Towards understanding crime dynamics in a heterogeneous environment: a mathematical approach.

K.A.Jane White[1], Eduard Campillo-Funollet[2], Farai Nyabadza[3], Davide Cusseddu[4], Christian Kasumo[5], Nancy Matendechere Imbusi[6], Victor Ogesa Juma[7], A.J. Meir[8], and Theresia Marijani.[9]

[1]Department of Mathematical Sciences, University of Bath, UK
[2]Department of Mathematics, & Genome Centre, University of Sussex, UK
[3]Department of Mathematics & Applied Mathematics, University of Johannesburg, South Africa
[4]Department of Mathematics, University of Sussex, UK & Instituto Gulbenkian de Ciencia, Oeiras, Portugal
[5]Department of Science and Mathematics, Mulungushi University, Zambia
[6]Mathematics Department, Kenyatta Univeresity, Kenya
[7]School of Mathematics, University of Nairobi, Kenya
[8]Department of Mathematics, Southern Methodist University, USA
[9]Department of Mathematics, University of Dar-es-Salam, Tanzania

Abstract

Crime data provides information on the nature and location of the crime but, in general, does not include information on the number of criminals operating in a region. By contrast, many approaches to crime reduction necessarily involve working with criminals or individuals at risk of engaging in criminal activity and so the dynamics of the criminal population is important. With this in mind, we develop a mechanistic, mathematical model which combines the number of crimes and number of criminals to create a dynamical system. Analysis of the model highlights a threshold for criminal efficiency, below which criminal numbers will settle to an equilibrium level that can be exploited to reduce crime through prevention. This efficiency measure arises from the initiation of new criminals in response to observation of criminal activity; other initiation routes - via opportunism or peer pressure - do not exhibit such thresholds although they do impact on the level of criminal activity observed. We used data from Cape Town, South Africa, to obtain parameter estimates and predicted that the number of criminals in the region is tending towards an equilibrium point but in a heterogeneous manner - a drop in the number of criminals from low crime neighbourhoods is being offset by an increase from high crime neighbourhoods.

1 Introduction

Crime dynamics and criminal behaviours are highly complex dynamic processes driven both by environmental conditions and individual responses to population behaviours ([11], [49], [8], [32]). Their interwoven relationship provides a mechanism to exploit crime data in order to gain insight into which

aspects of criminal behaviour dominate the observed criminal activity, and it is this relationship which motivates our study.
There is much written about individual motivation to engage in criminal activity ([16], [23], [25], [19], [18]). Throughout our work, we consider the three core motivators:

- Opportunism ([29]): An individual, independent of any external cues, engages in criminal activity;
- Observation ([5]): An individual observes the criminal activity of others in their environment and decides to engage because they perceive benefit in the activity;
- Peer pressure ([21]): An individual feels pressure to conform to behaviours in their environment associated with criminal activity and responds in a density-dependent manner i.e. the greater the number of criminals in their environment, the greater the chance that they will engage with criminal activity.

From this list, it is straightforward to see that crime prevention can be approached in different ways depending on which motivator we wish to affect. For example, increased security on property would reduce opportunistic motivation without direct intervention with individuals considering criminal activity. By contrast, to reduce the impact of observation or peer pressure would most likely involve education programmes aimed at changing individual perspective on, and desire to engage in, criminal activity.
Another feature of crime data is that it often highlights spatial heterogeneity in the distribution of crimes geographically ([2], [38], [35], [20], [26], [48]). Whilst this adds complexity to the problem, it also provides useful constraints on the system. For example, if hotspots for criminal activity are identified, crime prevention and control may be focussed around these areas. However, the problem is more complex than simply identifying hotspots since those engaged in criminal activity at the hotspot may not be domiciled close to that hotspot. Therefore, in our approach, we include partial movement of criminals between baseline and heightened crime regions.
Over the past decade, interest has grown significantly in using mathematical modelling as one tool in the arsenal needed to fully understand criminal activity and the potential for its control ([9], [7], [22], [47]). For example, Game Theoretic approaches have been used to understand criminal behaviour ([13], [43], [34], [1]); dynamical systems for the evolution of criminal behaviour ([37], [46], [28]); and novel reaction-diffusion systems have explored the emergence of spatial heterogeneity in criminal activity ([12], [42], [41],[30], [17], [33]). There is also an extensive literature that uses statistical modelling approaches to understand crime data ([44], [27], [4]) or agent based modelling to consider individual-level behaviour and/or activity ([10], [45], [31], [3]). In 2016, a special issue of the *European Journal of Applied Mathematics* was devoted to mathematical modelling of crime and security ([6]), further identifying the potential for mathematics to help our understanding of the complexity of criminal activity and behaviours.
In this paper we present a new model paradigm in which criminal activity and number of criminals are explicitly described. This allows us to parameterise and subsequently validate (using different data) model predictions whilst also setting the scene for future work on crime control strategy. In the next section we construct the model, clearly highlighting the model assumptions including the three distinct routes into criminal behaviour. Following on from this, we extract information from the model system, highlighting the importance of key model parameters (and parameter groupings) in the model dynamics. In particular, we identify a parameter grouping which we label **criminal efficiency** which

is an important indicator to determine whether a region is likely to experience unbounded growth in criminal activity. In Section 4, we use data from Cape Town, South Africa, firstly to parameterise our model for that region. Subsequently, using a different subset of the data, we validate the model and use the validation region to predict that the crime dynamics in Cape Town are tending towards an equilibrium point where criminal activity from low activity areas will be reduced from current levels but the activity in heightened activity regions will be higher.

In the conclusion, we highlight next stages for this modelling work, explaining how we think it might be used in initiatives around crime control and crime prevention.

2 Model formulation

We consider a model structure that couples two distinct regions for criminal activity which we label **baseline** and **heightened**. Every criminal is associated with one of these regions. Criminals from the baseline will only commit crimes within that region but criminals from the heightened region can commit crime within both regions.

At time t, $t \geq 0$ measured in years for consistency with data, we let $N_i(t)$, $i = A, B$, denote the number of criminals based in region i where A and B correspond to the baseline and heightened activity regions respectively. Assuming that the number of crimes committed is proportional to the number of criminals in each region at time t, then the number of crimes committed in each region, denoted $A(t)$ and $B(t)$ respectively, is given by:

$$\begin{aligned} A(t) &= \delta_A N_A(t) + q\delta_B N_B(t); \\ B(t) &= (1-q)\delta_B N_B(t). \end{aligned} \tag{1}$$

The parameters δ_i, $i = A, B$ denote the average number of crimes committed per criminal from region i and q denotes the fraction of crimes committed by criminals from region B in A. Whilst it is difficult to determine the accuracy of our assumptions about where criminals commit their crimes, our choice is driven by the desire for structural simplicity. The resulting model allows us to explore the impact of heterogeneity in behaviours and coupled regions on the scale and scope of criminal activity across both environments.

For the model equations governing the time evolution of criminal numbers from each region, we focussed on three distinct and recognised mechanisms that influence individuals to undertake criminal activity:

1. Self-initiation, independent of the amount of crime being committed and the number of criminals in the region. We interpret this mechanism as **intrinsic opportunism** since it is unrelated to any external influence. We represent this in the model with parameters λ_i, $i = A, B$ which give the number of new criminals per unit time in each region, independent of any influence from the amount of crime/criminals.

2. Observation of criminal activity and the associated reward (financial or otherwise). This process is analogous to an **efficiency measure** since it provides a conversion from crimes committed to new criminals. This is represented in the model by two distinct parameters: ϵ_i, $i = A, B$, is the number of new criminals per crime per unit time in each region; and ω which weighs how criminals based in region B respond to crimes in region A relative to crimes committed in B.

3. Peer pressure by which individuals are encouraged to engage in criminal activity because they are based in an environment where crime is normalised i.e. in a region of heightened activity. Following the literature, we assume that this is a nonlinear effect ([14]). In the model, there are two associated parameters: κ scales the impact of **peer pressure** to increase participation in criminal activity; and α is the nonlinear response constant to peer pressure.

Criminals may also cease to engage in criminal activity which we model as a simple linear decay term. We use the parameters σ_i^{-1}, $i = A, B$ to denote the average duration for a contiguous period of criminal activity in each region.
Combining these model assumptions leads to the governing equations for $N_i(t)$, $i = A, B$:

$$\begin{aligned}
\frac{dN_A}{dt} &= -\sigma_A N_A + \epsilon_A A(t) + \lambda_A; \\
\frac{dN_B}{dt} &= -\sigma_B N_B + \epsilon_B(\omega A(t) + B(t)) + \lambda_B(1 + \kappa {N_B}^{\alpha})
\end{aligned} \tag{2}$$

where t is again measured in years. In this work we set as $\alpha = 2$ for simplicity. To interpret this in a physical context, it means that the per capita rate of increase in criminal activity due to peer pressure in region B is directly proportional to the number of criminals in that region at any given time. Future work will explore the importance of the exponent α to the predictions made using this model structure.
The expressions given in (1) correspond to recorded data which is why we explicitly keep track of both $A(t)$ and $B(t)$ rather than simply the number of criminals based in a region (2) for which there is no data. However, to carry out the model analysis, we substitute (1) into (2), rearranging to highlight key parameter groupings, to give the coupled ODE system:

$$\begin{aligned}
\frac{dN_A}{dt} &= \sigma_A \left[m_A + (c_A - 1)N_A + c_A dq N_B\right] \\
\frac{dN_B}{dt} &= \sigma_B \left[m_B(1 + \kappa N_B^2) + (c_B(\omega q + (1-q)) - 1)N_B + \omega c_B d^{-1} N_A\right]
\end{aligned} \tag{3}$$

where, for $i = A, B$,

$$m_i = \frac{\lambda_i}{\sigma_i}, \qquad c_i = \frac{\epsilon_i \delta_i}{\sigma_i},$$

and $d = \dfrac{\delta_B}{\delta_A}$.
To fully specify the problem mathematically we provide initial values:

$$N_A(0) = N_A^0, \qquad N_B(0) = N_B^0, \qquad A(0) = A_0, \qquad B(0) = B_0.$$

Note that the form of (3) lends itself in a straightforward manner to non-dimensionalisation. We choose not to do that because we fit the model to data which is necessarily dimensional; rather the primary purpose of the rescaling was to identify the two key parameter groupings m_i and c_i, $i = A, B$.

3 Insights from model

The primary purpose of this work is to present a simple model structure to describe crime dynamics that can be parameterised and validated using data on the number of crimes committed as a proof of concept. Consequently, the analysis we undertake is simply to identify and interpret parameter constraints required to ensure that our model solutions are valid.

3.1 Parameter constraints for realistic model outcomes

Model predictions should always be physically realistic. In this case, the baseline requirement is that number of crimes/criminals should always be non-negative. Solving (1) for N_i, $i = A, B$ gives

$$\begin{aligned} N_A(t) &= \frac{1}{\delta_A(1-q)}\left((1-q)A(t) - qB(t)\right); \\ N_B(t) &= \frac{1}{\delta_B(1-q)}B(t). \end{aligned} \tag{4}$$

Using this expression, we see that $N_A \geq 0$ requires

$$(1-q)A(t) \geq qB(t)$$

or, alternatively

$$q \leq \frac{A(t)}{A(t)+B(t)}$$

which means that the fraction of crimes committed by criminals from B in region A must not exceed the fraction of all crimes (by any criminal) committed in region A. Based on our model set up, this is realistic assumption and we can use it alongside the data to provide finer bounds on the value of q (which is already restricted to lie in the interval $[0, 1]$).

Furthermore, we expect that the higher criminality levels in the B-type areas is reflected not only in the total number of crimes, but also in the number of criminals in the B areas. Therefore we expect that $N_A \leq N_B$, and in consequence, we use (4) to find that

$$\delta_A \geq \frac{\delta_B(1-q)A(t)}{B(t)} - \delta_B q.$$

3.2 Steady state analysis

Steady state solutions of (3), which solve

$$\frac{dN_A}{dt} = \frac{dN_B}{dt} = 0$$

are given by the coupled system

$$N_A^* = \frac{m_A + dqc_A N_B^*}{1-c_A} \tag{5}$$

where N_B^* is the solution of the quadratic equation:

$$m_B \kappa N_B^2 + N_B\left[\frac{c_B}{(1-c_A)}(\omega q + (1-q)(1-c_A)) - 1\right] + m_B + \frac{\omega c_B m_A}{d(1-c_A)} = 0 \tag{6}$$

whenever that solution is real and positive.

Since all model parameters are non-negative, the steady state for N_A given in (5) will only be feasible if

$$c_A < 1.$$

The parameter c_A measures the number of new criminals produced in the baseline region as the result of a single criminal in that region over a single period of criminal activity. It makes sense for this

value to be less than unity as a necessary condition for a steady state to exist since the region is characterised by low criminality and if criminal efficiency was above unity here, it would correspond to significant increase in criminal numbers over time in the baseline region. It is worth noting that, for $c_A > 1$, $dN_A/dt > 0$ for all t and the number of criminals in the baseline region would increase without bound.
We label c_i, $i = A, B$ **criminal efficiency**: the average number of new criminals resulting from the criminal activity of a single criminal within their region over a single period of criminal activity.
Having established the requirement for $c_A < 1$, we now consider the quadratic equation (6). Using the basic properties of a quadratic equation ($ax^2 + bx + c = 0$ with $a > 0$ has real positive roots iff $b < 0$ and $b^2 - 4ac > 0$), (6) will admit positive, and hence realistic, steady state solutions iff:

$$c_B < \frac{1 - c_A}{(1-q)(1-c_A) + \omega q} \quad (b < 0), \tag{7}$$

and

$$\left[\frac{c_B}{(1-c_A)}(\omega q + (1-q)(1-c_A)) - 1\right]^2 > 4 m_B \kappa \left[m_B + \frac{\omega c_B m_A}{d(1-c_A)}\right] \quad (b^2 - 4ac > 0). \tag{8}$$

Assuming that $c_B > c_A$ to reflect the model set up which identifies B as a region of heightened criminal activity, then (7) describes a bounded region in the $c_B - c_A$ plane as shown in Figure 1. A key observation here is that criminal efficiency in B may exceed unity and still produce a positive steady state solution.
Once (7) has been satisfied the inequality (8) can always be satisfied by suitable choice of the additional parameters in the following qualitative ways:

- Sufficiently low levels of criminal opportunism (m_i, $i = A, B$) in one or both of the regions;
- Sufficient difference in the number of crimes committed per criminal from the two regions (d);
- Sufficiently low peer pressure (κ) in region B.

The first and third of these conditions correspond to criminal environments in which we can identify restraint in criminal behaviour without which the model system would predict unbounded growth in the amount of criminal activity over time. The second condition links to the interplay between the two regions; one interpretation of this is to assume that the baseline region should exhibit very low levels of criminal activity, particularly in relation to the heightened region.
In the next section, we use the criteria presented in this section to help parameterise the model for criminal activity in the Cape Town vicinity of South Africa, thus demonstrating the potential for our model to help understand crime dynamics based on the criminal data available.

4 Case Study - Cape Town

To test the performance of the mathematical model, we study a dataset collected by the South African Police Service (SAPS) containing data about the crimes in South Africa in the period 2005 to 2016 [40]. The dataset contains the number of crimes in each police station, classified by crime types. SAPS also provides a separate dataset with the geographical data of each police station. We restrict our to the police stations in the Cape Town area, see Figure 2.

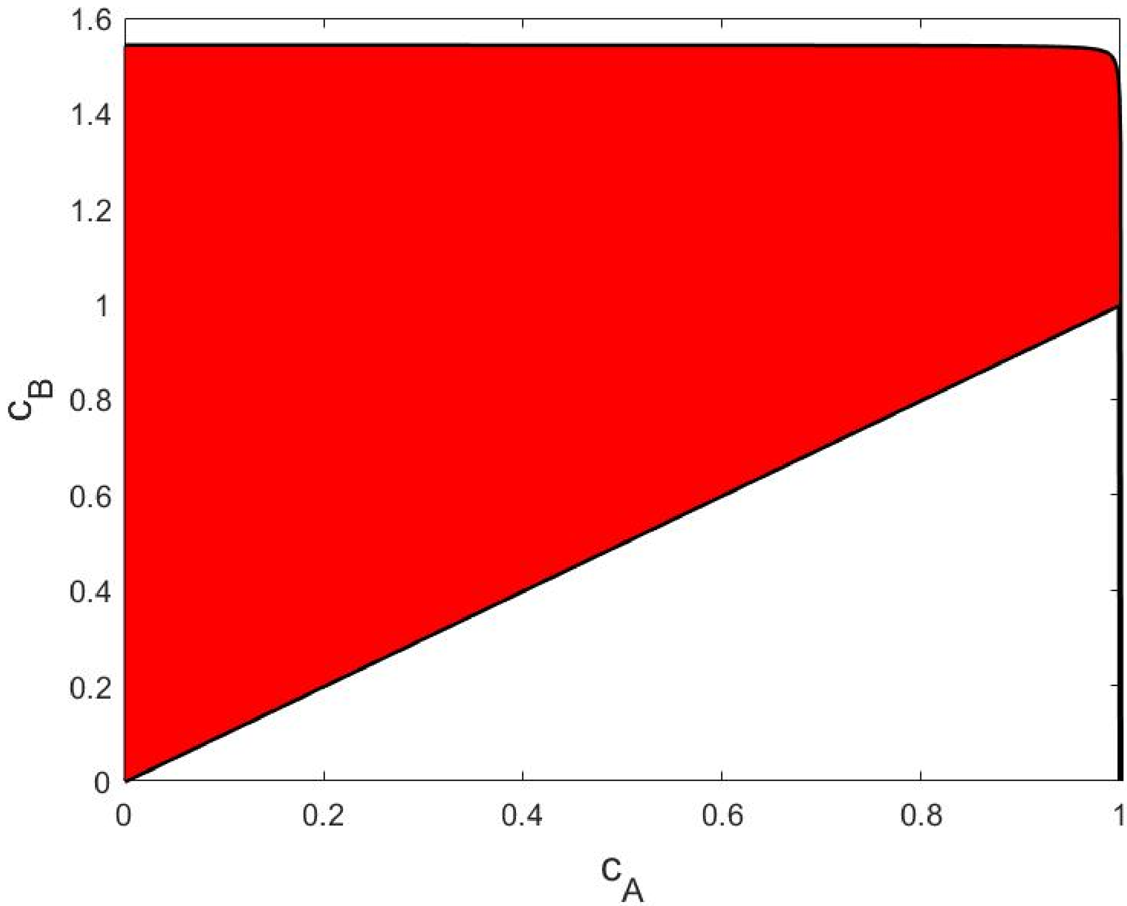

Figure 1: Graph of criminal efficiency in region B as a function of criminal efficiency in region A. The shaded area encloses the parameter space necessary for ensuring a positive steady state. It is given by the ntersection between condition (7) and the condition that $c_B > c_A$. Model parameters, $q = 0.351$ and $\omega = 2.793 \times 10^{-4}$, taken from Table 1 below, are the values estimated from fitting the model to data from the Cape Town area.

4.1 Classification of regions

In order to apply the mathematical model to the crime data, we need to determine which areas have a baseline criminality level (type A), and which areas exhibit a heightened criminal activity (type B). We will first proceed to classify each police station, and then we will aggregate the data to obtain an average dataset for each type.

There are many possible criteria to define areas of heightened criminal activity. One could use absolute crime numbers in each area, but some types of crime may not be linked to the overall criminal activity in a neighbourhood. For instance, domestic violence in relatively safe areas should not count towards a heightened criminal activity. Another problem with using absolute crime numbers is the total population differs from police station to police station, and in consequence more populated areas have more crimes. To avoid this problem, we use murder as a proxy for the criminal activity in a given area. According to [36], murder is a good proxy for violent crime, and hence we assume that heightened criminal activity areas will also have associated a higher level of violence.

We use the estimated population in each police station as provided by SAPS to compute the number of murders per inhabitant. We use only the data for year 2016 to perform the classification. This has the advantage of avoiding possible overfitting of the model, but limits the validity of the classification to areas that have not experimented significant changes in their overall criminality levels. To classify an area, we set a threshold of 1 murder for every 1000 inhabitants. Therefore, we considered areas with more than 1 murder for every 1000 inhabitants in 2016 to be areas of heightened criminal activity (type B), and other areas to be areas of baseline criminality (type A). Figure 2 depicts the classification of the police stations in the Cape Town area.

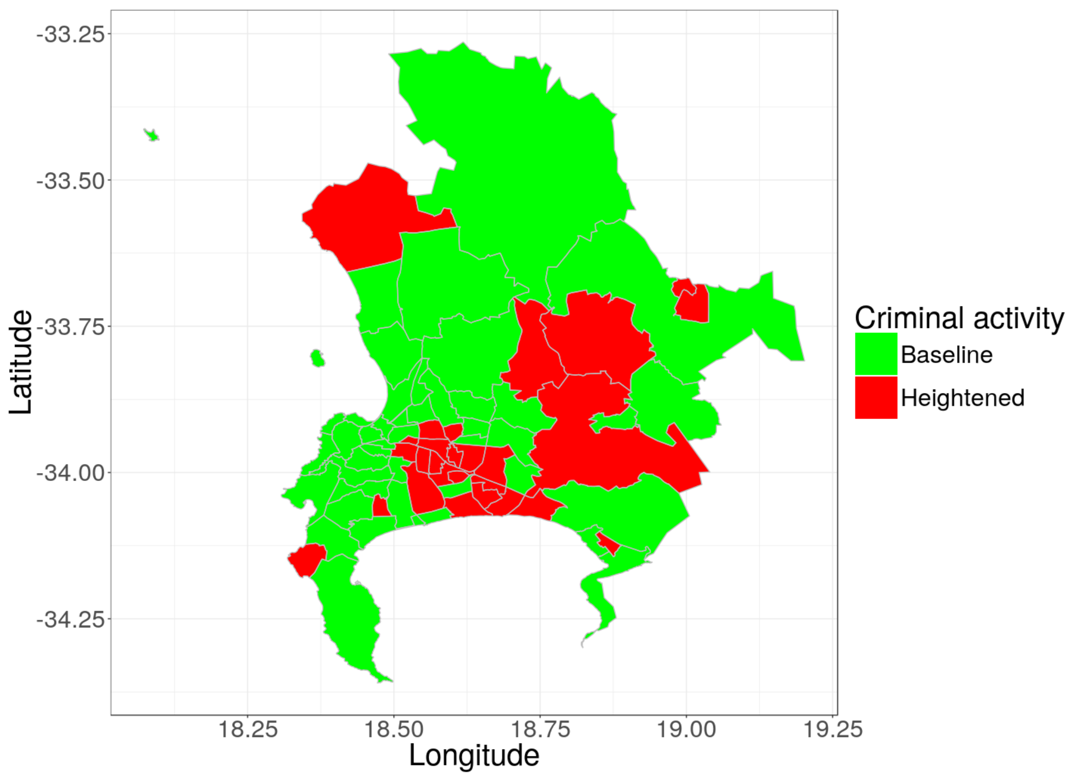

Figure 2: Map showing the break down of Cape Town by Police Station District. Each station has been classified as a region of baseline criminal activity or heightened criminal activity using the algorithm described in Section 4.1.

In the Cape Town area, we obtain 43 police stations in areas of baseline criminal activity, and 26 police stations in areas of heightened criminal activity. Most of the heightened criminality areas are located around the Cape Flats area and to the east. The distribution of the police stations with heightened criminal activity suggests a big area of high criminality, surrounded by baseline criminality areas. To apply the mathematical model, we consider the average number of crimes of all the police stations classified as baseline criminality as a single low criminality region, and similarly the police stations classified as heightened criminal activity as a single high criminality region.

Parameter	Definition	Estimated value	Units
δ_A	Average number of crimes per criminal in region A	4.477	Crimes/Criminal
δ_B	Average number of crimes per criminal in region B	22.20	Crimes/Criminal
q	Fraction of B-criminal crimes committed in A	0.3510	Non-dimensional
ϵ_A	Rate of initiation via observation in region A	0.001492	Criminals/Crime/Year
ϵ_B	Rate of initiation via observation in region B	1.1339	Criminals/Crime/Year
σ_A^{-1}	Average duration of criminal activity in region A	0.08953^{-1}	Years
σ_B^{-1}	Average duration of criminal activity in region B	16.42^{-1}	Years
λ_A	Rate of self-initiation in region A	0.07783	Criminals/Year
λ_B	Rate of self-initiation in region B	42.46	Criminals/Year
κ	Scaling of peer pressure	$1.5078 \cdot 10^{-7}$	Criminals $^{-1}$
ω	Scaling of observations in region A to region B	$2.793 \cdot 10^{-4}$	Non-dimensional

Table 1: Estimated parameters of the model (1)-(2). See Section 4.2 for details.

4.2 Methodology for parameter estimation

In order to compare the results of the mathematical model with the observed number of crimes, we need to estimate the parameters for the model. To estimate the parameters, we use a Bayesian framework [15]. We define the log-likelihood of a parameter as the square distance between the number of crimes predicted by the mathematical model (1) and (2) and the collected data, weighted by the standard deviation of the data. We prescribe uniform priors for all the parameters, imposing positivity constraints as well as the constraints derived in Section 3 to ensure that the baseline and heightened regions satisfy the modelling hypothesis. We run five independent Markov chains, of length 10^6. We check for convergence of each chain using the Geweke's Convergence Diagnostic [24], and we perform the usual burn-in and thinning procedures (see for instance [39]), to end up with 10^4 independent samples. Convergence in mean is finally confirmed by comparing the five independent chains.

We validate the model by splitting the data in two sets, the training dataset, and the test dataset. We use the average number of crimes for each area, from years 2005 to 2009 as a training dataset, and from 2010 to 2016 as a test. In particular, this means that the parameters are identified only using the information of the first five years.

4.3 Results

We report the estimated values for the parameters in Table 1. We observe that parameters related to criminal activity (for instance, δ_A, δ_B the average number of crimes per criminal), are much higher in the area of heightened criminal activity. The baseline criminality area also shows lower rates of initiation and self-initiation into criminal activity.

Using the estimated parameters, we can compare the results of the mathematical model with the observed data. Figure 3 depicts both the model result and the data. Although we use only the data from years 2005 to 2009 to fit the model parameters, the ODE solution is in good agreement with the data from 2010 onwards. The reason for this is better understood by looking at the phase plane of the number of criminals, depicted in Figure 4: the system is in a relatively straight orbit approaching the stable steady state, and it does not exhibit significant changes when time increases.

The phase plane of the system reveals also the dynamics of the number of criminals, which is not

Figure 3: Average number of crimes in the Cape Town area, for regions of baseline (A) and heightened (B) criminal activity. The ODE system is fitted using only the data from years 2005 to 2009 represented by the filled circles and triangles. The model is able to predict the data taken in the following 7 years, represented by the empty circles and triangles.

directly observable from the data. As shown in Figure 3, the average number of crimes is increasing both in the baseline and the heightened criminality areas, but we see in the phase plane (Figure 4) that the number of criminals is actually decreasing in the baseline criminality areas, and in consequence is the increase of the number of criminals in the heightened criminality areas that drives the increase in the number of crimes everywhere. Since we are studying only an average dataset, the actual predictive power of the present study is limited, but these results suggest that our modelling approach can be used to better allocate the resources to fight crime in Cape Town.

5 Conclusion

Our motivation to undertake the work presented in this paper was pragmatic: to develop a mathematical model that could provide a useful tool for colleagues working in the broad arena of crime prevention. This practical focus led us to create a model structure that combined both descriptions of the time evolution of number of crimes committed and the number of criminals. Heterogeneity

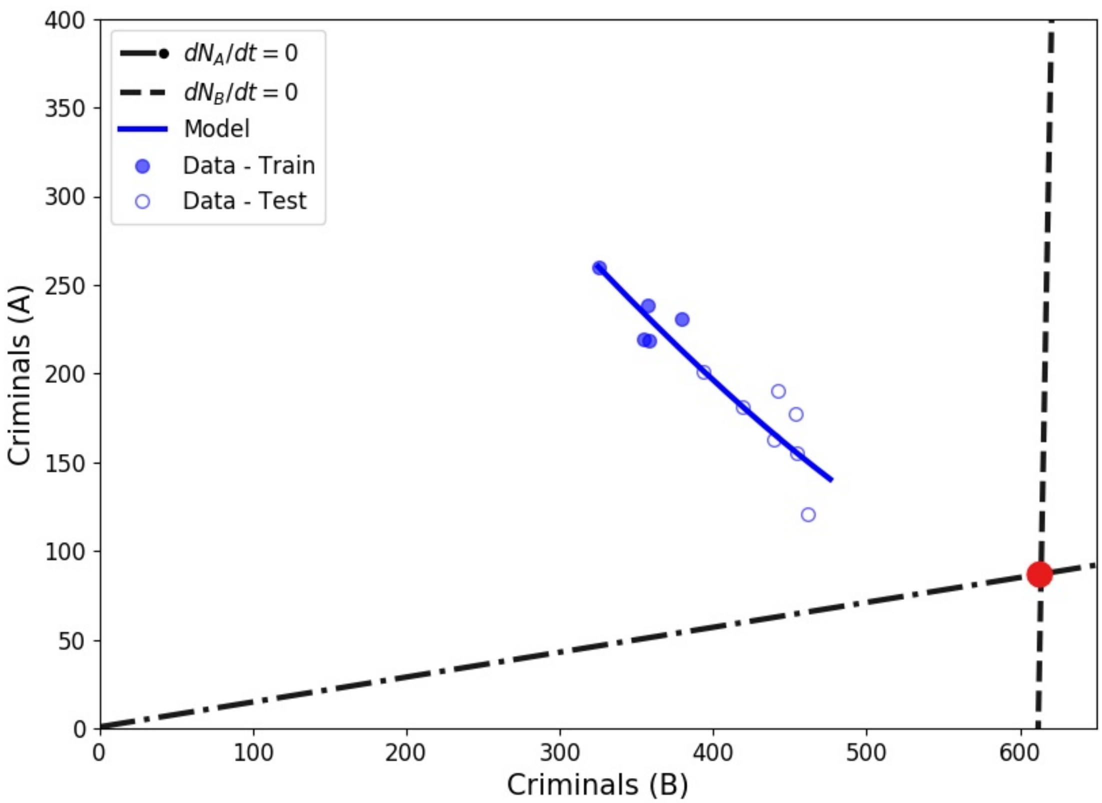

Figure 4: Phase plane of the number of criminals in each area. The solution of the model is moving towards the steady state (filled circle) given by the intersection of the nullclines $\frac{dN_A}{dt} = 0$ and $\frac{dN_B}{dt} = 0$.

in the amount of criminal activity recorded in neighbouring police districts also led us to consider a model that combines regions of low, baseline crime with regions where there is more crime and criminal activity. Having decided upon this structure, the model terms were simply chosen to describe the key, commonly-observed mechanisms for initiation into criminal activity, namely self-initiation (opportunism), observation and peer pressure. We combined these mechanisms with removal rates that we interpret as individuals engaging in periods of criminal activity interspersed with periods of very low or no activity (due, for example, to being arrested).

Despite its simplicity, the model form was helpful in identifying key parameter groupings which are important in the containment of criminal activity. Most important amongst these is the criminal efficiency which measures the impact of a single criminal on the initation of other criminals who observe the criminal activity (and gain). In the baseline regions, this measure must be maintained below unity in order to contain criminal activity; in the heightened regions, criminal efficiency must also be maintained at low levels although these can be higher than for the baseline region. Criminal efficiency has three components: average number of crimes committed per criminal; rate of initiation into criminal behaviour as a result of observing crime; and average period that an individual undertakes criminal activity. We suggest that this measure may be relevant to professionals involved in crime prevention through improved security, education, and sanction respectively. Initiation into criminal activity through opportunism or peer pressure are also important in the crime dynamics but they are dependent on the efficiency and do not have explicit thresholds associated with different population level outcomes.

Concerns over the challenges of parameter estimation were at the forefront of our thoughts as we developed the model structure because we wanted to create a useable tool and we were aware of the limited data available to us. The analysis undertaken in Section 3 was important in providing guidance on the region of parameter space that we were interested in exploring. Our prudent approach appears to be promising based on the case study presented in Section 4. From the data available to us, we used the oldest data points to estimate model parameters; the model was then validated using the most recent data (past 5 years). The decision to fix $\alpha = 2$ was, in part, driven by the lack of data and a desire to minimise the number of parameters that we needed to estimate from the data. Having said that, we do not anticipate that qualitatively different dynamics would be observed, and in particular the trajectory shown in the phase diagram in Figure 3, if the value of α had been different provided $\alpha \geq 1$. From our model parameters, we note that peer pressure is a weak component of the initiation into criminal behaviour and might better be represented using a simple linear function. Our analytical work on the model form will explore this further and will be reported in due course.

The trajectory shown in Figure 3 is interesting. It suggests that in the Cape Town region, there is likely to be an increase in the number of criminals from regions of heightened criminal activity offset by a reduction of the number in baseline regions. In principle, this prediction could be exploited to focus crime prevention activity in regions of heightened activity.

Going forward, in addition to undertaking a more rigorous treatment of the dynamical system to explore quasi steady state assumptions on the baseline regions for example, we intend to extend the model system to consider more complex spatial interactions. This will address the oversimplification that we make in assuming homogeneity of movement between regions A and B which does not take into consideration the spatial distribution of regions. Alongside those modifications, we are keen to exploit the model in the context of crime prevention and control, using our dynamical model for criminal activity within a control framework. In the meantime, we are pleased that such a simple, intuitive mechanistic model was able to provide useful insights into criminal activity and hope that this will encourage further exploitation of mechanistic models within the arena of crime prevention and, more generally, the exciting and rapidly growing field of criminology.

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