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### Comment on the paper by Y.Komura and Y.Okabe [arXiv:1011.3321]

Enter, Aernout C. D. van; Romano, Silvano; Zagrebnoy, Valentin A.

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Comment on the paper by  
Y. Komura and Y. Okabe, *J.Phys. A* **44**,  
015002, 2011

Aernout C.D. van Enter <sup>\*</sup>, Silvano Romano <sup>†</sup>  
and Valentin A. Zagrebnov <sup>‡</sup>

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**Abstract**

We point out that the claim of strong universality of [1] is incorrect, as it contradicts known rigorous results.

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**Keywords:** Generalised XY-model, Kosterlitz-Thouless versus first-order transition, nonuniversality.

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<sup>\*</sup>University of Groningen, Johann Bernoulli Institute of Mathematics and Computing Science, Postbus 407, 9700 AK Groningen, The Netherlands,

A.C.D.v.Enter@math.rug.nl,  
<http://www.math.rug.nl/~aenter/>

<sup>†</sup>University of Pavia, Dept of Physics, via A. Bassi 6, Pavia, Italy,  
Silvano.Romano@pv.infn.it,

<http://www.pv.infn.it/~romano/>

<sup>‡</sup>Université de la Méditerranée and Centre de Physique Théorique - UMR 6207 Luminy Case 977, 13288 Marseille, Cedex09, France

zagrebnov@cpt.univ-mrs.fr,  
<http://www.cpt.univ-mrs.fr/~zagrebno/zagrebnov.htm>

The generalised lattice ferromagnetic  $XY$ -model was introduced by two of us in [2]. Its simplest version involves 3-component unit spins on a  $D$ -dimensional lattice, parameterized by standard angles in the spherical coordinates and interacting according to the ferromagnetic nearest-neighbour Hamiltonian

$$H = -J \sum_{\langle i,j \rangle} (\sin\theta_i \sin\theta_j)^q \cos(\varphi_i - \varphi_j) , \quad J > 0 .$$

When  $D = 2$ , the model yields orientational disorder at all finite temperatures, and produces a transition to a low-temperature Berezinskiĭ–Kosterlitz–Thouless (BKT) phase, possessing slow decay of magnetic correlations and infinite susceptibility [2]. In turn, the transition is expected, and was found by simulation [3], to exhibit the usual BKT scenario, at least for small values of  $q$ . On the other hand, this scenario does not hold for *all* values of  $q$ , i.e. the BKT transitions are *not universal*.

Indeed, we *proved* in [4] that the named transition turns first-order for sufficiently large  $q$ . Recall that a hint of this behaviour was already suggested by numerical analysis [3].

Notice that the above *rigorous* results entail that both existence and the type of transition *depend* on the parameter  $q$ , i.e. they exclude universality.

The above-mentioned generalized  $XY$ -model was recently addressed by simulation for various values of  $q$  in Ref. [1]. Although the authors found only a certain evidence of the BKT criticality, they claim that this behaviour holds for *all values* of  $q$ . This claim can not be correct since it contradicts to our rigorous results [4], which seem to have been unknown to the authors of the Ref. [1].

## References

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