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Zhu, Xiang

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Analysis of dual sourcing strategies under supply disruptions

Stuart X. Zhu *

Department of Operations, University of Groningen, P.O. Box 800, 9700AV Groningen, The Netherlands

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ABSTRACT

We study a dual-sourcing problem of a firm in the face of supply disruptions from two suppliers: local and overseas. Under four different scenarios of disruption source and information availability, we characterize the optimal dynamic policy that simultaneously determines sourcing decisions to minimize the expected total discounted cost. Different from the previous dual-sourcing models without information availability in the literature, we develop a two-dimensional stochastic dynamic programming model to explicitly address this issue. Further, we analyze the impact of disruption source and information availability on cost performance. We find that (i) a supply disruption at the local source may cause a more remarkable deterioration of cost efficiency than a supply disruption at the overseas source; (ii) the information about the local source is more valuable than that about the overseas source; (iii) when a firm orders from both sources, the disruption information can achieve a significant cost saving. These findings contribute to the theory of strategic sourcing by demonstrating the value of information available at different sources. Moreover, they can also be used as a valuable guideline for managers to select an appropriate sourcing strategy in business practices.

1. Introduction

As a result of supply chain globalization, global sourcing has gained popularity as a way to exploit global efficiencies in the delivery of a product or service. As Johnson (2006) and Gereffi and Lee (2012) mention, it is a common business practice for a firm to have different supply sources that have different grades of reliability in terms of the quantity and quality of orders delivered. Having diversified suppliers with various supply uncertainties, a firm has to choose an effective and cost-efficient sourcing strategy in order to protect itself from potential supply disruptions, such as fire accidents (Nishiguchi and Beaudet, 1998; Latour, 2001) or earthquakes (Burrows, 1999). Chopra and Sodhi (2004) stress the importance of maintaining multiple sourcing opportunities and acquiring supply information for risk mitigation.

The above situations indicate that dual sourcing offers a solution for reducing the risk in the procurement process and closely matching supply with demand, and the information availability could be highly valuable for the procurement process. Facing different disruption scenarios, how do companies find an effective way of deciding how to source, when, and from whom? In terms of cost performance, does local disruption always have a more serious impact than overseas disruption? What is the cost saving of acquiring disruption information? What disruption information is more valuable? Those are the key issues faced by firms to manage supply risk in their supply chains. However, as Tang and Musa (2011) point out, quantitative models in the field of supply chain risk management are relatively lacking and information flow risk has received less attention.

To close the gap in the literature, our paper is the first to consider the combined impacts of disruption source, information availability, lead times, and costs for achieving an effective dual sourcing strategy. We assume that a firm, such as an original equipment manufacturer, can order a critical material or component from two sources: local and overseas. Disruption may occur at these two sources, i.e., the supply may be disrupted because of unpredictable emergences, such as political instability, natural disasters, dock strikes, or poor product quality. Additionally, if a supplier shares the disruption information with the firm in time, before the order is placed, then the firm can know whether its order can be fulfilled.

Besides different scenarios of disruption source and information availability, these two sources differ from each other with different lead times and variable costs. We assume that the local supply has no lead time while the overseas supply has one-period lead time. The above assumption is commonly used in the literature of dual sourcing, such as Anupindi and Akella (1993), and Yang et al. (2005). In fact, we can extend our results to the model in which the lead time of the local supply is a constant number

* Tel.: +31 503638617, fax: +31 503632032.
E-mail address: x.zhu@rug.nl

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and the lead time of the overseas supply is one period longer than that of the local supply. The only difference from the current model is that we use the inventory position to define the state variable instead of inventory level. The results of our models can still hold as long as the difference of the lead times between the local and overseas sources is one period. When the difference of lead times from these two sources is more than one period, order crossovers, i.e., replenishment orders are not received in the sequence they are ordered, will occur. The occurrence of order crossovers significantly complicates the analysis of the model. To achieve analytical tractability, our model focuses on one-period lead-time difference. We will give recommendations about how to address a general lead-time difference in the Conclusions section. Moreover, the variable costs of these two sources may be different. To survive in the global competition, the overseas supplier may offer a low variable cost due to low labor cost and relatively low capital investment.

The contributions of this study are twofold. First, our paper characterizes the optimal dynamic policy that simultaneously determines the order quantities from two sources to minimize the expected total discounted cost under four different scenarios of disruption source and information availability. We find that when the disruption information is unavailable, the optimal policy may be either a threshold type or a modified order-up-to type; when the information is available, the optimal policy is an order-up-to type. Second, to understand the impacts of disruption source and information, we perform both analytical and numerical comparison for those four models. We provide a careful explanation for these observations, leading to conclusions of managerial interest.

This paper is organized as follows. Section 2 reviews the related literature. In Section 3, we present a model description. In Section 4, for a situation with potential disruptions at both sources, we formulate a finite-period model by stochastic dynamic programming and develop the corresponding structure results of optimal policies. In Section 5, we study four models with different disruption source and information availability, and explore various structural properties of the optimal policies for these models. By numerical experiments, we investigate the impact of disruption source and information on cost performance in Section 6. We conclude the paper with discussion of the results and suggestions for possible future research in Section 7.

2. Literature review

Our work is closely related to three streams of research. The salient feature of the first stream of research is supply disruption. With supply disruption uncertainty, a supplier may experience two situations: if the disruption occurs, the supplier cannot fulfill any order; otherwise, the supplier can deliver an order in full and on time. Under the single-supplier scenario, Moinzadeh and Aggarwal (1997) and Parlar (1997) studied the optimal replenishment policy under the periodic review and continuous review, respectively. Both Parlar and Perry (1996) and Gürler and Parlar (1997) considered the dual-sourcing problem with constant demand. The difference is that Parlar and Perry (1996) assumed that inter-failure and repair times are exponentially distributed for both suppliers while Gürler and Parlar (1997) extended to Erlang-k inter-failure times and general repair time. Recently, under a newsvendor setting with unreliable supply, Gürler and Parlar (1997) investigated the influence that resource attributes, firm attributes, and product-portfolio attributes have on the attractiveness of various supply-chain structures that differ in their levels of mix flexibility and diversification. Tomlin (2006) considered a firm which faces one unreliable and cheap supplier with fixed capacity and one reliable and expensive supplier with flexible capacity. However, both suppliers had identical delivery lead-times. The author characterized the optimal disruption-management strategies and analyzed the impact of volume flexibility on the performance of the firm. Yu et al. (2009) studied the impact of supply disruption risk on the decision process of a two-stage supply chain consisting of two suppliers and a manufacturer. The authors examine the optimal sourcing strategy under the situation that the demand is price-sensitive and the market scale increases when supply disruption occurs. In a decentralized supply chain, Li et al. (2010) characterized the sourcing strategy of one retailer and the pricing strategies of two suppliers and studied the impact of the players' strategies on the supply chain performance. Altug and Muhammeroglu (2011) considered an inventory system with uncertain capacity from a single source. Using capacity forecasts as the advanced supply information, they obtained the optimal replenishment strategy. Atasoy et al. (2012) investigated the optimal inventory policy when the disruption information for the near future from a single supplier is available. Zhu (2013) studied pricing and inventory-control strategies of a sole-sourcing model with supply disruption of raw materials. The author obtained the structure of the optimal polices for raw-material replenishment, finished-goods production, and pricing. Different from the above papers, we focus on the impact of the disruption source on the replenishment strategy and cost performance of the firm under the dual-sourcing strategy.

There is extensive literature in the second stream of related research, which assumes either multiple suppliers or multiple delivery modes. Fukuda (1964) was the first to consider an inventory model with two delivery modes. The lead times of the two modes are varied by exactly one period and the faster one is more expensive. The optimal policy is shown to have the following form. There are two base-stock levels. If the initial stock level is below the smaller base-stock level, then an order with the faster mode is placed to bring the stock level up to that base-stock level. No order with the faster mode is placed otherwise. The system stock level is then brought up to the larger base-stock level by ordering with the slower mode. Anupindi and Akella (1993) studied the optimal replenishment policy for a dual-sourcing model with uncertain yield. Under the multiple delivery modes, Sethi et al. (2003) considered an inventory model with a fixed ordering cost and demand forecast updates. Recently, Yang et al. (2005) studied the firm's optimal sourcing strategy when the in-house production capacity is modeled by a Markov process. The authors proved that both the in-house production and the outsourcing policy are capacity-dependent. Feng et al. (2006) showed that when the number of modes is more than two, only the fastest two modes have optimal base stocks. Veeraraghavan and Scheller-Wolf (2008) extended the literature by allowing the difference between the lead times of the two modes to be more than one. Klosterhalfen et al. (2011) numerically investigated the cost performance of the constant-order policies (COP) and the dual-index policy for dual sourcing. The authors found that the COP performs quite well for specific practical situations. Riezebos and Zhu (2015) generalized theory on material requirements planning ordering by including the occurrence of dynamic lead-time variation and order crossovers in a multiple-supplier environment. Zhu (2015) is closely related to our work. This author studied the problem about how to use dual sourcing as a strategy to hedge the risk of the yield uncertainty of the in-house production process by considering the trade-off among different yields, costs, and leadtimes. The key insights from Zhu (2015) are the following: (i) the expected cost of a firm increases when the variation of the yield becomes large; (ii) cost saving achieved by the dual-sourcing strategy decreases in the variability of demand and yield. Different from Zhu (2015), we focus on the problem about how to achieve an effective and cost-efficient dual sourcing strategy by examining whether/which source to acquire information in face of potential supply disruptions from multiple suppliers. Note that none of the above-mentioned papers considers the impact of supply disruption source and information availability.

The third stream focuses on information sharing within the context of supply chain risk management. Wakolbinger and Cruz (2011) developed a supply chain network equilibrium model with information acquisition and information sharing. The authors also proposed risk-sharing contracts that distributed disruption risks among parties and evaluated the supply chain performances. Atasoy et al. (2012) studied the replenishment decision for a sole-sourcing periodic-review model with advanced information of potential supply disruption. Under the assumption of the all-or-nothing type of supply disruptions, the authors proved that a state-dependent (s,S) policy is optimal. Tang and Musa (2011) provided a comprehensive review of the recent research development in supply chain risk management between 1995 and 2009. Interesting readers can refer to Tang and Musa (2011) for more details.

3. Model description

We consider a single-item, periodic-review inventory model with two sources of supply: local and overseas. The local supply is instantaneous, while the overseas supply takes a lead time of one period for order processing, transportation, etc. Both sources face potential disruptions. Denote β_n and β_n^o as the probability that the local and overseas disruptions do not occur in period n, respectively. Note that β_n and β_n^o can be treated as the indicator of supply reliability in period n. The model is commonly used to model supply disruptions in the literature, such as Tomlin and Wang (2005). Further, we assume that the uncertainty of the local disruption is independent of that of the overseas disruption.

The decision sequence of each period is as follows. At the beginning of each period, based on the latest inventory level, the firm has to determine the order quantities from both the local and overseas sources for the current period. After that, the random demand is realized and fulfilled. Finally, at the end of each period, the inventory cost is counted and the unsatisfied demand is fully backlogged.

We also assume that the planning horizon is finite with N periods, with $1 \le N < \infty$. Demands in consecutive periods are non-stationary and independent. The demand in period n is given by a random variable D_n. In period n, the unit variable cost from the local supply is c_n and that from the overseas supply is ${c_n}^o$. Let $R_n(x)$ denote the holding and the backlog cost in period n when the ending inventory is z. We assume that $R_n(x)$ is convex in x, and $R_n(x) \to \infty$ as $|x| \to \infty$. A list of frequently used notation is given in Table 1.

Table 1
Frequently used notation.

q_n	Order quantity for the local source in period n
v_n	Order quantity for the overseas source in period n
c_n	Unit variable cost from the local source in period n
${c_n}^o$	Unit variable cost from the overseas source in period n
$R_n(x)$	The holding and the backlog cost in period n when the ending inventory is x
x	The initial inventory level at the beginning of a certain period
β_n	The probability that the local disruption does not occur in period n
${\beta_n}^o$	The probability that the overseas disruption does not occur in period n
D_n	The random demand in period n
α	The discount factor
N	The length of the planning horizon

4. Structural results of the optimal policies

In this section, we try to characterize the structure of the optimal policies for both the local and the overseas sources. Since the firm does not have any information of potential disruption and has to determine the order quantities before the overseas order placed in the previous period is realized, we need to define a two-dimension state space to record system state. Let $V_n(x,x^o)$ be the minimal expected total cost from period n to the end of the planning horizon given that the initial stock is x and the overseas quantity ordered in period $n-1$ is x^o. Then, from the standard theory of stochastic dynamic programming, the optimality equation is given by

$$\begin{aligned}V_n(x,x^o) = \min_{0\le q, 0\le v} \mathbf{E}\{&(1-\beta_n)(1-\beta_n^o)R_n(x-D_n)+(1-\beta_n)\beta_n^o R_n(x+x^o-D_n)\\ &+\beta_n(1-\beta_n^o)R_n(x+q-D_n)\\ &+\beta_n\beta_n^o R_n(x+x^o+q_n-D_n)\\ &+\alpha[(1-\beta_n)(1-\beta_n^o)V_{n+1}(x-D_n,v)\\ &+(1-\beta_n)\beta_n^o V_{n+1}(x+x^o-D_n,v)\\ &+\beta_n(1-\beta_n^o)V_{n+1}(x+q-D_n,v)+\beta_n\beta_n^o V_{n+1}(x+x^o+q-D_n,v)]\\ &+\beta_n c_n q+\beta_n^o c_n^o v\}. \end{aligned} \tag{1}$$

Without loss of generality, we assume that there is no cost for excess stock or shortage in period $N+1$. In other words, the terminal condition is 0, i.e., $V_{N+1}(x,x^o)=0$.

By rewriting (1), we denote

$$V_n(x,x^o) = \min_{0\le q, 0\le v} \tilde{J}_n(x,x^o,q,v).$$

Lemma 1. *$\tilde{J}_n(x,x^o,q,v)$ is jointly convex in (x,x^o,q,v) and $V_n(x,x^o)$ is jointly convex in (x,x^o).*

Because of the joint convexity of $\tilde{J}_n(x,x^o,q,v)$, we can first optimize q and then optimize v. Denote $y=x+q$. It is equivalent to optimize y and v instead of q and v.

Then, we have

$$\begin{aligned}V_n(x,x^o) = \min_{0\le v} J_n(x,x^o,v)-\beta_n c_n x+(1-\beta_n)\mathbf{E}[&(1-\beta_n^o)R_n(x-D_n)\\ &+\beta_n^o R_n(x+x^o-D_n)],\end{aligned}$$

where

$$\begin{aligned}J_n(x,x^o,v) &= \alpha\mathbf{E}[(1-\beta_n)(1-\beta_n^o)V_{n+1}(x-D_n,v)\\ &\quad +(1-\beta_n)\beta_n^o V_{n+1}(x+x^o-D_n,v)]+\beta_n^o c_n^o v+G_n(x,x^o,v),\\ G_n(x,x^o,v) &= \min_{x\le y} \mathbf{E}\{\beta_n(1-\beta_n^o)R_n(y-D_{N-1})+\beta_n\beta_n^o R_n(y+x^o-D_n)\\ &\quad +\beta_n c_n y+\alpha[\beta_n(1-\beta_n^o)V_{n+1}(y-D_n,v)\\ &\quad +\beta_n\beta_n^o V_{n+1}(y+x^o-D_n,v)]\}.\end{aligned}$$

Lemma 2.

(i) *$G_n(x,x^o,v)$ and $J_n(x,x^o,v)$ are jointly convex and supermodular in (x,x^o,v);*
(ii) *$V_n(x,x^o)$ is supermodular in (x,x^o).*

Define $(y_n(x,x^o),v_n(x,x^o)) = \arg\min_{x\le y, 0\le v}\tilde{J}_n(x,x^o,y,v)$.

Theorem 1.

(i) *For any given x, $v_n(x,x^o)$ is decreasing in x^o.*
(ii) *For any given x^o, $y_n(x,x^o)$ is increasing in x and $v_n(x,x^o)$ is decreasing in x.*

By (i) of Theorem 3, since $v_n(x,x^o)$ is decreasing in x^o for any given x, there exists a threshold $\bar{I}_n$, such that $\bar{I}_n = \arg\inf\{x^o : v_n(x,x^o)=0\}$. If $x^o<\bar{I}_n$, $v_n(x,x^o)>0$; otherwise, $v_n(x,x^o)=0$. Therefore, the optimal policy for the overseas supply is a threshold type for any given x. By following the similar argument for (ii) of Theorem 3, the optimal policy for the overseas supply is also a threshold type for any given x^o. However, the structure of the optimal policy for the local supply is more complicated. For instance, although $y_n(x,x^o)$ is increasing in x for any given x^o, it is still difficult to prove the monotonicity of $q_n(x,x^o)$ with respect to x since $q_n(x,x^o)=(y_n(x,x^o)-x)^+$.

5. Four models with different disruption sources and information availability

Based on the general model presented in Section 4, we intend to investigate the impact of disruption source and information availability on the optimal replenishment policy and the cost performance. According to different scenarios of disruption source and information availability, we study four models, as shown in Table 2. To distinguish the notation among the four models, we use the superscripts defined in Table 2. For example, the notation with superscript 'LN' is for Model I. However, when no confusion can arise, we often drop the superscript for simplicity. For Model I, we assume that the local supply may experience a disruption while the order from overseas is guaranteed to be delivered. When the firm places an order from the local supply in the current period, it is not sure whether the disruption occurs in the same period. For Model II, we assume that the order from the local source is guaranteed to be delivered while the overseas supply may experience a disruption. When the firm places an order from the overseas supply in the current period, the firm is not sure whether the disruption of the overseas order placed in the previous period occurs. Models III and IV are similar to Models I and II, respectively, except that the firm knows for certain whether the disruption occurs.

5.1. Model I: local disruption without information

Here, we assume that the local supply faces potential disruptions and the overseas supply is reliable, i.e., $b_n^o=1$. Since the overseas supply is always reliable, we only need one state variable instead of two state variables defined by (1). Let $V_n(x)$ be the minimal expected cost from period n to the end of the planning horizon given that the initial stock at the beginning of period n is x.

For Model I, the optimality equation is given by

$$V_n(x)=\min_{0\le q,0\le v}\mathbf{E}\{[(1-\beta_n)R_n(x-D_n)+\beta_nR_n(x+q-D_n)]+\beta_nc_nq$$
$$+c_n^ov+\alpha[(1-\beta_n)V_{n+1}(x+v-D_n)+\beta_nV_{n+1}(x+v+q-D_n)]\}.$$

Define

$$J_n(x,q)=\beta_n\mathbf{E}R_n(x+q-D_n)+\beta_nc_nq+G_n(x,q),$$
$$G_n(x,q)=\min_{0\le v}\{c_n^ov+\alpha\mathbf{E}[(1-\beta_n)V_{n+1}(x+v-D_n)+\beta_nV_n$$
$$+1(x+q+v-D_n)]\}.$$

Lemma 3. *For $n=1,\ldots,N$,*

(i) *$G_n(x,q)$ is jointly convex and supermodular in (x,q);*
(ii) *$J_n(x,q)$ is jointly convex and supermodular in (x,q).*

Table 2
Model analysis.

Disruption source & information availability	Without information	With information
Unreliable local	Model I (LN)	Model III (LI)
Unreliable overseas	Model II (ON)	Model IV (OI)

Denote $(q_n^{LN}(x),v_n^{LN}(x))$ as the optimal quantities in period n. Let $I_n^{LN}=\inf\{x:q_n^{LN}(x)=0\}$, and $\bar{I}_n^{LN}=\inf\{x:v_n^{LN}(x)=0\}$.

Theorem 2.

(i) *Both $q_n^{LN}(x)$ and $v_n^{LN}(x)$ are decreasing in x. $q_n^{LN}(x)=0$ as $x\to+\infty$ and $v_n^{LN}(x)=0$ as $x\to+\infty$.*
(ii) *The optimal policy for the local supply is a threshold policy, where ${I_n}^{LN}$ is the threshold value: order $q_n^{LN}(x)>0$ if $x<I_n^{LN}$, and do not order $q_n^{LN}(x)=0$ if $x\ge I_n^{LN}$.*
(iii) *The optimal policy for the overseas supply is a threshold policy, where $\bar{I}_n^{LN}$ is the threshold value: order $v_n^{LN}(x)>0$ if $x<\bar{I}_n^{LN}$, and do not order $v_n^{LN}(x)=0$ if $x\ge\bar{I}_n^{LN}$.*

Theorem 2 demonstrates that the optimal policies for both sources are threshold type. Next, let us examine the impact of the initial stock level on the optimal replenishment decisions.

From now on, for function $f(x)$, we assume that $d_xf(x)=f'(x)=df(x)/dx$ exists everywhere even though in reality there can be a countable number of points at which the corresponding derivative does not exist. Similarly, we denote $\partial_{x_1}f(x_1,x_2)=\partial f(x_1,x_2)/\partial x_1$.

Proposition 1. *Under the assumption that $\mathbf{E}R_n(x-D_n)$ is continuous and differentiable almost everywhere, we have*

(i) *$d_xv_n^{LN}(x)\in[-1,0)$;*
(ii) *$d_xq_n^{LN}(x)\in[-1,0)$;*
(iii) *when the firm orders from both sources, $d_x[q_n^{LN}(x)+v_n^{LN}(x)]\le-1$.*

Proposition 1 shows that with the disruption possibility at the local supply, when the initial stock is increased by one unit, the decrease in order quantity from both the local and the overseas supply is less than one unit. As such, the result is different from the result of the dual-sourcing model without supply uncertainty by Fukuda (1964), in which the one additional initial stock causes exactly one unit reduction of order quantity. We conjecture that the supply uncertainty weakens the impact of initial stock on order quantity. Consequently, the total amount of the replenishment is reduced by no less than one unit when the firm orders from both sources.

5.2. Model II: overseas disruption without information

We assume that the local supply is reliable and the overseas supply faces potential disruption, i.e., $\beta_n=1$. Simplifying (1) by substituting $\beta_n=1$, we obtain the optimality equation of Model II given by

$$V_n(x,x^o)=\min_{0\le q,0\le v}\mathbf{E}\{(1-\beta_n)R_n(x+q-D_n)+\beta_nR_n(x+x^o+q-D_n)$$
$$+\alpha[(1-\beta_n)V_{n+1}(x+q-D_n,v)+\beta_nV_{n+1}(x+x^o$$
$$+q-D_n,v)]+c_nq+\beta_nc_n^ov\}. \quad (2)$$

Since Model II can be treated as a special case of the model discussed in Section 4, the results by Lemma 1 and Theorem 1 are still held.

By the joint convexity of Lemma 1, we derive further property by redefining (2) as

$$V_n(x,x^o)=\min_{x\le y}J_n(y,x^o)-c_nx,$$

where $y=x+q$ and

$$J_n(y,x^o)=\mathbf{E}[(1-\beta_n)R_n(y-D_{N-1})+\beta_nR_n(y+x^o-D_n)]+c_ny+G_n(y,x^o), \quad (3)$$

$$G_n(y,x^o)=\min_{0\le v}\mathbf{E}\{\alpha(1-\beta_n)V_{n+1}(y-D_n,v)$$
$$+\alpha\beta_nV_{n+1}(y+x^o-D_n,v)+\beta_nc_n^ov\}. \quad (4)$$

Denote

$$y_n(x^o) = \arg\min_y J_n(y, x^o),$$

$$v_n(y, x^o) = \arg\min_v \{\mathbf{E}[\alpha(1-\beta_n)V_{n+1}(y-D_n, v) + \alpha\beta_n V_{n+1}(y+x^o-D_n, v)] + \beta_n c_n^o v\}.$$

Here, we define that $f(x_1, x_2)$ is diagonally dominant in x ($DD[x_1]$) if $d_{x_1} f(x_1+\delta, x_2-\delta)$ is increasing in δ. Similarly, $f(x_1, x_2)$ is $DD[x_2]$ if $d_{x_2} f(x_1-\delta, x_2+\delta)$ is increasing in δ.

Lemma 4.

(i) *Both $G_n(x^o, y)$ and $J_n(y, x^o)$ are supermodular in (y, x^o), $DD[y]$, and $DD[x^o]$.*
(ii) *$V_n(x, x^o)$ is supermodular in (x, x^o), $DD[x]$, and $DD[x^o]$.*
(iii) *$d_{x^o} y_n(x^o) \in [-1, 0)$, $\partial_y v_n(y, x^o) \in [-1, 0)$, and $\partial_{x^o} v_n(y, x^o) \in [-1, 0)$.*

Define $(q_n^{ON}(x, x^o), v_n^{ON}(x, x^o)) = \arg\min_{0 \le q, 0 \le v} \tilde{J}_n(x, x^o, q, v)$, $I_n^{ON}(x^o) = y_n(x^o)$, and $\bar{I}_n^{ON} = \arg\inf\{x : v_n^{ON}(x, x^o) = 0\}$.

Theorem 3. *$I_n^{ON}(x^o)$ is the order-up-to level for local replenishment and is decreasing in x^o. $q_n^{ON}(x, x^o) = (I_n^{ON}(x^o) - x)^+$.*

By Theorem 1, the optimal policy for the overseas supply is a threshold type. Theorem 3 shows that the optimal policy for the local supply is a modified order-up-to type and the order-up-to level depends on the previous overseas order. By part (iii) of Lemma 4, when the previous overseas order is increased by one unit, the order-up-to level is decreased by no more than one unit. The optimal policy for the overseas supply is a threshold type for any given x^o.

Proposition 2. *For $x \ge I_n^{ON}(x^o)$, $\partial_x q_n^{ON}(x, x^o) = \partial_{x^o} q_n^{ON}(x, x^o) = 0$. $\partial_x v_n^{ON}(x, x^o) \in [-1, 0)$ and $\partial_{x^o} v_n^{ON}(x, x^o) \in [-1, 0)$. Further, $\partial_x v_n^{ON}(x, x^o) \le \partial_{x^o} v_n^{ON}(x, x^o)$. For $x < I_n^{ON}(x^o)$, $\partial_x q_n^{ON}(x, x^o) = -1$, and $\partial_{x^o} q_n^{ON}(x, x^o) \in [-1, 0)$. $v_n^{ON}(x, x^o)$ is independent of x and $\partial_{x^o} v_n^{ON}(x, x^o) \in [-1, 1]$.*

Proposition 2 indicates that when $x \ge I_n^{ON}(x^o)$, the firm does not order from the local source. Thus, the local order quantity is not affected by the initial stock level and the previous overseas order quantity. The overseas order quantity in the current period is decreasing in the initial stock level and the previous overseas order quantity. Moreover, the increase of either one additional initial stock or one additional previous overseas quantity causes the decrease in the current overseas quantity to be less than one unit because of the supply uncertainty.

When $x < I_n^{ON}(x^o)$, the firm may order from both the local and the overseas sources. On the one hand, the increase of one additional initial stock decreases the local order quantity by one unit while the increase of one additional previous overseas quantity decreases the local order quantity by less than one unit, which shows that the initial stock level has a more significant impact on the local quantity than the previous overseas quantity. On the other hand, the current overseas quantity is independent of the initial stock level since the stock level is raised to the order-up-to level after the local replenishment under such a circumstance. It is interesting to find that the firm may either increase or decrease the size of the overseas order. The reason for this is that when the initial stock level is quite low, although the amount of the previous overseas quantity may be large, the firm may still increase the replenishment from overseas if the supply disruption is more likely to occur.

5.3. Model III: local disruption with information

Different from Model I, we assume that before the firm places orders in the current period, the firm is aware of whether the local disruption has occurred. Therefore, we have to add the other dimension into the state space of the model to indicate the state of disruption. We define that if $i=1$, the disruption does not occur; if $i=0$, it does. Here, the overseas source is still reliable. Further, we have $\beta_n^o = 1$.

Let $V_n(x, i)$ be the minimal expected total cost from period n to N with the initial stock x and the information state i. For this model, the optimality equation is given by

$$V_n(x, 1) = \min_{0 \le q, 0 \le v} \mathbf{E}\{R_n(x+q-D_n) + c_n q + c_n^o v + \alpha[(1-\beta_n)V_{n+1}(x+q+v-D_n, 0) + \beta_n V_{n+1}(x+q+v-D_n, 1)]\}, \tag{5}$$

$$V_n(x, 0) = \min_{0 \le v} \mathbf{E}\{R_n(x-D_n) + c_n^o v + \alpha[(1-\beta_n)V_{n+1}(x+v-D_n, 0) + \beta_n V_{n+1}(x+v-D_n, 1)]\}. \tag{6}$$

For $i=1$, because of the joint convexity, we can firstly optimize q and then v. Denote $y = x+q$ and $z = y+v$, which is equivalent to optimizing y and z. We can rewrite (5) as

$$V_n(x, 1) = -c_n x + \min_{x \le y}\{\mathbf{E}R_n(y-D_n) + (c_n - c_n^o)y + G_n(y)\},$$

$$G_n(y) = \min_{y \le z} \mathbf{E}\{c_n^o z + \alpha[(1-\beta_n)V_{n+1}(z-D_n, 0) + \beta_n V_{n+1}(z-D_n, 1)]\}.$$

Denote

$$I_n^{LI}(1) = \arg\min_y \mathbf{E}\{R_n(y-D_n) + (c_n - c_n^o)y + G_n(y)\}, \tag{7}$$

$$\bar{I}_n^{LI}(1) = \arg\min_z \mathbf{E}\{c_n^o z + \alpha(1-\beta_n)V_{n+1}(z-D_n, 0) + \alpha\beta_n V_{n+1}(z-D_n, 1)\}, \tag{8}$$

$$\bar{I}_n^{LI}(0) = \bar{I}_n^{LI}(1). \tag{9}$$

Denote $(q_n^{LI}(x, i), v_n^{LI}(x, i))$ as the optimal solution in period n with information state i.

Theorem 4. *For $i=0$, $v_n^{LI}(x, 0) = (\bar{I}_n^{LI}(0) - x)^+$; for $i=1$, $q_n^{LI}(x, 1) = (I_n^{LI}(1) - x)^+$, and $v_n^{LI}(x, 1) = (\bar{I}_n^{LI}(1) - I_n^{LI}(1) \vee x)^+$.*

Theorem 4 demonstrates that for $i=0$, the optimal policy for the local supply is an order-up-to type with the order-up-to level given by $\bar{I}_n^{LI}(0)$. For $i=1$, when $I_n^{LI}(1) < \bar{I}_n^{LI}(1)$, if $x < I_n^{LI}(1)$, the firm first uses the local supply to raise the inventory level to $I_n^{LI}(1)$ and then orders $\bar{I}_n^{LI}(1) - I_n^{LI}(1)$ from the overseas supply; if $I_n^{LI}(1) < x < \bar{I}_n^{LI}(1)$, the firm only orders $\bar{I}_n^{LI}(1) - I_n^{LI}(1)$ from the overseas supply; if $x \ge \bar{I}_n^{LI}(1)$, the firm does not place any order; if $I_n^{LI}(1) > \bar{I}_n^{LI}(1)$, the firm only uses the local supply and orders up to $I_n^{LI}(1)$ and never uses the overseas supply. Therefore, we conclude that the local replenishment follows an order-up-to policy with the level given by $I_n^{LI}(1)$ and the overseas replenishment also follows an order-up-to policy with the level given by $\bar{I}_n^{LI}(1)$.

For the model by Fukuda (1964), under the condition that $c_n^o \ge \alpha c_n$, the firm does not order from the overseas supply in period n since the overseas supply does not offer any cost saving and has a long lead time. For Model III, we find that when $i=1$, even if $c_n^o \ge \alpha c_n$, the firm may still order from the overseas supply. The reason for this is that although this order cannot yield cost and time advantages, the overseas supply is reliable and the firm may use the reliability to hedge the disruption risk of the local supply in future.

Proposition 3. *If the disruption distribution is stationary, i.e., $\beta_n = \beta$, we have:*

(i) *$I_n^{LI}(i)$, $\bar{I}_n^{LI}(i)$, $q_n^{LI}(x, i)$, and $v_n^{LI}(x, i)$ are decreasing in β;*
(ii) *$V_n^{LI'}(x, i)$ is increasing in β and $V_n^{LI}(x, i)$ is decreasing in β.*

Note that $(1-\beta)/\beta$ represents the squared coefficient of variation of the probability distribution of the disruption and is decreasing in β. Therefore, Proposition 3 means that (i) when the supply variability increases, the expected total cost becomes higher, the cost saving caused by one additional stock increases, and the firm orders more frequently in order to hedge against the risk caused by uncertain supply; and (ii) when the supply reliability increases, the expected total cost becomes lower, the cost saving caused by one additional stock decreases, and the firm orders less frequently. Compared with Model I, when the firm has no information of local disruption, the results in this proposition may not hold. For instance, under the situation that the firm does not have a sufficient amount of initial stock and the local disruption occurs with a high probability, the firm may increase the order quantities from both sources even when β increases.

Next, we compare Model III with Model I to show the value of the information when the local source faces a potential disruption.

Proposition 4. *Under the assumption that* $\mathbf{E}R_n(x-D_n)$ *is continuous and differentiable almost everywhere, we have*:

(i) $V_n^{LN\prime}(x) \le \beta_n V_n^{LI\prime}(x,1)+(1-\beta_n)V_n^{LI\prime}(x,0)$;
(ii) *the firm more likely orders from both the local and the overseas supply in Model I than in Model III, and* $v_n^{LN}(x) \ge v_n^{LI}(x,1)$.

Proposition 4 indicates that the expected marginal cost of one additional stock in Model I is lower than that of Model III, which means that one additional stock can yield a higher cost saving when the firm has no information about the supply disruption. In other words, the value of disruption information becomes less when there is sufficient stock available. This proposition also indicates that when the firm is not aware of the disruption information, the firm more likely orders from both the local and overseas supply compared with the situation that the firm has the disruption information. Moreover, the overseas order quantity without information is larger than that with information. The explanation of this is that when the disruption information is not available, the firm may order more frequently from the local and overseas sources in order to avoid stock shortage caused by the potential disruption.

5.4. Model IV: overseas disruption with information

Different from Model II, we assume that before the firm makes the replenishment decisions in the current period, it knows whether the disruption for the overseas order placed in the last period occurs. If $i=1$, the disruption does not occur; if $i=0$, it does. The local source is still reliable. Further, we have $\beta_n=1$. Similar to Model III, we also need to define a two-dimension state space. The optimality equation for Model IV is given by

$$\begin{aligned}V_n(x,1) = \min_{0\le q,0\le v} & \mathbf{E}\{R_n(x+q-D_n)+c_nq+c_n^o v \\ &+\alpha[(1-\beta_n^o)V_{n+1}(x+q+v-D_n,0) \\ &+\beta_n^o V_{n+1}(x+q+v-D_n,1)]\},\\ V_n(x,0) = \min_{0\le q} & \mathbf{E}\{R_n(x+q-D_n)+c_nq \\ &+\alpha[(1-\beta_n^o)V_{n+1}(x+q-D_n,0)+\beta_n^o V_{n+1}(x+q-D_n,1)]\}.\end{aligned}$$

Noting that $y=x+q$ and $z=y+v$, because of joint convexity, we can first optimize y and then z. Define

$$\begin{aligned}I_n^{OI}(0) &= \arg\min_y \mathbf{E}\{R_n(y-D_n)+c_ny \\ &\quad +\alpha(1-\beta_n^o)V_{n+1}(y-D_n,0)+\alpha\beta_n^o V_{n+1}(y-D_n,1)\},\\ I_n^{OI}(1) &= \arg\min_y \mathbf{E}\{R_n(y-D_n)+(c_n-c_n^o)y+G_n(y)\},\\ \bar{I}_n^{OI}(1) &= \arg\min_z \mathbf{E}\{c_n^o z+\alpha(1-\beta_n^o)V_{n+1}(z-D_n,0) \\ &\quad +\alpha\beta_n^o V_{n+1}(z-D_n,1)\} \qquad (10)\end{aligned}$$

where

$$G_n(y)=\min_{y\le z}\mathbf{E}\{c_n^o z+(1-\beta_n^o)V_{n+1}(z-D_n,0)+\alpha\beta_n^o V_{n+1}(z-D_n,1)\}.$$

Denote $(q_n^{OI}(x,i), v_n^{OI}(x,i))$ as the optimal solution in period n with information state i.

Theorem 5. *For* $i=0$, $q_n^{OI}(x,0)=(I_n^{OI}(0)-x)^+$. *For* $i=1$, $q_n^{OI}(x,1)=(I_n^{OI}(1)-x)^+$ *and* $v_n^{OI}(x,1)=(\bar{I}_n^{OI}(1)-I_n^{OI}(1)\vee x)^+$.

Theorem 5 shows that for Model IV, the structure of the optimal policy looks similar to that of Model III. For both sources, the optimal policies are order-up-to type. Further, we can show that if $c_n^o \ge \alpha c_n$, $v_n^{OI}(x,1)=0$. It is clear that when the uncertain overseas supply is no cheaper than the local supply, the firm has no incentive to order from it.

Similar to Proposition 3, we can prove that when the yield distribution is stationary, i.e., $\beta_n^o=\beta$, we obtain (i) both $I_n^{OI}(i)$ and $\bar{I}_n^{OI}(i)$ are decreasing in β and $q_n^{OI}(x,i)$ and $v_n^{OI}(x,i)$ are decreasing in β and (ii) $V_n^{OI\prime}(x,i)$ is increasing in β and $V_n^{OI}(x,i)$ is decreasing in β.

Next, we compare Model III with Model IV to show the impact of the disruption source.

Proposition 5. *Under the assumption that* $\mathbf{E}R_n(x-D_n)$ *is continuous and differentiable almost everywhere and* $\beta_n=\beta_n^o$, *we have*:

(i) $\beta_n V_n^{LI\prime}(x,1)+(1-\beta_n)V_n^{LI\prime}(x,0) \le \beta_n V_n^{OI\prime}(x,1)+(1-\beta_n)V_n^{OI\prime}(x,0)$;
(ii) *when the disruption information is available, the firm more likely orders from both the local and overseas supply in Model III than in Model IV, and* $q_n^{LI}(x,1) \ge q_n^{OI}(x,1)$, $v_n^{LI}(x,1) \ge v_n^{OI}(x,1)$.

Proposition 5 indicates that (i) the expected marginal cost of one additional stock with local disruption information is lower than that with the overseas disruption information, which means that the information of local disruption yields a higher contribution to cost saving than that of overseas disruption; (ii) compared with the overseas information, the firm orders more frequently and more amount from both sources with the local information.

6. Numerical study

In this section, we numerically investigate the impacts of disruption source and information availability on the cost efficiency. Demand is assumed to follow a normal distribution with mean equal to 100 and coefficient of variation (CV) equal to 0.2. We assume that $R_n(z)=h(x)^+ + b(-x)^+$, where h is the holding cost per unit and b is the backlog cost per unit. For parameter settings, we fix $h=1$, $c=2$, $b=8$, $c^o=1$, $\beta=\beta^o=0.9$, and $\alpha=1$. Such a setting may reflect a situation commonly faced by operations managers, such as component replenishment for automotive and medical-equipment manufacturers. Experiments are executed in the MATLAB software environment, version 7.13 (R2011b). During the computation, the limit of state space is set from -400 to 400. To avoid a non-zero probability of an actual negative demand, we use the truncated normal distribution. For the discretization of a normal distribution with mean μ and standard deviation σ, the interval around μ is considered, with the endpoints of the interval 0 and $\mu+3\sigma$.

First, we use a five-period model and compute the order-up-levels or thresholds of period 1 for four models as shown in Table 3. Note that for Models III and IV, when $i=0$, there is no order from the local supply and the overseas supply, respectively. Comparing Model I with Model II, we find that Model I has a lower threshold for the local supply and a higher threshold for the overseas supply. Comparing Model III with Model IV, we observe that if the disruption does not occur ($i=1$), the overall target stock

Table 3
Order-up-levels or thresholds for four models.

Model	Local supply	Overseas supply
I	128.91	96.34
II	143.55	63.21
III ($i=0$)	Not applicable	237.29
III ($i=1$)	155.58	94.34
IV ($i=0$)	158.77	Not applicable
IV ($i=1$)	152.12	92.81

Table 4
Comparison of models I (the benchmark model) and II (the measured model).

b	Gap(%)	c^o	Gap(%)	c	Gap(%)	β (β^o)	Gap(%)	CV	Gap(%)
10	12.70	2	18.36	3.5	7.39	0.95	5.89	0.25	13.17
8	11.03	1.5	13.81	3	8.23	0.9	11.03	0.2	11.03
6	9.55	1	11.03	2.5	9.33	0.85	16.07	0.15	9.39
4	8.23	0.5	9.17	2	11.03	0.8	21.92	0.1	6.31

Table 5
Comparison of models III (the benchmark model) and IV (the measured model).

b	Gap(%)	c^o	Gap(%)	c	Gap(%)	β(β^o)	Gap(%)	CV	Gap(%)
10	16.04	2	15.09	3.5	4.41	0.95	6.76	0.25	13.87
8	12.55	1.5	13.57	3	6.57	0.9	12.55	0.2	12.55
6	8.99	1	12.55	2.5	9.22	0.85	17.58	0.15	11.32
4	8.96	0.5	11.52	2	12.55	0.8	21.97	0.1	10.17

level of Model III (i.e., 249.92) is very close to that of Model IV (i.e., 244.93).

Next, we perform sensitivity analysis with respect to b, c^o, c, β, CV, and we only change one parameter at one time. Further, we change the initial inventory from -50 to 50 with an increment of 1. Each result reported in Tables 4–7 is the average of 101 problems with different amounts of the initial inventory. For each experiment, we consider a 20-period model and compute the expected discounted total cost. Here, we use the cost gap as the performance measure, i.e., the cost difference between the benchmark model and the measured model divided by the cost of the benchmark model. For example, in Table 4, to examine the impact of different disruption sources, we treat Models I and II as the benchmark and measured model, respectively.

First, Tables 4 and 5 demonstrate the impact of disruption source without and with information on cost efficiency, respectively. Both tables show that the total expected cost with the local disruption is higher than that with the overseas disruption, which means that the local disruption causes a more significant deterioration of cost performance than the overseas disruption. In particular, when the supply reliability is low, the gap could be more than 20%. Further, we observe that (i) the gap is increasing in both b and c^o but is decreasing in c; (ii) the gap is decreasing in β; (iii) the gap is increasing in CV. This can be explained as follows. For (i), with a large value of the shortage cost or the variable cost of the overseas supply, a firm prefers to order from the local supply over the overseas supply. Namely, the same degree of uncertainty at the local supply results in a more serious increase of the cost gap. However, when the variable cost of the local supply increases, the firm intends to order more from the overseas supply, which reduces the cost gap. For (ii), the smaller the value of β, the more uncertain the supply becomes, which causes an increase of the gap. For (iii), when the demand variability becomes higher, the potential disruption at the local supply has a more significant impact on the cost performance than that at the overseas supply.

Table 6
Comparison of models I (the benchmark model) and III (the measured model).

b	Gap(%)	c^o	Gap(%)	c	Gap(%)	β	Gap(%)	CV	Gap(%)
10	6.21	2	3.92	3.5	3.49	0.95	3.67	0.25	6.59
8	6.60	1.5	5.23	3	4.08	0.9	6.60	0.2	6.60
6	6.71	1	6.60	2.5	5.48	0.85	8.97	0.15	6.70
4	6.78	0.5	8.16	2	6.60	0.8	10.93	0.1	6.75

Table 7
Comparison of models II (the benchmark model) and IV (the measured model).

b	Gap(%)	c^o	Gap(%)	c	Gap(%)	β^o	Gap(%)	CV	Gap(%)
10	9.81	2	0.17	3.5	5.56	0.95	4.57	0.25	6.66
8	8.20	1.5	4.97	3	6.42	0.9	8.20	0.2	8.20
6	6.13	1	8.20	2.5	7.36	0.85	10.61	0.15	9.94
4	3.55	0.5	10.54	2	8.20	0.8	10.99	0.1	11.84

Second, Tables 6 and 7 demonstrate the impact of information at the local source and the overseas source on cost efficiency, respectively. Both tables show that the total expected cost without information is higher than that with information, which means that the information availability can create a cost saving. In particular, when the demand variability is high, the gap could be more than 11%. Further, we observe that (i) the gap is decreasing in c^o and c; (ii) for the local supply, the gap is decreasing in b while it is increasing in b for the overseas supply; (iii) the gap is decreasing in β; and (iv) the gap is decreasing in CV. This can be explained as follows. For (i), when c_o or c becomes small, the firm more likely orders from both sources. Then, the availability of disruption information affects the ordering decisions from these two sources, which results in a larger cost gap. For (ii), on the one hand, in Models I and III, since the information availability mainly affects the local order and the local supply is reliable with $\beta=0.9$, the value of information is relatively low, which yields a relatively stable cost gap. On the other hand, in Models II and IV, since the firm intends to order more from the overseas supply, the information about the overseas disruption helps reduce the risk of potential shortage, which is more valuable when b is larger. For (iii), the reason is the same as that for Tables 4 and 5. For (iv), although the availability of the disruption information results in the advantage of the reduction of the supply uncertainty, the increasing demand uncertainty may weaken such an advantage.

7. Conclusions

Our work addresses inventory replenishment strategies from two sources with different lead times, costs, and supply uncertainties. For four models with different scenarios of disruption source and information availability, we develop the corresponding optimal replenishment policies. For the local disruption without information, the optimal replenishment policies for both sources are a threshold type; for the overseas disruption without information, the optimal replenishment policy for the local source is a modified order-up-to type with the order-up-to level depending on the previous overseas order while the policy for the overseas source is a threshold type. For the other two models with information, optimal policies are both order-up-to type. Further, we investigate the impact of disruption source and information availability on the optimal policies and cost function by comparison with these models. The findings from numerical experiments show that (i) the supply disruption at the local source may cause a more remarkable deterioration of cost efficiency than that at the overseas source. For example, when the supply reliability is low, the cost gap could be more than 20%; (ii) the information of the local source is more valuable than that of the overseas source. In particular, when the demand variability is low, the cost gap with the local information could be half of that with the overseas information; (iii) when the firm more likely orders from both sources, the disruption information is quite valuable, namely is about 9% cost saving on average. Although the above insights are based on a limited setting of experiments, we believe that they can be used as a valuable reference for managers to select an appropriate sourcing strategy in business practices.

There are several possible extensions for future study. First, one key assumption of the current model is one unit time difference between the lead times of the overseas and local supplier. When the difference of lead times from these two suppliers is more than one period, order crossovers, i.e., replenishment orders are not received in the sequence they are ordered, will occur. The occurrence of order crossovers significantly complicates the analysis of the model since a multi-dimensional dynamic programming model has to been formulated to solve the model. Even if it is possible, it will be remarkably time-consuming to obtain the optimal solution. Hence, one possible way to overcome such a barrier is to develop a near-optimal heuristic. For example, Veeraraghavan and Scheller-Wolf (2008) propose a dual-index policy for making ordering decisions from two suppliers without supply disruption. Riezebos and Zhu (2015) develop some efficient heuristics to compute replenishment quantities for multiple sourcing with deterministic demand. Interesting readers may refer to these two articles as a first attempt and further explore how to make an extension for a more general scenario.

Second, we can consider the price-sensitive demand for the current models and determine the joint pricing and replenishment strategy under a dual-sourcing scenario. Third, as Wang and Gerchak (1996) point out that there are many factors that cause production capacity to be variable, such as unexpected breakdowns and unplanned maintenance, we can also integrate the variable capacity into our model and analyze the impact of the capacity uncertainty on the replenishment policy and the cost-efficiency.

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Appendix A

Proof of Lemma 1. For period N, because of $V_{N+1}=0$ and the convexity of R_N, it is clear that $\tilde{J}_N(x,x^o,q,v)$ is jointly convex in (x,x^o,q,v). Further, since minimization preserves convexity and $\{0\le q, 0\le v\}$ is a convex set, $V_N(x,x^o)$ is jointly convex in (x,x^o).

By induction, suppose that $V_{n+1}(x,x^o)$ is jointly convex in (x,x^o). Following the similar argument, we can also show that $\tilde{J}_n(x,x^o,q,v)$ is jointly convex in (x,x^o,q,v) and $V_n(x,x^o)$ is jointly convex in (x,x^o).□

Proof of Lemma 2. First, by Lemma 1, it is clear that $G_n(x,x^o,v)$ and $J_n(x,x^o,v)$ are jointly convex in (x,x^o,v).

Second, we prove the supermodularity by induction. For period N, it is straightforward that it is true due to the convexity of $R_N(z)$. Next, we will show that it also holds for period n. Denote

$$\begin{aligned}\tilde{G}_n(x,x^o,v)=\mathbf{E}\{&\beta_n(1-\beta_n^o)R_n(y-D_{N-1})+\beta_n\beta_n^oR_n(y+x^o-D_n)\\&+\beta_nc_ny+\alpha[\beta_n(1-\beta_n^o)V_{n+1}(y-D_n,v)\\&+\beta_n\beta_n^oV_{n+1}(y+x^o-D_n,v)]\}.\end{aligned} \tag{11}$$

Because of the supermodularity of V_{n+1} and the convexity of $R_n(z)$, it is clear that $\tilde{G}_n(x,x^o,v)$ is supermodular in (y,x^o,v). By following Lemma 3.2 in Chao et al. (2009), we can show that $G_n(x,x^o,v)$ is supermodular in (x,x^o,v). By induction, we obtain that $J_n(x,x^o,v)$ is also supermodular in (x,x^o,v). Further, we also have $V_n(x,x^o)$ is supermodular in (x,x^o).□

Proof of Theorem 1. By definition, we have $v_n(x,x^o)=\arg\min_{0\le v}J_n(x,x^o,v)$. By Lemma 2, we have $J_n(x,x^o,v)$ is supermodular in (x,x^o,v). By Theorem 2.8.2 in Topkis (1998, p. 77), $v_n(x,x^o)$ is decreasing in x for any given x^o and decreasing in x^o for any given x.

Next, for any x^o, due to the supermodularity of $\tilde{G}_n(x,x^o,v)$ by (11), we have $y_n(x^o,v)$ decreasing in v. Since $v_n(x,x^o)$ is decreasing in x for any given x^o, it is clear that $y_n(x,x^o)$ is increasing in x.□

Proof of Lemma 3. Denote

$$\tilde{G}_n(y,q)=c_n^oy+\alpha\mathbf{E}[\beta_nV_{n+1}(y+q-D_n)+(1-\beta_n)V_{n+1}(y-D_n)]. \tag{12}$$

Since V_{n+1} is convex, $\tilde{G}_n(y,q)$ is jointly convex in (y,q). By Lemma 2.6.2 in Topkis (1998, p. 50), $\tilde{G}_n(y,q)$ is supermodular in (y,q). Then, part (i) is simply an application of Lemma 3.2 in Chao et al. (2009). For part (ii), since $R_n(x+q-D_n)$ is jointly convex and supermodular in (x,q), by part (i), $J_n(x,q)$ is jointly convex and supermodular in (x,q).□.

Proof of Theorem 2. For (i), by part (ii) of Lemma 3, because of supermodularity, $q_n^{LN}(x)$ is decreasing in x. Denote

$$\hat{J}_n(x,v)=c_n^ov+\alpha(1-\beta_n)\mathbf{E}V_{n+1}(x+v-D_n)-c_nx+g_n(x,v),$$

where $g_n(x,v)$ is defined by

$$g_n(x,v)=\min_{x\le z}\mathbf{E}\{\beta_nR_n(z-D_n)+\beta c_nz+\alpha\beta_nV_{n+1}(z+v-D_n)\}. \tag{13}$$

Because V_{n+1} is convex and minimization preserves convexity, $g_n(x,v)$ is jointly convex in (x,v). By Lemma 2.6.2 in Topkis (1998, p. 50), $g_n(x,v)$ is supermodular in (x,v). Because of the convexity of $V_{n+1}(x)$ and the property of $g_n(x,v)$, $\hat{J}_n(x,v)$ is jointly convex and supermodular in (x,v).

Further, since $g_n(x,v)$ is supermodular in (x,v), $v_n^{LN}(x)$ is also decreasing in x. Since $J_n(x,q)$ is joint convex in (x,q) with a finite minimum, we have $J_n(x,0)\ge J_n(x,q)$ as $x\to+\infty$. Thus, $q_n^{LN}(x)=0$ as $x\to+\infty$. Similarly, we can show $v_n^{LN}(x)=0$ as $x\to+\infty$. (ii) and (iii) immediately follow by (i).□

Proof of Proposition 1. For (i), $v_n^{LN}(x)$ is given the first-order condition, i.e.,

$$c_n^o+\alpha(1-\beta_n)\mathbf{E}V'_{n+1}(x+v)+\partial_vg_n(x,v)=0, \tag{14}$$

where by (13), we have

$$\partial_v g_n(x,v)=\begin{cases}\alpha\beta_n\mathbf{E}V'_{n+1}(x+v), & (x\ge I_n^{LN});\\ \alpha\beta_n\mathbf{E}V'_{n+1}(z_n(v)+v) & \text{Otherwise}\end{cases}$$

For $x\ge I_n^{LN}$, it is clear that $d_x v_n^{LN}(x)=-1$.
For $x<I_n^{LN}$, $z_n(v)$ is given by

$$\beta_n\mathbf{E}\{R'_n(z-D_n)+\alpha V'_{n+1}(z+v-D_n)\}+c_n=0.$$

For $x<I_n^o<I_n$, based on the first-order condition, we have

$$d_v z_n(v)=-\frac{\alpha\mathbf{E}V''_{n+1}(z+v-D_n)}{\mathbf{E}\{R''_n(z-D_n)+\alpha V''_{n+1}(z+v-D_n)\}}.$$

Under the assumption that R_n is strictly convex, we can show that V_n is also strictly convex. Then, we have $d_v z_n(v)\in[-1,0)$. Namely, we have $-1\le d_v z_n(v)<0$. Then, we obtain that $z_n(v)+v$ is increasing in v. By the first-order condition (14), we have $-1\le d_x v_n^{LN}(x)<0$.

For (ii), when $x\ge I_n^{LN}$, $q_n^{LN}(x)=0$, which is independent of x. For $x<I_n^{LN}$, $q_n^{LN}(x)=z_n(v_n^{LN}(x))-x$. Then, we have

$$d_x q_n^{LN}(x)=d_v z_n(v)d_x v_n^{LN}(x)-1\ge-1,$$

where the inequality is true since $-1\le d_v z_n(v)<0$ and $-1\le d_x v_n^*(x)<0$.

Thus, we have $d_x q_n^{LN}(x)\in[-1,0)$.

For (iii), we need to consider two cases: $I_n^{LN}\ge\bar{I}_n^{LN}$ and $I_n<\bar{I}_n^{LN}$.

Case 1: $I_n^{LN}\ge\bar{I}_n^{LN}$. When $x\ge I_n^{LN}$, $q_n^{LN}(x)=v^{LN}(x)=0$. It is clear that $q_n^{LN}(x)+v_n^{LN}(x)$ is independent of x.

When $\bar{I}_n^{LN}\le x<I_n^{LN}$, $v_n^{LN}(x)=0$. Because of $d_x q_n^{LN}(x)\in[-1,0)$, we obtain that $q_n^{LN}(x)+v_n^{LN}(x)$ is decreasing in x.

When $x<\bar{I}_n^{LN}$, both $q_n^{LN}(x)$ and $v^{LN}(x)$ are positive. Then, we have

$$d_x(q_n^{LN}+v_n^{LN})=[d_v z_n(v)+1]d_x v_n^{LN}(x)-1\le-1,$$

where the equality is true since $q_n^{LN}(x)=z_n(v^{LN})-x$ and the inequality is true since $d_v z_n(v)\in[-1,0)$. In particular, if $d_v z_n(v)\in[-1,0)$ and $d_x v_n^{LN}(x)\in[-1,0)$, we have $d_x(q_n^{LN}+v_n^{LN})\le-1$.

Case 2: $I_n^{LN}<\bar{I}_n^{LN}$. The proof is similar to the argument in Case 1.□

Proof of Lemma 4. : We prove the lemma by induction. Let us start from the last period. We have

$$V_N(x,x^o)=\min_{0\le q}\mathbf{E}[(1-\beta_N)R_N(x+q-D_N)+\beta_N R_N(x+x^o+q-D_N)]+c_N q,$$

Because of the convexity of R_N, it is clear that the optimal policy of the in-house production follows an order-up-to policy for any given x^o. Thus, we have

$$V_N(x,x^o)=\begin{cases}\mathbf{E}[(1-\beta_N)R_N(x-D_N)+\beta_N R_N(x+x^o-D_N)] & \text{if } x\ge y_N(x^o);\\ \mathbf{E}[(1-\beta_N)R_N(y_N(x^o)-D_N)+\beta_N R_N(y_N(x^o)+x^o-D_N)] & \\ \quad+c_N(y_N(x^o)-x), & \text{O.W.}\end{cases}\tag{15}$$

By (15), it is obvious that $V_N(x,x^o)$ is supermodular in (x,x^o). Further, we have

$$\partial_x V_N(x,x^o)=\begin{cases}\mathbf{E}[(1-\beta_N)R'_N(x-D_N)+\beta_N R'_N(x+x^o-D_N)] & \text{if } x\ge y_N(x^o);\\ -c_N & \text{O.W.}\end{cases}\tag{16}$$

For any given δ, if $x+\delta<y_N(x^o-\delta)$, we have $\partial_x V_n(x+\delta,x^o-\delta)=-c_N$, which is independent of δ. If $x+\delta\ge y_N(x^o-\delta)$, we have

$$\partial_x V_N(x+\delta,x^o-\delta)=\mathbf{E}[(1-\beta_N)R'_N(x+\delta-D_N)+\beta_N R'_N(x+x^o-D_N)].$$

Because of the convexity, $\partial_x V_N(x+\delta,x^o-\delta)$ is increasing in δ.

And, we have

$$\partial_{x^o}V_N(x,x^o)=\begin{cases}\beta_N\mathbf{E}R'_N(x+x^o-D_N) & \text{if } x\ge y_N(x^o);\\ \beta_N\mathbf{E}R'_N(y_N(x^o)+x^o-D_N) & \text{O.W.}\end{cases}\tag{17}$$

If $x-\delta\ge y_N(x^o+\delta)$, we have $\partial_{x^o}V_N(x-\delta,x^o+\delta)=\beta_N^o\ \mathbf{E}R'_N(x+x^o-D_N)$, which is independent of δ. Otherwise, we have $\partial_{x^o}V_N(x-\delta,x^o+\delta)=\beta_N^o\mathbf{E}R'_N(y_N(x^o+\delta)+x^o+\delta-D_N)$.

By the first-order condition, we have

$$\mathbf{E}[(1-\beta_N)R'_N(y_N-D_N)+\beta_N R'_N(y_N+x^o-D_N)]+c_N=0.$$

It is clear that $d_{x^o}y_N(x^o)\in[-1,0]$. Then, $y_N(x^o)+x^o$ is increasing in x^o. Namely, $\partial_{x^o}V_N(x-\delta,x^o+\delta)$ is increasing in δ.

Now, we try to show that it is also true for period n. Firstly, we show that $G_n(y,x^o)$ is supermodular in (y,x^o), where $G_n(y,x^o)$ is given by (4). For $y_1\ge y_2$, we will prove that $G_n(y_1,x^o)-G_n(y_2,x^o)$ is increasing in x^o. By definition, because of the supermodularity of V_{n+1}, $v_n(y,x^o)$ is decreasing in y for any given x^o. We have $v_n(y_1,x^o)\le v_n(y_2,x^o)$. Then, we need to consider three cases.

Case (i): $v_n(y_2,x^o)<0$. Then, we have

$$\begin{aligned}G_n(y_1,x^o)-G_n(y_2,x^o)=&\mathbf{E}[(1-\beta_n)V_{n+1}(y_1-D_n,0)\\&+\beta_n V_{n+1}(y_1+x^o-D_n,0)]\\&-\mathbf{E}[(1-\beta_n)V_{n+1}(y_2-D_n,0)\\&+\beta_n V_{n+1}(y_2+x^o-D_n,0)].\end{aligned}$$

Because of the convexity of V_{n+1}, it is clear that $G_n(y_1,x_o)-G_n(y_2,x_o)$ is increasing in x^o.

Case (ii): $v_n(y_1,x^o)\le 0\le v_n(y_2,x^o)$. Then, we have

$$\begin{aligned}G_n(y_1,x^o)-G_n(y_2,x^o)=&\mathbf{E}[(1-\beta_n)V_{n+1}(y_1-D_n,0)\\&+\beta_n V_{n+1}(y_1+x^o-D_n,0)]\\&-\mathbf{E}[(1-\beta_n)V_{n+1}(y_2-D_n,v_n(y_2,x^o))\\&+\beta_n V_{n+1}(y_2+x^o\\&-D_n,v_n(y_2,x^o))]-\beta_n c_n^o v_n(y_2,x^o).\end{aligned}$$

Denote $y_1=y_2+\delta$. Differentiating the above equality with respect to x^o, we have

$$\begin{aligned}\partial_{x^o}[G_n(y_1,x^o)-G_n(y_2,x^o)]=&\beta_n\mathbf{E}[\partial_{x^o}V_{n+1}(y_1+x^o-D_n,0)\\&-\partial_{x^o}V_{n+1}(y_2+x^o-D_n,v_n(y_2,x^o))]\\\ge&\beta_n\mathbf{E}[\partial_{x^o}V_{n+1}(y_1+x^o-D_n,v_n(y_1,x^o))\\&-\partial_{x^o}V_{n+1}(y_2+x^o-D_n,v_n(y_2,x^o))]\\\ge&\beta_n\mathbf{E}[\partial_{x^o}V_{n+1}(y_2+\delta+x^o\\&-D_n,v_n(y_2,x^o)-\delta)-\partial_{x^o}V_{n+1}(y_2\\&+x^o-D_n,v_n(y_2,x^o))]\ge0,\end{aligned}$$

where the first inequality is true since $v_n(y_1,x^o)<0$ and the convexity of V_{n+1}. The second inequality is true since $v_n(y_1,x^o)\ge v_n(y_2,x^o)-\delta$. The reason is that by Lemma 3 in Yang and Qin (2007), because $V_{n+1}(x,x^o)$ is supermodular in (x,x^o), $DD[x]$, and $DD[x^o]$, we have $\partial_y v_n(y,x^o)\in[-1,0)$. Finally, the third inequality is true since $\partial_x V_{n+1}(x+\delta,x^o-\delta)$ is increasing in δ.

Case (iii): $v_n(y_1,x^o)>0$. Then, we have

$$\begin{aligned}G_n(y_1,x^o)-G_n(y_2,x^o)=&\mathbf{E}[(1-\beta_n)V_{n+1}(y_1-D_n,v_n(y_1,x^o))\\&+\beta_n V_{n+1}(y_1+x^o-D_n,v_n(y_1,x^o))]\\&-\mathbf{E}[(1-\beta_n)V_{n+1}(y_2-D_n,v_n(y_2,x^o))\\&+\beta_n V_{n+1}(y_2+x^o-D_n,v_n(y_2,x^o))]\end{aligned}$$

$$+\beta_n c_n^o v_n(y_1) - \beta_n c_n^o v_n(y_2, x^o).$$

Differentiating the above equality with respect to x^o, we have

$$\partial_{x^o}[G_n(y_1, x^o) - G_n(y_2, x^o)] = \beta_n \mathbf{E}[\partial_{x^o} V_{n+1}(y_1 + x^o - D_n, v_n(y_1, x^o)) - \partial_{x^o} V_{n+1}(y_2 + x^o - D_n, v_n(y_2, x^o))].$$

By following the similar argument as Case (ii), we can prove the statement.

In short, we have proven that $G_n(y, x^o)$ is supermodular in (y, x^o). Then, we will show that $G_n(y, x^o)$ is $DD[y]$ and $DD[x^o]$. For $\delta \ge 0$, if $v_n(y+\delta, x^o-\delta) < 0$, we have

$$\partial_y G_n(y+\delta, x^o-\delta) = \mathbf{E}[\alpha(1-\beta_n)\partial_y V_{n+1}(y+\delta-D_n, 0) + \alpha\beta_n \partial_y V_{n+1}(y+x^o-D_n, 0)].$$

Because of the convexity of V_{n+1}, $\partial_y G_n(y+\delta, x^o-\delta)$ is increasing in δ.

If $v_n(y+\delta, x^o-\delta) \ge 0$, we have to consider two cases: $v_n(y, x^o) \ge 0$ and $v_n(y, x^o) < 0$.

Case (i): $v_n(y, x^o) \ge 0$. We have

$$\begin{aligned}\partial_y G_n(y+\delta, x^o-\delta) - \partial_y G_n(y, x^o) &= \mathbf{E}[\alpha(1-\beta_n)\partial_y V_{n+1}(y+\delta - D_n, v_n(y+\delta, x^o-\delta)) \\ &\quad + \alpha\beta_n \partial_y V_{n+1}(y+x^o-D_n, v_n(y+\delta, x^o-\delta)] - \mathbf{E}[\alpha(1-\beta_n)\partial_y V_{n+1} \\ &\quad \times (y-D_n, v_n(y, x^o)) + \alpha\beta_n \partial_y V_{n+1} \\ &\quad \times (y+x^o-D_n, v_n(y, x^o))] \\ &\ge \mathbf{E}[\alpha(1-\beta_n)\partial_y V_{n+1}(y+\delta-D_n, v_n(y, x^o)-\delta) + \alpha\beta_n \partial_y V_{n+1}(y \\ &\quad + x^o - D_n, v_n(y, x^o)-\delta] \\ &\quad - \mathbf{E}[\alpha(1-\beta_n)\partial_y V_{n+1}(y-D_n, v_n(y, x^o)) + \alpha\beta_n \partial_y V_{n+1}(y+x^o \\ &\quad - D_n, v_n(y, x^o)] \ge 0,\end{aligned}$$

where the first inequality is true since $v_n(y+\delta, x^o-\delta) \ge v_n(y, x^o) - \delta$ and the second inequality is true since $\partial_x V_{n+1}(x+\delta, x^o-\delta)$ is increasing in δ.

Noting that by comparing the corresponding first order conditions, we have $v_n(y+\delta, x^o-\delta) \ge v_n(y, x^o)$.

Case (ii): $v_n(y, x^o) < 0$. We have

$$\begin{aligned}\partial_y G_n(y+\delta, x^o-\delta) - \partial_y G_n(y, x^o) &= \mathbf{E}[\alpha(1-\beta_n)\partial_y V_{n+1}(y+\delta-D_n, \\ v_n(y+\delta, x^o-\delta)) &+ \alpha\beta_n \partial_y V_{n+1}(y+x^o-D_n, v_n(y+\delta, x^o-\delta)] \\ &- \mathbf{E}[\alpha(1-\beta_n)\partial_y V_{n+1}(y-D_n, 0)) \\ &+ \alpha\beta_n \partial_y V_{n+1}(y+x^o-D_n, 0)] \ge 0,\end{aligned}$$

where the inequality is true due to the convexity and supermodularity of V_{n+1}.

Thus, we have $G_n(y, x^o)$ is $DD[y]$.

Next, we will show that $G_n(y, x^o)$ is $DD[x^o]$. If $v_n(y-\delta, x^o+\delta) < 0$, we have

$$\partial_{x^o} G_n(y-\delta, x^o+\delta) = \alpha\beta_n \mathbf{E}\partial_{x^o} V_{n+1}(y+x^o-D_n, 0).$$

It is obvious that $\partial_y G_n(y-\delta, x^o+\delta)$ is independent of δ. If $v_n(y-\delta, x^o+\delta) \ge 0$, we have

$$\begin{aligned}&\partial_{x^o} G_n(y-\delta, x^o+\delta) - \partial_{x^o} G_n(y, x^o) \\ &= \alpha\beta_n \mathbf{E}[\partial_{x^o} V_{n+1}(y+x^o-D_n, v_n(y-\delta, x^o+\delta)) \\ &\quad - \partial_{x^o} V_{n+1}(y+x^o-D_n, v_n(y, x^o) \vee 0)],\end{aligned}$$

where $v_n(y, x^o) \vee 0 = \max\{v_n(y, x^o), 0\}$.

Because of the supermodularity of V_{n+1} and $v_n(y-\delta, x^o+\delta) \ge 0$, we have $v_n(y-\delta, x^o+\delta) \ge v_n(y, x^o) \vee 0$. Then, it is clear that $\partial_{x^o} G_n(y-\delta, x^o+\delta) - \partial_{x^o} G_n(y, x^o) \ge 0$. Thus, $G_n(y, x^o)$ is $DD[y]$.

By definition, since $G_n(y, x^o)$ is $SP[y, x^o]$, $DD[y]$, and $DD[x^o]$, it is obvious that $J_n(y, x^o)$ is also $SP[y, x^o]$, $DD[y]$, and $DD[x^o]$.

Now, we show that $V_n(x, x^o)$ is supermodular in (x, x^o). In other words, for $x_1 \ge x_2$, we try to prove that $V_n(x_1, x^o) - V_n(x_2, x^o)$ is increasing in x^o. By definition, $y_n(x^o)$ is decreasing x^o. We need to consider three cases.

Case (i): $y_n(x^o) \le x_2$. Then, we have

$$\begin{aligned}V_n(x_1, x^o) - V_n(x_2, x^o) &= \mathbf{E}[(1-\beta_n)R_n(x_1-D_n) \\ &\quad + \beta_n R_n(x_1+x^o-D_n)] - \mathbf{E}[(1-\beta_n)R_n(x_2-D_n) \\ &\quad + \beta_n R_n(x_2+x^o-D_n)] + c_n x_1 + G_n(x_1, x^o) - c_n x_2 - G_n(x_2, x^o).\end{aligned}$$

Because of the convexity of R_n and the supermodularity of G_n, it is clear that the statement is true.

Case (ii): $x_2 < y_n(x^o) < x_1$. Then, we have

$$\begin{aligned}V_n(x_1, x^o) - V_n(x_2, x^o) &= \mathbf{E}[(1-\beta_n)R_n(x_1-D_n) + \beta_n R_n(x_1 \\ &\quad + x^o - D_n)] - \mathbf{E}[(1-\beta_n)R_n(y_n(x^o)-D_n) \\ &\quad + \beta_n R_n(y_n(x^o)+x^o-D_n)] + c_n x_1 + G_n(x_1, x^o) \\ &\quad - c_n y_n(x^o) - G_n(y_n(x^o), x^o).\end{aligned}$$

Differentiating the above equation with respect to x^o, we have

$$\begin{aligned}&\partial_{x^o}[V_n(x_1, x^o) - V_n(x_2, x^o)] \\ &= \beta_n \mathbf{E}[R_n'(x_1+x^o-D_n) - R_n'(y_n(x^o)+x^o-D_n)] \\ &\quad + \partial_{x^o} G_n(x_1, x^o) - \partial_{x^o} G_n(y_n(x^o), x^o).\end{aligned}$$

Because of $y_n(x^o) < x_1$ and the supermodularity of G_n, we have $\partial_{x^o}[V_n(x_1, x^o) - V_n(x_2, x^o)] \ge 0$.

Case (iii): $y_n(x^o) \ge x_1$. Then, we have $V_n(x_1, x^o) - V_n(x_2, x^o) = 0$. Thus, $V_n(x_1, x^o) - V_n(x_2, x^o)$ is independent of x^o.

In summary, we have shown that $V_n(x, x^o)$ is supermodular in (x, x^o). Now, we show that $V_n(x, x^o)$ is $DD[x]$ and $DD[x^o]$. For period n, we have

$$\partial_x V_n(x, x^o) = \begin{cases} \mathbf{E}[(1-\beta_n)R_n'(x-D_n) + \beta_n R_n'(x+x^o-D_n)] + \partial_x G_n(x, x^o) & \text{if } x \ge y_n(x^o); \\ -c_n & \text{O.W.} \end{cases} \tag{18}$$

By (18), if $x+\delta < y_n(x^o-\delta)$, we have $\partial_x V_n(x+\delta, x^o-\delta) = -c_n$, which is independent of δ. If $x+\delta \ge y_n(x^o-\delta)$, we have

$$\begin{aligned}\partial_x V_n(x+\delta, x^o-\delta) &= \mathbf{E}[(1-\beta_{n+1})R_{n+1}'(x+\delta-D_{n+1}) \\ &\quad + \beta_{n+1} R_{n+1}'(x+x^o-D_{n+1})] + \partial_x G_{n+1}(x+\delta, x^o-\delta).\end{aligned}$$

Since $G_{n+1}(x, x^o)$ is $DD[x]$ and R_n is convex, $\partial_x V_n(x+\delta, x^o-\delta)$ is increasing in δ. Namely, $V_n(x, x^o)$ is $DD[x]$. Similarly, we can prove that $V_n(x, x^o)$ is also $DD[x^o]$.

For (iii), the results immediately follow by Lemma 3 in Yang and Qin (2007).□

Proof of Theorem 3. For (i), the results immediately follow by part (i) of Lemma 4. For (ii), for any given x^o, we need to consider two cases. For $x < I_n^{ON}(x^o)$, since the order-up-to level is achieved and independent of x, $v_n^{ON}(x, x^o)$ is independent of x. For $x \ge I_n^{ON}(x^o)$, (iii) of Lemma 4, $v_n^{ON}(x, x^o)$ is decreasing in x. Thus, the optimal policy is a threshold type.□

Proof of Proposition 2. For $x \ge I_n^{ON}(x^o)$, we have $q_n^{ON}(x, x^o) = 0$. Thus, $\partial_x q_n^{ON}(x, x^o) = \partial_{x^o} q_n^{ON}(x, x^o) = 0$. Further, $v_n^{ON}(x, x^o)$ is obtained by

$$\min_{v \ge 0}\{\mathbf{E}[\alpha(1-\beta_n)V_{n+1}(x-D_n, v) + \alpha\beta_n V_{n+1}(x+x^o-D_n, v)] + \beta_n c_n^o v\}.$$

By part (ii) of Lemma 4, we have $\partial_x v_n^{ON}(x, x^o) \in [-1, 0)$ and

$\partial_{x^o} v_n^{ON}(x,x^o) \in [-1,0)$. From the first-order condition, we have $\partial_x v_n^{ON}(x,x^o) \le \partial_{x^o} v_n^{ON}(x,x^o)$.

For $x < I_n^{ON}(x^o)$, since $I_n^{ON}(x^o)$ is independent of x, $\partial_x q_n^{ON}(x,x^o) = -1$. By part (iii) of Lemma 4, we have $d_{x^o} I_n^{ON}(x^o) \in [-1,0)$. Then, $\partial_{x^o} q_n^{ON}(x,x^o) \in [-1,0)$.

Further, $v_n^{ON}(x,x^o)$ is also independent of x by

$$\min_{v \ge 0}\{\mathbf{E}[\alpha(1-\beta_n)V_{n+1}(y(x^o)-D_n,v) \\ +\alpha\beta_n V_{n+1}(y(x^o)+x^o-D_n,v)]+\beta_n c_n^o v\}.$$

By the first order condition and treating v_n as a function of x^o, we have

$$\beta_n c_n^o + \alpha\mathbf{E}[(1-\beta_n)\partial_v V_{n+1}(y_n(x^o)-D_n, v_n(x^o)) \\ +\beta_n \partial_v V_{n+1}(y_n(x^o)+x^o-D_n, v_n(x^o))] = 0.$$

Because of $y_n(x^o) \in [-1,0)$, if x^o is increased to $x^o+\delta$, $y_n(x^o)$ is decreased to $y_n(x^o)-\epsilon$, where $0<\epsilon\le\delta$. Then, we have

$$\beta_n c_n^o + \alpha\mathbf{E}[(1-\beta_n)\partial_v V_{n+1}(y_n(x^o)+\delta-\epsilon-D_n, v_n(x^o) \\ -(\delta-\epsilon))+\beta_n \partial_v V_{n+1}(y_n(x^o)+x^o-D_n+\delta-\epsilon, v_n(x^o)-(\delta-\epsilon))] < 0,$$

where the inequality is true since V_{n+1} is $DD[x^o]$. By the convexity of V_{n+1}, we have

$$v_n(x^o+\delta) \ge v_n(x^o)-(\delta-\epsilon) \Rightarrow \frac{v_n(x^o+\delta)-v_n(x^o)}{\delta-\epsilon} \ge -1 \\ \Rightarrow \frac{v_n(x^o+\delta)-v_n(x^o)}{\delta} \ge -1.$$

Similarly, we can show that $1 \ge [v_n(x^o+\delta)-v_n(x^o)]/(\delta-\epsilon) \ge [v_n(x^o+\delta)-v_n(x^o)]/\delta$. In summary, we have $d_{x^o} v_n(x,x^o) \in [-1,1]$.□

Proof of Theorem 4. Firstly, we can prove that $V_n(x,i)$ is convex in x by induction. Obviously, it is true for period $N+1$. Assuming that it holds for period $n+1$, we show that it is also true for period n. For $i=1$, by (5), because minimization preserves the convexity and $\{0\le q, 0\le v\}$ is a convex set, $V_n(x,1)$ is convex in x. For $i=0$, it is clear that $V_n(x,0)$ is convex in x by following the similar argument.

Secondly, for $i=0$, by (6), because of the convexity of $V_n(x,i)$, the optimal policy for the overseas supply is an order-up-to type with the order-up-to level given by (9). Thus, we obtain $v_n^{LI}(x,0) = (\bar{I}_n^{LI}(0)-x)^+$.

For $i=1$, by (7), because of the convexity of $V_n(x,i)$, it is clear that the optimal policy for the local supply is an order-up-to type with the order-up-to level given by (7). Thus, we obtain $q_n^{LI}(x,1) = (I_n^{LI}(1)-x)^+$.

For the overseas supply, we define

$$\hat{I}_n = \arg\min_y\{\mathbf{E}R_n(y-D_n)+(c_n-c_n^o)y\}. \tag{19}$$

Then, we need to consider two cases: $\hat{I}_n < \bar{I}_n^{LI}(1)$ and $\hat{I}_n \ge \bar{I}_n^{LI}(1)$, where $\bar{I}_n^{LI}(1)$ is given by (8). If $\hat{I}_n < \bar{I}_n^{LI}(1)$, we have $I_n^{LI}(1) = \hat{I}_n$. And, the interior point $\bar{I}_n^{LI}(1)$ is a feasible solution to $G_n(y)$. Therefore, when $x \ge \bar{I}_n^{LI}(1)$, $v_n^{LI}(x,1)=0$. When $x < \bar{I}_n^{LI}(1)$, $v_n^{LI}(x,1) = \bar{I}_n^{LI}(1)-\hat{I}_n \vee x >\ 0$. If $\hat{I}_n \ge \bar{I}_n^{LI}(1)$, by (6), Because of $y \le z$ and the convexity, we have $v_n^{LI}(x,1)=0$.

By combining these two cases, we obtain $v_n^{LI}(x,1) = (\bar{I}_n^{LI}(1)-I_n^{LI}(1)\vee x)^+$, where $\bar{I}_n^{LI}(1)$ is given by (8).□

Proof of Proposition 3. We prove it by induction. Since it is straightforward to show that it holds for period N. We need to show that it still holds for period n. Then, we need to consider two cases: $\bar{I}_n^{LI}(1) < \hat{I}_n$ and $\bar{I}_n^{LI}(1) \ge \hat{I}_n$, where $\hat{I}_n$ is given by (19).

Case 1: $\bar{I}_n^{LI}(1) < \hat{I}_n$. Note that $I_n^{LI}(1)$ is given by

$$\mathbf{E}[R_n'(I_n^{LI}(1)-D_n)+c_n+\alpha\beta_n V_{n+1}^{LI\prime}(I_n^{LI}(1)-D_n,1) \\ +\alpha(1-\beta_n)V_{n+1}^{LI\prime}(I_n^{LI}(1)-D_n,0)] = 0. \tag{20}$$

Since $V_{n+1}^{LI\prime}(x,0) \le V_{n+1}^{LI\prime}(x,1)$, $I_n^{LI}(1)$ is decreasing in β. By (8), $\bar{I}_n^{LI}(1)$ is decreasing in β. Note that $\bar{I}_n^{LI}(1) = \bar{I}_n^{LI}(0)$ $\bar{I}_n^{LI}(0)$ is decreasing in β. By Theorem 4, it is obvious that both $q_n^{LI}(x,i)$ and $v_n^{LI}(x,i)$ are decreasing in β.

For Model III, for $i=0$, we have

$$V_n^{LI\prime}(x,0) = \begin{cases} \mathbf{E}R_n'(x-D_n)-c_n^o, & x<\bar{I}_n^{LI}(0); \\ \mathbf{E}[R_n'(x-D_n)+\alpha\beta_n V_{n+1}^{LI\prime}(x-D_n,1) & \\ \quad +\alpha(1-\beta_n)V_{n+1}^{LI\prime}(x-D_n,0)], & x \ge \bar{I}_n^{LI}(0). \end{cases} \tag{21}$$

For $i=1$, we have

$$V_n^{LI\prime}(x,1) = \begin{cases} -c_n, & x<I_n^{LI}(1); \\ \mathbf{E}[R_n'(x-D_n)+\alpha\beta_n V_{n+1}^{LI\prime}(x-D_n,1) & \\ \quad +(1-\beta_n)V_{n+1}^{LI\prime}(x-D_n,0)], & x \ge I_n^{LI}(1), \end{cases} \tag{22}$$

Since $\hat{I}_n \ge I_n^{LI}(1)$ and $\bar{I}_n^{LI}(1) = \bar{I}_n^{LI}(0)$, we have $I_n^{LI}(1) \ge \bar{I}_n^{LI}(0)$. We have $V_n^{LI\prime}(x,1) \ge V_n^{LI\prime}(x,0)$. By (21) and (22), it is clear that $V_n'(x,i)$ is increasing in β. Since $V_n(x,1) \le V_n(x,0)$, we obtain that $V_n(x,i)$ is decreasing in β.

Case 2: $\bar{I}_n^{LI}(1) \ge \hat{I}_n$. We have $I_n^{LI}(1) = \hat{I}_n$ that is independent of β. By (8), $\bar{I}_n^{LI}(1)$ is decreasing in β. By Theorem 4, it is obvious that both $q_n^{LI}(x,i)$ and $v_n^{LI}(x,i)$ are decreasing in β.

For $i=0$, $V_n^{LI\prime}(x,1)$ is given by (21). For $i=1$, we have

$$V_n^{LI\prime}(x,1) = \begin{cases} -c_n, & x<\hat{I}_n; \\ \mathbf{E}R_n(x-D_n)-c_n^o, & \hat{I}_n < x < \bar{I}_n^{LI}(1); \\ \mathbf{E}[R_n'(x-D_n)+\alpha\beta_n V_{n+1}^{LI\prime}(x-D_n,1) & \\ \quad +\alpha(1-\beta_n)V_{n+1}^{LI\prime}(x-D_n,0)], & x \ge \bar{I}_n^{LI}(1), \end{cases} \tag{23}$$

Noting that $\bar{I}_n^{LI}(1) = \bar{I}_n^{LI}(0)$, We have $V_n^{LI\prime}(x,1) \ge V_n^{LI\prime}(x,0)$. By (21) and (23), it is clear that $V_n'(x,i)$ is increasing in β. Since $V_n(x,1) \le V_n(x,0)$, we obtain that $V_n(x,i)$ is decreasing in β.□

Proof of Proposition 4. We show this statement by induction. For period N, for Model I, we have

$$V_N^{LN\prime}(x) = \begin{cases} (1-\beta_N)\mathbf{E}R_N'(x-D_N)-\beta_N c_N, & x<I_N^{LN}; \\ \mathbf{E}R_N'(x-D_N), & x \ge I_N^{LN}. \end{cases}$$

For Model III, we have

$$V_N^{LI\prime}(x,1) = \begin{cases} -c_N, & x<I_N^{LI}(1); \\ \mathbf{E}R_N'(x-D_N), & x \ge I_N^{LI}(1). \end{cases} \tag{24}$$

and $V_N^{LI\prime}(x,0) = \mathbf{E}R_N'(x-D_N)$. Because of $I_N^{LN} = I_N^{LI}(1)$ and $v_n^{LN}(x) = v_n^{LI}(x,1) = 0$, both (i) and (ii) are true for period N.

Assuming that it holds for period $n+1$, we will show that it also holds for period n. Define $\hat{I}_n^o = \arg\min_z\{c_n^o z + \alpha\mathbf{E}V_{n+1}(z-D_n)\}$. By definition, because of the convexity and $V_{n+1}^{LN\prime}(x) \le \beta_{n+1}V_{n+1}^{LI\prime}(x,1) + (1-\beta_{n+1})V_{n+1}^{LI\prime}(x,0)$, we have $\hat{I}_n^o \ge \bar{I}_n^{LI}(1)$.

Then, we need to consider three cases: $\hat{I}_n \ge \hat{I}_n^o$, $\bar{I}_n^{LI}(1) < \hat{I}_n < \hat{I}_n^o$, $\hat{I}_n \le \bar{I}_n^{LI}(1)$.

Case 1: $\hat{I}_n \ge \hat{I}_n^o$. Since I_n^{LN} is uniquely given by $\mathbf{E}[R_n'(I_n^{LN}-D_n)+c_n+\alpha V_{n+1}^{LN\prime}(I_n^{LN}-D_n)]=0$ and $I_n^{LI}(1)$ is given by (20), we have $I_n^{LN} \ge I_n^{LI}(1)$ due to the convexity and $V_{n+1}^{LN\prime}(x) \le \beta_{n+1}V_{n+1}^{LI\prime}(x,1)+(1-\beta_{n+1})V_{n+1}^{LI\prime}(x,0)$.

Because $\hat{I}_n \geq I_n^{LI}(1)$, we have $v_n^{LI}(x,1)=0$. Namely, $v_n^{LN}(x) \geq v_n^{LI}(x,1)$. The firm more likely orders from both supplies in Model I.

For Model I, noting that $\bar{I}_n^{LN} \leq I_n^{LN}$, we have

$$V_n^{LN\prime}(x) = \begin{cases} (1-\beta_n)\mathbf{E}R_n'(x-D_n)+\beta_n\mathbf{E}R_n'(x+q_n^{LN}(x)-D_n)-c_n^o, & x \leq \bar{I}_n^{LN}; \\ (1-\beta_n)\mathbf{E}[R_n'(x-D_n)+\alpha V_{n+1}^{LN\prime}(x-D_n)]-\beta_n c_n, & \bar{I}_n^{LN} < x < I_n^{LN}; \\ \mathbf{E}[R_n'(x-D_n)+\alpha V_{n+1}^{LN\prime}(x-D_n)], & x \geq I_n^{LN}. \end{cases} \tag{25}$$

For Model III, for $i=0$, we have $V_n^{LI\prime}(x,0)$ and $V_n^{LI\prime}(x,1)$ given by (21) and (22), respectively. Since $\hat{I}_n \geq I_n^{LI}(1)$ and $\bar{I}_n^{LI}(1)=\bar{I}_n^{LI}(0)$, we have $I_n^{LI}(1) \geq \bar{I}_n^{LI}(0)$. Note that for Model I, when $x \leq I_n^{LN}$, $V_n^{LN\prime}(x) \leq (1-\beta_n)\mathbf{E}[R_n'(I_n^{LN}-D_n)+\alpha V_{n+1}^{LI\prime}(I_n^{LN}-D_n)]-\beta_n c_n$ due to the convexity. Then, by (25), (21), (22), we can show that $V_n^{LN\prime}(x) \leq \beta_n V_n^{LI\prime}(x,1)+(1-\beta_n)V_n^{LI\prime}(x,0)$.

Case 2: $\bar{I}_n^{LI}(1) < \hat{I}_n < \hat{I}_n^o$. Note that $I_n^{LN}=\hat{I}_n$ and $I_n^{LI}(1)$ is given by (20). According to the definition of $\hat{I}_n$ given by (19), we have $\hat{I}_n \geq I_n^{LI}(1)$. Thus, $I_n^{LN} \geq I_n^{LI}(1)$. For this case, because $\bar{I}_n^{LI}(1) < \hat{I}_n$, we still have $v_n^{LI}(x,1)=0$. Then, (ii) is also true for this case.

For Model I, we have $I_n^{LN}=\hat{I}_n$, $\bar{I}_n^{LN}=\hat{I}_n^o$, and

$$V_n^{LN\prime}(x) = \begin{cases} (1-\beta_n)\mathbf{E}R_n'(x-D_n)+\beta_n\mathbf{E}R_n'(x+q_n^{LN}(x)-D_n)-c_n^o, & x \leq \hat{I}_n; \\ \mathbf{E}R_n'(x-D_n)-c_n^o, & \hat{I}_n < x < \hat{I}_n^o; \\ \mathbf{E}[R_n'(x-D_n)+\alpha V_{n+1}^{LN\prime}(x-D_n)], & x \geq \hat{I}_n^o. \end{cases} \tag{26}$$

For Model III, for $i=0$ and $i=1$, $V_n'(x,0)$ is given by (21) and (22), respectively.

For Model I, because of the convexity, $V_n^{LN\prime}(x) \leq \mathbf{E}R_n(x-D_n)-c_n^o$ for $x \leq \hat{I}_n$. Because of $\bar{I}_n^{LI}(1) < \hat{I}_n$, we have $\hat{I}_n \geq I_n^{LI}(1) \geq \bar{I}_n^{LI}(1)$. Then, by (21), (22), (26), we can show that (i) is true.

Case 3: $\hat{I}_n \leq \bar{I}_n^{LI}(1)$. For that case, we have $I_n^{LN}=I_n^{LI}(1)=\hat{I}_n$. It is clear that for $x \geq \hat{I}_n$, $v_n^{LN}(x)=(\bar{I}_n^{LN}-x)^+$ and $v_n^{LI}(x,1)=(\bar{I}_n^{LI}(1)-x)^+$. Because of $\bar{I}_n^{LN}=\hat{I}_n^o$, $I_n^{LN} \geq \bar{I}_n^{LI}(1)$. Thus, $v_n^{LN}(x) \geq v_n^{LI}(x,1)$.

For Model I, $V_n^{LN\prime}(x)$ is given by (26). For Model III, for $i=0$, $V_n^{LI\prime}(x,0)$ is given by (21). For $i=1$, $V_n^{LI\prime}(x,1)$ is given by (23). For Model I, because of the convexity, $V_n^{LN\prime}(x) \leq \mathbf{E}R_n(x-D_n)-c_n^o$ for $x \leq \hat{I}_n$. Then, noting that $\bar{I}_n^{LI}(1)=\bar{I}_n^{LI}(0)$, by (21), (26), (23), we can show that (i) is true.□

Proof of Theorem 5. The proof follows the similar argument of Theorem 4.□

Proof of Proposition 5. Let us start from period N. For Model III, we obtain that $V_N^{LI\prime}(x,1)$ is given by (24) and $V_N^{LI\prime}(x,0)=\mathbf{E}R_N'(x-D_N)$.

For Model IV, we have

$$V_N^{OI\prime}(x,i) = \begin{cases} -c_N, & x < I_N^{OI}(i); \\ \mathbf{E}R_N'(x-D_N), & x \geq I_N^{OI}(i). \end{cases}$$

Because of $I_N^{LI}(1)=I_N^{OI}(i)$, we have $\beta_N V_N^{LI\prime}(x,1)+(1-\beta_N)V_N^{LI\prime}(x,0) \leq \beta_N V_N^{OI\prime}(x,1)+(1-\beta_N)V_N^{OI\prime}(x,0)$. And, (ii) is also true for period N.

Assuming that it holds for period $n+1$, we will show that it also holds for period n. By definition, because of the convexity and

$$\beta_{n+1}V_{n+1}^{LI\prime}(x,1)+(1-\beta_{n+1})V_{n+1}^{LI\prime}(x,0) \leq \beta_{n+1}V_{n+1}^{OI\prime}(x,1) + (1-\beta_{n+1})V_{n+1}^{OI\prime}(x,0),$$

we have $\bar{I}_n^{LI}(1) \geq \bar{I}_n^{OI}(1)$.

We need to consider three cases: $\hat{I}_n \geq \bar{I}_n^{LI}(1)$, $\bar{I}_n^{OI}(1) < \hat{I}_n < \bar{I}_n^{LI}(1)$, $\hat{I}_n \leq \bar{I}_n^{OI}(1)$.

Case 1: $\hat{I}_n \geq \bar{I}_n^{LI}(1)$. For this case, by definition, $I_n^{LI}(1)$ is given by (20). By (11), $I_n^{OI}(1)$ is given by

$$\mathbf{E}[R_n(I_n^{OI}(1)-D_n)+c_n+\alpha\beta_n V_{n+1}^{OI\prime}(I_n^{OI}(1)-D_n,1) + \alpha(1-\beta_n)V_{n+1}^{OI\prime}(I_n^{OI}(1)-D_n,0)]=0. \tag{27}$$

Thus, $I_n^{LI}(1) \geq I_n^{OI}(1)$. Because $q_n^{LI}(x,1)=(I_n^{LI}(1)-x)^+$ and $q_n^{OI}(x,1)=(I_n^{OI}(1)-x)^+$, we have $q_n^{LI}(x,1) \geq q_n^{OI}(x,1)$. For both models, there is no overseas order for $i=1$. Then, (ii) is true.

For Model III, $V_n^{LI\prime}(x,0)$ and $V_n^{LI\prime}(x,1)$ are given by (21) and (22), respectively.

For Model IV, because of $v_n^{OI}(x,i)=0$, we have

$$V_N^{OI\prime}(x,i) = \begin{cases} -c_n, & x < I_n^{OI}(i); \\ \mathbf{E}[R_n'(x-D_n)+\alpha\beta_n V_{n+1}^{OI\prime}(x-D_n,1), \\ \quad +\alpha(1-\beta_n)V_{n+1}^{OI\prime}(x-D_n,0)], & x \geq I_n^{OI}(i). \end{cases} \tag{28}$$

Noting that $I_n^{OI}(1)=I_n^{OI}(0)$, by comparing (21), (22), and (28), we can show that (i) is true.

Case 2: $\bar{I}_n^{OI}(1) < \hat{I}_n < \bar{I}_n^{LI}(1)$. For this case, by definition, $I_n^{LI}(1)=\hat{I}_n$. And, $I_n^{OI}(1)$ is still given by (27). Since $\bar{I}_n^{OI}(1) < \hat{I}_n$, we have $\hat{I}_n \geq I_n^{OI}(1)$. Thus, $I_n^{LI}(1) \geq I_n^{OI}(1)$.

For Model III, we have $v_n^{LI}(x,1) \geq 0$ while $v_n^{OI}(x,1)=0$ for Model IV. Thus, (ii) is true.

For Model III, $V_n^{LI\prime}(x,0)$ and $V_n^{LI\prime}(x,1)$ are given by (21) and (23), respectively. For Model IV, $V_n^{OI\prime}(x,i)$ is still given by (28). Noting that $\bar{I}_n^{LI}(1)=\bar{I}_n^{LI}(0)$ and $I_n^{OI}(1)=I_n^{OI}(0) \geq \bar{I}_n^{OI}(1)$. By comparing (21), (23), and (28), we can show that (i) is true.

Case 3: $\hat{I}_n \leq \bar{I}_n^{OI}(1)$. For this case, we have $I_n^{LI}(1)=I_n^{OI}(1)=\hat{I}_n$. And, we have $v_n^{LI}(x,1)=(\bar{I}_n^{LI}(1)-x)^+$ and $v_n^{OI}(x,1)=(\bar{I}_n^{OI}(1)-x)^+$. Because of $\bar{I}_n^{LI}(1) \geq \bar{I}_n^{OI}(1)$, it is obvious that $v_n^{LI}(x,1) \geq v_n^{OI}(x,1)$. Then, (ii) is true.

For Model III, $V_n^{LI\prime}(x,0)$ and $V_n^{LI\prime}(x,1)$ are given by (21) and (23), respectively. For Model IV, we have

$$V_n^{OI\prime}(x,0) = \begin{cases} -c_n, & x < I_n^{OI}(0); \\ \mathbf{E}[R_n'(x-D_n)+\alpha\beta_n V_{n+1}^{OI\prime}(x-D_n,1) \\ \quad +\alpha(1-\beta_n)V_{n+1}^{OI\prime}(x-D_n,0)], & x \geq I_n^{OI}(0). \end{cases} \tag{29}$$

and

$$V_n^{OI\prime}(x,1) = \begin{cases} -c_n, & x < \hat{I}_n; \\ \mathbf{E}R_n(x-D_n)-c_n^o, & \hat{I}_n < x < \bar{I}_n^{OI}(1); \\ \mathbf{E}[R_n'(x-D_n)+\alpha\beta_n V_{n+1}^{OI\prime}(x-D_n,1) \\ \quad +\alpha(1-\beta_n)V_{n+1}^{OI\prime}(x-D_n,0)], & x \geq \bar{I}_n^{OI}(1), \end{cases} \tag{30}$$

Since $\hat{I}_n \leq \bar{I}_n^{OI}(1)$, $\bar{I}_n^{OI}(1) \geq \bar{I}_n^{OI}(0)$. By comparing ((21), (23), (29), and (30)), we can show that (i) is true.□

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