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Underwater Acoustics: Multibeam Echo Sounding: Bathymetry and sediment Classification

Comparison of methods employed to extract information contained in seafloor backscatter

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Seabed maps are based on quantities extracted from measurements of the seafloor's acoustic response by sonar systems such as single-beam echo-sounders (SBES), multibeam echo-sounders (MBES) or sidescan sonars (SSS). In this paper, a comparison of various strategies to estimate the backscattering strength (BS) from recorded time-series, i.e. seabed echoes extracted from pings, is presented. The work hypotheses are based on processed data from a SBES designed to be tilted mechanically. Ideal survey conditions are taken into account and the seafloor is supposed to be rough so that BS is assumed to be equivalent to the Rayleigh probability density function parameter. Classical methods such as averaging corrected (sonar equation) backscattered single values over a set of pings to estimate BS are compared to other methods exploiting several time-samples being part of pings. Simulated data is considered to estimate BS in different situations (several estimators, natural/squared values, number of samples and pings). The best estimator to reach a 0.1dB uncertainty is proposed, and a formula governing the number of time-samples and pings needed to reach an accurate BS estimation according to the measurement conditions is derived.

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The seafloor acoustic backscattering strength (BS) is used for several applications such as seabed mapping, characterization or classification of seabed features, acoustic propagation models, etc. In most of these applications, it is defined as a single value processed for a given sounding[1] (in order to save memory space), a given angle[2,3] (BS curves), or a digital terrain model (DTM) cell[4] (reflectivity map). It is also a single value when associated to a frequency[5] such as in seabed classification algorithms.
In practice, this single BS value is estimated from received acoustic time-signals (pings), backscattered from the seabed. Each received ping can be considered as a list of samples scattered from the water column, the seafloor and other obstacles along the way. In our study, we only consider the samples backscattered from the seafloor, i.e. extracted from the seabed echo. During usual surveys, several pings are collected on different types of seafloor as the vessel follows survey lines that can be kilometers long.
Two questions arise from the definition of the BS as a single value: 1) how is a single value extracted from a time-samples list to obtain a BS estimate, and are there any existing methods employed to reduce the seafloor echo to a single value ? 2) How many samples and pings are needed to reach an accurate and useful estimate of the BS ?

1. CONTEXT AND HYPOTHESES

The practical context in which the present study takes place is that of a single-beam echo-sounder survey. The echo-sounder is tilted mechanically to reach a fixed grazing angle on the seafloor as shown in figure 1. The system is static - in practice it would be drifting slowly with any surface sea currents - to ensure that every ping is recorded on the same type of seafloor. The nature of the seafloor is not specified, but we consider that it is rough enough to guarantee the random nature of the received samples. Its acoustic response is defined as equivalent to the parameter σ^2 of the Rayleigh probability density function (PDF), i.e.:

$$BS = 10\log_{10}\left(\sigma^2\right) \tag{1}$$

with the Rayleigh PDF:

$$f(y;\sigma^2) = \frac{y}{\sigma^2}\exp\left(\frac{-y^2}{2\sigma^2}\right) \text{ for } y \in [0, +\infty[\tag{2}$$

The type of echo-sounder chosen has a large aperture and a small bandwidth. This is enough to make the assumption that the number of scatterers whose responses contribute to the received intensity of a time-sample is sufficient, and that each corrected time-sample from the seabed echo is a realisation of the Rayleigh distribution of parameter σ^2 (the angular dependence of the seabed response is neglected). In addition, stationary measurements force all seabed echoes of all pings to be made of corrected time-samples that are realisations of the same Rayleigh PDF of parameter σ^2.

A. LINK BETWEEN MEASUREMENT DATA AND MATHEMATICAL REPRESENTATION

Time-samples of every echoe from the seabed are assumed to have already been corrected using sonar equation parameters. Indeed, samples x shown in figure 1 come from raw measurements of the echo-sounder that were compensated from propagation loss, echo-sounder geometry and sensitivity, insonified area, etc.[6] so that their values are equivalent to seafloor indexes. These corrected time-samples j of ping i are simulated as realisations of the random variable X following a Rayleigh distribution and noted x_{ij} so that waterfall

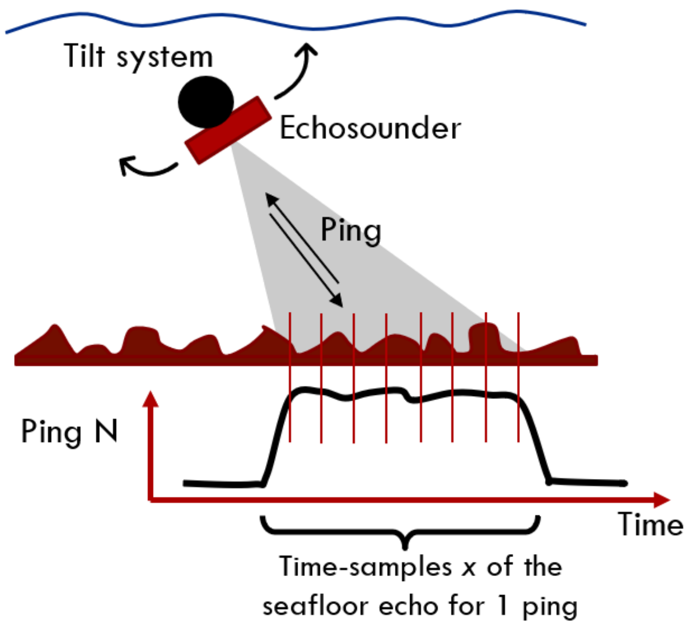

Figure 1: Description of the system over a rough seafloor, and schematic representation of the correspondence between the practical geometry and the signal received for one ping.

data correspond to the following matrix (equation 3):

$$\begin{array}{c} \\ \text{Ping 1} \\ \vdots \\ \text{Ping } j \\ \vdots \\ \text{Ping } N \end{array} \begin{array}{c} \begin{array}{ccc} \text{Sample 1} & \cdots & \text{Sample n} \end{array} \\ \begin{pmatrix} x_{11} & \cdots & x_{n1} \\ \cdots & \cdots & \cdots \\ x_{1j} & x_{ij} & x_{nj} \\ \cdots & \cdots & \cdots \\ x_{1N} & \cdots & x_{nN} \end{pmatrix} \end{array} \quad (3)$$

where N is the number of pings and n the number of time-samples in a seabed echo.

B. USUAL DESCRIPTORS

For each ping, the seabed echo (≈ snippet) is reduced to a single value through methods typically depending on the ultimate application. These reduced values are called descriptors in the following. Four descriptors frequently used in the literature are studied in this paper:

- the maximum value of the n seafloor echo samples[7] ;
- the median of the the n seafloor echo samples,[8] i.e. the value separating the higher half from the lower half of the n samples;
- the sample mean of the n seafloor echo samples;[8]
- the mean of the squared n seafloor echo samples,[8] abbreviated to MSS in the following for mean squared samples.

For each ping j, the descriptor is used to estimate the Rayleigh PDF parameter noted $\hat{\sigma}_j^2$.
Finally, to estimate the backscattering strength as the single value $\widehat{bs}$, all estimates $\hat{\sigma}_j^2$ of a number N of pings are averaged[9] such as:

$$\widehat{bs} = \frac{1}{N}\sum_{j=1}^{N}\hat{\sigma}_j^2 \tag{4}$$

This value can be written in decibels as:

$$\widehat{BS} = 10\log_{10}\left(\widehat{bs}\right) \tag{5}$$

2. SEAFLOOR BACKSCATTERING STRENGTH - ESTIMATOR COMPUTATION

In order to study and compare the different estimations of the backscattering strength under the Rayleigh assumption, we derive the probability density functions f of each descriptor according to the descriptor employed:

- The PDFs of the maximum and median of Rayleigh samples are derived from order statistics formulae[10,11] :

$$f_{\text{max}}(m;\sigma^2) = n\frac{m}{\sigma^2}e^{\frac{-m^2}{2\sigma^2}}\left[1-e^{\frac{-m^2}{2\sigma^2}}\right]^{n-1} \tag{6}$$

$$f_{\text{med}}(q;\sigma^2) = \frac{n!}{\left(\frac{n}{2}-1\right)!\left(\frac{n}{2}\right)!}\frac{q}{\sigma^2}e^{-\left(\frac{n}{2}+1\right)\frac{q^2}{2\sigma^2}}\left[1-e^{-\frac{q^2}{2\sigma^2}}\right]^{\frac{n}{2}-1} \tag{7}$$

  where m is the maximum value of n seabed echo time-samples, and q their median.

- PDFs of the sample mean and mean squared samples (MSS) of Rayleigh samples can be approximated[12–14] as:

$$f_{\text{mean}}(\mu;\sigma^2) = \frac{\mu^{2n-1}}{2^{n-1}(\sigma^2)^n\left[(2n-1)!!\right]\Gamma(n)}e^{\frac{-\mu^2 n^2}{2\sigma^2\left[(2n-1)!!\right]^{\frac{1}{n}}}} \tag{8}$$

$$f_{\text{MSS}}(r;\sigma^2) = r^{n-1}\frac{n^n}{(2\sigma^2)^n\Gamma(n)}e^{\frac{-rn}{2\sigma^2}} \tag{9}$$

  where μ is the sample mean of n seabed echo time-samples, r their mean squared sample (MSS), and $(2n-1)!! = (2n-1)\cdot(2n-3)\cdot\ldots\cdot3\cdot1$.

From these PDFs, the response σ^2 of the seabed for one ping is estimated by deriving the maximum likelihood estimator. This gives the estimate $\hat{\sigma}^2$ of the Rayleigh PDF parameter for each descriptor:

$$\hat{\sigma}^2_{\text{max}} = \frac{1}{2}\left(1-\frac{n-1}{e^{\frac{m^2}{2\hat{\sigma}^2_{\text{max}}}}-1}\right)m^2 \tag{10}$$

$$\hat{\sigma}^2_{\text{med}} = \frac{1}{2}\left[1+\frac{n}{2}+\left(1-\frac{n}{2}\right)\frac{1}{e^{-\frac{q^2}{2\hat{\sigma}^2_{\text{med}}}}-1}\right]q^2 \tag{11}$$

$$\hat{\sigma}^2_{\text{mean}} = \frac{n}{2}\left[(2n-1)!!\right]^{-\frac{1}{n}}\mu^2 \tag{12}$$

$$\hat{\sigma}^2_{\text{MSS}} = \frac{1}{2}r \tag{13}$$

Note that the mean squared sample descriptor gives an estimate of σ^2 equal to the maximum likelihood estimator (MLE) of the Rayleigh distribution:

$$\hat{\sigma}^2_{\text{MSS}} = \frac{1}{2}r = \frac{1}{2n}\sum_{i=1}^{n} x_i^2 = \hat{\sigma}^2_{\text{MLE}} \tag{14}$$

where x_i are samples following a Rayleigh PDF of parameter σ^2 (i.e. corrected seabed echo time-samples). This result proves that the mean squared sample descriptor gives an estimate of σ^2 asymptotically efficient[15] . However, depending on the echo-sounder data provided, the application and the measurements conditions, the other descriptors need to be taken into account because of their other advantages like the robustness to outliers[16,17] .

A. DESCRIPTOR VALUE VERSUS RAYLEIGH PARAMETER ESTIMATE

Corrected seabed echo time-samples have been simulated to study the behaviour of the descriptors. At first, we compare the expected value of the descriptor value (maximum, median, sample mean, or mean of squared seabed echo time-samples) with the expected value of the estimation of the seabed response $\hat{\sigma}^2$. In each simulation, the real seabed response BS is $BS = 10\log_{10}\left(\sigma^2\right) = -10$ dB. Results are given on figure 2 for $n = 80$ time-samples and $N = 200$ pings.

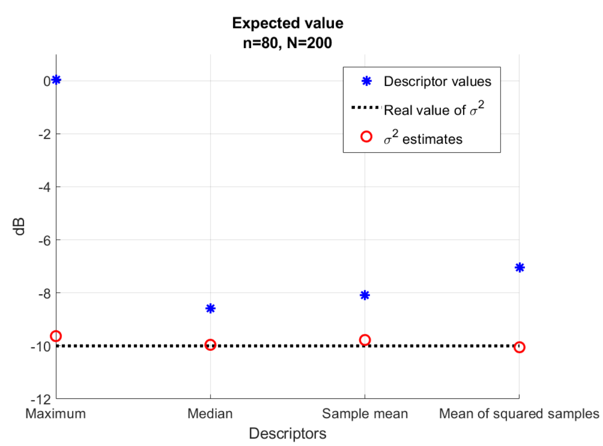

Figure 2: Comparison of the expected value of the descriptors with the expected value of the estimates $\hat{\sigma}^2$

We observe that all estimates of $\hat{\sigma}^2$ converge toward the real value of the seabed response, whilst the descriptors values are biased. This validates the simulation, and is also a reminder of good practice, in which unbiased estimations should be preferred.

B. FROM THE RAYLEIGH PARAMETER ESTIMATE OF EACH PING TO BACKSCATTERING STRENGTH ESTIMATES

After processing all estimations $\hat{\sigma}^2_j$ of each ping j, the backscattering strength estimate is computed as the average of all theses values (equations 4 and 5). It gives $\hat{bs}$ as a function of the number of time-samples n and the number of pings N:

$$\hat{bs} = \hat{bs}(n, N) \tag{15}$$

To compare backscattering strength estimates from all descriptors, we choose to analyse two characteristics of the variable $\widehat{bs}(n, N)$: its expected value and its variance (or standard deviation). By simulating a large number of realisations of the matrix shown in equation 3, the sample mean and sample variance of $\widehat{bs}(n, N)$ are estimated numerically as a function of n and N. These 2-D results can be represented as a plot, like figure 3.

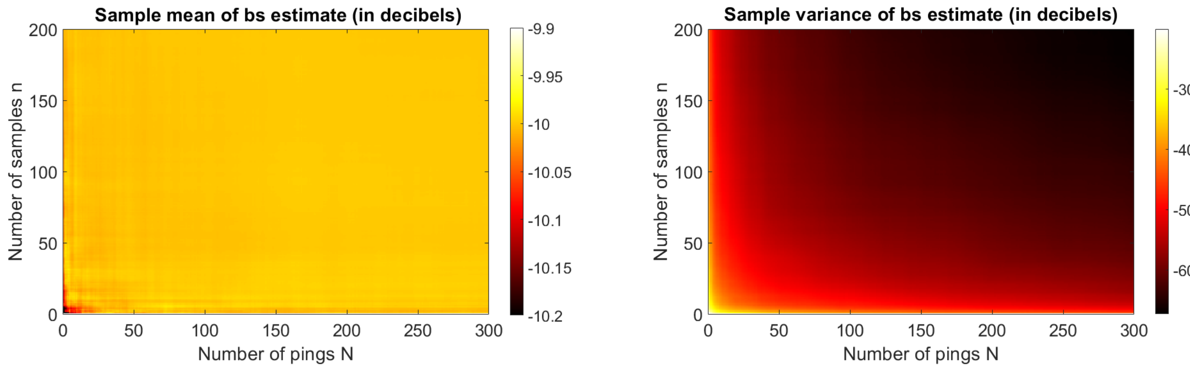

Figure 3: Computed sample mean (left) and sample variance (right) of $\widehat{bs}(n, N)$ in decibels for the descriptor mean of squared samples (MSS). Both are represented according to the number of time-samples n and the number of pings N taken into account (for 500 realisations of the matrix in equation 3 with $BS = 10\log_{10}(\sigma^2) = -10$ dB).

3. COMPARISON OF BACKSCATTERING STRENGTH ESTIMATORS

We define a backscattering strength estimator $\hat{bs}$ as *accurate* when it respects two criteria:

1. its expected value $E[\hat{bs}]$ is equal to the real value σ^2;
2. its variance is *negligible* according to the classification of BS uncertainties in Malik et. al (2018)[18] . Indeed, because we only compute here the uncertainty due to the random nature of the seafloor and because this study takes place in ideal conditions, the BS uncertainty should be *negligible*[18] , i.e. ≤ 0.1dB.

The backscattering strength estimator variance $var[\widehat{bs}]$ is linked to its uncertainty via its standard deviation, called $\widehat{\delta bs}$. The uncertainty is considered *negligible* by Malik et. al[18] when:

$$10\log_{10}\left(1 + \frac{\widehat{\delta bs}}{bs}\right) \leq 0.1\text{dB} \tag{16}$$

where bs is the true value of the seabed response, i.e. the true Rayleigh parameter σ^2.
From equation 16, we derive a condition on the standard deviation:

$$\widehat{\delta bs} \leq \sigma^2\left(10^{\frac{0.1}{10}} - 1\right) \tag{17}$$

Writing $\widehat{\delta bs} = \sqrt{var[\widehat{bs}]}$, we can compare the threshold of equation 17 to the backscattering strength variances computed in simulations (figure 3). The limit contour function of n and N for standard deviations higher or equal to the threshold is plotted in figure 4 for all estimators. These contours also represent the minimum number of time-samples or pings necessary to ensure an accurate BS estimation. Figure 4 shows that the BS estimator needing the less samples or pings to reach an accurate estimation is the one using the mean of the squared time-samples.

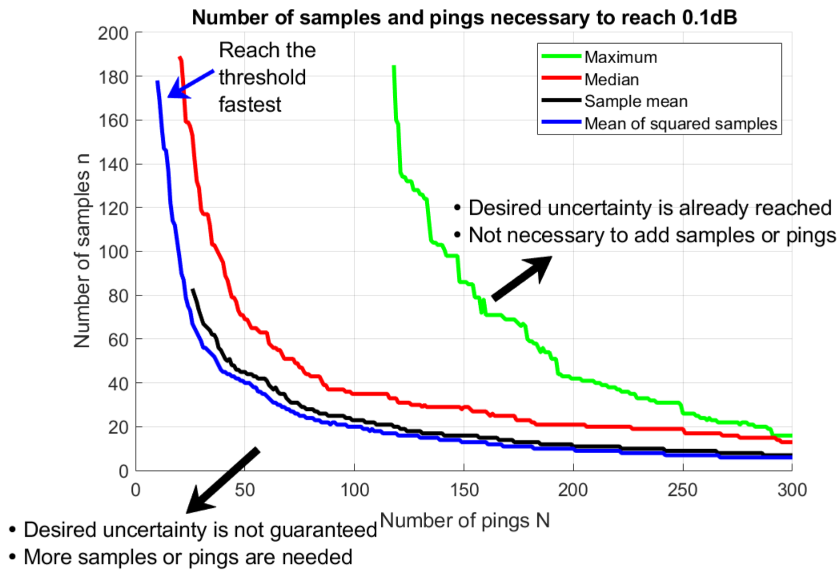

Figure 4: Minimum number of time-samples or pings necessary to reach the uncertainty threshold of 0.1 dB (for 500 realisations of the matrix in equation 3 with $BS = 10\log_{10}(\sigma^2) = -10$ ***dB).***

A. CONTOUR ANALYTICAL FORMULA

The limit contour function of n and N for the estimate using the mean squared samples descriptor $\hat{\sigma}^2_{\text{MSS}} = \frac{1}{2n}\sum_{i=1}^{n} x_i^2$ can be calculated analytically under the two BS estimation criteria. Indeed, the MSS of n random samples x, following a Rayleigh distribution of parameter σ^2, is distributed as a gamma probability density function of parameters $(n, \frac{2\sigma^2}{n})$.[12] Therefore:

$$\hat{\sigma}^2_{\text{MSS}} \sim \frac{1}{2}\gamma\left(n, \frac{2\sigma^2}{n}\right) = \gamma\left(n, \frac{\sigma^2}{n}\right) \tag{18}$$

The average of $\hat{\sigma}^2_{\text{MSS}}$ on N pings is therefore following a gamma PDF:

$$\frac{1}{N}\sum_{j=1}^{N}\hat{\sigma}^2_{\text{MSS}} = \widehat{bs}_{\text{MSS}} \sim \gamma\left(nN, \frac{\sigma^2}{nN}\right) \tag{19}$$

We can note here that, according to the efficiency theorem (Saporta, 2006[15] p.303), the average of $\hat{\sigma}^2_{\text{MSS}}$ following the gamma distribution $\gamma\left(n, \frac{\sigma^2}{n}\right)$ is an efficient and unbiased estimator of σ^2 i.e. $\widehat{bs}_{\text{MSS}}$ is an efficient and unbiased estimator of the backscattering strength.

Knowing the distribution of $\widehat{bs}_{\text{MSS}}$ allows us to derive its expected value and variance:

$$E\left[\widehat{bs}_{\text{MSS}}\right] = nN\frac{\sigma^2}{nN} = \sigma^2 \tag{20}$$

$$var\left[\widehat{bs}_{\text{MSS}}\right] = nN\left(\frac{\sigma^2}{nN}\right)^2 = \frac{(\sigma^2)^2}{nN} \tag{21}$$

Equation 20 show that, as required, the expected value of the backscattering strength estimator $\widehat{bs}_{\mathrm{MSS}}$ is the true acoustic response of the seabed $bs = \sigma^2$. This result ensures that the estimator is unbiased. Furthermore, the variance $\left(\sigma^2\right)^2 / nN$ is also the minimal variance[15] of the efficient estimator $\widehat{bs}_{\mathrm{MSS}}$.

Using equations 21 and 17, we can find an analytical formulation of the contour of the BS estimator using the mean of squared time-samples:

$$\sqrt{var\left[\widehat{bs}_{\mathrm{MSS}}\right]} \leq \sigma^2 \left(10^{\frac{0.1}{10}} - 1\right) \tag{22}$$

which gives the contour curve equation:

$$nN = \frac{1}{\left(10^{\frac{0.1}{10}} - 1\right)^2} \tag{23}$$

For this estimator, the minimal number of time-samples necessary to ensure an accurate BS estimation is therefore proportional to the number of pings taken into account. The proportionality coefficient depends on the magnitude of uncertainty the user allows on its measurements (here 0.1dB). Figure 5 shows a good match between the analytical formula of equation 23 and simulation results.

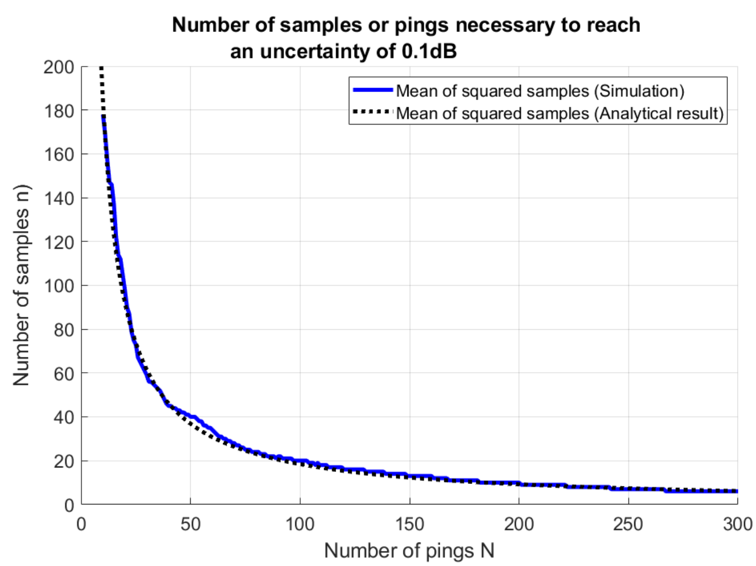

Figure 5: Minimum number of time-samples or pings necessary to reach a 0.1-dB uncertainty: comparison of analytical and simulation results for the estimation using the mean of squared time-samples (for 500 realisations of the matrix of equation 3 with $BS = 10\log_{10}(\sigma^2) = -10$ dB).

B. LOOK-UP TABLES

The calculations above focused on a maximum uncertainty of 0.1 dB (the *negligible* uncertainty mentioned in the seminal article by Malik et al.[18]). Depending on survey conditions and objectives, other levels of uncertainty can be selected[18] , like 0.1 - 1 dB (*small*), 1 - 3 dB (*moderate*), 3 - 6 dB (*high*) and higher (*prohibitive*). In fact, any threshold T (in dB) can be selected, and equation 23 then becomes:

$$nN = \frac{1}{\left(10^{\frac{T}{10}} - 1\right)^2} \tag{24}$$

T corresponds to the magnitude of BS uncertainty the user accepts to be subjected to. Some examples of contour curves are given for T ranging from 0.1 dB to 2 dB on figure 6.

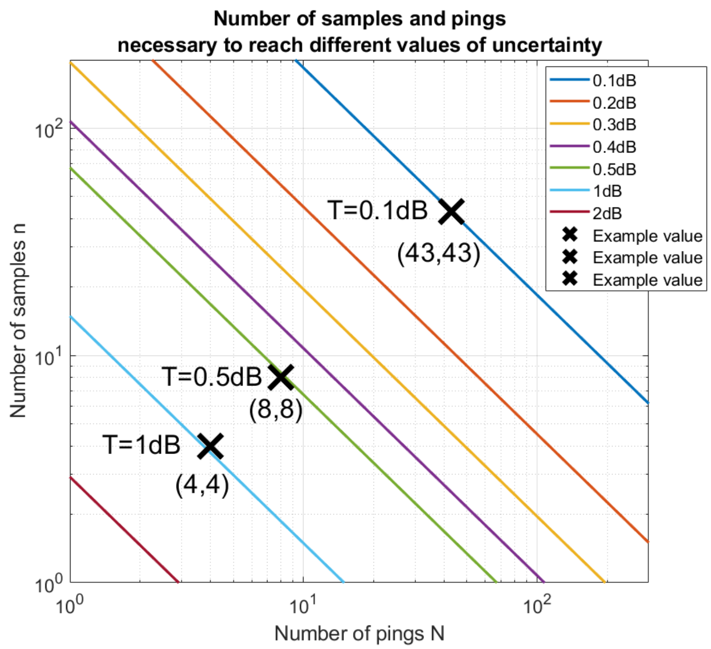

Figure 6: Minimum values of the pairs [number of time-samples, number of pings] necessary to reach an uncertainty of T, with $T = [0.1, 0.2, 0.3, 0.4, 0.5, 1, 2]$ dB. This look-up table has a log-log scale and some key example values are shown for $T = [0.1, 0.5, 1]$ dB.

4. CONCLUSION

Assuming a rough seafloor, ideal measurement conditions, and the equivalence of the backscattering strength BS with the Rayleigh PDF parameter σ^2, i.e. $BS = 10\log_{10}(\sigma^2)$, we derive four BS estimators from four descriptors of the received seabed echo. They correspond respectively to the maximum of the seabed echo time-samples (corrected from sonar equation beforehand), their median, their sample mean, and the mean of the squared time-samples. We show that the best BS estimator, in term of number of samples and pings needed to reach a 0.1-dB uncertainty on $\widehat{BS}$, is the mean of squared samples. For this estimator, the pairs [number of time-samples, number of pings] necessary to reach the threshold T (dB) aimed for is governed by the formula $nN = \left(10^{\frac{0.1}{10}} - 1\right)^{-2}$. Depending on the measurement conditions, users can employ this formula to ensure they are using enough pings or time-samples to obtain an accurate backscattering strength for the seafloor surveyed. The practical uses of this equation are therefore immediate, and allow optimising survey parameters depending on the desired accuracy, or constraining the maximum accuracy achievable for a given survey.

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