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Modelling the interdependence of spatial scales in urban systems

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Abstract

The multitude of interwoven spatial scales and their relevance for urban systems has been of interest to the complexity science of cities since its conception. Today, we are well aware that urban environments are being simultaneously shaped and organised through actions at all levels. However, the fundamental question of how to reveal and quantify the interdependence of processes in between various spatial and temporal scales is less often addressed. Deepening our theoretical understanding of the multiscale spatiotemporal complexity of urban systems demands a transdisciplinary framework and the deployment of novel and advanced mathematical models. This article performs a multiscale analysis of urban structures using a large dataset of rent price values in the Ruhr area, Germany. We argue that, due to their many interacting degrees of freedom, urban systems exhibit similar features as other strongly correlated systems, e.g., turbulent flows, notably the occurrence of extreme small-scale fluctuations. This analogy between urban and turbulent systems, which we support by empirical evidence, allows for the modelling of spatial structures on the basis of concepts and methods from turbulence theory. We demonstrate how by identifying the main turbulence-borrowed characteristics of an arbitrary two-dimensional urban field, it can be fully reproduced with a small number of prescribed points. Our findings have theoretical implications in the way we quantify and analyse scales in urban systems, model small-scale urban structures, as well as potential policy relevance on understanding the evolution and spatial organisation of cities.

Keywords

Multiscale framework, Strongly correlated systems, Urban analysis and modelling

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Introduction

As the surge of recent publications in the subject area attests, it is becoming increasingly hard to understand today's urban structures through the lens of traditional spatial and temporal scales (Bretagnolle et al. 2002; Rybski et al. 2019). Due to dynamical interactions and complex interdependencies between various urban dimensions (e.g. population, economy, infrastructure), conventional geographical but also disciplinary boundaries to explain urban phenomena are dissipating (Albeverio et al. 2007). It is now widely accepted that neighborhoods, cities and regions are being continuously shaped and reshaped through an accumulation of actions enforced at each scale (Haken and Portugali 1995; Batty 2013), whilst their closely intertwined development paths are further embedded in national and international environments. The realisation of any spatial unit is a function of its competitive and collaborative role within the hierarchical structure of the given system. Therefore, scales in cities have been of interest to numerous disciplines from the theoretical- and praxis-side alike. Fundamental questions ranging from the definition and delineation of urban areas, through complex multiscale descriptions of urban systems to how scales can be harnessed to promote more sustainable development paths are being extensively studied from diverging points of view (Cheng and Masser 2003; Batty 2005; Hayek et al. 2015; Li et al. 2019; Carra and Barthelemy 2019; Lemoy and Caruso 2020). Nevertheless, the task of capturing the profound interdependence in between processes at various spatial and temporal scales is in urgent need of further research and exploration. Multiscale phenomena are at the heart of our understanding of how urban settlements evolve and organize spatially and an enhanced insight into their main characteristics has the potential to improve the modelling and management of urban systems.

Following on from a brief overview of multiscale phenomena in cities, we suggest that a yet largely unexplored analogy between urban and turbulent systems may prove to be a beneficial extension to the overall quest for quantifying and modelling multiscale urban processes. After gathering empirical evidence on the viability of the latter analogy in the first part of this article, we demonstrate how findings are directly applicable for the modelling of urban structures in the second part. In the last section, we discuss the relevance of our results for both the description of multiscale phenomena in urban systems and the tackling of data-reconstruction challenges. Due to its complex settlement and socioeconomic structure (Wehling 2014), extraordinary development tendencies (Batty 2016) and fuzzy nature of scales (Parr 2004), the highly polycentric Ruhr area in western Germany is chosen as the case study for our current explorations.

Background

Scales in complexity science of cities have been evoked both as the target of the analysis and as means to gain further insights into urban characteristics (Manson 2007). Amongst others, latter concentrates on exploring inter- and intra-city spatial organization (Liu et al. 2018), morphological diversity (Lagarias and Prastacos 2020; Thomas et al. 2010; Ma et al. 2019), inter-scale linkages (Liu et al. 2018) and the dynamics of urban evolution (Chen and Jiang 2010). The former is often deployed to find universal features of both the urban form itself (Li et al. 2019) and its formation (Carra and Barthelemy 2019) whilst prominent methodological examples include urban scaling laws (Bettencourt 2013; Cottineau et al. 2017; Lemoy and Caruso 2020) and (multi-) fractal analysis (Batty and Longley 1994; Tannier and Pumain 2005). This article aims to integrate both of the above aspects: It deploys scales, first to derive universal characteristics of spatial structures, and then to directly apply these for multiscale urban modelling. More precisely, the here proposed methodology is closely linked to a longstanding tradition of spatio-temporal modelling of cities by concepts and methods from physics which can be initially dated back to the works by Kadanoff (1971). L. P. Kadanoff generalised J. W. Forrester's model of "Urban Dynamics" (Forrester 1970) and its theory of relative attractiveness to be applicable from one to any number of urban areas and thereby introduced a broader multiscale approach to emphasize how the efficacy of urban policy is strongly dependent on the scale of introduced measures (Kadanoff 1971). Similarly, Sociodynamics (SD) is an approach based on methods from statistical physics for understanding and modelling complex social systems, as devised by Haag and Weidlich (1984) (we also refer the reader to the monograph by Haag (2017)). Since proponents of SD argue that the dynamics of social systems entails both random and quasi-deterministic processes, it bases its mathematical methods on the theory of probabilistic systems and non-linear dynamics (Weidlich 2017). Over the past 50 years, SD has found various applications in the urban context for residential and employment migration (Weidlich 2005; Lengyel and Friedrich 2020), integrated land use transport modelling (Weidlich and Haag 1999), or for devising the evolution of urban and regional settlement structures (Weidlich 1999). More recently, advanced urban pollutant models (Soulhac et al. 2011) and methods aiming at classifying urban areas via remote sensing (Parrinello and Vaughan 2002) apply complex multiscalar statistical descriptions for quantifying diverse urban phenomena.

Finally, the above mentioned fractal formalism may also be evoked in this context as it uses the concept of self-similarity, or structures reproducible across scales, formulated with help of the statistics of scaling systems (Lauren 2000). Due to unequal probability of human activities across space and strong correlations between spatiotemporal dimensions, cities have been extended to be multi-scaling systems characterisable by the multifractal spectra (Hu et al. 2012). In this case a single fractal dimension is substituted by a continuous spectrum of scaling exponents to describe urban structures. For example, Chen and Wang (2013) employed the so-called multifractal geometry to model urban form and analyze urban growth in Beijing through time, whilst Hu et al. (2012) used multifractal analysis to characterize spatial distributions of land prices in Wuhan, China. Analogous to multifractal analysis, strong intrascale dependencies of urban structures are also to be reproduced by the methodology introduced here, however we will draw on the statistical description of turbulent flows to derive certain universal features. In particular, we expand methods that were originally devised for the modelling of turbulent velocity fields to a *spatial* interpolation of urban rent price fields. The proposed *stochastic interpolation* to model urban field quantities possesses two unique characteristics: *i.)* Unlike other interpolation methods (e.g., polynomial interpolations or kriging (Delhomme 1978; Cressie 1990)), it is capable to capture the *roughness* of urban structures, and *ii.)* due to the analogy to a turbulent system, it is able to recover strong small-scale fluctuations. Phenomenological models of turbulence have been successfully applied to a number of research areas (e.g., modelling wind fields (Mücke et al. 2011), airflow in urban environments (Kardan et al. 2018; Soulhac et al. 2011)) but never before to urban analytics of socio-economic phenomena. The viability and advantages of this analogy is the core subject of this contribution and will be inspected in great detail in the coming sections.

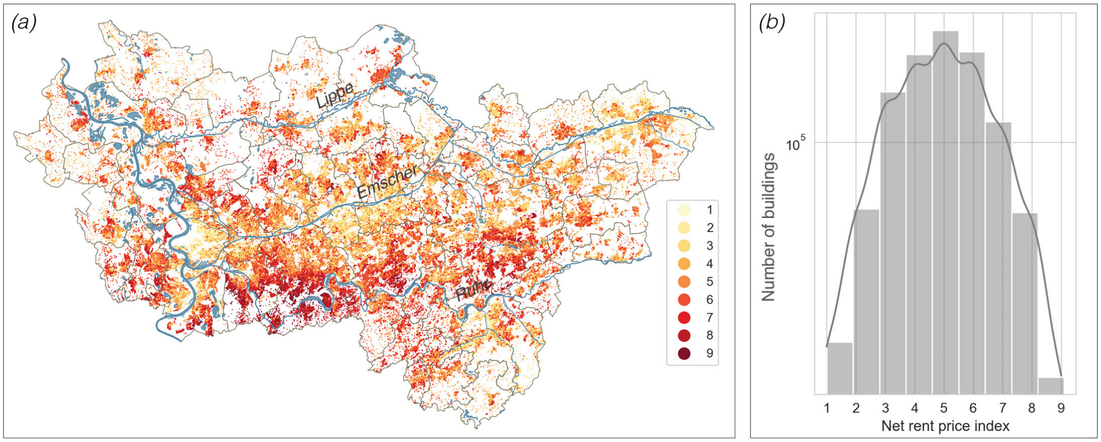


Figure 1. *(a)* Net rent price indices in 2016 ranging from one (very low) to nine (very high). Location of geo-referenced data points on the building level. *(b)* Distribution of net rent price indices. Values reflect modelled probabilities in accordance with General Data Protection Regulation (GDPR) guidelines of the EU.

Dataset

As shown in Figure 1, the dataset consists of *net rent price indices per square meter* ranging from one (very low) to nine (very high). Prices are obtained via dividing the monthly charge of the rental, excluding ancillary and heating costs, by the square meter of the property. Data were collected from the micro-dialog database of the German post for the year 2016 (Deutsche Post - Microdialog 2020) that takes advantage of ImmobilienScout24 (ImmobilienScout 2021), the largest online real estate portal in Germany, to calculate rental rates. In accordance with General Data Protection Regulation (GDPR) guidelines of the European Union, the microdialog works with modelled probabilities in order to protect the privacy of individual residents (see Figure 1). Spatial extension of the 1.05 million geo-referenced points covers all 53 cities and towns of the Ruhr metropolitan region (*Ruhrgebiet*), spreading over an area of 4435 km^2 with a population of just above 5.1 million (2019). Finally, data resolution is the building level, therefore we were able to use it both in its raw format in the first part of this paper and also in an aggregate set-up matching a 100x100 m^2 spatial grid which was suitable for modelling purposes in the second part (as shown in Figure 5(a)). For the tessellated data, we calculated the mean rent price index per grid cell, ranging again from one (very low) to nine (very high).

Developing the analogy between urban and turbulent systems

In this section, we propose a phenomenological description of multiscale urban processes on the basis of concepts and methods from the theory of turbulent systems. Our main aim thereby is to elucidate these concepts on concrete examples of multiscale phenomena occurring in urban environments. The next section will then illustrate how our gained insights may be used for the multiscale modeling of urban structures. We believe that drawing a parallel between urban and turbulent systems may be fruitful for mainly two reasons.

Multiscale transfer processes As it has already been discussed in the Introduction, when cities were first perceived as systems in the 1960s, they were suggested to be in, or at least, constantly thrive towards a state of equilibrium, whilst at the same time, being clearly discernible from their surroundings. Similarly as today, they were thought of having a large set of interacting elements, however back then these were regarded as being somehow governable from the "top-down". Today, due to the emergence of complexity theory, we assume that cities are no longer equilibrium systems. Instead, as a result of complex multiscale interactions that one encounters in such *open systems*, the urban dynamic is constantly *far from equilibrium* and must therefore be regarded as a non-equilibrium system (Batty 2013). Similarly, stirring of a turbulent fluid (e.g., in a coffee cup), results in an open system where kinetic energy is constantly fed in at large scales. In such systems, nonlinear interactions may give rise to strong and continuous correlations between its numerous spatial and temporal regimes (Friedrich 2021). Accordingly, one of the central notions of turbulence theory is the concept of the *turbulent energy cascade*, which emphasizes the fact that turbulent flows are essentially transport processes of certain quantities like energy or heat *in scale*. Here, large-scale stirring of a fluid is followed by a "cascade" of instabilities of vortical structures or eddies, meaning that large-scale vortices decay into vortices of smaller sizes until vortical structures reach a size where they are subject to viscous forces. Hence, via this cascade process, energy which was fed into the system at large scales is transferred to successively smaller scales and it is ultimately dissipated into heat. In the context of urban systems, we believe that similar transport processes may occur, e.g., in form of policies or various financial transactions. Therefore, we will now illustrate three main characteristics of the cascade process in more detail, that of *stationarity, local isotropy and universality*:

Firstly, as far as the temporal component of these processes is concerned, one assumes that the rate at which quantities are "injected" at large scales equals the rate at which they are "dissipated" at lower ones thus implying a *stationary* process. Whether or not this assumption of statistical stationarity in urban systems is entirely fulfilled is left for future work. Nonetheless, other complex systems, such as stock market prices seem to fulfill this assumption quite well (Ghashghaie et al. 1996). Note that certain quantities might also exhibit what is known as an "inverse cascade", where a transport process from small to large scales occurs, resulting in self-organized large-scale patterns (in the case of two-dimensional turbulence, for instance, vortices tend to develop in clusters and energy is transferred to large scales (Friedrich 2021)). Concepts and methods developed in the context of self-organizing systems have been applied rather extensively in the urban context for example as means to explain how cities evolve and form characteristic spatial patterns (Haken and Portugali 1995; Portugali 1997; Allen 2012). The second important assumption in the context of latter cascade processes is that interactions are assumed to be merely *local*, occurring from one scale to another. This effectively reduces the highly complex spatial statistics (or the joint probability distribution function) into individual transitions in between scales (so-called transition probabilities) (Friedrich and Peinke 1997; Friedrich et al. 2011). Third, statistical quantities are assumed to be *universal*, i.e., they are independent from large-scale features such as boundaries, etc. This immediately implies the existence of an *inertial range of scales* where statistical quantities are describable by only a few characteristic features. As it will be shown at the example of urban rent price fields, such an inertial range of scales can indeed be observed, which therefore underlines the importance of cascade phenomena for the modelling of urban structures. Nonetheless, before we further address this issue, we will put forth a second analogy between urban and turbulent systems, which is the empirically observed occurrence of extreme fluctuations at small scales.

Small-scale intermittency Due to its large number of constituents and strong correlations in between spatiotemporal dimensions, a turbulent system is proven to be non-self-similar in nature. The latter feature implies an increased probability for the occurrence of strong fluctuations with decreasing scale in comparison to large-scale statistics that are governed by close-to Gaussian (equilibrium) statistics (Friedrich and Peinke 1997; Frisch 1995). This phenomenon, which is referred to as small-scale intermittency, can also be encountered in other systems, e.g., stock market fluctuations (Ghashghaie et al. 1996) or hydraulic conductivity measurements (Meerschaert et al. 2004), and must be considered as one of the main signatures of strongly correlated systems. Hence, it is compelling to observe if this fundamental characteristic of small-scale intermittency in turbulence may also be found in urban settings. Similar presumptions of the urban environment (possessing strong spatial correlations) are supported by recent studies of the spatio-temporal distribution of land prices in an area near Wuhan City, China (Hu et al. 2012), as already indicated in the background section of this paper. In general, it is not hard to find examples for such phenomena in urban systems. In case of the Ruhr area for instance, one could argue that whenever the “Centro”, Germany’s largest shopping mall opened in Oberhausen Neue Mitte in 1996 (in the very heart of the Ruhr area), it significantly convulsed the local urban dynamic. On the one hand, it can be maintained that over time, businesses in the traditional city center could not keep up with the popularity of the new center and as one closed down after the other, urban decay was - further - exacerbated in the inner-city of Oberhausen (however the deterioration of the old city center may have had several other roots as well). On the other hand, one could regard the district of Neue Mitte to be in a complementary relationship with the “old” city center, targeting different audiences and establishing an - at the time - much needed center for entertainment, culture and recreational services in Oberhausen and in the Ruhr region in general, which could even provide a viable alternative to the neighboring large city centers (e.g., Düsseldorf (Stadt Oberhausen 2020)). No matter how one may look at the phenomenon, it can very well be considered an extreme event on the small scale. From a regional or even national perspective however, it could be regarded as “another shopping mall” constructed in the last decades of the twentieth century. As we will demonstrate, one of the main advantages of the here proposed modeling procedure is that such occurrences as the above example in Oberhausen are implicitly incorporated.

The original rent price field Let us now inspect this idea in more detail using the example of rent price indices on the building level in the Ruhr area. To this end, we perform a statistical analysis of spatial rent price increments comparable to the usual investigation of small-scale intermittency in turbulent flows. We start by defining the rent price field as $\Lambda(\mathbf{x})$ where $\mathbf{x}$ denotes a two-dimensional vector $\mathbf{x} = (x, y)$ with x and y denoting the projected coordinates of each building’s centroid. Second, we define the single-increment probability density function (PDF) $f(\lambda, r)$ which can be interpreted as the probability of encountering a rent price increment $\Lambda(\mathbf{x} + \mathbf{r}) - \Lambda(\mathbf{x})$ at a certain scale $r = \sqrt{x^2 + y^2}$. Here, it has to be stressed that we assumed a rotational and translational invariance of the single-increment PDF. The observed length scales r are ranging from 30 to 1200 meters and we applied a rolling buffer proportional to r in order to obtain the increments.

The occurrence of strong small-scale fluctuations can now be investigated by considering the evolution of the single-increment PDF in scale r as depicted in Figure 2. If rent price fields were to exhibit self-similar features, the single-increment PDF should have the explicit form

$$f(\lambda, r) = \frac{1}{r^H} g\left(\frac{\lambda}{r^H}\right) \;, \tag{1}$$

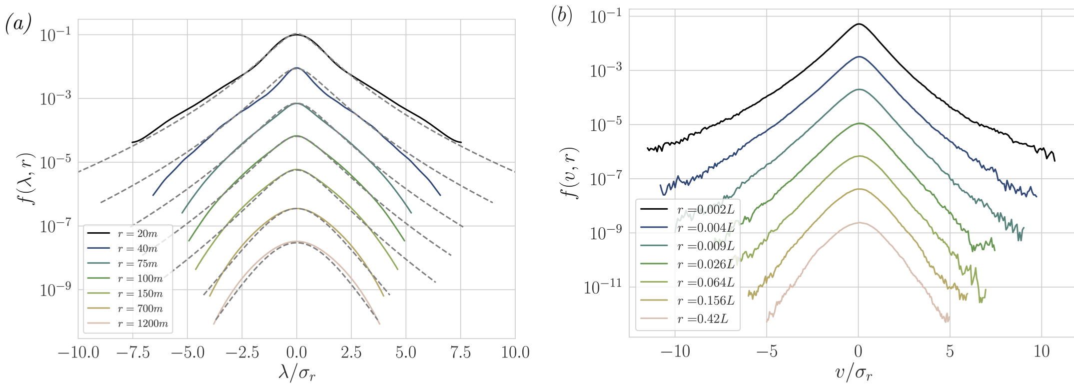


Figure 2. *(a)* Kernel density estimation of the probability density function (PDF) of rent price increments $f(\lambda, r)$ at different scales r where σ_r denotes the standard deviation of the increment at scale r. Whereas, large-scale statistics are close-to Gaussian, the PDFs exhibit non-Gaussian features at small scales manifesting themselves by an increased probability of strong small-scale increment fluctuations. Dashed lines indicate the PDFs of the K62 model of turbulence, see also Supplementary Material for further information. *(b)* The PDFs of velocity increments $f(v, r)$ in a von Kármán experiment using normal Helium. Similarly, to rent price fluctuations, velocity fluctuations exhibit non-Gaussian features at small scales. It is to be noted that, due to increased number of measurement points in the turbulence dataset in comparison to the rent price data, the PDFs extend further than in (a).

and the shape of the PDFs in Figure 2 should remain the same. Furthermore, Eq. (1) entails scaling of the moments of the rent price increments according to

$$S_n(r) = \langle [\Lambda(\mathbf{x}+\mathbf{r}) - \Lambda(\mathbf{x})]^n \rangle = \int \mathrm{d}\lambda \lambda^n f(\lambda, r) = C_n r^{nH}, \tag{2}$$

where the brackets denote spatial averaging over $\mathbf{x}$ and where we introduced the coefficients $C_n = \int \mathrm{d}\xi \xi^n g(\xi)$. Scaling laws similar to Eq. (2) indicate monofractal behavior and have been extensively discussed within the fractal theory of cities (Batty and Longley 1994; Tannier and Pumain 2005). Here, the Hurst exponent H is the roughness of the field and lies between 0 and 1, with $H = 1$ corresponding to completely smooth structures (Mandelbrot and Ness 1968; Grebenkov et al. 2015). However, as Figure 2(a) implies, rent price field increments are *non-self-similar* in scale, i.e., the scale evolution of the increment PDF from large to small scales deviates from Eq. (1), similarly to the phenomenon of small-scale intermittency in turbulent flows. The rent price field thus exhibits an increased probability for the occurrence of large (positive and negative) small-scale fluctuations in comparison to Gaussian statistics at large scales. This analogy is mainly supported by Figure 2(a), which bears a remarkable resemblance to Figure 2(b) depicting the PDFs of turbulent velocity increments obtained from hot wire anemometry in the superfluid high Reynolds von Kármán experiment (SHREK) at CEA-Grenoble (Rousset et al. 2014). Nonetheless, the figures also exhibit certain differences: Whereas the turbulent increment PDF is slightly skewed (the skewness can in fact be related to the rate at which energy is dissipated in the fluid), the rent price field seems to be symmetric. Furthermore, the convexity in Fig. 2 appears more pronounced in the turbulent case, however this difference may be partly attributed to insufficient statistics in the urban domain. At this point, further studies are needed in order to verify whether these differences in skewness and convexity are in fact intrinsic features of rent price fields.

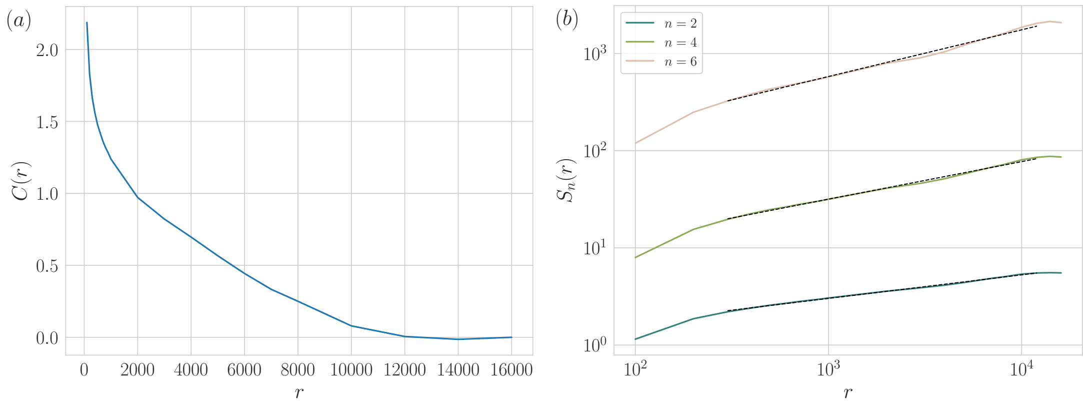


Figure 3. *(a)* Correlation function $C(r) = \langle \Lambda(\mathbf{x})\Lambda(\mathbf{x}+\mathbf{r})\rangle$ of the rent price field. The integral length scale L (see main text for further explanation of this quantity) can be obtained from an exponential fit according to $L = (3454 \pm 147)$m. *(b)* Structure functions $S_n(r) = \langle[\Lambda(\mathbf{x}+\mathbf{r}) - \Lambda(\mathbf{x})]^n\rangle$ of the rent price field. Dashed lines indicate power law fits $S_n(r) \sim r^{\zeta_n}$ in the inertial range of scales.

In the following, we perform a further analysis of the urban rent price field and determine certain characteristic quantities that are central to the phenomenological description of turbulent flows (Frisch 1995; Friedrich 2021; Monin and Yaglom 2007) and that are also indispensable for the modeling task in the second part of this paper. Figure 3(a) depicts the spatial correlation function $C(r) = \langle \Lambda(\mathbf{x})\Lambda(\mathbf{x}+\mathbf{r})\rangle$ for different values of the scale separation r. From this quantity, which is also heavily used in spatial analysis (Getis 2007; Myint and Lam 2005), we can infer the so-called integral length scale L, which can be interpreted as a large scale above which spatial correlations decay exponentially $C(r) \sim e^{-r/L}$. Hence, the usual definition of the integral length scale is $L = \int_0^\infty \mathrm{d}r \frac{C(r)}{C(0)}$. In case of the urban rent price field in the Ruhr area, we determined the integral length scale as $L = (3454 \pm 147)$m from an exponential fit in Figure 3(a). Latter implies that for our current case study, rent prices that are located at a distance larger than L are uncorrelated and do not significantly influence one another.

In order to quantify the intermittency behavior that is already visible in Figure 2(a) in more detail, we use the Kolmogorov-Oboukhov (K62) model of turbulence (Kolmogorov 1962; Obukhov 1962), which predicts anomalous scaling of the structure functions of order n (2) as $S_n(r) \sim r^{\zeta_n}$ with scaling exponents $\zeta_n = H(1+\frac{\mu}{2})n - H^2\frac{\mu}{2}n^2$. Here, setting the intermittency coefficient $\mu = 0$ results in the monofractal behavior given by Eq. (2). We displayed structure functions of the urban rent price field in log-log representation for orders $n = 2, 4, 6$ in Figure 3(b). We observe power law behavior in the inertial range, i.e., for $\eta \ll r \ll L$, where η denotes a small length scale, which, in turbulence theory is characteristic of smooth, dissipative field structures. Focusing now on this power law behavior in Figure 3(b), dashed lines correspond to fits r^{ζ_n} in the latter described inertial range. From the scaling exponents ζ_n we are able to determine the Hurst exponent $H = 0.121 \pm 0.013$ and the intermittency coefficient $\mu = 0.391 \pm 0.065$. The significance of these particular values for urban structures cannot be overstated: First, the Hurst exponent H can be interpreted as the level of self-similar fragmentation of the rent price field. The relatively low value that is found here (the self-similar theory of turbulence suggests a Hurst exponent of $H = 1/3$) indicates that the rent price field is extremely fragmented or *rough*. On the other hand, deviations from this self-similar behavior are also quite pronounced due to the relatively high value of the intermittency coefficient μ as already observed in Fig. 2(a). Here, we can estimate that the calculated probability for a rent price fluctuation $\lambda = 7\sigma_r$ (where σ_r denotes the standard deviation of this fluctuation) at a scale $r = 20$m is approximately 10^8 times higher than for a merely self-similar (Gaussian) PDF with $\mu = 0$. This implies that if one observes such level of fluctuation within a radius of 100m, one would have to take into account an area with a radius of 1000km to obtain the same level of fluctuation if the distribution were Gaussian. As it will be further discussed in the following section, such rather intricate intrascale dependencies of the rent price field can now be modeled on the basis of these two parameters.

Multiscale modelling of urban structures

In this section, we demonstrate how one may apply the above formulated analogy to the modelling of urban structures, or in this case, to generate a synthetic rent price field based on a small number of prescribed "real-world" points. To this end, we capture the rent price distribution in the following way:

Hot and cold spots Firstly, we assume that the rent price field can be decomposed into certain central locations or hot and cold-spots (centers of high and low rent prices). We define hot and cold spots via a simple spatial K-means clustering to find fifty meaningful spatial clusters and their centroids. Throughout numerous trials the algorithm assigns each observation to the cluster with the nearest mean, and the centroid point of these clusters are then our initial hot- and cold-spots (see Figure 5). The main goal thereby was to restrict ourselves to a minimal number of prescribed points that may still be able to preserve the large-scale characteristics of the rent price field (depletion of rent prices around the Emscher river and North-South differences). Here, we intentionally use clusters of highest and lowest regional values to minimise the number of points necessary for modelling, however as we will argue under the potential-applicability section, any georeferenced precise measurement may be fed into the model. Secondly, on the basis of the analysis put forth in the previous section, we introduce a modelling approach comprised of two main steps: *i.)* an embedding of the latter prescribed measurement points (in this example the 25 hot- and 25 cold-spots) into a random field, and *ii.)* an introduction of small-scale fluctuations into this random field that exactly follow the K62 model (dashed lines in Figure 2(a)) and thus approximate the intermittency properties of the original rent price field.

Modelling roughness As far as *i.)* is concerned, let us denote the prescribed measurements at points $\mathbf{x}_k$ as Λ_k (where in our case $k = 1, \ldots, 50$ stands for the 25 hot-and cold-spots). In order to interpolate between these points we consider a Gaussian random field $\Lambda(\mathbf{x})$ with monofractal scaling (2) known as fractional Brownian motion (fBm) (Mandelbrot and Ness 1968; Grebenkov et al. 2015). For better comparability, three typical realization of fBm of a one-dimensional fBm rent price field with different roughness (characterized by the Hurst exponent H) are depicted in Fig. 4(a). As it is further discussed in the Supplementary Material, we can construct a multipoint fractional Brownian bridge (Friedrich et al. 2020) from such random fields according to

$$\Lambda^B(\mathbf{x}) = \Lambda(\mathbf{x}) - \langle \Lambda(\mathbf{x})\Lambda(\mathbf{x}_j)\rangle \, \sigma_{jk}^{-1} [\Lambda(\mathbf{x}_k) - \Lambda_k] \;, \tag{3}$$

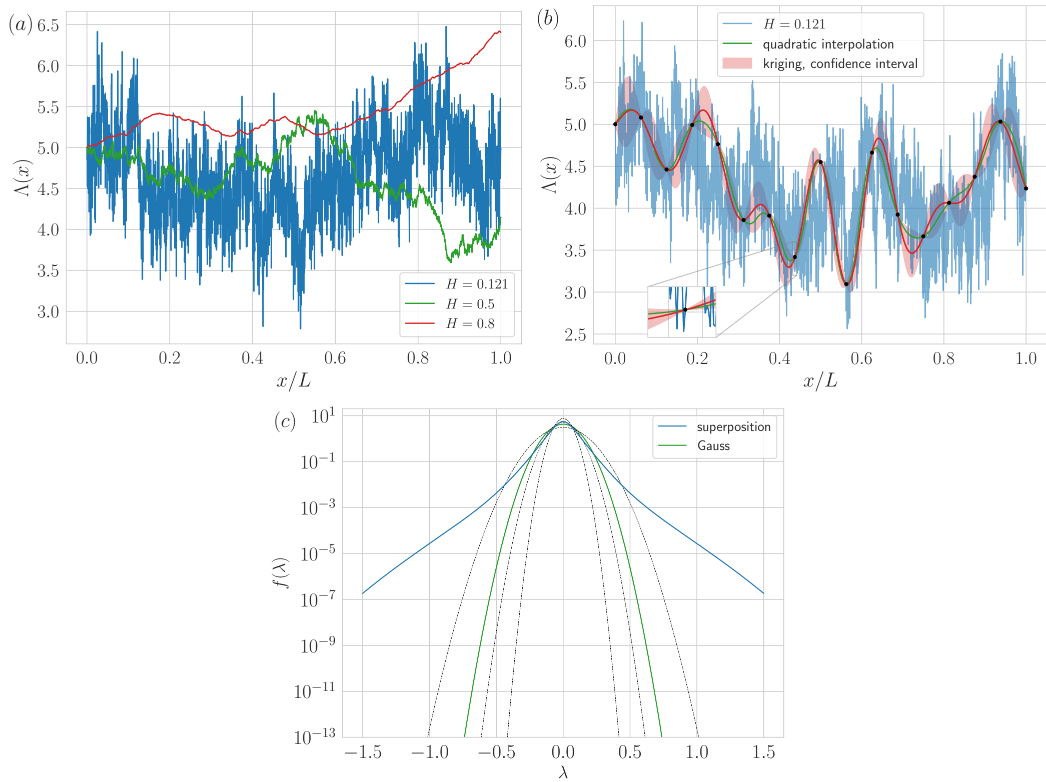


Figure 4. *(a)* Typical realizations of fractional Brownian motion (fBm) for varying Hurst parameters H. The "roughest" fBm with $H = 0.121$ corresponds to the monofractal behavior of the land price field which has been inferred from Figure 3. *(b)* Schematic comparison between spatial interpolation schemes. The black prescribed points (fBm with $H = 0.121$) are interpolated by a standard quadratic interpolation (orange), a standard kriging interpolation, and a multipoint fractional Brownian bridge in Eq. (3) with matching $H = 0.121$. *(c)* Schematic depiction of a Gaussian scale mixture. Several Gaussian distributions with varying variances (three examples are shown in dashed lines) are superposed leading to a probability density function with heavy tails (blue). For comparison, the green curve shows a Gaussian distribution with the same variance as the superposition (blue).

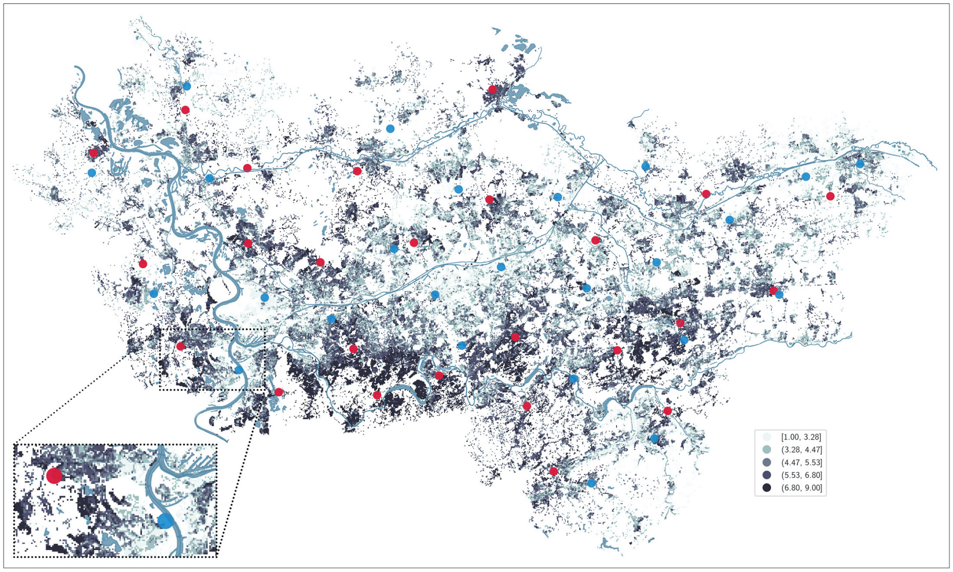
where we imply summation over identical indices and where σ_{jk} denotes the covariance matrix of the fBm, i.e., $\sigma_{jk} = \langle \Lambda(\mathbf{x}_j)\Lambda(\mathbf{x}_k)\rangle$. This construction now ensures that the bridge process exactly hits the prescribed points at $\mathbf{x}_k$, namely $\Lambda^B(\mathbf{x}_k) = \Lambda_k$ (see Supplemental Material for the derivation), and that it possesses monofractal scaling characterized by the Hurst exponent H. Hence, unlike other interpolation schemes, e.g, polynomial schemes or kriging that exhibit smooth features, the here-proposed method conserves the self-similar fragmentation/roughness of the original rent price field over an arbitrary range of scales. Figure 4 shows a schematic comparison of three different interpolation schemes for the points Λ_i (black): a standard polynomial interpolation (green), a simple kriging interpolation with optimized Gaussian variogram, and the multipoint fractional Brownian bridge. Here, the prescribed points Λ_i were drawn as fBm with $H = 0.121$, and the parameters of the covariance of the kriging interpolation (Cressie 1990) were optimized with respect to the covariance of the fBm. It is visible that neither the quadratic nor the simple kriging interpolation can achieve the roughness of the multipoint fractional Brownian bridge. As far as potential applications are concerned, one may for instance think of a scenario where the interpolated fields are used to derive other urban characteristics, e.g., in an agent-based residential model in which individual residents have to adapt to local rent price fluctuations (Lengyel and Friedrich 2020).

Modelling intermittency Turning to *ii.)*, we now present a quite general method which allows to sample fluctuations from a joint probability and is thus capable to introduce non-Gaussian features into the bridge construction discussed in *i.)*. One must emphasize that the core strength of the here deployed method is that it extends the conventionally used Gaussian *single-scale* (or monofractal) statistics in modelling to so-called *multiscale* (or multifractal) statistics via a superposition of multivariate Gaussian distributions (Friedrich et al. 2021). As it is further outlined in the Supplementary Material, non-Gaussian/intermittent properties (dashed lines in Fig. 2(a)) emerge due to fluctuations of the covariances that characterize each Gaussian distribution. This is schematically depicted in Figure 4(c) where a superposition of Gaussians with different variances (dashed lines) lead to heavy-tailed behavior (blue curve). As a result of this multivariate so-called Gaussian scale mixture, *intrascale spatial dependencies of the whole rent price field domain exert local influence at each modelled grid point*. Ultimately, it is exactly this modelling feature that allows us to embed complex multiscale spatial interconnectedness into our model, i.e., all scales are influencing the local results simultaneously.

Results The reconstructed rent price field is depicted in Fig. 5(b) and can be considered a rather good surrogate model for the original rent price field in Fig. 5(a). If we now inspect Figure 6(a) displaying the difference between the original and the synthetic rent price field per each grid cell, we may conclude the following: It is along the Emscher river where real values are below the regional average and are often at extreme ranges thus the model overestimates, and the inverse is true for the vicinity of the Ruhr river in the South. Latter charactersitic layered structure of the Ruhr area (also shown in in Fig. 5(a)) can be attributed to its historical development that both industrialisation and deindustrialisation followed a south to northwards path (Wehling 2014) and districts along the Emscher are still bearing heavy marks of the economic structural change. However, if we turn our attention to Figure 6(b) we derive that apart from a small number of exceptions along the Rhein river, there are no large-scale spatially continuous areas that are being systemically misestimated. More importantly, there appears to be no consistency between the location of our hot- and cold-spots and the distribution of cells that are being overly miscalculated. To summarize, the combination of *i.)* and *ii.)* allows for a surrogate model of urban rent price fields (or basically any spatial urban field quantity) by taking into account a certain large-scale configuration and filling it up by a random field that is characterized by the Hurst exponent H and the intermittency coefficient μ. Hence, the proposed surrogate model may be considered as a hybrid model between (incomplete) empirical observations and a rather advanced phenomenological model of complex spatial fields. Here we only highlighted and explained these main features in a broader sense however for a more profound proof of concept we refer the reader to the Supplementary Material.

Potential Applicability The immense reduction of degrees of freedom where we reconstructed the original 440572 spatial values with the help of only 50 points may have several areas of applicability for the urban analysis, modelling, planning and decision-making community. Firstly, the methodology is rather convenient for modelling the time evolution of large-scale urban structures, as demonstrated in the

(a)



(b)

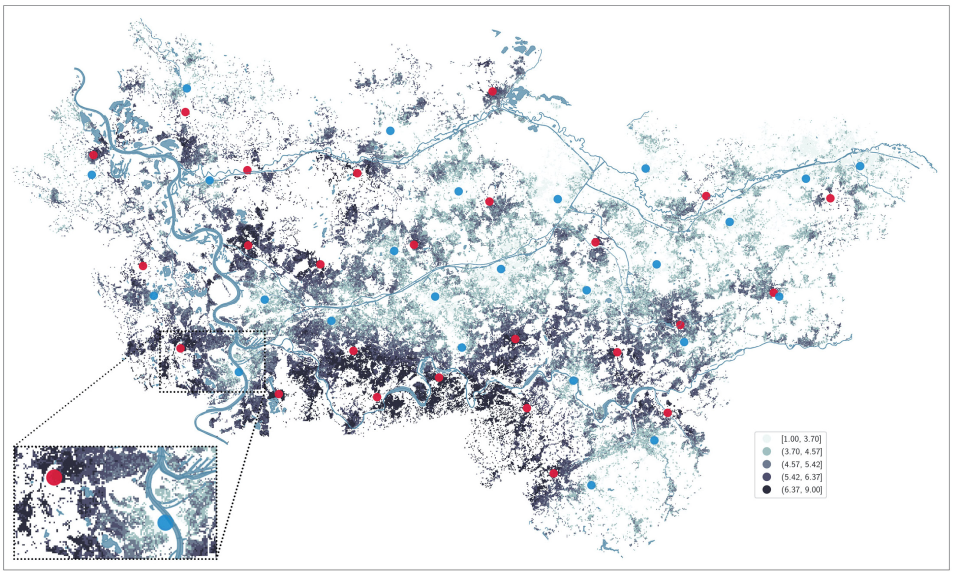


Figure 5. *(a)* Original rent price field in 2016: Average value of mean rent price indices on a 100 x 100 meters grid. The field consists of 440572 points. Mean = 4.969. *(b)* Synthetic rent price field in 2016: Average value of mean rent price indices on a 100 x 100 meters grid. The field is reconstructed from 50 sampling points and possesses the same mean as in (a). *Red dots*: Hot-spots. *Blue dots*: Cold-spots. *Inset*: Local housing market condition captured as an interplay between hot- and cold-spots in close vicinity.

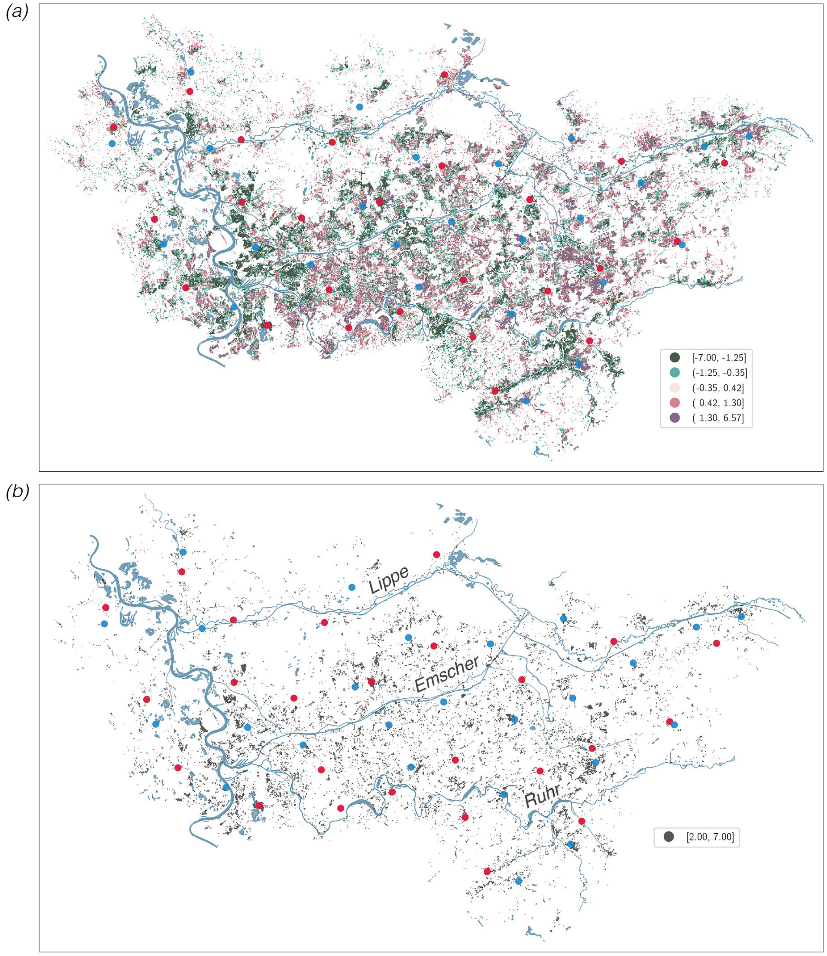


Figure 6. *(a)* Difference between the original and synthetic rent price field values pro grid cell (in 2016). Red dots: Hot-spots. Blue dots: Cold-spots. *(b)* Grey areas: cells with absolute difference larger than two. Red dots: Hot-spots. Blue dots: Cold-spots.

New Emscher Mobility (NEMO) project by Lengyel and Friedrich (2020). It is well known that land price greatly impacts land use and its development (Hu et al. 2012). Thus as an example, in the NEMO project embedded multiscale urban model (Lengyel and Friedrich 2020, 2021) the here developed synthetic field is being reconstructed each and every year depending on where firms and people are moving to (that being motivated by the master equation approach (Haag 2017)): If a certain number of employees (and people) move to a new area, then a novel rent price hot-spot emerges - and the inverse is happening for cold-spots. Thereby, rent price values which are not available in yet inhabited but potentially procurable areas are evaluated via the method of synthetic rent price fields and in course of the simulation, grid cells containing such areas can be unlocked for future expansions. It is easy to see how one could extend this methodology for the modelling of further urban subsystems such as morphological, social or economic ones. Moreover, due to its transparent mathematical description and computational efficiency the approach is well integrable with other methods from the urban modelling domain and it is also easily customisable for the testing of specific urban planning and design strategies.

Secondly, as the model's main feature is data-reconstruction it could be conveniently applied at data-scarce locations as means of informing urban decision-making processes (e.g., land use, land price, social-economic policy). Analysing the distribution of urban indicators through space, and when available through time, can help to define the location and scheduling of measures taken and thereby greatly increase efficacy. However, we must emphasize that the methodology is not compatible for the assignment of smaller scale values as a function of aggregate ones, i.e for data-downscaling. In contrast, our approach operates with precise georeferenced values and is able to interpolate between them at arbitrary spatial and temporal resolution. Thus, it may also prove to be helpful for the statistical spatial modelling of survey results e.g. generating micro-scale census data from sparse samples, or for the modelling of environmental and climate data (e.g., aerosol concentration or temperature) usually obtained via a small number of sensors at carefully selected locations. Finally, data-remodelling for better comparability across locations, such as the Ruhr area, may support cross-boundary decision making processes. In this data-remodelling context, one could further mention measures for the privacy-protection of individual residents.

To sum up, in the first part of this paper we observed how in both Figures 2 (a) and (b) - one derived from a turbulent the other from an urban system - the tails of the PDFs become increasingly pronounced at small scales, underlining the phenomena of higher fluctuations or more frequent extreme events at micro levels. These extreme fluctuations have been modeled with the help of a non-Gaussian bridge process in the second part.

Discussion and conclusions

Our paper empirically devised the remarkable similarities between urban and turbulent systems on the example of rent price values in the Ruhr area. This enabled us to capture two quintessential features of urban systems: the multiscale phenomena as the result of spatiotemporal transfer processes in scale and the so-called non-self-similarity. In more detail, we demonstrated how the analogy can be catered for deriving universal characteristics as the direct consequence of strong interdependence of spatial scales in urban systems. The use of turbulence theory to quantify this interdependency allowed us to borrow its corresponding numerical methods from statistical physics which led to a mathematically transparent and computationally efficient model of multiscale urban phenomena that is well integrable with other modelling approaches (as for instance with sociodynamics (Haag 2017; Lengyel and Friedrich 2020)) and may easily be customizable for specific urban research or policy questions. n terms of analysis results, it has to be stressed that we find here empirically that the self-similar part of the land price field, $H = 0.121$ is much *rougher* than that of a turbulent velocity field (the K41 model of turbulence suggests $H = 1/3$ (Frisch 1995)). Such degree of roughness is typically not captured by other interpolation methods, e.g., polynomial interpolation or kriging, but can be reproduced by the stochastic interpolation used here (Friedrich et al. 2020). However, both the analysis and modelling part remains to be tested for other urban subsystems, such as population or land use distribution, where the here addressed universal characteristics may somewhat differ and thus demand the introduction of further mathematical concepts. Note that even though in the current framework, multiscale properties have been modeled on the basis of the Hurst exponent H and the intermittency coefficient μ using a multiscale methodology, the two values were solely obtained on the basis of a single-scale analysis. A future task is to identify intrascale processes in more detail and introduce concepts and methods that already proved to be fruitful in the context of other complex systems (Friedrich and Peinke 1997; Friedrich et al. 2011). Finally, we argued that our modelling approach is applicable for data-reconstructing and data-remodelling efforts, however their further real-world testing still remains to be explored. In summary, we believe that devising similar rather advanced statistical descriptions of urban phenomena is a pressing and important research branch that aids our quantitative understanding of multiscale phenomena in urban systems a step further.

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