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# Regulatory Holidays and Optimal Network Expansion

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# Regulatory Holidays and Optimal Network Expansion\*

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## Abstract

We model the optimal regulation of continuous, irreversible, capacity expansion, in a model in which the regulated network firm has private information about its capacity costs, investments need to be financed out of the firm's cash flows from selling network access and demand is stochastic. If asymmetric information is large, the optimal mechanism consists of a regulatory holiday for low-cost firms, and a mark-up regime for higher-cost firms. With the regulatory holiday, a firm receives the full revenue of capacity sales, and expands capacity as if it were an unregulated monopolist. Under the mark-up regime, a firm receives only a fraction of the capacity revenues, and is obliged to expand capacity whenever the price for capacity reaches a threshold. The regulatory holiday is necessary to fund information rents to the most efficient firms, which invest relatively early, as direct investment subsidies are not feasible.

Keywords: regulatory holiday, real option value, asymmetric information, optimal contracts

**JEL classification:**D81, D82, L51

## 1 Introduction

Since the nineties many regulated network industries switched from cost-plus (or rate-of-return) to incentive regulation, often under some form of price cap regulation. This switch was motivated by the fear that cost-plus regulated firms would “gold-plate” their networks and over-invest in capital (Averch and Johnson, 1962) and the realization that a price cap provides high-powered incentives for cost-efficiency (Cabral and Riordan, 1989).<sup>1</sup> However, in recent years stakeholders have argued that with high powered incentive regulation, firms postpone socially efficient investments in durable assets, especially in risky environments, and that a different form of regulation is necessary. For instance, in response to large investment needs, the UK electricity and gas regulator OFGEM modernized its price cap mechanism by explicitly taking into account these investment needs.<sup>2</sup> European energy directives allow specific network investments to be exempted from regulation in order to foster investments if uncertainty is large in a regime which is often called a “regulatory holiday”.<sup>3</sup> For the

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<sup>1</sup>See for instance Sappington (2002) for an overview of the perceived drawbacks of rate of return regulation.

<sup>2</sup>Based on the work of Littlechild, the UK was one of the first countries to introduce the RPI-X price cap model (Beesley and Littlechild, 1989). In 2010, after a revision of the existing pricing regulation (the RPI-X@20 review), a new set of regulatory principles was introduced with the RIIO-model (Revenue = Incentives + Innovation + Output), which is still a form of price regulation (where prices are set for a period of 8 years) but which also includes output obligations for the firms, and additional funds for experimentation (OFGEM, 2010b).

<sup>3</sup>European regulation for both electricity and gas market (Regulation (EC) No.714/2009, Article 17 & Directive 2009/73/EC, Article 36) allow granting a regulatory holiday (“exemptions”) for new infrastructure investments under strict conditions. One of those conditions is that “the level of risk attached to the investment is such that the investment would not take place unless an exemption was granted”. The European Commission (2015) received 35 requests regarding exemption, most of which were (partially) granted.

telecom sector, ETNO, the industry association representing European telecom operators, recommends relaxing access regulation, as it sees it as the main reason for European infrastructure investments lagging those in the U.S. (Williamson, Lewin and Wood, 2016).<sup>4</sup> Also academic scholars recognize that implementing price cap regulation is challenging for durable investments and when uncertainty is important (Guthrie, 2006; Armstrong and Sappington, 2007).

In this paper we contribute to this debate by studying the optimal regulation of capacity investments in a dynamic setting in which investment prospects are uncertain. For this we consider a regulated private firm that has to gradually expand its network to cope with a growth in demand for network access, needs to fund its investments from operating profits and has superior information on investments costs. The regulator contracts with the firm about when it should expand capacity (and when it would be better to delay), at which price the capacity should be sold, and which fraction of its revenues it may keep. The regulator acts as a social planner and maximizes the expected discounted sum of consumers' surplus and the firm's profit.

In the optimal regulatory mechanism existing capacity is always used efficiently: prices for network access are equal to the short run marginal cost of transportation as long as there is spare capacity, and prices are above marginal cost, when there is congestion. Capacity is expanded, whenever the price for capacity reaches a threshold value. This price threshold increases with investment costs, and is always higher than under the first best symmetric information optimum with demand uncertainty. Hence, investments are delayed.<sup>5</sup>

As we assume that the firm does not receive subsidies, investment costs need to be paid from market revenues. However, any operating profits that remain after those costs have been paid for, could be taxed by the regulator. Under optimal regulation the regulator does *not* tax the firms which reveal to be relatively efficient, while the tax rate for the less efficient firms increases with their levels of inefficiency. Those low tax rates are necessary to provide the efficient firms with information rents.

If the information asymmetry between the regulator and the firm is large, then the relatively efficient firms will be allowed to invest as if they were unregulated monopolists, as this provides the largest possible information rents. Hence, a regulatory holiday is optimal for those firms. If the information asymmetry is small, then the regulator will bunch the more efficient firms and require identical investment levels for these. Hence, optimal regulation no longer results in an equilibrium with full separation of types.

In our model demand growth is not fully predictable (i.e. stochastic) and network investments are sunk. Hence the firm is continuously forecasting demand and balancing the benefits of expanding capacity now (and obtaining additional revenue) and delaying investments (and obtaining superior information about future demand). In other words, it needs to take into account the real option value of investments (Dixit and Pindyck, 1994). McDonald and Siegel (1986) show that an unregulated monopolist delays investments under uncertainty, and Pindyck (1988) extends this result to a continuous investment model. While also first-best investment involves a delay, under monopoly, this delay is longer. If a regulator would try to correct this situation with only the price cap instrument at its disposal, then the first best outcome cannot be reached (Dobbs, 2004), as one instrument is used for two goals: efficient investments ex-ante and optimal consumption ex-post. Building on Dobbs, but introducing scale economics for capacity expansion – in which case grouping investments across time is cost efficient – Evans and Guthrie (2012) show that the price cap should be lowered and that it might be efficient to allow some demand rationing to increase the size of subsequent expansions. Roques and Savva (2009) extend Dobbs' model to a Cournot duopoly with a price cap. Our paper also starts from Dobbs' model but includes asymmetric information, a self-financing constraint and assumes that the regulator has additional

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<sup>4</sup>The European Commission recognizes the challenges of building the Next Generation Access Networks (NGA), but, in contrast to the energy sector, does not allow regulatory holidays for the telecommunication sector (speech Kroes, 2012). Moreover, the European Commission has successfully appealed the decision of the German government to grant Deutsche Telekom a regulatory holiday for upgrading internet services to the VDSL standard. (Commission v. Germany 2009, case number C-424/07, the European Court of Justice.) Nevertheless, some leaked documents suggest that the German government is still considering softening the regulation on the incumbent to foster additional investments (Stupp, 2016).

<sup>5</sup>Note that the first best investment expansion plan already delays investments to take into account the real option value of network expansion.

instruments to enforce investments. As we assume constant returns to scale in capacity expansion, it is never optimal to group investments or to ration demand, in contrast to Evans and Guthrie (2012).

In order to model the interaction between the regulator and the firm, we rely on the assumption that the firm has superior information about its own investment costs as in the seminal paper by Baron and Myerson (1982). While most of those types of models allow for lump-sum transfers to the firm, we impose a self-financing constraint. In Baron and Myerson, the most efficient firm invests at the efficient level, i.e. there is “no distortion at the top”, and gains information rents by receiving a large lump-sum transfer. In our model, information rents can only be obtained by being more profitable in the market, and hence investment levels need to be distorted away from the efficient level. In fact, we find that if the information asymmetries are large, monopoly level investments are optimal for the most efficient firms. So we find what could be called “maximal distortion at the top”. If information asymmetries are small, we find a bunching equilibrium for the most efficient firms. Our results are similar to Gautier and Mitra (2006). They assume that the regulator can provide lump-sum transfers but is limited by its budget constraint, which is determined exogenously. They find bunching for efficient types and separation for less efficient types. In our model, the transfers that the firm can receive are determined endogenously by its investment decisions and the revenues those investments generate. Moreover, we find a separating equilibrium if information asymmetry is large.

One strand of the literature on incentive regulation and durable investments highlights the lack of commitment by the regulator.<sup>6</sup> If a regulator cannot commit to a price level for a sufficiently long period, it will lower prices once investment has taken place as those prices are ex-post efficient. This will lead to hold-up and lower investments ex-ante. In order to address this commitment problem Gans and King (2004) propose a regulatory holiday in which the regulator commits not to regulate prices for a limited duration, under the implicit assumption that it is easier to commit not to regulate than to commit to a high regulated price. We find that even if the regulator has full commitment power, it might be optimal to provide a regulatory holiday to the most efficient firms, as this provides information rents for firms who want to invest earlier and with larger quantities. Note that the regulatory holiday in our model is not unconditional. If a firm with a regulatory holiday invests too late, then a fraction of its operating profits should be taxed.

Many network industries are characterized by features similar to the ones of our model. They have capital intensive networks with relatively long-lived assets. Examples include the local-loop in telecommunications markets, low voltage distribution and high voltage transmission networks in power markets. Growing demand by network users both in volume and service quality require continuous upgrades and expansions of switches in local central offices, voltage transformers and new communication equipment in power networks.<sup>7</sup> Recent technological changes have put those investments requirements to the forefront. Video on demand and cloud computing creates additional pressure on telecommunication networks, the large scale introduction of renewable energy and decentralized production requires substantial upgrade of power networks. Many network firms have been privatized and governments are not keen on subsidizing investments.

The characterization of the telecom sector by the European association of telecom operators highlights many of our assumptions:

The bulk of the investment required to meet policy objectives for the Digital Single Market will need to come from private investment in Europe’s access networks. This private investment is a continuous and incremental, rather than a one-off, process. Investment decisions are constrained

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<sup>6</sup>Another strand of literature discusses the practical challenges for implementing price cap regulation when long term investments are necessary. Joskow (2008) reports problems with the correct measurement of the cost of capital in the energy sector and the difficulties for setting new prices at the end of a regulatory period. Evans and Guthrie (2005) extend Ramsey pricing to a dynamic setting and shows that the incentive regulation is not feasible if the regulator takes the value the investments at their replacement value.

<sup>7</sup>Instead of a growth of demand volume, a different interpretation of our model would be that demand for quality increases over time. Equivalently, instead of demand growth and infinitely lived assets, we could assume constant demand and network capacity which degrades over time and where the firm builds replacement capacity when necessary. Mathematically this model would be very similar.

by the annual cash flows generated by the businesses.... Market players are better placed to make efficient investment decisions than NRAs [National Regulatory Authorities] or governments. They have far more information on both the incremental costs of deploying new technologies and the incremental revenues which might flow from investing (Williamson, Lewin and Wood 2016, Pg.2-3).

In our model we assume that the regulator not only regulates the firm's revenue (by setting which fraction of operating profit the firm is allowed to keep), but also enforces the required investments levels. Such dual requirements are also found in practice. For instance under the new RIIO regulatory model for the UK energy markets, the regulator not only specifies a certain price level, but also agrees on specific output parameters. If firms are unable to reach those output targets they will loose their operating license (OFGEM, 2010*a*).

In our paper we do not consider one-off (lumpy) investments. Moreover, we consider information asymmetry on a static parameter (costs) and assume that the information asymmetry is not related to the stochastic demand realization, although those features might sometimes be present in practice. We refer to two companion papers for those aspects of regulation: Broer and Zwart (2013) assume that investments are lumpy and only occur once; Arve and Zwart (2014) assume asymmetric information with respect to stochastic parameters instead of static ones, and allow for lump sum transfers to the firm.

In telecommunication markets, access providers are often vertically integrated and compete with the buyers of network access in a downstream retail market. Those access providers might also decide to by-pass the existing access provider, by investing in a competing network. The regulator needs to take those aspects into account when designing access regulation. We neglect those additional considerations in our paper. Cambini and Jiang (2009) provide an excellent survey of this extended field of study.<sup>8</sup>

## 2 Model

We consider a continuous-time, continuous investment model of a principal, the regulator, contracting with a monopolist to make irreversible investments  $dQ(t)$  to expand network capacity  $Q(t)$ , as in Dobbs (2004). Capacity is a continuous variable, and capacity expansions come at a constant marginal cost  $c$ . There is no depreciation of capacity.<sup>9</sup> Initial capacity at  $t = 0$ , the time of contracting, is  $Q(0) = 0$ , i.e. we consider a greenfield project.<sup>10</sup> Marginal cost  $c$  is drawn from a cumulative distribution  $F(c)$  with full support  $[c_L, c_H]$ , and density  $f(c) > 0$ . A sufficient condition that the solution of the first-order conditions actually corresponds to a maximum, is that the density is downward sloping  $f'(c) \leq 0$  on its support  $[c_L, c_H]$ . We will assume this.<sup>11</sup>

At each moment in time, the capacity  $Q(t)$  is sold to users at a price  $p(t)$  (we will drop the  $t$ -dependence of price and other variables in subsequent notation).

The demand for network capacity has constant elasticity,

$$p = A Q^{-\gamma} \tag{1}$$

where  $0 < \gamma < 1$  is the inverse of demand elasticity. The associated flow of gross consumer surplus from using  $Q$  is then  $A Q^{-\gamma} / (1 - \gamma)$ .

The demand shift parameter  $A$  is stochastic, and satisfies a geometrical Brownian motion,

$$dA = \mu A dt + \sigma A dz, \tag{2}$$

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<sup>8</sup>The policy discussion is further complicated by the ‘‘Ladder of Investments’’ idea. Setting low access prices initially might attract entrants, who are then incentivized in a later stages by a higher access price to build their own networks.

<sup>9</sup>It would be a straightforward extension of the model to assume a constant depreciation rate.

<sup>10</sup>Alternatively, we demand that the regulator also needs to remunerate the monopolist for its existing investments in efficient capacity.

<sup>11</sup>Below we will show that our condition will only be relevant for costs above a certain (endogenously determined) threshold level. If local first order conditions are not sufficient, then the optimal contract will need additional ironing as in Guesnerie and Laffont (1984).



where  $\mu$  and  $\sigma > 0$  are the associated drift and volatility parameters. As  $A$  grows over time, demand for capacity will increase, making capacity investment more valuable. We will assume that demand  $A$  and capacity  $Q$  are observable and verifiable by the regulator, but the realization of investment cost  $c$  is private information.

The principal's objective is to maximize expected total welfare which is the difference of gross consumers' surplus and investment costs. The flow of total welfare, given costs  $c$ , equals

$$U(A, Q, t, c) dt = \frac{AQ^{1-\gamma}}{1-\gamma} - c dQ(t), \quad (3)$$

where  $dQ(t) \geq 0$  is the irreversible investment made at time  $t$ . The continuation value of total expected welfare at time  $t$  and capacity  $Q$  equals the expected discounted sum of these flows,

$$W(A, Q, t) = \int_{c_L}^{c_H} \mathbb{E}_{A(t)} \left[ \int_t^\infty e^{-r(\tau-t)} U(\tau, c) d\tau \right] dF(c) \quad (4)$$

where  $\mathbb{E}$  denotes the expectation over  $A(t)$ , and  $r$  the risk-free rate. We assume  $r > \mu$ .

The regulator contracts with the monopolist to achieve optimal investment. A contract specifies an investment rule, that depends on the observable parameters  $A$  (demand size) and  $Q$  (installed capacity), as well as a monetary transfer to the agent that remunerates him. We assume the following budget constraint: total expected remunerations  $T$  cannot exceed the total expected proceeds from the sale of capacity,

$$T \leq \mathbb{E} \left[ \int_{\tau=0}^\infty e^{-r\tau} pQ d\tau \right]. \quad (5)$$

In addition, we impose the participation constraint on the monopolist that total expected profits should be non-negative at the time of writing the contract, for any cost  $c$ :

$$\Pi = T - \mathbb{E} \left[ \int_{\tau=0}^\infty e^{-r\tau} c dQ(\tau) \right] \geq 0. \quad (6)$$

### 3 First-best and monopoly benchmarks

As a benchmark, we first explore the first-best outcome (as analysed in Pindyck, 1988). In the absence of asymmetric information, the principal will design an investment policy that specifies capacity additions as a function of the state variables  $A$ , the current value of the demand shift parameter, and  $Q$ , the current installed capacity: capacity will be expanded as soon as demand reaches some threshold value. We denote by  $\bar{A}(Q)$  the threshold value for  $A$ , given  $Q$ , at which investment occurs.<sup>12</sup>

With  $c$  known, the principal sets the threshold  $\bar{A}(Q)$  to optimize total continuation welfare

$$W(A, Q, t) = \mathbb{E} \left[ \int_t^\infty e^{-r(\tau-t)} U(A, Q, \tau, c) d\tau \right].$$

The standard method of solving for  $W$  is first to note that in the region  $A < \bar{A}(Q)$  where no investment occurs,  $W$  satisfies a Bellman equation (Dixit and Pindyck, 1994)<sup>13</sup>,

$$rW = \frac{AQ^{1-\gamma}}{1-\gamma} + \mu A \frac{\partial W}{\partial A} + \frac{1}{2} \sigma^2 A^2 \frac{\partial^2 W}{\partial A^2}.$$

Imposing the boundary condition that  $W$  vanishes when  $A \rightarrow 0$ , the general solution to this differential equation

<sup>12</sup>Note that since there is no exogenous dependence on time other than through  $A$ , the optimal policy, as well as total welfare, cannot explicitly depend on time  $t$ .

<sup>13</sup>This follows straightforwardly from an application of Ito's rule to  $W(A + dA, Q, t + dt)$ .

takes the form

$$W(A, Q) = \frac{AQ^{1-\gamma}}{(1-\gamma)(r-\mu)} + g(Q)A^\lambda, \quad (7)$$

where  $g(Q)$  is any function of  $Q$  and  $\lambda$  is the positive solution to the fundamental quadratic  $r = \mu\lambda + \frac{1}{2}\sigma^2\lambda(\lambda-1)$ . In this expression, the first term represents the expected present value from using existing capacity  $Q$  (without any future expansions), while the second term is the value of the option to expand capacity beyond its current level if demand rises.

Next, we solve for  $g(Q)$  by imposing the boundary condition at the point of investment  $\bar{A}(Q)$ , that the marginal benefit of increasing  $Q$  should equal the marginal cost of investment,

$$\frac{\partial W}{\partial Q}(\bar{A}(Q), Q) = c.$$

Substituting for  $W$ , we find a condition on the derivative of  $g(Q)$ ,

$$\frac{\partial g(Q)}{\partial Q} = \bar{A}(Q)^{-\lambda} \left( c - \frac{\bar{A}(Q)Q^{-\gamma}}{r-\mu} \right).$$

We impose that as  $Q$  goes to infinity, there is no longer an (option) value to further investment ( $g(Q) \rightarrow 0$ ), to find

$$g(Q) = \int_Q^\infty \bar{A}(q)^{-\lambda} \left( \frac{\bar{A}(q)q^{-\gamma}}{r-\mu} - c \right) dq, \quad (8)$$

which specifies, jointly with equation (7), total welfare  $W$ , given an investment threshold  $\bar{A}(Q)$ . Note that welfare  $W$  is an increasing function of  $g(Q)$ . The optimal investment threshold then follows from point-wise maximization of the integrand, and is given by

$$\bar{A}(Q)Q^{-\gamma} = \frac{\lambda}{\lambda-1}(r-\mu)c \equiv \bar{p}^c, \quad (9)$$

or in other words, investing whenever price  $p$  reaches the level  $\bar{p}^c$ . At  $t = 0$ , when capacity  $Q = 0$ , there will be a one-off investment  $Q_0 = (\frac{A_0}{\bar{p}^c})^{1/\gamma}$  to bring initial price to the threshold price.

The price at which optimal investment occurs exceeds the annualized costs by the factor  $\lambda/(\lambda-1) > 1$ , which itself depends on the parameters of the stochastic process and in particular grows as volatility  $\sigma$  increases. This is a reflection of the well-known option value of delaying investment (McDonald and Siegel, 1986).

As a second benchmark, it will be relevant to consider the investment rule that an unregulated monopolist receiving all revenues from selling capacity would choose. Define the firm's profit flow as

$$\pi(A, Q) dt = pQ dt - c dQ = AQ^{1-\gamma} dt - c dQ, \quad (10)$$

and the associated total expected continuation value of the firm as

$$V(A, Q) = \mathbb{E} \left[ \int_t^\infty e^{-r(\tau-t)} \pi(A, Q) d\tau \right]. \quad (11)$$

The unregulated monopolist will then choose an investment threshold  $\bar{A}(Q)$  that maximizes that expected value. The analysis is similar to the total welfare maximization, with firm value taking the form

$$V(A, Q) = \frac{AQ^{1-\gamma}}{r-\mu} + A^\lambda \int_Q^\infty \bar{A}(q)^{-\lambda} \left( \frac{\bar{A}(q)q^{-\gamma}(1-\gamma)}{r-\mu} - c \right) dq, \quad (12)$$

analogously to expressions (7, 8) for total welfare. We can again use point-wise maximization to find the profit

maximizing investment policy. This is to invest as soon as prices rise to the monopoly price level

$$\bar{p}^m = \frac{\lambda}{\lambda - 1} \frac{(r - \mu)c}{1 - \gamma}.$$

This expression differs from the welfare optimizing price  $\bar{p}^c$  by the  $1 - \gamma$  factor, representing the standard Lerner markup,  $(\bar{p}^m - \bar{p}^c)/\bar{p}^m = \gamma$ .

Finally, it is useful to evaluate total firm value  $V(A, Q)$  under the first-best investment rule, *i.e.* invest whenever price reaches  $\bar{p}^c$ . Substituting the corresponding threshold  $\bar{A}(Q)$  in the firm's value function (12), we find that the firm just breaks even, including the costs of the initial investment to bring capacity from  $Q = 0$  to a level consistent with the threshold price. In other words, with symmetric information on costs, the regulator can ask the firm to invest according to the first-best rule, and remunerate it using the proceeds of the capacity sales, hence satisfying both the budget constraint (5) and the firm's participation constraint (6). We summarize these benchmark results as

**Lemma 1.** *Compared to a welfare optimizing social planner, a monopolist delays investment in capacity. That is, it waits until demand has risen to higher levels before investing. Threshold prices that trigger investment are*

$$\bar{p}^m = \frac{\lambda}{\lambda - 1} \frac{r - \mu}{1 - \gamma} c, \quad (13)$$

$$\bar{p}^c = \frac{\lambda}{\lambda - 1} (r - \mu)c. \quad (14)$$

for the monopolist and the social planner respectively. When a greenfield firm invests at the competitive threshold  $\bar{p}^c$ , total expected revenues from selling capacity at market clearing prices equal total costs,

$$V(\bar{p}^c, Q(\bar{p}^c); \bar{p}^c) - cQ(\bar{p}^c) = 0. \quad (15)$$

## 4 Optimal regulation under adverse selection

In this section we turn our attention to regulation with asymmetric information on the firm's capacity expansion cost  $c$ . We consider the regulator offering the firm a (menu of) contracts for a capacity expansion schedule that may depend on demand (or price) realizations, which are observable and contractable. In return, the regulator offers a transfer fee  $T$  to the firm. The fee has to be financed out of the expected revenues of the capacity sale and therefore has to satisfy budget constraint (5). And the firm should earn a non-negative profit, as reflected by participation constraint (6). The regulator maximizes expected total welfare, which is the difference of gross consumers' surplus and investment costs.

Without asymmetric information on costs, we saw in section 3 that the regulator can achieve the first-best investment levels (invest when prices reach threshold price  $\bar{p}_c$ ), while respecting budget and participation constraints. With private information on costs, the contracts offered will need to respect incentive compatibility as well, and therefore will need to leave information rents to the firms. In view of the budget constraint, distorting the contracts from the first-best scheme is now optimal.

In analyzing the optimal scheme, we follow the standard procedure in optimal contract design, and focus without loss of generality on direct revelation, incentive compatible mechanisms. The regulator offers a menu of contracts to the firm, consisting of a transfer  $T(\hat{c})$ , and an investment threshold,  $\bar{A}(Q, \hat{c})$ , that depend on the firm's reported cost  $\hat{c}$ . The firm, by reporting costs  $\hat{c}$ , chooses the best option from this menu, which we design such that truthful reporting,  $\hat{c} = c$ , is optimal.

We saw that both first best and profit maximization require investing when prices reach threshold level,  $\bar{p}^c$  and  $\bar{p}^m$  respectively, where those thresholds did not depend on capacity. Here, for expositional simplicity, we assume likewise that the optimal threshold  $\bar{A}(Q, \hat{c})$  under adverse selection takes such a form, *i.e.* there will

exist a threshold price  $\bar{p}(\hat{c})$  such that  $\bar{A}(Q, \hat{c})Q^{-\gamma} = \bar{p}(\hat{c})$ . In the appendix we demonstrate that this is indeed optimal. We can then express the menu of contracts as a set of transfer fees and threshold prices,  $\{(T(\hat{c}), \bar{p}(\hat{c}))\}$ .

At current demand level  $A$ , a contract specifying price threshold  $\bar{p}(\hat{c})$  will require a greenfield firm to immediately invest capacity  $Q(\bar{p})$  to make current price equal the threshold price,  $Q = (\frac{A}{\bar{p}})^{1/\gamma}$ , so that  $A = \bar{A}(Q)$ . In addition the firm will have to increase capacity as demand  $A$  grows to ensure prices remain below the threshold. Total rents  $R$  of accepting the contract  $(T(\hat{c}), \bar{p}(\hat{c}))$ , for a firm with actual costs  $c$ , are then given by

$$\begin{aligned} R(A, c, \hat{c}) &= T(\hat{c}) - cQ(\bar{p}(\hat{c})) - \bar{A}(Q)^\lambda \int_{Q(\bar{p}(\hat{c}))}^{\infty} \bar{A}(q, \hat{c})^{-\lambda} c \, dq \\ &= T(\hat{c}) - cQ(\bar{p}(\hat{c})) \frac{\gamma\lambda}{\gamma\lambda - 1}. \end{aligned} \quad (16)$$

In the first line, we used the expected present value of the costs of future expansions, computed analogously to the cost component of the firm's continuation value  $V$  from (12). In the second line, we substituted  $\bar{A}(Q) = \bar{p}Q^\gamma$ , to evaluate the integral.

Incentive compatibility now requires that the firm optimizes this value if it truthfully reveals its costs,  $\hat{c} = c$ , choosing fee and threshold  $(T(c), \bar{p}(c))$  from the menu of contracts. Writing the resulting profits from this optimization as  $\Pi(A, c) = R(A, c, \hat{c} = c)$ , we derive the following necessary conditions for incentive compatibility.

**Lemma 2.** *Incentive compatibility requires that total profits  $\Pi$  vary with costs  $c$  as*

$$\frac{d\Pi}{dc} = -Q(\bar{p}(c)) \frac{\gamma\lambda}{\gamma\lambda - 1}, \quad (17)$$

and that the investment price threshold  $\bar{p}(c)$  is non-decreasing in costs  $c$ .

We will analyze the welfare optimizing choice of contracts under incentive compatibility constraint (17), as well as the budget and participation constraints, and ignore the monotonicity requirement on the threshold for the moment. After finding an optimal threshold, we will verify that monotonicity indeed holds. As a first step let us write the welfare function and the budget constraint in terms of the threshold price  $\bar{p}$ .

The regulator's objective is to maximize the total welfare, averaged over the possible realization of costs  $c$ . For a given cost  $c$  and threshold price  $\bar{p}$ , welfare includes the costs of a one-off lumpy investment  $Q$  to bring price to the threshold at the current value of the demand shift  $A$ , the expected welfare generated by this investment  $Q$ , as well as the expected additional welfare from future network expansions (i.e. real option value). From equation (7),

$$\begin{aligned} W(\bar{p}, c) &= -cQ + \frac{\bar{A}(Q)Q^{1-\gamma}}{(1-\gamma)(r-\mu)} + \bar{A}(Q)^\lambda \int_Q^\infty \bar{A}(q)^{-\lambda} \left( \frac{\bar{A}(q)q^{-\gamma}}{r-\mu} - c \right) dq, \\ &= \frac{\gamma\lambda}{\gamma\lambda - 1} Q \left( \frac{\bar{p}}{r-\mu} \frac{\lambda-1}{\lambda} \frac{1}{\gamma-1} - c \right), \end{aligned} \quad (18)$$

where again  $Q = (\frac{A}{\bar{p}})^{1/\gamma}$  and we used  $\bar{p} = \bar{A}(Q)Q^{-\gamma}$ .

The budget constraint is that for any cost  $c$ , total profits,  $\Pi$ , cannot exceed total revenues minus costs. Using the expression for continuation value of revenues and costs (equation 12) and again substituting  $\bar{p}$ , we

can write this as

$$\begin{aligned}
\Pi(c) &\leq -cQ + \frac{\bar{A}(Q)Q^{1-\gamma}}{r-\mu} + \bar{A}(Q)^\lambda \int_Q^\infty \bar{A}(q)^{-\lambda} \left( \frac{\bar{A}(q)q^{-\gamma}(1-\gamma)}{r-\mu} - c \right) dq, \\
&= \underbrace{\left( \frac{\gamma\lambda}{\gamma\lambda-1} \right) \left( \frac{\lambda-1}{\lambda} \right) \frac{\bar{p}}{r-\mu} Q}_{\text{Revenue}(\bar{p})} - \underbrace{\left( \frac{\gamma\lambda}{\gamma\lambda-1} \right) cQ}_{\text{Cost}(c,\bar{p})} \\
&= \frac{\gamma\lambda}{\gamma\lambda-1} Q \left( \frac{\bar{p}}{r-\mu} \frac{\lambda-1}{\lambda} - c \right). \tag{19}
\end{aligned}$$

Summing up, we can now state the regulator's optimization program in terms of an optimal control problem, with state variable  $\Pi(c)$  and control  $\bar{p}(c)$ , and the Hamiltonian

$$\mathcal{H}(c) = \frac{\gamma\lambda Q(\bar{p})}{\gamma\lambda-1} \left( f(c) \left[ \frac{\bar{p}(\lambda-1)}{(r-\mu)\lambda(1-\gamma)} - c \right] - \nu + \phi \left[ \frac{\bar{p}(\lambda-1)}{(r-\mu)\lambda} - c \right] \right) - \phi \Pi(c), \tag{20}$$

with  $f(c)$  the density of the distribution of costs, co-state variable  $\nu(c)$  the multiplier of the incentive constraint (17), and  $\phi(c)$  the multiplier for the budget constraint (19). The resulting first-order conditions for the optimum are

$$\frac{\partial \mathcal{H}}{\partial \bar{p}} = 0 \tag{21}$$

$$\frac{\partial \mathcal{H}}{\partial \Pi} = -\frac{d\nu}{dc} \tag{22}$$

At the upper boundary of the support of  $c$ , we impose that profits are zero,  $\Pi(c_H) = 0$ . At the lower boundary, we have either  $\nu(c_L) = 0$ , or price is at its monopoly level  $\bar{p}^m(c_L)$ .

The solution to these first-order equations is as follows

**Proposition 1.** *Optimal threshold prices fall in one of three regimes:*

- *Regime I, the markup regime: the budget constraint does not bind,  $\phi = 0$  and  $\nu$  is constant. In this regime,*

$$\bar{p}(c) = \frac{(r-\mu)\lambda}{\lambda-1} \left( c + \frac{\nu}{f(c)} \right). \tag{23}$$

- *Regime II, the bunching regime: the budget constraint binds,  $\bar{p}$  is a constant in between competitive and monopoly prices  $\bar{p}^m(c) \geq \bar{p} \geq \bar{p}^c(c)$ , and*

$$\nu(c) = \nu(c_0) \frac{\bar{p}^m(c_0) - \bar{p}}{\bar{p}^m(c) - \bar{p}} + \frac{1}{1-\gamma} \frac{\int_{c_0}^c (\bar{p} - \bar{p}^c(c')) f(c') dc'}{\bar{p}^m(c) - \bar{p}}. \tag{24}$$

for some  $c_0$  within the interval in which this regime holds.

- *Regime III, the monopoly regime: the budget constraint binds, price is at the monopoly level,*

$$\bar{p}(c) = \bar{p}^m(c), \text{ and } \frac{\nu(c)}{f(c)} = \frac{\gamma c}{1-\gamma}. \tag{25}$$

We see that as long as  $f(c)$  is non-increasing, prices are non-decreasing in each of the regimes. The optimal strategy is then a combination of two of the above regimes, joined together such that threshold price  $\bar{p}$ , co-state variable  $\nu$  and profit  $\Pi$  are continuous on the regime boundary:

**Proposition 2.** *Optimal regulation involves either monopoly pricing (regime III) for the lowest cost types, followed by the markup regime (Regime I) for higher types; or bunching at constant price (Regime II) for the lowest types, followed by the markup regime (Regime I).*

In the next section, we provide an illustration of the optimum strategy for the particular case of uniform cost distribution  $f(c)$ .

## 5 Example

For an illustration of the optimal regulation, we explore the case with uniform distribution on investment costs,  $f(c) = \frac{1}{c_H - c_L}$ . In that case, we can do the required integrations analytically, and use that to solve the model. As we will see, if the range of costs, measured by  $c_H/c_L$ , is not too large, we will have an optimum in which lower-cost firms bunch at a constant price (Regime II), while for higher costs firms, the budget constraint will not be binding and we are in the markup regime. Conversely, if cost uncertainty is large, the lowest cost types will end up being unregulated and setting monopoly prices instead.

To find the optimum we need to join together the various pricing regimes in a continuous fashion. We cannot have only the markup regime with constant  $\nu$ , since the boundary condition requires  $\nu(c_L) = 0$ , which would imply marginal cost pricing,  $\bar{p} = \bar{p}^c$  everywhere and would violate incentive compatibility. It is also easy to see that the only possibilities are to have either the monopoly regime at low costs, and the markup regime for higher costs, or the bunching regime at low cost, with again the markup regime for higher costs.

Let us first examine when the monopoly regime can be part of the solution, with monopoly prices for costs  $c_L \leq c \leq c_m$  for some  $c_m$ , and constant markup  $\nu/f$  for  $c > c_m$ . At  $c_m$ , we have monopoly price  $\bar{p}(c_m) = \bar{p}^m(c_m)$  and co-state parameter  $\frac{\nu(c_m)}{f} = \frac{\gamma c_m}{1-\gamma}$ . For  $c > c_m$ ,  $\nu$  remains constant at this level, and prices retain their constant markup above the competitive level. To find the appropriate transition point  $c_m$ , we use the facts that  $\Pi(c_H) = 0$ ,  $\Pi(c_m)$  is at its monopoly value, and incentive compatibility (17) fixes the slope of the profit function  $d\Pi/dc$ . Doing the integration leads to the following

**Proposition 3.** *With uniform cost distribution and a large support  $\frac{c_L}{c_H} \leq \xi \equiv (1-\gamma)(\gamma^{\frac{\gamma}{\gamma-1}} - \gamma)^{-1}$ , we have monopoly pricing for  $c \in [c_L, c_m]$ , and a constant markup on competitive prices for  $c \in [c_m, c_H]$ , with  $c_m = c_H \xi$ .*

If the range of costs is small, so that  $c_L > c_H \xi$ , we do not have a monopoly regime but instead have bunching at constant price for low cost levels,  $c < c_b$ , while again we find constant markups for  $c > c_b$ . To find  $c_b$ , as well as the price and markup levels, we can again use continuity of  $\Pi(c)$ ,  $\bar{p}(c)$  and  $\nu(c)$ , and combine the incentive compatibility equation (17) with the expression for  $\nu$  in the bunching regime, (24). The resulting conditions on the transition level  $c_b$  and the bunching price are as in the following proposition.

**Proposition 4.** *With uniform cost distribution and a small support  $\frac{c_L}{c_H} \geq \xi$ , we have bunching for  $c \in [c_L, c_b]$ , and the markup regime (with constant markup) for  $c \in [c_b, c_H]$ , with transition point  $c_b$  and bunching price  $\bar{p}_b \equiv \frac{(r-\mu)\lambda}{\lambda-1} \tilde{p}_b$  determined jointly by the solution to*

$$\begin{aligned} \tilde{p}_b - c_b &= \frac{\tilde{p}_b(c_b - c_L) - \frac{1}{2}(c_b^2 - c_L^2)}{c_b - \tilde{p}_b(1-\gamma)} \\ \tilde{p}_b^{-\frac{1}{\gamma}}(\tilde{p}_b - c_b) &= \frac{\gamma}{\gamma-1} \left( (\tilde{p}_b - c_b + c_H)^{\frac{\gamma-1}{\gamma}} - \tilde{p}_b^{\frac{\gamma-1}{\gamma}} \right). \end{aligned}$$

The first equation in the proposition follows from integrating  $\nu$  between  $c_L$  and  $c_b$ , and requiring that its end value equals the markup in the markup regime. The second equation follows from making sure the profits at that transition point (where the budget constraint holds with equality) coincide with the integral of the incentive compatibility constraint, and profits at  $c_H$  are zero.

We plot the results of the two solutions, one for high cost uncertainty (high  $c_H/c_L$ ), figure 1, and one for low cost uncertainty ( $c_H/c_L$  nearer to one), figure 2. In the first case, we have monopoly pricing up to  $c_m$ , and in the second we see bunching at constant price for low realizations of costs.

With a direct revelation mechanism, the regulator offers the menu of contracts  $\{(\Pi(c), \bar{p}(c))\}$  and the firm truthfully announces its type  $c$ . Alternatively, the regulator could offer a menu of contracts  $\{\tau(\bar{p})\}$  in which

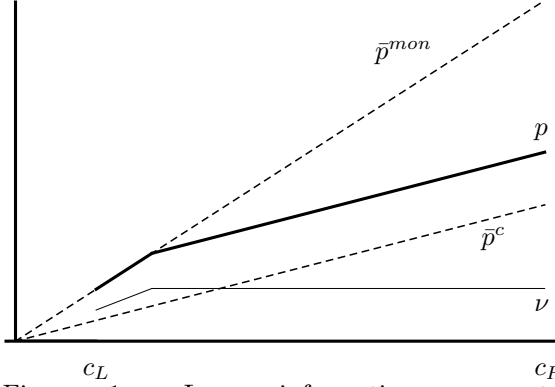


Figure 1: Large information asymmetry: Monopoly and mark-up regime

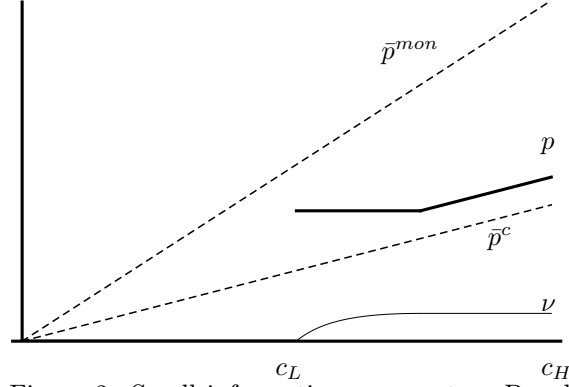


Figure 2: Small information asymmetry: Bunching regime and markup regime

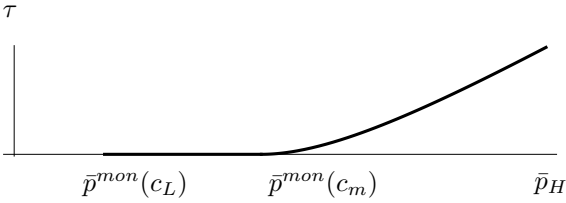


Figure 3: Taxes versus price threshold with monopoly regime

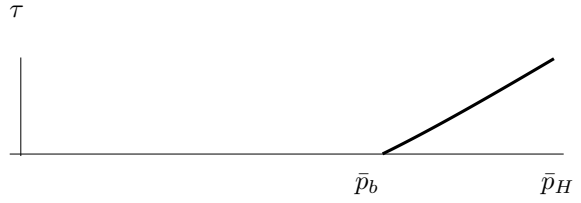


Figure 4: Taxes versus price threshold with bunching regime

the firm announces at which maximum threshold price level  $\bar{p}$  it will invest, and the regulator taxes a fraction  $\tau(\bar{p})$  of the revenue of selling capacity. Given an optimal menu of contracts  $\{(\Pi(c), \bar{p}(c))\}$ , the tax rate  $\tau(\bar{p})$  is determined implicitly by the following condition:

$$\Pi(c) = [1 - \tau(\bar{p}(c))] \text{Revenue}(\bar{p}(c)) - \text{Costs}(c, \bar{p}(c)),$$

where  $\text{Revenue}(\bar{p})$  and  $\text{Cost}(c, \bar{p})$  are defined by equation (19). Such a  $\tau(\bar{p})$  exists as  $\bar{p}(c)$  is strictly increasing in the mark-up regime and  $\tau$  is zero in the two other regimes as the budget constraint binds and hence the firm gets all revenues from sales. For the markup regime we find

$$\tau(\bar{p}) = \frac{\bar{p}(c_H) - \bar{p}^c(c_H)}{\bar{p}(c)} - \frac{\gamma}{1 - \gamma} \left( 1 - \left( \frac{\bar{p}(c)}{\bar{p}(c_H)} \right)^{\frac{1-\gamma}{\gamma}} \right),$$

matching to  $\tau = 0$  at the transition point with the monopoly or bunching regimes. Figures 3 and 4 plot the resulting set of pairs of threshold prices  $\bar{p}$  versus required taxes  $\tau$ . With large cost asymmetry  $c_H/c_L$ , we have the range of monopoly prices for low costs, accompanied with zero taxation. With smaller  $c_H/c_L$ , we have the single bunching price for the lower cost realizations. To benefit from the zero tax rate the firm needs to invest early, at a relatively low threshold price. Alternatively, the firm could invest later, which implies accepting a higher tax rate.

## 6 Discussion and conclusion

In this paper we have derived the optimal regulation of network expansion, by combining the real option and principal-agent literature. We show that a regulatory holiday might sometimes be optimal, not to prevent

hold-up problems created by a lack of commitment power by the regulator as in Gans and King (2004), but because of the combination of a self-financing constraint and information asymmetry: The information rents of efficient cost types need to be collected in the market, which require higher prices for network access and delayed investments. However, such a regulatory holiday is not a blanket authorization for the firm to invest whenever it feels fit. It implies the requirement, in accordance with the relatively low expansion costs of the efficient firm, for sufficiently early investment. If information asymmetries are small, the efficient firm requires less information rents and a regulatory holiday is no longer optimal. Instead the regulator will bunch the regulatory contracts for the most efficient firms, obliging them to invest at a price below the monopoly price.

Note that whether a regulatory holiday is socially optimal, does *not* depend on the level of demand uncertainty, and the riskiness of investments.<sup>14</sup> This stands in contrast with the requirement in EU energy markets, that the *risk* should be too high for investments to incur without exemption. In our model, the regulatory holiday is a reward for the low cost firm, who invests earlier than the high cost firm.

In the optimal regulation, inefficient firms are subject to an investment requirement, *i.e.* they are obliged to invest whenever the price for capacity reaches a threshold level. It is well known from the literature that when demand is stochastic, a price cap cannot be used to both limit the rents of the regulated firm and to incentivize timely investments. Instead the regulator needs to rely on a combination of instruments such as for instance in the UK where the new regulation sets a price cap but also sets an output obligation on the firms. Alternatively, as we show, the regulator can penalize a firm investing late by increasing its tax level.

Formally, our model considers greenfield investments. The same analysis holds, however, when regulated firms are allowed by law to recoup sunk investments cost of previously built assets, as long as these are at or below the regulated level. Such a principle of no regulatory takings is common. If, on the other hand, some of the investments have already been fully paid off at the time of contracting, the participation constraint is relaxed, and less information rents need to be paid to investors. The regulatory holiday regime is then less likely to be optimal. We also assumed that capacity does not depreciate. This could easily be adjusted by appropriate shifts in  $\mu$  and  $r$ , as in Dobbs (2004).

In telecom markets, the upstream network access provider often competes with multiple access buyers in a downstream market. In our paper we do not study the additional effects of providing access to downstream competitors. In that case, regulatory price setting in the retail market should also take into account the relaxation of the budget constraint in the network investment problem.

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<sup>14</sup>Recall that the firm is perfectly informed about its own investment costs, so the only source of risk is demand uncertainty.



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## A Proofs

**Proof of lemma 1** The expressions for  $\bar{p}^m$  and  $\bar{p}^c$  (equations 13 and 14) are derived in the text. The zero profit result for a greenfield firm (equation 15), follows from substituting  $A = \bar{p}^c Q^\gamma$  and  $\bar{A}(q) = \bar{p}^c q^\gamma$  in the expression for firm value, equation (12), and integrating. We then find

$$V - cQ = \frac{\gamma\lambda Q}{\gamma\lambda - 1} \left( \frac{\bar{p}^c(\lambda - 1)}{(r - \mu)\lambda} - c \right),$$

and by the definition of  $\bar{p}^c$  (14) this vanishes.

**Proof of lemma 2** We have

$$R(c, \hat{c}) = T(\hat{c}) - cQ(\bar{p}(\hat{c})) \frac{\gamma\lambda}{\gamma\lambda - 1},$$

and  $\Pi(c) = R(c, \hat{c} = c)$ . If  $\hat{c} = c$  optimizes  $R$ , we can use the envelope theorem to find

$$\frac{d\Pi(c)}{dc} = \left. \frac{\partial R(c, \hat{c})}{\partial c} \right|_{\hat{c}=c} = -Q(\bar{p}(c)) \frac{\gamma\lambda}{\gamma\lambda - 1}.$$

To verify that  $\bar{p}(c)$  is non-decreasing in  $c$ , we note that truthful revelation for a firm with type  $c$  requires that  $R(c, c) - R(c, \hat{c}) \geq 0$  for any  $\hat{c}$ . Equivalently, for a firm with type  $\hat{c}$  it must be that  $R(\hat{c}, \hat{c}) - R(\hat{c}, c) \geq 0$ . Hence, combining both expressions, for any  $c, \hat{c}$ , we must have that:

$$(R(c, c) - R(\hat{c}, c)) - (R(c, \hat{c}) - R(\hat{c}, \hat{c})) \geq 0,$$

or equivalently:

$$\int_{\hat{c}}^c \left( \frac{\partial R}{\partial c'}(c', c) - \frac{\partial R}{\partial c'}(c', \hat{c}) \right) dc' = \frac{\gamma\lambda}{\gamma\lambda - 1} [Q(\bar{p}(\hat{c})) - Q(\bar{p}(c))] (c - \hat{c}) \geq 0.$$

It then follows that for  $\hat{c} < c$ ,  $Q(\bar{p}(\hat{c})) \geq Q(\bar{p}(c))$ , or as demand is downward sloping  $\bar{p}(\hat{c}) \leq \bar{p}(c)$ .

**Proof of proposition 1** The first-order condition of the Hamiltonian for  $\Pi$  (22) gives the dynamics for  $\nu(c)$ ,

$$\frac{d\nu}{dc} = \phi.$$

In cost-regions where the budget constraint (19) does not bind, its multiplier is zero,  $\phi = 0$ , and hence  $\nu$  is constant. Using the short-hand

$$\tilde{p} = \frac{\bar{p}(\lambda - 1)}{(r - \mu)\lambda},$$

we can write the first-order condition of the Hamiltonian for  $\bar{p}$  (21) as:

$$f(c)(\tilde{p} - c) - \nu + \phi(\tilde{p}(1 - \gamma) - c) = 0.$$

With  $\phi = 0$  and  $\nu$  constant, this leads to

$$\tilde{p} = c + \frac{\nu}{f}$$

in this *markup regime I*.

In the cost-regions where the budget constraint (19) does bind, the firm's profit is equal to the revenue from capacity sales  $\Pi = \frac{\gamma\lambda Q(\tilde{p})}{\gamma\lambda - 1}(\tilde{p} - c)$ . From this we can derive the total derivative of profits as a function of the firm's type  $c$ :

$$\frac{d\Pi}{dc} = \frac{\partial \Pi}{\partial \tilde{p}} \frac{d\tilde{p}}{dc} - \frac{\gamma\lambda Q(\tilde{p})}{\gamma\lambda - 1}$$

Given the incentive compatibility condition (17), the first term on the right hand side is zero, we must then have that either  $\tilde{p}$  is constant, or  $\frac{\partial \Pi}{\partial \tilde{p}} = 0$ , and hence monopoly pricing.

In the case of constant  $\tilde{p}$ , we are in the bunching regime II. From the first order condition on  $\tilde{p}$ , with  $d\nu/dc = \phi$ , we then have

$$f(c)(\tilde{p} - c) - \nu + \frac{d\nu}{dc}(\tilde{p}(1 - \gamma) - c) = 0,$$

with constant  $\tilde{p}$ . This is a differential equation for  $\nu(c)$ , for which the solution is equation (24).

Finally, in the *monopoly regime III*, we have  $\tilde{p}^m = \frac{c}{1-\gamma}$ , so that the first-order equation reduces to

$$\frac{\nu(c)}{f(c)} = \tilde{p}^m - c = \frac{\gamma c}{\gamma - 1}. \quad (26)$$

**Proof of proposition 2** With  $f(c)$  non-increasing,  $\frac{\nu(c)}{f(c)}$  is non-decreasing in all three regimes. For the highest cost realization  $c_H$ , we have that the information rents are zero,  $\Pi(c_H) = 0$ , and therefore a non-binding budget regime ( $\phi = 0$ ), so we will have the markup regime. Suppose that for lower cost levels, we have a region  $[\underline{c}, \bar{c}]$  in which the bunching regime at a constant price  $\hat{p}$  applies,  $c_H > \bar{c} > \underline{c} > c_L$ . As the lower boundary is assumed to be strictly larger than  $c_L$ , the bunching regime is connected from below to one of the other regimes (monopoly or markup regime). Hence, at this point  $c = \bar{c}$ , we have  $\hat{p} - \underline{c} = \frac{\nu(\bar{c})}{f(\bar{c})}$ , since that relation holds in both monopoly and markup regimes. Similarly, the bunching regime is connected to the monopoly or markup regime from above, so we must have  $\hat{p} - \bar{c} = \frac{\nu(\bar{c})}{f(\bar{c})}$ . But this leads to a contradiction since  $\bar{c} > \underline{c}$  and  $\nu(c)/f(c)$  is non-decreasing. Hence we cannot have  $\underline{c} > c_L$ , and hence if the bunching regime occurs it is optimal for certain cost level, it is also the case for all lower cost levels.

**Proof of proposition 3** Consider a monopoly regime in the region  $[c_L, c_m]$ . At the boundary  $c_m$ , we need to match price  $\tilde{p}(c_m)$ , co-state variable  $\nu(c_m)$  and profit  $\Pi(c_m)$  to a constant markup regime on  $[c_m, c_H]$  on which  $\nu = \nu^{cte}$  is a constant, and where profit  $\Pi(c)$  is determined by the boundary condition  $\Pi(c_H) = 0$  and the incentive compatibility condition which determined  $d\Pi/dc$ . Matching  $\nu/f$  of both regimes at  $c_m$  gives

$$\frac{\nu^{cte}}{f} = \frac{\gamma c_m}{1 - \gamma},$$

which determines  $\nu^{cte}$  as a function of  $c_m$ . The price in the mark-up regime is then determined by  $\tilde{p} = c + \frac{\nu^{cte}}{f}$ , where  $\tilde{p} = \frac{\lambda-1}{\lambda(r-\mu)}\bar{p}$ . We now use incentive compatibility on  $\Pi$  to write

$$\Pi(c_m) = \frac{\gamma\lambda}{\gamma\lambda - 1} \int_{c_m}^{c_H} Q((\tilde{p}(c'))) dc'$$

With, by continuity of profits,  $\Pi(c_m)$  also equaling the monopoly profit at that point, we can then work out the required value of  $c_m$ .

**Proof of proposition 4** In the alternative case of the bunching regime for low cost  $c \in [c_L, c_b]$  and the markup regime for higher costs  $c \in [c_b, c_H]$ , we again find the boundary point  $c_b$  by matching price  $\tilde{p}$ , co-stage variable  $\nu$  and profit  $\Pi$ . The co-state variable  $\nu$  is defined in bunching region by a differential equation and the boundary condition  $\nu(c_L) = 0$ . Profit in the mark-up regime is defined by the incentive compatibility and the boundary condition  $\Pi(c_H) = 0$ .

We first compute  $\nu(c_b)$  from the differential equation for  $\nu$  in the bunching region, with boundary condition

$\nu(c_L) = 0$ . Doing the integration, this gives

$$\frac{\nu(c_b)}{f} = \frac{\tilde{p}_b(c_b - c_L) + \frac{1}{2}(c_L^2 - c_b^2)}{c_b - \tilde{p}_b(1 - \gamma)}$$

and this should equal  $\tilde{p}_b - c_b = \frac{\nu^{cte}}{f}$  by matching to the markup region. This is the first equation of the proposition.

From incentive compatibility in the mark-up regime we find

$$\Pi(c_b) = \frac{\gamma\lambda}{\gamma\lambda - 1} \int_{c_m}^{c_H} Q((\tilde{p}(c')) dc',$$

where  $\tilde{p}(c) = c + \frac{\nu^{cte}}{f} = c + \tilde{p}_b - c_b$ . Matching this to the profits at  $c_b$  from the binding budget constraint gives the second equation of the proposition.

## B Constant price thresholds

In this appendix we establish that also in the adverse selection case, the investment threshold occurs at constant price,  $\bar{p} = \bar{A}(Q)Q^{-\gamma}$ . In terms of  $\bar{A}(Q)$ , we have welfare

$$W(\bar{A}(Q), c) = -cQ + \frac{AQ^{1-\gamma}}{(1-\gamma)(r-\mu)} + A^\lambda \int_Q^\infty \bar{A}(q)^{-\lambda} \left( \frac{\bar{A}(q)q^{-\gamma}}{r-\mu} - c \right) dq,$$

with  $Q$  defined by  $\bar{A}(Q) = A$ . Similarly, the incentive and budget constraints are

$$\begin{aligned} \frac{d\Pi}{dc} &= -Q - A^\lambda \int_Q^\infty \bar{A}(q, \hat{c})^{-\lambda} dq, \\ \Pi(c) &\leq -cQ + \frac{AQ^{1-\gamma}}{r-\mu} + A^\lambda \int_Q^\infty \bar{A}(q)^{-\lambda} \left( \frac{\bar{A}(q)q^{-\gamma}(1-\gamma)}{r-\mu} - c \right) dq. \end{aligned}$$

Combining these, we can then write the Hamiltonian

$$\begin{aligned} \mathcal{H} &= \frac{AQ^{1-\gamma}}{(1-\gamma)(r-\mu)} (f + (1-\gamma)\phi) \\ &\quad + A^\lambda \int_Q^\infty \bar{A}(q)^{-\lambda} \left( \frac{\bar{A}(q)q^{-\gamma}}{r-\mu} (f + (1-\gamma)\phi) - (fc + \nu + \phi c) \right) dq \\ &\quad - (fc + \nu + \phi c)Q - \phi(c)\Pi. \end{aligned}$$

To optimize, we now need to use variational calculus on the function  $\bar{A}(Q)$ . Such a variation also induces a concomitant variation  $\delta Q$  so as to keep  $\bar{A}(Q) = A$  verified. It is now straightforward to see that the  $\delta Q$  terms in the variation vanish, leaving us only with the integral,

$$A^\lambda \int_Q^\infty \delta \bar{A}(q) \bar{A}(q)^{-\lambda-1} \left[ (1-\lambda) \frac{\bar{A}(q)q^{-\gamma}}{r-\mu} (f + (1-\gamma)\phi) + \lambda(fc + \nu + \phi c) \right] dq = 0$$

Since this holds for any variation  $\delta \bar{A}(Q)$ , we see that  $\bar{A}(Q)Q^{-\gamma}$  is independent from  $Q$ , and we regain the first-order equations from the main text.



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