



Citation for published version:

Kisialiou, Y, Gribkovskaia, I & Laporte, G 2021, 'Periodic supply vessel planning under demand and weather uncertainty', *INFOR*, vol. 59, no. 3, pp. 411-438. <https://doi.org/10.1080/03155986.2021.1907872>

DOI:

[10.1080/03155986.2021.1907872](https://doi.org/10.1080/03155986.2021.1907872)

Publication date:

2021

Document Version

Peer reviewed version

[Link to publication](#)

This is an Accepted Manuscript of an article published by Taylor & Francis in *INFOR: Information Systems and Operational Research* on 19/5/2021 available online:
<https://www.tandfonline.com/doi/full/10.1080/03155986.2021.1907872>

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Periodic supply vessel planning under demand and weather uncertainty

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February 20, 2021

Abstract

We solve a periodic supply vessel planning problem under demand and weather uncertainty, arising in offshore of oil and gas production. Our study is motivated by the case of the Norwegian energy operator Equinor which supplied us with data. The aim is to determine an optimal fleet composition and a least-cost vessel schedule under uncertain demand at the installations and uncertain weather conditions. We present a methodology incorporating a metaheuristic within a discrete-event simulation model which, applied iteratively for the increasing values of reliability level parameters, yields a vessel schedule of least expected cost.

Keywords: supply vessel planning; offshore logistics; uncertainty; reliable vessel schedules; metaheuristic; discrete-event simulation; recourse.

1 Introduction

We solve a periodic supply vessel planning problem under demand and weather uncertainty, arising in the operations of oil and gas production offshore. Our study is motivated by the case of the Norwegian energy operator Equinor which supplied us with data. It is, however, of general applicability. Upstream offshore oil and gas supply chain involves several layers of a large number of suppliers from which the cargo is transported by different transportation modes (truck, rail and sea) to supply bases. At the supply bases, the goods are consolidated for subsequent shipment to the production and exploration installations located offshore. The transportation of cargo to the installations and the return of cargo (rented equipment, empty load carriers, waste, etc.) are performed by specialized vessels. The installations are usually located at large distances from the shore, resulting in intensive long-haul maritime transportation activities. In general the offshore installations have a very limited storage capacity, and to ensure the continuous supply of equipment and materials, the deliveries to the installations should be performed frequently and require careful planning.

The supply is affected by a number of uncertain factors, like weather conditions, which often result in delivery delays. Various equipment failures and other unforeseen events may require ad hoc deliveries. Furthermore, the demand for supplies at the installations, especially during the exploration phase, varies significantly and is characterized by uncertain volumes and due dates. Timeliness is vital for the installations activities as missing supplies may in the worst case cause delays in drilling operations, or a temporary reduction or a shutdown of oil and gas production. Taking into account the fact that the supply process takes place in a highly uncertain environment and is associated with high transportation and shortage costs, logistics planners seek ways of ensuring a reliable and effective supply.

Each supply base has a predefined set of installations to serve and hosts a fleet of chartered supply vessels hired on a long-term basis to deliver the cargo. The vessel schedule is built for a given planning period (a week in our case) and is performed repetitively over an execution horizon (several months or a season). The execution of the schedule is often disrupted by uncertain demand at the installations and by fluctuating weather conditions. To account for disruptions, logistics planners perform operational modifications to the schedule and, in the worst case, may hire spot vessels on a short-term basis. The schedule should therefore be constructed so as to minimize its expected cost over the execution horizon after the operational modifications have been implemented. Deterministic versions of the problem were recently studied by Shyshou et al. (2012), Borthen et al. (2017) and Kisialiou et al. (2018a). Its stochastic variant under uncertain weather conditions was investigated by Halvorsen-Weare and Fagerholt (2011), Norlund et al. (2015), Norlund and Gribkovskaia (2017) and Kisialiou et al. (2018b), and the uncertain demand

was treated by Kisialiou et al. (2019). The problem in which the demand and weather uncertainties are dealt with simultaneously is more complicated yet more realistic than when only one type of uncertainty is considered and poses a significant methodological challenge. In this study we solve this case for the first time. We introduce a methodology enabling the generation of vessel schedules of least expected cost given uncertain demand and weather conditions.

1.1 The periodic supply vessel planning problem

We focus on the problem of tactical vessel planning for the fleet of supply vessels which is known as the Periodic Supply Vessel Planning Problem (PSVPP). Fig. 1 depicts the locations of the offshore installations on the Norwegian continental shelf served from the supply base located in Mongstad (marked by a red circle). Each installation for a given *planning period* (one week in our case) has an estimated cargo delivery demand and a delivery frequency. The total weekly cargo demand is assumed to be evenly distributed across the deliveries. The delivery of the cargo to the installations is performed according to a planned *vessel schedule* constructed for the planning period.

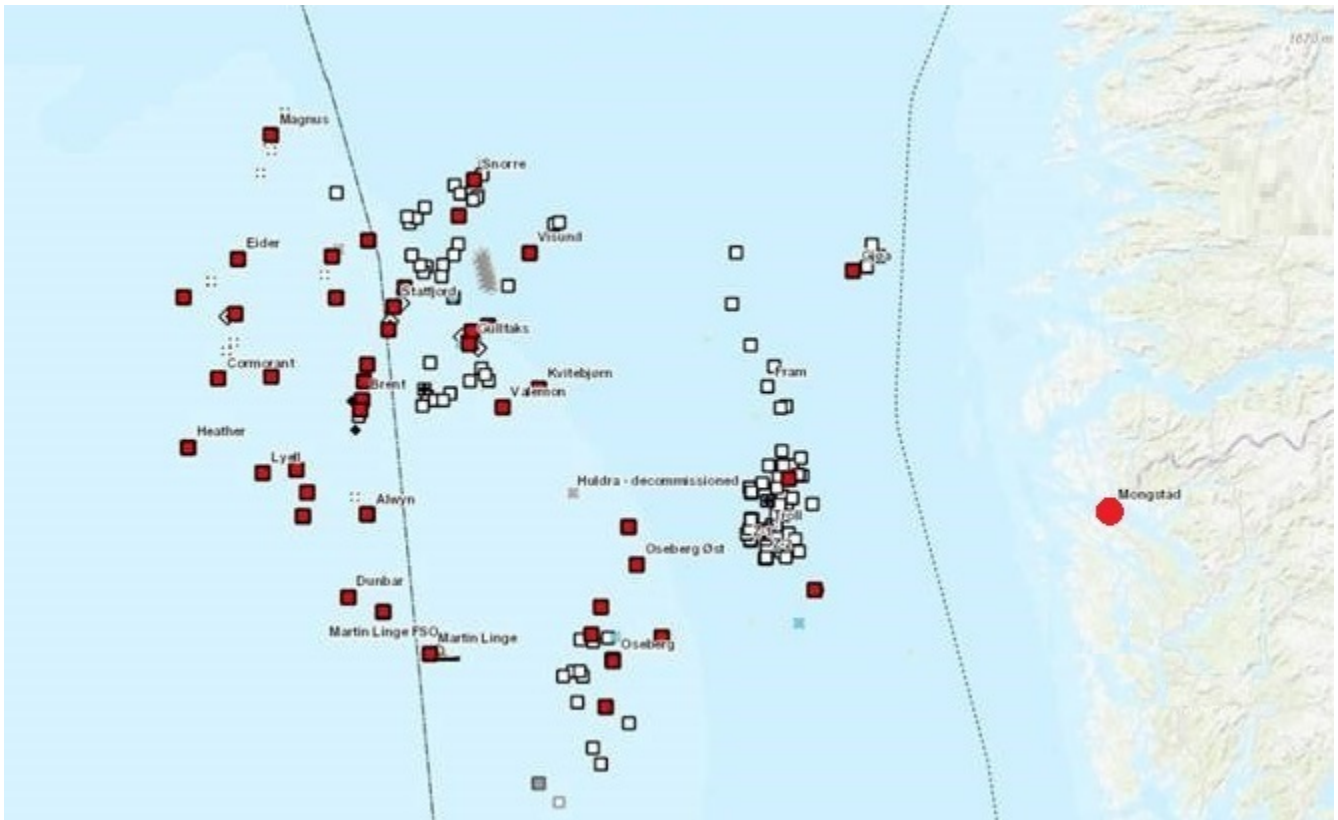


Fig. 1. Map showing the offshore installations locations on the Norwegian continental shelf and the supply base.

The unloading time an installation depends on the amount of cargo to be delivered. Some of the installations may be night-closed, and therefore the service can be only performed during the installation’s opening hours. Similarly, the supply base has opening hours during which the vessels may perform the loading operations. Each supply vessel is associated with a daily *charter cost* which is the major cost contributor, and has an economic sailing speed which is used for the computation of sailing times during the construction of the vessel schedule. The deck capacity of a vessel is limited and the total demand of the installations visited during a voyage performed by the vessel should not be exceeded. In addition, the vessels have different fuel consumption rates when sailing, servicing and waiting at an installation. The turnaround time at the base, which includes waiting and loading, is also vessel dependent. Each vessel has a set of possible departure times from the base during the day. The specificity of the loading operations at the base imposes restrictions such as a maximum number of vessel departures from the base during the day (base capacity constraint). The requested delivery lead time to the installations visited on a voyage limits the maximum voyage duration. In order to provide a continuous supply at an installation, the departures of vessels visiting this installation should be evenly spread throughout the planning period. A vessel schedule is performed repeatedly over several months or a season, called the *execution horizon*, as both the installations and the supply base require the predictability of vessels departures to the installations in order to manage a continuous flow of cargo along the supply chain. For planning purposes, the installation’s total weekly demand is assumed to be constant and is defined as an average over the execution horizon.

Fig. 2 provides an example of a weekly vessel schedule performed by a fleet of vessels (marked V1, V2, V3), each having a set of planned voyages to be performed within the week. Each voyage starts at the supply base according to the planned date and time, and has a set of installations (each marked with three letters) to be visited in a given sequence. Each voyage starts from vessel loading at the base (the turnaround time is marked as “B”), followed by the vessel departure. Due to the repetitive nature of the schedule, the set of days in the planning period is circular, meaning that a vessel may start its voyage at the end of the planning period (for example, the last voyage of vessel V1) and finish it at the beginning of the next period (on Monday of the following week). Here, for simplicity, each day is subdivided into three eight-hour time slots and for each time slot, line 2 gives the elapsed time in hours. Each voyage has a planned end time at the base, and during the schedule construction, so-called voyage non-overlap restrictions are imposed for each vessel to ensure that it cannot start a new voyage until it has finished the previous one.

	Monday			Tuesday			Wednesday			Thursday			Friday			Saturday			Sunday									
	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168							
	0-8	8-16	16-24	0-8	8-16	16-24	0-8	8-16	16-24	0-8	8-16	16-24	0-8	8-16	16-24	0-8	8-16	16-24	0-8	8-16	16-24							
V1	DSA			B	CPR	COI	WVE	TRB	TRC	SDO		B	SDO	COI	WVE	TRB	TRO	CPR		B		GFC	GFA	GFB	STC	STA	STB	DSA
V2		B	SDO	COI	WVE	CPR	TRO			B		GFC	GFA	GFB	STC	STA	STB	DSA			B		SDO	COI	WVE	TRC	TRO	CPR
V3				B			GFC	GFA	GFB	STC	STA	STB	DSA							B		CPR	WVE	GFA	GFB	GFC	COI	SDO

Fig. 2. Example of a vessel schedule.

Below we describe some important characteristics of a vessel schedule. We define a voyage TW as the length of time between the voyage start time and the start of loading operations for the next voyage. Fig. 3 depicts a vessel with two voyages and the beginning of the third one. We define slacks within a voyage (called intra-voyage slacks) and slacks between voyages (called inter-voyage slacks). The intra-voyage slack is defined by the time span between the service completion at an installation and the end of its TW. The inter-voyage slack is the time interval between the end of voyage (arrival at the base) and the end of its TW (beginning of loading operations for the next voyage).

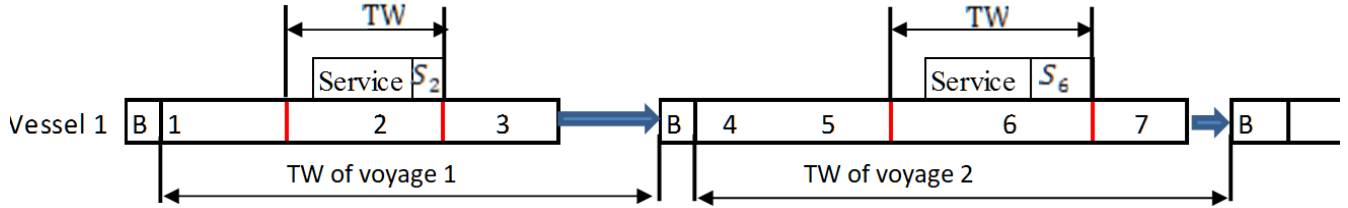


Fig. 3. Two voyages performed by vessel 1, showing the base (B) and the installations served. The TW of each voyage is shown at the bottom. The inter-voyage slack is represented by a thick arrow. The TWs of installations 2 and 6 are shown at the top. One can see that service time at these installations is less than the TW length and hence there is some intra-voyage slack (denoted by S_2 and S_6).

1.2 Demand and weather uncertainty

The demand at the installations varies from week to week and from departure to departure. The weather also varies over the schedule's execution horizon. Uncertain demand and weather conditions may render the execution of some of the scheduled vessel voyages impossible due to the violation of the vessel capacity and of the planned voyage duration. Cargo not delivered as planned may result in the postponement of the start of the planned activities at the installations and in the impossibility of carrying out production and exploration as planned. To guarantee that the departure of the cargo to the installations is performed according to the vessel schedule, the planners must perform daily modifications of the planned voyages and, if necessary, plan additional voyages using extra resources. It is therefore critical, when constructing a vessel schedule, to enforce measures in order to reduce the probability of schedule infeasibility and hence minimize the use of extra resources.

The demand fluctuations over time are relatively high for some installations (drilling rigs) due to the large volumes of cargo needed at different stages of the exploration and production activities. Moreover, the start and the duration of activities at the installations may deviate from the planned schedule. In addition, some ad hoc unplanned deliveries often take place. With such demand variability, there is always a risk that the total demand of all installations visited on a voyage will exceed the capacity of the vessel. The probability of capacity violation on a voyage depends on the level of actual demands relative to the planned demands at the installations visited on the voyage, and on the capacity of the vessel performing the voyage. In addition, the uncertain installation demands result in fluctuations of service times at the installations since service duration depends on the amount of delivered cargo. Surges in demand may therefore result in voyage TW violations.

Another major uncertain factor lies in the weather conditions which may seriously impact the sailing and service times and often lead to serious schedule disruptions. Weather conditions are usually described by factors such as wind speed, wave direction and wave height, where the latter is considered to be one of the most significant. It affects both the sailing speed and the service time at the installations, which may lead to a violation of the voyage TW. It may even happen that the waves are so high that the service at an installation is temporarily interrupted. A situation in which a vessel should wait for better weather conditions to be able to start or continue the service at an installation is called “Wait on Weather” (WOW) by the logistics planners. When the WOW conditions last for one or several days, the departures of vessels on voyages scheduled on these days are canceled until the weather improves. In this case the cargo of the canceled voyages must be delivered on the days following the end of the WOW period. The arrival time at an installation depends on the sequence of installations previously visited on the voyage and on the day and time of the voyage start. This makes the sailing and service times to be the time- and location-dependent uncertain parameters in the PSVPP.

In practice, a violation of a voyage TW may result from the increased service time caused by surges in demand, or by the increased sailing and service times due to the deterioration of weather conditions, or due to a combination of both factors. Such situations are highly likely when there are night-closed installations in the voyage. The late arrival of a vessel at a night-closed installation may result in a 12-hour delay of the service start since the vessel should wait for the opening of the installation TW on the next day. In case of the violation of a voyage TW or of the vessel capacity constraint, the schedule’s infeasibility is eliminated by applying *operational modifications* consisting of visit relocations between voyages. Logistics planners differentiate between planned voyages performed according to the schedule, and unplanned additional voyages performed either by a free charter vessel or by a hired spot vessel to ensure the required service level. The goal of

visit relocations is to reload part of the demand from the infeasible voyages to another planned or unplanned feasible voyage. There are several ways of relocating visits. The primary relocation option is to redistribute part of the demand of infeasible voyages to feasible planned voyages departing either on the same day or on the next day. Another option is to deliver part of the demand by an unplanned voyage performed by a charter vessel, provided that it is available and that this voyage does not overlap with the other planned voyages of this vessel. Otherwise, the excess demand may be delivered by a spot vessel hired on the short term. It worth noting that modifications of the planned voyages are performed so that their number and departure times remain unchanged. These relocation options may be applied separately or in the various combinations, depending on the situation. Voyage modifications are applied on a daily basis in case of infeasibility during the schedule execution, and generate additional costs, the largest being the hiring of spot vessels due to their very high rates.

The objective of the PSVPP with demand and weather uncertainty is to minimize the expected schedule cost computed over the execution horizon after the operational modifications have been implemented.

1.3 Positioning within the scientific literature

Our problem belongs to the general area of supply chain resilience (Hosseini et al., 2019), and to the family of Periodic Vehicle Routing Problems (PVRPs) (Francis et al., 2008). The literature on stochastic PVRPs is rather scarce. The algorithms developed for the stochastic PVRP are similar to those for the widely studied Stochastic Vehicle Routing Problems (SVRPs), see for example Gendreau et al. (2016). Most studies on SVRPs consider only one uncertain factor, such as stochastic demand, stochastic customers, stochastic travel time, or stochastic service time. The solution approaches to SVRPs are subdivided into two-stage stochastic programming with recourse, reoptimization, robust optimization, and chance-constrained programming. There exist only a few studies on to the PSVPP with uncertainty (Halvorsen-Weare and Fagerholt, 2011 and Norlund et al., 2015), and none has ever considered both demand and weather uncertainty.

1.3.1 Solution approaches to stochastic routing problems

Most algorithms for SVRPs use two-stage stochastic programming with recourse where, in the first stage, routes are constructed before the information on the stochastic parameters is revealed and, in the second stage, the first-stage solution is modified when information on uncertainty becomes available. This corrective action is called a recourse. The expected cost of recourse is computed assuming that stochastic parameters follow probability distributions yielding simple convolutions

(Laporte et al., 2002), although there exist some methods incorporating Monte Carlo sampling (Kleywegt et al., 2001; Kenyon and Morton, 2003; Linderoth et al., 2006 and Rei et al., 2010). The reoptimization paradigm implies recomputing the routing decisions while the information on uncertain factors is dynamically revealed during the route execution. Being a proactive approach, reoptimization has an advantage over a priori optimization in terms of efficiency, but this may come at the expense of added computational complexity (Gendreau et al., 2016). Robust optimization (Ben-Tal and Nemirovski, 1999; Bertsimas and Sim, 2004; Bertsimas et al., 2011) tries to optimize against the worst-case outcome defined by an uncertainty set. Robust optimization does not rely on probability distributions and does not use recourse actions as in the two-stage stochastic programming. Chance-constrained programming is similar to robust optimization but relies on the probability distribution of uncertain parameters. It ensures that the probability of satisfying constraints with uncertain parameters lies above a certain *reliability level* (Birge and Louveaux, 1997). Such an approach results in a more consistent service level and in less need for complex recourse actions to be taken. In practice, however, the problem of defining general distributions is computationally challenging. Analytical approximations (Nemirovski and Shapiro, 2006 and Geletu et al., 2014) and Monte Carlo sampling (Calafiore and Campi, 2005) are often used to approximate general distributions.

A direct application of any of the above mentioned approaches to the stochastic PSVPP is likely to be inefficient. The most popular two-stage optimization with recourse algorithms applied to the VRP perform relatively well assuming that the probability distributions yield simple convolutions, which is rarely the case. Moreover, usually only one classical recourse action is applied, which does not significantly increase the computational complexity of the second-stage problem. In our case, however, when the demands at installations follow different probability distributions, it is challenging to compute convolutions. Approximating the cost of recourse with several available recourse actions involves solving a computationally hard combinatorial problem in which all options of recourse actions implementation are assessed. Furthermore, the service and sailing times cannot be modelled analytically (being time- and location-dependent) and hence simulation modeling is the only practical way of approximating sailing and service times. Pure applications of robust and chance-constrained optimization are not suitable to our problem because these methodologies do not apply any recourse actions and may yield rather costly solutions.

1.3.2 Periodic Supply Vessel Planning Problem with uncertainty

Kisialiou et al. (2019) recently developed a heuristic for the PSVPP under uncertain demand which combines chance-constrained optimization to control reliability level with respect to the vessel capacity, and discrete-event simulation to compute average expected cost of solution

after modifications. In this heuristic, multiple vessel schedules with different reliability levels are generated by means of an adaptive large neighbourhood search (ALNS) metaheuristic with incorporated chance constraints. Discrete-event simulation is subsequently used to assess the average expected cost of schedule performance over the execution horizon after the application of modifications. The combination of optimization and discrete-event simulation also has been used in other stochastic logistics contexts, such as the Vehicle Routing Problem (Sörensen and Sevaux, 2009) and the Orienteering Problem (Bian and Liu, 2018), but in different ways.

There exist several studies dedicated to the robust PSVPP under weather uncertainty. Halvorsen-Weare and Fagerholt (2011) have introduced a two-phase algorithm based on the set partitioning formulation of the PSVPP, where in the voyage generation phase a robustness measure is computed for each feasible voyage by simulation. Norlund et al. (2015) used a different two-phase methodology in which the set partitioning model is solved for a set of voyages selected after simulation according to a given robustness requirement. Both methods enable the generation of solutions for small- and medium-size instances. Kisialiou et al. (2018b) developed an ALNS metaheuristic allowing for the construction of solutions for large real-life instances incorporating dynamic voyage slacks and thus ensuring some protection against voyage duration infeasibility. In this approach, the slack duration is controlled by a robustness parameter, and discrete-event simulation is used to assess the service level of schedules generated under different robustness requirements.

1.4 Scientific contribution of the paper

Our aim is to develop a methodology for the supply vessel routing and scheduling problem with uncertain demands, travel and service times. We modify the ALNS metaheuristic for the deterministic PSVPP presented in Kisialiou et al. (2018a) to control voyage duration infeasibility under uncertain weather conditions by incorporating dynamic inter- and intra-voyage slacks as proposed in Kisialiou et al. (2018b). To control the reliability level under uncertain demand, we apply chance-constrained programming to ensure that the probability of satisfying the capacity constraint is above a certain level, as proposed in Kisialiou et al. (2019), in order to calculate the expected cost of the schedule under uncertain demand and weather conditions. We develop a simulation model and hence assess the schedule performance over its execution horizon. The simulation model incorporates several visit relocation options applied in different combinations at the operational level, thus ensuring the schedule feasibility in case of violation of a voyage capacity or duration constraints during the simulation. We introduce a post-optimization procedure incorporating a score function that accounts for the trade-off between the schedule cost and its reliability. The score function allows selecting from the solutions with slightly different costs a

solution with the higher potential of yielding the lowest expected cost. The ALNS metaheuristic and the simulation model are integrated into a single optimization-simulation decision support tool enabling the construction of schedules of least expected cost for large-size instances under demand and weather uncertainty. We demonstrate the performance of the developed tool on several real instances provided by Equinor.

1.5 Organization of the paper

The remainder of this paper is organized as follows. In Section 2 we present a metaheuristic containing mechanisms to control the reliability level against demand and weather uncertainty. This algorithm is followed by the description of the discrete-event simulation model for the computation of the expected cost after modifications. In Section 3 we describe the test instances with input data, parameter tuning, and results of computational experiments. Conclusions are presented in Section 4.

2 Optimization-simulation algorithm

We present a new methodology for the periodic supply vessel planning with combined uncertain demand and weather conditions. We first describe the mechanisms developed to ensure a preset reliability level in the vessel schedules followed by a metaheuristic that incorporates these mechanisms. We introduce a special score function accounting for the schedule’s planned cost and reliability level, and used in the post-optimization procedure incorporated in the ALNS metaheuristic. We further develop a discrete-event simulation model to assess the schedule performance over the execution horizon after application of operational modifications, and to compute the schedule’s average expected total cost. Finally, we present an iterative optimization-simulation algorithm which generates solutions with given reliability levels and computes their expected cost.

2.1 Reliability level control

We differentiate between the voyage capacity feasibility and its TW feasibility. The capacity feasibility of a voyage in a context of uncertain demand depends on the actual demand realization and on the vessel capacity. In fact, one knows before the voyage starts whether vessel capacity will be violated or not. Regarding the uncertain weather conditions, the voyage TW feasibility is not known in advance. For this reason, to ensure a certain reliability level of the schedule with two uncertain variables, we need to account for them during the schedule construction using two

different approaches. In what follows we describe the mechanisms applied to control reliability under uncertain demand and weather conditions during the schedule construction.

2.1.1 Reliability control against uncertain demand

To ensure a certain reliability level against the uncertain demand in the vessel schedule, we modify the capacity constraint of voyage v into its chance-constrained counterpart:

$$P(D_v \leq C_v) \geq p, \quad (1)$$

which sets a lower limit p on the probability that the voyage total demand D_v will not exceed the vessel capacity C_v . In reality, the demands at the installations follow distributions belonging to the different families (see Kisialiou et al., 2019). To compute the convolution of installation demands on a voyage and to define the demand cumulative distribution function (CDF) F_v , we use the fast Fourier transform algorithm and the property of the convolution theorem (Broughton and Bryan, 2008). During the schedule construction we impose the capacity chance constraints for all voyages:

$$F_v(C_v) \geq p, v \in V, \quad (2)$$

where V is the set of all voyages in a schedule. The parameter p is further referred to as the *demand reliability* parameter.

2.1.2 Reliability control against uncertain weather conditions

Imposing chance constraints to the voyage TWs is impossible since the travel and service times are time- and location-dependent uncertain parameters and the duration of the voyage TW is a variable. As shown by Kisialiou et al. (2018b), the reliability of a voyage relative to uncertain weather conditions depends on inter- and intra-voyage slacks which serve as time buffers in case of delays accumulated along the voyage. Longer slacks ensure a higher reliability level. In this study, in order to control the intra- and inter-voyage slack durations during the voyage construction, we adopt a robust approach and introduce a reliability parameter α enforcing the condition

$$s_i/t_i \geq \alpha, i \in N_v^{TW} \quad (3)$$

for the set N_v^{TW} of all installations with TWs visited on voyage v and the base (see Kisialiou et al., 2018b for more details). We use the notation s_i for the intra-voyage slack at installation i with a TW, and for the inter-voyage slack between voyage v and the next voyage of the same vessel. The

parameter t_i is calculated for the intra-voyage slack at installation i with TW as the deterministic travel time between the departure time from the previous installation with a TW visited on the voyage (or from the departure time from the base) and the end of service at installation i . For the inter-voyage slack, t_i is calculated as the travel time between the end of service at the last installation i with TW visited on the voyage, and the arrival time at the base. For a voyage without installations with TWs, condition (3) would take the form $s/t \geq \alpha$, where s is the inter-voyage slack and t is the deterministic voyage duration. The parameter α is user-defined and guarantees a certain minimal slack duration. The motivation behind this approach is that longer voyages require larger inter- and intra-voyage slacks. The duration of each slack takes into account the duration of the slacks at the installations with TW already visited on the voyage. We further refer to α as the *weather reliability* parameter. The approach with slacks control implements in some way the idea of robust optimization.

2.2 Metaheuristic algorithm

Below we describe an ALNS metaheuristic algorithm for the construction of vessel schedules with certain values of demand and weather reliability parameters. The algorithm is an extension of the algorithm developed in Kisialiou et al. (2018a) for the deterministic version of the problem.

2.2.1 Construction of robust vessel schedules

Algorithm 1 provides a brief pseudo-code of the ALNS metaheuristic. This algorithm is applied for n restarts of η iterations each. The values of the demand and weather reliability parameters p and α are used as inputs. At each restart an initial feasible solution z^0 satisfying conditions (2) and (3) is randomly generated. Solution z is defined as the current solution and, at the beginning of each restart, is equal to z^0 . The best known solution found during the search is defined as z^* . We further define solution ω^* yielding the highest value Q^* of the score function accounting for the trade-off between schedule cost and its reliability level (see Section 2.2.2 for the score function description). Further, an attempt is made to improve solution z^0 within η iterations. At the start of each iteration the current solution z is partially destroyed by removing q visits from it. The number of visits to be removed is randomly chosen within a certain interval. The removal of visits is performed by a destroy operator which is selected from a set ψ according to a discrete probability distribution based on its past performance (at the beginning of a restart the probabilities are equal). Three operators are used for partial destroying the solution: *Shaw removal* (see Shaw, 1997), *worst removal* and *random voyage removal*. Each of the operators removes visits from solution z according to its own logic. The visits removed by a destroy operator are inserted

in a pool S and the destroyed solution is defined as z'' . An attempt is then made to repair solution z'' by reinserting visits from pool S back into z'' . Insertions are made by a repair operator selected from a set φ . Three repair operators are used: *2-regret insertion*, *3-regret insertion* and *deep greedy insertion*. In the case of a successful reinsertion of all visits from S back into z'' , a new solution z' is obtained. By means of application of the destroy and repair operators a move from the space of neighbourhood solutions $N(z)$ to the neighbourhood space $N(z')$ of the solution z' is performed. Further, a set of improvement operators is applied to the solution z' in an attempt to find a better solution in its neighbourhood. These operators are applied in a given sequence while the cost of the solution decreases. The primary aim of the improvement operators is either to reduce the fleet size or to find a less costly fleet composition (for details see Kisialiou et al., 2018a). The selection probability of each destroy and repair operator is recalculated after a certain number of iterations (user-defined), based on its performance.

To ensure a certain reliability level defined by parameters p and α , conditions (2) and (3) are checked (lines 4, 8 and 11) whenever a voyage is constructed or modified (a visit is added or reassigned to another vessel). Such checks are implemented for each operator. We denote by $c(z)$ the cost of solution z . After the application of the improvement operators, the cost $c(z')$ is compared with that of the best solution $c(z^*)$ and with that of the current solution $c(z)$. We denote by Ω the set of solutions whose cost is higher than $c(z^*)$ by at most Δ (a user-defined threshold value). If the best solution is updated (line 14), the set Ω is revised (line 15) to satisfy the condition $c(\omega) \leq c(z^*) + \Delta$, $\omega \in \Omega$. If the solution has not improved, a simulated annealing acceptance criterion is applied (line 18) which accepts solution z' with a certain probability. After the last iteration of the last restart, the algorithm calculates the score Q_ω (see Section 2.2.2) for each solution in Ω . Finally, the algorithm returns a solution ω^* yielding the highest score Q^* . For more details on the ALNS heuristic for the supply vessel problem, see Kisialiou et al. (2018a).

Algorithm 1 ALNS(α, p) for the PSVPP with uncertain demand and weather conditions

```
1: Set the cost of the best known solution  $c(z^*) = \infty$ ;
2: Set the value of the highest score  $Q^* = 0$ ;
3: for  $n$  restarts do
4:   Construct initial solution  $z^0$  satisfying (2) and (3);
5:    $z^* \leftarrow z^0$ ;  $z \leftarrow z^0$ ;  $c^* \leftarrow c(z^*)$ ;
6:   for  $\eta$  iterations do
7:      $z'' \leftarrow \psi(z, q, S)$ , remove  $q$  visits;
8:      $z' \leftarrow \varphi(z'', q, S)$ , insert  $q$  visits while satisfying (2) and (3);
9:     if  $S = \emptyset$  and  $z'$  is feasible then
10:      while  $z'$  improves do
11:        Run the set of improvement operators while satisfying (2) and (3);
12:      end while
13:      if  $c(z') \leq c(z^*)$  then
14:         $z^* \leftarrow z'$ ;  $z \leftarrow z'$ ;
15:        Revise candidate solutions ( $\Omega$ );
16:      else if  $c(z') \leq c(z)$  then
17:         $z \leftarrow z'$ ;
18:      else if  $\text{accept}(z, z')$  then
19:         $z \leftarrow z'$ ;
20:      end if
21:      if  $c(z') \leq c(z^*) + \Delta$  then
22:         $\Omega \leftarrow \Omega \cup z'$ ;
23:      end if
24:    end if
25:  end for
26: end for
27: for each schedule  $\omega$  in  $\Omega$  do
28:    $Q_\omega \leftarrow \text{calculate score}(\omega)$ ;
29:   if  $Q^* \leq Q_\omega$  then
30:      $Q^* \leftarrow Q_\omega$ ;  $\omega^* \leftarrow \omega$ ;
31:   end if
32: end for
33: return  $\omega^*$ ;
```

2.2.2 Score function

In this section we present the motivation for the introduction of the score function and the logic behind its parameters. Conditions (2) and (3) account for the control of reliability (or probability of feasibility) for single voyages. We define the schedule probability of feasibility as the average probability of both voyage TW and the capacity-feasibility of it voyages. To compute the probability of voyage TW-feasibility we simulate the voyage under uncertain demand and weather

conditions since the weather impact, the sailing time, and both the weather and the demand affect the service time.

Preliminary experiments with the optimization-simulation heuristic have shown that the schedules generated with the same given levels of reliability parameters α and p , may have a small cost difference (below 0.1%) while having a relatively large difference in probability of schedule feasibility (4% to 6%). We have discovered that schedules with approximately the same and high probability of feasibility have a higher likelihood of yielding the lowest expected cost. When the variability is high, the bottlenecks (in voyage capacity and TW) in the schedule make it more difficult to restore feasibility. For this reason we propose a post-optimization procedure aimed at selecting from the schedules with small costs deviations from $c(z^*)$ a schedule having a higher probability of feasibility.

We calculate the score based on the schedule cost, the average probability of capacity-feasibility and TW-feasibility, and the mean absolute deviation (MAD) of voyages feasibility probabilities (both capacity and TW) from the average feasibility probabilities. The score function $Q(\omega)$ of schedule ω is defined as

$$Q(\omega) = CF \cdot S^{CF} + TF \cdot S^{TF} - \overline{CF} \cdot S^{\overline{CF}} - \overline{TF} \cdot S^{\overline{TF}} - c(\omega) \cdot S^{c(\omega)}, \quad (4)$$

where

CF is the average schedule probability of capacity-feasibility, computed as $\sum_{v \in V} CF_v / |V|$;

TF is the average schedule probability of TW-feasibility, computed as $\sum_{v \in V} TF_v / |V|$;

CF_v is the probability of voyage $v \in V$ capacity-feasibility;

TF_v is the probability of voyage $v \in V$ TW-feasibility;

\overline{CF} is the MAD of CF_v from CF , computed as $\sum_{v \in V} (|CF_v - CF|) / |V|$;

\overline{TF} is the MAD of TF_v from TF , computed as $\sum_{v \in V} (|TF_v - TF|) / |V|$;

$c(\omega)$ is the cost of solution ω ;

S^{CF} , $S^{\overline{CF}}$, S^{TF} , $S^{\overline{TF}}$ and $S^{c(\omega)}$ - the weights for CF , \overline{CF} , TF , \overline{TF} , $c(\omega)$ respectively. The values of weights are defined experimentally. For the computation of CF_v values we use the Fourier transform algorithm (Bian and Liu, 2018), and TF_v values are found after simulation.

2.3 Simulation modelling

We now describe how we generate the demand and weather scenarios, and explain how we use discrete-event simulation (1) to calculate the voyage TW-feasibility probability used in the post-optimization procedure in the ALNS heuristic, and (2) to compute the expected cost of the

schedule after implementation of the modifications aimed at eliminating the infeasibility. The simulation is performed for N consecutive schedule planning periods (one week in our case) which constitute the schedule execution horizon. We consider the following types of discrete events while simulating a voyage and a vessel schedule over the execution horizon: changes of weather conditions (within a certain time interval), the vessel departure from the base, the vessel arrival at an installation, the start of service at an installation, the vessel departure from an installation, and the vessel arrival at the base.

2.3.1 Data modelling

The weather conditions are summarized by three estimates: significant wave height, wind speed, and wave direction. The modeling of weather data is based on the approach used by Maisiuk and Gribkovskaia (2014) and Kisialiou et al. (2018b). Prior to the start of the voyage or of the schedule simulation, the generated weather data are analyzed for WOW situations. We assume that no service can take place in WOW situations when servicing an installation; the vessel should then wait until better weather conditions materialize. If the total number of WOW hours exceeds the historical maximum, the weather data at this location for this period are regenerated. We use Monte Carlo simulation to generate scenarios for the demands of all installations visited in a voyage.

2.3.2 Voyage simulation

To compute the probability of voyage TW-feasibility needed for calculation of the schedule score value, we run simulations for all voyages in the schedule over the schedule execution horizon. The simulation is performed for a certain number of replications, in each of which the actual voyage duration for each planning period is defined, and the voyage TW-feasibility is checked. The probability of voyage TW-feasibility is computed as the ratio of the number of simulation replications for which the voyage was feasible to the total number of replications.

A voyage may be rerouted, which is performed by enumerating the sequences of visits using a recursive branch-and-bound procedure (as in Kisialiou et al., 2018a) and calculating the voyage duration for the revealed demand and weather scenarios. The service times at the installations are adjusted with respect to the revealed demands and weather conditions, and the sailing times are calculated based on the speed loss for every three-hour interval. The voyage with the sequence of visits yielding the shortest voyage duration is then selected. In fact, rerouting may reduce the probability of voyage infeasibility after the demand and weather conditions become known.

2.3.3 Schedule simulation

To compute the expected cost of a schedule, we simulate its performance over its execution horizon and apply modifications in case of infeasibility. We recall that modifications are performed daily so that the number of the planned voyages for each day and their planned departure times are not changed (to keep the supply predictability to the installations), although the planned voyages may be modified and new voyages may appear. We define two types of schedule cost: the planned schedule cost (the cost of the schedule generated by the ALNS heuristic), and the expected schedule cost calculated as the average cost over all simulation replications. Further, we distinguish between the planned and the simulated sailing times, the service time and the demand of the installations. The planned sailing time corresponds to the sailing time under economic speed. The planned service time and the planned demand are taken as an average according to the historical data for each installation. It is assumed that the installations' demands become known one day before the voyage departure, and the weather conditions are known for the whole voyage duration.

Algorithm 2 presents the pseudo-code describing the general logic of the schedule simulation model. As an input, the algorithm uses the modeled demand and the simulated weather data. The simulation of the schedule is performed for ρ replications. We denote by S a pool of visits removed from the infeasible voyages departing on the consecutive days which were impossible to reinsert due to bad weather and limited base capacity. At each replication, the schedule $z(\alpha, p)$ generated by the ALNS heuristic for given values of α and p is simulated for N consecutive planning periods. For each day $d = 1, \dots, D$ of the planning period $n = 1, \dots, N$, the demand and weather data are revealed for the voyages departing on days d and $(d+1) \bmod D$. The weather scenario for a voyage is taken from the set of pregenerated scenarios. The demand for each visit of an installation on the voyage is sampled from the corresponding probability distribution. For each planning period n , a copy $z_n^{mdf}(\alpha, p)$ of schedule $z(\alpha, p)$ is created where the modification of voyages and creation of new ones will take place. At the beginning of the execution horizon ($n = 1$ and $d = 1$), after the demand and weather conditions have been revealed, the service times for the voyages departing on days 1 and 2 are adjusted and the voyages are possibly rerouted. For the other days of the execution horizon, the adjustment of the service times and rerouting is done only for voyages departing on day $(d+1) \bmod D$. Since each planning period is circular, the voyages departing on day 1 of period n may be modified while ensuring the feasibility of voyages departing on day D of the previous period $n-1$. In this case the voyages of the schedule $z_n^{mdf}(\alpha, p)$ departing on day 1 of period n are updated according to the modifications performed in period $n-1$.

The set of all voyages departing on day d that are capacity- or TW-infeasible is defined as V^d . Further, an attempt is made to eliminate the infeasibility of voyages in the set V^d by relocating visits from these voyages using one of the following three options: relocation into the feasible

planned voyages, relocation into unplanned new voyages performed by either an available charter vessel or by a hired spot vessel (see Kisialiou et al., 2019). An attempt is made first to relocate all visits from pool S , and only then from infeasible voyages in V^d . The visit relocation procedure is provided in Algorithm 3. The visits are relocated into feasible planned and unplanned voyages departing on days d and $(d + 1) \bmod D$, further referred to as target voyages. The non-relocated visits can be stored in pool S for at most three days. Rerouting and visits relocation, applied for the simulation of each period $n = 1, \dots, N$, yield the schedule $z_n^{mdf}(\alpha, p)$ with the modified voyages having cost c_n . After each replication $i = 1, \dots, \rho$, the average cost c_i^{av} of schedules $z_n^{mdf}(\alpha, p)$, $n = 1, \dots, N$ is computed. After the last replication the algorithm returns the expected cost $c^{\text{exp}}(\alpha, p)$ of solution $z(\alpha, p)$.

Algorithm 2 Schedule simulation

```
1: Simulate weather data for the schedule execution horizon;
2: Model demand for the schedule execution horizon;
3: for  $i = 1$  to  $\rho$  do
4:   Pool  $S = \emptyset$ ;
5:   for  $n = 1$  to  $N$  do
6:     Schedule  $z(\alpha, p)$ ;
7:     Let  $z_n^{mdf}(\alpha, p)$  be the modified schedule  $z(\alpha, p)$ ;
8:     for  $d = 1$  to  $D$  do
9:       if  $n = 1$  and  $d = 1$  then
10:        Simulate demand for each visit on voyages started on days  $d$  and  $(d + 1) \bmod D$ 
        and adjust service times;
11:         $z_n^{mdf}(\alpha, p) \leftarrow$  Reroute voyages on days  $d$  and  $(d + 1) \bmod D$  based on adjusted
        service times and weather scenario;
12:      else
13:        if  $n > 1$  and  $d = 1$  then
14:          Update voyages of day  $d$  in  $z_n^{mdf}(\alpha, p)$  according to modifications done on day
           $D$  in period  $n - 1$ ;
15:        end if
16:        Simulate demand for each visit on voyages started on day  $(d + 1) \bmod D$  and
        adjust service times;
17:         $z_n^{mdf}(\alpha, p) \leftarrow$  Reroute voyages on day  $(d + 1) \bmod D$  based on adjusted service
        times and weather scenario;
18:      end if
19:       $V^d \leftarrow$  Set of infeasible voyages departing on day  $d$ ;
20:      if  $V^d \neq \emptyset$  then
21:         $z_n^{mdf}(\alpha, p) \leftarrow$  perform visits relocation  $(V^d, S)$  (Algorithm 3);
22:      end if
23:    end for
24:    Calculate cost  $c_n$  of schedule  $z_n^{mdf}(\alpha, p)$ ;
25:  end for
26:  Calculate  $c_i^{av} = \sum_{n=1}^N c_n / N$ ;
27: end for
28: return  $c^{exp}(\alpha, p) = \sum_{i=1}^{\rho} c_i^{av} / \rho$ 
```

2.3.4 Visit relocations

For visits relocations we modified the algorithm of Kisialiou et al. (2019) to account for weather uncertainty (see Algorithm 3). The main idea of the algorithm is to relocate visits from infeasible voyages departing on day d to other feasible voyages of day d and $(d + 1) \bmod D$. First, for all infeasible voyages of day d included in set V^d , the set U of all possible combinations of visits for removal is generated. Combinations are generated so that for each infeasible voyage the minimal

possible number of visits are removed in order to eliminate the infeasibility. The set of feasible planned voyages departing on days d and $(d + 1) \bmod D$ is defined as V^f . Further, the algorithm uses the set V^{ch} of voyages of available charter vessels able to depart on day d . The set of voyages of spot vessels available for the departure on day d is denoted by V^{sp} . The sets V^f , V^{ch} and V^{sp} are included in the set V^{target} of target voyages.

An attempt is first made to insert *all* visits from pool S in voyages from set V^{target} . The set of all insertion options of visits from S into voyages of V^{target} is defined as O^S . Visit insertion options are generated in all possible combinations so that the total number of planned and unplanned voyages considered for departure on day d do not violate the base capacity. For each insertion option $o \in O^S$ we define the insertion cost $c(o)$ and the total overlap $\lambda(o)$ of voyages (measured in hours) in case of voyage TW-violation. The insertion option yielding the least overlap λ^{min} among all insertion options is defined as o^{min} . If a feasible insertion option of all visits from S (i.e., $\lambda^{\text{min}} = 0$) is identified, the insertion option o^* yielding the lowest insertion cost $c(o^*)$ is implemented. Otherwise, the insertion option o^{min} yielding the minimal overlap λ^{min} is implemented. In this case, to ensure the feasibility of voyages in V^{target} after the visit relocation, the procedure aims at eliminating overlaps is applied to each voyage in $V^{\text{inf}} \subseteq V^{\text{target}}$ containing an overlap. The aim of the procedure is to iteratively remove from the voyage those visits yielding the largest voyage duration decrease while an overlap remains (the procedure is based on the worst removal operator of Ropke and Pisinger, 2006). The removed visits are stored in the pool S . The procedure first removes the planned visits and, if the overlap has not yet been eliminated, it continues to remove from S visits that were currently inserted. This is done to reduce the delay of visits stored in S , so that the unperformed visits from the voyages departed on the previous days will be included in the voyages departing on day d , while some visits from the voyages planned for departure on day d may be performed later.

An attempt is further made to relocate visits from the planned infeasible voyages of set V^d departing on day d into voyages of set V^{target} according to one of the visit removal combinations in U (Algorithm 4). For each combination of visits identified for removal $u \in U$, the set of all possible removal-insertion options into voyages of V^{target} is defined as O^u . For each removal-insertion option $o^u \in O^u$, each voyage in V^{target} is checked for feasibility, and the inserted visits causing infeasibility are gradually removed (one at a time, in non-decreasing order of their demand) until the voyage becomes feasible. The total number of such removed visits for option o^u is defined as $q(o^u)$. The cost of the removal-insertion option o^u is denoted as $c(o^u)$. The least number of non-inserted visits for all removal-insertion options in O^u , $u \in U$, is defined as q^* . The least cost among the costs of all removal-insertion options with the least number of non-inserted visits q^* is defined as c^* . Thus, by enumerating of all removal-insertion options $o^u \in O^u$ for all removal combinations $u \in U$, the

removal-insertion option o^{u*} yielding the least number of non-inserted visits q^* and the least cost c^* is identified. Finally, the visits from infeasible voyages in V^d are relocated according to the removal-insertion option o^{u*} , and the non-inserted visits (if any) are inserted in pool S .

Algorithm 3 Visits relocation into voyages departing on day d

- 1: $V^f \leftarrow$ Find feasible planned voyages departing on day d and $(d + 1) \bmod D$;
 - 2: $V^{ch} \leftarrow$ Create empty voyages of available charter vessels;
 - 3: $V^{sp} \leftarrow$ Create empty voyages of available spot vessels;
 - 4: Define set of target voyages $V^{target} \leftarrow V^f \cup V^{ch} \cup V^{sp}$;
 - 5: $O^S \leftarrow$ Generate set of all possible insertion options of visits from S into voyages in V^{target} ;
 - 6: o^* - the least cost feasible insertion option of all visits from S ;
 - 7: $c(o^*) = \infty$ - cost of insertion option o^* ;
 - 8: **if** $O^S \neq \emptyset$ **then**
 - 9: **for** each $o \in O^S$ **do**
 - 10: Try to relocate visits according to insertion option o into voyages in V^{target} ;
 - 11: Calculate total voyage overlap $\lambda(o)$ for insertion option o ;
 - 12: **if** $c(o) < c(o^*)$ **then**
 - 13: $o^* \leftarrow o$;
 - 14: $c(o^*) \leftarrow c(o)$;
 - 15: **end if**
 - 16: **end for**
 - 17: o^{min} - insertion option with minimal overlap $\lambda^{min} = \min_o \lambda(o)$;
 - 18: **if** $\lambda^{min} = 0$ **then**
 - 19: Implement visits relocation according to insertion option o^* ;
 - 20: **else**
 - 21: Implement visits relocation according to insertion option o^{min} ;
 - 22: $V^{inf} \leftarrow$ set of voyages in V^{target} with overlap;
 - 23: **for** each $v \in V^{inf}$ **do**
 - 24: $S \leftarrow \text{eliminate overlap}(v)$;
 - 25: **end for**
 - 26: **end if**
 - 27: **end if**
 - 28: Perform visits relocation from planned infeasible voyages (V^d, S) (Algorithm 4);
-

2.3.5 Algorithm for the PSVPP planning under uncertain demand and weather conditions

Algorithm 5 describes our optimization-simulation algorithm for the construction of PSVPP schedules with different levels (α, p) of reliability and enabling the construction of a schedule of a minimal expected cost $c^{\text{exp}}(\alpha, p)$. The algorithm takes as an input the sets P and A of the values of demand and weather reliability parameters, the parameter ρ defining the number

Algorithm 4 Visits relocation from planned infeasible voyages departing on day d

- 1: Generate set U of all possible visit removal combinations u from voyages in set V^d ;
 - 2: $q^* = \infty$ - the least number of non-inserted visits among all removal-insertion relocations;
 - 3: $c^* = \infty$ - the least cost for the removal-insertion options with the least number of non-inserted visits q^* ;
 - 4: o^{u^*} - removal-insertion option yielding the least number of non-inserted visits q^* and the least cost of insertion c^* ;
 - 5: **for** each $u \in U$ **do**
 - 6: Generate set O^u of removal-insertion options o^u for visit removal combination u ;
 - 7: **for** each $o^u \in O^u$ **do**
 - 8: $c(o^u)$ - cost of removal-insertion option o^u ;
 - 9: $q(o^u)$ - number of non-inserted visits o^u ;
 - 10: **if** $q(o^u) < q^*$ or $(q(o^u) = q^*$ and $c(o^u) < c^*)$ **then**
 - 11: $o^{u^*} \leftarrow o^u$;
 - 12: $q^* \leftarrow q(o^u)$;
 - 13: $c^* \leftarrow c(o^u)$;
 - 14: **end if**
 - 15: **end for**
 - 16: **end for**
 - 17: Relocate visits from voyages in set V^d according to removal-insertion option o^{u^*} ;
 - 18: $S \leftarrow$ non-inserted visits for option o^{u^*} ;
-

of simulation replications, and the parameter N defining the length of the simulation horizon (measured in the number of planning periods). The ALNS heuristic constructs solutions $z(\alpha, p)$ for all combinations of α and p values contained in the sets A and P . All such solutions are saved in a list $R = \{z(\alpha, p) : \alpha \in A, p \in P\}$. After the schedule simulation, the expected schedule cost $c^{\text{exp}}(\alpha, p)$ is saved in the list of expected costs $C = \{c^{\text{exp}}(\alpha, p) : \alpha \in A, p \in P\}$. The algorithm returns the lists R and C .

Algorithm 5 Optimization-simulation algorithm

- 1: Define sets A and P of α and p values;
 - 2: ρ - the number of simulation replications;
 - 3: N - the number of schedule executions during the simulation horizon;
 - 4: **for** each $p \in P$ **do**
 - 5: **for** each $\alpha \in A$ **do**
 - 6: $z(\alpha, p) \leftarrow$ ALNS(α, p);
 - 7: $R \leftarrow z(\alpha, p)$;
 - 8: Simulate schedule $(z(\alpha, p), \rho, N)$;
 - 9: Save $C \leftarrow c^{\text{exp}}(\alpha, p)$;
 - 10: **end for**
 - 11: **end for**
 - 12: **return** R and C ;
-

3 Computational experiments

Here we describe the input data to the algorithm, the parameter tuning and the computational experiments performed to analyze the behavior of our optimization-simulation algorithm. Both the ALNS metaheuristic and the simulation model were coded in the C# programming language and integrated in a single application. The experiments were conducted on a computer with 16 GB RAM, Pentium 6 core processor of 3.2 GHz and 3GB graphic processor GeForce GTX 1060, under the Windows operating system.

3.1 Input data and parameters

The input data are subdivided into two main parts: the data for the ALNS metaheuristic and the data for the simulation model. The instances of the PSVPP used for the experiments were provided by Equinor and contain data on the installations, the fleets of vessels and the supply base. For the experiments we used two instances with 14 and 26 installations and up to 10 vessels, including three spot vessels. The schedule is constructed for a one-week planning period. For each installation, we defined the demand CDF based on the historical data provided by Equinor for the period corresponding to the simulation execution horizon (winter 2016-2017). For the installations with insufficient demand observations we assumed that the demands follow a PERT probability distribution.

The simulation of weather data is done as in Maisiuk and Gribkovskaia (2014) and Kisialiou et al. (2018b) for the PSVPP with uncertain weather conditions. It is based on a time series analysis of the data provided by the Norwegian Meteorological Institute (MET). The weather data include the following three sea state estimates: significant wave height, wave direction (measured in degrees), and wind speed. The modelling of the weather data is based on the generation of time-series for the three estimates using a bootstrapping technique based on the data for the three estimates. For more details on weather modelling, calculation of service and sailing durations (based on simulated weather estimates), see Maisiuk and Gribkovskaia (2014) and Kisialiou et al. (2018b). The main input parameters to the optimization-simulation algorithm are summarized in Table 1.

Table 1. The main input parameters

Parameter	Description	Value
n	Number of restarts	3
η	Number of iterations	800
p	Set P of p values	[0.1, 0.2, ..., 0.9]
α	Set A of α values	[0.0, 0.05, ..., 0.25]
ρ	Number of simulation replications	1000
N	Number of schedule executions within the simulation horizon	12

3.2 Training instances and parameter tuning

We tuned the values of the score function weight using 10 instances of different sizes with nine, 10, 12, 14, 16, 18, 20, 22, 24 and 26 installations among those provided by Equinor. To tune the parameters we followed the procedure described by Ropke and Pisinger (2006) where initially the values of parameters are defined through an ad hoc trial-and-error phase and then allowing one parameter to take a number of values while keeping the rest fixed. The algorithm was run 15 times for each instance to compute the average minimal expected cost. The list of parameters and their tuned values are provided in Table 2.

Table 2. Parameters tuning results

Parameter	Description	Experimental value
S^{CF}	Weight for the average schedule probability CF of capacity-feasibility	100
S^{TF}	Weight for the average schedule probability TF of TW-feasibility	100
$S^{\overline{CF}}$	Weight for the MAD \overline{CF} of CF_v from CF	50
$S^{\overline{TF}}$	Weight for the MAD \overline{TF} of TF_v from TF	50
$S^{c(\omega)}$	Weight for the cost $c(\omega)$ of solution ω	0.000002
Δ	Threshold value	300000

3.3 Experiments

We first assess the efficiency of the post-optimization analysis based on the developed score function. For this experiment we used the same instances as for the parameter tuning. The algorithm was run 15 times for each instance to collect statistics. For each combination of α and p values we computed the schedules z^* and ω^* yielding the minimal cost $C(z^*)$ and the highest score $Q(\omega^*)$. Each schedule was simulated using Algorithm 2 to compute the expected costs $c^{\text{exp}}(z^*(\alpha, p))$ and $c^{\text{exp}}(\omega^*(\alpha, p))$. Table 3 summarizes the results for each instance. The first column gives the

name of the instance, where the numbers separated with hyphens are the number of installations, the number of visits per week and the number of vessels available. The second column F^{z^*}/F^{ω^*} provides the ratio of the average reliability levels for solutions z^* and ω^* , where F^{z^*} and F^{ω^*} are computed as the average of their respective values $(CF + TF)/2$ over all runs. The third column $\overline{F^{z^*}}/\overline{F^{\omega^*}}$ provides the ratio of the average of MAD of the voyage feasibility probability for schedules z^* and ω^* . The values of $\overline{F^{z^*}}$ and $\overline{F^{\omega^*}}$ are defined as the average of their respective values $(\overline{CF} + \overline{TF})/2$ over all runs. The last column provides the ratio of the average expected costs $c^{\text{exp}}(z^*(\alpha, p))$ and $c^{\text{exp}}(\omega^*(\alpha, p))$. From Table 3 we can see that for each problem instance the schedule $\omega^*(\alpha, p)$ yields a higher average probability of feasibility and a lower average variability of voyages feasibility probability and, as we had anticipated, a lower expected cost. On average, the schedules with the highest score yield 2.7% a lower expected cost.

Table 3. Score function efficiency evaluation results

Parameter	F^{z^*}/F^{ω^*}	$\overline{F^{z^*}}/\overline{F^{\omega^*}}$	$c^{\text{exp}}(z^*(\alpha, p))/c^{\text{exp}}(\omega^*(\alpha, p))$
9-41-4	0.92	1.25	1.03
10-43-4	0.93	1.15	1.023
12-51-4	0.929	1.18	1.031
14-59-6	0.944	1.21	1.019
16-64-6	0.975	1.17	1.024
18-67-6	0.943	1.14	1.011
20-71-8	0.956	1.16	1.035
22-76-8	0.958	1.21	1.022
22-81-8	0.948	1.11	1.028
24-81-10	0.961	1.09	1.034
26-85-10	0.971	1.16	1.027
Average	0.948	1.17	1.027

We now analyze the behavior of the schedule's planned and the expected costs depending on the reliability level. Figures 4 and 5 depict the mean (over 60 runs) of planned and expected schedule costs depending on the p and α values for the instances with 14 and 26 installations, respectively. From both figures we can see the increasing trend of the planned cost and the decreasing trend of the expected cost while the average reliability level grows. Both costs converge, and at some point the expected cost reaches its minimum. The minimal expected cost for the instance with 14 installations is 3,300,000 with p and α values of 0.7 and 0.1 respectively. The minimal expected cost for the instance with 26 installations is 7,100,000 with $p = 0.7$ and $\alpha = 0.15$. High surges in the planned cost (around 700,000 NOK) for both instances correspond to a fleet size increase. Up to the point where the expected cost reaches its minimum we can observe a strong negative correlation between the planned and expected costs. The larger planned schedule cost corresponds to a higher

reliability level and to a lower expected cost, and vice versa. After the minimum expected cost is reached, both the planned cost and the expected cost grow together with the fleet size increase and still converge. The high cyclical cost surges for the instance with 26 installations (Fig. 5) occur when for each value of p (up to 0.6) the algorithm imposes a higher requirement on the reliability level against weather uncertainty (by increasing the value of α). Whereas for the instance with 14 installations such cost fluctuations are minor, for the instance with 26 installations they are rather high, resulting in a fleet size increase or decrease by one vessel. Declines in the planned cost take place when the algorithm proceeds to the next solution with a higher reliability with respect to demand uncertainty and no reliability requirements with respect to weather. The difference in the degree of planned costs fluctuations (for p values below 0.7) between the two instances can be explained by the larger natural inter-voyage slacks in the schedules for the 14-installation instance. The schedules for this instance are not as tight as those for the 26-installation instance, and hence have a higher feasibility probability than the required minimum set by α and p .

Since both the planned and expected costs were estimated taking into account simultaneous influence of the demand and weather variability we further analyze their separate contribution to costs (see Figure 5). Planned cost with $p = 0.1$ and $\alpha = 0$ corresponds to the cost without reliability requirements imposed (planned cost with $p = 0.0$ is the same as with $p = 0.1$). The planned cost with perfect weather i.e. with only demand reliability requirement is shown by the green line which connects points where weather reliability parameter α is zero. As previously, we see that the planned cost increases with higher reliability level p . The light green line depicts the expected cost with perfect weather. We see that there are no those costs surges caused by imposed weather reliability requirements and both lines are smooth. From the figure we can observe that imposing weather reliability requirements (in addition to demand reliability requirements) leads to the expected cost reduction. Regarding the minimal expected cost, the difference amounts to 100,000 and for the case with perfect weather reaches the minimum for $p = 0.7$. Such relatively small difference is explained by the fact that the schedule generated with some reliability level against demand uncertainty is reliable against weather uncertainty as well because reliability is achieved by reducing the number of visits on the voyage. Although, when imposing reliability requirements against weather uncertainty not only the number of visits on the voyage matters (as for the demand reliability level) but in addition the sequence of installations on the voyages with time windows. In fact, which requirement is more restrictive and leads to a higher expected costs depends on the variability level (standard deviation for example) and depends on a particular case.

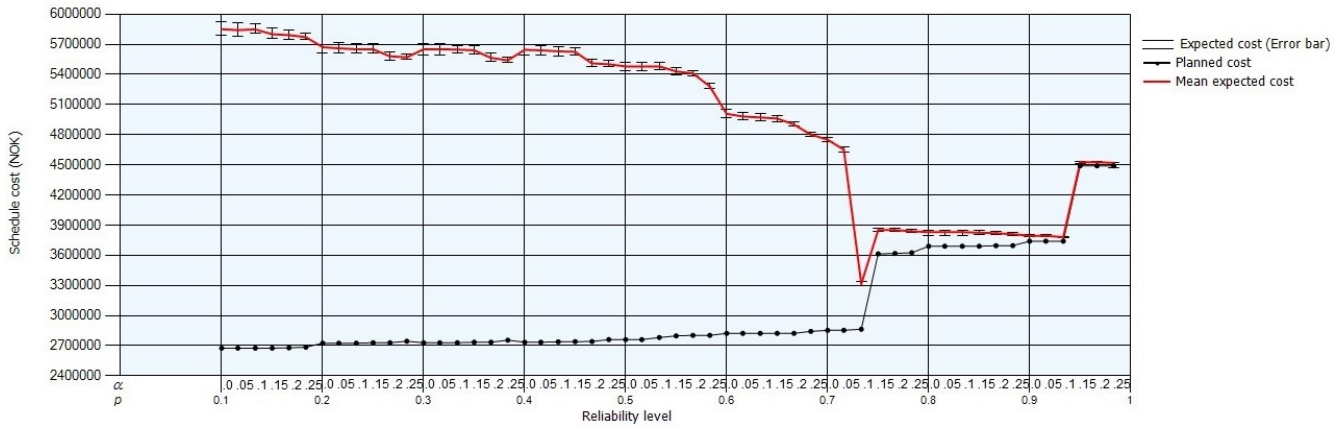


Fig. 4. Planned schedule costs and corresponding expected costs after simulation for the 14-installation instance

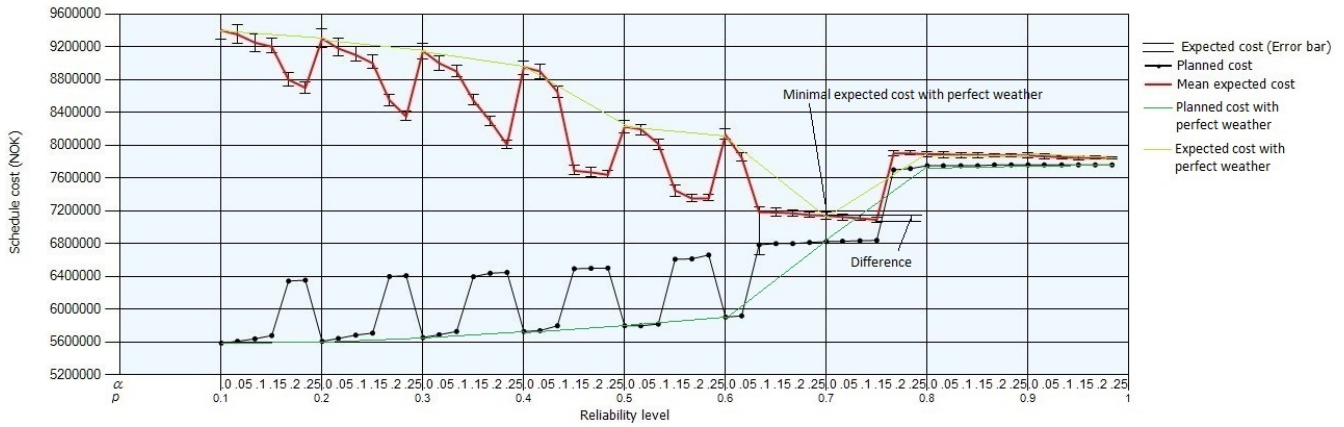


Fig. 5. Planned schedule costs and corresponding expected costs after simulation for the 26-installation instance

The average computational time for each combination of α and p , which is composed of the computational times of the ALNS algorithm and the simulation model, is provided in Fig. 6. As can be seen, the total computational time goes down as the reliability level grows. Although the ALNS computational time reduces slightly as reliability increases, we can observe an almost 10-fold simulation time reduction for the schedule with the minimal reliability level, compared with the schedule with the maximal reliability level. Such a reduction is due to the small number of required recourse actions for the schedules with a higher reliability level. The total computational time for all p and α combinations for the instances with 14 and 26 installations is 272 and 1,850 minutes, respectively. Experiments showed that the minimum expected cost for both instances was achieved for p values in excess of 0.6. For this reason, if the initial p value is set at 0.5, the total computational time for the instances goes down to 71% and 65%, respectively, corresponding to 94 and 532 minutes.

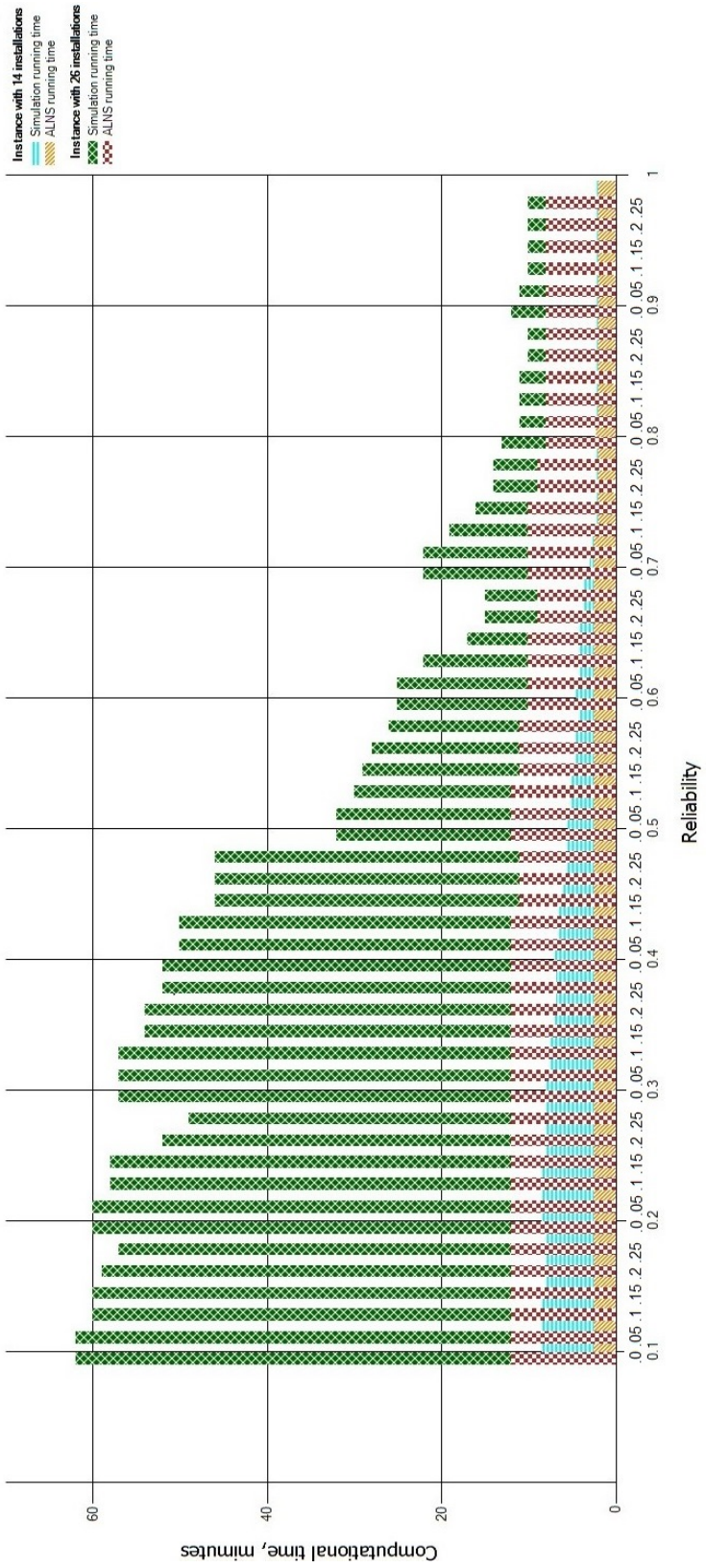


Fig. 6. Computational time as a function of the reliability level

4 Conclusions

We have introduced a methodology enabling the solution of large-size instances of a real-life supply vessel planning problem with combined uncertain demands and weather conditions. Frequent schedule disruptions may result in modifications of the planned voyages, and in additional unplanned voyages of charter vessels or spot vessels hired on the short-term basis. While uncertainty results in a reduced quality of service, the steps taken to eliminate disruptions generate extra costs. We have imposed reliability requirements accounting for both demand and weather uncertainty during the schedule construction. Our ALNS metaheuristic was used to generate a set of vessel schedules with different levels of reliability, and incorporates a score function that accounts for the trade-off between schedule cost and reliability level. We have further developed a discrete-event simulation model to construct a schedule of least average expected cost, after operational modifications applied to ensure feasibility. These modifications involve several options of visit relocations which may be used in various combinations over the simulation horizon, based on the revealed weather and demand data. We have performed extensive computational experiments in order to assess the efficiency of our methodology and to illustrate its performance on real problem instances provided by the Norwegian energy operator Equinor. The proposed methodology enables the construction of vessel schedules of least expected cost for large-size instances under demand and weather uncertainty.

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