Probability and confirmation in logical empiricism

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PROBABILITY IN LOGICAL EMPIRICISM

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While logical empiricism was not a single doctrine, what brought its representatives together was a commitment to empiricism, even if understood differently. Around 1929–30, their empiricism focused on a criterion that required that meaningful statements were only the ones that could in principle be verified by observation or deduction. A sentence is verifiable when there is, in principle, a procedure that would determine whether the sentence is true or false.

Sooner or later, anyone who submitted to this idea had to concede that strict verificationism is impossible to uphold: actual science uses terms that mix empirical with theoretical content, or terms that refer to dispositions, as well as universally quantified statements. Hence, this strict notion of verification had to be replaced with something that would reflect this more complicated reality: confirmation.

Confirmation and probability

Confirmation is a relation between evidence and a hypothesis. Through this concept, we recognize the evidential support that the evidence provides to the hypothesis without straightforwardly verifying or falsifying it. Observing more and more black ravens gives more weight to the hypothesis that all ravens are black—confirms it to higher and higher degree—but no finite number of observations can fully verify this sentence. The shift of focus towards confirmation emphasizes the essentially inductive character of scientific reasoning.

Just as in this raven example, confirmation is easily seen as a gradable notion. Degrees of confirmation can be thought of as probabilities: more or better evidence confirms a given hypothesis to a higher degree, which means making it more probable that the hypothesis is true. However, not everybody within the logical empiricism movement considered confirmation and probability to be so inextricably linked. For instance, Hempel explicitly detached confirmation from probability and focused on a qualitative concept of confirmation (see CH. 25). Carnap and Reichenbach, the main figures of the movement who engaged with the topic and whose proposals are the focus of this chapter, did explicate confirmation in terms of probability. Others, like Richard von Mises or Friedrich Waismann, while working with similar interpretations of probability as, respectively, Reichenbach and Carnap, did not model confirmation in terms of probability.

Connecting confirmation to probability does not automatically lead to a clear account of confirmation: one still needs a clear conception of probability. There were two main interpretations

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of probability within the logical empiricism movement: the frequentist one, developed most intensively by Reichenbach, and the logical one, which Carnap focused on. The frequentist one derives probabilities from sequences of observations, and the logical one sees them as meaning relations between sentences.

Reichenbach's approach to the impossibility of full verification was a radical reconsidering of all semantics, disposing with the notion of truth altogether and replacing it with a probabilistic continuum of degrees of truth. He chose the underlying notion of probability to be the limiting relative frequency in infinite sequences of events. This choice was largely motivated by his work on causality and his rejection of the epistemic views on probability early on in his career. Probability was taken to be a property of sequences of events; it applied to the observed world and not to the language that the world was described in.

Carnap's approach was in line with the rest of his work at the time, with his focus on syntax and semantics of the scientific language. Carnap took confirmation to be a purely semantic notion, with sentences that state the degree to which one statement confirms another coming out as analytic. In this view, sentences about degrees of confirmation do not describe facts about the relations between observed events, but rather objective meaning relations between propositions. The notion of probability that Carnap chose with which to explicate this notion of confirmation was the logical probability.

Probability in the Vienna Circle

Those two main views came about in a specific historical context. Probability was an important topic in the Vienna Circle from its beginning. The main focus of their probability discussions at the time was the logical conception of probability, explored by Wittgenstein and Waismann. Wittgenstein's brief remarks on probability are located in the proposition 5.15 of the *Tractatus*. There, he sketched a simple picture of conditional probability as a relation between ranges of propositions: the proportion of the number of cases (worlds, states) that make the propositions true. In 1929, Waismann gave a few talks about probability to Schlick's Circle. His work on the topic was an elaboration and elucidation of Wittgenstein's basic idea. At the same time, Waismann did not completely shun the frequentist conception, but rather called for a future account of logical probability that will also clarify its relationship with the frequentist one (1930).

In the spring of 1929, Eino Kaila visited Vienna (see CH. 33). Kaila and Carnap met a number of times during that visit to discuss the *Aufbau*. An important part of Kaila's critique of Carnap's book was the lack of any treatment of probability. The criticism did push Carnap to consider the issue more seriously, which eventually led to "Testability and Meaning." While Carnap introduced the concept of confirmation there, he did not yet draw the full connection between confirmation and probability. Instead, he wrote that he considered it as rather impossible to explicate degree of confirmation as "the degree of probability in the strict sense which this concept has in the calculus of probability, i.e. as the limit of relative frequency" (1936–7: 427). It was only at the beginning of the 1940s that he started to work on explicating degrees of confirmation using the concept of probability, but this time choosing the logical rather than the frequentist conception.

While Carnap was almost coerced to working on confirmation and probability—by Kaila's criticisms, as well as the very nature of his interest in the language of science—Reichenbach's way to it was more straightforward. He was interested in the topic from the beginning of his career, with his doctoral dissertation being on the use of the concept of probability in scientific descriptions of the world (1915). When it came to his own explication of probability, his main influence was his Berlin colleague Richard von Mises, who developed a strictly frequentist conception of probability based on random sequences.

Besides Waismann, Carnap, and Reichenbach, other members of the movement spent parts or the whole of their careers working on confirmation and probability. Herbert Feigl's 1927 doctoral dissertation investigated the role of induction and probability in science. He argued that inductive reasoning cannot be shown to be valid, but should rather be sought to be pragmatically vindicated. Janina Hosiasson made a number of contributions to confirmation theory and the logic of inductive reasoning (1931, 1940), as well as a criticism of Reichenbach's probability logic (1936). Ernest Nagel also criticized Reichenbach's conception and argued for a truth-frequency theory of probability, focused on sequences of sentences rather than events (1939).

Two concepts of probability

While there were some efforts to offer alternatives to the two dominant positions, as well as reconciliations, in practice most of the work on probability and confirmation within the logical empiricism movement revolved around the frequentist and the logical conceptions of probability. Extreme subjectivism in Ramsey's and de Finetti's style, while developed roughly at the same time, did not properly enter those discussions until the 1950s, when it was popularized by Leonard Savage.

This dualism of conceptions was summarized by Carnap in his distinction between the two concepts of probability: probability$_1$ and probability$_2$. Probability$_2$ is the physical probability: probability manifested in sequences of observations and explicated using the frequency conception. Probability$_1$ is the logical, semantic one, explicated as the degree of confirmation. Sentences about probability$_1$ are analytic and express a logical relation between evidence and hypothesis. (However, one needs to tread with caution here when it comes to this objective, analytic character of the concept: this is post-*Syntax* Carnap, who assumes implicit relativization to a conceptual framework.) Later on, Carnap elaborated on this idea, writing that statements about probability$_2$ values occur within science, and statements about probability$_1$, belong to inductive logic, which provides rules for operations on the statements within science (1953: 192).

According to Carnap, the two concepts are not incompatible. It is a mistake to insist that only one of them is the correct probability concept, trying to dismiss or reduce the other interpretation. Both concepts—both explicanda—are legitimate objects of formal explication efforts, simply used in different contexts and for different purposes. Reichenbach's stance on that distinction was very different. He maintained that in every context where there is talk of probability, the frequency interpretation can be used. It was part of his program to show how this can be done in the epistemic context.

In spite of this "monist" attitude of Reichenbach, Carnap was positive about his work, although they did not interact on the topic a lot. By the time Carnap was working on probability full time, Reichenbach's theory was fully developed. Moreover, Reichenbach's premature death in the early 1950s prevented him from seeing Carnap's later work.

Carnap's probability: the logical interpretation

In "Testability and Meaning," Carnap was still skeptical about the possibility of having a quantitative explication of the concept of confirmation. In the five years following that paper, he seems to have changed his mind. In his diaries from the time, there are multiple reports of working on "weight" and "confirmation." Inspired by the Vienna Circle discussions, in early 1941 he was led to re-examine the work of other logical probability authors like J. M. Keynes and H. Jeffreys. Soon, an idea for a large monograph was formed, which was published in 1950 as *Logical Foundations of Probability*. From then on, Carnap spent almost all of his time on developing his inductive logic; that is, the theory of logical probability.

The probability that Carnap aims to explicate in *Foundations* is probability_1: the objective, semantic concept of quantitative confirmation. He proceeds according to his own explication procedure (1950: 3–8). First, one specifies the explicandum, choosing those meanings or uses of the original concept that will be the target of the explication. Only when that is done is the formal explicatum constructed. The descriptions of the explicandum that Carnap initially focused on were probability as: measure of evidential support, fair betting quotient, and estimate of relative frequency (i.e., estimate of probability_2). Carnapian inductive logic was a normative project, aimed to provide standards of rationality for beliefs which are influenced by evidence.

Carnap's formal explicata are the confirmation functions, or c-functions. They are functions of pairs of sentences, one representing the available evidence, and the other the hypothesis. The value of the c-function for a given pair of evidence and hypothesis statements represents the degree to which the evidence confirms the hypothesis. The formal languages that the c-functions are defined on are first-order languages with unary predicates, which restricted the applicability of inductive logic.

The c-functions are conditional probability functions, defined in the standard way on the basis of unconditional probability. The latter is in turn defined as a measure on the space of models for the language. However, the axioms of probability calculus alone do not provide specific numerical values for contingent propositions: any such sentence can in principle have any probability assigned to it by a particular function. Carnap considered the resulting theory too weak to be useful as a theory of confirmation: for successful applications, it had to offer the scientist specific confirmation, workable values for the kind of statements that she would be interested in, i.e., empirical rather than purely logical ones.

Hence, Carnap proposed further axioms, regularity and symmetry. Regularity ensures, roughly, that logically possible sentences are assigned positive probability. Symmetry requires a priori probabilities for every atomic sentence to be equal; it is a version of the infamous principle of indifference, which says that in absence of reasons to do otherwise, one should assume equal probabilities for all possible outcomes. It was Carnap's continuous reliance on versions of this principle—and his insistence that they were rationally required—that earned him the bulk of the criticism that his inductive logic received over the years. He continued to defend his use of the principle as applicable in the kind of situation he was modeling, i.e., under the assumption of the lack of any knowledge beyond the knowledge of the structure of the object language. Any further updates from this initial position of complete ignorance were to be made by conditionalizing on all of the information received. The latter condition is known as the requirement of total evidence, which was often criticized as unrealistic.

After *Foundations*, Carnap focused on the search for axiomatic representations of further classes of rationally admissible c-functions, with the additional axioms justified by considerations concerning the rationality of inductive reasoning. At first, the class of admissible confirmation functions was parametrized using a single real-valued parameter λ (1952). The λ expresses the rate at which a c-function is influenced by the observations, as opposed to being tied to the a priori assumptions about the possible observations. In the limit, with more and more observations, the confirmation values for observed properties converge to the empirical frequencies.

The last stage of the development of Carnap's inductive logic was the *Basic System of Inductive Logic* (1971a, 1980). Instead of proposing a single parametrized family, Carnap considered a range of possibilities in terms of new axioms and parameters. He relaxed the previous symmetry requirement, allowing for the prior probabilities to not be distributed evenly across all predicates of the language. He also explored ways to formalize possible statistical dependencies between different predicates.

Another development within the inductive logic program which sparked a lot of debate, was the bringing in of a new description of probability$_1$ in terms of rational decision making. The new description was of probability$_1$ as the rational probability value to be used when calculating rational expected utility of an action; Carnap eventually dropped the confirmation theory angle entirely. This change was likely brought in under the influence of John Kemeny, who was Carnap's close collaborator in the early 1950s and introduced him to the developments in the subjective epistemic probability interpretation.

Carnapian inductive logic received a lot of criticism over the years. At first it was because Carnap was mistakenly thought to have endorsed a single rational confirmation function, which was perceived as far too strict and aprioristic. Additionally, his continuing reliance on different versions of the principle of indifference was never accepted. Finally, as Carnap investigated wider and wider classes of admissible confirmation functions, his project was interpreted as having moved from explicating an objective to a subjective concept of probability—with Carnap himself consistently denying this interpretation, insisting that nothing that he ever said made his concept of probability "subjectivistic" (see, e.g., 1963: 972).

Reichenbach's probability: the frequency interpretation

After having rejected both the subjective and objective epistemic interpretations of probability in his doctoral dissertation, Reichenbach turned to an account informed primarily by scientific practice. His first book-length treatment of probability was *Wahrscheinlichkeitslehre* (1935; revised for the English translation of 1949). It was not received uncritically, with prominent figures like C. I. Lewis and Russell expressing serious disagreement. However, the basic conceptual features of his interpretation of probability, as well as his pragmatic justification of induction, have secured a lasting place in the history of philosophy and shaped many of the subsequent discussions.

Reichenbach's approach to probability is expressly empirical. Instead of coming up with rationally justified or self-evident first principles of confirmation, we are to look at our most successful inductive practices and make their methods explicit, by formalizing their assumptions into an axiomatic system. Such an approach cohered with his account of deductive logic, which was a formalist one: he saw logic as an axiomatic calculus, with the degree to which the axioms correspond to the features of the real world as a matter of "coordination," which was not an a priori issue. Reichenbach insisted that in every context where the concept of probability is used, it can be modeled using his frequency conception; this was to be true also of epistemic contexts, or contexts in which we talk about probabilities of general sentences, like the scientific laws.

As an objective feature of the world, probability is considered by Reichenbach to be a property of sequences of events: the probability of an event is the limiting relative frequency of events of its type in an infinite sequence of events. While this formulation is simple, it is a challenge to clearly spell out the details. First of all, what kind of infinite sequences of events are ones that determine probabilities? They cannot be just any sequences, but they have to be random in some way. Compare the following sequence of coin tosses: H(eads)T(ails)HTHTHT. . ., where the relative frequency of each outcome approaches 0.5, but intuitively, the outcome of the next toss becomes almost certain as the sequence grows. Reichenbach, however, was skeptical about the possibility of a successful definition of truly random sequences, something which von Mises aspired to. Hence, Reichenbach restricted himself to so-called normal sequences of events, which is a weaker concept than random sequences.

Regardless of their exact specification, in real life we do not have access to infinite sequences of observed events. This means that the probability values used in practice must be approximations, or estimations, of the ones provided by idealized, infinite sequences of events. The method of approximation that Reichenbach proposes is the Straight Rule, which says that as the probability value, one should take the empirical frequency in the actually observed sequence of events. The Straight Rule provides us with workable basic probability values for contingent statements, in the same way the symmetry axiom gave Carnap some basic probability values to start with, so to speak.

The Straight Rule needed justification. Reichenbach's argument for it was that it converges to the limiting relative frequency in the long run, i.e., that it gives values that can be arbitrarily close to the actual probability, as the number of observations increases (provided that there is a limit). However, not only does this hold for many methods other than the Straight Rule, but it does not guarantee uniform convergence, which in turn means that in any particular case we cannot know how many observations (at most) it will take to get close to the limiting frequency.

Finally, there is the issue of what events count as events of the same type: this is the reference class problem. This problem comes up especially strongly when it comes to calculating probabilities of one-off, or singular, events, for which there is no natural sequence of previous observations, like the death of a particular person or the occurrence of a previously unknown disease. According to Reichenbach, probabilities are assigned to such events in an extended sense, as posits. We are to choose the narrowest reference class of events similar to the one in question, for which we have stable statistics, and posit the frequency in that class as the one-off probability. For instance, we can assign the person in question the class of people of the same age who suffer from the same diseases, and check the death rates in that class. This simple idea, however, is problematically circular: the availability of "stable statistics" presupposes an inductive procedure to have been there already in the first place.

Posits are divided into blind and appraised ones, depending on whether there was any data to go by—in the form of statistics concerning an adequate reference class—when deciding on the value for a posit. When more information becomes available, blind posits become appraised through integrating this information into the new probability estimation. The same kind of dynamics occurs when we move from primitive to appraised knowledge, as more and longer sequences of observations become available as bases for probability estimates. Hence, the knowledge of probabilities for Reichenbach is deeply rooted in the available experience, and grows together with it.

As Reichenbach took the idea that no empirical statements can be fully verified or falsified to its logical end, he disposed of the classical notion of truth. As a result, the binary truth-values were replaced by a range of values, and the new probability logic was continuum-valued. Under the frequency interpretation, probabilities are assigned to sequences of events. However, the probability logic is defined on a formal language, which requires a propositional representation of the sequences of events. This is achieved by creating sequences of propositions describing the events in the sequence, creating an isomorphic sequence within the language.

Reichenbach's probability logic is the logic of those sequences of propositions. It is based on classical deductive logic, with additional axioms that capture the idea that probabilities are properties of infinite sequences. These axioms were: univocality, normalization, addition, and multiplication. They form, in Reichenbach's own language, an axiomatization of finitely additive probability. The final element is the Straight Rule, or rule of induction, which allows for the derivation of specific numerical values for probabilities of contingent propositions. The exact details of the syntax and semantics of the actual probability logic were left unclear or underdeveloped in many places, which makes it hard to evaluate the extent to which Reichenbach achieved formally what he was aiming at conceptually.

Apriorism, theories, induction

Reichenbach distanced himself explicitly from any kind of apriorism, both in logic and in epistemology. Hence, in his conception, specific degrees of probability can be known only a posteriori, based on enumerative induction on observations, formulated as the Straight Rule. Carnap's logical probability, on the other hand, was explicitly aprioristic, as he sought to explicate degree of confirmation as a purely semantic relation between propositions. The justification that Carnap offered for those of his axioms that extended the basic probability calculus was based on normative considerations of rationality. These two conceptions of probability lead to different stances on important topics within epistemology and philosophy of science, just as they emerge from different conceptions of semantics or logic.

The problem of single case probabilities arises differently for the two interpretations. For frequentism, this issue is immediate, since probability itself is defined as a property of sequences of events. I explained earlier how Reichenbach's solution to assign probability to singular events in a special, extended way led to the reference class problem. Carnap did not address this problem explicitly, because it did not arise so starkly for his approach. His inductive logic was syntax-based in the sense that (with some exceptions in the Basic System where he considers partially interpreted languages) he did not focus at all on what the basic predicates of the object language were supposed to mean, and whether the individual constants referred to any specific individuals that did not belong to any more general types. His inductive logic provided a priori probability values for any kind of event, as long as it could be described using one of the predicates of the object language, regardless of how singular the events could have been.

Assigning confirmation values to scientific theories, as opposed to simple observation statements, poses a significant challenge for both conceptions, albeit for different reasons. In Carnap's inductive logic, universally quantified sentences always have zero probability, which is a straightforward consequence of his measure-theoretic approach to unconditional probabilities. This means that, under his interpretation, no scientific theory could be confirmed to a positive degree. He chose to bite the bullet on this issue and to focus on one-step predictive probability: confirmation functions tell us the probability of the next observed object being of a certain kind, rather than specifying the probability for any object being of that kind. He argued that in ordinary discourse, when confirmation of theories is discussed, it is meant that they hold only of a finite, rather than infinite, number of instances (1950, §110G).

When it comes to the impact that the observations have on scientific theories, or hypotheses, Reichenbach's view was Bayesian: the a posteriori probabilities of theories were to be calculated according to the Bayes formula. The prior probabilities were calculated objectively from the empirical frequencies using the Straight Rule. However, under the frequency interpretation, it only makes sense to ascribe probability values when there is an appropriate sequence of events available. This means that to assign probabilities, or degrees of confirmation, to scientific theories, one needs some sort of sequence of theories "of the same kind," and an interpretation of how the truth frequency in such a sequence would be determined. Reichenbach suggested a solution along those lines, and the exact details were worked out by Wesley Salmon (1967). The kind of procedure envisioned by Reichenbach turned out to be essentially the same as the modern hierarchical Bayesian picture.

A full theory of confirmation and the effect that evidence has on theories requires a general justification of inductive reasoning, explaining why observations can be rationally expected to have a predictive value over the future ones. As the answer to that problem, Reichenbach offered his pragmatist vindication of induction in terms of success: if the world is such that any prediction method can succeed in it, then his inductive method will be successful (see CH. 24).

Carnap's relationship with this problem was more complicated. In *Foundations* (1950: §41f), he offered an unsatisfactory treatment of the problem of induction, claiming that in his inductive logic one can analytically prove the sentence that states that conditional on our experience, the degree of uniformity of the world is high (which is a presupposition of induction). After being repeatedly criticized on this, he continued working on a better answer, which was never published (for background details, see Carus 2017).

Later, he argued that justifying the inductive method reduces to the question of justification of the axioms for confirmation functions, which is a higher-level issue than choosing a particular function from the set of admissible confirmation functions. In the 1960s Carnap offered two kind of answers to that problem. On the one hand, he attempted to justify his axioms in terms of rationality considerations about decisions that the beliefs following those axioms would lead to (1971b). On the other hand, he made what detractors considered a controversial claim about inductive intuition: that it allows us to directly "see" the correctness of the basic axioms of inductive logic in the same way that we "see" the correctness of the basic axioms of deductive logic (1968). Defenders argue that no special faculty was meant to be invoked (Wagner 2011).

After the big two

The philosophical study of probability is currently dominated by subjective Bayesianism, which is somewhat distant from Carnap's more aprioristic approach focused on finding more and more formal constraints on rational credences. Carnap's program was continued after his death, among others by his long-time collaborator Richard Jeffrey (1973) (see also Zabell 2011). Carnapian confirmation functions were also shown by Brian Skyrms to be natural counterparts of certain classes of Bayesian priors (1996). Currently, the inductive logic program is also continued under the label of objective Bayesianism (Williamson 2016); there is also work done in the pure inductive logic tradition, which focuses on extending Carnap's functions to richer, but still uninterpreted, languages (Paris and Vencovská 2015).

Reichenbach should be given credit for integrating the frequency conception of probability into a full philosophical system and constructing a complete epistemology that took seriously the idea that all knowledge is eventually probabilistic (Eberhardt and Glymour 2011). Several of Reichenbach's own ideas were elaborated on by his students like Salmon (1967), but his probability logic did not enjoy the kind of following that Carnap's did, in terms of an organized research program continuing for decades afterwards.

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