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G-PFEM: an open access numerical tool for the simulation of spudcan penetration in clays

G-PFEM: un outil numérique en libre accès pour la simulation de la pénétration de spudcan dans les argiles

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ABSTRACT: Penetration depth of Spudcan foundations for offshore mobile Jack-up rigs is typically predicted by considering a wished-in-place foundation at different depths and following traditional bearing capacity approaches. However, the large penetration depths involved, stress redistributions and the flow of the material around the spudcan are some example of features which make the wished-in-place assumption quite unrealistic. This paper presents an open access application package developed to simulate and hence predict load penetration curves of spudcan installation in multi-layered clay profiles. The numerical tool adopts the recently developed particle finite element method for geotechnical applications (G-PFEM). The potential of this large strain particle finite element application is demonstrated by simulating field data from the literature. The results show that load-penetration curves obtained using G-PFEM capture more efficiently the field results with respect to other numerical and analytical methods. It is also shown how the G-PFEM automatically captures cavity infill. In the paper the computational cost for the simulation of penetration up to 40m on a standard desktop are shown to be low compared to other commercial software.

RÉSUMÉ : La profondeur de pénétration des fondations Spudcan pour les plates-formes élévatrices mobiles offshore est généralement prévue en considérant une fondation en place souhaitée à différentes profondeurs et en suivant les approches traditionnelles de capacité portante. Cependant, les grandes profondeurs de pénétration impliquées, les redistributions des contraintes et l'écoulement du matériau autour du spudcan sont quelques exemples de caractéristiques qui rendent l'hypothèse souhaitée sur place assez irréaliste. Cet article présente un package d'application en libre accès développé pour simuler et donc prédire les courbes de pénétration de charge d'une installation de spudcan dans des profils d'argile multicouches. L'outil numérique adopte la méthode des éléments finis de particules récemment développée pour les applications géotechniques (G-PFEM). Le potentiel de cette application d'éléments finis à particules de grande déformation est démontré en simulant des données de terrain tirées de la littérature. Les résultats montrent que les courbes de pénétration de charge obtenues à l'aide du G-PFEM capturent plus efficacement les résultats sur le terrain par rapport à d'autres méthodes numériques et analytiques. Il est également montré comment le G-PFEM capture automatiquement le remplissage de la cavité. Dans l'article, le coût de calcul pour la simulation d'une pénétration jusqu'à 40 m sur un ordinateur de bureau standard s'avère faible par rapport à d'autres logiciels commerciaux.

KEYWORDS: Spudcan penetration, clay, numerical modelling, particle finite element method.

1 INTRODUCTION

The offshore industry has been growing exponentially thanks to solid demand for energy. Mobile jack-up rigs are key tools for the offshore industry due to their flexibility, mobility and cost-effectiveness in all weather conditions (Randolph & Gourvenec, 2017). Leg penetration of jack-up rigs during installation should be carefully analysed to optimise rig selection and avoid any sudden uncontrolled motions, a feature of layered soil profiles with potentially catastrophic consequences. This problem has attracted a great deal of engineering attention, and various analytical and numerical solutions to predict spudcan penetration process are in use. Although analytical solutions (e.g. based on bearing capacity formulae) have intrinsic limitations and numerical modelling is potentially more versatile, it has been argued that input parameter uncertainty is typically so large that it may dwarf the difference in precision achieved by using more elaborate computational models (Menzies et al, 2018). An interesting avenue to overcome that limitation is to incorporate uncertainty into the simulation, for instance by using a random finite element method (Yi et al. 2020). This kind of countermeasure had to pay the price of a very heavy computational load. There is thus room for improvement in this respect and it is therefore interesting to examine the possibilities of newer numerical simulation techniques to address the spudcan installation problem.

In this work the load displacement curve of a spudcan during the pre-load phase of a mobile jack up rig installation process is predicted using four different methods. The first uses the analytical bearing capacity method suggested by the ISO 19905-1 (2016) guideline, the second one is a small strain Finite Element Method (FEM) wished-in-place simulations, the third one used also small-strain but simulating installation effects and the last one used a large strain approach. The results are then compared with filed test data available from the literature. In this work the commercial FE package, PLAXIS (Brinkgreve et al, 2018) was used for the small strain simulations while the PFEM implemented into the Kratos Multiphysics framework (Dadvand et al, 2010) was used for the full installation large strain analyses.

2 ANALYTICAL AND NUMERICAL APPROACHES

Consider a spudcan leg pushed into the ground during the preload phase of a mobile jack up unit installation. At any penetration depth D, the total vertical leg load V is sustained by the available spudcan reaction. This reaction is provided by the ground, which, because of the large displacements involved (up to 40m) is at failure. The failure mechanisms occurring within the soil continuously evolve with penetration depth and the collapse of the cavity formed above the spudcan may also occur. All these features should be considered for a proper estimation of the V–D curve which is needed for a safe design of the preload phase. As detailed in Figure 1a, V will be the resultant force of the spudcan-soil system that includes the bearing capacity of the ground Q, the spudcan and leg weights (W_{SPUD} and W_{LEGS}) and the eventual weight of the soil filling the cavity ($W_{BACKFILL}$).

$$V = Q - (W_{SPUD} + W_{LEGS}) - W_{BACKFILL} \tag{1}$$

As clearly detailed in the existing guidelines SNAME (2008) and ISO 19905-1(2016), the simplest method to estimate the V-D curve is to calculate the spudcan vertical bearing capacity at various depths using bearing capacity formulae. The guidelines address various failure mechanisms such as punch through and squeezing that may occur in variable strength and/or layered soil beds. Figure 1b presents a schematic showing how the available structural spudcan reaction (V_L) is calculated in the guidelines. First the vertical bearing capacity (Q_V) of an open hole circular flat base foundation for various depths D is determined using closed form bearing capacity solutions. This value is then corrected to account for the soil buoyancy of the spudcan below the bearing area (B_S) to obtain he available structural spudcan reaction V_L. The backfill weight W_{BF} (which also accounts for the weight of the top part of the spudcan) is then used to correct V_L.

$$V_L = Q_V + B_S - W_{BF} \tag{2}$$

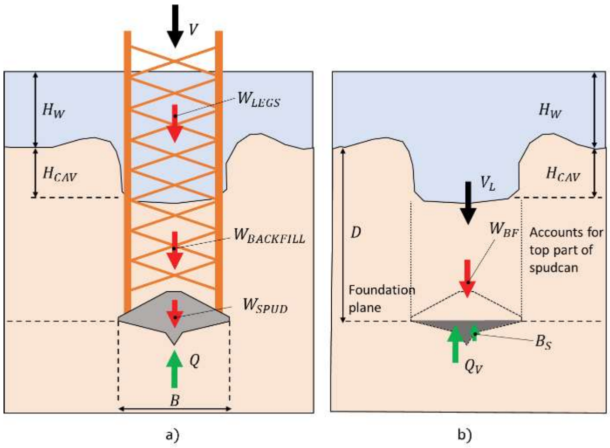


Figure 1. a) Schematic of a penetrating spud can foundation and b) geometrical and loads to consider as reported in the ISO 19905-1(2016)

The main difference between SNAME and ISO is the determination of the depth of the stable cavity height (H_{CAV}) that is necessary order to calculate the backfill weight. The SNAME method is based on the static hole stability (Meyerhof 1972, Britto & Kusakabe 1982) while the ISO approach uses an empirical fit proposed by Hossain and Randolph (2009).

$$\frac{H_{cav}}{B} = \left[\frac{s_{uH}}{\gamma' B}\right]^{0.55} - \frac{1}{4}\left[\frac{s_{uH}}{\gamma' B}\right] \tag{3}$$

Eq. (3) was determined by means of centrifuge and large strain numerical analyses. Herein the Hossain and Randolph method adopted by ISO will be used.

Finite element modelling is becoming increasingly popular in offshore geotechnical Engineering design and consequently the use of numerical approaches to determine the V-U curve is nowadays very common. There are two ways in which commercial traditional FE software can be used to for this purpose: the wished in place *(WIP)* approach, that doesn't consider installation effects and the press and replace (*PR*) method that allows to model the full installation process whilst using a small strain FE formulation. These are briefly described here below.

2.1 *Small strain FEM wished in place (WIP) method*

By running displacement controlled spudcan penetration analyses starting from various penetration depths, the yielding point of the spudcan foundation can be identified. The analysis is usually performed prescribed a nodal displacement of an element of the spudcan. The reaction force on this node will directly return the structural spudcan reaction (V_L). The V-U curve is hence inferred in post processing phase by joining these yielding points and properly accounting for backfill and spudcan and leg weights.

2.2 *The press and replace (PR) method*

The PR method is a numerical procedure proposed by Engin et al. (2015) to simulate jacked pile installation. By means of step-wise procedure based on geometry update of small deformation phases the continuous penetration of an object is modelled while preserving the initial mesh during the entire process. The PR technique presents a step-wise geometry update comprising of numerous paired phases: *press phase* which represents the penetration of the object (spudcan in this study) and the straining of the soil material, and *replace phase* which represents the geometry update (Figure 2). At every press phase, displacement boundary conditions are prescribed to model the penetration and to mobilize the soil resistance. Following the *press phase*, the zone of soil displaced by the object is replaced by the material of the penetrating object in the replace phase (geometry update). Geometry update results in modification of the global stiffness matrix at the beginning of every replace phase. Thanks to this procedure, installation effects can be accounted without distorting the mesh. Hereto, the reaction force on the node used to impose the spudcan displacements coincides with the V_L.

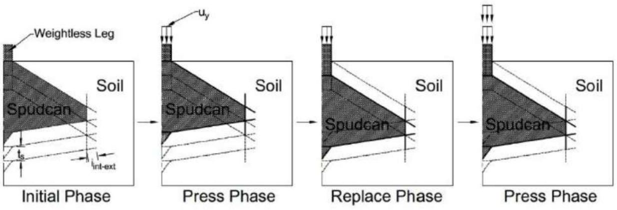


Figure 2. PRM scheme adopted for the spudcan penetration analysis.

To model the soil-spudcan interaction, interface elements are used shown as dark continuous lines in Figure 2. For the interface elements along the spudcan periphery, interface strength reduction factor is set to 0, to model a smooth interface. In addition, interface extensions at the sharp edges of the spudcan having equal length to the slice thickness are employed to avoid numerical stress fluctuations.

2.3 *Large deformation method*

The application of FEM-related numerical techniques to model a large strain problem such as spudcan penetration include the Arbitrary Lagrangian Eulerian (ALE) method (van den Berg et al. 1996) the Material Point Method (Sulsky et al, 1994) and the Point in Cell Method (Harlow, 1964). During the last decade the Particle Finite Element method (PFEM) has been developed into a viable alternative to deal with large strain problems. PFEM is a Lagrangian particle method supported by a finite element mesh (Oñate et al, 2004). During the computation, the mesh is constantly rebuilt using the current position of nodes. The typical solution algorithm of PFEM is conceptually illustrated in Figure 3. Given a *collection of particles or cloud of nodes* (*C*) belonging to the analysis domain, we can define a *mesh* (*M*) discretizing the domain with finite elements considering a predefined boundary representing the *volume* (*V*) of the computing domain.

The G-PFEM (geotechnical particle finite element method) is an open source numerical code developed for the analysis of large strain insertion problems in geomechanics (Monforte et al, 2017, 2018). G-PFEM has a rich library of constitutive models for clays/sands (Hauser & Schweiger, 2021) and soft rocks (Monforte et al, 2019, Oliynyk. et al, 2021). More details of the current potential of G-PFEM are given by Carbonell et al. (2021).

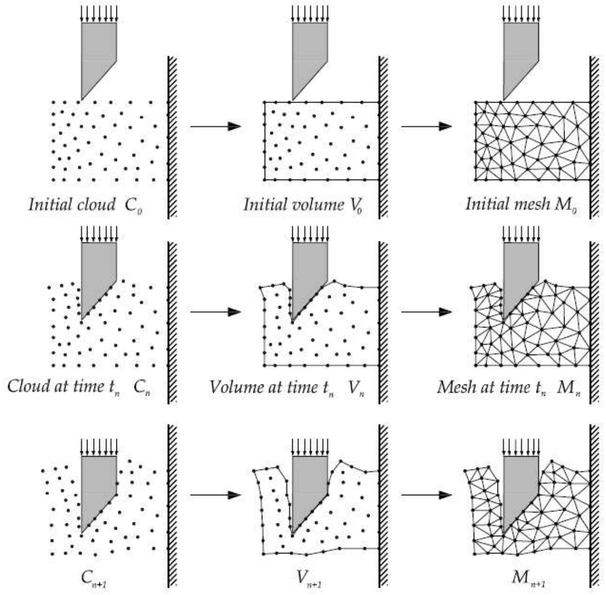


Figure 3. Sequence of steps to update in time a “cloud” of nodes representing a soil mass that is progressively penetrated by the action of an external structure using the PFEM (Carbonell et al. 2021).

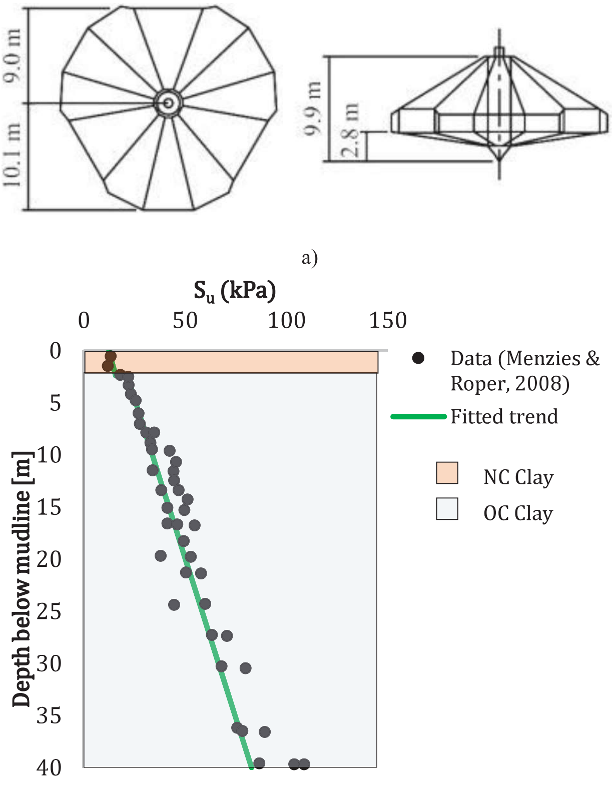
In this work G-PFEM will be used to simulate spudcan penetration. Due to the differences in stiffness between the soil and the structure, the spudcan is considered completely rigid; contact constraints are imposed to the solution using a penalty parameter and the shape and motion are pre-defined. In all simulations reported here a completely smooth interface is considered.

3 PENETRATION CURVE PREDICTION EXCERCISE

In this section the analytical and numerical methods presented in section 2 are used to simulate the spudcan load-penetration curve of a MLT 224-C Super Gorilla jack-up rig installed in the Gulf of Mexico. Each spudcan features an asymmetrical shape, having 9 meters from center to short side and 10.1 meters to long side. For the numerical analysis a symmetrical shape with an equivalent diameter of 19.8 m is assumed based on the maximum cross-sectional area (Figure 4a). The ground consists of a normally consolidated (NC) soft clay overlying slightly over-consolidated (OC) clay. The soil profile including the test results as reported by Menzies and Roper (2008) is depicted in Figure 4b. Table 1 summaries the parameters used to describe the fitted trend line represented in the figure. Further details of this case study may be found in Menzies and Roper (2008) and Van Dijk and Yetginer (2015).

Table 1. Properties of the clay layers.

Clay Layers	Thickness $[m]$	Submerged Unit Weight, γ' $[kN/m^3]$	Shear Strength, s_{ut} $[kPa]$	Rate of Increase, ρ $[kPa/m]$	Stiffness Ratio	Clay Type
Soft Clay	2.40	5.70	12.50	1.37	300	NC
Stiff Clay	42.70	6.45	21.60	1.63	500	OC



a)

b)

Figure 4. Problem geometry and ground conditions. a) MLT 224-C Spudcan geometry (Van Dijk and Yetginer, 2015) and b) undrained shear strength profile. Data adapted from Menzies & Roper (2008)

3.1 *Bearing capacity analytical method prediction*

Figure 5 reports the Spudcan lad reaction curve obtained following the bearing capacity method as reported in the ISO guidelines. The figure also reports the spudcan buoyancy (B_S) correction, the backfill weight (W_{BF}) and the bearing capacity curve (Q_V). Comparing the V_L curve with the field test results shows that the capacity at 35 m of penetration is overestimated by 61%. As discussed in Menzies and Roper (2008) one of the reasons that could explain this difference is the non-symmetry of the spudcan. Nonetheless even if the capacity is factored by 0.85 to account for shape, the preload capacity is still overestimated by 37%. The filed data indicates that more backfill material may have collapsed in the cavity after installation causing an extra 3 m of spudcan penetration. Van Dijk & Yetginer (2015) suggest that other factors -such as the intrinsic limitations of a non-softening Tresca model for the soil may be also at play. For coherence, these limitations (axysimmetric geometry, Tresca model) have been also maintained in the numerical methods used below.

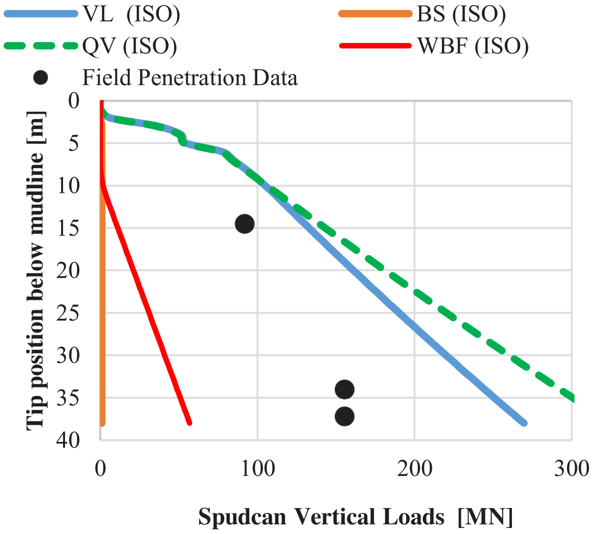


Figure 5. ISO prediction of the structural spudcan reaction load.

3.2 *Numerical predictions*

To determine a spudcan *V-U* curve numerically a total stress approach will be used, since the penetration during pre-load is fast and undrained conditions can be assumed. The soil behaviour is hence described by a quasi-incompressible elastic model (a Poison's ratio of 0.49 is employed) and a perfectly plastic model Tresca model. All the analysis reported here assume a rigidity index, $I_r = G/S_u$, equal to 300 and 500 for the NC and OC clay respectively. These values fall within the range commonly adopted for soft clays and stiff clays (Knappett and Craig, 2012). Prior to spudcan penetration, a K_0=1 geostatic stress field is assumed. For all the numerical simulations a monophase material with a unit weight of 6.45 kN/m^3 (submerged unit weight of the clay) was considered. The model does not represent the jack-up leg (i.e. $W_{LEGS} = W_{SPUDCAN} = 0$) hence the nodal reaction in the vertical direction of the displaced is equal to the structural spudcan reaction (V_L). The WIP and PR PLAXIS simulations require a manual application of the backfill material if it is deemed necessary. By themselves the small-strain FE models predict a stable cavity in this case. The G-PFEM model does not require manual activation of backfill as this will be captured automatically during the simulation. However, as shown in the next section, the cavity of the G-PFEM model in this case remained also stable up to 35m of penetration.

Figure 6 reports the WIP FEM spudcan penetration curve. In the figure the variable initial depth penetration analyses used to retrieve the spudcan load penetration curve are also represented. The numerical method overestimates the field result at 35 m of penetration by 68% (more than the ISO prediction). Even if the reaction is factored by 0.85 to account for the non-circular base the WIP method overestimate the field result by 43%.

Figure 7 shows the penetration curve obtained with the PR method. In this case the filed data is overpredicted by 54% and 31% with and without shape factor correction respectively. This slight improvement in the prediction indicated that installation effects can slightly improve the model prediction ability. This is confirmed by the large strain G-PFEM simulation results reported in Figure 8 which are similar to the PR ones. The -PFEM model overpredict the field test data by 53%. This value reduces to 33% when considering the spudcan non-uniform area effect. The simulation time required to reach the final depth for the WIP, PR and G-PFEM models were of 15 min, 40 min and 3 hrs on a standard desktop respectively.

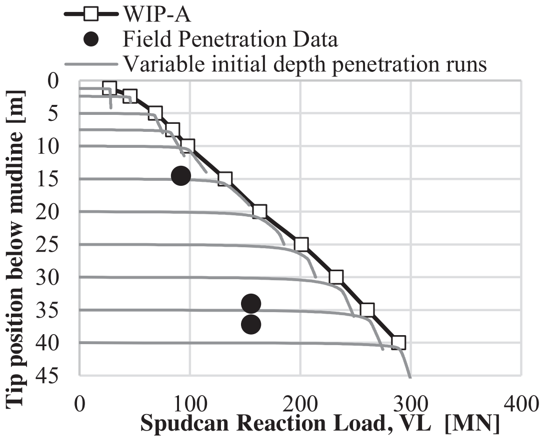


Figure 6. FE numerical evaluation of the spudcan reaction lad curve using the WIP approach.

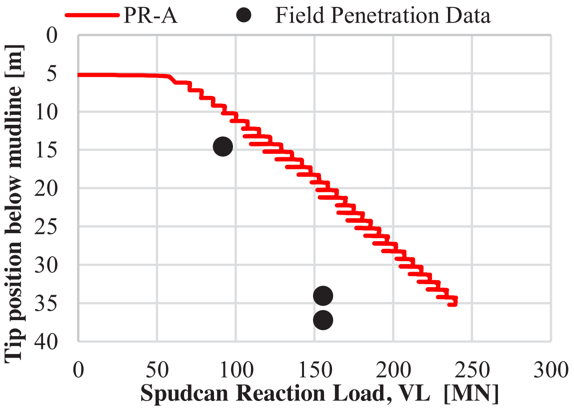


Figure 7. PR FEM numerical prediction of the spudcan reaction load curve

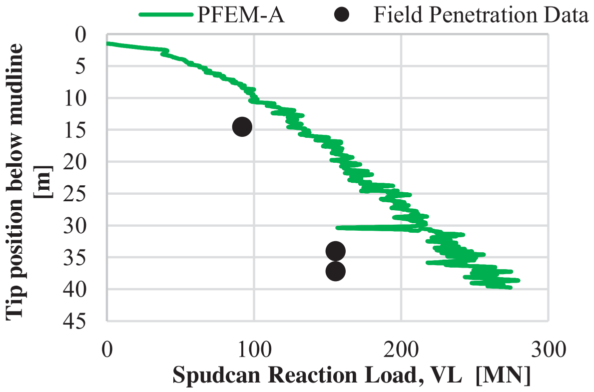


Figure 8. G-PFEM numerical prediction of the spudcan reaction load curve

3.3 *Results comparison*

Figure 9 compares all the penetration curves determined in the previous section. Although the analytical and numerical curves appear to follow the same response, the WIP model predicts values 4% larger than the ISO while the PR and G-PFEM are 4.5% and 5.1% lower than the ISO. From a practical engineering perspective however, such variations are not negligible hence suggesting that taking into account the full installation process may result in a safer design.

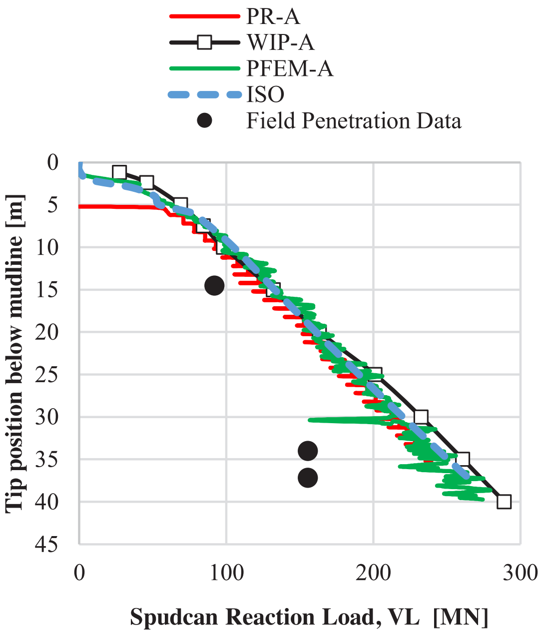


Figure 9. Spudcan reaction load curve. Comparison of prediction performance between the ISO analytical approach, the FE MIP method, the PR method and large strain G-PFEM simulation.

4 PR VS G-PFEM

Given that the PR and G-PFEM resulted in a very similar result, this section will compare with some more detail the results from these two analyses. Figure 11 shows the stress path of a point in the soil 15 meters below the mudline and 15 meters away from the axis centre. Hereto the results from the PR model and G-PFEM are very similar. Whilst it was somehow expected that using a total stress approach and a Tresca model for the soil the load penetration curve would return a similar response, the similarity between the stress paths was more of a surprise. Both models identify the increase in mean stress induced by the passing of the spudcan tip and show that the point remains in plastic flow since the spudcan tip is at 10 m.

Figure 11 compares the incremental displacements of the two simulations at three depths. Interestingly the flow mechanisms are also very similar despite the PR uses a simplistic approach to replicate installation process. As shown in the figure both models clearly show the transition from a shallow failure mechanism to a deep flow like failure. A more detailed analysis of the G-PFEM shows that the cavity is starting to be filled. Such feature cannot be captured with the PR method. Indeed, for this particular case Figure 12 shows a G-PFEM model where instead of using the submerged weight the total weight is employed. Here the cavity collapses over the spudcan in a manner similar to what was observed in the centrifuge experiments by Hossain et al. (2005). Moreover, the depth at which backfill appears is that predicted by ISO (10 m). The automatic capturing of backfill when using a rigorous large strain FE simulation such as the G-PFEM appears to be the major advantage. Using total weight in this kind of analysis is, nevertheless, a departure from ISO, and the implications are not pursued here.

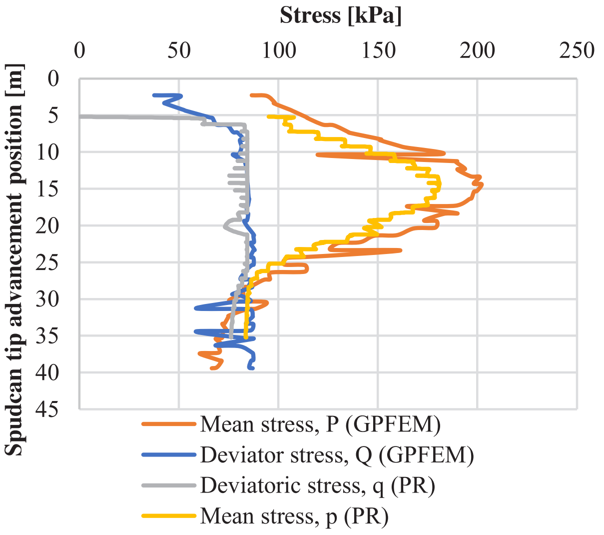
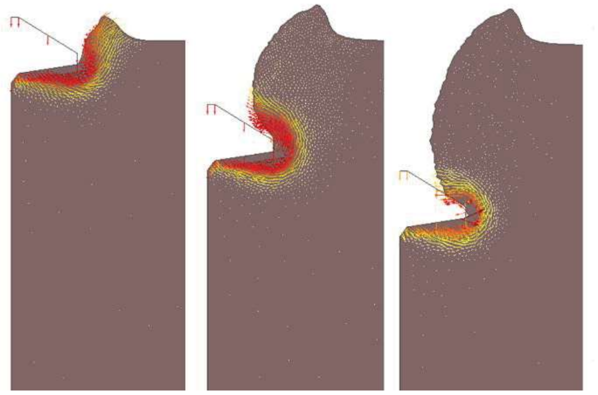
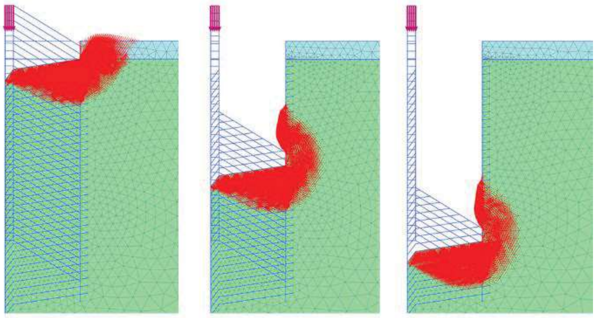


Figure 10. Comparison of G-PFEM and PR modelled stress paths as the spudcan penetration advances of a point in the soil located 15m below mudline and 15 m away from the centre of symmetry.



a) G-PFEM analysis



b) PLAXIS PR model

Figure 11. Soil flow showing transition from shallow to deep failure mechanism. The snapshots represent only small portion of model domain

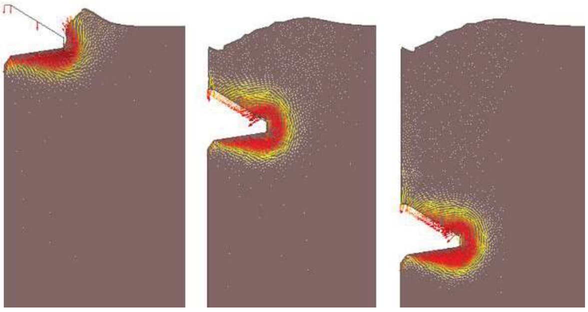


Figure 12. Soil flow showing cavity infill for G-PFEM model with saturated unit weight. The snapshots represent only small portion of model domain.

5 CONCLUSIONS

This paper presented four different ways to estimate reaction load for a spudcan foundation in clay. Namely the ISO guideline (based on bearing capacity analytical equations), two small strain FE analyses and one large strain GPFEM numerical simulation replicating the full installation process. Using the field test data from a real case study of a spudcan installation in the Gulf of Mexico, the predictive performance of the four different methods was tested. The main conclusion that can be drwn from the sudy are:

i) When using an undrained total stress approach, the spudcan reaction load curve predicted by the 3 FEM based numerical models are very similar (within $\mp 15MN$ at 35m of depth) to what obtained with the ISO bearing capacity analytical method.
ii) The WIP FE method predicts slightly higher reaction loads with respect to the ISO, but most interestingly the PR method and the G-PFEM model (which consider installation effects) predict lower values (up to 5% less) with respect to the ISO. This means that the ISO method might overestimate the penetration response
iii) The PR method was observed to perform very similarly to a large strain G-PFEM model. It appears to be a practical numerical approach to incorporate installation effects, although limited in its ability to capture potential cavity filling.

6 ACKNOWLEDGEMENTS

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