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Distributed Dynamic Event-Based Control for Nonlinear Multi-Agent Systems

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***Abstract*—This brief studies the synchronization problem for a class of QUAD and interconnected nonlinear multi-agent systems (MASs). A dynamic event-based control scheme is designed, and two event-based synchronization conditions are constructed by utilizing stability theory. Moreover, the Zeno-behavior can be excluded in the MASs. An example and its simulation are given to verify the applicability of the designed dynamic event-based protocol for MASs.**

***Index Terms*—Nonlinear multi-agent systems, dynamic event-based scheme, distributed control, synchronization.**

I. Introduction

The past decade has witnessed the fast development of modeling, analysis and control for MASs due to their wide applications in the fields of social networks [1], mobile robots [2], sensor networks [3], smart grids [4] and so on. The distributed cooperative control for MASs guides a team of agents in a distributed manner to accomplish a global task. Various distributed control schemes have been introduced for the synchronization of MASs, such as distributed impulse control [5], [6], adaptive control [7], and sampling control [8].

In practice, the communication among agents and the data update between sensors all consume computational resources of agents. However, most agents in the networks are only equipped with digital microprocessors due to cost constraints, and as a result the computation and communication powers of these agents are strictly constrained. Reducing the communication frequency or input updating rate are efficient schemes to maintain the control performance while satisfying the hardware constraints. Event-based control as a low resource consumption scheme is utilized to solve the synchronization of MASs, which has been widely studied [7], [9]–[13].

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Unlike the other sampling control schemes, the state of agents is guided back into a desired trajectory with relatively fewer communication or controller updates under the event-based strategy [14]–[16]. Such intermittent communication or discontinuous update methods have been used in, e.g., the multi-vehicle formation [17], and set stabilization of Boolean networks [14]. Many reported results show that the control performance can be maintained while the communication between agents can be reduced if the selected control is combined with the event-triggered strategy. For instance, Tan *et al.* discussed the distributed event-based impulsive consensus of MASs in [5], [6], in which the results have shown that the event-based impulsive schemes can effectively accelerate consensus speed.

However, most of these reported results depend on the knowledge of the Laplacian matrix (or its smallest nonzero eigenvalue) of the network typology. For example, the linear matrix inequities (LMI) of the criteria in [5], [6], [9] rely on the Laplacian matrix, where the computation load of the criteria will increase when the number of agents grows. Recently, many researchers try to establish the consensus criteria via the event-based strategy without relying on the information of the Laplacian matrix [12], but it seems that these methods are only suitable for some special models such as the first/second-order model. It is hard to be extended to deal with some more generalized models. So, how to design an effective algorithm for these MASs to realize the two goals remains a big challenge.

Motivated by the above observations, this brief aims at reducing the computation load and resource consumption of a class of nonlinear MASs by designing a distributed dynamics event-based control framework, where the dynamics of the agents satisfy the QUAD and some network connectivity conditions. We stress that the control protocol, the triggering conditions, and the synchronization criteria in this brief do not rely on the Laplacian matrix and its eigenvalues. The obtained synchronization conditions for a class of nonlinear MASs only depend on the number of agents, which can reduce the computation load. Under the designed distributed dynamic event-based control, the asymptotic and exponential synchronization of the MASs are achieved, and it ensures that the Zeno-behavior is excluded in the closed-loop systems.

This brief is organized as below. Section II introduces the model of a nonlinear MASs, several assumptions and the synchronization definition, respectively. The synchronization criteria are given in Section III. An example is given in Section IV to verify the results. Conclusion is drawn in Section V.

Notations: $\mathbb{N}_+$, $\mathbb{R}^n$, and $\mathbb{R}^{n\times m}$ are the sets of non-negative integers, n-dimensional real vectors, and $(n \times m)$-dimensional real matrices, respectively. For real matrices R and P, $P \otimes R$ refers to the Kronecker product of P and R; $\lambda_{\max}(R)/\lambda_{\min}(R)$ means the maximal/minimal eigenvalue of R. I_n is the $(n \times n)$-dimensional identity matrix, and $\text{diag}\{d_1, d_2, \ldots, d_n\}$ is a diagonal matrix with diagonal elements d_i, $i = 1, 2, \ldots, n$. We use X^T and $\|X\|$ to denote the transpose and the norm of a vector or matrix X, respectively.

II. Problem Formulation and Preliminaries

Consider the topology of the MASs is characterized by an undirected *connected* graph $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \hat{A})$, where $\mathscr{V} = \{1, 2, \ldots, N\}$ is the vertex set with N nodes, $\mathscr{E} = \{(i,j)|$ if there is an edge between agents i and $j\} \subseteq \mathscr{V} \times \mathscr{V}$ denotes the edge set, and $\hat{A} = [a_1^T, a_2^T, \ldots, a_N^T]^T$ is the adjacency matrix where $a_i = [a_{i1}, a_{i2}, \ldots, a_{iN}], i \in \mathscr{V}$. For $\forall i, j \in \mathscr{V}$, agent j is a neighbor of i, if $(i,j) \in \mathscr{E}$ with the binary indicator a_{ij}, where $a_{ij} = a_{ji} = 1$ if and only if $(i,j) \in \mathscr{E}$; otherwise, $a_{ij} = 0$. Let $\mathcal{N}_i = \{j \in \mathscr{V}| \ (i,j) \in \mathscr{E}\}$ denote the neighbor set of agent i. We require $i \notin \mathcal{N}_i$ and $(i,i) \notin \mathscr{E}$. The Laplacian matrix of $\mathscr{G}$ is $L = [l_1^T, l_2^T, \ldots, l_N^T]^T$, where $l_i = [l_{i1}, l_{i2}, \ldots, l_{iN}]$, $i \in \mathscr{V}$ where $l_{ij} = -a_{ij}$ if $i \neq j$; otherwise, $l_{ii} = \sum_{j \in \mathcal{N}_i} a_{ij}$. The eigenvalues of L satisfy $0 = \lambda_1(L) < \lambda_2(L) \leq \cdots \leq \lambda_N(L)$, and $\lambda_2(L)$ is the algebraic connectivity of $\mathscr{G}$ satisfying the property stipulated in the lemma as below.

Lemma 1 [10]: For an unweighted and undirected connected graph with N nodes, the algebraic connectivity $\lambda_2(L)$ of its Laplacian matrix $L \in \mathbb{R}^{N \times N}$ satisfies $\lambda_2(L) \geq \frac{4}{N(N-1)}$.

Consider the dynamics of MASs as follows:

$$\dot{x}_i(t) = Ax_i(t) + f(x_i(t)) + u_i(t), \tag{1}$$

where $A \in \mathbb{R}^{n \times n}$, $x_i(t) \in \mathbb{R}^n$ is the state of agent i and its input is $u_i(t) \in \mathbb{R}^n$ where $i = 1, 2, \ldots, N$. The nonlinear function $f : \mathbb{R}^n \to \mathbb{R}^n$ satisfies the following assumptions.

Assumption 1: For the vector field f in (1), there exists a constant ω such that $(\tilde{\theta}_1 - \tilde{\theta}_2)^T(f(\tilde{\theta}_1) - f(\tilde{\theta}_2)) \leq \omega\|\tilde{\theta}_1 - \tilde{\theta}_2\|^2, \forall \tilde{\theta}_1, \tilde{\theta}_2 \in \mathcal{D}$.

Assumption 2: The vector field f in (1) is quadratically inner-bounded in $\mathbb{R}^n$, i.e., there are $\omega_1, \omega_2 \in \mathbb{R}$ such that $\|f(\tilde{\theta}_1) - f(\tilde{\theta}_2)\|^2 \leq \omega_1(\tilde{\theta}_1 - \tilde{\theta}_2)^T(f(\tilde{\theta}_1) - f(\tilde{\theta}_2)) + \omega_2\|\tilde{\theta}_1 - \tilde{\theta}_2\|^2, \forall \tilde{\theta}_1, \tilde{\theta}_2 \in \mathcal{D}$.

Remark 1: Assumption 1 is the QUAD (or one-sided Lipschitz) condition, which is very mild and weaker than being Lipschitz continuous. Any locally Lipschitz function is QUAD and quadratically inner-bounded, but the converse is not true [18].

For convenience, two definitions are given below.

Definition 1: $D^+f(x) = \limsup_{\Delta \to 0^+} \frac{f(x+\Delta) - f(x)}{\Delta}$ if $f(x)$ is differentiable from its right side.

Definition 2: For any agents $i, j \in \mathscr{V}$ with any initial states, the MASs (1) is synchronization if $\lim_{t \to \infty} \|x_i(t) - x_j(t)\| = 0$.

To achieve the synchronization, the control input $u_i(t)$ in (1) is designed to be

$$u_i(t) = c_i \sum_{j \in \mathcal{N}_i} a_{ij}(x_j(t_k^i) - x_i(t_k^i)), t \in [t_k^i, t_{k+1}^i), \tag{2}$$

where c_i is the input gain. In addition, the k-th event-triggered instant of agent i is t_k^i, which is defined iteratively by

$$t_{k+1}^i = \inf\Big\{t : t > t_k^i, \\ \eta_i(t) + \beta_i(\mu_i\|q_i(t_k^i)\|_P - \|\delta_i(t)\|_P) \leq 0\Big\} \tag{3}$$

where $\beta_i > 0$, $1 \geq \mu_i \geq 0$, $\|q_i(t_k^i)\|_P = q_i^T(t_k^i)Pq_i(t_k^i)$, $\|\delta_i(t)\|_P = \delta_i^T(t)P\delta_i(t)$, $q_i(t) = \sum_{j \in \mathcal{N}_i} a_{ij}(x_j(t) - x_i(t))$, and $\delta_i(t) = q_i(t_k^i) - q_i(t)$. Let the initial value $\eta_i(t_0) > 0$ and $\eta_i(t)$ take the form

$$D^+\eta_i(t) = -\xi_i\eta_i(t) + \theta_i(\mu_i\|q_i(t_k^i)\|_P - \|\delta_i(t)\|_P), \tag{4}$$

$\xi_i > 0$ and $\theta_i > 0$. We can adjust the triggering frequency by choosing parameters of this triggering condition. One can infer that $\eta_i(t) \geq \eta_i(t_0)\exp(-\zeta_i(t - t_0))$ with $\zeta_i = \xi_i + \frac{\theta_i}{\beta_i}$. For analysis purposes, merge all the time sequences in the chronological order, i.e., $\{t_k\} = \bigcup_{i=1}^N \{t_k^i, k \in \mathbb{N}_+\}$, where t_k satisfy $t_0 < t_1 < \cdots < t_k < t_{k+1} < \cdots$, $\lim_{k \to \infty} t_k = +\infty$. This property will be proved later.

III. Synchronization Analysis

The synchronization analysis for MASs (1) under the control laws (2)-(4) will be given in this section. The proof of excluding Zeno-behavior is first discussed as follows.

Theorem 1: Under the event-triggered rules (3) and (4), the Zeno-behavior does not appearer in the system (1).

Proof: Let $\tilde{V}(t) = \frac{1}{2}\delta_i^T(t)\delta_i(t), t \in [t_k^i, t_{k+1}^i), \forall k \in \mathbb{N}_+$. There are constants $\tilde{\varepsilon}_{i0}, \tilde{\varepsilon}_{i1}, \tilde{\varepsilon}_{i2} > 0$ such that

$$\begin{aligned} D^+\tilde{V}(t) &= \delta_i^T(t)\Bigg(\sum_{j=1}^N a_{ij}\Bigg(A(x_j(t) - x_i(t)) \\ &\quad + (f(x_j(t)) - f(x_i(t))) + (u_j(t_k^j) - u_i(t_k^i))\Bigg)\Bigg) \\ &\leq \frac{1}{2}\sum_{l=0}^2 \tilde{\varepsilon}_{il}\|\delta_i(t)\|^2 + \frac{1}{2\tilde{\varepsilon}_{i0}}\|\sum_{j=1}^N a_{ij}A(x_j(t) - x_i(t))\|^2 \\ &\quad + \frac{1}{2\tilde{\varepsilon}_{i1}}\|\sum_{j=1}^N a_{ij}(u_j(t_k^j) - u_i(t_k^i))\|^2 \\ &\quad + \frac{1}{2\tilde{\varepsilon}_{i2}}\|\sum_{j=1}^N a_{ij}(f(x_j(t)) - f(x_i(t)))\|^2. \end{aligned}$$

According to Assumptions 1 and 2, one can infer that $\|\sum_{j=1}^N a_{ij}(f(x_j(t)) - f(x_i(t)))\|^2 \leq \lambda_{\max}(a_i^Ta_i)|\omega_1\omega + \omega_2|\sum_{j=1}^N \|(x_j(t) - x_i(t))\|^2$.

Should Zeno-behavior take place, there would have been a positive constant t^* such that $\lim_{k \to +\infty} t_{k+1}^i = t^* < +\infty$. We know that, for any $i \in \{1, 2, \ldots, N\}$, $x_i(t)$ is continuous in the interval $[t_0^i, t_{k+1}^i] \subset [t_0^i, t^*], \forall k \in \mathbb{N}_+$. Thus, for any $t \in [t_k^i, t_{k+1}^i) \subset [t_0^i, t^*], \forall k \in \mathbb{N}_+$, there exists $M > 0$ such that $\sum_{j=1}^N \|(x_j(t) - x_i(t))\|^2 \leq M$ due to the fact that $x_j(t) - x_i(t)$ is continuous in the interval $[t_0^i, t_{k+1}^i), \forall k \in \mathbb{N}_+$. Let $\mathcal{M} = (\frac{\lambda_{\max}^2(A)}{2\tilde{\varepsilon}_{i0}}\lambda_{\max}(a_i^Ta_i) + \frac{1}{2\tilde{\varepsilon}_{i2}}\lambda_{\max}(a_i^Ta_i)|\omega_1\omega + \omega_2|)M + \frac{1}{2\tilde{\varepsilon}_{i1}}\|\sum_{j=1}^N a_{ij}(u_j(t_k^j) - u_i(t_k^i))\|^2$. One infers that

$$D^+\tilde{V}(t) \leq \tfrac{1}{2}\textstyle\sum_{l=0}^2 \tilde{\varepsilon}_{il}\|\delta_i(t)\|^2 + \mathcal{M}, t \in [t_k^i, t_{k+1}^i), \forall k \in \mathbb{N}_+.$$

Since $\|\delta_i(t_k^i)\|^2 = 0$, we have $\|\delta_i(t)\|^2 \leq \hat{\mathcal{M}}(\exp(\sum_{l=0}^2 \tilde{\varepsilon}_{il}(t - t_k^i)) - 1)$ where $\hat{\mathcal{M}} = \mathcal{M}/\sum_{l=0}^2 \tilde{\varepsilon}_{il}$. Thus, for a positive definite symmetric matrix P, one has

$$\delta_i^T(t)P\delta_i(t) \le \hat{\mathcal{M}}\lambda_{\max}(P)\Big(e^{(\sum_{l=0}^{2}\tilde{\varepsilon}_{il})(t-t_k^i)}-1\Big). \tag{5}$$

Let $\tilde{\delta}(t) := \hat{\mathcal{M}}\lambda_{\max}(P)(\exp(\sum_{l=0}^{2}\tilde{\varepsilon}_{il}(t-t_k^i))-1)$ replace the error $\delta_i^T(t)P\delta_i(t)$ of the triggering condition (3). If the next event occurs at $t_{k+1}^{i'}$ under the error $\tilde{\delta}(t)$, then

$$\tilde{\delta}(t_{k+1}^{i'}) = \frac{1}{\beta_i}\eta_i(t_{k+1}^{i'}) + \mu_i q_i^T(t_k^i)Pq_i(t_k^i).$$

Letting $T_k^i = t_{k+1}^{i'} - t_k^i$, one has

$$\frac{\eta_i(t_0)}{\beta_i}e^{-\zeta_i T_k^i} < \hat{\mathcal{M}}\lambda_{\max}(P)e^{\zeta_i(t^*-t_0)}\Big(e^{(\sum_{l=0}^{2}\tilde{\varepsilon}_{il})T_k^i}-1\Big).$$

Since $\lim_{k\to+\infty} t_k^i = t^* < +\infty$, we know that $T_k^i \to 0$ as $k \to +\infty$. As a result, we obtain that $\frac{\eta_i(t_0)}{\beta_i} \le 0$ which contradicts $\eta_i(t_0) > 0$ and $\beta_i > 0$. Since $\tilde{\varepsilon}_i = \max\{\sum_{l=0}^{2}\tilde{\varepsilon}_{il}, \zeta_i\}$, we have

$$\frac{\eta_i(t_0)}{\beta_i}e^{-\zeta_i T_k^i} < \hat{\mathcal{M}}\lambda_{\max}(P)\Big(e^{\tilde{\varepsilon}_i(t_{k+1}^{i'}-t_0)} - e^{\tilde{\varepsilon}_i(t_k^i-t_0)}\Big).$$

This means that the constant $T^* := \inf_{k\in\mathbb{N}_+}\{T_k^i\} > 0$ for any $i \in \mathscr{V}$. From (5), we know that $t_{k+1}^i - t_k^i \ge T_k^i$, i.e., $\hat{T} := \inf_{k\in\mathbb{N}_+}\{t_{k+1}^i - t_k^i\} > 0$ such that $\hat{T} \ge T^*$. This implies that the Zeno-behavior is successfully excluded. ■

Remark 2: An internal dynamic variable $\eta_i(t)$ is introduced into the triggering condition. This is a generalized form of the traditional scheme where (3) becomes a traditional static triggering condition if $\beta_i \to +\infty$. In addition, since $\eta_i(t) \ge 0$, the minimum inter-event time of the triggering condition (3) is greater than that resulting from the traditional static event-triggered strategy.

The synchronization analysis for the system (1) will be given as the following.

Theorem 2: Under Assumptions 1-2, consider the system (1) with the controller (2) and the triggering rules (3) and (4) where $\xi_i > 0$, $\beta_i > 0$, $1 \ge \mu_i \ge 0$, $\theta_i > 0$, and the input gain $c_i = \mu_i(\theta_i + \xi_i\beta_i)$, $i = 1, 2, \ldots, N$. Then the system (1) is asymptotical synchronization, if there are constants $\hat{c} = \min\{c_i\}$, $\alpha > 0$, and positive definite matrices P, Q such that

$$A^TP + PA + Q - \frac{1}{\alpha}P - \frac{4\hat{c}}{(N-1)N}P \le 0, \tag{6}$$

$$\lambda_{\max}(P)[(1+\omega_2) + \omega(2\alpha + \omega_1)] < \alpha\lambda_{\min}(Q), \tag{7}$$

$$2\alpha + \omega_1 > 0, \tag{8}$$

$$\lambda_{\max}(P)(\alpha^2 - 1) - \alpha^2\lambda_{\min}(P) < 0. \tag{9}$$

Proof: Let $V(t) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}e_{ij}^T(t)Pe_{ij}(t) + \sum_{i=1}^{N}\eta_i(t)$ where $e_{ij}(t) = x_j(t) - x_i(t)$ and P is a positive definite matrix. For any $t \in [t_k, t_{k+1})$, $\forall k \in \mathbb{N}_+$, one has

$$\begin{aligned} D^+V(t)|_{(1)} &= \sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\dot{e}_{ij}^T(t)Pe_{ij}(t) + \sum_{i=1}^{N}D^+\eta_i(t) \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\Big[\frac{1}{2}e_{ij}^T(t)(A^TP+PA)e_{ij}(t) \\ &\quad + (f(x_j(t)) - f(x_i(t)))^TPe_{ij}(t) \\ &\quad + (u_j(t) - u_i(t))^TPe_{ij}(t)\Big] + \sum_{i=1}^{N}D^+\eta_i(t). \end{aligned}$$

According to Assumptions 1 and 2, there is $\alpha > 0$ such that $2e_{ij}^T(t)P(f(x_j(t)) - f(x_i(t))) \le \frac{\gamma_1}{\alpha}\|e_{ij}(t)\|^2 - \frac{1}{\alpha}e_{ij}^T(t)Pe_{ij}(t) + \frac{\gamma_2}{\alpha}\|f(x_j(t)) - f(x_i(t))\|^2$ where $\gamma_1 = \lambda_{\max}(P)[(1+\omega_2) + \omega(2\alpha + \omega_1)]$, and $\gamma_2 = \lambda_{\max}(P)(\alpha^2-1) - \alpha^2\lambda_{\min}(P) < 0$. Thus

$$\begin{aligned} &D^+V(t)|_{(1)} \\ &\le \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\Big\{e_{ij}^T(t)(A^TP + PA + \frac{\gamma_1}{\alpha}I_n - \frac{1}{\alpha}P)e_{ij}(t)\Big\} \\ &\quad + \sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\Big[u_j^T(t)Pe_{ij}(t) - u_i^T(t)Pe_{ij}(t)\Big] + \sum_{i=1}^{N}D^+\eta_i(t). \end{aligned}$$

Because $a_{ij} = a_{ji}$, one has $\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}u_j^T(t)Pe_{ij}(t) = -\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}u_i^T(t)Pe_{ij}(t)$. Thus

$$\begin{aligned} &D^+V(t)|_{(1)} \\ &\le \sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\Big\{\frac{1}{2}e_{ij}^T(t)(A^TP + PA + \frac{\gamma_1}{\alpha}I_n - \frac{1}{\alpha}P)e_{ij}(t) \\ &\qquad - 2u_i^T(t)Pe_{ij}(t)\Big\} + \sum_{i=1}^{N}D^+\eta_i(t). \end{aligned}$$

Let $\bar{x}(t) = \frac{1}{N}\sum_{j=1}^{N}x_j(t)$, $e_i(t) = x_i(t) - \bar{x}(t)$, $\rho_i(t) = e_i(t_k^i) - e_i(t)$, $e(t) = [e_1^T(t), \ldots, e_N^T(t)]^T$, and $\rho(t) = [\rho_1^T(t), \ldots, \rho_N^T(t)]^T$. We know that $e_{ij}(t) = e_j(t) - e_i(t)$, $q_i(t) = (l_i \otimes I_n)e(t)$, $\delta_i(t) = (l_i \otimes I_n)\rho(t)$. Based on (2) and $e_i(t_k^i) = \rho_i(t) + e_i(t)$, for any $t \in [t_k^i, t_{k+1}^i)$, we have $u_i(t) = c_i(l_i \otimes I_n)[e(t) + \rho(t)]$. Thus

$$\begin{aligned} D^+V(t)|_{(1)} &\le e^TL \otimes (A^TP + PA + \frac{\gamma_1}{\alpha}I_n - \frac{1}{\alpha}P)e(t) \\ &\quad + \sum_{i=1}^{N}\Big\{-2c_ie^T(t)(l_i^Tl_i \otimes P)e(t) \\ &\qquad - 2c_i\rho^T(t)(l_i^Tl_i \otimes P)e(t) + D^+\eta_i(t)\Big\}. \end{aligned} \tag{10}$$

According to equation (3), we have

$$\begin{aligned} D^+\eta_i(t) &\le (\theta_i + \xi_i\beta_i)\Big\{\mu_i\Big[e^T(t)(l_i^Tl_i \otimes P)e(t) \\ &\quad + e^T(t)(l_i^Tl_i \otimes P)\rho(t) + \rho^T(t)(l_i^Tl_i \otimes P)e(t)\Big] \\ &\quad - (1-\mu_i)\rho^T(t)(l_i^Tl_i \otimes P)\rho(t)\Big\}. \end{aligned} \tag{11}$$

Let $c_i = \mu_i(\theta_i + \xi_i\beta_i)$, $0 \le \mu_i \le 1$, and $\hat{c} = \min\{c_i\}$, $i = 1, 2, \ldots, N$. Combining (10) and (11), one obtains

$$\begin{aligned} D^+V(t)|_{(1)} &\le e^T[L \otimes (A^TP + PA + \frac{\gamma_1}{\alpha}I_n - \frac{1}{\alpha}P) \\ &\quad - \hat{c}L^TL \otimes P]e(t). \end{aligned} \tag{12}$$

The Laplacian matrix L is symmetric because the graph $\mathscr{G}$ is connected and undirected. There is an orthogonal matrix $\Gamma = (\frac{1}{\sqrt{N}}\mathbf{1}_N, \Xi) \in \mathbb{R}^{N\times N}$ such that $L = \Gamma\Lambda\Gamma^T$ where $\Xi \in \mathbb{R}^{N\times N-1}$, and $\Lambda = \mathrm{diag}\{0, \lambda_2(L), \ldots, \lambda_n(L)\}$ $(\lambda_i(L) > 0, i = 2, 3, \ldots, N)$. Let $\tilde{e}(t) = (\Gamma^T \otimes I_n)e(t)$. According to (6) and the fact that $\lambda_i(L) \geq \lambda_2(L) \geq \frac{4}{(N-1)N}$, $i = 3, 4, \ldots, N$, if $A^T P + PA + Q - \frac{1}{\alpha}P - \frac{4\hat{c}}{(N-1)N}P \leq 0$, then

$$
\begin{aligned}
&D^+V(t)|_{(1)} \\
&\leq \tilde{e}^T[\Lambda \otimes (A^T P + PA + \frac{\gamma_1}{\alpha}I_n - \frac{1}{\alpha}P) - \hat{c}\Lambda^T\Lambda \otimes P]\tilde{e}(t) \\
&\leq \sum_{i=1}^{N} \lambda_i(L)\tilde{e}_i^T[A^T P + PA + \frac{\gamma_1}{\alpha}I_n - (\frac{1}{\alpha} + \hat{c}\lambda_2(L))P]\tilde{e}_i(t) \\
&\leq (\frac{\gamma_1}{\alpha} - \lambda_{\min}(Q)) \sum_{i=1}^{N} \lambda_i(L)\tilde{e}_i^T\tilde{e}_i(t). \qquad (13)
\end{aligned}
$$

We know that $D^+V(t)|_{(1)} \leq 0$ due to $\gamma_1 < \alpha\lambda_{\min}(Q)$. Then, $V(t_{k+1}^-) \leq V(t) \leq V(t_k)$ for any $t \in [t_k, t_{k+1})$, $\forall k \in \mathbb{N}_+$.

Let $\Upsilon(\epsilon_k) = V(t_{k+1}^-) - \epsilon_k V(t_k)$, $\epsilon_k \in (0, 1)$, $\forall k \in \mathbb{N}_+$. We know that $\Upsilon(\epsilon_k)$ is continuous in ϵ_k where $\epsilon_k \in (0, 1), k = 1, 2, \ldots,$, and $\Upsilon(0) > 0$, $\Upsilon(1) < 0$. There exists $\epsilon_k \in (0, 1), k = 1, 2, \ldots$, such that $\Upsilon(\epsilon_k) = 0$, i.e., $V(t_{k+1}^-) = \epsilon_k V(t_k)$. In addition, we know that $V(t_{k+1}^-) = V(t_{k+1})$ as $V(t)$ is continuous at t_{k+1}. Thus, for any $t \in [t_k, t_{k+1})$, we have $V(t) = \epsilon_k V(t_k) = \epsilon_k\epsilon_{k-1}V(t_{k-1}) = \cdots = \prod_{j=0}^{k}\epsilon_j V(t_0) \leq \hat{\epsilon}^{k+1}V(t_0)$ where $\hat{\epsilon} = \max\{\epsilon_j\} < 1, j = 1, 2, \ldots, k$. Thus, $V(t) \to 0$ as $k \to +\infty$, i.e., $\forall i, j \in \mathscr{V}$, $\|x_i(t) - x_j(t)\| \to 0$ as $t \to +\infty$. Then the nonlinear MASs achieve the asymptotic synchronization. ■

Theorem 3: Under Assumptions 1-2, consider the system (1) with the controller (2) and the triggering rules (3) and (4) where $\xi_i > 0$, $\beta_i > 0$, $1 \geq \mu_i \geq 0$, $\theta_i > 0$, and the control gain $c_i = \mu_i\theta_i$, $i = 1, 2, \ldots, N$. Then the system (1) is exponential synchronization, if there are constants $\hat{c} = \min\{c_i\}$, $\hat{\alpha} > 0$, $\alpha > 0$ and positive definite matrices P and Q such that

$$A^T P + PA + Q - \frac{1}{\alpha}P - \frac{4\hat{c}}{(N-1)N}P \leq 0, \qquad (14)$$

$$\lambda_{\max}(P)[(1+\omega_2) + \omega(2\alpha + \omega_1)] + \hat{\alpha}\alpha < \alpha\lambda_{\min}(Q), \qquad (15)$$

$$2\alpha + \omega_1 > 0, \qquad (16)$$

$$\lambda_{\max}(P)(\alpha^2 - 1) - \alpha^2\lambda_{\min}(P) < 0. \qquad (17)$$

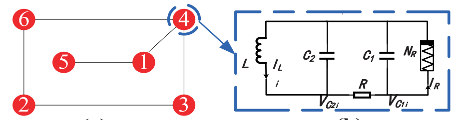
Proof: Let $V(t) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}e_{ij}^T(t)Pe_{ij}(t) + \sum_{i=1}^{N}\eta_i(t)$ where, $e_{ij}(t)$ is given in Theorem 2, and P is a positive definite matrix. Obviously, $V(t)$ can be rewritten as $V(t) = e^T(t)L \otimes Pe(t) + \sum_{i=1}^{N}\eta_i(t)$.

According to formula (10) and (11), for any $t \in [t_k, t_{k+1})$, $\forall k \in \mathbb{N}_+$, we have

$$
\begin{aligned}
D^+V(t)|_{(1)} \leq\, & e^T(t)L \otimes (A^T P + PA + \frac{\gamma_1}{\alpha}I_n - \frac{1}{\alpha}P)e(t) \\
& - \sum_{i=1}^{N}\Big\{(2c_i - \mu_i\theta_i)e^T(t)(l_i^T l_i \otimes P)e(t) \\
& + 2(c_i - \mu_i\theta_i)\rho^T(t)(l_i^T l_i \otimes P)e(t) \\
& + \theta_i(1-\mu_i)\rho^T(t)(l_i^T l_i \otimes P)\rho(t) + \xi_i\eta_i(t)\Big\}.
\end{aligned}
$$

Let $c_i = \mu_i\theta_i$, $0 \leq \mu_i \leq 1$, and $\hat{c} = \min\{c_i\}$, $i = 1, 2, \ldots, N$. Similar with (12), one has

$$
\begin{aligned}
D^+V(t)|_{(1)} \leq\, & e^T[L \otimes (A^T P + PA + \frac{\gamma_1}{\alpha}I_n - \frac{1}{\alpha}P) \\
& - \hat{c}L^T L \otimes P]e(t) - \sum_{i=1}^{N}\xi_i\eta_i(t).
\end{aligned}
$$



(a) (b)

Fig. 1. The communication topology of agent networks.

Let $\tilde{e}(t) = (\Gamma^T \otimes I_n)e(t)$. According to formula (14) and $\lambda_i(L) \geq \lambda_2(L) \geq \frac{4}{(N-1)N}$, $i = 3, 4, \ldots, N$, if $\frac{\gamma_1}{\alpha} + \hat{\alpha} - \lambda_{\min}(Q) \leq 0$, then similar with (13), we obtain that

$$D^+V(t)|_{(1)} \leq -2\hat{\alpha}\lambda_{\min}(P^{-1})e^T(t)L \otimes Pe(t) - \sum_{i=1}^{N}\xi_i\eta_i(t).$$

Letting $\varepsilon = \min\{2\hat{\alpha}\lambda_{\min}(P^{-1}), \xi_1, \xi_1, \ldots, \xi_N\}$, we have $D^+V(t)|_{(1)} \leq -\varepsilon V(t)$. Then $V(t) \leq V(t_k)e^{-\varepsilon(t-t_k)}$, for any $t \in [t_k, t_{k+1}), \forall k \in \mathbb{N}_+$. Similarly, we know that $V(t_{k+1}^-) = V(t_{k+1})$ as $V(t)$ is continuous at t_{k+1} for any $k \in \mathbb{N}_+$. Thus, $V(t) \leq V(t_k)e^{-\varepsilon(t-t_k)} \leq V(t_{k-1})e^{-\varepsilon(t-t_k)}e^{-\varepsilon(t_k-t_{k-1})} \leq \cdots \leq V(t_0)e^{-\varepsilon(t-t_0)}$. Then the system (1) is exponential synchronization with the convergence rate ε. ■

Remark 3: In this method, the controller gain c_i depend on the triggering parameters, i.e., $c_i = \mu_i(\theta_i + \xi_i\beta_i)$ (or $c_i = \mu_i(\xi_i\beta_i)$). The controller has been designed when we design the triggering rule. In addition, this method can reduce the conservatism in designing the control of the MASs. The reason is that no inequality technique is utilized when we obtain the controller gain c_i.

Remark 4: For any agent, only the local information related to its neighbors is utilized in the control protocol and the triggering conditions. Unlike the results in [5], [6], Theorems 2 and 3 in this brief just involve the number of the nodes and the parameters of the triggering conditions, which is helpful for reducing the algorithm complexity and computation load.

IV. Numerical Simulation

In this section, an example is given to verify the obtained results. The topology of the MASs is shown in Fig. 1.

As depicted in Fig. 1 (b), each agent is described by the Chua's circuit

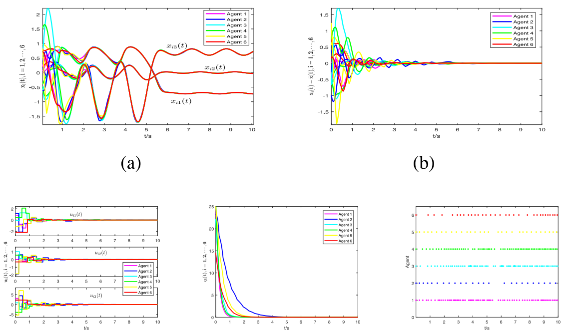
$$\dot{x}_i(t) = Ax_i(t) + f(x_i(t)) + u_i(t), \quad i = 1, 2, \ldots, 6$$

where $A = \begin{bmatrix} -\frac{1}{C_1 R} + \frac{G_0}{C_1} & \frac{1}{C_1 R} & 0 \\ \frac{1}{C_2 R} & -\frac{1}{C_2 R} & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & 0 \end{bmatrix}$, $x_i(t) = [x_{i1}(t), x_{i2}(t), x_{i3}(t)]^T$, $f(x_i(t)) = [\phi(x_{i1}(t)), 0, 0]^T$, $\phi(x_{i1}(t)) = -\frac{K}{C_1}x_{i1}^3(t)$ refers to the $I - V$ characteristic of the nonlinear resistor R_N. Let $x_{i1}(t)$, $x_{i2}(t)$ and $x_{i3}(t)$ be the voltage across C_{1i}, the voltage across C_{2i} and the current through L_i, respectively, where $C_1 = 0.1$, $C_2 = 1$, $L = 0.0625$, $R = 1$, $G_0 = 0.168$ and $K = 1$. $u_i(t) = c_i\sum_{i=1}^{N}a_{ij}(x_j(t_k^i) - x_i(t_k^i))$ are agent i's input where $c_i = \mu_i(\theta_i + \xi_i\beta_i)$, $k = 1, 2, \ldots$

It is easy to verify that $\omega = 0$, $\omega_1 = -0.5$, $\omega_2 = -1$ satisfy the Assumptions 1 and 2. To ensure $2\alpha + \omega_1 > 0$, we take $\alpha = 0.5$. The initial state $x_i(t_0)$ and $\eta_i(t_0)$ are randomly generated from $[-1, 1]$ and $(0, 30]$, respectively, and the parameters of the triggering condition (3) and (4) are given in Table I.

TABLE I
THE PARAMETERS OF THE TRIGGERING CONDITION

Agent i	1	2	3	4	5	6
ξ_i	4.50	1.00	3.00	4.00	2.00	2.00
β_i	0.48	0.64	0.32	0.96	0.80	0.48
μ_i	0.40	0.80	0.60	0.40	1.00	0.80
θ_i	1.00	1.10	0.75	0.75	0.75	1.25



(a) (b) (c) (d) (e)

Fig. 2. The simulation of the MASs (1) under the strategies (2)-(4).

TABLE II
THE COMPARISON OF THE TRIGGERING NUMBER IN [0, 10]

Agent i	1	2	3	4	5	6	Average
Theorem 2	107	27	75	78	33	39	69.5
ETC [9]	104	108	111	109	109	104	107.5

By LMI (6), we obtain that $P = \begin{pmatrix} 3.2190 & 7.1144 & 1.1394 \\ 7.1144 & 68.1165 & -0.2114 \\ 1.1394 & -0.2114 & 4.9658 \end{pmatrix}$ and $Q = \begin{pmatrix} 20.7086 & -5.5842 & 1.6567 \\ -5.5842 & 15.6180 & -0.2031 \\ 1.6567 & -0.2031 & 1.8174 \end{pmatrix}$. It is easy to verify that the formulae (7)-(9) hold. Obviously, the algorithm complexity and computation load of our results are lower than those from the general event-based control such as [5], [6], [9]. Similarly, Theorem 3 can be verified, which is omitted here. The simulation is shown in Fig. 2, where (a) and (b) show the evolutions of the state $x_i(t)$ and the state error $x_i(t) - \bar{x}(t)$, which verified that the network achieves synchronization. The input of each agent converges to 0, and have is shown in Fig. 2 (c). Fig. 2 (d) depicts the state $\eta_i(t)$, and the triggering instants of the agents are shown in Fig. 2 (e), which verifies that the Zeno-behavior does not appearer in the MASs.

In general, the update frequency of our method is lower than that of the edge-based triggering scheme [8] where the triggering law needs to be designed for each neighbor of the agents; it is also lower than that of the general event-based scheme [9] (see Table II), which shows that our method has advantage in terms of reducing requirement for resources.

V. CONCLUSION

In this brief, a distributed dynamic event-based protocol has been designed for a class of nonlinear MASs, in which the nonlinear function satisfies QUAD characteristic and quadratic inner-bounded condition. Because the obtained results only rely on the number of the graph, the algorithm complexity and computation load can be reduced. Hence, this method has great potential in practical applications. Moreover, the Zeno-behavior is also excluded. Finally, an example has been given to verify the effectiveness of the obtained results. Since the presented results hold only when the network graph is undirected and unweighted, we are interested in working on solving the fully-distributed dynamic event-based synchronization for nonlinear MASs with directed/weighted graphs.

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