Efficient Stochastic Analysis of Transmission Signal Integrity for Remote Sensing Applications

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***Abstract*—This paper presents a statistical evaluation of the transmission signal integrity at the intra-system level of remote sensing devices. The signal integrity is evaluated using the crosstalk induced to the signal. Due to the vague knowledge of the characteristics of the signal cable, the crosstalk becomes a random quantity and needs to be described using statistics. In this paper, the *stochastic reduced order model* (SROM) method is applied to obtain the statistics of crosstalk in the presence of the random cable configuration. By comparing to the Monte-Carlo method, it is found that the SROM method is much more efficient and accurate. Due to the generality of the SROM method, it may be inspiring to solve other stochastic electromagnetic compatibility problems of remote sensing devices.**

***Keywords*— cable crosstalk; electromagnetic compatibility; remote sensing; statistical analysis; stochastic reduced order models**

I. Introduction

Remote sensing technologies acquire the information about an object of interest without the need of physical contact. The acquired information can be used in various domains, such as the geographic study and environmental monitoring. Typically, remote sensing devices non-intrusively probe the environment via the use of radar, antennas, and sensors. However, inside the remote sensing device, the detected signal from the environment is transmitted between different processing modules through cables. Taking the frequency-modulated sensing radar for example, the crosstalk could typically occur between the transmitting and receiving branches, thus degrading the system performance. One reason of crosstalk is the wires corded in close proximity in the cable. As a result, crosstalk can be introduced to the wires and degrade the signal integrity, causing malfunctions in the signal processing units. Therefore, the crosstalk of the wire is an important electromagnetic compatibility (EMC) aspect of the system, and needs to be evaluated at the early stage.

The twisted-wire pair (TWP) is widely used to transmit signals between components, due to its good immunity to electromagnetic disturbances. Therefore, the TWP coupling problem is of great importance to engineers. The general methodology of cable crosstalk prediction was described in [1]. A transmission-line model for predicting the crosstalk involving TWP was developed in [2]. This prediction model was simplified under the low-frequency assumption in [3], to provide insightful understanding of the coupling mechanism in the TWP case. In [4], experiment was performed to validate the model of the TWP in [1]-[3].

The model for predicting the crosstalk in the TWP case described in [1]-[3] is based on the deterministic analysis. That is, the variables characterising the model were assumed to take nominal values that are believed to truly reflect the reality. The result of crosstalk is single-valued from the deterministic analysis. However, in reality, uncertainty is inevitably introduced to the cable system. Due to uncertainty, the electrical property and geometry of the cable are always different from the nominal in an unpredicted way, making the crosstalk a random quantity as well. Therefore, the result of the deterministic prediction is unconvincing, as it fails to capture the variability of crosstalk. Clearly, it is more sensible to statistically describe the crosstalk using statistical methods.

The conventional statistical approach is the Monte-Carlo (MC) method [5]. The MC method takes into account the uncertainty by repeating the simulation with different input values for a sufficient number of times. This exhaustive evaluation of all the possible cases makes the statistical analysis very consuming, especially in the presence of a large number of random variables. To increase the efficiency of statistical analysis, novel statistical methods were proposed, such as the polynomial chaos expansion (PCE) method [6], the stochastic collocation (SC) method [7], and the *stochastic reduced order model* (SROM) method [8]. The aim of the PCE and SC methods is the same: to derive the analytical relationship between the output and the random input variables. After the analytical relationship is acquired, the statistics of the output can be obtained using the mathematical transformation techniques described in [9]. Please see [10] and [11] for applications of the PCE and SC methods on the uncertainty quantification of cable crosstalk. The SROM method was recently proposed in [8] as a non-intrusive and efficient statistical approach. Compared to the MC method, the SROM method uses much less computational cost without losing accuracy. The first application of the SROM method on electromagnetic compatibility (EMC) problems can be found in [12].

This paper is aimed to present the application of the SROM method on the prediction of the crosstalk statistics in the TWP scenario. In this paper, the cable is modelled using a single

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conductor and a TWP above the ground plane. The randomness is introduced to the geometry of the wire position in the cross section of the cable. The subsequent sections of the paper are structured as follows: Section II presents an overview of the SROM method. In Section III, the model of the cable structure is introduced. Section IV presents the application of the SROM method to predict the probabilistic laws of crosstalk subject to the random cross section geometry. The conclusion of the paper is given in Section V.

II. Stochastic Reduced Order Model (SROM) Method

The idea of the SROM method is to select a very small number of samples with assigned probabilities to accurately approximate the statistics of the random input variables. Thus, the deterministic solver only needs to evaluate these selected input samples to reduce computational cost.

Let us assume that the number of random variables in the problem of interest is D. The uncertain input space can be represented by a D-dimensional random variable $\boldsymbol{X} = [X_1, X_2, \ldots, X_D]$. Each dimension of $\boldsymbol{X}$ is dedicated to describe a random input variable. By default, the statistics of the i-th dimension X_i ($1 \leq i \leq D$) are already known in terms of the cumulative distribution function $F_i(\theta)$ and the q-th order moment $\mu_i(q)$, defined as [13]:

$$F_i(\theta) = P(X_i \leq \theta)$$

$$\mu_i(q) = E(X_i^q)$$

where $F_i(\theta)$ represents the probability of X_i taking a value not greater than θ. The symbol $E(\cdot)$ is the calculation of the mean value. In addition to (1) and (2), another statistical property of $\boldsymbol{X}$ is the correlation function given by:

$$\boldsymbol{r} = E[\boldsymbol{X}\boldsymbol{X}^T]$$

A. Definition of SROM

A SROM $\widetilde{\boldsymbol{X}}$ is a statistical representation of $\boldsymbol{X}$. The SROM $\widetilde{\boldsymbol{X}}$ consists of a sample set $\tilde{\boldsymbol{x}} = \{\tilde{\boldsymbol{x}}^{(1)}, \ldots, \tilde{\boldsymbol{x}}^{(m)}\}$ and a probability set $\{p^{(1)}, \ldots, p^{(m)}\}$. The choice of the sample size m is based on the requirement of the computational cost and accuracy. Each sample $\tilde{\boldsymbol{x}}^{(k)}$ in $\tilde{\boldsymbol{x}}$ contains a set of coordinates $(\tilde{x}_1^{(k)}, \ldots, \tilde{x}_D^{(k)})$ for each dimension in $\boldsymbol{X}$. The statistics of $\widetilde{\boldsymbol{X}}$ can be defined similarly as the counterparts in (1) - (3):

$$\tilde{F}_i(\theta) = P(\tilde{X}_i \leq \theta) = \sum_{k=1}^{m} p^{(k)} 1(\tilde{x}_i^{(k)} \leq \theta)$$

$$\tilde{\mu}_i(q) = E(\tilde{X}_i^q) = \sum_{k=1}^{m} p^{(k)} (\tilde{x}_i^{(k)})^q$$

$$\tilde{r}_{ij} = E[\tilde{X}_i \tilde{X}_j] = \sum_{k=1}^{m} p^{(k)} \tilde{x}_i^{(k)} \tilde{x}_j^{(k)}$$

where $1(A)$ is the indicator function, that is, $1(A) = 1$ if A is true and $1(A) = 0$ if A is false.

B. Construction of SROM

Due to the loose definition of $\widetilde{\boldsymbol{X}}$, there could exist many candidate models $\widetilde{\boldsymbol{X}}$ for $\boldsymbol{X}$. However, the optimal $\widetilde{\boldsymbol{X}}$ minimising the difference between the statistics of $\boldsymbol{X}$ and $\widetilde{\boldsymbol{X}}$ must be used to accurately propagate the uncertainty from the input $\boldsymbol{X}$ to the output $\boldsymbol{Y}$. The methodology of finding the optimal $\widetilde{\boldsymbol{X}}$ is detailed in [12].

C. Uncertainty Propagation Using SROM

After determining the optimal $\widetilde{\boldsymbol{X}}$, the SROM-based output $\widetilde{\boldsymbol{Y}}$ can be obtained using the deterministic solver. The model $\widetilde{\boldsymbol{Y}}$ also consists of a sample set $\tilde{\boldsymbol{y}}$ and a probability set $\boldsymbol{p}_y$. The samples $\{\tilde{\boldsymbol{y}}^{(1)}, \ldots, \tilde{\boldsymbol{y}}^{(m)}\}$ in $\tilde{\boldsymbol{y}}$ are obtained by evaluating the samples $\{\tilde{\boldsymbol{x}}^{(1)}, \ldots, \tilde{\boldsymbol{x}}^{(m)}\}$ in $\widetilde{\boldsymbol{X}}$ using the deterministic solver. The probabilities $\{p_y^{(1)}, \ldots, p_y^{(m)}\}$ in $\boldsymbol{p}_y$ are given by: $p_y^{(k)} = p^{(k)}$, $1 \leq k \leq m$. The cumulative distribution function (CDF), q-th order moment, and standard deviation of $\widetilde{\boldsymbol{Y}}$ can be obtained by:

$$P(\widetilde{\boldsymbol{Y}} \leq \xi) = \sum_{k=1}^{m} p^{(k)} \boldsymbol{I}(\tilde{\boldsymbol{y}}^{(k)} \leq \xi)$$

$$E(\widetilde{\boldsymbol{Y}}^q) = \sum_{k=1}^{m} p^{(k)} (\tilde{\boldsymbol{y}}^{(k)})^q$$

$$\sigma(\widetilde{\boldsymbol{Y}}) = \sum_{k=1}^{m} p^{(k)} [\tilde{\boldsymbol{y}}^{(k)} - E(\widetilde{\boldsymbol{Y}})]^2$$

respectively. The statistics of the real output $\boldsymbol{Y}$ are approximated by those of the model $\widetilde{\boldsymbol{Y}}$ in (7) – (9).

III. Cable Model

In this section, the cable configuration is introduced, and the input variables and the output response of the coupling problem are defined. Specifically, the cable configuration contains a single wire and a TWP consisting of two wires twisted together, placed in parallel above the ground plane. The schematic of the cable configuration is shown in Fig. 1. In this study, each wire is modelled using a lossless conductor with the length L.

A. Input Variables

As shown in Fig. 1, the generator circuit is formed by terminating the single wire to the ground plane via the voltage source V_S with impedance Z_{0G} at the near-end and the load impedance Z_{LG} at the far-end. In the receptor circuit, the two twisted wires are connected at the near-end and far-end with the loads Z_{0R} and Z_{LR}, respectively, and terminated to the ground plane at the near-end. The ground plane acts as the current return path for the generator and receptor circuits.

As shown in Fig. 2, the cable configuration is characterised

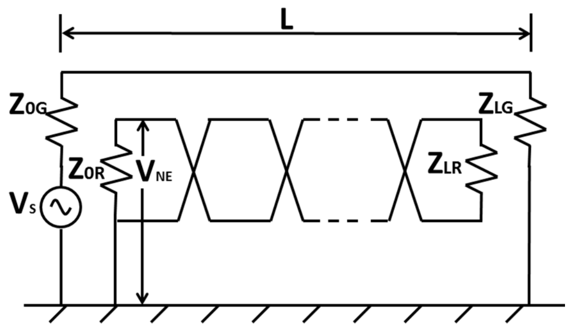

Fig. 1. The model of the cable configuration.

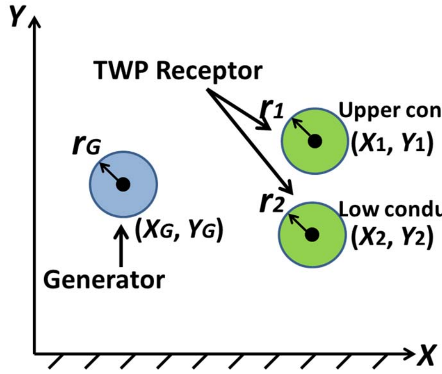

Fig. 2. Cross section view of the cable configuration.

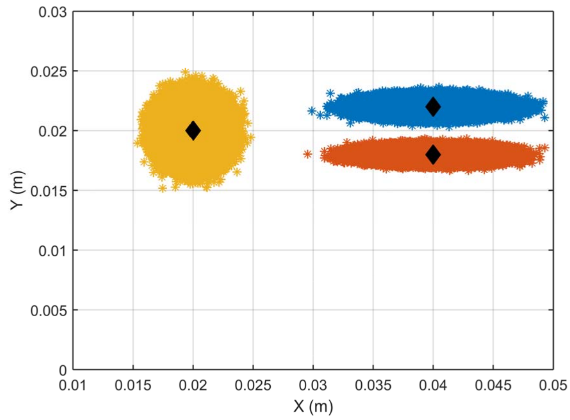

Fig. 3. Nominal wire positions in the uncertain regions.

by the following geometric variables: the radii r_1, r_2, and r_G of the two twisted wires and the generator wire, respectively. The centre of the generator conductor is denoted by the coordinates (X_G, Y_G) in the *x*-*y* plane. In the longitudinal direction, the TWP is seen as a cascade of *N* loops. Each loop is assumed to be rectangular, which means the two twisted wires are parallel in each loop and exchange positions at the interface between any two loops. The centres of the upper conductor and lower conductor in each loop are marked by the coordinates (X_1, Y_1) and (X_2, Y_2), respectively.

B. Output Response

When switching on the voltage source V_S in the generator circuit, the *near-end* terminal of the TWP receives the coupled voltage V_{NE} across the load Z_{0R}. The *near-end* crosstalk (*NEXT*) in this study is defined as the ratio of the coupled voltage V_{NE} to the voltage source V_S as:

$$NEXT = \frac{V_{NE}}{V_S}$$

The output from the deterministic solver is *NEXT* in this paper.

C. Deterministic Solver

The prediction model developed in [3] is used as the deterministic solver to calculate *NEXT* based on the values of the input variables.

IV. Numeircal Analysis

In this section, the SROM method is applied to obtain the statistics of the crosstalk (*NEXT*) subject to the random geometry of the cable configuration. Specifically, all the nine variables characterising the wire radii and positions are assumed to be random and follow Gaussian distributions. The detailed statistics of each random geometric variable are listed in Table I.

TABLE I
Statistics of the Random Input Variables

Input Variable	Mean (m)	Standard deviation (m)
X_1	0.04	0.002
X_2	0.04	0.002
X_G	0.02	0.001
Y_1	0.022	3.3×10-4
Y_2	0.018	2.7×10-4
Y_G	0.02	0.001
r_1	4.064×10-4	2.032×10-5
r_2	4.064×10-4	2.032×10-5
r_G	4.064×10-4	2.032×10-5

In Fig. 3, the nominal position of each wire is depicted with respect to 10,000 possible positions of that wire due to the aforementioned probabilistic laws. Specifically, the black diamond-shaped markers denote the nominal positions of the generator wire and the two twisted wires. The yellow, blue, and red shaded areas represent the uncertain regions of the generator conductor, and the upper and lower conductors of the TWP in the cross section of each loop, respectively. As can be seen, the uncertain region of the centre of each wire is chosen to be large to adequately validate the efficacy of the SROM method.

Other input variables are assumed to take deterministic values. Specifically, the four termination loads in Fig. 1 take the following values: $Z_{0G} = 0\ \Omega$, $Z_{LG} = 1000\ \Omega$, $Z_{0R} = 1000\ \Omega$, and $Z_{LR} = 1000\ \Omega$. The TWP consists of 225 loops with a total length of $L = 4.7$ m. As the prediction model developed in [3] is valid up to 1MHz, the frequency of the voltage source V_S is set to 1 MHz. The result from 1,000,000 MC simulations is used as the benchmark to evaluate the accuracy of the SROM result.

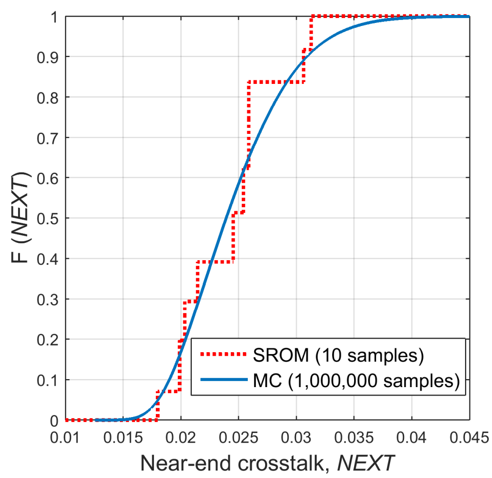

Fig. 4. Reference CDF of *NEXT* and the CDF approximated by the SROM-based output $\widehat{NEXT}$ with the sample size of 10.

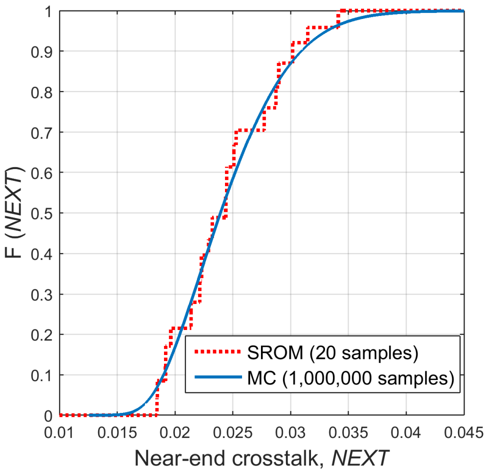

Fig. 5. Reference CDF of *NEXT* and the CDF approximated by the SROM-based output $\widehat{NEXT}$ with the sample size of 20.

First, the optimal SROM-based input $\tilde{\boldsymbol{X}}$ for the 9-D random variable $\boldsymbol{X}$ = [X_1, X_2, X_G, Y_1, Y_2, Y_G, r_1, r_2, r_G] needs to be constructed using the methodology described in [12], in order to represent the statistics of the uncertain input space. After choosing the optimal $\tilde{\boldsymbol{X}}$, the SROM-based solution $\widehat{NEXT}$ can be obtained using the deterministic solver. In the last step, the statistics of $\widehat{NEXT}$ are calculated using (7)-(9) to approximate the statistics of the real response *NEXT*.

Fig. 4 shows the derived CDF of the *near-end* crosstalk *NEXT* using the SROM method with only 10 samples, in comparison with the reference CDF given by 1,000,000 MC simulations. It is clear that the model $\widehat{NEXT}$ with the sample size of 10 is able to depict the general shape of the reference CDF. As shown in Fig. 5, by increase the sample size of $\widehat{NEXT}$ to 20, the difference between the reference CDF and the recovered CDF using the SROM method is further reduced. Clearly, the accurate CDF of *NEXT* can be approximated using the SROM method with only 20 samples.

Fig. 6 shows the convergence rates of the SROM method to obtain the accurate mean value and standard deviation of the output response *NEXT*. Here, the relative error is used to

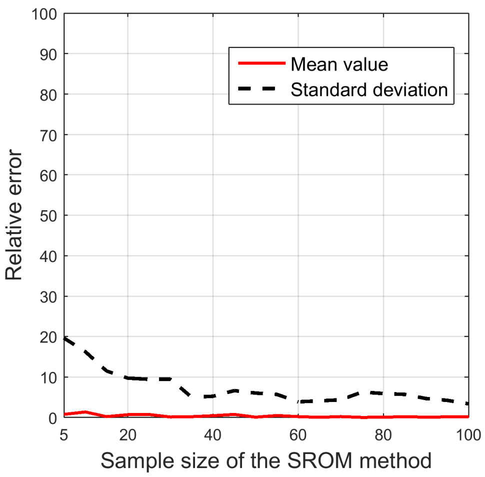

Fig. 6. Convergence rates of the SROM method.

measure the accuracy of the SROM result, and is defined as:

$$error = \frac{|Obtained\ result - Reference\ result|}{Reference\ result}. \quad (11)$$

As can be seen in Fig. 6, the convergence rates of the SROM method for the mean value and standard deviation are different. Specifically, the SROM method produces very accurate mean value from the sample size of 5 onwards, and needs 20 samples to converge within the error of 10% for the standard deviation. It is worth noting that the convergence rates of the MC method are neglected in Fig. 6. This is because the performance of the MC method is unstable at such small sample numbers, and therefore unrepeatable. Specifically, the MC method needs at least 400 samples to converge to the accuracy of the SROM result using 20 samples. Therefore, in this study, the SROM method is able to accelerate the statistical analysis by at least a factor of 400/20=20, compared to the MC method.

Clearly, due to the hazardous effect of crosstalk on remote sensing performance, it is necessary to estimate the crosstalk at the design stage. As explicitly demonstrated in Section IV, the computational load to obtain the statistics of crosstalk due to environmental uncertainty can be reduced significantly using the SROM technique.

V. Conclusions

In this paper, a statistical evaluation of the signal integrity at the intra-system level of remote sensing devices has been performed. Despite the large number of uncertainty sources in the cable configuration, the statistics of the cable crosstalk have been efficiently and accurately predicted using the SROM method. Compared with the traditional MC method, the computational cost required by the accurate statistics of crosstalk has been reduced by 20 times using the SROM method. Given the superior efficiency of the SROM method demonstrated in this study, it is hoped that the SROM method can be used to capture the variability of the system response in other stochastic EMC problems.

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