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The symmetries of the Carroll superparticle

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Abstract

Motivated by recent applications of Carroll symmetries we investigate, using the method of nonlinear realizations, the geometry of flat and curved (AdS) Carroll space and the symmetries of a particle moving in such a space both in the bosonic as well as in the supersymmetric case. In the bosonic case we find that the Carroll particle possesses an infinite-dimensional symmetry which only in the flat case includes dilatations. The duality between the Bargmann and Carroll algebra, relevant for the flat case, does not extend to the curved case. In the supersymmetric case we study the dynamics of the $\mathcal{N} = 1$ AdS Carroll superparticle. Only in the flat limit we find that the action is invariant under an infinite-dimensional symmetry that includes a supersymmetric extension of the Lifshitz Carroll algebra with dynamical exponent $z = 0$. We also discuss in the flat case the extension to $\mathcal{N} = 2$ supersymmetry and show that the flat $\mathcal{N} = 2$ superparticle is equivalent to the (non-moving) $\mathcal{N} = 1$ superparticle and that therefore it is not BPS unlike its Galilei counterpart. This is due to the fact that in this case kappa-symmetry eliminates the linearized supersymmetry. In an appendix we discuss the $\mathcal{N} = 2$ curved case in three-dimensions only and show that there are two $\mathcal{N} = 2$ theories that are physically different.

Keywords: superparticle, Carroll algebra, kappa-symmetry

1. Introduction

Space–time symmetries have played a central role in the understanding of various physical theories such as Newtonian gravity, Maxwell's electromagnetism, special relativity, general relativity, strings and supergravity. Most of these models are based on relativistic symmetries.

An example of a model with non-relativistic symmetries is Newtonian gravity which is based on the Galilei symmetries. Such non-relativistic symmetries arise when the velocity of light is sent to infinity.

A formulation of non-relativistic gravity that is invariant under diffeomorphisms was introduced by Cartan [1], see also [2–6]. This so-called Newton–Cartan gravity can be reformulated as a gauge theory of the Bargmann algebra [7, 8]. The interest in Galilean-invariant theories with diffeomorphism invariance has increased recently due to their relation with condensed matter systems [9–11], see also [12, 13] and references therein. Galilean-invariant theories have also appeared recently in studies of Lifshitz holography [14, 15].

Other non-relativistic theories such as non-relativistic superstrings and superbranes have been studied as special points in the parameter space of *M*-theory [16, 17]. Non-relativistic strings have also attracted attention due to the fact that they appear as a possible soluble sector within string theory or *M*-theory [18, 19].

A less well known example of a non-relativistic symmetry are the Carroll symmetries which arise when the velocity of light is sent to zero [20]. In this sense the Carroll symmetries are the opposite to the Galilei symmetries. This can also be seen by looking at the light cone which in the Carroll case, at each point of spacetime, collapses to the time axis whereas in the Galilei case it coincides with the space axis.

In physical systems Carroll symmetries appeared for the first time in the strong coupling limit of general relativity when a flat spacetime solution is considered [21]. Recently, it has been proven that the asymptotic symmetry of asymptotically flat spacetimes, the so-called BMS symmetry [22–24], is isomorphic to a conformal extension of the Carroll symmetries [25]. In particular, it has been shown that the Carroll group can be viewed as a subgroup of the Poincaré group in $(D + 1, 1)$ dimensions. The same reference also studies non-Einsteinian electrodynamics in the Carroll limit and gives an example where both Galilean and Carrollian symmetries coexist in a non-relativistic *D*-dimensional Chaplygin gas. More recently, it has been proposed that the BMS group is relevant to understand the holography in asymptotically flat space times [26–30].

Some studies have shown that the Carroll group plays an essential role in the phenomenon of tachyon condensates in string theory [31]. Sen [32] had a remarkable insight into the nature of the open bosonic string tachyon. He pointed out that a space-filling *D*-brane is unstable in the bosonic theory, as it does not carry any conserved charge, and he suggested that the open bosonic string tachyon should be interpreted as the instability mode of the *D*-brane. This led him to conjecture that the open string field theory could be used to precisely determine a new vacuum for the open string, namely one in which the *D*-brane is annihilated through condensation of the tachyonic unstable mode [33]. It turns out that the open string excitations are subject to a time-like half line collapsed lightcone which corresponds to the Carroll limit [34].

Other recent developments are that Carroll symmetries allow to build a fully covariant formalism for warped conformal field theories (WCFTs), i.e. the simplest field theories without Lorentz invariance that can be described holographically, in curved spaces [35]. A procedure to gauge the Carroll algebra is given in [35, 36], and in [36–38] one can find constructions of more general Carroll geometries and associated Carroll structures that are out of the scope of this paper.

A systematic investigation of the possible relativity groups[4] was initiated by Bacry and Lévy-Leblond [39]. They showed that all these groups can be obtained by a contraction of the

[4] A relativity group is an invariance group of a physical theory that contains the generators of special relativity: time translations, spatial translations, boosts and spatial rotations.

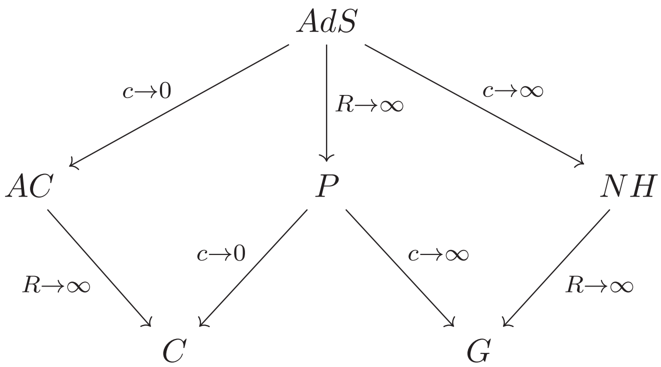

Figure 1. The figure displays the different contractions of the AdS group. The different abbreviations are explained in the text.

Table 1. This table gives an overview of the algebras of the relativity groups that we consider in this paper.

	$[P_a, K_b]$	$[H, K_a]$	$[H, P_a]$	$[P_a, P_b]$	$[K_a, K_b]$
AdS	$\delta_{ab}H$	P_a	$-\frac{1}{R^2}K_a$	$\frac{1}{R^2}M_{ab}$	M_{ab}
Poincaré	$\delta_{ab}H$	P_a	0	0	M_{ab}
Newton–Hooke	$\delta_{ab}Z$	P_a	$-\frac{1}{R^2}K_a$	0	0
AdS-Carroll	$\delta_{ab}H$	0	$-\frac{1}{R^2}K_a$	$\frac{1}{R^2}M_{ab}$	0
Galilei	$\delta_{ab}Z$	P_a	0	0	0
Carroll	$\delta_{ab}H$	0	0	0	0

anti-de Sitter (AdS) and de Sitter (dS) groups[5]. As figure 1 shows there are three different types of contractions: the non-relativistic limit $c \to \infty$ of the AdS group leads to the Newton–Hooke (NH) group. The flat limit $R \to \infty$ leads to the Poincaré (P) group and the ultra-relativistic limit $c \to 0$ leads to the AdS-Carroll (AC) group [20][6]. In a second stage, the flat limit of the AC group and the ultra-relativistic limit of the Poincaré group leads to the Carroll (C) group while the non-relativistic limit of the Poincaré group and the flat limit of the NH group leads to the Galilean (G) group.

All the algebras corresponding to the groups given in figure 1 contain the same commutators involving spatial rotations. These commutators are given by

$$[M_{ab}, M_{cd}] = 2\eta_{a[c}M_{d]b} - 2\eta_{b[c}M_{d]a}, \tag{1.1}$$

$$[M_{ab}, P_c] = 2\delta_{c[b}P_{a]}, \qquad [M_{ab}, K_c] = 2\delta_{c[b}K_{a]}, \tag{1.2}$$

where $a = 1, \ldots, D - 1$, for a D-dimensional space–time. The Galilean algebra can extended with a central charge generator Z to the so-called Bargmann algebra [40]. It has been recently shown that there is a duality between this Bargmann algebra and the Carroll algebra by the exchange of Z and the generator of time translations H [37]. Note that this duality does not extend to a duality between the NH and AC algebras. This is due to the expression for the commutator $[P_a, P_b]$, see table 1.

[5] In this paper we will only consider the AdS case.
[6] Bacry and Lévy-Leblond [39] call this algebra the para-Poincaré algebra.

Table 2. In this table we give the (anti-)commutators of the $\mathcal{N}=1$ Newton–Hooke and AC superalgebras that involve the generators Q of supersymmetry. Note that here is no duality between the two algebras.

$\mathcal{N}=1$	$[M_{ab}, Q]$	$[P_a, Q]$	$\{Q_\alpha, Q_\beta\}$
Newton–Hooke	$-\frac{1}{2}\gamma_{ab}Q$	0	$2\delta_{\alpha\beta}Z$
AdS-Carroll	$-\frac{1}{2}\gamma_{ab}Q$	$\frac{1}{2R}\gamma_a Q$	$[\gamma^0 C^{-1}]_{\alpha\beta}H + \frac{2}{R}[\gamma^{a0}C^{-1}]_{\alpha\beta}K_a$

The aim of this paper is to study the general structure of the Carroll symmetries along the same lines as this has been done for the Galilean symmetries. This will be done in two stages. As a first step we will study the geometry of the empty Carroll space considering the coset $G/H = \mathrm{AC}/\mathrm{HomAC}$, where Hom AC is the homogeneous part of the AC algebra. In a second step we will put a particle in this Carroll space and construct an action describing its dynamics.

More specifically, in the first part of this paper we consider the bosonic AC algebra. In particular, we will construct the action of a particle invariant under the symmetries corresponding to this algebra using the method of nonlinear realizations [41, 42]. This so-called AC particle reduces, in the limit that the AdS radius goes to infinity, to the Carroll particle that we studied in our previous paper [43]. A characteristic feature of the free Carroll particle is that it does not move [37, 43, 44][7]. As we will see the AC particle does not move, but unlike the Carroll particle the momenta are not a constant of motion as a consequence of the AC symmetry. Another difference with the Carroll particle is that the mass-shell constraint depends on the coordinates of the AC space, therefore the AC particle 'sees' the geometry. This is different from the Carroll case where the energy of the particle is equal to plus or minus the mass [37, 43]. We find that only in the massless limit the mass-shell constraint coincides with the flat Carroll case. Using the AC particle action we will construct the Killing equations for the AC space. We find that the solution of the Killing equations produces an infinite-dimensional algebra that contains the symmetries of the AC algebra. The Lifshitz dilatations are not included in these symmetries. Only in the flat case the dilatations with $z = 0$ are part of the infinite dimensional algebra.

In the second part of this paper we consider the supersymmetric extension of the Carroll algebras[8]. We first construct the $\mathcal{N}=1$ AC superalgebra in any dimension (see tables 1 and 2, where Q stands for the generator of supersymmetry). A difference with respect to the supersymmetric NH case is that we have a conventional supersymmetry algebra, where the energy and boost generators appear in the anti-commutator of the supersymmetries. The AC superalgebra in the flat limit contains the supersymmetric extension of the 'Lifshitz boost extended Carroll algebra' introduced in appendix B of [45]. We construct the AC superparticle action both as the non-relativistic limit of the relativistic massive superparticle [46, 47] as well as by applying the nonlinear realization technique. As we will see the $\mathcal{N}=1$ AC superparticle like in the Relativistic and Galilean case is non-BPS, i.e. the supersymmetries are nonlinearly realized. We will study the super-Killing equations and we find in general an infinite-dimensional algebra of symmetries thereby extending the finite $\mathcal{N}=1$ super AC transformations.

7 If we consider two particles or a particle interacting with Carroll gauge fields the dynamics is non-trivial. The same phenomenon occurs in tachyon condensation when the tachyon interacts with a gauge field [31].

8 A first attempt in this direction was done in the unpublished notes [44].

Table 3. In this table we give the (anti-)commutators of the $\mathcal{N}=2$ Galilei and Carroll supersymmetry algebras. Note that there is no duality between these two algebras.

$\mathcal{N}=2$	$[M_{ab}, Q^{\pm}]$	$[K_a, Q^{+}]$	$\{Q^{+}_{\alpha}, Q^{+}_{\beta}\}$	$\{Q^{+}_{\alpha}, Q^{-}_{\beta}\}$	$\{Q^{-}_{\alpha}, Q^{-}_{\beta}\}$
Galilei	$-\frac{1}{2}\gamma_{ab}Q^{\pm}$	$-\frac{1}{2}\gamma_{a0}Q^{-}$	$[\gamma^0 C^{-1}]_{\alpha\beta}H$	$[\gamma^a C^{-1}]_{\alpha\beta}P_a$	$2[\gamma^0 C^{-1}]_{\alpha\beta}Z$
Carroll	$-\frac{1}{2}\gamma_{ab}Q^{\pm}$	0	$\frac{1}{2}[\gamma^0 C^{-1}]_{\alpha\beta}(H+2Z)$	0	$\frac{1}{2}[\gamma^0 C^{-1}]_{\alpha\beta}(H-2Z)$

Inspired by the relativistic and Galilei case we will investigate whether the $\mathcal{N}=2$ Carroll superparticle is BPS or not. For simplicity we restrict to the three-dimensional case. We first construct the $\mathcal{N}=2$ Carroll superalgebra as a contraction of the $\mathcal{N}=2$ Poincaré superalgebra. This leads to the result given in table 3. We see that, unlike in the bosonic case, there is no duality in the supersymmetric case. Next, we construct the action for the $\mathcal{N}=2$ Carroll superparticle. This action has two terms, one of them is a Wess–Zumino term. If we properly choose the coefficients of the two terms we find a so-called kappa gauge symmetry [48, 49] that kills half of the fermions. This gauge symmetry has the form of a Stückelberg symmetry, similar to what we found in the Galilean case [18, 50]. We find that after fixing the kappa-symmetry the super-Carroll action reduces to the action we found in the $\mathcal{N}=1$ case. The linearly realized supersymmetry acts trivially on all the fields and therefore the $\mathcal{N}=2$ Carroll superparticle reduces to the $\mathcal{N}=1$ Carroll superparticle and hence is not BPS. This is rather different from the $\mathcal{N}=2$ super-Galilei case were BPS particles do exist. The main difference between the super-Carroll and super-Galilei cases comes from the kappa symmetry transformations, in the former case it eliminates the linearized supersymmetry and it the last case it does not.

In a separate appendix we extend our investigations to the $\mathcal{N}=2$ curved case and consider the Carroll contraction of the so-called (p, q) AdS superalgebras [51] for the particular cases of $(p, q) = (2, 0)$ and $(p, q) = (1, 1)$. The (2, 0) and (1, 1) AC algebras are not isomorphic. We find that the associated particle actions are rather different. While in the (2, 0) case we have kappa-symmetry, we find that this is not the case in the (1, 1) case. The two models have different degrees of freedom.

This paper is organized as follows. In section 2 we discuss the bosonic free AC particle thereby extending our previous analysis [43] to the curved case. In particular, we construct the action and investigate the Killing equations. In section 3 we consider the $\mathcal{N}=1$ AC superparticle. At the end of this section we discuss the flat limit. Finally, in section 4 we investigate the $\mathcal{N}=2$ super Carroll particle. Our conclusions are presented in section 5. Some technical details and the extension of the $\mathcal{N}=2$ super Carroll particle to the curved case, for three-dimensions only, are given in three appendices.

2. The free AC particle

Before discussing the supersymmetric case we will first study in this section different aspects of the free AC particle.

2.1. The AC algebra

In order to write the commutators corresponding to the AC algebra, we will start with the contraction of the D-dimensional AdS algebra. The basic commutators are given by $(A = 0, 1, \ldots, D-1)$

$$[M_{AB},\, M_{CD}] = 2\eta_{A[C}M_{D]B} - 2\eta_{B[C}M_{D]A}, \tag{2.1}$$

$$[M_{AB},\, P_C] = 2\eta_{C[B}P_{A]}, \qquad [P_A,\, P_B] = \frac{1}{R^2}M_{AB}, \tag{2.2}$$

where R is the AdS radius. Here P_A and M_{AB} are the (anti-hermitian) generators of space–time translations and Lorentz rotations, respectively.

To make the Carroll contraction we rescale the generators with a parameter ω as follows [20, 39]:

$$P_0 = \frac{\omega}{2}H, \qquad M_{a0} = \omega K_a. \tag{2.3}$$

Taking the limit $\omega \to \infty$ we find that the commutators corresponding to the D-dimensional AC algebra are given by ($a = 1,\ldots,D-1$):

$$[M_{ab},\, M_{cd}] = 2\eta_{a[c}M_{d]b} - 2\eta_{b[c}M_{d]a}, \qquad [M_{ab},\, K_c] = 2\delta_{c[b}K_{a]}, \tag{2.4}$$

$$[M_{ab},\, P_c] = 2\delta_{c[b}P_{a]}, \qquad [P_a,\, K_b] = \frac{1}{2}\delta_{ab}H, \tag{2.5}$$

$$[P_a,\, P_b] = \frac{1}{R^2}M_{ab}, \qquad [P_a,\, H] = \frac{2}{R^2}K_a. \tag{2.6}$$

Notice that the commutation relations of space–time translation coincide with the same commutation relations of the AdS algebra. The difference between the AdS and AC algebra is in the different commutation relations that involve the boost generators. Note that this is not the case for the NH algebras.

The AC algebra can be expressed in terms of the left invariant Maurer–Cartan one-forms L^a, which satisfy the Maurer–Cartan equations $\mathrm{d}L^C - \frac{1}{2}f^C{}_{AB}L^BL^A = 0$. Explicitly, these equations read

$$\mathrm{d}L_H + \frac{1}{2}L_P^a\, L_K^a = 0, \qquad \mathrm{d}L_P^a - 2L_P^b\, L_M^{ab} = 0, \tag{2.7}$$

$$\mathrm{d}L_K^a - 2L_K^b\, L_M^{ab} = \frac{2}{R^2}L_H L_P{}^a, \qquad \mathrm{d}L_M^{ab} - 2L_M^{ca}\, L_M^{cb} = \frac{1}{2R^2}L_P{}^b L_P{}^a. \tag{2.8}$$

2.2. Nonlinear realizations

In this subsection we apply the method of nonlinear realizations [41, 42] and use the algebra (2.4) to construct the action of the AC particle.

We consider the coset $G/H = \mathrm{AC}/\mathrm{SO}(D-1)$ and the coset element $g = g_0\, U$, where $g_0 = e^{Ht}e^{P_a x^a}$ is the coset representing the AC space and $U = e^{K_a v^a}$ is a general Carroll boost. The x^a ($a = 1, \ldots D-1$) are the Goldstone bosons of broken translations, t is the Goldstone boson of the unbroken time translation[9] and U is parametrized by the Goldstone bosons of the broken Carroll boost transformations.

The reason to consider the coset element in terms of g_0 and U is because in this way we have that for a general symmetric space–time g_0 is the coset element representing the 'empty' space–time, while U represents the broken symmetries that are due to the presence of a dynamical object, in our case a particle, in the 'empty' space–time. For the case of a particle U is given by the general rotation that mixes the 'longitudinal' time direction with the

[9] The unbroken translation P_0 generates via a right action [52, 53] a transformation which is equivalent to the world-line diffeomorphisms.

'transverse' space directions, i.e. the Carroll boosts. If we would like to consider as a dynamical object a p-brane, we should consider as U the general rotations that mix the longitudinal and tranverse directions [53].

Returning to the AC particle, it is interesting to write out the Maurer–Cartan form Ω_0 associated to the AC space

$$\Omega_0 = g_0^{-1}\mathrm{d}g_0 = He^0 + P_a e^a + K_a\omega^{a0} + M_{ab}\omega^{ab}, \tag{2.9}$$

where (e^0, e^a) and $(\omega^{a0}, \omega^{ab})$ are the space and time components of the Vielbein and spin connection one-forms of the AdS space, respectively. If we parametrize the AdS space as $e^{Ht}e^{P_a x^a}$, the Vielbein and spin-connection one-forms corresponding to the AC space are given by

$$\begin{aligned}
e^0 &= \mathrm{d}t\cosh\frac{x}{R},\\
e^a &= \frac{R}{x}\mathrm{d}x^a\sinh\frac{x}{R} + \frac{1}{x^2}x^a x^b\mathrm{d}x_b\left(1 - \frac{R}{x}\sinh\frac{x}{R}\right),\\
\omega^{a0} &= -\frac{2}{xR}\mathrm{d}tx^a\sinh\frac{x}{R},\\
\omega^{ab} &= \frac{1}{2x^2}(x^b\mathrm{d}x^a - x^a\mathrm{d}x^b)\left(\cosh\frac{x}{R} - 1\right).
\end{aligned} \tag{2.10}$$

These one-forms satisfy the structure equations

$$\mathrm{d}e^0 + \frac{1}{2}e^{\,a}\,\omega^{\,a0} = 0, \qquad \mathrm{d}e^{\,a} - 2e^{\,b}\,\omega^{\,ab} = 0, \tag{2.11}$$

$$\mathrm{d}\omega^{\,a0} - 2\omega^{\,b0}\,\omega^{\,ab} = \frac{2}{R^2}e^0e^a, \qquad \mathrm{d}\omega^{\,ab} - 2\omega^{\,ca}\,\omega^{\,cb} = \frac{1}{2R^2}e^be^a. \tag{2.12}$$

We see that the the AC space, like the ancestor AdS space, has constant negative spatial curvature, i.e. the left side of the equation on the right of equation (2.12) is non-zero.

We now insert a particle in the empty AC space and consider the Maurer–Cartan form of the combined system:

$$\Omega = g^{-1}dg = U^{-1}\Omega_0 U + U^{-1}\mathrm{d}U. \tag{2.13}$$

In order to derive an expression for Ω we need to know how the space–time translation generators and the boost generators transform under a general Carroll boost:

$$\begin{aligned}
U^{-1}\,H\,U &= H + \frac{1}{2}v^a P_a,\\
U^{-1}\,P_a\,U &= P_a,\\
U^{-1}\,K_a\,U &= K_a,\\
U^{-1}\,M_{ab}\,U &= M_{ab} + v_b K_a - v_a K_b.
\end{aligned} \tag{2.14}$$

We have also $U^{-1}\mathrm{d}U = \mathrm{d}v^a K_a$. Using these formulae we find that the Maurer–Cartan form Ω is given by

$$\begin{aligned} L_H &= e^0 + \frac{1}{2}v_a e^a, \\ L_P{}^a &= e^a, \\ L_K{}^a &= \omega^{0a} + \mathrm{d}v^a + 2v_b\,\omega^{ab}, \\ L_M{}^{ab} &= \omega^{ab}. \end{aligned} \tag{2.15}$$

We note that that the Maurer–Cartan forms of space–time translations can be written in matrix-form as follows:

$$\left(L_H,\ \ L_P{}^a\right) = \left(e^0,\ \ e^a\right)\begin{pmatrix} 1 & 0 \\ \frac{1}{2}v_a & 1 \end{pmatrix}. \tag{2.16}$$

The matrix appearing at the right-hand side is the most general Carroll boost in the vector representation.

We now proceed with the construction of an action of the AC particle. An action with the lowest number of derivatives is obtained by taking the pull-back of all the L's that are invariant under rotations, see for example [53]. In this way we obtain the following action:

$$\begin{aligned} S &= M\int (L_H)^* = M\int \left(e^0 + \frac{1}{2}v_a e^a\right)^* \\ &= M\int \mathrm{d}\tau\left(\dot{t}\cosh\frac{x}{R} + \frac{R}{2x}v_a\dot{x}^a\sinh\frac{x}{R} + \frac{1}{2x^2}x^b v_b x_a\dot{x}^a\left(1 - \frac{R}{x}\sinh\frac{x}{R}\right)\right). \end{aligned} \tag{2.17}$$

This action is invariant under the following transformation rules with constant parameters $(\zeta,\, a^i,\, \lambda^i,\, \lambda^i_j)$ corresponding to time translations, spatial translations, boosts and spatial rotations, respectively:

$$\begin{aligned} \delta t &= -\zeta + \frac{R}{2x}\lambda^k x_k\tanh\frac{x}{R} + \frac{t}{Rx}a^k x_k\tanh\frac{x}{R}, \\ \delta x^i &= -\frac{1}{x^2}\left(x^i a^k x_k - \frac{x}{R}\coth\frac{x}{R}(x^i a^k x_k - a^i x^2)\right) - 2\lambda^i{}_k\, x^k, \\ \delta v^i &= -\lambda^i - \frac{1}{x^2}\lambda^k x_k x^i\,\mathrm{sech}\,\frac{x}{R}\left(1 - \cosh\frac{x}{R}\right) - 2\lambda^i{}_j\, v^j - \frac{2t}{R^2}a^i \\ &\quad - \frac{2t}{R^2x^2}x^i a^k x_k\,\mathrm{sech}\,\frac{x}{R}\left(1 - \cosh\frac{x}{R}\right) + \frac{2}{Rx}v_b a^{[i}x^{b]}\mathrm{csch}\frac{x}{R}\left(1 - \cosh\frac{x}{R}\right). \end{aligned} \tag{2.18}$$

The equations of motion for t, x^a and v^a read

$$\begin{aligned} 0 &= \frac{1}{xR}x^a\dot{x}_a\sinh\frac{x}{R}, \\ 0 &= -\frac{R}{2x}\dot{x}^a\sinh\frac{x}{R} - \frac{1}{2x^2}x^a x_b\dot{x}^b\left(1 - \frac{R}{x}\sinh\frac{x}{R}\right), \\ 0 &= \frac{R}{2x}\dot{v}_a\sinh\frac{x}{R} - \frac{1}{xR}\dot{t}x_a\sinh\frac{x}{R} + \frac{1}{2x^2}x_a x^b\dot{v}_b\left(1 - \frac{R}{x}\sinh\frac{x}{R}\right) \\ &\quad + \frac{\dot{x}^b}{2x^2}(v_a x^b - x_a v_b)\left(\cosh\frac{x}{R} - 1\right). \end{aligned} \tag{2.19}$$

These equations imply that

$$\dot{x}^a = 0,$$
$$\frac{1}{xR}\dot{t}x_a \sinh\frac{x}{R} = \frac{R}{2x}\dot{v}_a \sinh\frac{x}{R} + \frac{1}{2x^2}x_a x^b \dot{v}_b\left(1 - \frac{R}{x}\sinh\frac{x}{R}\right). \tag{2.20}$$

Notice that the evolution of v^a is non-trivial. If we take the limit $R \to \infty$ we recover the flat bosonic equations of motion $\dot{x}_a = \dot{v}_a = 0$ and therefore a trivial dynamics for both x^a, v^a [43].

The energy and spatial momenta of the free AC particle are given by

$$E = -\frac{\partial \mathcal{L}}{\partial \dot{t}} = -M\cosh\frac{x}{R},$$
$$p_a = \frac{\partial \mathcal{L}}{\partial \dot{x}_a} = M\left[\frac{R}{2x}v_a \sinh\frac{x}{R} + \frac{1}{2x^2}x_a x^b v_b\left(1 - \frac{R}{x}\sinh\frac{x}{R}\right)\right]. \tag{2.21}$$

They satisfy the constraint

$$E^2 - M^2\cosh^2\frac{x}{R} = 0. \tag{2.22}$$

The canonical action of the AC particle is given by[10]

$$S = \int \mathrm{d}\tau\left[-E\dot{t} + p_a\dot{x}^a - \frac{e}{2}\left(E^2 - M^2\cosh^2\frac{x}{R}\right)\right]. \tag{2.23}$$

Note that if we calculate $\dot{p}_a$ and impose both equations of motion (2.20) we obtain

$$\dot{p}_a = \frac{M}{Rx}\dot{t}x_a \sinh\frac{x}{R} = \frac{eM^2}{Rx}x_a\cosh\frac{x}{R}\sinh\frac{x}{R}. \tag{2.24}$$

In the last step we have used that $\dot{t} = -eE = eM\cosh\frac{x}{R}$, see equation (2.26). This is the same result one finds using the Hamiltonian form given in equation (A.5).

2.3. The Killing equations of the AC particle

In order to find the Killing symmetries of the AC space, it is convenient to consider the symmetries of the canonical action (2.23). The basic Poisson brackets of the canonical variables occurring in the action (2.23) are given by

$$\{E, t\} = 1, \qquad \{e, \pi_e\} = 1, \qquad \{x_i, p_j\} = \delta_{ij}. \tag{2.25}$$

This leads to the following equations of motion:

$$\dot{t} = -eE, \quad \dot{x}^i = 0, \quad \dot{E} = 0, \quad \dot{p}^i = \frac{eM^2}{2Rx}x^i\sinh\frac{2x}{R},$$
$$\dot{\pi}_e = -\frac{1}{2}\left(E^2 - M^2\cosh^2\frac{x}{R}\right), \qquad \dot{e} = \lambda. \tag{2.26}$$

Here $\lambda = \lambda(\tau)$ is an arbitrary function and π_e is constrained by $\dot{\pi}_e = 0$.

We take as the generator of canonical transformations

$$G = -E\xi^0(t, \vec{x}, e) + p_i\,\xi^i(t, \vec{x}, e) + \gamma(t, \vec{x}, e)\pi_e, \tag{2.27}$$

[10] Alternatively, we can obtain this action by taking the Carroll limit of the canonical action of a massive particle in AdS, see appendix A.

where $\xi^0 = \xi^0(t, \vec{x}, e)$, $\xi^i = \xi^i(t, \vec{x}, e)$ and $\gamma = \gamma(t, \vec{x}, e)$. The condition that this generator generates a Noether symmetry is that it is a constant of motion and it leads to the following restrictions:

$$\begin{aligned}\dot{G} = 0 &= -E(\dot{t}\partial_t\xi^0 + \dot{e}\partial_e\xi^0) + \dot{p}_i\xi^i + p_i(\dot{t}\partial_t\xi^i + \dot{e}\partial_e\xi^i) + \gamma\dot{\pi}_e \\ &= eE^2\partial_t\xi^0 - \lambda E\partial_e\xi^0 + \frac{eM^2}{2Rx}x_i\xi^i \sinh\frac{2x}{R} \\ &\quad - eEp_i\partial_t\xi^i + \lambda p_i\partial_e\xi^i - \frac{\gamma}{2}\left(E^2 - M^2\cosh^2\frac{x}{R}\right).\end{aligned} \tag{2.28}$$

From this equation we deduce the following equations describing the symmetries of the AC space:

$$\begin{aligned}&\partial_e\xi^0 = 0, \qquad \partial_e\xi^i = 0, \qquad \partial_t\xi^i = 0,\\ &\gamma = 2e\partial_t\xi^0, \qquad \frac{e}{xR}x_i\xi^i\sinh\frac{x}{R} + \frac{1}{2}\gamma\cosh\frac{x}{R} = 0.\end{aligned} \tag{2.29}$$

The last two equations can be combined into the single condition

$$\partial_t\xi^0 = -\frac{1}{xR}x_i\xi^i\tanh\frac{x}{R}. \tag{2.30}$$

The generator G is given by

$$G = -E\xi^0(t, \vec{x}) + p_i\,\xi^i(\vec{x}) + \gamma(t, \vec{x}, e)\pi_e. \tag{2.31}$$

From the variation of the momenta we can obtain the transformation rules for v_i as follows. First, we use that

$$\begin{aligned}\delta p_i &= \{p_i, G\} = \{p_i, -E\xi^0(t, \vec{x}) + p_i\,\xi^i(\vec{x}) + 2e\partial_t\xi^0(t, \vec{x})\pi_e\} \\ &= E\partial_i\xi^0 - p_k\,\partial_i\xi^k - 2e\partial_t\partial_i\xi^0\pi_e.\end{aligned} \tag{2.32}$$

Next, using equation (2.21) and $\pi_e = 0$ we obtain

$$\delta p_i = -M\cosh\frac{x}{R}\partial_i\xi^0 - M\left[\frac{R}{2x}v_i\sinh\frac{x}{R} + \frac{1}{2x^2}x_ix^bv_b\left(1 - \frac{R}{x}\sinh\frac{x}{R}\right)\right]\partial_i\xi^k. \tag{2.33}$$

Finally, using the expression for p_i given in equation (2.21), we obtain the following transformations of the variables v_i:

$$\begin{aligned}\delta v_i = &-\frac{2x}{R}\partial_i\xi^0\coth\frac{x}{R} - v^a\partial_i\xi_a - \frac{1}{Rx}v_bx^bx_k\,\partial_i\xi^k\left(1 - \frac{R}{x}\sinh\frac{x}{R}\right) \\ &+ \frac{2}{Rx}\coth\frac{x}{R}\left(1 - \frac{R}{x}\sinh\frac{x}{R}\right)x_ix_a\partial^a\xi^0 + \frac{1}{Rx}\left(\frac{R}{x} - \coth\frac{x}{R}\right)v_ix_b\xi^b \\ &- \frac{1}{Rx}\left(\frac{R}{x} - \frac{R^2}{x^2}\sinh\frac{x}{R}\right)\left(x_ix_b\xi^b - \frac{1}{x^2}x_ix_a\partial^a\xi_kv^k\right) \\ &+ \frac{1}{Rx^3}\mathrm{csch}\frac{x}{R}\left(-\frac{R}{x}\sinh\frac{x}{R} - \frac{R^2}{x^2}\sinh^2\frac{x}{R} + 1 + \cosh\frac{x}{R}\right)x_ix_b\xi^bx_kv^k\end{aligned}$$

$$
+\frac{1}{Rx}\mathrm{csch}\frac{x}{R}\left(-2\frac{R}{x}\sinh\frac{x}{R}-\frac{R^2}{x^2}\sinh^2\frac{x}{R}+1+\cosh\frac{x}{R}\right)x_i x_b \xi^b x_k v^k
+\frac{1}{Rx}\mathrm{csch}\frac{x}{R}\left(\frac{R}{x}\sinh\frac{x}{R}-1\right)\xi_i x_b v^b. \tag{2.34}
$$

We see that the free Carroll particle in an AdS background has an infinite-dimensional symmetry. A possible solution to these equations is given by equation (2.18) which are the symmetry transformations of the Carroll group. We do not find any Lifshitz dilatations in this case i.e., a transformation with parameters $\xi^i = x^i$, $\xi^0 = zt$.

2.3.1. The massles limit. Using the canonical action

$$
S=\int \mathrm{d}\tau\left[-E\dot{t}+p_a\dot{x}^a-\frac{e}{2}\left(E^2-M^2\cosh^2\frac{x}{R}\right)\right], \tag{2.35}
$$

it is straightforward to take the massless limit $M \to 0$ and obtain the action

$$
S=\int \mathrm{d}\tau\left(-E\dot{t}+p_a\dot{x}^a-\frac{e}{2}E^2\right). \tag{2.36}
$$

We see that in the massless limit the R-dependence of the AC particle has disappeared. This means that the *massive* Carroll particles are affected by the geometry but the *massless* Carroll particles are not. Consequently, in the massless limit there is no difference between particles in an AdS or flat background. Furthermore, the isometries should be given by the most general conformal Carroll group as it was analyzed in [43]. In this case dilatations are included i.e., with parameters $\xi^i = x^i$, $\xi^0 = zt$.

3. The $\mathcal{N}=1$ AC superparticle

In this section we extend our investigations to the $\mathcal{N}=1$ supersymmetric case and consider the AC superparticle.

3.1. The $\mathcal{N}=1$ AC superalgebra

We start by taking the contraction of the D-dimensional $\mathcal{N}=1$ AdS algebra. The basic commutators are given by ($A = 0, 1, \ldots, D-1$)

$$
\begin{aligned}
&[M_{AB}, M_{CD}] = 2\eta_{A[C}M_{D]B} - 2\eta_{B[C}M_{D]A},\\
&[M_{AB}, P_C] = 2\eta_{C[B}P_{A]}, \qquad [P_A, P_B] = 4x^2 M_{AB},\\
&[M_{AB}, Q] = -\frac{1}{2}\gamma_{AB}Q, \qquad [P_A, Q] = \frac{1}{2R}\gamma_A Q,\\
&\{Q_\alpha, Q_\beta\} = 2[\gamma^A C^{-1}]_{\alpha\beta}P_A + \frac{1}{R}[\gamma^{AB}C^{-1}]_{\alpha\beta}M_{AB},
\end{aligned} \tag{3.1}
$$

where R is the AdS radius and P_A, M_{AB} and Q_α are the generators of space–time translations, Lorentz rotations, and supersymmetry transformations, respectively. The bosonic generators P_A and M_{AB} are anti-hermitian while de fermionic generator Q_α is hermitian.

To make the Carroll contraction we rescale the generators with a parameter ω as follows:

$$P_0 = \frac{\omega}{2} H, \qquad M_{a0} = \omega K_a, \qquad Q = \sqrt{\omega}\tilde{Q}; \qquad a = 1, 2, \ldots, D-1. \tag{3.2}$$

Taking the limit $\omega \to \infty$ and dropping the tildes on the Q we get the following $\mathcal{N} = 1$ AC superalgebra:

$$\begin{aligned}
&[M_{ab}, P_c] = 2\delta_{c[b}P_{a]}, \qquad [M_{ab}, K_c] = 2\delta_{c[b}K_{a]},\\
&[P_a, P_b] = \frac{1}{R^2}M_{ab}, \qquad [P_a, K_b] = \frac{1}{2}\delta_{ab}H, \qquad [P_a, H] = \frac{2}{R^2}K_a\\
&[P_a, Q] = \frac{1}{2R}\gamma_a Q, \qquad [M_{ab}, Q] = -\frac{1}{2}\gamma_{ab}Q,\\
&\{Q_\alpha, Q_\beta\} = [\gamma^0 C^{-1}]_{\alpha\beta}H + \frac{2}{R}[\gamma^{a0}C^{-1}]_{\alpha\beta}K_a.
\end{aligned} \tag{3.3}$$

The Maurer–Cartan equation $\mathrm{d}L^C - \frac{1}{2}f^C{}_{AB}L^B L^A = 0$ in components reads

$$\begin{aligned}
&\mathrm{d}L_H = -\frac{1}{2}L_P^a\, L_K^a - \frac{1}{2}\bar{L}_Q\gamma^0 L_Q, \qquad \mathrm{d}L_P^a = 2L_P^b\, L_M^{ab},\\
&\mathrm{d}L_K^a = 2L_K^b\, L_M^{ab} + \frac{2}{R^2}L_H L_P{}^a - \frac{1}{R}\bar{L}_Q\gamma^{a0}L_Q, \qquad \mathrm{d}L_M^{ab} = 2L_M^{ca}\, L_M^{cb} + \frac{1}{2R^2}L_P{}^b L_P{}^a,\\
&\mathrm{d}L_Q = \frac{1}{2}\gamma_{ab}L_Q\, L_M^{ab} - \frac{1}{2R}\gamma_a L_Q L_P{}^a.
\end{aligned} \tag{3.4}$$

3.2. Superparticle action

We now use the algebra (3.3) to construct the action of the AC superparticle with the coset

$$\frac{G}{H} = \frac{\mathcal{N} = 1 \text{ AdS Carroll}}{\mathrm{SO}(D-1)} \tag{3.5}$$

that is locally parametrized as $g = g_0\, U$, where $g_0 = e^{Ht}e^{P_a x^a}e^{Q_\alpha \theta^\alpha}$ is the coset representing the 'empty' curved AC Carroll superspace and $U = e^{K_a v^a}$ is a general Carroll boost representing the particle inserted in the empty space. The Maurer–Cartan form Ω_0 associated to the empty AC superspace is given by

$$\Omega_0 = g_0^{-1}\mathrm{d}g_0 = HE^0 + P_a E^a + K_a \omega^{a0} + M_{ab}\omega^{ab} - \bar{Q}E, \tag{3.6}$$

where (E^0, E^a, E_α) and $(\omega^{a0}, \omega^{ab})$ are the time and space components of the supervielbein and the spin connection of super-AdS if we parametrize the AdS superspace as $e^{Ht}e^{P_a x^a}e^{Q_\alpha \theta^\alpha}$. The explicit expressions for these components are given by

$$
\begin{aligned}
E^0 &= \mathrm{d}t\cosh\frac{x}{R} - \frac{1}{2}\bar{\theta}\gamma^0\mathrm{d}\theta - \frac{1}{2}\omega^{ab}\bar{\theta}\gamma_{ab}\gamma^0\theta,\\
E^a &= \frac{R}{x}\mathrm{d}x^a\sinh\frac{x}{R} + \frac{1}{x^2}x^a x^b\mathrm{d}x_b\left(1 - \frac{R}{x}\sinh\frac{x}{R}\right),\\
\omega^{a0} &= -\frac{2}{xR}\mathrm{d}tx^a\sinh\frac{x}{R} - \frac{1}{R}\bar{\theta}\gamma^{a0}\mathrm{d}\theta - \frac{1}{2R^2}\bar{\theta}\gamma_{ab}\gamma^0\theta E^b,\\
\omega^{ab} &= \frac{1}{2x^2}(x^b\mathrm{d}x^a - x^a\mathrm{d}x^b)\left(\cosh\frac{x}{R} - 1\right),\\
E_\alpha &= \mathrm{d}\theta_\alpha - \frac{1}{2R}[\gamma_a\theta]_\alpha E^a + \frac{1}{2}\omega^{ab}[\gamma_{ab}\theta]_\alpha.
\end{aligned}
\tag{3.7}
$$

In this case we have torsion given by $T_0 = -\frac{1}{2}\bar{E}^\alpha\gamma^0 E_\alpha$ and a non-vanishing spin connection. The Maurer–Cartan form for the $\mathcal{N} = 1$ AC superparticle inserted in the AC superspace is given by

$$
\Omega = g^{-1}dg = U^{-1}\Omega_0 U + U^{-1}\mathrm{d}U,
\tag{3.8}
$$

where

$$
\begin{aligned}
L_H &= E^0 + \frac{1}{2}v_a E^a,\\
L_P{}^a &= E^a,\\
L_K{}^a &= \omega^{a0} + \mathrm{d}v^a + 2v_b\,\omega^{ab},\\
L_M{}^{ab} &= \omega^{ab},\\
L_{Q_\alpha} &= E_\alpha.
\end{aligned}
\tag{3.9}
$$

Note that the Maurer–Cartan forms of the spacetime supertranslations can be written in matrix form in terms of the Supervielbein components of the AC superspace as follows:

$$
\left(L_H,\;\; L_P{}^a,\;\; L_{Q_\alpha}\right) = \left(E^0,\;\; E^a,\;\; E_\alpha\right)\begin{pmatrix} 1 & 0 & 0\\ \frac{1}{2}v_a & 1 & 0\\ 0 & 0 & 1\end{pmatrix}.
\tag{3.10}
$$

Like in the bosonic case the Maurer–Cartan forms of the supertranslations of the AC superparticle can be obtained from the Maurer–Cartan forms of the AC superspace by a matrix representation of the Carroll boost.

The action of the $\mathcal{N} = 1$ AC superparticle is given by the pull-back of all the L's that are invariant under rotations:

$$
\begin{aligned}
S &= M\int (L_H)^* = M\int\left(E^0 + \frac{1}{2}v_a E^a\right)^*\\
&= M\int\mathrm{d}\tau\left(\dot{t}\cosh\frac{x}{R} + \frac{R}{2x}v_a\dot{x}^a\sinh\frac{x}{R} + \frac{1}{2x^2}x^b v_b x_a\dot{x}^a\left(1 - \frac{R}{x}\sinh\frac{x}{R}\right) - \frac{1}{2}\bar{\theta}\gamma^0\dot{\theta}\right.\\
&\quad\left. - \frac{1}{4x^2}x^b\dot{x}^a\bar{\theta}\gamma_{ab}\gamma^0\theta\left(\cosh\frac{x}{R} - 1\right)\right).
\end{aligned}
\tag{3.11}
$$

The equations of motion corresponding to this action can be written as follows

$$\dot{x}^i = 0, \qquad \dot{\theta} = 0,$$
$$\frac{1}{xR}\dot{t}x_a \sinh\frac{x}{R} = \frac{R}{2x}\dot{v}_a \sinh\frac{x}{R} + \frac{1}{2x^2}x_a x^b \dot{v}_b\left(1 - \frac{R}{x}\sinh\frac{x}{R}\right). \tag{3.12}$$

We can write a Hamiltonian version of this action with the momenta given by

$$\begin{aligned}
p_t &= M\cosh\frac{x}{R},\\
p_a &= M\left[\frac{R}{2x}v_a\sinh\frac{x}{R} + \frac{1}{2x^2}x_a x^b v_b\left(1 - \frac{R}{x}\sinh\frac{x}{R}\right) - \frac{1}{4x^2}x^b\bar{\theta}\gamma_{ab}\gamma^0\theta\left(\cosh\frac{x}{R} - 1\right)\right],\\
\bar{P}_\theta &= \frac{M}{2}\bar{\theta}\gamma^0.
\end{aligned} \tag{3.13}$$

Then, the canonical form of (3.11) is

$$S = \int \mathrm{d}\tau\left[-\dot{t}E + \dot{x}_a p^a + \dot{\bar{\theta}}P_\theta - \frac{e}{2}\left(E^2 - M^2\cosh^2\frac{x}{R}\right) - \left(\bar{P}_\theta\cosh\frac{x}{R} + \frac{1}{2}E\bar{\theta}\gamma^0\right)\rho\right]. \tag{3.14}$$

The bosonic transformation rules for the coordinates with constant parameters (ζ, a^i, λ^i, λ^i_j) corresponding to time translations, spatial translations, boosts and rotations, respectively, are given by

$$\begin{aligned}
\delta t &= -\zeta + \frac{R}{2x}\lambda^k x_k\tanh\frac{x}{R} + \frac{t}{Rx}a^k x_k\tanh\frac{x}{R},\\
\delta x^i &= -\frac{1}{x^2}\left(x^i a^k x_k - \frac{x}{R}\coth\frac{x}{R}(x^i a^k x_k - a^i x^2)\right) - 2\lambda^i{}_k\, x^k,\\
\delta v^i &= -\lambda^i - \frac{1}{x^2}\lambda^k x_k x^i\,\mathrm{sech}\frac{x}{R}\left(1 - \cosh\frac{x}{R}\right) - 2\lambda^i{}_j\, v^j - \frac{2t}{R^2}a^i\\
&\quad - \frac{2t}{R^2x^2}x^i a^k x_k\,\mathrm{sech}\frac{x}{R}\left(1 - \cosh\frac{x}{R}\right) + \frac{2}{Rx}v_b a^{[i}x^{b]}\mathrm{csch}\frac{x}{R}\left(1 - \cosh\frac{x}{R}\right),\\
\delta\theta &= -\frac{1}{2}\lambda^{ab}\gamma_{ab}\theta + \frac{1}{2Rx}a^k x^b\gamma_{kb}\theta\,\mathrm{csch}\frac{x}{R}\left(1 - \cosh\frac{x}{R}\right).
\end{aligned} \tag{3.15}$$

The fermionic transformation rules with constant parameter ϵ corresponding to the supersymmetry transformation are given by

$$\begin{aligned}
\delta t &= \frac{1}{2}\bar{\epsilon}\gamma^0\theta\,\mathrm{sech}\frac{x}{R}\cosh\frac{x}{2R} - \frac{1}{2x}x^k\bar{\epsilon}\gamma^{k0}\theta\,\mathrm{sech}\frac{x}{R}\sinh\frac{x}{2R},\\
\delta x^i &= 0,\\
\delta v^i &= \frac{1}{Rx}x^i\tanh\frac{x}{R}\left(\bar{\epsilon}\gamma^0\theta\cosh\frac{x}{2R} - \frac{1}{x}x^k\bar{\epsilon}\gamma^{k0}\theta\sinh\frac{x}{2R}\right)\\
&\quad - \frac{1}{Rx}x^i\bar{\epsilon}\gamma^0\theta\sinh\frac{x}{2R} + \frac{1}{R}\bar{\epsilon}\gamma^{i0}\theta\cosh\frac{x}{2R} + \frac{1}{Rx}x^k\bar{\epsilon}\gamma^{ik0}\theta\sinh\frac{x}{2R},\\
\delta\theta &= \epsilon\cosh\frac{x}{2R} + \frac{1}{x}x^k\gamma_k\epsilon\sinh\frac{x}{2R}.
\end{aligned} \tag{3.16}$$

3.3. The super Killing equations

The basic Poisson brackets of the canonical variables are given by

$$\{E, t\} = 1, \qquad \{e, \pi_e\} = 1, \qquad \{x_i, p_j\} = \delta_{ij},$$
$$\{P_\theta^\alpha, \theta_\beta\} = -\delta_\beta^\alpha, \qquad \{\Pi_\rho^\alpha, \rho_\beta\} = -\delta_\beta^\alpha, \tag{3.17}$$

and the corresponding Dirac Hamiltonian of the action (3.14) is given by

$$H_D = \frac{e}{2}\left(E^2 - M^2\cosh^2\frac{x}{R}\right) + \lambda\pi_e + \left(\bar{P}_\theta\cosh\frac{x}{R} + \frac{1}{2}E\bar{\theta}\gamma^0\right)\rho + \bar{\pi}_\rho\Lambda, \tag{3.18}$$

$\pi_e = 0$ and $\Pi_\rho = 0$ are the primary constraints, $\lambda = \lambda(\tau)$ and $\Lambda = \Lambda(\tau)$ are arbitrary functions. The corresponding primary Hamiltonian equations of motion are given by

$$\dot{t} = -\,eE - \frac{1}{2}\bar{\theta}\gamma^0\rho, \qquad \dot{x}^i = 0, \qquad \dot{E} = 0,$$
$$\dot{p}^i = \frac{eM^2}{xR}x^i\cosh\frac{x}{R}\sinh\frac{x}{R} - \frac{1}{xR}x^i\sinh\frac{x}{R}\bar{P}_\theta\rho,$$
$$\dot{\pi}_e = -\frac{1}{2}\left(E^2 - M^2\cosh^2\frac{x}{R}\right), \qquad \dot{e} = \lambda,$$
$$\dot{\theta} = -\,\rho\cosh\frac{x}{R}, \qquad \dot{\bar{P}}_\theta = -\frac{1}{2}E\bar{\rho}\gamma^0, \qquad \dot{\rho} = -\Lambda, \qquad \dot{\bar{\Pi}}_\rho = \bar{P}_\theta\cosh\frac{x}{R} + \frac{1}{2}E\bar{\theta}\gamma^0. \tag{3.19}$$

The stability of primary constraints give as secondary constraint the mass-shell condition $E^2 - M^2\cosh^2\frac{x}{R} = 0$ and the fermionic constraint $\bar{P}_\theta\cosh\frac{x}{R} + \frac{1}{2}E\bar{\theta}\gamma^0 = 0$. If we require the stability of the secondary constraints we get $\rho = 0$. Substituting this into (3.19) and using the canonical momenta (3.13) we obtain equation (3.12).

The generator of canonical transformations has a bosonic and a fermionic part given by

$$G = -E\xi^0(t, \vec{x}, \theta) + p_i\,\xi^i(t, \vec{x}, \theta) + \gamma(t, \vec{x}, \theta)\pi_e - \bar{P}_\theta\chi(t, \vec{x}, \theta) + \bar{\Pi}_\rho\Gamma(t, \vec{x}, \theta), \tag{3.20}$$

the parameters $\xi^0 = \xi^0(t, \vec{x}, \theta)$, $\xi^i = \xi^i(t, \vec{x}, \theta)$, $\chi = \chi(t, \vec{x}, \theta)$, $\gamma = \gamma(t, \vec{x}, \theta)$ have the following restrictions

$$\begin{aligned}
0 &= \dot{G}\\
&= -\,E(\dot{t}\partial_t\xi^0 + \partial_\theta\xi^0\dot{\theta}) + \dot{p}_i\xi^i + p_i(\dot{t}\partial_t\xi^i + \partial_\theta\xi^i\dot{\theta})\\
&\quad + \gamma\dot{\pi}_e - \dot{\bar{P}}_\theta\chi - \bar{P}_\theta(\partial_t\chi\dot{t} + \partial_\theta\chi\dot{\theta}) + \dot{\bar{\Pi}}_\rho\Gamma\\
&= eE^2\partial_t\xi^0 + \frac{1}{2}E\partial_t\xi^0\bar{\theta}\gamma^0\rho + E\partial_\theta\xi^0\rho\cosh\frac{x}{R} + \frac{eM^2}{xR}x^i\xi_i\cosh\frac{x}{R}\sinh\frac{x}{R}\\
&\quad - \frac{1}{xR}x^i\xi_i\sinh\frac{x}{R}\bar{P}_\theta\rho - eEp_i\partial_t\xi^i - \frac{1}{2}p_i\partial_t\xi^i\bar{\theta}\gamma^0\rho - p_i\partial_\theta\xi^i\rho\cosh\frac{x}{R}\\
&\quad - \frac{1}{2}\gamma\left(E^2 - M^2\cosh^2\frac{x}{R}\right) + \frac{E}{2}\bar{\rho}\gamma^0\chi + eE\bar{P}_\theta\partial_t\chi\\
&\quad + \frac{1}{2}\bar{P}_\theta\partial_t\chi\bar{\theta}\gamma^0\rho + \bar{P}_\theta\partial_\theta\chi\rho\cosh\frac{x}{R}\\
&\quad + \bar{P}_\theta\Gamma\cosh\frac{x}{R} + \frac{E}{2}\bar{\theta}\gamma^0\Gamma.
\end{aligned} \tag{3.21}$$

From this equation we derive the super-Killing equations

$$\gamma = 2e\partial_t\xi^0, \qquad \Gamma = -\partial_\theta\chi\rho + \frac{1}{xR}x^i\xi_i \tanh\frac{x}{R}\rho,$$
$$\partial_t\xi^0 = -\frac{1}{xR}x^i\xi_i \tanh\frac{x}{R}, \qquad \partial_\theta\xi^0 = \frac{1}{2}\bar{\chi}\gamma^0 \operatorname{sech}\frac{x}{R} + \frac{1}{2}\bar{\theta}\gamma^0\partial_\theta\chi \operatorname{sech}\frac{x}{R},$$
$$\partial_t\xi^i = 0, \qquad \partial_\theta\xi^i = 0, \qquad \partial_t\chi = 0. \tag{3.22}$$

The solution to this equations is given by equations (3.15) and (3.16) with the symmetry generator G given by

$$G = -E\xi^0(\vec{x},\theta) + p_i\,\xi^i(t,\vec{x}) + 2e\partial_t\xi^0(\vec{x},\theta)\pi_e - \bar{P}_\theta\chi(\vec{x},\theta) + \bar{\Pi}_\rho\left(-\partial_\theta\chi(\vec{x},\theta)\rho + \frac{1}{xR}x^i\xi_i(\vec{x},\theta)\tanh\frac{x}{R}\rho\right). \tag{3.23}$$

Then, the $\mathcal{N}=1$ AC superparticle has an infinite dimensional algebra with the transformation rules given by (3.15) and (3.16).

3.4. The flat limit

We end this section with some comments on the flat limit ($R \to \infty$) which can be taken directly from the AC curved case in order to obtain the dynamics and symmetries of the $\mathcal{N}=1$ flat Carroll superparticle. In this case, the time and space components of the supervielbein simplify to

$$E^0 = \mathrm{d}t - \frac{1}{2}\bar{\theta}\gamma^0\mathrm{d}\theta, \qquad E^a = \mathrm{d}x^a, \qquad E_\alpha = \mathrm{d}\theta_\alpha. \tag{3.24}$$

In the $R \to \infty$ limit, the torsion becomes $T_0 = -\frac{1}{2}\mathrm{d}\bar{\theta}\gamma^0\mathrm{d}\theta$ and since we are studying the flat case, the spin connection vanishes. The supertranslations can be again written in terms of the supervielbein in matrix form as in (3.10) and the action is given by

$$S = M\int\left(E^0 + \frac{1}{2}v_aE^a\right)^* = M\int\mathrm{d}\tau\left(\dot{t} - \frac{1}{2}\bar{\theta}\gamma^0\dot{\theta} + \frac{1}{2}v_a\dot{x}^a\right). \tag{3.25}$$

The equations of motion that follow from this action are:

$$\dot{\vec{x}} = \dot{\vec{v}} = \dot{\theta} = 0. \tag{3.26}$$

Therefore, the superparticle does not move. The transformation rules of the different variables are given by

$$\delta t = -\zeta + \frac{1}{2}\lambda^i x_i + \frac{1}{2}\bar{\epsilon}\gamma^0\theta, \qquad \delta x^i = -a^i - 2\lambda^i{}_j\,x^j,$$
$$\delta v^i = -\lambda^i - 2\lambda^i{}_j\,v^j, \qquad \delta\theta = -\frac{1}{2}\lambda_{ij}\gamma^{ij}\theta + \epsilon. \tag{3.27}$$

As we can see from the transformation of θ the $\mathcal{N}=1$ Carroll superparticle is not BPS like in the relativistic and Galilean case.

If we rewrite the action (3.25) in Hamiltonian form

$$S = \int\mathrm{d}\tau\left[-\dot{t}E + \dot{x}_ap^a + \dot{\bar{\theta}}P_\theta - \frac{e}{2}(E^2 - M^2) - \left(\bar{P}_\theta + \frac{1}{2}E\bar{\theta}\gamma^0\right)\rho\right], \tag{3.28}$$

it turns out that the super-Killing equations can be obtained as the flat limit of the equations (3.22)

$$\gamma = 0, \quad \Gamma = -\partial_\theta \chi \rho, \quad \partial_t \xi^0 = 0, \quad \partial_t \xi^i = 0, \quad \partial_\theta \xi^i = 0, \quad \partial_t \chi = 0,$$
$$\partial_\theta \xi^0 = \frac{1}{2}\bar{\chi}\gamma^0 + \frac{1}{2}\bar{\theta}\gamma^0 \partial_\theta \chi, \tag{3.29}$$

where the symmetry generator G is

$$G = -E\xi^0(\vec{x}, \theta) + p_i\, \xi^i(\vec{x}) - \bar{P}_\theta \chi(\vec{x}, \theta) - \bar{\Pi}_\rho \partial_\theta \chi(\vec{x}, \theta)\rho. \tag{3.30}$$

From the variation of the momenta

$$\delta p_i = \{p_i, G\} = E\partial_i \xi^0 - p_k \partial_i \xi^k + \bar{P}_\theta \partial_i \chi \tag{3.31}$$

and using that the energy, the spatial momenta and the fermionic momenta are given by

$$E = -M, \qquad p_i = \frac{M}{2} v_i, \qquad \bar{P}_\theta = \frac{M}{2}\bar{\theta}\gamma^0, \tag{3.32}$$

we find that the transformation rule of v^i

$$\delta v_i = -2\partial_i \xi^0 - v_k \partial_i \xi^k + \bar{\theta}\gamma^0 \partial_i \chi. \tag{3.33}$$

Note that the above symmetries include the dilatations given by

$$\delta t = 0,\ \delta x^a = x^a,\ \delta\theta = 0,\ \delta v^a = -v^a. \tag{3.34}$$

These dilatations, together with the super-Carroll transformations, form a supersymmetric extension of the Lifshitz Carroll algebra [45] with dynamical exponent $z = 0$. The Lifshitz Carroll algebra with $z = 0$ has appeared in a recent study of WCFTs [35].

4. The $\mathcal{N} = 2$ flat Carroll superparticle

In this section we extend our investigations to the $\mathcal{N} = 2$ supersymmetric case. The flat case is discussed in this section while the curved case will be dealt with in appendix C.

4.1. The $\mathcal{N} = 2$ Carroll superalgebra

Our starting point is the $\mathcal{N} = 2$ super-Poincaré algebra. For simplicity, we consider 3D only. The basic commutators are ($A = 0, 1, 2$; $i = 1, 2$)

$$\begin{aligned}
[M_{AB}, M_{CD}] &= 2\eta_{A[C}M_{D]B} - 2\eta_{B[C}M_{D]A},\\
[M_{AB}, P_C] &= 2\eta_{C[B}P_{A]},\\
[M_{AB}, Q^i] &= -\frac{1}{2}\gamma_{AB}Q^i,\\
\{Q^i_\alpha, Q^j_\beta\} &= 2[\gamma^A C^{-1}]_{\alpha\beta} P_A \delta^{ij} + 2[C^{-1}]_{\alpha\beta}\epsilon^{ij} Z.
\end{aligned} \tag{4.1}$$

To make the Carroll contraction we define new supersymmetry charges by

$$Q^\pm_\alpha = \frac{1}{2}(Q^1_\alpha \pm \gamma_0 Q^2_\alpha) \tag{4.2}$$

and rescale the different symmetry generators with a parameter ω as follows:

$$\begin{aligned}
P_0 = \frac{\omega}{2}H, \qquad M_{a0} = \omega K_a, \qquad Z = \omega\tilde{Z},\\
Q^+ = \sqrt{\omega}\tilde{Q}^+, \qquad Q^- = \sqrt{\omega}\tilde{Q}^-.
\end{aligned} \tag{4.3}$$

Taking the limit $\omega \to \infty$ we obtain the following 3D Carroll algebra

$$
\begin{aligned}
&[M_{ab}, K_c] = 2\delta_{c[b}K_{a]}, && [M_{ab}, P_c] = 2\delta_{c[b}P_{a]},\\
&[K_a, P_b] = -\frac{1}{2}\delta_{ab}H, && [M_{ab}, \tilde{Q}^{\pm}] = -\frac{1}{2}\gamma_{ab}\tilde{Q}^{\pm}\\
&\{\tilde{Q}^{+}_{\alpha}, \tilde{Q}^{+}_{\beta}\} = [\gamma^0 C^{-1}]_{\alpha\beta}\left(\frac{1}{2}H + \tilde{Z}\right), && \{\tilde{Q}^{-}_{\alpha}, \tilde{Q}^{-}_{\beta}\} = [\gamma^0 C^{-1}]_{\alpha\beta}\left(\frac{1}{2}H - \tilde{Z}\right).
\end{aligned}
\tag{4.4}
$$

The Maurer–Cartan equation $\mathrm{d}L^C - \frac{1}{2}f^C_{AB}L^B \wedge L^A = 0$ in components reads:

$$
\begin{aligned}
&\mathrm{d}L_H = -\frac{1}{2}L^a_P\, L^a_K - \frac{1}{4}\bar{L}_-\gamma^0 L_- - \frac{1}{4}\bar{L}_+\gamma^0 L_+, && \mathrm{d}L^a_P = 2L^b_P\, L^{ab}_M,\\
&\mathrm{d}L_Z = -\frac{1}{2}\bar{L}_+\gamma^0 L_+ + \frac{1}{2}\bar{L}_-\gamma^0 L_-, && \mathrm{d}L^a_K = 2L^b_K\, L^{ab}_M,\\
&\mathrm{d}L_- = \frac{1}{2}\gamma_{ab}L_-\, L^{ab}_M, && \mathrm{d}L_+ = \frac{1}{2}\gamma_{ab}L_+\, L^{ab}_M,\\
&\mathrm{d}L^{ab}_M = 2L^{ca}_M\, L^{cb}_M.
\end{aligned}
\tag{4.5}
$$

4.2. Superparticle action and kappa symmetry

To construct the action of the $\mathcal{N} = 2$ Carrollian superparticle we consider the following coset:

$$
\frac{G}{H} = \frac{\mathcal{N} = 2 \text{ super Carroll}}{\mathrm{SO}(D-1)}.
\tag{4.6}
$$

The coset element is given by $g = g_0\, U$, where $g_0 = e^{Ht}e^{P_a x^a}e^{Q^-_\alpha\theta^\alpha_-}e^{Q^+_\alpha\theta^\alpha_+}e^{Zs}$ is the coset representing the 'empty' $\mathcal{N} = 2$ Carroll superspace with a central charge extension and $U = e^{K_a v^a}$ is a general Carroll boost representing the insertion of the particle.

The Maurer–Cartan form associated to the super-Carroll space is given by

$$
\Omega_0 = (g_0)^{-1}\mathrm{d}g_0 = HE^0 + P_a E^a - \bar{Q}^- E_- - \bar{Q}^+ E_+ + ZE_Z,
\tag{4.7}
$$

where $(E^0, E^a, E_{-\alpha}, E_{+\alpha}, E_Z)$ are the supervielbein components of the Carroll superspace given explicitly by

$$
\begin{aligned}
&E^0 = \mathrm{d}t - \frac{1}{4}\bar{\theta}_-\gamma^0\mathrm{d}\theta_- - \frac{1}{4}\bar{\theta}_+\gamma^0\mathrm{d}\theta_+, && E^a = \mathrm{d}x^a,\\
&E_{-\alpha} = \mathrm{d}\theta_{-\alpha}, && E_{+\alpha} = \mathrm{d}\theta_{+\alpha},\\
&E_Z = \mathrm{d}s + \frac{1}{2}\bar{\theta}_-\gamma^0\mathrm{d}\theta_- - \frac{1}{2}\bar{\theta}_+\gamma^0\mathrm{d}\theta_+.
\end{aligned}
\tag{4.8}
$$

In terms of the supervielbein the Maurer–Cartan form of the $\mathcal{N} = 2$ Carroll superparticle is given by

$$
\begin{aligned}
&L_H = E^0 + \frac{1}{2}v_a E^a, && L^a_P = E^a,\\
&L_Z = E_Z, && L^a_K = \mathrm{d}v^a,\\
&L_{-\alpha} = E_{-\alpha}, && L_{+\alpha} = E_{+\alpha}.
\end{aligned}
\tag{4.9}
$$

As before, we can write the space–time super-translations in matrix form in terms of the Vielbein of Carroll superspace as follows:

$$\left(L_H,\ \ L_P{}^a,\ \ L_{-\alpha},\ \ L_{+\alpha},\ \ L_Z\right)=\left(E^0,\ \ E^a,\ \ E_{-\alpha},\ \ E_{+\alpha},\ \ E_Z\right)\begin{pmatrix}1 & 0 & 0 & 0 & 0\\ \frac{1}{2}v_a & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1\end{pmatrix}. \tag{4.10}$$

The action of the $\mathcal{N}=2$ Carrollian superparticle is given by the pull-back of all the L's that are invariant under rotations:

$$\begin{aligned} S &= a\int (L_H)^* + b\int (L_Z)^* \\ &= a\int \mathrm{d}\tau\left(\dot{t}-\frac{1}{4}\bar{\theta}_-\gamma^0\dot{\theta}_- - \frac{1}{4}\bar{\theta}_+\gamma^0\dot{\theta}_+ + \frac{1}{2}v_a\dot{x}^a\right) \\ &\quad + b\int \mathrm{d}\tau\left(\dot{s}+\frac{1}{2}\bar{\theta}_-\gamma^0\dot{\theta}_- - \frac{1}{2}\bar{\theta}_+\gamma^0\dot{\theta}_+\right). \end{aligned} \tag{4.11}$$

The equations of motion corresponding to this action are given by

$$\dot{x}_a=0,\quad \dot{v}_a=0,\quad \dot{\theta}_-=0,\quad \dot{\theta}_+=0. \tag{4.12}$$

The transformation rules for the coordinates with constant parameters (ζ, η, a^i, λ^i, λ^i_j, ϵ_+, ϵ_-) corresponding to time translations, Z transformations, spatial translations, boosts, rotations and supersymmetry transformations, respectively, are given by

$$\begin{aligned} \delta t &= -\zeta+\frac{1}{2}\lambda^i x_i+\frac{1}{4}\bar{\epsilon}_-\gamma^0\theta_- + \frac{1}{4}\bar{\epsilon}_+\gamma^0\theta_+, & \delta x^i &= -a^i-2\lambda^i{}_j\,x^j, \\ \delta s &= -\eta-\frac{1}{2}\bar{\epsilon}_-\gamma^0\theta_- + \frac{1}{2}\bar{\epsilon}_+\gamma^0\theta_+, & \delta v^i &= -\lambda^i-2\lambda^i{}_j\,v^j, \\ \delta\theta_+ &= -\frac{1}{2}\lambda^{ab}\gamma_{ab}\theta_+ + \epsilon_+, & \delta\theta_- &= -\frac{1}{2}\lambda^{ab}\gamma_{ab}\theta_- + \epsilon_-. \end{aligned} \tag{4.13}$$

To derive an action that is invariant under additional κ-transformations we need to find a fermionic gauge-transformation that leaves L_H and/or L_Z invariant. The variation of L_H and L_Z under gauge-transformations is given by

$$\begin{aligned} \delta L_H &= \mathrm{d}([\delta z_H]) + \frac{1}{2}L_P^a[\delta z_K^a] + \frac{1}{2}L_K^a[\delta z_P^a] + \frac{1}{2}\bar{L}_-\gamma^0[\delta z_-] + \frac{1}{2}\bar{L}_+\gamma^0[\delta z_+], \\ \delta L_Z &= \mathrm{d}([\delta z_Z]) - \bar{L}_-\gamma^0[\delta z_-] + \bar{L}_+\gamma^0[\delta z_+]. \end{aligned} \tag{4.14}$$

Where $[\delta z_K^a]$ is obtained from L_H by changing the one-forms $\mathrm{d}t$, $\mathrm{d}\theta_+$, $\mathrm{d}\theta_-$ with the transformations δt, $\delta\theta_+$, $\delta\theta_-$. In analogous way we can construct the other terms appearing in (4.14).

For κ-transformations, $[\delta z_H]=0$, $[\delta z_K^a]=0$, $[\delta z_P^a]=0$,

$$\begin{aligned} 0 &= \delta L_H = \frac{1}{2}\delta\bar{\theta}_-\gamma^0[\delta z_-] + \frac{1}{2}\delta\bar{\theta}_+\gamma^0[\delta z_+], \\ 0 &= \delta L_Z = -\,\delta\bar{\theta}_-\gamma^0[\delta z_-] + \delta\bar{\theta}_+\gamma^0[\delta z_+]. \end{aligned} \tag{4.15}$$

It follows that to obtain a κ-symmetric action we need to take $b=\pm\frac{1}{2}a$. We focus here on the case $b=-\frac{1}{2}a$. With this choice the action and κ-symmetry rules are given by

$$S = a\int\left(L_H - \frac{1}{2}L_Z\right)^*, \qquad [\delta z_+] = \kappa, \qquad [\delta z_-] = 0, \tag{4.16}$$

where $\kappa = \kappa(\tau)$ is an arbitrary local parameter. Using this we find the following κ-transformations of the coordinates

$$\begin{aligned} \delta t &= \frac{1}{4}\bar\theta_+\gamma^0\kappa, & \delta x^a &= 0, & \delta\theta_+ &= \kappa, \\ \delta s &= \frac{1}{2}\bar\theta_+\gamma^0\kappa, & \delta v_a &= 0, & \delta\theta_- &= 0. \end{aligned} \tag{4.17}$$

After fixing the κ-symmetry, by imposing the gauge condition $\theta_+ = 0$, the action reduces to

$$S = a\int \mathrm{d}\tau\left(\dot t - \frac{1}{2}\dot s - \frac{1}{2}\bar\theta_-\gamma^0\dot\theta_- + \frac{1}{2}v_a\dot x^a\right). \tag{4.18}$$

The residual transformations that leave this action invariant are given by

$$\begin{aligned} \delta t &= -\zeta + \frac{1}{2}\lambda^i x_i + \frac{1}{4}\bar\epsilon_-\gamma^0\theta_-, & \delta x^i &= -a^i - 2\lambda^i{}_j\, x^j, \\ \delta s &= -\eta - \frac{1}{2}\bar\epsilon_-\gamma^0\theta_-, & \delta v^i &= -\lambda^i - 2\lambda^i{}_j\, v^j, \\ \delta\theta_- &= -\frac{1}{2}\lambda^{ab}\gamma_{ab}\theta_- + \epsilon_-. \end{aligned} \tag{4.19}$$

The linearly realized supersymmetry acts trivially on all the fields and therefore the $\mathcal{N} = 2$ super Carroll particle reduces to the $\mathcal{N} = 1$ super Carroll particle and hence is not BPS since the kappa-symmetry eliminates the linearized supersymmetry. This is different from the $\mathcal{N} = 2$ super Galilei case were BPS particles do exist.

5. Discussion and outlook

In this paper we have investigated, using the method of nonlinear realizations, the geometry of the flat and curved (AdS) Carroll space both in the bosonic as well as in the supersymmetric case. We furthermore have analyzed the symmetries of a particle moving in such a space. In the bosonic case we constructed the Vielbein and spin connection of the AC space which shows that this space has constant (negative) spatial curvature. We constructed the action of a massive particle moving in this space thereby extending the flat case analysis of [43]. Like in the flat case, we found that the AC particle does not move. However, in the curved case the momenta are not conserved. Particles moving in a Carroll space, whether flat or curved, do not have a relation among their velocities and momenta.

Using the symmetries of the AC particle we have computed the Killing equations of the AC space. We found that these Killing equations allow an infinite-dimensional algebra of symmetries that, unlike in the flat case, does not include dilatations. Another difference with the flat case is that there is no duality between the NH and AC algebras. Furthermore, in the curved case the mass-shell constraint depends on the coordinates of the AC space.

In the second part of this paper we have extended our investigations to the supersymmetric case. Unlike the bosonic case, the $\mathcal{N} = 1$ AC superspace has torsion with constant curvature due to the presence of fermions. Like in the bosonic case, we found that the $\mathcal{N} = 1$ AC superparticle does not move and the momenta are conserved. We have constructed the super-Killing equations and showed that the symmetries form an infinite dimensional superalgebra. After taking the flat limit we found that among the symmetries of the $\mathcal{N} = 1$

Carroll superparticle we have a supersymmetric extension of the Lifshitz Carroll algebra [45] with dynamical exponent $z = 0$. The bosonic part of this algebra has appeared as a symmetry of WCFTs [35].

We also showed that the $\mathcal{N} = 2$ Carroll superparticle has a fermionic kappa-symmetry such that, when this gauge symmetry is fixed, the $\mathcal{N} = 2$ Carroll superparticle reduces to the $\mathcal{N} = 1$ Carroll superparticle. Apparently, in flat Carroll superspace the number of supersymmetries is not physically relevant. This is due to the fact that the kappa gauge symmetry neutralizes the extra linear supersymmetries beyond $\mathcal{N} = 1$. Unlike the bosonic case, there is no duality between the $\mathcal{N} = 2$ super Galilei and super Carroll algebras.

In a separate appendix we investigated the $\mathcal{N} = 2$ AC superparticle[11]. We studied the so-called (2, 0) and (1, 1) super-Carroll spaces and the corresponding superparticles. Physically, the (2, 0) and (1, 1) cases are different, they have unequal degrees of freedom. For instance, only the (2, 0) superparticle has a kappa-symmetry. Apparently, for the AC superparticle the type of supersymmetry one considers does make a difference.

We note that, both in the bosonic and in the supersymmetric case, the equations of motion that follow from our (super-)particle actions have an interesting geometrical interpretation, i.e. they can be viewed as the geodesic equation for a suitable Carrollian connection[12].

As a possible continuation to the ideas presented in this paper it would be interesting to find the coupling of the AC particle, and the corresponding superparticle, to the (super) AdS gauge fields. Like in the flat Carroll case [43] we expect that the (super) particle will have a non-trivial dynamics.

Finally, it would be interesting to study if one could construct the corresponding Carroll (super) gravity theory. There are two approaches to this issue. One approach is to gauge the (super) Carroll algebra and/or the Lifhsitz Carroll algebra with $z = 0$. In this respect we note that the gauging of the Carroll algebra as performed in [43] can be improved by imposing curvature constraints that allow to set some of the spin-connection fields equal to zero, like in [35], instead of trying to solve for all of the spin-connection fields. It would be interesting to apply this improved gauging technique to the other algebras as well. A second alternative approach would be to try to define an ultra-relativistic limit of relativistic (super-)gravity similar to the non-relativistic limit.

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Appendix A. The Carroll action as a limit of the AdS action

In this appendix we show how to obtain the action of the D-dimensional free AC particle starting from the massive particle moving in an D-dimensional AdS

[11] For simplicity we did only consider the 3D case.
[12] We thank the referee for pointing this out.

spacetime and to take the Carroll limit. The canonical form of the action before taking the limit is given by

$$S = \int \mathrm{d}\tau \,[p^\mu \dot{x}_\mu - \frac{\tilde{e}}{2}(g_{\mu\nu}p^\mu p^\nu + m^2)], \tag{A.1}$$

where τ is the evolution parameter, $g_{\mu\nu}$ is the metric of an AdS space and $\tilde{e}$ is a Lagrange multiplier. We use that the signature of the metric is $(-, +, +, \cdots)$ and that the AdS line element is given by

$$\mathrm{d}s^2 = -\cosh^2\frac{x}{R}(\mathrm{d}x^0)^2 + \frac{R^2}{x^2}\sinh^2\frac{x}{R}(\mathrm{d}x^a)^2 - \left(\frac{R^2}{x^2}\sinh^2\frac{x}{R} - 1\right)(\mathrm{d}x)^2, \tag{A.2}$$

where $x = \sqrt{x_a x^a}$. To take the Carroll limit we first consider a re-scaling of the variables

$$x^0 = \frac{t}{\omega}, \quad p^0 = \omega E, \quad m = \omega M, \quad \tilde{e} = -\frac{e}{\omega^2}, \tag{A.3}$$

and next take the limit $\omega \to \infty$ to obtain

$$S = \int \mathrm{d}\tau \left[-E\dot{t} + p^a \dot{x}_a - \frac{e}{2}\left(E^2 - M^2\cosh^2\frac{x}{R}\right)\right]. \tag{A.4}$$

The equations of motion are given by

$$\begin{aligned} &\dot{t} = -\, eE, \quad \dot{E} = 0, \\ &\dot{x}^a = 0, \qquad \dot{p}^a = \frac{eM^2}{Rx}x^a\cosh\frac{x}{R}\sinh\frac{x}{R}, \\ &\dot{e} = \lambda, \qquad \pi_e = -\frac{1}{2}\left(E^2 - M^2\cosh^2\frac{x}{R}\right). \end{aligned} \tag{A.5}$$

Note that although the dynamics of x is trivial, i.e. $\dot{x}^a = 0$ (the particle is not changing its position), the momentum is changing over τ because $\dot{p}^a \neq 0$. In the flat limit (the limit when $R \to \infty$) the particle is at rest and does not move.

Finally, the mass-shell constraint reads

$$E^2 - M^2\cosh^2\frac{x}{R} = 0. \tag{A.6}$$

Appendix B. The super-AC action as a limit of the super-AdS action

In the supersymmetric case, we obtain the action of the free AC superparticle starting from the massive superparticle moving in an AdS spacetime whose action is given by

$$S = \int \mathrm{d}\tau \,[\dot{x}_\mu p^\mu + \dot{\bar{\phi}}P_\phi - \frac{\tilde{e}}{2}(g_{\mu\nu}p^\mu p^\nu + m^2) + (\bar{P}_\phi + g_{\mu\nu}p^\mu\bar{\phi}\gamma^\nu)\lambda], \tag{B.1}$$

where $g_{\mu\nu}$ is the AdS metric with line element given by equation (A.2). Rescaling the variables as

$$\begin{aligned} &x^0 = \frac{t}{\omega}, \quad p^0 = \omega E, \quad m = \omega M, \quad \tilde{e} = -\frac{e}{\omega^2}, \\ &\phi = \frac{1}{\sqrt{\omega}}\theta, \quad P_\phi = \sqrt{\omega}\,P_\theta, \quad \lambda = \frac{1}{\sqrt{\omega}}\rho, \end{aligned} \tag{B.2}$$

allows us to take the Carroll limit with $\omega \to \infty$ to obtain

$$S = \int \mathrm{d}\tau \left[-\dot{t}E + \dot{x}_a p^a + \dot{\bar{\theta}} P_\theta - \frac{e}{2}\left(E^2 - M^2 \cosh^2 \frac{x}{R}\right) + \left(\bar{P}_\theta \cosh \frac{x}{R} + E\, \bar{\theta}\gamma^0\right)\rho \right]. \tag{B.3}$$

The primary equations of motion are

$$\begin{aligned}
&\dot{t} = -\, eE - \bar{\theta}\gamma^0\rho, && \dot{E} = 0,\\
&\dot{x}^a = 0, && \dot{p}^a = \frac{eM^2}{Rx} x^a \cosh \frac{x}{R} \sinh \frac{x}{R} - \frac{1}{xR} x^a \sinh \frac{x}{R} \bar{P}_\theta \rho,\\
&\dot{e} = \lambda, && \pi_e = -\frac{1}{2}\left(E^2 - M^2 \cosh^2 \frac{x}{R}\right),\\
&\dot{\theta} = -\, \cosh \frac{x}{R}\rho, && \dot{\bar{P}}_\theta = -E\, \bar{\rho}\gamma^0,\\
&\dot{\rho} = -\, \Lambda, && \bar{\Pi}_\rho = \bar{P}_\theta \cosh \frac{x}{R} + E\, \bar{\theta}\gamma^0.
\end{aligned} \tag{B.4}$$

After requiring the stability of all the constraints we obtain the equations of motion (3.12). Like in the bosonic case we find that the dynamics of x is trivial, $\dot{x}^a = 0$ (the particle is not changing its position), but that the momentum is changing over τ because $\dot{p}^a \neq 0$.

Appendix C. The 3D $\mathcal{N} = 2$ AC superparticle

There are two independent versions of the 3D $\mathcal{N} = 2$ AdS algebra, the so-called $\mathcal{N} = (1, 1)$ and $\mathcal{N} = (2, 0)$ algebras. Correspondingly, there are two possible $\mathcal{N} = 2$ AC superalgebras which we consider below.

C.1. The $\mathcal{N} = (2, 0)$ AC superalgebra

We will start with the contraction of the 3D $\mathcal{N} = (2, 0)$ AdS algebra. The basic commutators are given by ($A = 0, 1, 2$; $i = 1, 2$)

$$\begin{aligned}
&[M_{AB}, M_{CD}] = 2\eta_{A[C}M_{D]B} - 2\eta_{B[C}M_{D]A}, && [M_{AB}, Q^i] = -\frac{1}{2}\gamma_{AB}Q^i,\\
&[M_{AB}, P_C] = 2\eta_{C[B}P_{A]}, && [P_A, Q^i] = x\gamma_A Q^i,\\
&[P_A, P_B] = 4x^2 M_{AB}, && [\mathcal{R}, Q^i] = 2x\epsilon^{ij}Q^j,\\
&\{Q^i_\alpha, Q^j_\beta\} = 2[\gamma^A C^{-1}]_{\alpha\beta} P_A \delta^{ij} + 2x[\gamma^{AB}C^{-1}]_{\alpha\beta} M_{AB}\delta^{ij} + 2[C^{-1}]_{\alpha\beta}\epsilon^{ij}\mathcal{R}. &&
\end{aligned} \tag{C.1}$$

Here P_A, M_{AB}, $\mathcal{R}$ and Q^i_α are the generators of space–time translations, Lorentz rotations, SO(2) R-symmetry transformations and supersymmetry transformations, respectively. The bosonic generators P_A, M_{AB} and $\mathcal{R}$ are anti-hermitian while de fermionic generators Q^i_α are hermitian. The parameter $x = 1/(2R)$, with R being the AdS radius. Note that the generator of the SO(2) R-symmetry becomes the central element of the Poincaré algebra in the flat limit $x \to 0$.

To take the Carroll contraction we define new supersymmetry charges by

$$Q^\pm_\alpha = \frac{1}{2}(Q^1_\alpha \pm \gamma_0 Q^2_\alpha) \tag{C.2}$$

and rescale the generators with a parameter ω as follows:

$$P_0 = \frac{\omega}{2} H, \qquad \mathcal{R} = \omega Z, \qquad M_{a0} = \omega K_a, \qquad Q^{\pm} = \sqrt{\omega}\tilde{Q}^{\pm}. \tag{C.3}$$

Taking the limit $\omega \to \infty$ and dropping the tildes on the $Q^{\pm}$ we get the following 3D $\mathcal{N} = (2, 0)$ Carroll superalgebra:

$$\begin{aligned}
&[M_{ab}, P_c] = 2\delta_{c[b}P_{a]}, \qquad [M_{ab}, K_c] = 2\delta_{c[b}K_{a]},\\
&[P_a, P_b] = \frac{1}{R^2}M_{ab}, \qquad [P_a, K_b] = \frac{1}{2}\delta_{ab}H, \qquad [P_a, H] = \frac{2}{R^2}K_a\\
&[P_a, Q^{\pm}] = \frac{1}{2R}\gamma_a Q^{\mp}, \qquad [M_{ab}, Q^{\pm}] = -\frac{1}{2}\gamma_{ab}Q^{\pm},\\
&\{Q^+_\alpha, Q^+_\beta\} = \frac{1}{2}[\gamma^0 C^{-1}]_{\alpha\beta}(H + 2Z), \qquad \{Q^-_\alpha, Q^-_\beta\} = \frac{1}{2}[\gamma^0 C^{-1}]_{\alpha\beta}(H - 2Z),\\
&\{Q^+_\alpha, Q^-_\beta\} = \frac{1}{R}[\gamma^{a0}C^{-1}]_{\alpha\beta}K_a.
\end{aligned} \tag{C.4}$$

In components the Maurer–Cartan equation $\mathrm{d}L^C - \frac{1}{2}f^C{}_{AB}L^B L^A = 0$ reads as follows:

$$\begin{aligned}
&\mathrm{d}L_H = -\frac{1}{2}L_P^a L_K^a - \frac{1}{4}\bar{L}_-\gamma^0 L_- - \frac{1}{4}\bar{L}_+\gamma^0 L_+, \qquad \mathrm{d}L_P^a = 2L_P^b L_M^{ab},\\
&\mathrm{d}L_K^a = 2L_K^b L_M^{ab} + \frac{2}{R^2}L_H L_P{}^a - \frac{1}{R}\bar{L}_-\gamma^{a0}L_+, \qquad \mathrm{d}L_Z = -\frac{1}{2}\bar{L}_+\gamma^0 L_+ + \frac{1}{2}\bar{L}_-\gamma^0 L_-,\\
&\mathrm{d}L_- = \frac{1}{2}\gamma_{ab}L_- L_M^{ab} - \frac{1}{2R}\gamma_a L_+ L_P{}^a, \qquad \mathrm{d}L_+ = \frac{1}{2}\gamma_{ab}L_+ L_M^{ab} - \frac{1}{2R}\gamma_a L_- L_P{}^a,\\
&\mathrm{d}L_M^{ab} = 2L_M^{ca}L_M^{cb} + \frac{1}{2R^2}L_P{}^b L_P{}^a.
\end{aligned} \tag{C.5}$$

C.2. Superparticle action

We use the algebra (C.4) to construct the action of the $\mathcal{N} = 2$ Carrollian superparticle. The coset that we will consider is

$$\frac{G}{H} = \frac{\mathcal{N} = (2, 0)\,\mathrm{AdS\ Carroll}}{\mathrm{SO}(D - 1)}, \tag{C.6}$$

with the coset element $g = g_0\, U$, where $g_0 = e^{Ht}e^{P_a x^a}e^{Q^-_\alpha \theta^\alpha_-}e^{Q^+_\alpha \theta^\alpha_+}e^{Zs}$ is the coset representing the $\mathcal{N} = (2, 0)$ Carroll superspace with a central charge extension and $U = e^{K_a v^a}$ is a general Carroll boost that represents the superparticle.

The Maurer–Cartan form associated to the super-Carroll space is given by

$$\Omega_0 = (g_0)^{-1}\mathrm{d}g_0 = HE^0 + P_a E^a + K_a\omega^{a0} + M_{ab}\omega^{ab} - \bar{Q}^- E_- - \bar{Q}^+ E_+ + ZE_Z, \tag{C.7}$$

where $(E^0, E^a, E_{-\alpha}, E_{+\alpha}, E_Z)$ and $(\omega^{a0}, \omega^{ab})$ are the supervielbein and the spin connection of the Carroll superspace which are given explicitly by

$$
\begin{aligned}
E^0 &= \mathrm{d}t\cosh\frac{x}{R} - \frac{1}{4}(\bar{\theta}_-\gamma^0\mathrm{d}\theta_- + \bar{\theta}_+\gamma^0\mathrm{d}\theta_+) - \frac{1}{4}\omega^{ab}(\bar{\theta}_+\gamma_{ab}\gamma^0\theta_+ + \bar{\theta}_-\gamma_{ab}\gamma^0\theta_-)\\
&\quad + \frac{1}{4R}\bar{\theta}_-\gamma^{a0}\theta_+E^a,\\
E^a &= \frac{R}{x}\mathrm{d}x^a\sinh\frac{x}{R} + \frac{1}{x^2}x^a x^b\mathrm{d}x_b\left(1 - \frac{R}{x}\sinh\frac{x}{R}\right),\\
\omega^{a0} &= -\frac{2}{xR}\mathrm{d}tx^a\sinh\frac{x}{R} - \frac{1}{R}\bar{\theta}_+\gamma^{a0}\mathrm{d}\theta_- - \frac{1}{4R^2}(\bar{\theta}_+\gamma_{ab}\gamma^0\theta_+ + \bar{\theta}_-\gamma_{ab}\gamma^0\theta_-)E^b\\
&\quad - \frac{1}{R}\omega^{bc}\bar{\theta}_-\gamma_{bc}\gamma^{a0}\theta_+,\\
\omega^{ab} &= \frac{1}{2x^2}(x^b\mathrm{d}x^a - x^a\mathrm{d}x^b)\left(\cosh\frac{x}{R} - 1\right),\\
E_{-\alpha} &= [\mathrm{d}\theta_-]_\alpha - \frac{1}{2R}[\gamma_a\theta_+]_\alpha E^a + \frac{1}{2}\omega^{ab}[\gamma_{ab}\theta_-]_\alpha,\\
E_{+\alpha} &= [\mathrm{d}\theta_+]_\alpha - \frac{1}{2R}[\gamma_a\theta_-]_\alpha E^a + \frac{1}{2}\omega^{ab}[\gamma_{ab}\theta_+]_\alpha,\\
E_Z &= ds + \frac{1}{2}\bar{\theta}_-\gamma^0\mathrm{d}\theta_- - \frac{1}{2}\bar{\theta}_+\gamma^0\mathrm{d}\theta_+\\
&\quad - \frac{1}{2}\omega^{ab}(\bar{\theta}_+\gamma_{ab}\gamma^0\theta_+ + \bar{\theta}_-\gamma_{ab}\gamma^0\theta_-) + \frac{1}{2R}\bar{\theta}_-\gamma^{a0}\theta_+E^a.
\end{aligned}
\tag{C.8}
$$

We can use the supervielbein to write the Maurer–Cartan form of the $\mathcal{N} = (2, 0)$ Carroll superparticle as follows:

$$
\begin{aligned}
L_H &= E^0 + \frac{1}{2}v_a E^a, & L_P^a &= E^a,\\
L_K^a &= \omega^{a0} + \mathrm{d}v^a + 2v_b\,\omega^{ab}, & L_Z &= E_Z,\\
L_{-\alpha} &= E_{-\alpha}, & L_{+\alpha} &= E_{+\alpha}.
\end{aligned}
\tag{C.9}
$$

C.3. Global symmetries and kappa symmetry

The action of the Carrollian superparticle is given by the pull-back of all L's that are invariant under rotations:

$$
\begin{aligned}
S &= a\int (L_H)^* + b\int (L_Z)^*\\
&= a\int \mathrm{d}\tau\Bigg(\dot{t}\cosh\frac{x}{R} + \frac{R}{2x}v_a\dot{x}^a\sinh\frac{x}{R} + \frac{1}{2x^2}x^b v_b x_a\dot{x}^a\left(1 - \frac{R}{x}\sinh\frac{x}{R}\right)\\
&\quad - \frac{1}{4}\bar{\theta}_-\gamma^0\dot{\theta}_- - \frac{1}{4}\bar{\theta}_+\gamma^0\dot{\theta}_+ - \frac{1}{8x^2}x^b\dot{x}^a(\bar{\theta}_+\gamma_{ab}\gamma^0\theta_+ + \bar{\theta}_-\gamma_{ab}\gamma^0\theta_-)\left(\cosh\frac{x}{R} - 1\right)\\
&\quad + \frac{1}{4x}\bar{\theta}_-\gamma^{a0}\theta_+\left[\dot{x}_a\sinh\frac{x}{R} + \frac{1}{Rx}x_a x_b\dot{x}^b\left(1 - \frac{R}{x}\sinh\frac{x}{R}\right)\right]\Bigg)\\
&\quad + b\int \mathrm{d}\tau\Bigg(\dot{s} + \frac{1}{2}\bar{\theta}_-\gamma^0\dot{\theta}_- - \frac{1}{2}\bar{\theta}_+\gamma^0\dot{\theta}_+\\
&\quad - \frac{1}{4x^2}x^b\dot{x}^a(\bar{\theta}_+\gamma_{ab}\gamma^0\theta_+ - \bar{\theta}_-\gamma_{ab}\gamma^0\theta_-)\left(\cosh\frac{x}{R} - 1\right)
\end{aligned}
$$

$$+\frac{1}{2x}\bar{\theta}_-\gamma^{a0}\theta_+\left[\dot{x}_a\sinh\frac{x}{R}+\frac{1}{Rx}x_ax_b\dot{x}^b\left(1-\frac{R}{x}\sinh\frac{x}{R}\right)\right]\Bigg), \tag{C.10}$$

which is invariant under the following bosonic transformation rules for the coordinates with constant parameters (ζ, η, a^i, λ^i, λ^i_j) corresponding to time translations, Z transformations, spatial translations, boosts, rotations, respectively

$$\begin{aligned}
\delta t &= -\zeta+\frac{R}{2x}\lambda^k x_k\tanh\frac{x}{R}+\frac{t}{Rx}a^kx_k\tanh\frac{x}{R},\\
\delta x^i &= -\frac{1}{x^2}\left(x^ia^kx_k-\frac{x}{R}\coth\frac{x}{R}(x^ia^kx_k-a^ix^2)\right)-2\lambda^i{}_k\,x^k,\\
\delta s &= -\eta,\\
\delta v^i &= -\lambda^i-\frac{1}{x^2}\lambda^kx_kx^i\,\mathrm{sech}\frac{x}{R}\left(1-\cosh\frac{x}{R}\right)-2\lambda^i{}_j\,v^j\\
&\quad-\frac{2t}{R^2}a^i-\frac{2t}{R^2x^2}x^ia^kx_k\,\mathrm{sech}\frac{x}{R}\left(1-\cosh\frac{x}{R}\right)+\frac{2}{Rx}v_ba^{[i}x^{b]}\mathrm{csch}\frac{x}{R}\left(1-\cosh\frac{x}{R}\right),\\
\delta\theta_+ &= -\frac{1}{2}\lambda^{ab}\gamma_{ab}\theta_++\frac{1}{2Rx}a^kx^b\gamma_{kb}\theta_-\mathrm{csch}\frac{x}{R}\left(1-\cosh\frac{x}{R}\right),\\
\delta\theta_- &= -\frac{1}{2}\lambda^{ab}\gamma_{ab}\theta_-+\frac{1}{2Rx}a^kx^b\gamma_{kb}\theta_+\mathrm{csch}\frac{x}{R}\left(1-\cosh\frac{x}{R}\right).
\end{aligned}\tag{C.11}$$

The same action is invariant under fermionic transformation rules with constant parameters (ϵ_+, ϵ_-) corresponding to the supersymmetry transformations

$$\begin{aligned}
\delta t &= \frac{1}{4}\bar{\epsilon}_+\gamma^0\theta_+\mathrm{sech}\frac{x}{R}\cosh\frac{x}{2R}-\frac{1}{4x}x^k\bar{\epsilon}_+\gamma^{k0}\theta_-\mathrm{sech}\frac{x}{R}\sinh\frac{x}{2R}\\
&\quad+\frac{1}{4}\bar{\epsilon}_-\gamma^0\theta_-\mathrm{sech}\frac{x}{R}\cosh\frac{x}{2R}-\frac{1}{4x}x^k\bar{\epsilon}_-\gamma^{k0}\theta_+\mathrm{sech}\frac{x}{R}\sinh\frac{x}{2R},\\
\delta x^i &= 0,\\
\delta v^i &= \frac{1}{Rx}x^i\bar{\epsilon}_+\gamma^0\theta_+\left(\frac{1}{2}\tanh\frac{x}{R}\cosh\frac{x}{2R}-2\sinh\frac{x}{2R}\right)+\frac{1}{R}\bar{\epsilon}_+\gamma^{i0}\theta_-\cosh\frac{x}{2R}\\
&\quad-\frac{1}{2Rx^2}x^ix^k\bar{\epsilon}_+\gamma^{k0}\theta_-\tanh\frac{x}{R}\sinh\frac{x}{2R}\\
&\quad+\frac{1}{2Rx}x^i\bar{\epsilon}_-\gamma^0\theta_-\tanh\frac{x}{R}\cosh\frac{x}{2R}-\frac{1}{Rx}x^b\bar{\epsilon}_-\gamma_b\gamma^{i0}\theta_-\sinh\frac{x}{2R}\\
&\quad-\frac{1}{2Rx^2}x^ix^k\bar{\epsilon}_-\gamma^{k0}\theta_+\tanh\frac{x}{R}\sinh\frac{x}{2R},\\
\delta s &= \frac{1}{2}\bar{\epsilon}_+\gamma^0\theta_+\cosh\frac{x}{2R}+\frac{1}{2x}x^k\bar{\epsilon}_+\gamma^{k0}\theta_-\sinh\frac{x}{2R}\\
&\quad-\frac{1}{2}\bar{\epsilon}_-\gamma^0\theta_-\cosh\frac{x}{2R}-\frac{1}{2x}x^k\bar{\epsilon}_-\gamma^{k0}\theta_+\sinh\frac{x}{2R},\\
\delta\theta_+ &= \epsilon_+\cosh\frac{x}{2R}+\frac{1}{x}x^k\gamma_k\,\epsilon_-\sinh\frac{x}{2R},\\
\delta\theta_- &= \epsilon_-\cosh\frac{x}{2R}+\frac{1}{x}x^k\gamma_k\,\epsilon_+\sinh\frac{x}{2R}.
\end{aligned}\tag{C.12}$$

To derive an action that is invariant under κ-transformations we need to find a fermionic gauge-transformation that leaves L_H and/or L_Z invariant. The variation of L_H and L_Z under gauge-transformations are given by

$$\begin{aligned}\delta L_H &= \mathrm{d}([\delta z_H]) + \frac{1}{2}L_P^a[\delta z_K^a] + \frac{1}{2}L_K^a[\delta z_P^a] + \frac{1}{2}\bar{L}_-\gamma^0[\delta z_-] + \frac{1}{2}\bar{L}_+\gamma^0[\delta z_+],\\ \delta L_Z &= \mathrm{d}([\delta z_Z]) - \bar{L}_-\gamma^0[\delta z_-] + \bar{L}_+\gamma^0[\delta z_+],\end{aligned} \tag{C.13}$$

where, for example, $[\delta z_K^a]$ is obtained from L_H by changing the one-forms $\mathrm{d}t$, $\mathrm{d}\theta_+$, $\mathrm{d}\theta_-$ with the transformations δt, $\delta\theta_+$, $\delta\theta_-$. For κ-transformations we have $[\delta z_H] = 0$, $[\delta z_K^a] = 0$, $[\delta z_P^a] = 0$ and hence we find

$$\begin{aligned}\delta L_H &= \frac{1}{2}\delta\bar{\theta}_-\gamma^0[\delta z_-] + \frac{1}{2}\delta\bar{\theta}_+\gamma^0[\delta z_+],\\ \delta L_Z &= -\delta\bar{\theta}_-\gamma^0[\delta z_-] + \delta\bar{\theta}_+\gamma^0[\delta z_+].\end{aligned} \tag{C.14}$$

It follows that to obtain a κ-symmetric action we need to take the pull-back of either L_H or L_Z, with $b = \pm\frac{1}{2}a$. We focus here on the case $b = -\frac{1}{2}a$. For this choice the action and κ-symmetry rules are given by

$$S = a\int\left(L_H - \frac{1}{2}L_Z\right)^*, \qquad [\delta z_+] = \kappa, \qquad [\delta z_-] = 0, \tag{C.15}$$

where $\kappa = \kappa(\tau)$ is an arbitrary local parameter. Using this we find the following κ-transformations of the coordinates

$$\begin{aligned}&\delta t = \frac{1}{4}\,\mathrm{sech}\,\frac{x}{R}\bar{\theta}_+\gamma^0\kappa, && \delta x^a = 0, && \delta\theta_+ = \kappa,\\ &\delta s = \frac{1}{2}\bar{\theta}_+\gamma^0\kappa, && \delta v_a = \frac{1}{2Rx}x^a\bar{\theta}_+\gamma^0\kappa\tanh\frac{x}{R}, && \delta\theta_- = 0.\end{aligned} \tag{C.16}$$

After κ-gauge fixing (setting $\theta_+ = 0$) the action reads

$$\begin{aligned}S = a\int \mathrm{d}\tau\Bigg(&\dot{t}\cosh\frac{x}{R} - \frac{1}{2}\dot{s} + \frac{R}{2x}v_a\dot{x}^a\sinh\frac{x}{R} + \frac{1}{2x^2}x^b v_b x_a\dot{x}^a\left(1 - \frac{R}{x}\sinh\frac{x}{R}\right)\\ &- \frac{1}{2}\bar{\theta}_-\gamma^0\dot{\theta}_- - \frac{1}{4x^2}x^b\dot{x}^a\bar{\theta}_-\gamma_{ab}\gamma^0\theta_-\left(\cosh\frac{x}{R} - 1\right)\Bigg).\end{aligned} \tag{C.17}$$

This action is invariant under the following transformation rules

$$\begin{aligned}\delta t &= -\frac{1}{4x}x^k\bar{\epsilon}_+\gamma^{k0}\theta_-\mathrm{sech}\,\frac{x}{R}\sinh\frac{x}{2R} + \frac{1}{4}\bar{\epsilon}_-\gamma^0\theta_-\mathrm{sech}\,\frac{x}{R}\cosh\frac{x}{2R},\\ \delta x^i &= 0,\\ \delta v^i &= \frac{1}{R}\bar{\epsilon}_+\gamma^{i0}\theta_-\cosh\frac{x}{2R} - \frac{1}{2Rx^2}x^i x^k\bar{\epsilon}_+\gamma^{k0}\theta_-\tanh\frac{x}{R}\sinh\frac{x}{2R}\\ &\quad + \frac{1}{2Rx}x^i\bar{\epsilon}_-\gamma^0\theta_-\tanh\frac{x}{R}\cosh\frac{x}{2R} - \frac{1}{Rx}x^b\bar{\epsilon}_-\gamma_b\gamma^{i0}\theta_-\sinh\frac{x}{2R}\\ \delta s &= -\frac{1}{2x}x^k\bar{\epsilon}_+\gamma^{k0}\theta_-\sinh\frac{x}{2R} - \frac{1}{2}\bar{\epsilon}_-\gamma^0\theta_-\cosh\frac{x}{2R},\\ \delta\theta_- &= \epsilon_-\cosh\frac{x}{2R} + \frac{1}{x}x^k\gamma_k\epsilon_+\sinh\frac{x}{2R}.\end{aligned} \tag{C.18}$$

C.4. The $\mathcal{N} = (1, 1)$ AC superalgebra

We now consider the 3*D* $\mathcal{N} = (1, 1)$ AdS algebra which is given by

$$\begin{aligned}
&[M_{AB}, M_{CD}] = 2\eta_{A[C}M_{D]B} - 2\eta_{B[C}M_{D]A}, && [M_{AB}, Q^{\pm}] = -\frac{1}{2}\gamma_{AB}Q^{\pm},\\
&[M_{AB}, P_C] = 2\eta_{C[B}P_{A]}, && [P_A, Q^{\pm}] = \pm x\gamma_A Q^{\pm},\\
&\{Q_\alpha^{\pm}, Q_\beta^{\pm}\} = 4[\gamma^A C^{-1}]_{\alpha\beta}P_A \pm 4x[\gamma^{AB}C^{-1}]_{\alpha\beta}M_{AB}, && [P_A, P_B] = 4x^2 M_{AB}.
\end{aligned} \tag{C.19}$$

Here P_A, M_{AB} and $Q_\alpha^{\pm}$ are the generators of space–time translations, Lorentz rotations and supersymmetry transformations, respectively. The bosonic generators P_A and M_{AB} are anti-hermitian while de fermionic generators $Q_\alpha^{\pm}$ are hermitian. Like in the previous case, the parameter $x = 1/(2R)$ is a contraction parameter.

To make the Carroll contraction we rescale the generators with a parameter ω as follows:

$$P_0 = \frac{\omega}{2}H, \qquad M_{a0} = \omega K_a, \qquad Q^{\pm} = \sqrt{\omega}\tilde{Q}^{\pm}. \tag{C.20}$$

Taking the limit $\omega \to \infty$ and dropping the tildes on the $Q^{\pm}$ we get the following 3D $\mathcal{N} = (1, 1)$ Carroll superalgebra:

$$\begin{aligned}
&[M_{ab}, P_c] = 2\delta_{c[b}P_{a]}, \qquad [M_{ab}, K_c] = 2\delta_{c[b}K_{a]},\\
&[P_a, P_b] = \frac{1}{R^2}M_{ab}, \qquad [P_a, K_b] = \frac{1}{2}\delta_{ab}H, \qquad [P_a, H] = \frac{2}{R^2}K_a\\
&[P_a, Q^{\pm}] = \pm\frac{1}{2R}\gamma_a Q^{\pm}, \qquad [M_{ab}, Q^{\pm}] = -\frac{1}{2}\gamma_{ab}Q^{\pm},\\
&\{Q_\alpha^{\pm}, Q_\beta^{\pm}\} = 2[\gamma^0 C^{-1}]H \pm \frac{4}{R}[\gamma^{a0}C^{-1}]_{\alpha\beta}K_a.
\end{aligned} \tag{C.21}$$

The corresponing componetns of the Maurer–Cartan equation $\mathrm{d}L^C - \frac{1}{2}f^C{}_{AB}L^B L^A = 0$ are given by

$$\begin{aligned}
\mathrm{d}L_H &= -\frac{1}{2}L_P^a L_K^a - \bar{L}_+\gamma^0 L_+ - \bar{L}_-\gamma^0 L_-,\\
\mathrm{d}L_P^a &= 2L_P^b L_M^{ab},\\
\mathrm{d}L_K^a &= 2L_K^b L_M^{ab} + \frac{2}{R^2}L_H L_P{}^a - \frac{2}{R}\bar{L}_+\gamma^{a0}L_+ + \frac{2}{R}\bar{L}_-\gamma^{a0}L_-,\\
\mathrm{d}L_M^{ab} &= 2L_M^{ca} L_M^{cb} + \frac{1}{2R^2}L_P{}^b L_P{}^a,\\
\mathrm{d}L_+ &= \frac{1}{2}\gamma_{ab}L_+ L_M^{ab} - \frac{1}{2R}\gamma_a L_+ L_P{}^a,\\
\mathrm{d}L_- &= \frac{1}{2}\gamma_{ab}L_- L_M^{ab} + \frac{1}{2R}\gamma_a L_- L_P{}^a.
\end{aligned} \tag{C.22}$$

C.5. Superparticle action

Taking the algebra (C.21) we consider the following coset

$$\frac{G}{H} = \frac{\mathcal{N} = (1, 1)\,\mathrm{AdS\ Carroll}}{\mathrm{SO}(D - 1)}. \tag{C.23}$$

The coset element is $g = g_0\, U$, where $g_0 = e^{Ht} e^{P_a x^a} e^{Q_\alpha^- \theta_-^\alpha} e^{Q_\alpha^+ \theta_+^\alpha}$ is the coset representing the $\mathcal{N} = (1,\,1)$ Carroll superspace and $U = e^{K_a v^a}$ is a general Carroll boost representing the insertion of the superparticle.

The Maurer–Cartan form associated to the super-Carroll space is given by

$$\Omega_0 = (g_0)^{-1}\mathrm{d}g_0 = HE^0 + P_a E^a + K_a \omega^{a0} + M_{ab}\omega^{ab} - \bar{Q}^- E_- - \bar{Q}^+ E_+, \tag{C.24}$$

where $(E^0,\, E^a,\, E_{-\alpha},\, E_{+\alpha})$ and $(\omega^{a0},\, \omega^{ab})$ are the supervielbein and the spin connection of the Carroll superspace:

$$\begin{aligned}
E^0 &= \mathrm{d}t \cosh\frac{x}{R} - \bar{\theta}_- \gamma^0 \mathrm{d}\theta_- - \bar{\theta}_+ \gamma^0 \mathrm{d}\theta_+ - \omega^{ab}(\bar{\theta}_+ \gamma_{ab}\gamma^0\theta_+ + \bar{\theta}_- \gamma_{ab}\gamma^0\theta_-),\\
E^a &= \frac{R}{x}\mathrm{d}x^a \sinh\frac{x}{R} + \frac{1}{x^2}x^a x^b \mathrm{d}x_b\left(1 - \frac{R}{x}\sinh\frac{x}{R}\right),\\
\omega^{a0} &= -\frac{2}{xR}\mathrm{d}t x^a \sinh\frac{x}{R} - \frac{1}{R^2}(\bar{\theta}_+ \gamma_{ab}\gamma^0\theta_+ + \bar{\theta}_- \gamma_{ab}\gamma^0\theta_-)E^b\\
&\quad - \frac{2}{R}(\bar{\theta}_+ \gamma^{a0}\mathrm{d}\theta_+ - \bar{\theta}_- \gamma^{a0}\mathrm{d}\theta_-),\\
\omega^{ab} &= \frac{1}{2x^2}(x^b \mathrm{d}x^a - x^a \mathrm{d}x^b)\left(\cosh\frac{x}{R} - 1\right),\\
E_{-\alpha} &= [\mathrm{d}\theta_-]_\alpha + \frac{1}{2R}[\gamma_a\theta_-]_\alpha E^a + \frac{1}{2}\omega^{ab}[\gamma_{ab}\theta_-]_\alpha,\\
E_{+\alpha} &= [\mathrm{d}\theta_+]_\alpha - \frac{1}{2R}[\gamma_a\theta_+]_\alpha E^a + \frac{1}{2}\omega^{ab}[\gamma_{ab}\theta_+]_\alpha.
\end{aligned} \tag{C.25}$$

We can use the supervielbein to write the Maurer–Cartan form of the $\mathcal{N} = (1,\,1)$ Carroll superparticle as follows:

$$\begin{aligned}
L_H &= E^0 + \frac{1}{2}v_a E^a, & L_P^a &= E^a,\\
L_K^a &= \omega^{a0} + \mathrm{d}v^a + 2v_b\,\omega^{ab}, & &\\
L_{-\alpha} &= E_{-\alpha}, & L_{+\alpha} &= E_{+\alpha}.
\end{aligned} \tag{C.26}$$

C.6. Global symmetries

The action of the Carrollian superparticle is given by the pull-back of all L's that are invariant under rotations:

$$\begin{aligned}
S &= M\int (L_H)^*\\
&= M\int \mathrm{d}\tau\Bigg(\dot{t}\cosh\frac{x}{R} + \frac{R}{2x}v_a\dot{x}^a\sinh\frac{x}{R} + \frac{1}{2x^2}x^b v_b x_a \dot{x}^a\left(1 - \frac{R}{x}\sinh\frac{x}{R}\right)\\
&\quad - \bar{\theta}_-\gamma^0\dot{\theta}_- - \bar{\theta}_+\gamma^0\dot{\theta}_+ - \frac{1}{2x^2}x^b\dot{x}^a(\bar{\theta}_+\gamma_{ab}\gamma^0\theta_+ + \bar{\theta}_-\gamma_{ab}\gamma^0\theta_-)\left(\cosh\frac{x}{R} - 1\right)\Bigg).
\end{aligned} \tag{C.27}$$

This action is invariant under the following bosonic transformation rules for the coordinates with constant parameters $(\zeta,\, a^i,\, \lambda^i,\, \lambda^i_j)$ corresponding to time translations, spatial translations, boosts and rotations, respectively

$$
\begin{aligned}
\delta t =& -\zeta + \frac{R}{2x}\lambda^k x_k \tanh\frac{x}{R} + \frac{t}{Rx}a^k x_k \tanh\frac{x}{R},\\
\delta x^i =& -\frac{1}{x^2}\left(x^i a^k x_k - \frac{x}{R}\coth\frac{x}{R}(x^i a^k x_k - a^i x^2)\right) - 2\lambda^i{}_k\, x^k,\\
\delta v^i =& -\lambda^i - \frac{1}{x^2}\lambda^k x_k x^i \operatorname{sech}\frac{x}{R}\left(1 - \cosh\frac{x}{R}\right) - 2\lambda^i{}_j\, v^j\\
& - \frac{2t}{R^2}a^i - \frac{2t}{R^2x^2}x^i a^k x_k \operatorname{sech}\frac{x}{R}\left(1 - \cosh\frac{x}{R}\right)\\
& + \frac{2}{Rx}v_b a^{[i}x^{b]}\operatorname{csch}\frac{x}{R}\left(1 - \cosh\frac{x}{R}\right),\\
\delta\theta_+ =& -\frac{1}{2}\lambda^{ab}\gamma_{ab}\theta_+ + \frac{1}{2Rx}a^k x^b\gamma_{kb}\theta_+ \operatorname{csch}\frac{x}{R}\left(1 - \cosh\frac{x}{R}\right),\\
\delta\theta_- =& -\frac{1}{2}\lambda^{ab}\gamma_{ab}\theta_- + \frac{1}{2Rx}a^k x^b\gamma_{kb}\theta_- \operatorname{csch}\frac{x}{R}\left(1 - \cosh\frac{x}{R}\right).
\end{aligned} \tag{C.28}
$$

The same action is invariant inder the following fermionic transformation rules with constant parameters (ϵ_+, ϵ_-) corresponding to supersymmetry transformations:

$$
\begin{aligned}
\delta t =& \bar\epsilon_+\gamma^0\theta_+ \operatorname{sech}\frac{x}{R}\cosh\frac{x}{2R} - \frac{1}{x}x^k\bar\epsilon_+\gamma^{k0}\theta_+ \operatorname{sech}\frac{x}{R}\sinh\frac{x}{2R}\\
& + \bar\epsilon_-\gamma^0\theta_-\operatorname{sech}\frac{x}{R}\cosh\frac{x}{2R} + \frac{1}{x}x^k\bar\epsilon_-\gamma^{k0}\theta_-\operatorname{sech}\frac{x}{R}\sinh\frac{x}{2R},\\
\delta x^i =& 0,\\
\delta v^i =& \frac{2}{R}\bar\epsilon_+\gamma^{i0}\theta_+\cosh\frac{x}{2R} - \frac{2}{Rx}x^k\bar\epsilon_+\gamma_k\gamma^{i0}\theta_+\sinh\frac{x}{2R}\\
& + \frac{2}{xR}x^i\tanh\frac{x}{R}\left(\bar\epsilon_+\gamma^0\theta_+\cosh\frac{x}{2R} - \frac{1}{x}x^k\bar\epsilon_+\gamma^{k0}\theta_+\sinh\frac{x}{2R}\right)\\
& - \frac{2}{R}\bar\epsilon_-\gamma^{i0}\theta_-\cosh\frac{x}{2R} - \frac{2}{Rx}x^k\bar\epsilon_-\gamma_k\gamma^{i0}\theta_-\sinh\frac{x}{2R}\\
& + \frac{2}{xR}x^i\tanh\frac{x}{R}\left(\bar\epsilon_-\gamma^0\theta_-\cosh\frac{x}{2R} + \frac{1}{x}x^k\bar\epsilon_-\gamma^{k0}\theta_-\sinh\frac{x}{2R}\right)\\
\delta\theta_+ =& \epsilon_+\cosh\frac{x}{2R} + \frac{1}{x}x^k\gamma_k\epsilon_+\sinh\frac{x}{2R},\\
\delta\theta_- =& \epsilon_-\cosh\frac{x}{2R} - \frac{1}{x}x^k\gamma_k\epsilon_-\sinh\frac{x}{2R}.
\end{aligned} \tag{C.29}
$$

References

[1] Cartan E 1923 Sur les variétés à connexion affine et la théorie de la relativité généralisée (première partie) *Ann. Sci. Ecole Norm. Sup.* **40** 325–412

[2] Havas P 1964 Four-dimensional formulations of Newtonian mechanics and their relation to the special and the general theory of relativity *Rev. Mod. Phys.* **36** 938–65

[3] Anderson J 1967 *Principles of Relativity Physics* (New York: Academic)

[4] Trautman A 1965 *Theories of Space: Time and Gravitation* (*Lectures on General Relativity*) ed S Deser and K Ford (Englewood Cliffs, NJ: Prentice-Hall)

[5] Künzle H P 1972 Galilei and Lorentz structures on space–time: comparison of the corresponding geometry and physics *Ann. Inst. Henri Poincare* A **17** 337–62

[6] Kuchař K 1980 Gravitation, geometry, and nonrelativistic quantum theory *Phys. Rev.* D **22** 1285–99

[7] De Pietri R, Lusanna L and Pauri M 1995 Standard and generalized Newtonian gravities as 'gauge' theories of the extended Galilei group: I. The standard theory *Class. Quantum Grav.* **12** 219–54

[8] Andringa R, Bergshoeff E, Panda S and de Roo M 2011 Newtonian gravity and the Bargmann algebra *Class. Quantum Grav.* **28** 105011

[9] Son D and Wingate M 2006 General coordinate invariance and conformal invariance in nonrelativistic physics: unitary Fermi gas *Ann. Phys.* **321** 197–224

[10] Son D T 2013 Newton–Cartan geometry and the quantum Hall effect arXiv:1306.0638

[11] Geracie M, Son D T, Wu C and Wu S-F 2015 Space–time symmetries of the quantum Hall effect *Phys. Rev.* D **91** 045030

[12] Jensen K and Karch A 2014 Revisiting non-relativistic limits *J. High Energy Phys.* JHEP04(2015)155

[13] Banerjee R, Mitra A and Mukherjee P 2015 General algorithm for non-relativistic diffeomorphism invariance *Phys. Rev.* D **91** 084021

[14] Christensen M H, Hartong J, Obers N A and Rollier B 2014 Boundary stress–energy tensor and Newton–Cartan geometry in Lifshitz holography *J. High Energy Phys.* JHEP01(2014)057

[15] Hartong J, Kiritsis E and Obers N A 2015 Field theory on Newton–Cartan backgrounds and symmetries of the Lifshitz vacuum *J. High Energy Phys.* JHEP08(2015)006

[16] Gomis J and Ooguri H 2001 Nonrelativistic closed string theory *J. Math. Phys.* **42** 3127–51

[17] Danielsson U H, Guijosa A and Kruczenski M 2000 IIA/B, wound and wrapped *J. High Energy Phys.* JHEP10(2000)020

[18] Gomis J, Kamimura K and Townsend P K 2004 Non-relativistic superbranes *J. High Energy Phys.* JHEP11(2004)051

[19] Gomis J, Gomis J and Kamimura K 2005 Non-relativistic superstrings: a new soluble sector of AdS(5) $x\ S^{**}5$ *J. High Energy Phys.* JHEP12(2005)024

[20] Lévy-Leblond J-M 1965 Une nouvelle limite non-relativiste du groupe de Poincaré *Ann. Inst. Henri Poincare* A **3** 1–2

[21] Henneaux M 1979 Geometry of zero signature space–times *Bull. Soc. Math. Belg.* **31** 47–63

[22] Bondi H, van der Burg M G J and Metzner A W K 1962 Gravitational waves in general relativity: VII. Waves from axisymmetric isolated systems *Proc. R. Soc.* A **269** 21–52

[23] Sachs R K 1962 Gravitational waves in general relativity. VIII. Waves in asymptotically flat space–times *Proc. R. Soc.* A **270** 103–26

[24] Sachs R 1962 Asymptotic symmetries in gravitational theory *Phys. Rev.* **128** 2851–64

[25] Duval C, Gibbons G W and Horvathy P A 2014 Conformal Carroll groups and BMS symmetry *Class. Quantum Grav.* **31** 092001

[26] Banks T 2003 A critique of pure string theory: heterodox opinions of diverse dimensions arXiv: hep-th/0306074

[27] de Boer J and Solodukhin S N 2003 A holographic reduction of Minkowski space–time *Nucl. Phys.* B **665** 545–93

[28] Arcioni G and Dappiaggi C 2003 Exploring the holographic principle in asymptotically flat space–times via the BMS group *Nucl. Phys.* B **674** 553–92

[29] Barnich G and Troessaert C 2010 Aspects of the BMS/CFT correspondence *J. High Energy Phys.* JHEP05(2010)062

[30] Hartong J 2015 Holographic reconstruction of 3D flat space–time arXiv:1511.01387 [hep-th]

[31] Gibbons G, Hashimoto K and Yi P 2002 Tachyon condensates Carrollian contraction of Lorentz group, and fundamental strings *J. High Energy Phys.* JHEP09(2002)061

[32] Sen A 1999 Universality of the tachyon potential *J. High Energy Phys.* JHEP12(1999)027

[33] Taylor W 2003 Lectures on *D*-branes, tachyon condensation, and string field theory *(Lectures on quantum gravity) Proc. School of Quantum Gravity (Valdivia, Chile, 4-14 January 2002)* pp 151–206

[34] Hashimoto K and Terashima S 2004 Brane decay and death of open strings *J. High Energy Phys.* JHEP06(2004)048

[35] Hofman D M and Rollier B 2015 Warped conformal field theory as lower spin gravity *Nucl. Phys.* B **897** 1–38

[36] Hartong J 2015 Gauging the Carroll algebra and ultra-relativistic gravity *J. High Energy Phys.* JHEP08(2015)069

[37] Duval C, Gibbons G, Horvathy P and Zhang P 2014 Carroll versus Newton and Galilei: two dual non-Einsteinian concepts of time *Class. Quantum Grav.* **31** 085016
[38] Bekaert X and Morand K 2015 Connections and dynamical trajectories in generalised Newton–Cartan gravity: II. An ambient perspective arXiv:1505.03739
[39] Bacry H and Levy-Leblond J 1968 Possible kinematics *J. Math. Phys.* **9** 1605–14
[40] Bargmann V 1954 On unitary ray representations of continuous groups *Ann. Math.* **59** 1–46
[41] Coleman S R, Wess J and Zumino B 1969 Structure of phenomenological Lagrangians: I *Phys. Rev.* **177** 2239–47
[42] Callan J, Curtis G, Coleman S R, Wess J and Zumino B 1969 Structure of phenomenological Lagrangians: II *Phys. Rev.* **177** 2247–50
[43] Bergshoeff E, Gomis J and Longhi G 2014 Dynamics of Carroll particles *Class. Quantum Grav.* **31** 205009
[44] Gomis J and Passerini F 2005 unpublished notes
[45] Gibbons G, Gomis J and Pope C 2010 Deforming the Maxwell–Sim algebra *Phys. Rev.* D **82** 065002
[46] Casalbuoni R 1976 The classical mechanics for Bose–Fermi systems *Nuovo Cimento* A **33** 389
[47] Brink L and Schwarz J 1981 Quantum superspace *Phys. Lett.* B **100** 310–2
[48] de Azcarraga J A and Lukierski J 1982 Supersymmetric particles with internal symmetries and central charges *Phys. Lett.* B **113** 170
[49] Siegel W 1983 Hidden local supersymmetry in the supersymmetric particle action *Phys. Lett.* B **128** 397
[50] Bergshoeff E, Gomis J, Kovacevic M, Parra L, Rosseel J *et al* 2014 Nonrelativistic superparticle in a curved background *Phys. Rev.* D **90** 065006
[51] Achucarro A and Townsend P 1986 A Chern–Simons action for three-dimensional anti-de Sitter supergravity theories *Phys. Lett.* B **180** 89
[52] McArthur I N 2000 Kappa symmetry of Green–Schwarz actions in coset superspaces *Nucl. Phys.* B **573** 811–29
[53] Gomis J, Kamimura K and West P C 2006 The Construction of brane and superbrane actions using non-linear realisations *Class. Quantum Grav.* **23** 7369–82