

University of Groningen

The evolution of altruism in spatial threshold public goods games via an insurance mechanism

Zhang, Jianlei; Zhang, Chunyan

Published in:
Journal of statistical mechanics-Theory and experiment

DOI:
[10.1088/1742-5468/2015/05/P05001](https://doi.org/10.1088/1742-5468/2015/05/P05001)

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2015

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Zhang, J., & Zhang, C. (2015). The evolution of altruism in spatial threshold public goods games via an insurance mechanism. *Journal of statistical mechanics-Theory and experiment*, 2015(5), [P05001]. <https://doi.org/10.1088/1742-5468/2015/05/P05001>

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

The evolution of altruism in spatial threshold public goods games via an insurance mechanism

Jianlei Zhang^{1,2,3} and Chunyan Zhang¹

¹ Department of Automation, College of Computer and Control Engineering, Nankai University, Tianjin, 300071, People's Republic of China

² Theoretical Biology Group, Groningen Institute for Evolutionary Life Sciences, University of Groningen, 9712 CP Groningen, The Netherlands

³ Network Analysis and Control Group, Engineering and Technology Institute Groningen, University of Groningen, 9712 CP Groningen, The Netherlands
E-mail: jianlei.zhang@rug.nl

Received 13 November 2014

Accepted for publication 7 April 2015

Published 7 May 2015



Online at stacks.iop.org/JSTAT/2015/P05001

[doi:10.1088/1742-5468/2015/05/P05001](https://doi.org/10.1088/1742-5468/2015/05/P05001)

Abstract. The persistence of cooperation in public goods situations has become an important puzzle for researchers. This paper considers the threshold public goods games where the option of insurance is provided for players from the standpoint of diversification of risk, envisaging the possibility of multiple strategies in such scenarios. In this setting, the provision point is defined in terms of the minimum number of contributors in one threshold public goods game, below which the game fails. In the presence of risk and insurance, more contributions are motivated if (1) only cooperators can opt to be insured and thus their contribution loss in the aborted games can be (partly or full) covered by the insurance; (2) insured cooperators obtain larger compensation, at lower values of the threshold point (the required minimum number of contributors). Moreover, results suggest the dominance of insured defectors who get a better promotion by more profitable benefits from insurance. We provide results of extensive computer simulations in the realm of spatial games (random regular networks and scale-free networks here), and support this study with analytical results for well-mixed populations. Our study is expected to establish a causal link between the widespread altruistic behaviors and the existing insurance system.

Keywords: game-theory (theory), applications to game theory and mathematical economics

Contents

1. Introduction	2
2. The game model	3
3. General analysis	5
3.1. Results in Scenario 1	5
3.2. Results in Scenario 2	6
4. Evolutionary outcomes in structured populations	7
4.1. Results in Scenario 1	10
4.2. Results in Scenario 2	12
5. Conclusion	15
Acknowledgments	16
References	16

1. Introduction

Social dilemmas in which the self-interest may clash with the collective interest constitute a significant form of societal problems. Evolutionary game theory is an interdisciplinary mathematical tool which seems to embody several relevant features of the problem and, as such, is employed in a vast amount of cooperation-oriented study [1–6]. As a heuristic framework, the oft-cited public goods game [7–11] is a paradigm example for investigating the root of cooperative behaviors in spite of the fact that self-interest seems to dictate defective behaviors. Previous research has identified a series of solutions to cooperation dilemma problems, including the ‘five rules’ (kin selection, direct reciprocity, indirect reciprocity, network reciprocity and group selection) introduced by [12, 13]. Further, a variety of studies suggest that the population structure [14, 15] and the coevolution [16], closely related with complex networks, are also relevant factors to take into account in since they may enhance strong altruism. Other resolutions of this type of conflicts depicted by public goods game include punishment [17–19], social diversity [20–22], voluntary participation [8, 23], dynamic group size [24], social preferences [25], kinship [26] and positive interactions [27], which seems to also offer a way out for cooperation to emerge in public goods dilemma scenarios.

Models of the public goods game nicely capture the dominating features of most cooperative phenomena in nature. However, the model of PGG may no longer be feasible in many collective action scenarios and can be more appropriately described by the so-called threshold public goods game (TPGG) [28–31]. In this version of the public goods game, a

successful cooperative effort is achieved only if the number of cooperators equal or exceed a required threshold; otherwise, no public goods is provided. For instance, building a flood-resistant dam requires a minimum amount of contributors for the project to be successfully built. If not, flood will probably inundate the whole village due to the unaccomplished and nonfunctional dam. This version of PGG has been widely studied as a suitable model for the confrontation between cooperative and selfish behaviors in such circumstances. So far, several factors to foster cooperation and solve the social dilemma in an efficient way have been proposed and tested, such as incomplete information, identifiability of individual contributions, and so on [32–36].

Past works on PGG have assumed binary situations in which two strategies are available: either choose cooperation (C) in order to serve the public interest, or choose alternative defection (D), which serves the immediate private interest. And yet, the simple two-strategy profile can be extended and made more realistic in a variety of ways. Note that ample examples of insurance behaviours take place in human societies, which can insure people's life or benefits against accident or some risks. For example, the whole point of a universal health insurance system is that everyone pays in, even if they are currently healthy, and in return everyone has insurance coverage if and when they need it. The everyday phenomena moreover indicate that the risk preferences, age or education level of the individual variables have significant differences to the choices on the insurance among populations. Risk-averse players conservatively prefer to transfer their (possible) future losses to the insurance companies. Oppositely, risk-seeking players don't contemplate paying any insurance cost and prefer the high-risk high-yield behaviors, irrespective of the potential risks.

Therefore, in an attempt to explore a more realistic scenario, we now broaden the two-strategy profile and employ multiple strategies in the threshold public goods games. Our previous work [37] introduces speculation against punishment and studies the effects of such speculation for defectors in public goods games. Here we focus on the possible effectiveness of insurance behaviors in promoting contributive behaviors, when participators face the risk of joining a failed threshold public goods game finally. And more remarkably, our interest is primarily in the capacity of agents to contribute and produce the public goods when they are confronted by ambiguous losses, and also the proposed insurance. In this threshold public goods system, agents can purchase an insurance that sequentially covers part or all of their potential losses.

The rest of this paper is organized as indicated below. Section 2 presents the basic framework of model with insurance in full detail. Section 3 provides the general analysis in infinite populations. Section 4 is devoted to the presentation of main findings in structured populations, and section 5 concludes.

2. The game model

This section first sets up our model of a threshold public goods game with multiple strategies led by the introduced insurance. Herein N agents play TPGGs, and each player independently decides how much (between a constant value c and none here) of her endowment to contribute to the public goods. The accumulated contribution is enlarged

by the enhancement factor r , and then evenly distributed to all the individuals, only if the number of contributors in the group attains a certain threshold T , saying that the public goods is successfully provided. If the number of contributors is less than the required threshold, the public goods game fails and contributions are not returned to the players. As mentioned, in the present study we set the threshold point as the minimum number of contributors in one public goods game, not the threshold of contributions as previous studies [38].

Here we explore two scenarios to reproduce those possible real-world setups about insurance. Scenario 1: only cooperators face the choice of insurance, by considering that free riders may have no incentive to insure their zero contributions. Scenario 2: all the players can opt to be insured if willing to pay the insurance cost. Hence, we propose two additional strategies named as insured cooperation and insured defection respectively, besides cooperation and defection. It is worth reminding that both cooperators and insured cooperators are the contributors for the common pool of public goods. Except for the different strategy profiles, these two scenarios share other identical rules (e.g. the strategy updating) as game proceeds.

Next, the earnings of an individual depend upon her strategy and the combination of the strategies adopted by her opponents. For the group of size N , let N_d specify the number of the defectors in a group of size N , thus N_c , N_{ic} and N_{id} the numbers of cooperators, insured cooperators and insured defectors respectively. If the game succeeds ($N_c + N_{ic} \geq T$), each player receives an equal amount of benefit from the resulted public goods, minus her contribution to the common pool and the insurance cost (if she is insured). The net payoffs of the involved roles under these two scenarios are thus severally given by

Scenario 1:

$$\begin{cases} P_c = \frac{rc(N-N_d)}{N} - c \\ P_d = \frac{rc(N-N_d)}{N} \\ P_{ic} = \frac{rc(N-N_d)}{N} - c - \lambda \end{cases}, \quad (1)$$

Scenario 2:

$$\begin{cases} P_c = \frac{rc(N-N_d-N_{id})}{N} - c \\ P_d = \frac{rc(N-N_d-N_{id})}{N} \\ P_{ic} = \frac{rc(N-N_d-N_{id})}{N} - c - \lambda \\ P_{id} = \frac{rc(N-N_d-N_{id})}{N} - \lambda \end{cases}. \quad (2)$$

Here, the collected sum is enhanced by a factor r ($r > 1$), and is then redistributed to all the participants, irrespective of their strategies. And, the assured player will join the insurance guarantee by paying a cost of λ .

If the number of contributors is less than the threshold, i.e. $N_c + N_{ic} < T$, the contributors lose their contributions and the game fails. Thus, the net payoffs of the strategies under these two scenarios respectively read as

Scenario 1:

$$\begin{cases} P_c = -c \\ P_d = 0 \\ P_{ic} = \varepsilon - c - \lambda \end{cases}, \quad (3)$$

Scenario 2:

$$\begin{cases} P_c = -c \\ P_d = 0 \\ P_{ic} = \varepsilon - c - \lambda \\ P_{id} = \varepsilon - \lambda \end{cases} \quad (4)$$

As previously mentioned, the insured individual will be compensated with ε ($\varepsilon > 0$), provided by the insurance system if the game fails. The presence of the compensation denoted by parameter ε may act as a focal point for cooperation and thus is worthy of our investigation.

3. General analysis

3.1. Results in Scenario 1

We then combine game theory and population dynamics in a replicator equation. We first perform the approximate calculations about the evolutionary results in well-mixed and infinite populations, a fraction f_d of which is composed of defectors. From time to time, $k + 1$ agents are randomly chosen from this mixed and large population according to the Binomial probability function. Notably, the probability that two players in large population ever meet again can be neglected.

We hence assume that each game is played by $k + 1$ participants, to maintain consistency with simulations in RR networks in the following section (where k is the degree). In addition, the roles of cooperator clusters in spatial games are ignored in theoretical calculation here.

In such groups, the probability that there are m contributors among the k other agents in the sample population in which a given player (may acting as the role of C , D or IC) finds herself, is determined by

$$\binom{k}{m} f_d^{k-m} (1 - f_d)^m. \quad (5)$$

This probability is independent of whether the agent is a contributor or a defector.

As an additional simplification but without loss of generality, the cost c of contribution is set to 1. By introducing $\eta = r/(k + 1)$, the expected payoff for a defector in such a population is

$$P_d = \sum_{m=T}^k \eta m \binom{k}{m} f_d^{k-m} (1 - f_d)^m. \quad (6)$$

The payoff of a cooperator is given by

$$P_c = \sum_{m=T-1}^k [\eta(m + 1) - 1] \binom{k}{m} f_d^{k-m} (1 - f_d)^m + \sum_{m=0}^{T-2} (-1) \binom{k}{m} f_d^{k-m} (1 - f_d)^m. \quad (7)$$

The payoff of an insured cooperator will thus be

$$P_{ic} = \sum_{m=T-1}^k [\eta(m + 1) - 1 - \lambda] \binom{k}{m} f_d^{k-m} (1 - f_d)^m + \sum_{m=0}^{T-2} (\varepsilon - 1 - \lambda) \binom{k}{m} f_d^{k-m} (1 - f_d)^m. \quad (8)$$

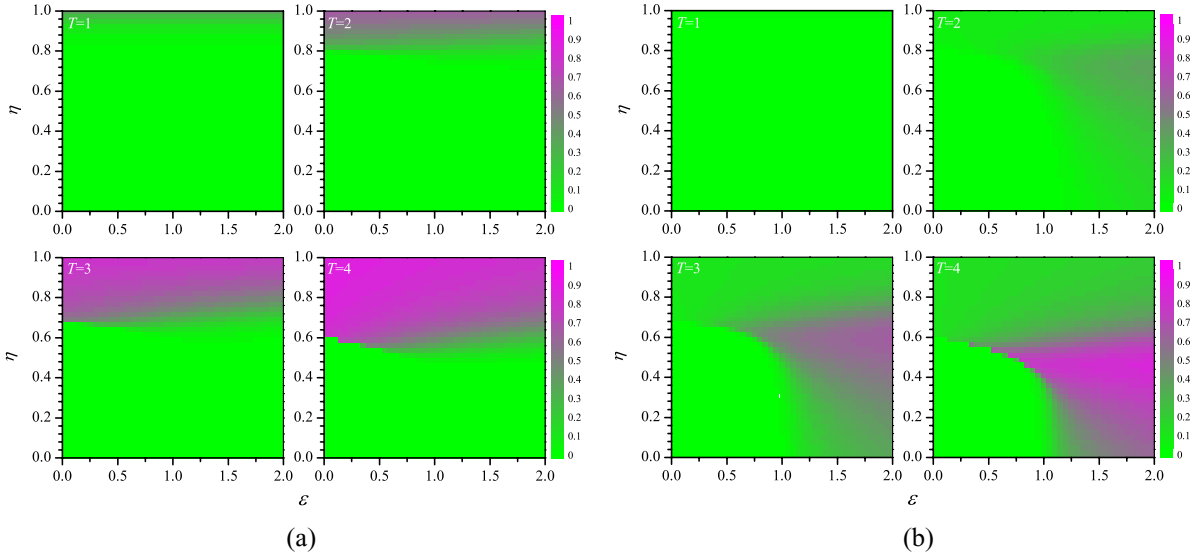


Figure 1. Scenario 1: Final fractions of uninsured cooperators and insured cooperators by theoretical analysis in well-mixed populations. (a) Scenario 1: Fraction of uninsured cooperators in the well-mixed population by theoretical predictions. (b) Scenario 1: Fraction of insured cooperators in the well-mixed population by theoretical predictions.

Evolutionary game theory assumes that a strategy's payoff determines the growth rate of its fraction in the population. In the continuous time model, the evolution of the frequency f_g of the strategy g is given by the reduced differential equation

$$\dot{f}_g = f_g(P_g - \bar{P}), \quad (9)$$

where $g \in (C, D, IC)$, and $\bar{P} = f_c P_c + f_d P_d + f_{ic} P_{ic}$. We can obtain the evolutionary results as summarized in figure 1, where both the outcomes of uninsured cooperators and insured cooperators are given. Results suggest that insurance for cooperators when collective dilemma games fail can induce a recovery of contributions, and the larger payoff yielded by the insured cooperation option the more obvious this effect is. Results also show that cooperators and insured cooperators can prevail over defectors for suitable values of T .

3.2. Results in Scenario 2

Let us also assume a sufficiently large population, at the mean field level, groups of $k + 1$ individuals are formed randomly according to the binomial sampling.

For a given player (playing the strategy of C , D , IS or ID), the probability of finding, among the k other players in the sample, m contributors (including C and IC), is given by

$$\binom{k}{m} (f_d + f_{id})^{k-m} (1 - f_d - f_{id})^m, \quad (10)$$

where f_d and f_{id} denotes the fraction of defectors and insured defectors in the population.

Therefore, the expected payoff for a defector in this population in Scenario 2 is specified as

$$P_d = \sum_{m=T}^k \eta m \binom{k}{m} (f_d + f_{id})^{k-m} (1 - f_d - f_{id})^m. \quad (11)$$

In the same manner, the cooperator receives the following expected payoffs

$$P_c = \sum_{m=T-1}^k [\eta(m+1) - 1] \binom{k}{m} (f_d + f_{id})^{k-m} (1 - f_d - f_{id})^m + \sum_{m=0}^{T-2} (-1) \binom{k}{m} (f_d + f_{id})^{k-m} (1 - f_d - f_{id})^m. \quad (12)$$

The payoff of an insured cooperator will thus be

$$P_{ic} = \sum_{m=T-1}^k [\eta(m+1) - 1 - \lambda] \binom{k}{m} (f_d + f_{id})^{k-m} (1 - f_d - f_{id})^m + \sum_{m=0}^{T-2} (\varepsilon - 1 - \lambda) \binom{k}{m} (f_d + f_{id})^{k-m} (1 - f_d - f_{id})^m. \quad (13)$$

The payoff of an insured defector will be

$$P_{id} = \sum_{m=T}^k (\eta m - \lambda) \binom{k}{m} (f_d + f_{id})^{k-m} (1 - f_d - f_{id})^m + \sum_{m=0}^{T-1} (\varepsilon - \lambda) \binom{k}{m} (f_d + f_{id})^{k-m} (1 - f_d - f_{id})^m. \quad (14)$$

According to the replicator equation

$$\dot{f}_g = f_g (P_g - \bar{P}), \quad (15)$$

where g can be strategy C, D, IC, ID , and $\bar{P} = f_c P_c + f_d P_d + f_{ic} P_{ic} + f_{id} P_{id}$.

The competition results among the four strategies are summarized in figure 2. We observe the existence of decreases of contributive behaviors in this case, especially when insured defectors could gain profitable benefits from insurance. We have also checked that the same behavior arises for a very wide range of values of parameters ε and η , and hence in this sense the relation between evolution dynamics and insurance rules is robust.

4. Evolutionary outcomes in structured populations

Our analytical results in well-mixed and infinite players do not directly apply to an agent population using a heterogeneous network as its iteration topology. To supplement the analytical approximation results above, we then show the observations through simulations under more realistic assumptions, summarized by figures 3–6 respectively.

We consider a random regular (RR) network as the simplest of networks to situate the involved population [39, 40], as well as a Barabási–Albert scale-free (BASF) network, which is likely a more apt model for realistic social networks [41–43]. A RR network is a

The evolution of altruism in spatial threshold public goods games via an insurance mechanism

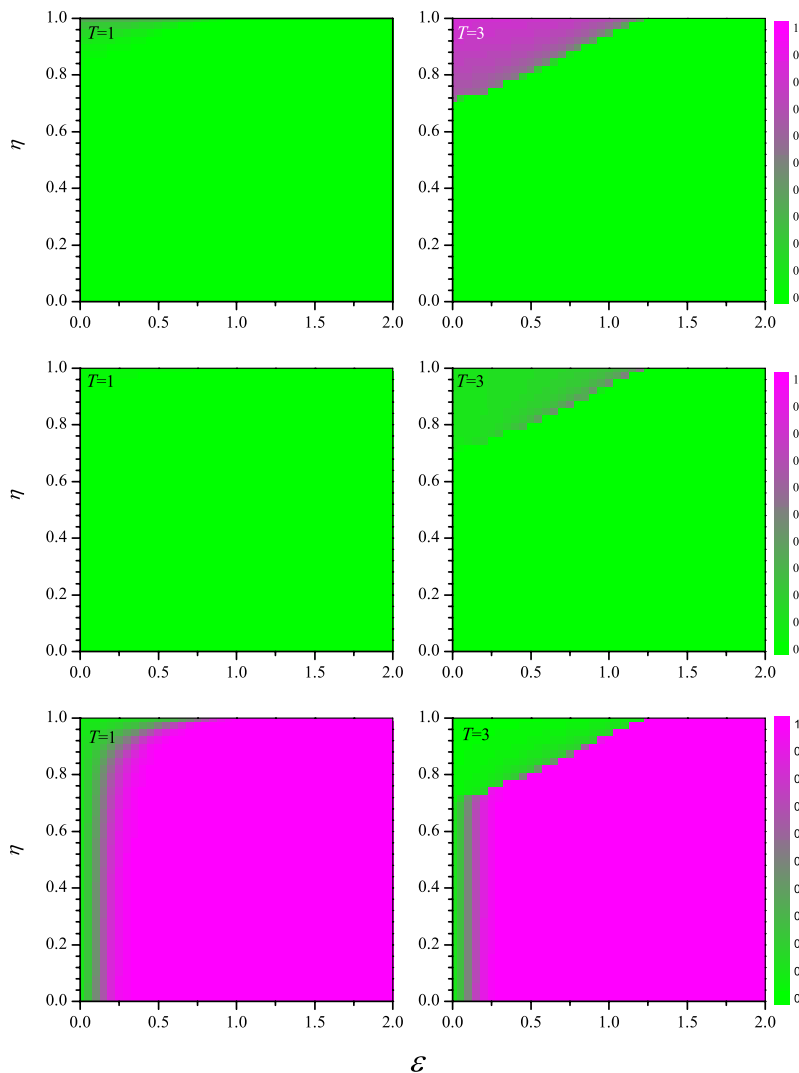


Figure 2. Scenario 2: The theoretical predication about the evolutionary outcomes in unstructured populations under Scenario 2. Upper panel: the fraction of cooperators; Middle panel: the fraction of insured cooperators; Lower panel: the average fraction of insured defectors.

network whose links are randomly generated but where every node has the same degree k (i.e. the same number of neighbors). The average degree is held constant ($k = 4$) to remove the effects of average degrees on contribution levels. We moreover assume that every vertex is occupied, and by one agent only. Two individuals can interact only if they are connected by an edge of the graph, whereby self-interactions, duplicate-interactions and isolated individuals are excluded.

Additionally, diversity associated with the number and the size of the PGG has been verified to promote strong cooperation in selfish populations [20]. Inspired by this striking idea, here we still focus on the consideration that one player joins in multiple games organized by herself and her close neighbors. Specifically, a given player x acts as an organizer of the common pool x with size $k_x + 1$, where there occurs the TPGG involving

x herself and her k_x neighboring players. Besides the TPGG organized by herself, player x also engages in other k_x TPGGs organized by her neighbors.

As game proceeds, if contributing, player x will equally contribute a fixed endowment c to each TPGG that she engages in. Then the resulting goods of each TPGG is equally divided to all participants in the group if the game succeeds. In this case, the payoff P_{xy} of player x associated with the neighborhood centered at neighboring agent y can be expressed as

$$P_{xy} = \frac{r \sum_{i=0}^{k_y} c \xi_i}{k_y + 1} - C_x. \quad (16)$$

As a convenience, c is set to 1. Here, $i = 0$ stands for y , and k_y is the degree of player y . Moreover, $\xi_i = 1$ if the adopted strategy is C or IC , and $\xi_i = 0$ if the chosen strategy is D or ID . C_x is the cost paid by player x . Particularly, $C_x = 1$ if x is a cooperator, $C_x = 1 + \lambda$ if x is an insured cooperator, $C_x = 0$ if x is a defector and $C_x = \lambda$ if x is an insured defector. And, we also employ $\eta = r/(k + 1)$ as a renormalized enhancement factor on the public goods such that $\eta = r/(k + 1)$ values in $(0, 1)$.

However if the game fails, the corresponding payoff P_{xy} of player x associated with the neighborhood centered at agent y is given by

$$P_{xy} = 0 - C_x, \quad (17)$$

where C_x is the cost associated with the strategy of the player x as mentioned.

Irrespective the success or failure of the game, the accumulated payoff P_x of player x is the sum of gains from all interactions in which her participates,

$$P_x = \sum_{y \in \Omega_x} P_{xy}, \quad (18)$$

where Ω_x denotes the community of player x 's connecting neighbors plus itself.

More notably, the initial strategies of the participating players are equivalently distributed on the adopted networks. As is typical for agent-based simulations of spatial games, an individual shows one behavioral strategy at a given time, which is experienced by all of her gaming partners. This determines her overall payoff which is compared with that the neighboring individuals gain at the time, when these are chosen to play with their neighbors.

Then, each player executes the potential strategy updating after playing games with their neighbors. The following dynamics serves to allow each player to evolve and choose a more successful character as the game proceeds, either its previous strategy or a different one of neighboring agents. In particular, evolution of the two strategies is performed according to a pairwise comparison rule, and the Fermi function [44] is adopted here. We restrict attention to that all players make strategy updating simultaneously, and the gained conclusions remain effective for asynchronous updating, as proved by additional simulations. First the payoffs of the focal player x and one of her randomly chosen neighbor y are calculated, according to their strategies and that of their gaming partners. Next, the probability that player x will adopt the strategy s_y of her neighbor y depends on the payoffs P_x and P_y of both players in the light of

$$q(s_x \leftarrow s_y) = \frac{1}{1 + e^{\beta(P_x - P_y)}}, \quad (19)$$

where the magnitude of β characterizes the uncertainty related to the strategy update. For finite positive values of β , strategies performing worse may also be adopted based on unpredictable variations in payoffs or errors in the decision making. To focus on the key issue at hand, we set β to 1 in the main body of the paper, but this assumption can be relaxed. Moreover, we have verified that the main conclusions remain qualitatively unaffected for other β values (e.g. $\beta = 0.01, 0.1, 10$).

We maintain and evolve a population composed of 10^3 players distributed on the nodes of the adopted networks. Thus, the following results are simulated on the two representative network structures with the same average degree $k = 4$. Initially, all the strategies are randomly distributed among the population. We conduct a systematic analysis of the model's parameter space. Each simulation consists of 10^4 time steps, and we report the average results per time step during the last 10^3 time steps. And, each datum is an ensemble average over 100 independent realizations of both the networks and the initial conditions, to reduce the variability of our statistics to an acceptable level.

4.1. Results in Scenario 1

First we show the results of average fractions of strategies distributed on the BASF networks when the system evolves to a steady state, depends on the combination of game parameters in our system. Larger values of T will lead to a smaller amount of cooperators in the population, for example results for $T = 4$ in figure 3. The large threshold T means a hard condition for the TPGG to be successfully achieved, where the successful achievement of TPGG requires a large number of contributors in the gaming group. In this case, the cooperative agents are more frequently defeated by other agents who have higher payoffs. However, it is not the case for the insured cooperators. Especially when the potential compensation ε from insurance is not constrained and rises, $\varepsilon > 1 + \lambda$ will promote a larger number of insured cooperators to occur. In these conditions the uninsured cooperators and defectors will both be defeated by an overwhelming majority or even dominance of insured cooperators, as shown in figure 3(b). This is not surprising given that higher values of ε mean that there is a relatively stronger payoff advantage of insured cooperation over free-riding strategy when the TPGG fails. In this case, it is also informative to note that varying the compensation ε has a weaker and unobtrusive impact on the cooperation levels in the gaming system.

Therefore, for the larger threshold T , the amounts of cooperators and defectors are likely to decrease rather than thrive as insured cooperators do, due to the increased risk of encountering failing TPGGs. A higher threshold T and larger compensation ε will encourage insured cooperators to survive and spread. In this case, insurance for contributors will eliminate the risk of losing one's all contributions when the participating TPGG fails. The insurance tends to eliminate the fear of needlessly losing all the contributions, inspiring others with an incentive to contribute with insurance. Reducing the threshold T , which provides a relaxed condition for the success achievement of TPGG, can better promote the survival of cooperators in the gaming population. By comparing figures 3(a) and (b), we can see that the majority of population are contributors (i.e. cooperators and insured cooperators in this study) and defectors are in a decided minority, if the enhancement factor η increases and exceeds some critical values. Concretely, the effect of renormalized enhancement rate η is also clearly depicted by figure 3. The higher

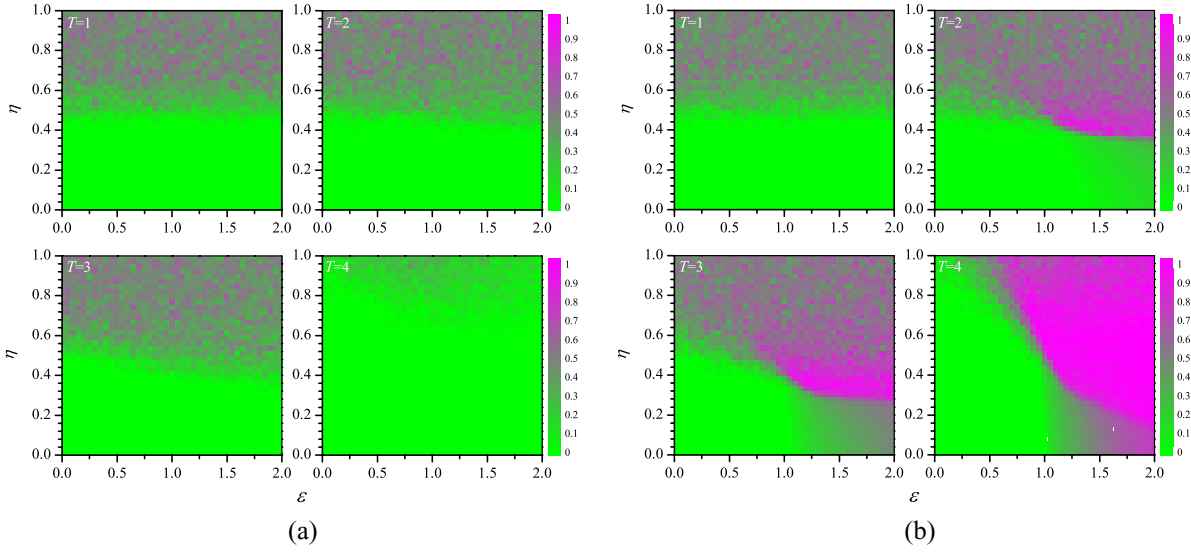


Figure 3. Scenario 1: the population consists of cooperators, defectors and insured cooperators. Evolutionary outcomes affected by the threshold T , compensation value ε from insurance and the renormalized multiplication rate η as indicated. T is the minimum number of contributors in one TPGG, below which the game will fail. Results were obtained by setting $N = 10^3$, $\lambda = 0.1$, average degree $k = 4$ and $\eta = r/(k + 1)$. Larger T can better facilitate the coexistence of the contributors (cooperators and insured cooperators), while higher T and larger ε (e.g. see lower panel in figure 3(b)) will promote the spreading and prosperity of insured cooperation strategy. (a) Scenario 1: Fraction of cooperators in the population embedded on Barabási–Albert scale-free (BASF) networks. (b) Scenario 1: Fraction of insured cooperators in the population embedded on Barabási–Albert scale-free (BASF) networks.

the value of the public goods multiplier, the increasing probability that more cooperators gain chances for survival in this situation. The cooperation levels drop significantly with smaller values of η denoting harder cooperation conditions.

Then we shift our attention to check the generality of the results if the network topology is changed to the RR networks, as summarized in figure 4. Compared with the RR network, the case of the BASF network (figure 3) leads to a larger region of uninsured cooperative behaviors in the gaming population. When being embedded on heterogeneous networks, cooperators will be likely to form clusters between themselves more than defectors, as the latter tend to follow cooperators instead of clustering between themselves. Meanwhile, defectors at the border of a cooperator cluster will probably imitating the strategy of cooperators, thus extending the cooperator clusters. Different with cases of uninsured cooperators, a wider range of ε and T is suitable for enhancing the insured cooperators on RR networks than that on BASF networks (see figure 4(b)). Thus, the general conclusion one can draw from these plots is that the topological property is also a key point which affects the outcome of the competing strategies here.

Besides, some common features demonstrated by the figures 3–4 are: (1) cooperators go extinct in smaller η , grow and maintain a certain level for larger η . It can be seen that increasing the value of η leads to an expansion of the region where cooperation is

The evolution of altruism in spatial threshold public goods games via an insurance mechanism

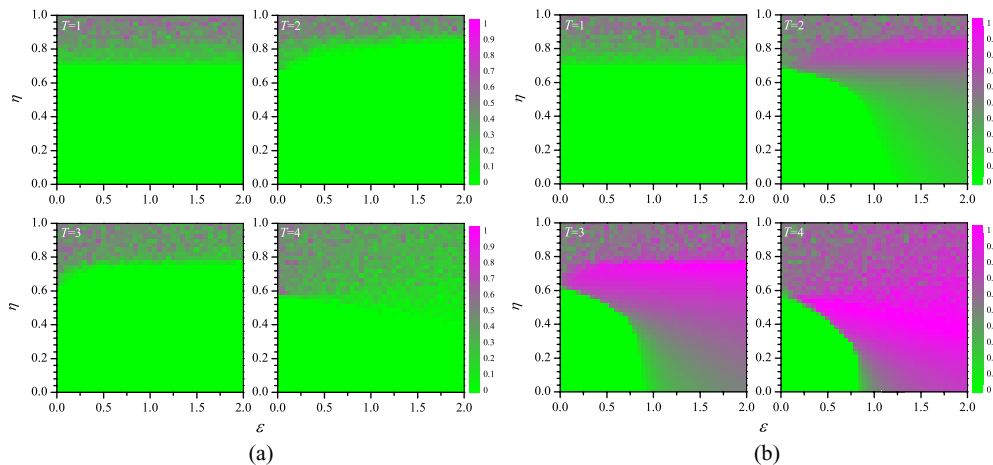


Figure 4. Scenario 1: Evolutionary outcomes affected by the threshold T , the renormalized multiplication rate η and compensation ε from insurance. Results were obtained by setting $N = 10^3$, $\lambda = 0.1$, average degree $k = 4$ and $\eta = r/(k + 1)$. Compared with figure 3, the heterogeneity of interactions, marked by the BASF networks employed in this study, fosters the survival and maintenance of uninsured cooperators in the TPGG, however, it is not the case for insured cooperators. Strategy of insured cooperation can be better promoted in failing games, resulted by the homogeneity of networks, or larger threshold T in the model setting. Compared with figure 1, the gained results suggest the theoretical validity of the results affected by the insurance mechanism in computer simulations. (a) Scenario 1: Fraction of cooperators in the population situated on Random Regular (RR) networks. (b) Scenario 1: Fraction of insured cooperators in the population situated on Random Regular (RR) networks.

observed. This is not surprising given that higher values of η denotes easier situations for cooperative behaviors to emerge. (2) the variations in ε significantly influence the evolutionary outcomes of insured cooperators and, to a smaller extent, the uninsured cooperation levels. (3) the insured cooperation is best enhanced with large threshold T and high compensation ε in the TPGG.

The design of evolution dynamics on RR networks allows for approximately testing the theoretical predictions above. Comparison of simulation results (figure 4) to the theoretical predictions (figure 1), indicates that the analytical approximations are in agreement with simulation based results in RR networks. It is worthy noting that the discrepancy brought about by these two methods indicate that our theoretical predication does not fully consider the influences of spatial structures and limited local interactions, especially the possible appearance of clusters formed by cooperators. By virtue of two methods, we can safely draw a conclusion that the rise of cooperation in our model hinges on the existence of collective dilemma risk and the insurance choice provided for players.

4.2. Results in Scenario 2

Further, we extend our three-strategy profile by adding a new strategy named as insured defection, as mentioned in the Model description. It may be plausible and irrecusable

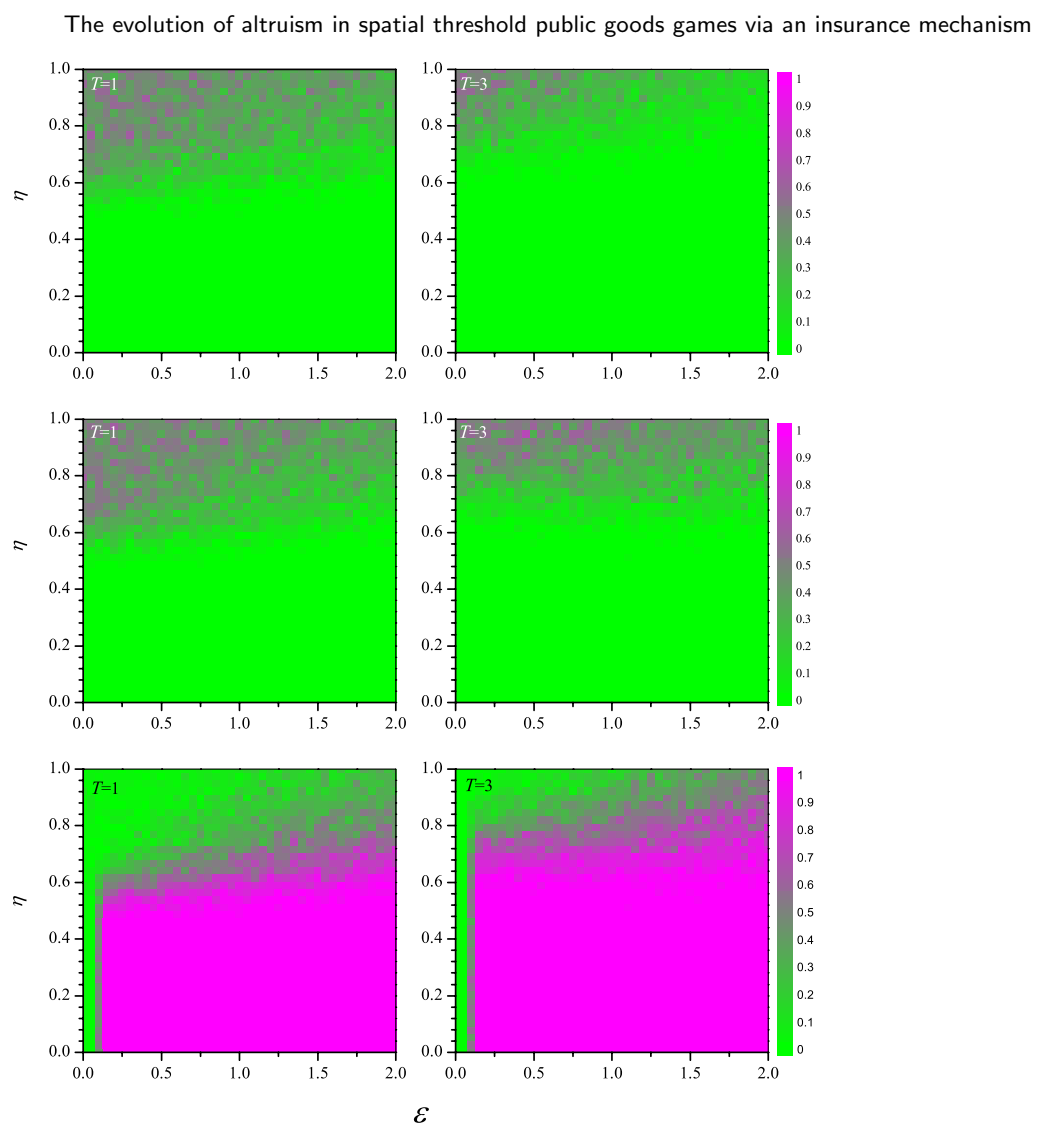


Figure 5. Scenario 2: the gaming population consists of cooperators, defectors, insured cooperators and insured defectors situating on BASF networks. Evolutionary outcomes affected by the threshold T , the renormalized multiplication coefficient η and compensation ε from insurance. Parameters employed here are: $N = 10^3$, $\lambda = 0.1$, average degree $k = 4$ and $\eta = r/(k + 1)$. Upper panel: the average fraction of cooperators; Middle panel: the average fraction of insured cooperators; Lower panel: the average fraction of insured defectors. Results indicate a strong dominance of insured defectors in the gaming population.

that free riders also want to join the insurance and get compensation when the TPGG fails to distribute public goods to the group members. We analyze this issue numerically in figures 5 and 6, which display a fine description of the dynamic outcomes when the population situate on the BASF network and the RR network respectively.

We can immediately see that this four strategy profile triggers the domination of insured defectors for a very wide range of values ε and T , when ε exceeds some critical values. A particular result of dynamics is the small region of x -axis ($\varepsilon < \lambda$ and $\lambda = 0.1$ in

The evolution of altruism in spatial threshold public goods games via an insurance mechanism

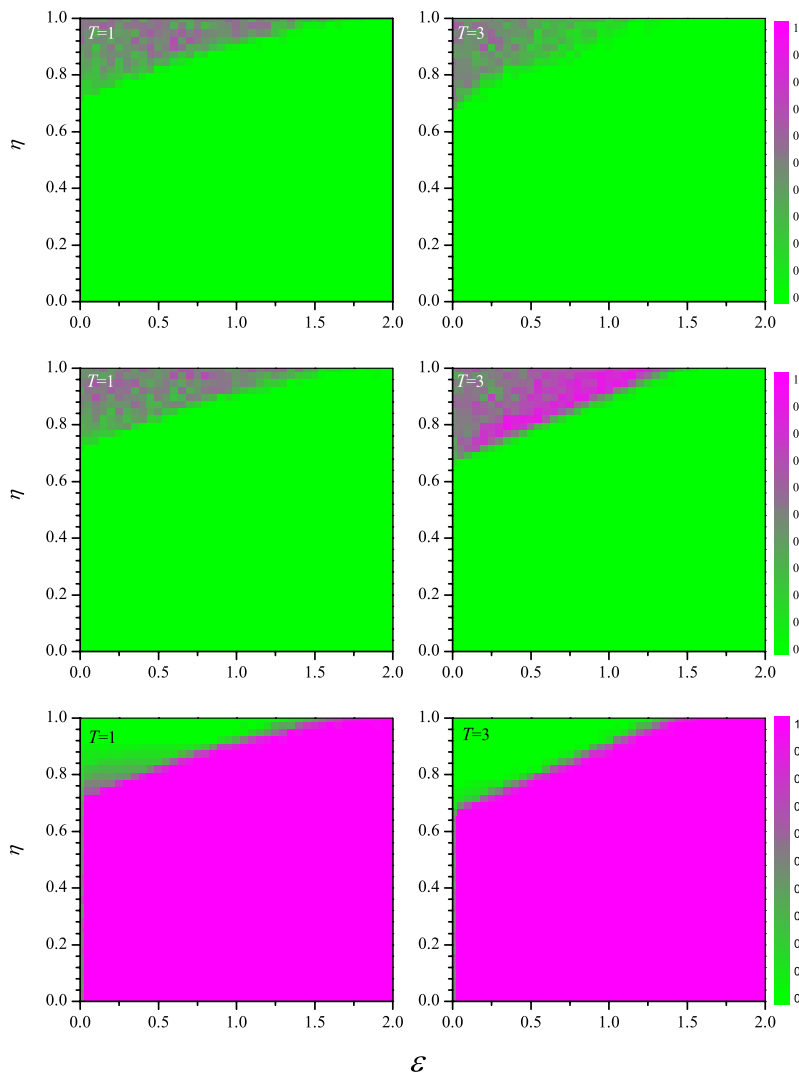


Figure 6. Scenario 2: Evolutionary outcomes affected by the threshold T , the renormalized multiplication rate η and compensation ε from insurance, where the individual interactions are characterized by RR network. Parameters employed here are: $N = 10^3$, $\lambda = 0.1$, average degree $k = 4$ and $\eta = r/(k + 1)$. Upper panel: the average fraction of cooperators; Middle panel: the average fraction of insured cooperators; Lower panel: the average fraction of insured defectors. The homogeneity of interactions (as characterized by the RR network here) intensifies the dominance of insured defectors in the population. The main conclusions in well-mixed populations (gained in figure 2) are qualitatively consistent with that observed by simulations in RR networks.

figure 5) among which insured defectors situating on the RR networks can reach a majority within the population. When the compensation ε from insurance is smaller than the cost λ , insured defectors will get unprofitable benefits from insurance. However, the payoff advantage over other strategies still help them to gain the dominance of the population.

Results indicate that, the insurance for all players is not sufficient to counteract the general decay of the altruistic behaviors (including uninsured cooperation and insured

cooperation) in the TPGG. This phenomenon is even more notable when the population situate on the RR networks. In this respect, the observed qualitatively explosion of insured defectors is rooted in the fact that they prevail over others with relatively higher payoffs, where cooperators and insured cooperators can only struggle to survive by the aid of large η and low ε . Therefore, it considerably hinders the contributive behaviors in the TPGG, when defectors can choose to be insured and even gain profitable compensation demanded for the public goods loss. For larger value $\varepsilon - \lambda$, there is a higher probability that more selfish agents imitate the insured defectors and overpower the spreading of cooperation. When $\varepsilon > \lambda$, players probably quit contributing for the collective goods games as soon as the insured defection is more attractive, instead of contributing with insurance as players under Scenario 1 usually do. In this case, the insurance mechanism significantly influences the final dynamic outcomes in the steady state as evidenced in the two scenarios.

As depicted by analytical predictions in the well-mixed situations (see figure 2), there appear to be a clear upward trend in free-riding behaviors when defectors can also be insured. Figure 6 led by simulations in RR networks provides visual support for this conclusion, which details a dramatic decreases of contributive behaviors when the strategy of insured defection is added, especially when they could gain profitable benefits from insurance. The payoff-maximization incentive will foster the spreading and almost domination of this strategy, thus inhibiting the successful public goods provision and contribution levels. In brief, the presented theoretical and simulation results convey persuasively that the compensation from insurance is of crucial importance for the survival and persistence of cooperators in the competing population.

5. Conclusion

This paper proposes and studies an insurance mechanism for players, where cooperative dilemma is modeled as a threshold public goods game. We focus on our analysis on the threshold public goods game with the provision point defined by the minimum numbers of contributors in the game. With regard to the population structure, we explore the individual-based model with networked populations, by employing the Random Regular (RR) networks and the Barabási–Albert scale-free (BASF) networks here from comparison. Specifically, two kinds of scenarios about insurance are also considered and investigated.

First, three candidate strategies are provided for the players: cooperation, defection, and insured cooperation. Our results afford evidences that the insurance for cooperators, not for all players, in threshold public goods games has positive effect on fostering players' contributions. In our view the main possible implication is that, when players choose the insured cooperation strategy, it can ensure them a certain compensation and thus protect the potential contributive incentives. It is worthy reminding that larger compensation ε , and higher threshold T (suggesting that public goods game can hardly expect to succeed), can help generate sustained insured cooperation levels. The referred parameters can trigger a revival of contributive behaviors in the hard cooperation situations by the aid of insurance. At this point, we argue that the insurance for cooperators is a potent and crucial factor that able to boost the willingness to contribute, thus representing a viable escape hatch out of collective goods stalemate.

Second, considering the large range covered by the insurance where all individuals can coequally opt to be insured, we further extend the three-strategy profile by adding the fourth candidate strategy: insured defection. The dramatic decreases of contributive behaviors have been observed in this case, especially when insured defectors could gain profitable benefits from insurance. Some important implications from our results are how to design the insurance policy, and the selection of insurance objects play an important role in affecting the contributive behaviors.

Finally, uninsured cooperation can be promoted by the heterogeneity of agent connections when populations are embedded on the BASF network, while, the emergence and maintenance of insured cooperators can be better safeguarded by the homogeneity of RR network. Employing different networks thus provides us some hints on the relation between insurance and contributive behaviors.

In sum we provide a simple mechanism which explains how insurance could affect altruistic behaviors in public goods games, and many interesting insights could be gained from it. Future research should further explore the role insurance function plays in the evolution of strategies.

Acknowledgments

We appreciate the anonymous referees for their thoughtful and thorough review, and the constructive comments and suggestions which helped us improve our manuscript. Financial support from the Dutch Organization for Scientific Research (NWO) (82301006) and the European Research Council (ERC-StG-307207) is gratefully acknowledged.

References

- [1] Hamilton W D 1963 The evolution of altruistic behavior *Am. Nat.* **97** 354–6
- [2] Boyd R and Lorberbaum J P 1987 No pure strategy is evolutionarily stable in the repeated prisoner's dilemma game *Nature* **327** 58–9
- [3] Burtsev M and Turchin P 2006 Evolution of cooperative strategies from first principles *Nature* **440** 1041–4
- [4] Colman A M 2006 The puzzle of cooperation *Nature* **440** 744–5
- [5] Henrich J 2006 Cooperation, punishment, and the evolution of human institutions *Science* **312** 60–1
- [6] Sinatra R, Iranzo J, Gómez-Gardeñes J, Floría L M, Latora V and Moreno Y 2009 The ultimatum game in complex networks *J. Stat. Mech.* P09012
- [7] Hardin G 1968 The tragedy of the commons *Science* **162** 1243–8
- [8] Hauert C, Monte S D, Hofbauer J and Sigmund K 2002 Volunteering as Red Queen mechanism for cooperation in public goods game *Science* **296** 1129–32
- [9] Kümmerli R, Burton-Chellew M N, Gillespie A R and West S A 2010 Resistance to extreme strategies, rather than prosocial preferences, can explain human cooperation in public goods games *Proc. Natl Acad. Sci. USA* **107** 10125–30
- [10] Kurokawa S and Ihara Y 2009 Emergence of cooperation in public goods game *Proc. R. Soc. B* **276** 1379–84
- [11] Rankin D J, Bargum K and Kokko H 2007 The tragedy of the commons in evolutionary biology *Trends Ecol. Evol.* **22** 643–51
- [12] Nowak M A 2006 Five rules for the evolution of cooperation *Science* **314** 1560–3
- [13] Pinheiro F, Vasconcelos V V, Santos F C and Pacheco J M 2014 Evolution of all-or-none strategies in repeated public goods dilemmas *PLoS Comput. Biol.* **10** e1003945
- [14] Szabó G and Fáth G 2007 Evolutionary games on graphs *Phys. Rep.* **446** 97–216
- [15] Pinheiro F L, Pacheco J M and Santos F C 2012 From local to global dilemmas in social networks *PLoS One* **7** e32114

- [16] Perc M and Szolnoki A 2010 Coevolutionary gamesa mini review *BioSystems* **99** 109–25
- [17] O’Gorman R, Henrich J and Vugt M V 2009 Constraining free riding in public goods games: designated solitary punishers can sustain human cooperation *Proc. R. Soc. B* **276** 323–9
- [18] Brandt H, Hauert C and Sigmund K 2003 Punishment and reputation in spatial public goods games *Proc. R. Soc. B* **270** 1099–104
- [19] Fehr E and Gächter S 2000 Cooperation and punishment in public goods experiments *Am. Econ. Rev.* **90** 980–94
- [20] Santos F C, Santos M D and Pacheco J M 2008 Social diversity promotes the emergence of cooperation in public goods games *Nature* **454** 213–6
- [21] Perc M and Szolnoki A 2008 Social diversity and promotion of cooperation in the spatial prisoner’s dilemma game *Phys. Rev. E* **77** 011904
- [22] Santos F C, Pinheiro F L, Lenaerts T and Pacheco J M 2012 The role of diversity in the evolution of cooperation *J. Theor. Biol.* **299** 88–96
- [23] My K B and Chavignac B 2010 Voluntary participation and cooperation in a collective-good game *J. Econ. Psychol.* **31** 705–18
- [24] Janssen M A and Goldstone R L 2006 Dynamic-persistence of cooperation in public good games when group size dynamic *J. Theor. Biol.* **243** 134–42
- [25] Janssen M A and Ahn T K 2006 Learning, signaling, and social preferences in public-good games *Ecol. Soc.* **11** 21
- [26] Krupp D B, Debruine L M and Barclay P 2008 A cue of kinship promotes cooperation for the public good *Evol. Hum. Behav.* **29** 49–55
- [27] Rand D G, Dreber A, Ellingsen T, Fudenberg D and Nowak M A 2009 Positive interactions promote public cooperation *Science* **325** 1272–5
- [28] Cadsby C B and Maynes E 1998 Gender and free riding in a threshold public goods game: experimental evidence *J. Econ. Behav. Org.* **34** 603–20
- [29] Cadsby C B, Hamaguchi Y, Kawagoe T, Maynes E and Song F 2007 Cross-national gender differences in behavior in a threshold public goods game: Japan versus Canada *J. Econ. Psychol.* **28** 242–60
- [30] Cadsby C B, Croson R, Marks M and Maynes E 2008 Step return versus net reward in the voluntary provision of a threshold public good: an adversarial collaboration *Public Choice* **135** 277–89
- [31] Croson R T A and Marks M B 2000 Step returns in threshold public goods: a meta-and experimental analysis *Exper. Econ.* **2** 239–59
- [32] Croson R and Marks M 1998 Identifiability of individual contributions in a threshold public goods experiment *J. Math. Psychol.* **42** 167–90
- [33] Marks M and Croson R 1998 Alternative rebate rules in the provision of a threshold public good: an experimental investigation *J. Public Econ.* **67** 195–220
- [34] Marks M B and Croson R T A 1999 The effect of incomplete information in a threshold public goods experiment *Public Choice* **99** 103–18
- [35] Santos F C and Pacheco J M 2011 Risk of collective failure provides an escape from the tragedy of the commons *Proc. Natl Acad. Sci. USA* **108** 10421–5
- [36] Santos M D, Pinheiro F L, Santos F C and Pacheco J M 2012 Dynamics of n-person snowdrift games in structured populations *J. Theor. Biol.* **315** 81–6
- [37] Zhang J, Chu T and Weissing F J 2013 Does insurance against punishment undermine cooperation in the evolution of public goods games? *J. Theor. Biol.* **321** 78–82
- [38] Cadsby C B and Maynes E 1999 Voluntary provision of threshold public goods with continuous contributions: experimental evidence *J. Public Econ.* **71** 53–73
- [39] Wormald N C 1999 *Models of Random Regular Graphs London Math. Soc. Lecture Note Ser.* **267** 239–98
- [40] Bollobás B 2001 How mutation affects evolutionary games on graphs *J. Theor. Biol.* **73** 97–105
- [41] Costa L F, Rodrigues F A, Travieso G and Boas P R V 2007 Characterization of complex networks: A survey of measurements *Adv. Phys.* **56** 167–242
- [42] Erdős P and Rényi A 1960 On the evolution of random graphs *Publ. Math. Inst. Hung. Acad. Sci.* **5** 17–61
- [43] Barabási A L and Albert R 1999 Emergence of scaling in random networks *Science* **286** 509
- [44] Traulsen A, Pacheco J M and Nowak M A 2007 Pairwise comparison and selection temperature in evolutionary game dynamics *J. Theor. Biol.* **246** 522–9