programming which makes use of the powerful mathematical digital technologies that are widely used in society and the workplace'.
- Digital tools are already playing a vastly **expanded role in novel applications** and increasingly in **constructing mathematics knowledge** which, in turn, should impact the school curriculum. The school

curriculum, however, remains stubbornly resistant to change and rather backward looking; more influenced by well-established mathematics and traditional methods, than by exciting new ideas and new technologies.
- An emerging body of research highlights the role of digital technologies in **bridging** between mathematics and art, linguistics, humanities, social sciences, and other areas of human activities (see, e.g, Fenyvesi et al., 2017). Such connections offer new possibilities for wider appreciation of mathematics and technology, better cross-curricular links and supporting greater diversity.

Digital tools currently available include, in overlapping categories by broad purpose:

- **Generic digital platforms** for teachers that enable remote teaching and learning: email, webinar software, VLEs, perhaps drawing on a selection of the tools below. Might support capture of live teaching/discussion, including with subtitles, or pre-record: in either case, events/resources are then available flexibly for revisiting. Might allow uploading of student work for discussion and/or assessment. Such platforms also allow 'bringing (shared) expertise into the classroom' by increasing connectivity with other learners or with expertise not otherwise physically available, and so widen opportunities for collaboration.
- **Tools for presentation**, either in person or remotely: might simply mimic a whiteboard but might be further developed with hyperlinks, embedded Apps etc., might be teacher-developed or freely downloadable from web or a bought-into package or website, might be stand-alone presentation software used for teaching. Teacher or students might 'capture' boardwork for later reference, thus offering learners additional tools for developing resilience and self-direction.
- **Stand-alone 'Whole teaching' lessons or packages** that might conceivably replace, complement or support the teacher (e.g. DfE-funded 'Oak Academy', MEI's Integral). To date these have typically had limited opportunity for or responsiveness to student input but that is in some cases improving, e.g. Desmos.
- Stand-alone **generic tools for mathematical exploration/concept development or problem solving**, e.g. spreadsheets and more refined data handling (analysis/presentation) software, programming languages, data loggers such as motion detectors and GPS
- **Computational devices/software** (e.g. calculators, statistical software) that 'outsources' procedural aspects of problems to enable greater attention to be given to problem solving and modelling, pattern spotting, global behaviour, invariants, etc. A risk here is that the procedures and assumptions (e.g. in statistical software) can get hidden and so be not understood or appreciated (e.g. Bakker et al., 2006). The same occurs when outsourcing with digital technologies or outsourcing arguments in mathematics more generally.
- Stand-alone **mathematical software used for mathematical exploration/concept development** or problem solving, e.g., graphing software, dynamic geometry package (Geogebra), CAS, programming languages developed specifically for mathematical purposes such as Logo, Scratch. These can support exploration and conjecture, identification of key relationships between different representations, development of algorithmic thinking - as well as outsourcing computations or arguments.
- **Digital manipulative representations** (overlapping with e.g. graphing packages). These are thought to be often equally, but differently, powerful support for learning compared with physical representations (Hunt et al. 2011).

Other tools:

- Apps or small pieces of software designed for specific purposes, e.g. data loggers such as motion detectors and GPS.
- Interactive subject-specific software designed to support development of particular (sets of) mathematical concepts, e.g. Cornerstone Mathematics.

- Simulation software, modelling specific types of situations that involve a number of manipulable variables. Such software, in conjunction with other mentioned such as programming languages, allow creation and exploration of ‘microworlds’.
- Software designed to support practice/experience with (and perhaps assessment) of core knowledge and skills, e.g. MyMaths. These might also have a ‘teaching’ element.
- Digital versions of printed independent learning and practice tools, e.g. textbooks. Might be enhanced by particular digital affordances such as hyperlinked solutions or apps.
- Images, videos or podcasts or…. with potential for stimulating mathematical questions, models, etc.

Some key issues

- Tools matter (they are not neutral): they both mediate and can disrupt the relationship between learner and concept, and they shape learning activity. More needs to be known about the way and the underlying principles in which activities with digital tools mediate the learning in a fruitful way, so that such potential can fully exploited (Monaghan, Trouche, & Borwein, 2016). Related to this, the differences in the learner psychology of mathematics-students and *mathematics-students-with-mobiles* are yet to be understood (Borba et al., 2017).
- Sinclair and Yurita (2008) discuss how using a dynamic geometry tool changes classroom discourse, with significant differences in talk around geometric objects, use of visual artefacts and modelling of geometric reasoning. Sometimes, especially remotely, use of a digital tool brings absence of a ‘familiar curriculum script’ (Ruthven, 2009). Such work requires an explicit focus on design – of tasks, digital tools, feedback and evaluation – as well as the investigation and identification of what might be different goals for school mathematics in the light of the available technological infrastructure.
- Student appropriation of a mathematical digital tool for a specific purpose requires the co-emergence of technical and conceptual knowledge, and so skilled teaching.
- Teachers therefore require ‘Technological pedagogical content knowledge’ (‘TPACK’) (Fig 1) where the ‘content’ is the target mathematics. They may also suffer from: limited conviction about the potential of digital technologies for the teaching and learning of mathematics, limited confidence with digital technologies or resolving problems with the technology; concern about knowing less than their learners and limited access to digital technologies or suitable bandwidth. They might have inappropriate training, time pressures, and technology use poorly embedded into schemes of work (NCETM, 2010). However, digital technologies also offer opportunities for teacher development, with advantages of time and cost, though drawbacks of more limited physical contact.

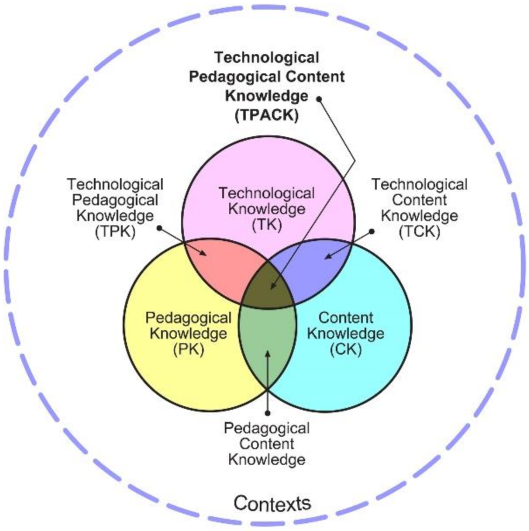

Figure 1:The TPACK framework (Mishra & Koehler 2006)

- Access to mobile technologies disrupts the traditional flow of mathematics knowledge between teacher and learner, in ways not well understood from a research perspective (Borba et al., 2017). For example, we know little about the potential of MOOCs to affect access to, quality of, and hierarchy in mathematics education; about the pedagogical design of the resources freely available on the internet that attracts students maybe before they turn to teachers, and how that impacts on the quality of mathematics learning; or about the effectiveness of different models used for blended learning that can make the classroom a place of extension, enrichment, challenge and elaboration rather than primarily of direct instruction.

What use of digital tools is actually made, or promoted, in UK classrooms?

The above uses can all be seen in and around some mathematics classrooms in England and other parts of the UK, but such use has always been variable, spearheaded by enthusiasts, and the detailed evidence for benefits largely based on small-scale studies. Indeed, some larger studies show just how difficult it is to scale up and embed good use of technology for mathematical purposes[i]. The Royal Society Vision report (2014, p51) confidently states 'There is a strong expectation, therefore, that the new digital technologies will have a profound impact on young people's education' but progress appears slow, and the recommendations of the JMC report on digital technologies and mathematics education (2011), reproduced in the Appendix, are far from being met. The last Ofsted mathematics report (2012) in England makes very little mention of digital tools: of its 226 paragraphs, #65 says 'the potential of ICT to develop learning in mathematics continues to be underdeveloped' and there follow 4 examples of good use in both primary and secondary schools on pp29-30. #71 gives an example that uses graphing software, and the promoted scheme of work in #190 mentions ICT.

International comparisons: In PISA 2018, Headteachers in England on average reported a greater availability of internet-enabled devices, and greater teacher technical and pedagogical preparedness for their use, than in much of the rest of the OECD, though this was not broken down by subject taught. In TIMSS19 in England, about one-fifth of pupils participated in at least monthly activities on computers in both mathematics and science (lower than that found in the majority of comparator countries), though there was no association between average attainment and year 9 pupils' access to computers during mathematics and science lessons.

Curriculum integration: It is important to understand what is currently valued in curriculum and assessment. The issue has a low profile in the current English National Curriculum (2014), which in its preamble says 'Calculators should not be used as a substitute for good written and mental arithmetic. They should therefore only be introduced near the end of key stage 2 (*age 11*) to support pupils' conceptual understanding and exploration of more complex number problems, if written and mental arithmetic are secure. In both primary and secondary schools, teachers should use their judgement about when ICT tools should be used'. No further mention is made, even in setting out what data-related work students should cover. In Mathematics A Level, the specifications from 2017 include mandated engagement with a 'large data set' using technology, but some emerging evidence suggests many teachers and students remain ill-prepared for the data handling technology, and the requirements were being widely ignored; also that while some students make use active use of statistical, graphing, or other modelling software, others access little more than a scientific calculator for mathematical purposes. The mathematics curriculum in Scotland, for example, is rather more embracing of the potential of digital tools to enhanced the learning of mathematics.

The recent pandemic has shown schools and families to be very variably prepared, or equipped, to move to online learning, and such issues need to be addressed. A range of emerging systematic evidence of the uses to which digital technologies have been put during this period suggests teachers and students have often become more adept at using technology for meetings, presentations and sharing work, or accessing pre-prepared digital packages, but that use for mathematical exploration, modelling, analysis, etc. has not yet experienced similar uptake.

This brief overview suggests that there is much work to be done to realise the potential of digital technologies to support mathematics curricula, pedagogies, and related assessments, and to ensure that their development is appropriate for and supportive of future mathematical needs, and equitably accessible.

References

Bakker, A., Hoyles, C., Kent, P., & Noss, R. (2006). *Improving work processes by making the invisible visible*. Journal of Education and Work, 19, 343-361.

Borba, M. C., Askar, P., Engelbrecht, J., Gadanidis, G., Llinares, S., & Aguilar, M. S. (2017). Digital technology in mathematics education: Research over the last decade. In Proceedings of the 13th International Congress on Mathematical Education (pp. 221-233). Springer, Cham.

Davis, P., Hersh, R., & Marchisotto, E. A. (2011). The mathematical experience. Springer Science & Business Media.

Drijvers, P. (2018b). Tools and taxonomies: a response to Hoyles, *Research in Mathematics Education, 20*:3, 229-235. DOI: 10.1080/14794802.2018.1522269

Drijvers, P. (2018a). Empirical evidence for benefit? Reviewing quantitative research on the use of digital tools in mathematics education. In L. Ball, P. Drijvers, S. Ladel, H.-S. Siller, M. Tabach, & C. Vale (Eds.), *Uses of technology in primary and secondary mathematics education; tools, topics and trends* (161–178). Cham: Springer International Publishing.

Fenyvesi, K., Lähdesmäki, T., & Verlag, B. (Eds.). (2017). Aesthetics of interdisciplinarity: art and mathematics. Cham: Springer International Publishing.

Golding, J., Barrow, E., & Grima, G. (2021). *Power Maths: implementation, response and learning in a pandemic Autumn.* London: Pearson UK.

Gowers, T. (2002). Mathematics: A very short introduction. Oxford, UK: Oxford University Press.

Gudmundsdottir, S. (1990). Values in pedagogical content knowledge.

Hoyles, C. (2018). Transforming the mathematical practices of learners and teachers through digital technology, *Research in Mathematics Education, 20*:3, 209-228, DOI: 10.1080/14794802.2018.1484799

Hunt, A.W., Nipper, K.L., & Nash, L.E. (2011). Virtual vs. concrete manipulatives in mathematics teacher education: is one type more effective than the other? *Current Issues in Middle School Education 16*:2, 1-6.

Khait, A. (2005). The definition of mathematics: philosophical and pedagogical aspects. Science & Education, 14(2), 137-159.

Lyakhova, S., Joubert, M., Capraro, M. M., & Capraro, R. M. (2019). Designing a curriculum based on four purposes: let mathematics speak for itself. *Journal of Curriculum Studies*, 51(4), 1–17.

Mishra P. & Koehler, M.J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record, 108*:6, 1017–1054.

Monaghan, J., Trouche, L., & Borwein, J.M. (2016). *Tools and Mathematics: Instruments for Learning*. *Springer.*

National Centre for Excellence in the Teaching of Mathematics (2010). *Mathematics and digital technologies: New Beginnings*.

Ruthven, K., Deaney, R. & Hennessy, S. (2009). Using graphing software to teach about algebraic forms: a study of technology-supported practice in secondary-school mathematics. Educ Stud Math 71, 279–297.

Sinclair, N. & Yurita, V. (2008). To be or to become: How dynamic geometry changes discourse, Research in Mathematics Education, 10:2, 135-150, DOI: 10.1090/14794800802233670.

Appendix

'Digital technologies and mathematics education' (JMC, 2011) recommendations:

Recommendation 1 *(For policy makers and teachers):* School and college mathematics should acknowledge the significant use of digital technologies for expressive and analytic purposes both in mathematical practice outside the school and college and in the everyday lives of young people.

Recommendation 2 (For policy makers): Curriculum and assessment in school mathematics should explicitly require that all young people become proficient in using digital technologies for mathematical purposes.

Recommendation 3 *(For policy makers):* High-stakes assessment needs to change in order to encourage the creative use of digital technologies in mathematics classes in schools and colleges.

Recommendation 4 *(For policy makers and school leaders):* As the development of a technologically enriched student learning experience occurs at the level of the classroom, such change has to be supported by school leaders and accompanied by sustained professional development opportunities for teachers.

Recommendation 5 *(For policy makers):* The UK Departments for Education and for Business, Innovation and Skills should establish a Task Force to take the lead in bringing together various parties with appropriate expertise to take forward the recommendations of this report and advise the Departments on required policy initiatives.