Coexistence of competing strategies in evolutionary games

Zhang, Jianlei

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Chapter 1

General Introduction and Thesis Outline

Cooperation, where one incurs a cost to confer a benefit on another, is regarded as a fundamental building block of all the life in nature and particularly human societies. However, it is hard to understand this seemingly altruistic behavior from the perspective of natural selection rule where maximizing one's benefits stems from a survival instinct of individual organisms in nature. Evolutionary game theory analyzes models of rational and selfish individuals acting in their own best interests, models of competition and cooperation between game players. The complex network theory, one of the big concerns for engineering and computer science researchers, also pays its attention towards networks that depict the gaming connections of populations. Combining the evolutionary game theory with the complex network theory yields an interdiscipline direction which has received a rapidly increasing amount of attention in recent years.

Moreover, individual heterogeneity and biological or social diversity are also well-known phenomena in nature and in social society of humans. It is a main focus whether and how biodiversity affects the emergence and transmission of strategy, disease, information, opinion and so on. The potential difficulties brought by individual heterogeneity in mathematical modeling, raise important challenges for existing theoretical models which have only considered simple individuals in games. However, many more studies concerning with the individual heterogeneity or diversity and their possible coexistence, in the framework evolutionary game theory, are expected in the near future. Only in this way could we gain more hints on cracking a series of perplexing puzzles about cooperative phenomena in the real social society.

In this dissertation, I apply the approaches from mathematics, statistical physics, computer science and engineering to explore the competing dynamics in the related populations involved in social dilemma situations. Herein I present a collection of evolutionary game theoretic models that help to explore questions related to the origins and maintenance of cooperative actions in nature. The basic line of this thesis is addressing the role of individual heterogeneity in promoting cooperation. In this thesis, I aim to contribute to our understanding about the interplay between the individual heterogeneity and human cooperation, by the aid of establishing effective theoretical models in the framework of evolutionary game theory. First, I endow the players with switching probabilities between strategies, and study whether and (if the answer is yes) how different switching probabilities affect the strategy evolution dynamics in the gaming population (Chapters 2 and 3). Second, I investigate the individual difference on the time scales in strategy updating process, hoping to shed light on how cooperation can be influenced by the individual diversity or heterogeneity (Chapter 4). Third and finally, I investigate the effects of insurance on the evolution of cooperation in two scenarios (Chapters 5 and 6).

This chapter serves as a brief introduction to the evolutionary problem of cooperative behaviors among selfish populations, complex networks employed as the structure of the gaming populations, and our motivation. Finally, I will give an overview of the aims and contents of this thesis.

1.1 Background and framework

Here, we first introduce some background information and motivation for the research work in this thesis.

• The puzzle of altruism (cooperation)

In biology, altruism can be defined as an individual performing an action which is harmful or at a cost to itself (e.g., pleasure and quality of life, time, probability of survival or reproduction), but benefits, either directly or indirectly, another third-party individual, without the expectation of reciprocity or compensation for that action (Moran 1962, Hamilton 1963, Axelrod 1984, Mukherji et al. 1996, Axelrod and

Dion 1988). In this sense, cooperation is deemed as the process where groups of organisms work or act altogether for the common or mutual benefits of the groups. It is commonly defined as any adaptation that has evolved, at least in part, to increase the reproductive success of the actor's social partners (Gardner et al. 2009). For instance, territorial choruses by male lions discourage intruders and then probably benefit all the contributors in the group (Clutton-Brock 2009a). This process contrasts with the intragroup competition where agents work against each other for some selfish reasons. The diversity of taxa that exhibits cooperation is quite large, ranging from zebra herds to pied babblers to African elephants. Many animal and plant species cooperate with both members of their own species and with members of other species.

In spite of the diversity of different contexts in which agents cooperate, researchers from multi-disciplinary areas incline to focus their studies on situations in which the benefits of an individual are opposed to the interests of the collective group. Under this kind of social dilemma situations, cooperative action means a cost for the actor and benefits others. From an evolutionary perspective, cooperative behavior is puzzling due to the fact that selfish individuals help others at their own cost or expense, and hence there is the potential or temptation for exploitation of cooperative individuals by free riders, or defectors who profit at others's expense. Theoretical analysis predicts that rational individuals, who aim to maximize their payoffs or benefits in games, should behave selfishly in such circumstances. However, cooperative situations are so widespread in real-life situations such as the animal kingdom and human societies.

How cooperation among non-relatives can persist in the face of cheating and the cruel rule of 'survival of the fittest' driven by natural selection, remains a puzzling, fascinating and broad-ranging unsolved question in evolutionary biology. Moreover, this interdisciplinary topic has also obtained plenty of attention, interest and research across disciplines, i.e. social sciences, behavioral sciences, psychology, physics, computer science, engineering and so on. Explaining the cooperation evolution is not only an issue of central importance to evolutionary biology but also one of hot interdisciplinary topics so far, since it is commonplace throughout all levels of the natural world.

Human societies are founded on cooperation, and psychologists and economists have explored what and how personal factors motivate agents to cooperate. It is plausible that the individual heterogeneity cannot be neglected when studying the cooperation of real agents in real social societies. And, the details of individual personal factors are so complicated that needs much more attention in the future study. These empirical studies complement a large body of theoretical work from evolutionary game theory by proposing some effective mechanisms. However, there still need much more effort to establish (possibly complicated) mathematical models involving the individual personal factors, also to verify the effectiveness of experimental results in the related works. This is also the focus and motivation of this thesis, to dig into the complex issues of cooperation that are overwhelming us from the perspective of individual heterogeneity, by the aid of mathematical analysis and agent based evolutionary simulations in computer science and engineering.

• Evolutionary game models

As mentioned, altruism refers to a costly behavior that benefits others. However, mutual cooperation is often found in nature even when selfish behavior is apparently rational for individuals. Thus, social dilemmas are situations in which the optimal decision of an individual contrasts with the optimal decision for the group. Why and under what circumstances, presumptively selfish agents cooperate is a question of longstanding interest to multidisciplinary research (Nash 1950b, Nash 1951, Axelrod 1980, Axelrod and Hamilton 1981, Axelrod 1984, Dawkins 1976, Axelrod and Dion 1988, Alexander and Irvine 1987, Colman 2006, Diggle et al. 2007, Doebeli et al. 2004, Bendor and Swistak 1995). Game theory is one of the key paradigms behind many scientific disciplines from biology to behavioral sciences to economics.

Past studies used simple game theory models, such as the classic prisoner's dilemma models, to determine decisions made by individuals in cooperative interactions. However, complicated interactions between individuals need more complicated concepts such as the Nash equilibrium (Nash 1950a, Nash 1951, Nash 1950b). The Nash equilibrium is frequently used in a type of non-cooperative game theory that assumes an individual's decision is affected by its knowledge of the strategies of other individuals. This theory is novel in considering the higher cognitive capabilities of gaming individuals. The evolutionarily stable strategy (Maynard Smith and Price 1973) is a refined version of the Nash equilibrium in that it assumes strategies are heritable and are subject to natural selection. Established by Maynard Smith and Price (Maynard Smith 1978, Maynard Smith 1979), evolutionary game theory provides a competent theoretical framework to address the subtleties of cooperation among selfish and unrelated individuals. Moreover, evolutionary game theory is an interdisciplinary mathematical tool which seems to be able to embody several relevant features of the problem and, as such, is used in much cooperation-oriented research. By the aid of evolutionary game theory, vast theoretical or experimental mechanisms for emergence and maintenance of cooperation in social dilemma games have been reported thus far (Clutton-Brock 2009b, dos Santos et al. 2011, Eldakar and Wilson 2008).

The referred social dilemmas are described as the situation where individual benefits are opposed to that of the group. In investigating the social dilemma problem, the standard framework utilized is evolutionary game theory together with its extensions involving evolutionary context. Since in this thesis I will not restrict the work to one specific form of social dilemma model, here I list some of them as follows for reference. In existing studies, the prisoner's dilemma game is unrivaled in popularity when it comes to studying the evolution of cooperation through pairwise interactions (Axelrod 1980, André and Day 2007, Nowak and Sigmund 1989, Andreoni and Varian 1999, Ashlock et al. 1996, Zhang, Chen, Zhang, Wang and Chu 2010a). The game promises a defector the highest payoff if encountering a cooperator. Meanwhile, the exploited cooperator is worse than a defector playing with another defector. In line with the principles of Darwinian's natural selection, defection will be the dominating strategy of the population.

Relaxing the inevitability of a social downfall resulted by the well-mixed prisoner's dilemma game is the snowdrift game or hawk-dove games (Ahmed and Elgazzar 2000). Other interesting games also constitute powerful metaphors to describe conflicting situations often encountered in natural and social sciences. For example, trust game (Anderhub et al. 2002), volunteer's dilemma (Archetti 2009b), donor-recipient games (Berger 2009), stag hunt dilemmas (Pacheco et al. 2009a, Pestelacci and Tomassini 2008, Skyrms 2004), predator–prey game (Abbott 2010) and so on. Whenever collective action of groups of individuals is at stake, N-

person games are appropriate. Many previous investigations of cooperation have employed the N-person prisoner's dilemma games in the form of public goods game (Andreoni 1995, Archetti 2009a, Barclay 2004, Zhang, Zhang and Chu 2011) to study the possibility of emergence of cooperation among groups of interacting agents. Chapter 2, entitled Strategy updating for evolution in interaction networks, employs the prisoner's dilemma game and snowdrift game for model setting and an effective comparison. Chapters 3 and 5 adopt the prisoner's dilemma game, snowdrift game and stag hunt game for a systematic study to ask whether the specific dilemma model affects the evolution dynamics of the populations. Chapter 4 uses the public goods game, a classical n-person paradigm for recurring game interactions, to investigate the four competing strategies in such settings.

- **Competing and coexistence of strategies**

Cooperation and defection are the two strategies that are at the heart or core of each social dilemma. Cooperators make contribution to the collective benefit at a personal cost or damage, while defectors make no contribution and take advantage of others' contributions. Since individual heterogeneity is a common phenomenon in nature and society, and real agents always face multiple strategy choices in the competition with others involved in social dilemma situations. This is particulary true in the context of human cooperation where human decision making is probably shaped by a wealth of individual factors.

Based on these considerations, aside from these two classical strategies which indicate obligatory participation, many different strategies (e.g., loner and punishment) have also been proposed to investigate their potential roles in resolving the cooperation dilemma problems. Voluntary participation (Hauert et al. 2002b, Hauert et al. 2002a) allows players to adopt a risk-aversion strategy, named as loner. A loner refuses to join in an unpromising public enterprise and instead relies on a small but fixed payoff. Cooperation can also be stabilized by punishment. In theory and in experiments, punishment has turned out to be a simple but effective mechanism to prevent cheating. There is now a rich literature on whether and how various forms of punishment are effective in bringing about cooperation (van den

Berg et al. 2012, Perc and Szolnoki 2010), peer punishment (Boyd et al. 2003, Hauert et al. 2007), pool punishment (Szolnoki et al. 2011), and anti-social punishment (Rand and Nowak 2011). Besides, our previous work proposes another role in game playing, named as insurance against punishment, enriching the potential strategy profiles for players (Zhang et al. 2013).

As for the cooperation problem, a major challenge for the involved researchers is to explain how cooperation is maintained or even dominates in a selfish population, by the aid of effective mechanisms which help the competition, invasion or domination of cooperators. However, reality suggests that a typical characteristic for the real societies and nature is the individual heterogeneity, social diversity and coexistence of competing partners. Such a society, from the perspective of evolution, is a society where the competition and cooperation coexist, and thus is a complicated system. Even if our genes may instruct us to be selfish, we are not necessarily compelled to obey them all our lives (Dawkins 1976). And importantly, the coexistence, not only competition, of multiple competing decisions indeed can be observed in real society and nature. Actually, agents often adopt multiple choices in decision making due to the internal personality factors or intervention of external factors, especially when facing the complicated cooperative dilemma situation. It may be reasonable and meaningful to share or split some attention to the coexistence of competing roles, when we rack our brains trying to figure out how cooperation can defeat all the other strategies, and dominate the population finally. This is also the starting point of my thesis.

Modeling the additional strategy options found in several real-life systems, has also evolved into a mushrooming avenue of research. Probing into more strategies not only stems from the need to provide new ways of fostering cooperation in situations constituting social dilemma, but also help us understanding the coexistence of multiple competing strategies or social diversity in nature. Inspired by this fact, we propose a new strategy named as speculator and comprehensively examine four kinds of strategies (cooperation, defection, loner, and speculators) in Chapter 4. These model settings are inspired by the existing insurance behaviors in economic systems. To fill in the gap between theoretical model and economic behaviors in real social society, we establish the mathematical model and focus on the evolution of evolutionary dynamics in this setting. It is remarkable that proposing more competing strategies in the gaming populations is still widely open to research and thus holds promises of exciting new discoveries. Moreover, it is worthy noting that we also relax the limitation of uniform players in the field of switching probability (Chapters 2 and 3) and time scales (Chapter 5), to study the competition or coexistence of different players in the games.

• History of cooperation research

One of the first references to animal cooperation was made by Charles Darwin, who noted it as a potential problem for his natural selection theory (Darwin 1859, Darwin 1871). He proposed many mechanisms that could help explain why cooperation could be favored over selfish behaviors. Nowadays, the mechanisms introduced by Darwin are still at the core stage of research for solving the puzzle of cooperation evolution. Prominent biologists, such as E.O. Wilson, and W.D. Hamilton, have also found the evolution of cooperation fascinating because natural selection favors those who obtains the greatest reproductive success while cooperative behavior often decreases or inhibits the reproductive success of the individual performing the cooperative action (Clutton-Brock 2009a). Additionally, some species have been found to perform cooperative behaviors that may even be detrimental or harmful to their own evolutionary fitness or survival benefits. For example, when a ground squirrel sounds an alarm call to warn other group members of a nearby coyote, it attracts the coyote's attention to itself and meanwhile increases its own risk of being eaten and death (Sherman 1977). Therefore, cooperation poses a fundamental puzzle to the traditional theory of natural selection, which rests on the assumption that individuals selfishly and vehemently compete to survive and maximize their reproductive successes in nature.

'How did cooperative behavior evolve' was regarded as one of the top 25 big questions facing science over the next quarter-century, proposed for celebrating the journal of *SCIENCE*'s 125 anniversary in 1995. And, Robert May began his last presidential address to the Royal Society on 30 November 2005 by saying:" The most important unanswered question in evolutionary biology, and more generally in the social sciences, is how cooperative behavior evolved and can be maintained in human or other animal groups and societies" (Colman 2006). In this sense, cooperation problem has already been an issue of central importance to many disciplines, including the active members of engineering, physics and computer science. More importantly and meaningfully, achieving a satisfactory understanding of the evolution of cooperation in social dilemmas is fundamental for elucidating many important problems in social sciences, such as the sustainability of social diversity, information or strategy spreading, public resources consumption, public project provision and so on.

Since widespread cooperation is crucial for the prosperity of society and is frequently encountered in nature, many studies and new approaches aimed towards resolving the social dilemma have been spawned. It is worthwhile to highlight some of them here to acknowledge their contributions, and also since some of them will be referred in the discussions of this thesis. For example, the kin selection theory focuses on cooperation among individuals that are genetically related. Darwin recognised that reciprocity may lead to or foster cooperation: when individuals interact with each other repeatedly, a cooperative action may be returned later when the games proceed. Theories of direct reciprocity thus focus on the selfish incentives for cooperation in bilateral long-term interactions (Hamilton 1964, Clutton-Brock 2002, Nowak 2006, Ohtsuki and Nowak 2007, Pacheco et al. 2008). The theories of indirect reciprocity and signalling show how cooperation in larger groups can emerge when the cooperators can build a reputation (Nowak and Sigmund 2005, Brandt and Sigmund 2005). Other effective mechanisms or rules see (Gross and Blasius 2008a, Perc and Szolnoki 2010) for a comprehensive review, and references therein.

Particularly vibrant in recent years has been the subject of evolutionary games on complex networks. The ubiquity and importance of complex networks raised quite naturally the question of how natural selection works on top of different kinds of network topologies of agents. It is well known that the evolution of cooperative behavior is dependant upon certain environmental conditions. And, in realistic multi-player systems players do not interact with all other players. One such condition that has been extensively studied is the use of a spatially structured population (Alonso-Sanz 2009, Amaral et al. 2000, Arapaki 2009, Barabási and Albert 1999, Zhang et al. 2014). The key concept of spatially structured populations is: agents are assigned to the vertices of a network, which can be a regular lattice or has a more complex structure. The edges denote links between players in terms of game dynamical interactions. Then, agents are constrained to interact only with their adjacent neighbors to play evolutionary games in which more successful strategies spread on the system, if a social dilemma is embedded in a richer game theoretical structure.

The preceding transitions from well-mixed populations to spatial grids and further to complex networks, and particularly their success in promoting the evolution of cooperation, invite further extensions of the theoretical framework. Further, a variety of studies suggest that coevolution, including migration, is also a relevant factor to take into account in as much as they may enhance strong altruism (Szabó and Fáth 2007, Ohtsuki et al. 2006, Zhang, Zhang and Chu 2011). And the evolution and coevolutiion of dynamics in multi-layer complex networks has added a new wrinkle to this transatlantic research on cooperation.

Here is a very brief introduction about the complex network and the networked gaming populations:

- Node: the node is the principle unit of the network. A networks consists of a number of nodes connected by edges. In a typical setup of spatial evolutionary games, agents are assigned to the nodes of the network.

- Neighbors: two nodes are said to be neighbors if they are connected by a link or edge.

- Link: a link is a connection between two nodes in the networks. In the common setup of spatial evolutionary games, the edges denote links between the corresponding players in terms of game dynamical interactions.

- Degree: the degree of a node is the number of closest neighbors to which a node is interacted with. The average degree of the network is the mean of the individual degrees of all the nodes in the network.

- Dynamics: depending on the context, the word dynamics is used in the literature to refer to a temporal change of either the state or the topology of a network. In the common setup of spatial evolutionary games, it denotes the evolutionary game dynamics occurring on the interactions, being subject to the specific strategy updating rules or the introduced coevolution dynamics between networks and strategies.

The integration of the microscopic patterns of interactions among the agents composing a large population into the evolutionary setting provides a way out for cooperation to survive in paradigmatic scenarios. This is also an extremely hot topic in recent years, and attracts plenty of attention of researchers especially from engineering and computer science. The body of literature devoted to this topic is extensive, from game dynamics on static networks to evolving complex networks, from regular lattice network to complex real-world networks. Along this booming line, many more studies concerning with the individual heterogeneity or diversity on complex network are expected in the near future. The most often employed networks are: random regular network (Wormald 1981), lattice network (Nowak et al. 1994), small-world networks (Watts and Strogatz 1998, Watts 1999, Newman and Watts 1999), scale-free graphs (Barabási and Albert 1999), evolving networks (Skyrms and Pemantle 2000) and so on. Based on this consideration, in Chapters 2 and 3 of this thesis, the random regular graph and BA scale-free networks are both employed for investigating the competing strategies among the structured populations.

• **Dynamics of evolution**

A model in evolutionary game theory is made complete by postulating the game dynamics, i.e., the rules that describe the update of strategies in the population. Depending on the actual problem, different kinds of dynamics can be appropriate. The game dynamics can be continuous or discrete, deterministic or stochastic, and within these major categories a large number of different rules can be formulated depending on the situation under investigation. On the macroscopic level, by far the most studied continuous evolutionary dynamics is the replicator dynamics. It was originally introduced by Taylor and Jonker (Taylor and Jonker 1978), and it has exceptional status in the models of biological evolution. On the phenomenological level the replicator dynamics can be postulated directly by the reasonable assumption that the per capita growth rate $\dot{\rho}_i/\rho_i$ of a given strategy type is proportional to the fitness difference (Szabó and Fáth 2007).

$$\frac{\dot{\rho}_i}{\rho_i} = \text{fitness of type i - average fitness}$$

The fitness is the individual's evolutionary success, i.e., in the game theory context the payoff of the game.

A large number of different population-level dynamics are discussed in the game theory literature. These can be either derived rigorously from microscopic (i.e., agent-based) strategy update rules in the large population limit, or they are simply posited as the starting point of the analysis on the aggregate (population, macro) level. Many of these share important properties with the replicator dynamics, others behave quite differently. An excellent review on the various game dynamics in (Hofbauer and Sigmund 2003).

Evolutionary game dynamics generally involve how players update their strategies as time evolves. The updating rules are therefore crucial and, until now, most of them are based on replication and imitation (Nowak and Sigmund 2004, Schlag 1999). The essence of replication rules is that a strategy with better performances has a higher replication rate. Imitation rules assume that a player can imitate her opponents' strategy with a probability when interacting with individuals having obtained higher payoffs.

One much studied approach to spatial games is based on a more detailed modelling of the networks of interacting players. Considering the simplest case, players situate at the nodes of a given lattice (Hofbauer and Sigmund 2003). At each of the (discrete) time steps t, each agent k participates in pairwise interactions with each of the partners l from some neighborhood $N(k)$. Each game yields a payoff $P(k,l)$, and player k's total payoff is determined by $P(k) := \Sigma_{l \neq k \in N(k)} P(k,l)$. Next, players make strategy updating for larger payoffs through some imitation rule. For example, player k compares payoff with all her neighbors $l \in N(k)$ and finally adopts the strategy of the best performer. Again, many variants are possible: in particular, the set of k's potential role models could be distinct from $N(k)$, the imitation rule could be stochastic rather than deterministic, the updating of the strategy could occur at different times for different players, the neighborhood lattice could evolve in time, etc. It is worth noting that Chapter 5 in this thesis proposes multiple time scales in strategy updating in theoretical game model, aiming to enhance our understanding of cooperation.

Hence, to apply such update rules, players have to know in general the exact magnitudes of the payoffs of all her opponents. In Chapters 2 and 3 of this thesis, we aim at decreasing the need for specific information by introducing switching probability endowed with players, and study the corresponding evolution dynamics underlying game theory.

1.2 Related approaches

1.2.1 Infinite populations

For infinite populations, the main analysis tools are the Lotka-Volterra equations. The competitive Lotka-Volterra equations, proposed by Lotka (Lotka 1922, Lotka 1925) and Volterra (Volterra 1926), are a simple model of the population dynamics of species competing for some common resource.

Here x represents the number of preys, and y denotes the number of predators. The Lotka-Volterra equations often takes the following form:

$$\begin{aligned}\frac{dx}{dt} &= x(\alpha - \beta y)\\ \frac{dy}{dt} &= -y(\gamma - \delta x),\end{aligned}$$

where α denotes the birth rate of preys, β represents the effect predators have on the population of preys, γ represents the death rate of predators, and δ means the the effect preys have on the population of predators.

1.2.2 Finite populations

It is plausible that the size of populations in real societies and nature are often finite. Stochastic noise will occur when employing the finite population with the deterministic dynamics equation, for example the above mentioned Lotka-Volterra equations. For finite populations, stochastic models often seem more realistic than deterministic ones. For computer simulations of multi-agent systems, this is the only natural approach to model the dynamics. In this case, the stochastic processes theory is effective in the dynamic analysis of evolutionary games in finite populations. The most often used examples for stochastic evolutionary game dynamics are Pairwise comparison process, Moran process and Wright-Fisher process (Traulsen and Hauert 2009). Here we give a brief introduction about these mentioned process.

- Moran process

  Assuming that A and B are the two available strategies in the gaming population. we focus on a population of size N, consisting of i A players and $N - i$ B players. The probability of an A encounters with another A is $i/(N-1)$, and the probability of A encounters with a B is $(N-i)/(N-1)$. For an agent B, its probability of encountering with an A is $i/(N-1)$, and the probability of encountering with another B is $(N-i-1)/(N-1)$. Thus, the expected payoffs of player A and B are respectively given by

$$\begin{aligned} F_i &= \frac{a(i-1)+b(N-i)}{N-1} \\ G_i &= \frac{ci+d(N-i-1)}{N-1}. \end{aligned}$$

  In the above equations, F_i and G_i are the expected payoffs of player A and B when there are i A players in the investigated population. The fitness of agent A and B under natural selection are respectively described by

$$\begin{aligned} f_i &= 1-\beta+\beta F_i \\ g_i &= 1-\beta+\beta G_i, \end{aligned}$$

  where the constant β is called the intensity of selection since $\beta \to \infty$ leads to strong selection where the probability for selecting fitter individual is 1 and when $\beta \ll 1$, the update reduces to the Moran process under weak selection (Nowak et al. 2004).

- Pairwise comparison process

In this kind of dynamics, two agents, a focal individual and a role model, are sampled randomly from the large population. The focal one imitates the strategy of the role model with probability p, which depends on the corresponding payoff comparison. If both individuals gain the same payoff, the focal individual randomizes between the two strategies. One common choice of a nonlinear function of the payoff difference for p is the Fermi function from statistical mechanisms, given by

$$p(s_x \leftarrow s_y) = \frac{1}{1 + \exp[(P_x - P_y)/\omega]}, \tag{1.1}$$

where the magnitude of ω characterizes the uncertainty related to the strategy update. P_x and P_y are the payoffs of agent x and y respectively. For finite positive values of ω, strategies performing worse may also be adopted based on unpredictable variations in payoffs or errors in the decision making. For weak selection $\omega \ll 1$, the probability p reduces to a linear function of the payoff difference. For strong selection $\omega \to \infty$, this process converges to the imitation dynamics. In this case, p becomes a step function being positive for $P_x < P_y$ and negative for $P_x > P_y$.

- Wright-Fisher process

  The Wright-Fisher process is also rooted in population genetics. Different from the selection dynamics in the Moran process, where only one individual reproduces at a time, the Wright-Fisher process represents discrete generations. In each generation, each member of the population with size N produce a large number of offsprings, proportional to their fitness. From the large offspring members, a new generation of size N will be sampled at random. In this situation, the population composition can change or update much faster. The population could go back to a single ancestor in a single generation. This suggests the fact that the Wright-Fisher process is a more general Markov process.

The main approaches we employed in the thesis are Discrete-time Markov chain, Stochastic process theory, Pair approximation analysis, Mean-field analysis, and Monte Carlo simulations. By the aid of them, this thesis mainly focuses on the coexistence of competing strategies, and the main factors in evolutionary game theory such as strategies and time scales, in evolutionary games.

1.3 Scope of this thesis

The objective of this thesis is manifold, it contains:

- the introduction of evolutionary game theory and the cooperative behaviors in structured populations,
- a new method based on that switching probability of competing strategies between players in structured populations,
- introduction of the expanding strategy profile in the evolutionary game theory and
- introduction of diversity of time scales in strategy updating process.

Individual heterogeneity is one of a most often observed phenomenon in realistic systems. Explaining the competition and coexistence of individual diversity is an open question. There are several ways how this feature can be built into a model. Henceforth we present an extensive, systematic study concentrating on the potential heterogeneity of individual behaviors. The proposed individual differences refer to strategy decisions, time scales, transfer probabilities of strategies. Depending on the microscopic details these features can either decrease or increase the frequency of cooperators in the gaming population, indeed influencing the evolutionary dynamic outcomes. Nonetheless, the investigation of evolutionary games on these topics is still widely open to research, and will lead to the exploration of new phenomena and thus raise a number of interesting questions.

It is worth recalling that some basic information (e.g., payoffs at least), are required in the strategy updating rules or dynamics. For example, players will imitate the strategy of those neighboring players (including themselves) who has scored the highest payoff. However, from the viewpoint of real societies, the traditional assumption is often unrealistic. Even in simple interactions between two individuals A and B, it is not easy to obtain full information of partners' decision making, as individuals usually acquire rather limited or even wrong information about the gaming partners or other reference objects. Simply stated, the information acquisition ability and results vary among different real social agents. In **Chapter 2** we tackle the problem of payoffs dependency issue in strategy updating, we introduce a new strategy changing updating rule, an intriguing feature of which is the absence of related payoff information. More specially, we propose the switching probability between competing strategies and employ them for strategy updating, and this novel approach can be successfully used in various specific gaming models. The results presented in this chapter have been published in (Zhang, Zhang, Cao and Weissing 2015)

Chapter 3 further extends the proposed switching probabilities to a more general case. Herein strategy switch happens among all the individuals, not only the restrictive case for the competing strategists (cooperator and defector). Therefore, we introduce and analyze an alternative way of establishing the strategy renewal for interacting players. The work may be helpful in reflecting the real phenomenon in social systems.

The time scales of gaming and strategy updating are also a crucial concept and feature responsible for the cooperative phenomena. **Chapter 4** of this thesis focuses on the multiple time scales in strategy updating. The corresponding work gives mathematical evidence that heterogeneity in time scales enriches the evolutionary dynamics and under simplifying conditions, the possible outcomes can be effectively predicted under suitable situations.

The above three chapters focus on updating rules. We show that details in updating rules, for example the number of neighbours for updating and the multiple time scales in updating, have significant effects on the evolution of strategies.

In the traditional settings of classical (rational) game theory, players have two options to choose from which are called cooperation and defection. For instance, people face frequently the situation of prisoner's dilemmas in real life when they have to choose between to be selfish or altruistic, to keep the ethical norms or not, to work hard or lazy, etc. However, multiple strategy choices resulted in the complex decision making process are also notable reality in human society that can not be overlooked. Examples in previous studies include punisher, loner, and so on. Introducing more strategies combining the individual characteristics, will meaningfully help our understanding about how altruistic behavior occurs in many naturally occurring dilemma situations. In **Chapter 5** of this thesis, we introduce the insurance for cooperators into threshold publics goods games. We analyzed the conditions with different initial states and parameters. We find some scenarios where contribution to the public pool is promoted. The results presented in this chapter have been published in (Zhang, Zhang and Cao 2015)

In **Chapter 6** of this thesis, we will discuss the possibilities and conditions under which cooperative behavior can subsist in multi-agent models, with multiple strategies (cooperation, defection, loner and speculation) capable of representing a remarkably rich variety of decision choices in games. Our aim is to study the competition and coexistence of competing strategies in this productive framework.

1.4 Outline and contributions

This section briefly states the outline of the thesis and the topics of the chapters. The chapters are organized as follows:

Chapter 1 briefly introduces the background of cooperative dilemma problems and the gained research results, including the hot topic of cooperation study in complex-structured populations these years. It is the preliminary for the thesis work.

Chapter 2 provides a new approach to investigate strategy updating process in the framework of evolutionary games. In this work for two-strategy evolutionary games in structured populations, we remove the requirement for explicit information about exact payoffs, by encoding the payoffs into the willingness of any player to switch from her current strategy to the competing one. Moreover, the robustness of the proposed methods is verified in different types of game models such as the prisoner's dilemma game, snowdrift game and stag hunt game.

- Theoretical computations and numerical simulations indicate that the evolutionary dynamics are intrinsically regulated by contact relationships specified by the network topologies of the populations. More precisely, when each player plays simultaneously against more than one neighbor, strategies can easily coexist even when one strategy dominates the other in each base game. The results further reveal that the frequencies of the coexisting strategies can be calculated analytically.

- This work provides a new analysis tool in analyzing the competing dynamics of different strategies. And the results help us to find a viable escape hatch out of evolutionary stalemate.

Chapter 3 extends the individual player's switching probabilities between players, relaxing the restriction that strategy switch occurs between competing strategies. The critical ingredient that enables us discover new mechanisms for coexistence of strategies based on players' contact patterns, is exactly each player's probability of switching strategies that we have just described.

In previous studies, one of most-often used assumption is that natural selection acts on individuals at the same time scale, i.e. players renovate their strategies with the same frequency. Everyday phenomenon reminds us of the variation in learning rates within populations. Thus, evolutionary game theory may not necessarily be restricted to uniform time scales associated with the game interaction and strategy adaption evolution. In **Chapter 4** we focus our attention on a more realistic model where the population update strategies at non-uniform time scales. The basic message from results is that heterogeneity in time scales of individuals' updating will drastically enrich collective evolutionary dynamics.

- We remove the assumption of uniform time scales by dividing the population into fast and slow groups according to the players' updating frequencies. We aim to investigate how different strategy compositions of one group influence the evolutionary outcome of the other's fixation probabilities. Analytical analysis and numerical calculations are established to study the evolution dynamics of strategies in some typical classes of two-player games (Prisoner's dilemma game, snowdrift game and stag-hunt game here).

- Results show that heterogeneity in strategy-update time scales dramatically affects the dynamics of strategies. We provide a proximation formula of the fixation probability of mutant types in finite populations and study the evolution outcomes under weak selection. This work shows that heterogeneity in time scales enriches the evolutionary dynamics and under simplifying conditions, the more complicated possible outcomes can be effectively predicted in the premise that the population composition and payoff parameters are known.

Our previous work found scenarios where speculation either leads to the reduction of the basin of attraction of the cooperative equilibrium or even the loss of stability of this equilibrium, if the insurance costs are lower than the expected fines on defectors. In **Chapter 5** we extend the common binary-strategy combination of cooperation and defection by adding a third strategy, called insured cooperation, which corresponds to buying an insurance covering the potential loss resulted from the unsuccessful public goods game. We analyze the dynamics in such a three strategy system and find that insurance enhances the cooperation.

As an extension of our study proposing speculation strategy (Zhang et al. 2013), in **Chapter 6** we restrict our attention to the the analysis of replicator dynamics competed by four competing strategies: C (cooperators), D (defectors), S (speculators) and L (loners, i.e. nonparticipants). Our main interest is to probe into effective mechanisms for cooperation to get supported, when players face multiple decisions or choices. Moreover, we hope to gain more insight into the competition and coexistence of multiple strategies in nature, by the aid of this model settings.

- Results show that the evolutionary dynamic outcomes of the gaming population are closely related to the model parameters. Initialized from a three-strategy state, the system will evolve into the observed domination of some strategy or a rock-paper-scissors type of cycle, suggesting that the additional strategic options can radically alter the evolution of cooperation. And, larger multiplication factor and punishment on defectors can facilitate cooperation to be a dominant strategy in the absence of speculation. Results suggest that the option to abstain from the joint enterprise offers an escape from the social trap, leading to the decline of exploiters and allows the reemergence of cooperators.

- Moreover, public goods cooperation can also be fostered to be an equilibrium under moderate values of punishment and cost of insurance in the absence of loner. Further, cooperation fails to dominate the population in the competition with speculation and loner strategy, even though in the absence of defection. And, when the initial state consists of the four strategies, at least one strategy will go extinction within the evolution.

Finally, **Chapter 7** presents a concluding summary of the research and a collection of ideas for future work and investigation.