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#### Decline of the Calculators in Paris c. 1500: Humanism and Print

#### **Richard Oosterhoff**

#### Introduction\*

A usual account of why the calculatory tradition barely survived past the early sixteenth century turns to humanism. When the Calculators are mentioned by grand humanists such as Bruni, Petrarch, Erasmus, Barbaro, and Vives, they are in for a good kicking.<sup>1</sup> The mood is set by Thomas More's letter to Martin Dorp of 1518, in which he decried the 'supersophistical trifles' of modern logic. He identified a better future in the textbooks of the eminent Parisian scholar Jacques Lefèvre d'Étaples (c. 1455–1536).<sup>2</sup> And indeed, Lefèvre often presented his textbooks as an antidote to the bad philosophizing of sophists. Moreover, the Calculators emblematized the worst kinds of pathological thought:

What about this tramp who, like some sort of trickster or fraud, long ago was rightly exiled from Italy and found shelter among us – who calls herself 'calculation' while perverting all rational calculating? First off, remove the too rough and unlearned absurdity of speech, which confounds men and angels with asses and mingles gods above with realms below. The absurdity propagates every day in the poisonous manner of weeds; indeed they ensnare and ruin tender wits, spoiling the bread of all teaching.<sup>3</sup>

<sup>\*</sup> My thanks to Daniel Di Liscia for sharing many kind suggestions and material over the past few years, and for kindly enabling my remote participation in the conference leading to this volume, even when I could not be there in person. I also owe thanks to both Daniel and Edith Sylla for thoughtful and generous suggestions on an earlier draft of this chapter.

Studies of this reputation include C. Dionisotti, 'Ermolao Barbaro e la Fortuna di Suiseth,' Medioevo e Rinascimento: Studi in onore di Bruno Nardi, Florence 1955, 1: 219–253; C. Vasoli, Studi sulla cultura del rinascimento, Manduria 1968 (Biblioteca de studi moderni, 5), 139–177; D. A. Di Liscia, 'Kalkulierte Ethik: Vives und die "Zerstörer" der Moralphilosophie (Le Maistre, Cranston und Almain),' in S. Ebbersmeyer and E. Keßler (eds.), Ethik: Wissenschaft oder Lebenskunst? Modelle der Normenbegründung von der Antike bis zur frühen Neuzeit, Münster 2007, 75–105.

<sup>&</sup>lt;sup>2</sup> T. More, 'Letter to Martin Dorp,' ed. D. Kinney, New Haven, CT, 1986 (The Complete Works of St. Thomas More, 15), 22–23. Basic studies of Lefèvre include A. Renaudet, *Préréforme et humanisme à Paris pendant les premières guerres d'Italie, 1494–1517*, 2<sup>nd</sup> ed., Paris 1953; E. F. Rice, Jr. (ed.), *The Prefatory Epistles of Jacques Lefèvre d'Étaples and Related Texts*, New York 1972.

<sup>&</sup>lt;sup>3</sup> J. Lefèvre d'Étaples (ed.), Egregii patris et clari theologi Ricardi [Sancti Victoris] De superdivina Trinitate theologicum opus hexade librorum distinctum. Commentarius artificio analytico, Paris 1510, sig. a<sup>2r</sup> (ed. Rice, The Prefatory Epistles, 226): "Quid quod inculcatoria quae pridem tamquam circulatrix quaedam et subdola iure ab Italis exulat apud nostros invenit asylum et se calculatoriam nominat quae omnem rationis pervertit calculum? Tolle insuper rudem nimis et indoctam sermonis

By taking aim at the Calculators, Lefèvre echoed a standard critique of certain tendencies in medieval university thought. The critique targeted a genre that began in the undergraduate exercises of disputation known as *sophismata*, devoted to fast-paced unknotting of logical problems.<sup>4</sup> Students often deployed calculatory techniques in the course of disputing sophisms, as Edith Sylla once pointed out.<sup>5</sup> In the works of William Heytesbury, Richard Swineshead, and Thomas Bradwardine, many *sophismata* aimed to apply mathematical models to physical change, using what John Murdoch called "languages of analysis" that drew on proportions, the intension and remission of forms, first and last instants, and maxima and minima.<sup>6</sup> Over the fourteenth century, at Oxford, and then at Paris and elsewhere, these techniques and exercises became popular.<sup>7</sup> Although the word *sophista* could simply describe students engaged in such exercises, without reprobation, the term's pejorative meanings were convenient for university critics.

These mathematical *sophismata* already seemed pernicious to late medieval critics within the university. In fact, despite the angle of humanist attack, it is misleading to see it as coming from outside the university, for it was university men who set the trajectory of the humanist critique – a first hint that the usual humanist story is oversimplified. The critique had been given particular force by Jean Gerson, chancellor of the University of Paris in the years around 1500. He worried that even theology had become, over the last generation, filled with unhealthy habits, so that theologians were jostling for position in logical puzzles rather than preparing to serve a public in need of pious teachers. Gerson used the word "sophist", Zénon Kaluza has suggested, both to describe students engaged in certain disputations as well as those espousing the "English" style of inquiry after Bradwardine, Heytesbury, etc.<sup>8</sup> In Gerson's view, the problem was partly that students gained bad habits, valuing obscurity over common sense, a kind of "curiosity" that hungered after novelty more than truth. The problem was also partly one of boundary crossing. Rather than leaving these techniques in logic

absurditatem quae homines et angelos cum asinis confundit et superos Acherontaque miscet, loliorum more sese in dies noxie propagantem et tenella implicantem immo perdentem ingenia, omnis doctrinae panem inficientem."

<sup>&</sup>lt;sup>4</sup> E.g. the recent study of S. Ebbesen and F. Goubier, *A Catalogue of 13<sup>th</sup>-Century Sophismata*, I: *Introduction and Indices*; II: *Catalogue of* Sophismata, Paris 2010 (Sic et non).

<sup>&</sup>lt;sup>5</sup> E. D. Sylla, 'The Oxford Calculators,' in N. Kretzmann, A. Kenny, J. Pinborg and E. Stump (eds.), *The Cambridge History of Later Medieval Philosophy. From the Rediscovery of Aristotle to the Disintegration of Scholasticism, 1100–1600*, Cambridge 1982, 540–563, esp. 542–547; E. D. Sylla, 'The Oxford Calculators in Context,' *Science in Context* 1 (1987), 257–279.

<sup>&</sup>lt;sup>6</sup> J. E. Murdoch, 'From Social into Intellectual Factors: An Aspect of the Unitary Character of Late Medieval Learning,' in J. E. Murdoch and E. D. Sylla (eds.), *The Cultural Context of Medieval Learning*, Dordrecht 1975 (Boston Studies in the Philosophy of Science, 26), 271–348.

On this expanding popularity, see J. E. Murdoch, 'Mathesis in philosophiam scholasticam introducta. The Rise and Developement of the Application of Mathematics in Fourteenth-Century Philosophy and Theology,' in Arts libéraux et philosophie au Moyen Âge. Actes du quatrième Congrès international de philosophie médiévale, Montréal 1967, Montreal 1969, 215–254, esp. 216–227; Sylla, 'The Oxford Calculators in Context'; D. A. Di Liscia, Zwischen Geometrie und Naturphilosophie: die Entwicklung der Formlatitudenlehre im deutschen Sprachraum, unpublished Ph.D. dissertation, University of Munich, 2003.

<sup>&</sup>lt;sup>8</sup> Z. Kaluza, Les querelles doctrinales à Paris: nominalistes et réalistes aux confins du XIV<sup>e</sup> et du XV<sup>e</sup> siècles, Bergamo 1988 (Quodlibet, 2), 43–45.

or mathematics, he thought students carried them into domains of theology where other forms of reasoning should prevail. At Paris, such criticisms can be traced through the works of university reformers from Gerson to Jean Standonck at the Collège de Montaigue. As this rhetoric also became a standard line among critics who took up stances outside the university, these charges have set "scholasticism" against "humanism", opposing an ideal of philosophy as driven by technical school-logic to one rooted in common-sense language.<sup>9</sup>

But did this rhetorical opposition cause the decline of the Calculators' project? Quite rightly, historians have noted that calculatory techniques received a burst of interest in Paris shortly after 1500, in the circle around John Mair at the Collège de Montaigu.<sup>10</sup> A key figure here is Alvarus Thomas, who published in 1509 the most sophisticated contribution to the calculatory tradition of the sixteenth century.<sup>11</sup> Others included Luis Coronel, Gaspar Lax, Pedro Ciruelo, Juan Martínez Silíceo, and Juan de Celeya – who all wrote on mathematics, and would go on to teach and publish mathematical works in Spain, nourishing the mathematical culture that encouraged Domingo de Soto to put the mean-speed theorem to work on falling bodies, much as Galileo would a couple generations later.

In this picture, one missing piece is the pedagogical scene. What relation did these analytical languages have to usual physics teaching on the *cursus*? How far, in other words, was Alvarus Thomas from the reach of most students? Was this burst of interest in Calculators a sudden flash among a few high-powered intellects or was it part of everyday intellectual experience at Paris? In this chapter, I argue that, to understand the demise of the calculatory tradition, we need to take account of how this discipline was framed in university textbooks, as well as the changing mathematical culture of the early sixteenth century.

<sup>&</sup>lt;sup>9</sup> This definition of humanism is given by L. W. Nauta, 'Latin as a Common Language: The Coherence of Lorenzo Valla's Humanist Program,' *Renaissance Quarterly* 71 (2018), 1–32; C. S. Celenza, *The Intellectual World of the Italian Renaissance: Language, Philosophy, and the Search for Meaning,* New York 2018.

<sup>&</sup>lt;sup>10</sup> Perhaps the most forceful view has been advanced by William Wallace, following Pierre Duhem, that this was a key link between the Calculators and Galileo: W. A. Wallace, 'The Calculatores in Early Sixteenth-Century Physics,' The British Journal for the History of Science 4 (1969), 221–232 (republished in Id., Prelude to Galileo: Essays on Medieval and Sixteenth-Century Sources of Galileo's Thought, Dordrecht 1981 [Boston Studies in the Philosophy and History of Science, 62], 78–90). On this generation, see H. Élie, 'Quelques maîtres de l'Université de Paris vers l'an 1500,' Archives d'histoire doctrinale et littéraire du Moyen Âge 18 (1950–1951), 193–243; A. Broadie, The Circle of John Mair. Logic and Logicians in Pre-Reformation Scotland, Oxford 1985; Di Liscia, 'Kalkulierte Ethik.'

<sup>11</sup> A. Thomas, Liber de triplici motu proportionibus annexis magistri Alvari Thome Ulixbonensis Suisseth calculationes ex parte declarans, Paris 1509. Besides E. D. Sylla, 'Alvarus Thomas and the Role of Logic and Calculations in Sixteenth-Century Natural Philosophy,' in S. Caroti (ed.), Studies in Medieval Natural Philosophy, Florence 1989 (Biblioteca di Nuncius, 1), 257-298, see now S. P. Trzeciok, Alvarus Thomas und sein Liber de triplici motu, 1: Naturphilosophie an der Pariser Artistenfakultät; 2: Bearbeiteter Text und Faksimile, Berlin 2016, URL = <https://edition-opensources.org/media/sources/7/Sources7.pdf>. See also Edith Sylla's contribution to the present volume.

#### 1. Parisian Printed Textbooks c. 1500

Notwithstanding the strong words he would aim at the Calculators, Lefèvre himself wrote one of two new contributions to the genre to be printed in Paris in the 1490s: his *Dialogus difficilium physicalium introductorius (Introductory Dialogue on More Difficult Physics*).<sup>12</sup> (The other was a collection of extracts from various sources, by a young Scotist theologian in training, Pierre Tartaret.<sup>13</sup>) Lefèvre's contribution concluded his first publication, a textbook of Aristotelian natural philosophy intended to help undergraduate students through all the topics they needed to navigate natural philosophy at Paris. The book began with a paraphrase of Aristotle's *Physics*, continued through a survey of *On the Heavens, On Generation and Corruption*, the *Meteorology, On the Soul*, and a selection from the *Parva naturalia*. The volume closed with two dialogues: the first introduced the basic terms of the *Physics*, preparing the reader to approach the second, "more difficult" dialogue, which introduced techniques for reasoning about latitudes of forms, maxima and minima, and so on.<sup>14</sup>

My task will be to see what the context of printed textbooks at Paris may suggest about the place of such studies in regular teaching. As Edith Sylla pointed out, attitudes about and interest in the texts of the Oxford Calculators varied considerably from place to place; Daniel Di Liscia has uncovered important evidence that the study of the *latitudines formarum* retained a place in the curriculum of the universities of Cologne and Vienna.<sup>15</sup> A fuller answer for Paris would have to take account of manuscripts; but in 1490s Paris, university teachers were beginning to experiment with new printed books, a fact which would swiftly reshape the university curriculum over the next decade.

In 1492, then, a printed handbook to natural philosophy was new. There had been medieval *summae naturales*, such as those by Robert Grosseteste and (ps.) Albert, which covered much of the same ground.<sup>16</sup> But even though rhetorics and classics were some

<sup>&</sup>lt;sup>12</sup> J. Lefèvre d'Étaples, Dialogus difficilium physicalium introductorius, Paris 1492. The present study grows out of an account of this work in R. J. Oosterhoff, Making Mathematical Culture: University and Print in the Circle of Lefèvre d'Étaples, Oxford 2018 (Oxford-Warburg Studies), 181–199. I have expanded on the concerns of genre around these works in R. J. Oosterhoff, 'The Dialogue of Ingenuous Students: Early Printed Textbooks at Paris,' in S. Berger and D. Garber (eds.), Teaching Philosophy in the Seventeenth Century: Text and Image, Dordrecht 2021 (forthcoming).

<sup>&</sup>lt;sup>13</sup> P. Tartaret, *Tractatus de intensione, rarefactione et condensatione formarum utilis ad totam phisicam intelligendam extractus a Gregorio de Herimino et aliis doctoribus*, Paris 1493.

<sup>&</sup>lt;sup>14</sup> J. Lefèvre d'Étaples, *Totius Aristotelis philosophiae naturalis paraphrases*, Paris 1492.

E. D. Sylla, 'The Fate of the Oxford Calculatory Tradition,' in Ch. Wénin (ed.), L'homme et son univers au Moyen Âge. Actes du septième congrès international de philosophie médiévale (30 août – 4 septembre 1982), Louvain-la-Neuve 1986 (Philosophes médiévaux, 26), 2: 692–698; Di Liscia, Zwischen Geometrie und Naturphilosophie, 173–212; D. A. Di Liscia, 'The "Latitudines breves" and Late Medieval University Teaching,' SCIAMVS 17 (2016), 55–120.

<sup>&</sup>lt;sup>16</sup> The classic study is M. Grabmann, *Methoden und Hilfsmittel des Aristotelesstudiums im Mittelalter*, München 1939 (Sitzungsberichte der Bayerischen Akademie der Wissenschaften, Philosophischhistorische Abteilung, 1939/5); see also A. Cunningham and S. Kusukawa, 'Translators' Introduction,' in A. Cunningham and S. Kusukawa (eds.), *Natural Philosophy Epitomised: Books 8–11 of Gregor Reisch's* Philosophical Pearl (*1503*), Farnham 2010, ix–lxxiv; D. A. Lines, 'Teaching Physics in Louvain and Bologna: Franz Titelmans and Ulysse Aldrovandi,' in E. Campi, S. De Angelis, A.-S. Goeing and A. Grafton (eds.), *Scholarly Knowledge: Textbooks in Early Modern Europe*, Geneva 2008 (Travaux d'humanisme et Renaissance, 447), 183–203. For examples of late fifteenth-century Parisian

of the first books to be printed in Paris, followed quickly by theological works, it took decades for the usual university textbooks of logic and natural philosophy to appear from the new presses.<sup>17</sup> For whatever reason, it seems that those *libraires* to whom the university had given copying privileges did not have the first printing presses. This would explain why in the 1480s such introductions were first published outside of Paris, notably Thomas Bricot's abbreviations of Aristotle's natural philosophy, which he had interspersed with *quaestiones* from George of Brussels.<sup>18</sup> Gradually, the Parisian market opened up. In the late 1480s, two editions of Bricot's abbreviations of logic were printed in Paris.<sup>19</sup> In 1491 Wolfgang Hopyl published Bricot's edition of *quaestiones* on physics, and then in 1494 Hopyl and his new partner Johann Higmann printed another edition with Bricot's abbreviations too.<sup>20</sup> Early in 1495, a new printer, André Bocard, also published a similar Expositio totius philosophiae nec non metaphysicae Aristotelis by Pierre Tartaret, who would stand alongside Lefèvre and Bricot (with George of Brussels) as the main textbook authors printed for the University of Paris over the next two decades.<sup>21</sup> When Hopyl and Higmann printed Lefèvre's new introduction and dialogues in 1492, it was part of this first wave of printed handbooks to Aristotelian natural philosophy.

In the 1490s, however, Lefèvre's dialogues represent the high point of the calculatory techniques. The combined handbook by Thomas Bricot and George of Brussels did not mention such techniques at any length, or in any focused way; neither did Pierre Tartaret's list of *quaestiones* cover the same ground. Only Lefèvre's textbook dealt with these topics – and then, only within a distinct dialogue, rather than integrated into the flow of his introduction to physics. The one other Paris example, Pierre Tartaret's

handbooks of Aristotelian natural philosophy, see P. J. J. M. Bakker, 'Natural Philosophy and Metaphysics in Late Fifteenth-Century Paris. I: The Commentaries on Aristotle by Johannes Hennon,' *Bulletin de philosophie médiévale* 47 (2005), 125–155; P. J. J. M. Bakker, 'Natural Philosophy and Metaphysics in Late Fifteenth-Century Paris. II: The Commentaries on Aristotle by Johannes le Damoisiau,' *Bulletin de philosophie médiévale* 48 (2006), 209–228; P. J. J. M. Bakker, 'Natural Philosophy and Metaphysics in Late Fifteenth-Century Paris. III: The Commentaries on Aristotle by Johannes de Caulaincourt (alias Johannes de Magistris,' *Bulletin de philosophie médiévale* 49 (2007), 195–237. For Louvain, see B. Bartocci, S. Masolini and R. L. Friedman, 'Reading Aristotle at the University of Louvain in the Fifteenth Century: A First Survey of Petrus de Rivo's Commentaries on Aristotle (I),' *Bulletin de philosophie médiévale* 55 (2013), 133–175; B. Bartocci and S. Masolini, 'Reading Aristotle at the University of Louvain in the Fifteenth Century: A First Survey of Petrus de Rivo's Commentaries on Aristotle (II),' *Bulletin de philosophie médiévale* 56 (2014), 281–383.

<sup>&</sup>lt;sup>17</sup> On the omnipresent *summulae*, absent in print until the 1480s, see S. Corsten, 'Universities and Early Printing,' *Bibliography and the Study of 15<sup>th</sup>-Century Civilization* 5 (1987) 83–123.

<sup>&</sup>lt;sup>18</sup> Thomas Bricot and George of Brussels, *Textus abbreviatus in cursum totius physices et metaphysicorum Aristotelis*, Lyon 1486, ISTC ib01202000.

<sup>&</sup>lt;sup>19</sup> Thomas Bricot, *Textus abbreviatus in cursum totius logices Aristotelis*, Paris 1489, ISTC ib01199400; Paris 1489, ISTC ib01199500.

<sup>&</sup>lt;sup>20</sup> George of Brussels, *Cursus quaestionum super philosophiam Aristotelis*, ed. Thomas Bricot, Paris 1491, ISTC ig00146900.

P. Tartaret, Expositio totius philosophiae nec non metaphysicae Aristotelis, Paris 1495, ISTC it00044000. As for Bricot, the first edition of Tartaret was published outside Paris, in Poitiers in 1493 – Bocard, who was not yet a libraire-juré with university privileges, seems to have published the first Paris edition for Jacques Bezanceau in Poitiers. An immediate measure of the place of Bricot, Tartaret, and Lefèvre can be taken from comparing their entries in the ISTC and USTC, which confirms the claims by Renaudet, Préréforme et humanisme, 95–99 and passim.

extracts from earlier philosophers, similarly treats the intension and remission of forms as a topic outside the usual introduction to physics.<sup>22</sup> On this evidence, it seems that the latitude of forms was taught at Paris, as Daniel Di Liscia has shown for Cologne and Vienna. But when the students of Paris did encounter such techniques as a distinct genre, they encountered it – unlike, it would seem, at Cologne and Vienna – apart from the rest of the natural philosophy course.

The format of these experimental printed textbooks shaped how a student encountered these calculatory techniques. At a glance, Lefèvre's version remained unusual in comparison to competitors.<sup>23</sup> It offered a single column of text printed on pages in small quarto format, reminiscent of the first classical texts printed at Paris and Venice. All other textbooks of physics printed at Paris were in larger quartos and folios. These typographical choices were deliberate: all of Lefèvre's works used the same format of single columns – even when students later added commentary to new editions. And when Lefèvre's printers, Hopyl and Higmann, printed the works of Bricot and George of Brussels, for example, they immediately switched format. Moreover, Lefèvre's dialogues on physics were – like his other books – full of visual aids that other books on these topics did not include, such as printed diagrams, lines labelled with letters. A student who picked up these books would immediately notice the difference in size, format, and visual layout.

These differences went beyond the surface, as a dense paratextual apparatus guided students into the subject.<sup>24</sup> Lefèvre prefaced each of his introductory works by condensing the longer paraphrase into a shorter list of theses; this was itself further condensed into a prefatory diagram of the text's main elements. Therefore the reader, from the outset, was coaxed into visualising the text at multiple levels of abstraction, first catching a birds-eye view of terms before focusing on general statements, and finally zooming down to the level of specific arguments. Indeed, as discussed below, Lefèvre's first dialogue on physics opened with an example of a student using such an introductory diagram to memorize the main lineaments of the discipline, before discussing specific claims with his teachers. Such visual pedagogy remained a constant presence in textbooks Lefèvre published in later years.

The reader also navigated layers of more discursive notes to the reader, poems, and dedicatory notes – these were like the prefatory figures, lists of theses, and diagrams, simply absent from Tartaret and Bricot's introductions to natural philosophy, and from Tartaret's brief introduction to the intension and remission of forms. Lefèvre and his circle were sharply aware of the novelty of their prefatory texts. They used these textual fragments to critique current practices in the university, and to offer an alternative vision of how intellectual discourse should unfold.<sup>25</sup>

<sup>&</sup>lt;sup>22</sup> Tartaret, *Tractatus de intensione*. One might also compare Alvarus Thomas, *Liber de triplici motu*, which is pitched as a specialist study, well beyond a beginning student.

<sup>&</sup>lt;sup>23</sup> For an overview of the subgenres found within Lefèvre's textbooks, see D. A. Lines, 'Lefèvre and French Aristotelianism on the Eve of the Sixteenth Century,' in G. Frank and A. Speer (eds.), *Der Aristotelismus in der Frühen Neuzeit: Kontinuität oder Wiederangeignung?*, Wiesbaden 2007, 273– 290.

<sup>&</sup>lt;sup>24</sup> This use of prefatory elements is analysed at length in Oosterhoff, *Making Mathematical Culture*, esp. ch. 4.

<sup>&</sup>lt;sup>25</sup> The next two pages are lightly revised from Oosterhoff, *Making Mathematical Culture*, 184–189.

The medium was intended to perform a message. In fact, the various prefatory notes and explicits of Lefèvre's books are strikingly chatty: they are by, to, or about a whole community of students and colleagues. This display of the friendship binding this community together underpinned the pedagogical claim driving these works. Lefèvre wrote the preface of the 1492 volume to a certain Stephanus, describing their longstanding friendship. This "holy oath of friendship" held together reform-minded insiders against bad philosophizers. "Outsiders may marvel," Lefèvre exclaimed, "at how great the goodwill is among those who cultivate the liberal arts in our Paris studium, where this experience is well known."<sup>26</sup> Those outsiders, Lefèvre went on to clarify, are "envious, malevolent men who cut each other up with their teeth," and therefore are dogs, not lovers of wisdom who merit the title philosopher.<sup>27</sup> Friendship was not simply a conceit to frame the production of books (though it was that); in those books, intellectual friendship exemplified what good philosophy was all about. Aristotle had pointed out that real friendship was characterised by shared goodwill (mutua benevolentia).<sup>28</sup> Therefore, what could good philosophers share more deeply than the love of wisdom?29

What rescued all this talk of friendship from being mere self-indulgent flattery was the way it informed the dialogues on physics. The choice of dialogue matched pedagogical goals. Lefèvre explained this choice. Guillaume Gontier, a student who accompanied him to Rome, had suggested dialogues because "if you do, you will advise those who are learning how they should ask questions and how they should answer them; at the same time you will usefully counsel both student and teacher."<sup>30</sup> Lefèvre later recalled how when visiting Rome he had marvelled at two youths pleasantly disputing in a mode of dialogue they evidently had learned from George of Trebizond's rhetoric.<sup>31</sup> The dialogues on physics therefore present a model of how discussion *ought* to happen. The

<sup>&</sup>lt;sup>26</sup> Lefèvre d'Étaples, Totius Aristotelis philosophiae naturalis paraphrases, sig. J viii<sup>r</sup> (ed. Rice, The Prefatory Epistles, 15): "Mutua nos multos annos astrinximus benevolentia. Carissime Stephane, quanta sit animorum benevolentia inter liberalium artium cultores in hoc nostro Parisio studio (ubi res cognita esset) exteri mirarentur." Presumably "hoc nostrum studium" refers to their Collège du Cardinal Lemoine.

<sup>&</sup>lt;sup>27</sup> Lefèvre d'Étaples, *Totius Aristotelis philosophiae naturalis paraphrases*, sig. J viii<sup>r</sup> (ed. Rice, *The Prefatory Epistles*, 16): "Quo fit ut cum primum invidos, malevolos et sese mutuis dentibus lacerantes sentiunt, non amplius philosophos reputent, sed eos pro flagiti conditione in Pythagoreos canes versos putant." The reference to "Pythagorean dogs" is surely to transmigrated souls; philosophers of such ill will are canine souls in human bodies.

<sup>&</sup>lt;sup>28</sup> E.g. Aristotle, Nicomachean Ethics VIII.2, 1155b30–1156a5; J. Lefèvre d'Étaples and J. Clichtove, Artificialis introductio per modum Epitomatis in decem libros Ethicorum Aristotelis adiectis elucidata [Jodoci Clichtovei] commentariis, Paris 1502, 31<sup>v</sup>.

<sup>&</sup>lt;sup>29</sup> I have also tried to come to terms with these themes in R. J. Oosterhoff, 'Lovers in Paratexts: Oronce Fine's Republic of Mathematics,' *Nuncius* 31/3 (2016), 549–583. See further U. Langer, *Perfect Friendship: Studies in Literature and Moral Philosophy from Boccaccio to Corneille*, Geneva 1994 (Histoire des idées et critique littéraire, 331).

<sup>&</sup>lt;sup>30</sup> J. Lefèvre d'Étaples, *Introductio in metaphysicorum libros Aristotelis*, ed. J. Clichtove, Paris 1494 (Johann Higmann), sig. b1<sup>v</sup> (ed. Rice, *The Prefatory Epistles*, 22): "Si ita feceris, [Gontier] inquit, admonebis qui docturi erunt quo pacto interrogare debeant, interrogataque docere, et simul utiliter discipulo consules et docenti."

<sup>&</sup>lt;sup>31</sup> J. Lefèvre d'Étaples (ed.), *Georgii Trapezontii dialectica*, Paris 1508, sig. a1<sup>v</sup>-a2<sup>r</sup> (ed. Rice, *The Prefatory Epistles*, 190–191).

sorites and sophisms of adversarial disputation were to be contrasted with a sociable search for the truth.

The two dialogues self-consciously exemplify this ethos of philosophical friendship. The interlocutors bear names that match their function. Thus the first dialogue is led by the teachers Hermeneus ("interpreter") and Oneropolus ("conjector"), who respectively interpret the subject and offer conjectures for reasoning about it. Enantius ("contrarian") presents arguments from the opposing viewpoint, while Homophron ("harmonizer") suggests ways to find common ground between all the perspectives on offer: the *personae* perform a kind of concordance of opposites.

The two dialogues take different approaches. In the first dialogue, the father Polypragmon places his son Epiponus in the care of several teachers. Lefèvre translates the father's name as *negociator*; we might see him as a "busybody" or man of action, who regrets that his business ties leave him unable to pursue the greater contemplative wisdom he wishes for his "intellectual" son.<sup>32</sup> As the teachers talk with Epiponus, they set him the task of reading a book of introductory physics much like Lefèvre's own: "he should read it three or four times over again, and set it in his memory. Meanwhile, we will go for a walk as we wait" until he finishes.<sup>33</sup>

On his return, the teachers point to a diagram of circles – the same diagram introducing Lefèvre's own paraphrases on physics: "Do you see this figure placed at the beginning of our introduction?"<sup>34</sup> And so the lesson begins, glossing the various parts of the diagram: *Natura*, *causa*, *motus*, *infinitum*, *locus*, *vacuum*, and *tempus*.

Sometimes the student prods his teachers for glimpses of future delights. He conjectures about how the union of form and matter can be compared to a line and a surface, which fuse together to make an object. Hermeneus swiftly intervenes:

Hermeneus: "Don't tease the boy with analogies!"

Epiponus [student]: "Do you mean then that all natural things arise whole and composite from lines and surfaces, since they are fused from matter and form?" Hermeneus: "See whether or not I'm right to think you're teasing the boy." Oneropolus: "I speak, boy, but I do not speak as you suppose, for now you cannot understand. But keep on and you will understand eventually."<sup>35</sup>

At such points in the dialogue, mathematics threatens to enter but is successfully rebuffed –until the end of this first dialogue, where the discussion turns to time. Aristotle

<sup>&</sup>lt;sup>32</sup> The term *polypragmon* has an uncomplimentary history: M. Leigh, *From Polypragmon to Curiosus: Ancient Concepts of Curious and Meddlesome*, Oxford 2013.

<sup>&</sup>lt;sup>33</sup> J. Lefèvre d'Étaples and J. Clichtove, *Totius philosophiae naturalis paraphrases, adiecto commentario [Clichtovei]*, Paris 1502, 119<sup>v</sup>: "Noere, huic coaequali tuo Physicam introductionem procura. Quam ter quarterve repetitis vicibus legat, memoriaeque mandet. Nos obiter expectantes deambulabimus."

<sup>&</sup>lt;sup>34</sup> Lefèvre d'Étaples and Clichtove, *Totius philosophiae naturalis paraphrases*, 119<sup>v</sup>: "Figura igitur huic nostrae introductioni praepositam vides?"

<sup>&</sup>lt;sup>35</sup> Lefèvre d'Étaples and Clichtove, *Totius philosophiae naturalis paraphrases*, 121<sup>v</sup>: "Her. Noli o Oneropole puerum analogiis ludere. Epi. Visne ergo rem naturalem omnem ex lineis et superficiebus consurgere perfectam, compositamque esse, quandoquidem ex materia et forma conflata sit? Her. Vide an ne non recte sentirem te puerum ludere. One. Dico fili, et non dico ut concipis, nunc autem intelligere non possis. Sed tu aliquando intellecturus reserva."

had described time as an example of continuous magnitude, since it can be properly considered as a continuous series of instants, just as a line is a series of points.<sup>36</sup> Together, Oneropolus and Epiponus labour through a list of analogies. It is here that we find the intensification and remission of forms: just as extension characterises change in the quantity of objects, so qualities such as whiteness change by *intensio* or intensification; so also, the succession of time is *latio*. Here also, we see Lefèvre's editorial favouring of diagrams. Oneropolus teaches his student how to visualise the change of time on a line from A (past) to E (future).

The second dialogue – which now takes these calculatory techniques as its focus – is framed as a "difficult" one, beginning with its title: Dialogus difficilium physicalium introductorius. The student is no longer the puer Epiponus, but now the adolescens Neanias ("youth"). The dialogue's difficulty is thematised in a pair of tantalising notae – which also explain the choice of dialogue as genre. First, Proteus is presented as an ancient sage who would only willingly foretell the truth if overcome and bound. Second, Milo of Croton is the Greek wrestler who "killed a bull with a barehanded blow in an olympic contest, carried it for a hundred yards, and then ate the whole thing that very day."<sup>37</sup> Both stories would become powerful metaphors for the difficulty of pursuing natural knowledge. Over the sixteenth century, the metaphor of binding Proteus would become a telling image of nature itself, to be wrestled into revealing its secrets, or else of the investigator, chasing down barehanded experience.<sup>38</sup> Here in Lefèvre's dialogue, however, Proteus is a friendly conversationalist. Near the end of the dialogue Oneropolus claims that he has finished the subject, but Enantius and Homophron together plead with him to continue. "Just as Aristeus once bound up the varied and multiformed truth-telling Proteus with arms and chains, and so compelled him to tell the truth, so Homophron and I bind you in our arms, and we compel you to teach us what we asked."<sup>39</sup> Oneropolus

<sup>&</sup>lt;sup>36</sup> E.g. Physica IV.13, 222a28–b7. This misses the finely nuanced debates on the relation of indivisibles to continua worked out in the fourteenth century. See inter alia E. D. Sylla, 'Thomas Bradwardine's De continuo and the Structure of Fourteenth-Century Learning,' in E. D. Sylla and M. R. McVaugh (eds.), Texts and Contexts in Ancient and Medieval Science. Studies on the Occasion of John E. Murdoch's Seventieth Birthday, Leiden 1997 (Brill's Studies in Intellectual History, 78), 148–186.

<sup>&</sup>lt;sup>37</sup> Lefèvre d'Étaples and Clichtove, *Totius philosophiae naturalis paraphrases*, 127<sup>r</sup>: 'Proteus filius Oceani et Tethyos, vates maximus, nonnisi coactus, victusque volens ora veridica soluere. Milo Crotonensis atheleta fortissimus, qui nude manus ictu in certamine olympico taurum interfecit, stadio uno spiritu retento portavit, quem totum die illo comedit.'

<sup>&</sup>lt;sup>38</sup> Proteus appeared in the Odyssey 4.382–569; Plato, Euthydemus, 288b; Virgil, Georgics 4.387–529; Ovid, Metamorphoses 8.731, 11.221–56; Diodorus Siculus, Antiquities of Egypt 1.62. The identification of Proteus as a vates maximus suggests that Lefèvre had Diodorus at least partly in mind, while a later reference to Aristaeus reveals Virgil as another source. For the early modern period, see inter alia E. Wind, Pagan Mysteries in the Renaissance. An Exploration of Philosophical and Mystical Sources of Iconography in Renaissance Art, London 1958, 158–175; W. E. Burns, "A Proverb of Versatile Mutability": Proteus and Natural Knowledge in Early Modern Britain,' The Sixteenth Century Journal 32 (2001), 969–980; P. Pesic, 'Shapes of Proteus in Renaissance Art,' Huntington Library Quarterly 73 (2010), 57–82.

<sup>&</sup>lt;sup>39</sup> Lefèvre d'Étaples and Clichtove, *Totius philosophiae naturalis paraphrases*, 148<sup>v</sup>–149<sup>r</sup>: "Ut Aristaeus olim varium, multiformemque veridicum tamen Protea brachiis et vinculis implicuit, compulitque verum fateri, ita quoque ego et Homophron brachiis te implicabimus, cogemusque quod quaerimus te nos docere. One. Comis es o Enanti, et pulchre me cogis, emerita Milonis brachia timentem, vestris petitionibus acquiescere."

acquiesces: "you're a friend, Enantius, and nobly you compel me – fearing the worthy arms of Milo – to assent to your requests." The toil of knowing, the wrestling of the natural philosopher is not in the savage cut and parry of disputatious one-upmanship, but the encouraging conversation of friends who share the goal of knowledge.<sup>40</sup> Friendship does not eliminate struggle. Rather, friendship makes the struggle for truth *fruitful*. Throughout the dialogues, then, the motif of friendship counters the association of calculatory techniques with the putative pathologies of sophisms.

Wrapped in layers of paratext, the techniques of the Calculators could be taught safely – perhaps too safely. We might suppose that such ginger handling would put off some readers. A master or student looking for strategies to deploy in disputations would have found Lefèvre's dialogue unnecessary work, since it did not straightforwardly set out arguments *pro* and *contra* on given *quaestiones*. What other options were available to a young scholar interested in calculatory techniques?

The limited number of introductory printed options may help explain why such techniques became less prominent over the next decades. As already mentioned, Tartaret's excerpts on the intention and remission of forms (1493) was far from basic, and did not gain enough readers to be reprinted; similarly, Alvarus Thomas's Liber de triplici motu (1509) was sophisticated and difficult.<sup>41</sup> Jacques Almain's Embammata phisicalia (1506) seems more promising, at first glance. It was published under conditions much closer to those of Lefèvre's textbooks: one of Almain's students, Peter Heymeric, published the book in 1506, when Almain was an arts regent before he became a doctor in the faculty of theology in 1512.<sup>42</sup> Though printed in the traditional double columns, it shares with Lefèvre's books a generous apparatus, including dedicatory poems, Almain's own preface thanking his student-cum-editor, and a list of arguments at the end. The first two pages make some introductory moves, quickly surveying the theory of proportions. But the contents of the book swiftly swing into a very different direction. Instead of gradually building up a familiarity with the contents of Aristotle's *Physics*, the bulk of the 228 quarto pages survey a series of philosophical "difficulties" thrown up by these analytical languages, picking up themes from Book I and Book III of the Physics, on the divisibility of points, the difference between mathematical and physical accounts of

Elsewhere, Lefèvre, Clichtove, and Alain de Varennes expand on the Aristotelian view that true friendship involves mutual goodwill around shared aims: Oosterhoff, 'Lovers in Paratexts,' 565–566.
Sula 'Alvarus Thomas'

<sup>&</sup>lt;sup>41</sup> Sylla, 'Alvarus Thomas.'

<sup>&</sup>lt;sup>42</sup> For an overview of Almain's life, see J. K. Farge, *Biographical Register of Paris Doctors of Theology*, *1500–1536*, Toronto, ON, 1980 (Subsidia mediaevalia, 10), 15–17. An incomplete manuscript of the *Embammata* was copied by one Antonius of Toledo (Lugdunensis), now in Sevilla, Biblioteca Capitular y Colombina, Ms. 7–6–12. For this reference, my thanks to Daniel Di Liscia, who has rediscovered Almain's work; see the first fruits in D. A. Di Liscia, 'Velocidad *quo ad effectus* y velocidad *quo ad causas*: La tradición de los calculadores y la metodología aristotélica,' in D. A. Di Liscia, E. Kessler and C. Methuen (eds.), *Method and Order in Renaissance Philosophy of Nature. The Aristotle Commentary Tradition*, Aldershot 1997, 143–176, and D. A. Di Liscia, 'La conclusio pulchra, *mirabilis et bona*: una ingeniosa demostración atribuible a Nicole Oresme,' Mediaevalia. Textos e *estudos* 37 (2018), 139–168, at 163–167. We can anticipate an edition of work by Almain from Di Liscia and Sabine Rommevaux-Tani.

magnitude, and the maximum and the minimum. Each difficulty is argued from several angles, and ultimately resolved from a nominalist perspective.<sup>43</sup>

Just how much the book differs from Lefèvre's introduction can be measured from Almain's preface, where he explains the enigmatic title *Embammata* (literally, "sauces"). The letter begins in a posture of humility often found in Lefèvre's letters. It is addressed to his student-editor, Peter Heymeric, and stresses Peter's role in bringing the work to print: Almain had abandoned these indigesta long before, but Peter had compelled him to offer them for public consumption. In fact, like Lefèvre, Almain describes philosophical love as a motivation for Heymeric's labours – but with a different emphasis. "You were so filled with love of philosophy that, unless you had taken a spiked stick to drive out the shackles and chains from the various flowers of physics, you would have died with Diodorus, who they say ended his life over the shame of a philosophical question."<sup>44</sup> Like Lefèvre's letters, this note is alert to the passions in philosophical debate; but it draws a different lesson about pride and shame. Almain does not present a solution to harmful passions, but rather gives a charmingly self-deprecating account of his "sauces". He observes that, although drawn from his "pained breast, moved by cares – like a tender kid goat from its mother's teats," nevertheless his work may not nourish like fresh milk. But, "even if the taste does not please, perhaps it may find favour in sight or smell. For the *libanotis* (as they call the plant growing on the seaside), even though it does nothing for the tongue, is pleasing to see and smell when green and watered, and so is often gathered."45 These sauces are a sensual pleasure. They may not nourish, but like a bouquet of flowers they are still good to have around.

Even more than Lefèvre's dialogues – which, recall, are outside the main paraphrases of the textbook – Almain's *Embammata*, like Alvarus Thomas's *Liber de triplici motu*, is not so much a main course as a pleasure for particular appetites, an elective study that was self-consciously distinct from the regular cursus in natural philosophy. In the 1510s, two books did begin to frame calculatory techniques in pedagogically sensitive ways: the *Physice perscrutationes* of Luis Coronel (1511) and Juan

<sup>&</sup>lt;sup>43</sup> E.g. on the maximum and minimum, see J. Almain, *Embammata phisicalia*, Paris 1506, 23<sup>v</sup>-42<sup>v</sup>. Oddly enough, the table of theses appended to the end includes intriguing topics such as "Bertha is the mother of two, who endure even when Bertha herself no longer exists as their mother" (sig. a iii<sup>v</sup>: "Bertha est mater duorum et ipsis manentibus cum Bertha ipsa illorum duorum non erit amplius mater").

<sup>&</sup>lt;sup>44</sup> Almain to Peter Heymeric, preface to *Embammata phisicalia*, sig. a2<sup>r</sup>: "Presertim cum tantu philosophie amore infiltratus fueris, ut nisi ferrula variis physices flosculis dentata numellas torquesque ipsos elim[in]averis, cum Dyodoro (quem aiunt) philosophice questionis pudore vitam terminasse." The story about Diodorus was passed on by Diogenes Laertius, *Lives of the Eminent Philosophers*, § 111.

<sup>&</sup>lt;sup>45</sup> Almain, *Embammata phisicalia*, sig. a2<sup>r</sup>: "At ubi expetitas nostrorum quamvis immaturorum videris Embammatum taleas, se motis tristi a pectore curis (tanquam caper tenellus ad matris ubera) covagies. Quas quidem (ut spero) tantis tamquam frequentibus ictulis saturaturus mulgebis, ut si non lacteos emanarint liquores; attamen nescio quid stillantis succi avidus exhauries. Aut si non gustulum voluptaverint forsitan visum vel nares ipsas confovebunt. Libanotis enim (quem rorem marinum appellitant) quamquam linguam non decorat, olfactum tum visum placent viredine irrigando, componere solet."

de Celeya's *Expositio in octo libros Physicorum* (1517).<sup>46</sup> But the two previous decades seem to have set in place a printed textbook tradition that undercut the role of the calculatory techniques among the introductions to natural philosophy.

### 2. A Changing Mathematical Culture

The rather spotty presence of the Calculators in the printed textbook tradition that came to dominate instruction at Paris around 1500 surely goes some way towards explaining the decline of their analytical tools as an intellectual tradition. But it may be even more significant that these techniques were not a priority – barely even present – in the later generations of mathematically minded Parisian practitioners who built on Lefèvre's mathematical teaching. These included the theologians Juan Martínez Silíceo and Pedro Ciruelo, as well as the arts masters Oronce Fine and Jean Fernel. Considering why these figures did not make calculatory techniques central, despite their fascination with mathematics and its potential for modeling nature, may let us hazard two alternative answers.

The first answer is that the mathematical interests of these scholars were in fact somewhat disconnected from calculatory techniques. They framed their mathematical projects more in relation to the pedagogy of liberal arts than in relation to natural philosophy. Wallace cited Silíceo and Ciruelo among the group of mathematically sophisticated members of Mair's circle, pointing out that they would go on to play a crucial role in setting up the mathematical culture of sixteenth-century Spain, and so "are entitled to be called the first Spanish 'Calculatores'."<sup>47</sup> Both Silíceo and Ciruelo did publish relevant works by Swineshead and Bradwardine.<sup>48</sup> But both more thoroughly connected their mathematical expertise to the tradition begun at Cardinal Lemoine. Lefèvre had published, with students such as Josse Clichtove, David Laux, Charles de Bovelles and others, introductions to mathematics as liberal arts, the Boethian quadrivium. His main works, therefore, included epitomes of Boethius's arithmetic, a reworking of Jordanus's *Arithmetica*, the astronomy of Sacrobosco, a brief theorica, and eventually a new edition of Euclid.<sup>49</sup>

This vision captivated Silíceo and Ciruelo, and formed the basis of their *translatio* of Parisian mathematics to the Spanish universities in the 1510s. In 1513, Silíceo published his *Arithmetica practica*, with the evocative subtitle "very useful for astrologers, physicists, and calculators" (*Liber arithmetice practice astrologis, physicis, et calculatoribus admodum utilis*). But the contents of the book itself make no reference to the calculatory tradition in physics. Instead, Silíceo dedicates the book to the bishop of his hometown in Portugal with a long list of commonplaces about the value of arithmetic

<sup>&</sup>lt;sup>46</sup> L. Coronel, Physice perscrutationes, Paris 1511; J. de Celaya, Expositio in octo libros Physicorum Aristotelis, cum questionibus eiusdem secundum triplicem viam beati Thomae, realium, et nominalium, Paris 1517.

<sup>&</sup>lt;sup>47</sup> Wallace, 'The Calculatores,' 225. See now also V. Navarro-Brotons, Disciplinas, saberes y prácticas: Filiosofía natural, matemáticas y astronomía en la sociedad española de la época moderna, Valencia 2014, 26–28, 58–72.

<sup>&</sup>lt;sup>48</sup> Thomas Bradwardine, *Arithmetica speculativa*, ed. P. Ciruelo, Paris 1495; Richard Swineshead, *Calculatoris sublime et prope divinum opus*, ed. J. M. Silíceo, Salamanca 1520.

<sup>&</sup>lt;sup>49</sup> For more details, see bibliography in Rice (ed.), *The Prefatory Epistles*.

for civic rulers. The context for this exercise becomes clearer in the preface, which gives a genealogy of the discipline that begins with Pythagoras and ends with Lefèvre d'Étaples and Josse Clichtove: "I would say that all of these things are rightly indebted to these men."<sup>50</sup> The book closes with a poem addressed to Clichtove, praising Silíceo's work as the latest eminent contribution to the same enterprise: "But no elegance seems in our age, I think, more polished than the art of Silíceo; A brighter flash of lightning consumes the smoke, just as a great river exists from the smallest streams."<sup>51</sup> The *aemulatio* makes clear just which tradition Silíceo saw himself working within.

Ciruelo's books engaged the work of Lefèvre and his colleagues even more directly. Ciruelo was a talented mathematical author in his own right, with editions and writings ranging across the breadth of late medieval mathematical culture.<sup>52</sup> His first publication was an edition of Bradwardine's theoretical work on proportions (1495), followed three years later by an edition of Pierre d'Ailly's questions on Sacrobosco's Sphere, with his own commentary. Already then, however, Ciruelo cited Lefèvre's own commentary on the Sphere, and he concluded the work with a dialogue in much the same mode as Lefèvre's dialogues on physics, emphasizing the role of philosophy as the pursuit of truth, and critiquing "our Parisian philosophers" who tended to ignore the views of other universities.<sup>53</sup> Ciruelo's engagement with the mathematical tradition of Lefèvre's circle intensified when Ciruelo left Paris to join the new wave of humanistic studies associated with Elio Antonio de Nebrija at Salamanca: his signal contributions to curricular reform were new introductions that were either excerpted from or explicitly modelled on Lefèvre, Clichtove, and Bovelles's mathematical works.<sup>54</sup> As a doctor of theology, Ciruelo was well placed to make methodological pronunciations across the disciplines, and indeed one of his most striking works was a defense of astrology (in light of Pico's reverberating critique), which emphasised the closeness between physics and astronomy.<sup>55</sup> At various points in his works, Ciruelo referred to Lefèvre's works as an example of a philosophy of mathematics that might show the physical implications of mathematics.<sup>56</sup> But – despite helping make available some of the Calculators – Ciruelo himself did not try to expound the calculator's version of a mathematical natural philosophy.

It makes some sense that Silíceo and Ciruelo, theologians who worked at mathematics within a reformist plan for university teaching, would perhaps overlook the opportunity to integrate calculatory techniques more systematically into the *cursus* of arts. But we might expect something else from mathematical practitioners, those who

<sup>&</sup>lt;sup>50</sup> J. Martínez Silíceo, Liber arithmetices practice Astrologis phisicis et calculatoribus admodum utilis, Paris 1513, sig. Aii<sup>r</sup>: "... quos debito iure omnium horarum viros esse dixerim" (reading dixerim for diverim).

<sup>&</sup>lt;sup>51</sup> Silíceo, *Liber arithmetices practice*, sig. fiv<sup>r</sup>. "Nulla tamen nostro visa est facundia seclo | Martini solidi cultior arte puto | Candidiora dabit consumpto fulgura fumo | Grande velut minimo flumine flumen adest."

<sup>&</sup>lt;sup>52</sup> An overview of recent literature T. M. C. Lanuza-Navarro, 'Astrology in Court: The Spanish Inquisition, Authority, and Expertise,' *History of Science* 55 (2017), 187–209, at 195.

<sup>&</sup>lt;sup>53</sup> P. Ciruelo, Uberrimum sphere mundi commentum intersertis etiam questionibus domini Petri de Aliaco, Paris 1498, sig. n iiii<sup>v</sup>-n vii<sup>r</sup>.

<sup>&</sup>lt;sup>54</sup> P. Ciruelo, *Cursus quatuor mathematicarum artium liberalium*, Alcalá 1516.

<sup>&</sup>lt;sup>55</sup> P. Ciruelo, *Apotelesmata astrologiae christianae*, Alcalá 1521.

<sup>&</sup>lt;sup>56</sup> Oosterhoff, *Making Mathematical Culture*, 161–162, 205, 211–212.

made their reputation and money chiefly through teaching, publishing, and advising on mathematical matters. The chief example would be Oronce Fine (1594–1555), who came to define the profile of Parisian mathematics in the sixteenth century.<sup>57</sup> While teaching arts at the Collège de Navarre and the Collège de Maître Gervais, Fine was also active in the print trade, and his work as a mathematician was informed by his work as an editor and designer of mathematical woodcuts, especially for Lefèvre's collaborating printer Henri Estiennes the Elder and his successor Simon de Colines. He edited Silíceo's *Arithmetica* in 1519, and was responsible for repeated editions of Lefèvre's mathematical works through the 1520s and 1530s. Thanks in part to Guillaume Budé's support, in 1532 Fine was made the professor of mathematics in Francis I's new Collège Royal, on the strength of his publications in arithmetic, geometry, cosmography, optics, and his skill in crafting maps, astronomical instruments, and dialls.

Fine's works reveal a deep interest in the relation of mathematics to nature's structures, yet I have not found calculatory techniques anywhere in his oeuvre. Angela Axworthy has analysed at length Fine's claims about the value of geometry and arithmetic for explaining nature: geometry "has taught us the quantities of all bodies, their shapes, movements, bounds, and positions."58 Despite arguing strenuously, and over decades, that mathematics was the path to all philosophy, Fine in the main argued that mathematics was best for training the student's mind. Even more than Silíceo and Ciruelo (and like Lefèvre) Fine downplayed the technical, mechanical insights that mathematics might offer philosophy. In his most public statements, he presented the sciences of measuring as beneficial, but chiefly to a public man, engaged in the arts of ruling. Therefore, as part of the liberal arts, mathematical practitioners best defended their place in public by claiming how mathematics would render the *esprits* and *ingenia* of youth ready for noble service. A mistrusted mathematical culture began to bid for greater prestige, but in the context of a Parisian clerical elite that oriented university education increasingly towards the court of Francis I, it found less time for the analytical languages of the Calculators. This first explanation, then, for declining interest in intension and remission of forms, first and last instances, etc., is that adepts like Fine (and to some extent Silíceo and Ciruelo) had a different professional trajectory, needing little from natural philosophy.

A second explanation may be that Paris humanists like Fine had turned to other philosophical linkages between mathematics and physics. A line of analysis might consider the term *physiologia*. Since antiquity, the main meaning of *physiologia* was as an inkhorn Graecism for "natural philosophy", a study of the properties and changes of natural objects – as Cicero observed, the *ratio naturae* was "what the Greeks called physiology."<sup>59</sup> But the term was not widely used until the late sixteenth century, when popular textbooks such as Johannes Magirus's *Physiologia peripatetica* (first published at Frankfurt, 1597) made it stand in for the main subjects of natural philosophy. Thus it

<sup>&</sup>lt;sup>57</sup> The starting point is now A. Axworthy, *Le mathématicien renaissant et son savoir. Le statut des mathématiques selon Oronce Fine*, Paris 2016 (Histoire et philosophie des sciences, 11), and A. Marr (ed), *The Worlds of Oronce Fine. Mathematics, Instruments and Print in Renaissance France*, Donington 2009.

O. Fine, *Epistre exhortative*, Paris 1531, sig. B1<sup>v</sup>, ed. Axworthy, *Le mathématicien renaissant*, 365–380, at 374.

<sup>&</sup>lt;sup>59</sup> Cic. *Nat. de*. 1.8.20.

showed up in the subtitle of Robert Hooke's famous work, *Micrographia, or, some physiological descriptions of minute bodies made by magnifying glasses* (London, 1665).<sup>60</sup> In these works, mathematics often featured as a preliminary study for natural philosophy. The dedicatory letter to Magirus proclaims the value of "mathematics and physiology, namely natural science."<sup>61</sup> Whereas mathematics measures natural bodies, physiology studies their motions.

This doublet makes more sense in light of the rehabilitation of the term physiologia one century earlier. Giorgio Valla's encyclopaedic De expetendis et fugiendis rebus, published posthumously by a student in 1501, tried to refound the entire cycle of education on a Platonic account of mathematics. Valla had helped to circulate and translate late antique Platonist texts such as Proclus's Commentary on the First Book of *Euclid* – extracts were silently incorporated into the body of *De expetendis*.<sup>62</sup> As a whole, the two large folio volumes approximate a progression from theory to practice: they begin with the mathematical quadrivium, move through its applications in natural philosophy and medicine, and take up the social disciplines of grammar, logic, and poetics, to their applications in public life, in *oeconomia* and *politica*. For us the important thing is the placement of natural philosophy: it comes under the name physiologia, popular among contemporary Greek scholars such as Ficino and Poliziano. Physiologia follows directly after astronomy. Valla presents this astronomy (or astrology) as a study of heavenly structures that chiefly has use in medicine. Therefore the four books on astrology map the configuration of the heavenly macrocosm onto the structure of the human microcosm. Indeed, Book II of Valla's Astrologia is simply twenty-five pages of tables correlating body parts with longitudes and latitudes of heavenly bodies.<sup>63</sup> This account also deeply marks Valla's four books on *physiologia*. The order of nature and its movements in fact are inescapably the working out of divine "metaphysical seeds" (semina metaphysices) that Valla finds strewn throughout the frame of nature. The relation of macrocosm and microcosm explains why Valla begins this natural philosophy with the human soul and *daemones*, before considering elemental principles.<sup>64</sup>

<sup>&</sup>lt;sup>60</sup> Cf. G. Vossius, De theologia gentili et physiologia Christiana, Amsterdam 1641; M. F. Wendelin, Contemplationum physicarum sectio I. qua physiologia generalis, De principiis et affectionibus Corporibus naturalis, Cambridge 1648; W. Charleton, Physiologia Epicuro-Gassendo-Charltoniana, London 1654; T. Hobbes, Decameron physiologicum, or, Ten Dialogues of Natural Philosophy, London 1678.

<sup>&</sup>lt;sup>61</sup> Conrad Nebenius, dedicatory letter to J. Magirus, *Physiologia peripatetica*, Frankfurt am Main 1619, 6: "... Mathesis, aut Physiologia seu scientia naturalis est." The word *mathesis* clearly refers to mathematics, since the next three paragraphs are a paean on the dignity and use of this discipline, *circa corporum dimensiones occupata paedia est*.

 <sup>&</sup>lt;sup>62</sup> G. Valla, *De expetendis et fugiendis rebus*, Venice 1501. For context see A. A. Raschieri, 'Giorgio Valla, Editor and Translator of Ancient Scientific Texts,' in: P. Olmos (ed), *Greek Science in the Long Run: Essays on the Greek Scientific Tradition (4<sup>th</sup> c. BCE–17<sup>th</sup> c. CE)*, Cambridge 2012, 127–149; P. L. Rose, 'Bartolomeo Zamberti's Funeral Oration for the Humanist Encyclopedist Giorgio Valla,' in C. H. Clough, *Cultural Aspects of the Italian Renaissance: Essays in Honour of Paul Oskar Kristeller*, Manchester 1976, 299–310. A foundational resource is J. L. Heiberg, *Beiträge zur Geschichte Georg Vallas und seiner Bibliothek*, Leipzig 1896.

<sup>&</sup>lt;sup>63</sup> Valla, *De expetendis et fugiendis rebus*, sig. dd 2<sup>r</sup>–ee 6<sup>r</sup>.

<sup>&</sup>lt;sup>64</sup> Valla, *De expetendis et fugiendis rebus*, at sig. ii 8<sup>r</sup>, and sig. hh 4<sup>r</sup>.

There is no need for calculatory technique in such an account. Natural change is explained not by a succession of different qualities, sensed through an Aristotelian process of abstraction. For the Aristotelian, mathematics merely models the experience of change. For Valla, mathematics carries the mind into the eternal seeds (*semina*) that exert power over physical form. Sometimes described as a *spiritus*, this force is itself actively rational – therefore it should be explained by reasoning about principles, not by a posteriori reasoning about experience. As a result, we might suppose that Valla's vision of philosophy, though interested in the mathematical principles of natural philosophy, depends on a very different concept of 'principle' than most Aristotelian accounts.

Valla's account had close, attentive readers in early sixteenth-century Paris, particularly among those reflecting on the power of mathematics in Oronce Fine's circle. It is clear that Fine himself was influenced by this account of the disciplines, for his arguments about geometry closely follow Proclus – only occasionally rewording the translation of Valla in *De expetendis et fugiendis rebus*.<sup>65</sup> Although Fine may not have been sharply aware of this divergence from the Aristotelian doctrine of abstraction, his account of intuition of mathematical principles was influential among followers at the Collège Royal such as Antoine Mizauld, and eventually Peter Ramus.<sup>66</sup>

Fine's student Jean Fernel is a particularly interesting corroborating case. As a physician teaching at Paris, Fernel has long been understood as one of the most influential and subtle medical theorists of the sixteenth century, a Galenist counterpoint to Vesalius, whose *Physiologia* is usually cited as the first use of the term as a domain of medical study.<sup>67</sup> But before he devoted himself to medicine, Fernel's first career choice had been mathematics, and he worked closely with Fine in the 1520s. Some have seen Fernel's move away from mathematics as a kind of intellectual conversion.<sup>68</sup> But in fact the title of the *Physiologia* hints at ways that Fernel's approach to mathematics in the 1520s continued to animate his work. The work itself followed a conception of the human body's relation to seeds and principles – what he famously called *spiritus*. The sources of such pneumatic ideas are surely many: Fernel himself cited Galen, ancient Stoics, Aristotle's own claims about *entelechia*, and various representatives of the Platonic tradition.<sup>69</sup> Fernel's own decision to call this a *physiologia* surely also reflects his wish to

<sup>&</sup>lt;sup>65</sup> Axworthy, *Le mathématicien renaissant*, 60–64, 101–104.

<sup>&</sup>lt;sup>66</sup> On Ramus' intuitive mathematics, see R. Goulding, *Defending Hypatia: Ramus, Savile, and the Renaissance Rediscovery of Mathematical History*, New York 2010 (Archimedes, New Studies in the History of Science and Technology, 25). In forthcoming work, Goulding also address Ramus' efforts to apply such intuition to natural philosophy.

<sup>&</sup>lt;sup>67</sup> For some support of this view, see A. Cunningham, 'The Pen and the Sword: Recovering the Disciplinary Identity of Physiology and Anatomy before 1800: I: Old Physiology—the Pen,' *Studies in History and Philosophy of Biological and Biomedical Sciences* 33/4 (2002), 631–665. An argument for more continuity is V. Nutton, 'Physiologia from Galen to Jacob Bording,' in M. Horstmanshoff, H. King and C. Zittel (eds.), *Blood, Sweat and Tears: The Changing Concepts of Physiology from Antiquity into Early Modern Europe*, Leiden 2012 (Intersections, 25), 27–40.

 <sup>&</sup>lt;sup>68</sup> C. Sherrington, *The Endeavour of Jean Fernel, With a List of the Editions of his Writings*, London 1946;
J. Henry, "Mathematics Made No Contribution to the Public Weal": Why Jean Fernel (1497–1558)
Became a Physician, *Centaurus* 53 (2011), 193–220.

<sup>&</sup>lt;sup>69</sup> See the notes to J. M. Forrester and J. Henry (eds.), *The 'Physiologia' of Jean Fernel (1567)*, Philadephia, PA, 2003 (Transactions of the American Philosophical Society, 93/1). For some relevant framework of body-earth-cosmos analogies, see L. Taub, 'Physiological Analogies and Metaphors in

use the classicizing vocabulary of Pseudo-Galen.<sup>70</sup> But the architectonic claim that this discipline was a *physiologia* becomes more intelligible in light of Valla's physiology, where the structure of the cosmos is linked to the human body's functions, so that reasoning about planetary powers informs medical remedies. Likewise, Fernel titled *Physiologia* that part of medicine that lay between a study of the body's anatomy and the study of remedies. Valla presented reasoning from one to the other as a deductive process – it is mathematical or scientific reasoning, not merely prudential craft. Likewise, Fernel explained that physiology proceeded not by abstraction from anatomical experience, but on demonstrative reasoning:

So, since the human body is already divided up by anatomy into the parts open to sense, the next stage must be to move across to those that are learned by thought alone, and to track down further the elements that contribute to each part, and their elemental mixture, their temperament, the powers and faculties that lie hidden in them, and the spirit and heat by which they are maintained. When these matters have been discovered and grasped by analysis, then it will become clear from the sequence of their composition what the efficient causes of everything are, what humors are generated by them, what the functions of individual parts are, and what the natural management of everything is. In this way a comprehensive physiology will be put together, that establishes the natural study of man by the force of demonstration.<sup>71</sup>

A serious evaluation of Fernel's potential debt to Valla must await another time. But there is at least a family resemblance in their insistance on physiology as a study of combinations of elements upheld by inborn powers, built around the analogy of macrocosm and microcosm. And for the purposes of this chapter, the implications in either case for the study of natural philosophy are clear: Valla and Fernel represent a *mathesis* in which "natural principles" meant something different than they meant to the Calculators.<sup>72</sup> In the previous generation, Lefèvre and Ciruelo had been familiar with such principles – Ciruelo described them as belonging to the "magicians", and offered a

Explanations of the Earth and the Cosmos,' in M. Horstmanshoff, H. King and C. Zittel (eds.), *Blood, Sweat and Tears: The Changing Concepts of Physiology from Antiquity into Early Modern Europe,* Leiden 2012 (Intersections, 25), 41–61. On Fernel in the context of Lefèvre's interests, see J. J. Bono, 'Reform and the Languages of Renaissance Theoretical Medicine: Harvey versus Fernel,' Journal of *the History of Biology* 23/3 (1990), 341–387.

<sup>&</sup>lt;sup>70</sup> Nutton, 'Physiologia,' 29–30.

<sup>&</sup>lt;sup>71</sup> Forrester and Henry (eds.), *The 'Physiologia*, 180–182: "Quum igitur humanum corpus anatome in partes sensu conspicuas dissolutum iam sit, ab his deinceps ad ea transeundum, quae cogitatione sola discuntur, et altiùs investigandum ex quibus elementis pars vnaquaeque condita sit, et quae sit elementorum permistio, quae temperatio, quae vires ac facultates lateant in illis, et quo spiritu quóve calore serventor. Quum analysi haec inuenta perceptáque fuerint, compositionis dein ordine conspicuum fiet, quae sint effectrices omnium causae, qui ab his gignantur humores, quae partium singularum functiones sint, et quae naturalis omnium administratio. Sic universa contrahetur physiologia, quae naturalem de homine contemplationem demonstrationis vi constituit" (English translation 181–183).

<sup>&</sup>lt;sup>72</sup> This leads to questions that have long absorbed historians of later natural philosophy: J. Henry, 'Occult Qualities and the Experimental Philosophy: Active Principles in Pre-Newtonian Matter Theory,' *History of Science*, 24 (1986), 335–381.

sidelong reference to Lefèvre as an example.<sup>73</sup> But they remained responsible teachers of the Aristotelian cursus, and therefore recognised an important place for calculatory techniques. As practitioners of other *métiers*, Fine and Fernel adopted another architectonic account of mathematics in which such analytical languages seemed unnecessary.

#### Conclusion

Why did these calculatory techniques lose the attractions that had made them such a popular language of analysis in the fourteenth century, in John Murdoch's words? One line of argument has depended on the rhetoric that set apart humanism and scholasticism. This is less convincing on closer examination. Recent studies of these cultures have been alert to the fact that those usually identified as either scholastics or humanists in fact depended on the same institutions, training, career paths, and indeed skills.<sup>74</sup> Moreover, some 'scholastics' such as Gerson railed at *anglicanes subtilitates*, while some 'humanists' like Lefèvre wrote introductions for them. The division of humanism and scholasticism often says more about historians than the history.

Instead, I have argued that the rise of the printed textbook helps us to better explain the decline of calculatory techniques as a genre. After the 1490s (in Paris, but likely elsewhere too), students increasingly encountered natural philosophy in printed textbooks. It would be interesting to find (though hard to prove) that students preferred to buy a cheap book rather than engage in the live disputation that helped train students in the skills necessary for the Calculators' project. Certainly, what counted as peripatetic physics was constrained by what those textbooks contained – and most of them did not include calculatory techniques. Between the introduction of printed textbooks in Paris in 1492 and 1511, there was only one introductory account of Aristotle's natural philosophy that also included the basics on intension and remission of forms, maxima and minima, first and last instances, and so on, and that was Lefèvre's Totius Aristotelis philosophiae naturalis paraphrases (1492). And it - adopting a line of university reform that is traceable back to Gerson – was hardly a straightforward example of how to deploy such techniques in university disputations. It seems that an unintended consequence of the two decades it took to come to terms with printed textbooks, the calculatory analytical languages dropped out of regular teaching.

By looking at mathematics in the generations after Lefèvre, I have suggested, we can see two further reasons for the declining interest. First, what we might call the "professional" trajectory of mathematical interests developed apart from natural philosophy, among those who claimed special expertise in mathematics. Already Ciruelo

<sup>&</sup>lt;sup>73</sup> Oosterhoff, *Making Mathematical Culture*, 211–212.

<sup>&</sup>lt;sup>74</sup> E.g. D. A. Lines, 'Humanism and the Italian Universities,' in C. S. Celenza and K. Gouwens (eds.), *Humanism and Creativity in the Renaissance. Essays in Honor of Ronald G. Witt*, Leiden 2006 (Brill's Studies in Intellectual History, 136), 327–346; D. Hobbins, *Authorship and Publicity Before Print: Jean Gerson and the Transformation of Late Medieval Learning*, Philadelphia, PA, 2009 (Middle Ages Series); A. Broadie, 'John Mair's *Dialogus de Materia Theologo Tractanda*. Introduction, Text and Translation,' in A. A. MacDonald, Z. R. W. M. von Martels and J. R. Veenstra (eds.), *Christian Humanism: Essays in Honour of Arjo Vanderjagt*, Leiden 2009 (Studies in Medieval and Reformation Traditions, 142), 419–430.

and Silíceo, picking up Lefèvre's project, did not feel responsible for linking their reform of mathematical teaching to the natural philosophical curriculum. And clearly, for the *lecteur royal* Oronce Fine (and, we might extrapolate, the young Jean Fernel), the identity of the mathematical practitioner – although bolstered by claims of what mathematics might do for philosophers – did not entail publishing textbooks on natural philosophy or calculatory techniques. Even within the university, the growing profile of the mathematical practitioner was linked to astronomy (especially via astrology's place in medicine), mapping, and increasingly machinery, but not natural philosophy per se.<sup>75</sup>

Finally, the philosophical foundations of mathematical practice had subtly shifted in this new generation, with implications for the Calculators' tradition. Efforts to reimagine the entire cycle of arts on a mathematical foundation, as Giorgio Valla had done a little earlier in Venice, focused on mathematical principles as Platonic ideas, and were less interested in harmonizing with Aristotelian accounts of abstraction. As a result, the analytical languages of the Calculators seemed unnecessary. Put crudely: it was unnecessary because the deductive model held out the promise that – eventually – one might deduce mathematical causes. What need then for merely approximate models?

On the mathematical practitioner, see R. S. Westman, 'Humanism and Scientific Roles in the Sixteenth Century,' in F. Krafft and R. Schmitz (eds.), *Humanismus und Naturwissenschaften*, Boppard am Rhein 1980 (Beiträge zur Humanismusforschung, 6), 83–100; M. Biagioli, 'The Social Status of Italian Mathematicians,' *History of Science* 27 (1989), 41–95; I. Hantsche, *Der 'Mathematicus': Zur Entwicklung und Bedeutung einer neuen Berufsgruppe in der Zeit Gerhard Mercators*, Bochum 1996 (Duisburger Mercator-Studien, 4). For a later period, L. B. Cormack, S. A. Walton and J. A. Schuster (eds.), *Mathematical Practitioners and the Transformation of Natural Knowledge in Early Modern Europe*, Cham, Switzerland, 2017 (Studies in History and Philosophy of Science, 45).