University of Groningen

The college gender gap reversal

Reijnders, L.S.M.

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2014

Link to publication in University of Groningen/UMCG research database

Citation for published version (APA):
Reijnders, L. S. M. (2014). *The college gender gap reversal: Insights from a life-cycle perspective*. (SOM Research Reports; Vol. 14006-EEF). University of Groningen, SOM research school.

Copyright
Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment.

Take-down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): http://www.rug.nl/research/portal. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

Download date: 13-02-2023

university of groningen / faculty of economics and business

14006-EEF

The college gender gap reversal: Insights from a life-cycle perspective

Laurie S.M. Reijnders

SOM is the research institute of the Faculty of Economics & Business at the University of Groningen. SOM has six programmes:
- Economics, Econometrics and Finance
- Global Economics & Management
- Human Resource Management & Organizational Behaviour
- Innovation & Organization
- Marketing
- Operations Management & Operations Research

Research Institute SOM
Faculty of Economics & Business
University of Groningen

Visiting address:
Nettelbosje 2
9747 AE Groningen
The Netherlands

Postal address:
P.O. Box 800
9700 AV Groningen
The Netherlands

T +31 50 363 7068/3815

www.rug.nl/feb/research

The college gender gap reversal: Insights from a life-cycle perspective

Laurie S.M. Reijnders
University of Groningen
l.s.m.reijnders@rug.nl

The college gender gap reversal: insights from a life-cycle perspective*

Laurie S. M. Reijnders#

University of Groningen; Netspar

February 2014

Abstract

Why have women surpassed men in terms of educational attainment, even though they appear to have less incentives to go to college? The aim of this paper is to set up a basic theoretical life-cycle model in order to study the potential role of gender differences in the benefit of education in explaining the college gender gap reversal. Its main contribution is to show under which conditions the model can generate a reversal in college graduation rates, and to highlight the importance of the curvature of the utility function and the presence of subsistence constraints in this respect. In particular, I show that the labour market benefit of education for women can be higher than for men even if they have the same college wage premium if the elasticity of the marginal utility of wealth is greater than unity or there are fixed costs. Initially this might be dominated by a lower marriage market return, but a decrease in the probability of marriage can induce women to overtake men in educational attainment.

Keywords: College gender gap, education, life cycle, marriage market
JEL: D91, I24, J12, J16

1 Introduction

Over the last decades women have caught up with men in many domains, but nowhere has the change been so striking as for college education. Not only did they manage to close the gap, nowadays women even graduate in larger numbers in most developed countries. This is known as the 'college gender gap reversal', see for example Goldin et al. (2006). To illustrate this phenomenon for the US, Figure 1(i) plots the fraction of females and males who completed 4 years of college or more at the age of 40 by birth year. Whereas in the 1950 cohort about 30% of the men obtained a college education versus 25% of the women, by 1970 the fraction of educated women had surpassed that of men. The same pattern shows up in the enrollment rates for tertiary education, see Figure 1(ii). Both in the US and for the EU countries the ratio of female to male enrollment has increased over time and nowadays exceeds 100%.

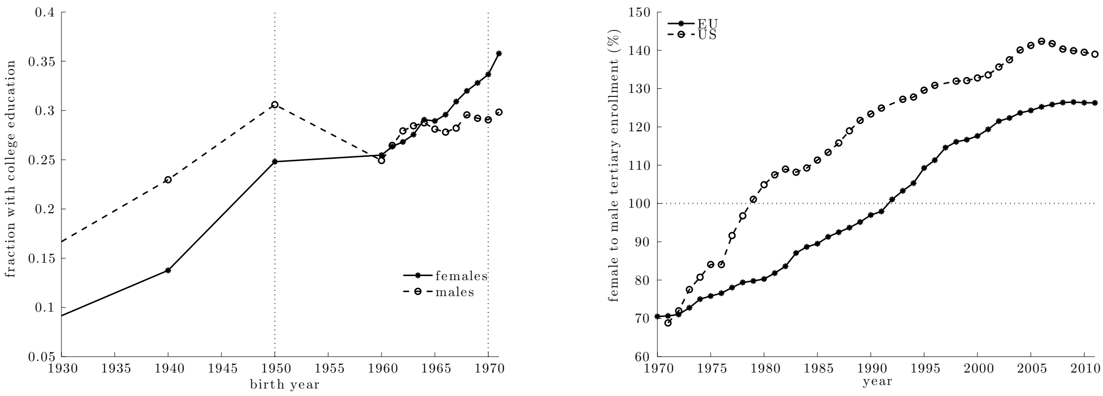
*I thank Ben Heijdra, Pieter Woltjer and Petros Milionis for valuable comments.

#University of Groningen, Faculty of Economics and Business, Nettelbosje 2, 9747 AE Groningen, The Netherlands. Phone: +31 50 363 4001. Email: `l.s.m.reijnders@rug.nl`.

Figure 1: The college gender gap reversal

(i) College graduates in US

(ii) Tertiary enrollment in US and EU



Source: Panel (i): Integrated Public Use Microdata Series (IPUMS) for 1970-2010. Panel (ii): United Nations Educational, Scientific, and Cultural Organization (UNESCO) Institute for Statistics.

There are two main strands of literature that attempt to explain gender differences in educational attainment. The first assumes that parents make education decisions for their children. This may be particularly relevant in early stages of economic development or when the existence of borrowing constraints makes family income an important source of college funding. For example, Echevarria and Merlo (1999) develop a model in which men and women bargain over a binding prenuptial agreement which specifies the investment in education of children conditional on gender. As long as the time cost of child bearing is positive, girls receive less education than their brothers.

The second approach postulates that individuals make their own decision about whether or not to obtain a college degree. When education is viewed as a pure investment decision, the college gender gap reversal is all the more puzzling since women appear to have fewer incentives to invest. On average they earn less in the labour market and spend more time on household work and child care than men, which lowers the return on their human capital. Some authors have attempted to resolve this conundrum by claiming that the relative wage of educated versus uneducated workers (the college wage premium) is actually higher for women, see for example Dougherty (2005). Others point to the returns to education that extend beyond the labour market, which include a higher probability of marriage and a greater marital surplus (Chiappori et al. (2009), DiPrete and Buchmann (2006)). According to Becker et al. (2010) the benefit of education is still higher for men despite these considerations. Instead they argue that the solution to the puzzle lies in differences in the distribution of non-cognitive skills (such as self-motivation and discipline). If women have a higher level of these skills on average and the variability among them is lower, then the supply of female college graduates is more responsive to changes in the economic environment that increase the payoff of a college education.

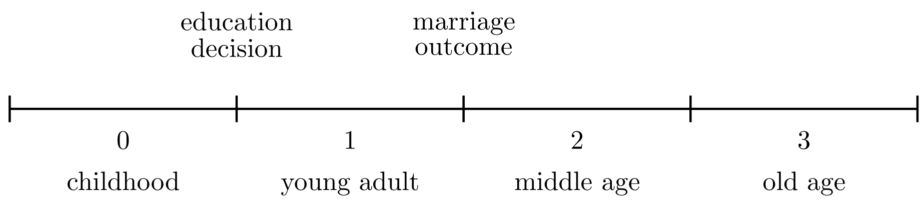
This paper is written in the spirit of the second approach, which seems more relevant for developed countries. The aim is to set up a basic theoretical life-cycle model in order to study the potential role of gender differences in the benefit of education in explaining the college gender gap reversal. The main contribution of this paper is to show under which conditions the model can generate a reversal in college graduation rates, and to highlight the importance of the curvature of the utility function and the presence of subsistence constraints in this respect. In particular, I show that the labour market benefit of education for women can be higher than for men even if they have the same college wage premium if the elasticity of the marginal utility of wealth is greater than unity or there are fixed costs. Initially this might be dominated by a lower marriage market return, but a decrease in the probability of marriage can induce women to overtake men in educational attainment.

The remainder of this paper is organized as follows. Section 2 discusses the benefits and costs of a college education and the general set-up of the model. Section 3 describes a fully specified example, which will be used to study gender differences in education choices in Section 4. In Section 5 I parameterize the model using US census data and illustrate how it can account for the college gender gap reversal. The last section concludes.

2 The benefit of education

From the perspective of an individual, education can be seen as an investment in human capital. The costs of this investment are incurred early in life, while the benefits are likely to materialize later. To study the intertemporal trade-offs pertaining to (tertiary) education I divide the life-cycle into four periods, see Figure 2. The first of these (period 0) is spent passively in the household of the parents and is ignored here. The remaining three constitute the life span of an adult individual. He or she makes the education decision at the start of period 1. It is modelled here as a binary choice: either a person obtains a college degree ($E^j = 1$) or not ($E^j = 0$).

Figure 2: Life cycle



The pecuniary benefits and cost of a college education are easy to observe. There are costs in terms of tuition fees that have to be paid and wages that are foregone by delaying entry into the labour market. On the benefit side, a college graduate can earn a higher wage. At first glance, the (net) benefit of education is that it increases the present value of wages earned over the life-cycle (known as human capital). That is, $H_1^j(1) > H_1^j(0)$ where:

$$H_1^j(E^j) = w_1^j(E^j)[1 - \epsilon E^j] + \frac{w_2^j(E^j)}{1+r} + \frac{w_3^j(E^j)[1-R]}{(1+r)^2}, \tag{1}$$

with $w_t^j(E^j)$ the wage rate of a person of gender $j \in \{f, m\}$ with education level $E^j \in \{0, 1\}$ in period $t \in \{1, 2, 3\}$ and r the interest rate. The time cost of schooling is a fraction ϵ of the time endowment in period 1 and R is the (exogenous) portion of the last period spent in retirement.

An individual might choose to obtain education if the increase in wages more than compensates for the tuition fee υ, that is if $\upsilon < H_1^j(1) - H_1^j(0)$.

However, this definition of the benefit of education does not take into account that educated and uneducated individuals make different life-cycle choices, for example regarding how much to consume in every period. What matters is therefore not the amount of resources but the level of welfare that can be attained with them. A more comprehensive definition of the benefit of education would be that it increases the discounted utility of consumption. That is, $\hat{\mathcal{L}}_1^j(1) > \hat{\mathcal{L}}_1^j(0)$ where:

$$\hat{\mathcal{L}}_1^j(E^j) = \max_{\{c_t^j\}_{t=1}^3} \sum_{t=1}^{3} \left(\frac{1}{1+\rho}\right)^{t-1} u(c_t^j, 0) \quad \text{s.t.} \quad \sum_{t=1}^{3} \left(\frac{1}{1+r}\right)^{t-1} c_t^j = H_1^j(E^j) - \delta\bar{c} - \upsilon E^j, \tag{2}$$

with c_t^j denoting consumption in period t and ρ the rate of time preference. I assume that there is a fixed cost $\bar{c}$ in every period and define δ to be the cumulative three-period interest discount factor.[1] An individual might choose to obtain education if the increase in utility from consumption more than compensates for the utility cost of schooling θ^j, that is if $\theta^j < \hat{\mathcal{L}}^j(1) - \hat{\mathcal{L}}^j(0)$. This non-pecuniary or 'psychic' cost is inversely related to an individual's aptitude for learning.

Yet one important aspect of the benefit of education is still missing. As individuals might spent a large part of their life together with someone else, expectations about marriage and the characteristics of a future spouse play a role. What is crucial about the education decision in this respect, is that it is generally made individually and non-cooperatively as the spouse-to-be has not yet been met. In order to understand how this affects the trade-off of benefits and costs I will first discuss the differences between singles and married couples (Section 2.1) and the assumptions I make about the marriage market (Section 2.2). Subsequently I will define the notion of an equilibrium in this model (Section 2.3) and decompose the benefit of education into its constituent parts (Section 2.4).

2.1 Singles versus married couples

For now I only impose a few mild restrictions on preferences and the nature of marriage.[2] First, I assume that utility is time separable. In each period the individual derives felicity $u(c, b)$ from private consumption goods c and the number of children b (which is a public good within the household). For simplicity labour supply is taken to be exogenous here, although the framework can easily be extended to allow for felicity from leisure. A second restriction is that only married couples can have children and only in period 2. As I will be comparing the welfare of singles and married individuals I need to impose the condition that children are not a necessary 'good', meaning that $u(c, 0)$ is well-defined for any $c > 0$. Finally, I assume that marriage can only take

[1] The parameter δ is defined as:

$$\delta \equiv 1 + \frac{1}{1+r} + \frac{1}{(1+r)^2}$$

[2] For the purpose of this paper, marriage and cohabitation are equivalent. The term marriage is used by itself throughout for convenience.

place at the start of period 2 and that all relevant information about an individual at that moment in time can be can be summarized by his or her education E^j and accumulated savings a_1^j alone.

Let $\mathcal{M}_2^j(E^j, E^{-j}, a_1^j, a_1^{-j})$ denote remaining life-time utility (the 'value function') of a married individual of gender j at the start of period 2, where $\{E^j, a_1^j\}$ represent the own state variables and $\{E^{-j}, a_1^{-j}\}$ those of the spouse. At this point I am agnostic about how this utility from marriage is defined or what kind of household decision-making process has given rise to it. Similarly, let $\mathcal{S}_2^j(E^j, a_1^j)$ be the value function of a single individual in period 2. This person remains alone for the rest of his or her life such that:

$$\begin{aligned} \mathcal{S}_2^j(E^j, a_1^j) &= \max_{c_2^j, c_3^j} \left\{ u(c_2^j, 0) + \frac{u(c_3^j, 0)}{1+\rho} \right\} \\ \text{s.t.} \quad 0 &= (1+r)a_1^j + H_2^j(E^j) - c_2^j - \frac{c_3^j}{1+r} - \frac{2+r}{1+r}\bar{c} \end{aligned} \tag{3}$$

where $H_2^j(E^j)$ is the net present value of wage income from the perspective of period 2:

$$H_2^j(E^j) = w_2^j(E^j) + \frac{w_3^j(E^j)[1-R]}{1+r}. \tag{4}$$

2.2 The marriage market

As stated above, I assume that it is only possible to get married at the start of period 2. This leaves open the question of who will marry whom. I make the simplifying assumptions that (i) all matching probabilities are exogenously given (which is another way of saying that marriage decisions are driven by factors outside the model), (ii) the probability of getting married q is independent of education level, and (iii) there is no divorce. Each of these assumptions is clearly at odds with reality, but they will allow me to concentrate on life-cycle choices (the focus of this paper) and to obtain more analytical solutions.

Let $\pi^j(E^j)$ denote the fraction of individuals of gender $j \in \{f, m\}$ with education level $E^j \in \{0, 1\}$. Naturally $\pi^j(0)+\pi^j(1) = 1$. Write $\pi(E^f, E^m)$ for the probability of observing a match in which the female has education E^f and the male education E^m. Despite the popular saying that 'opposites attract', most people tend to get married to someone with a similar level of education.[3] This type of marital sorting is known as positive assortative matching, and it might arise because of complementarities between spouses or simply because individuals who are alike are more prone to meet and fall in love. To allow for this kind of behaviour I define:

$$\pi(1,1) = \pi^f(1)\pi^m(1) + \lambda\Big[\min\big\{\pi^f(1), \pi^m(1)\big\} - \pi^f(1)\pi^m(1)\Big], \tag{5}$$

where the parameter λ is an index of the degree of marital sorting. If $\lambda = 0$ then $\pi(1,1) = \pi^f(1)\pi^m(1)$ and matching is random. If $\lambda = 1$ then $\pi(1,1) = \min\{\pi^f(1), \pi^m(1)\}$ and matching is perfectly positively assortative, with the observed frequency of the match determined by the short side of the market. In the special case that $\pi^f(1) = \pi^m(1)$, λ equals the correlation coefficient

[3] See for example the matching patterns from the data in Section 5.3.

between female and male education (as in Fernández and Rogerson (2001)). The expression for $\pi(0,0)$ is similar, and the cross probabilities follow (see Figure 3).

Figure 3: Matching probabilities

		E^f		
		0	1	
E^m	0	$\pi(0,0)$	$\pi(1,0)$	$\pi^m(0)$
	1	$\pi(0,1)$	$\pi(1,1)$	$\pi^m(1)$
		$\pi^f(0)$	$\pi^f(1)$	1

Finally, I define $\pi_f^m(E^m|E^f)$ as the conditional probability that a woman with education E^f is matched to a man with education E^m (and vice versa for $\pi_m^f(E^f|E^m)$). By Bayes' Rule:

$$\pi_f^m(E^m|E^f) = \frac{\pi(E^f, E^m)}{\pi^f(E^f)}. \tag{6}$$

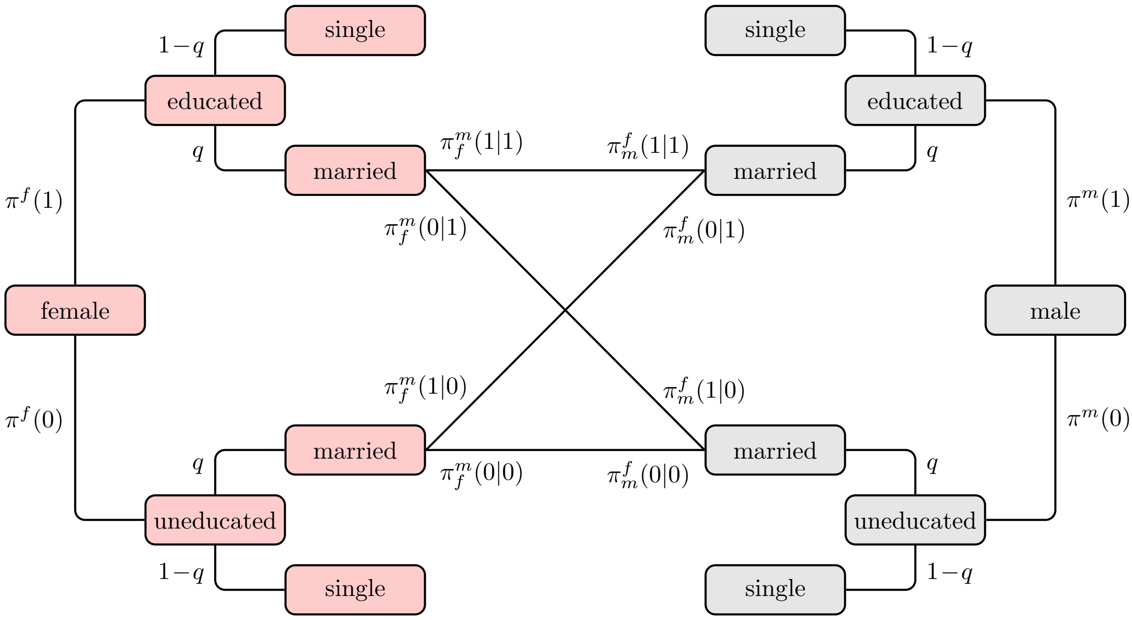
If $0 < \lambda \leq 1$ then the probability of being matched to an educated husband is greater for an educated woman than for an uneducated one, that is $\pi_f^m(1|1) > \pi_f^m(1|0)$.

Figure 4 provides a schematic overview of the matching process. Starting from the sides, males and females choose to become educated or not. A fixed fraction then gets married while the remainder stays single. Given marriage, the conditional probabilities determine which type of female ends up with which type of male. Note that although the matching probabilities are exogenous, they have to be consistent with the actual number of educated and uneducated men and women.

2.3 Equilibrium

How does the prospect of marriage affect the pre-marital education and savings decisions of individuals? For tractability I assume that the psychic cost of education is a pure utility cost which only affects the education choice and no other life-cycle decisions. This implies that although there is a distribution of utility costs for each gender $\theta^j \sim F^j$, at the end of period 1 there are only four different types of individuals in a given cohort: educated females, uneducated females, educated males, and uneducated males. The optimal level of savings in period 1 depends on both education and gender and is denoted by $a_1^j(E^j)$.

Figure 4: Matching process



The individual's value function at the start of adult life can be written as:

$$\mathcal{S}_1^j(\theta^j|\mathcal{P},\mathcal{A}^{-j}) = \max_{E^j,c_1^j,a_1^j} \Big\{ u(c_1^j,0) - \theta^j E^j + \frac{1}{1+\rho}\Big[(1-q)\mathcal{S}_2^j(E^j,a_1^j)$$

$$+ q\Big[\pi_j^{-j}(0|E^j)\mathcal{M}_2^j(E^j,0,a_1^j,a_1^{-j}(0)) + \pi_j^{-j}(1|E^j)\mathcal{M}_2^j(E^j,1,a_1^j,a_1^{-j}(1))\Big]\Big]\Big\}$$

$$\text{s.t.} \quad a_1^j = w_1^j(E^j)[1-\epsilon E^j] - c_1^j - \bar{c} - \upsilon E^j > -\frac{1}{1+r}\Big[H_2^j(E^j) - \frac{2+r}{1+r}\bar{c}\Big] \tag{7}$$

The first two terms are the immediate felicity from consumption and the utility cost of education in period 1. The remaining terms capture the expected discounted utility from period 2 onward. With probability $1-q$ the individual remains single and has value function $\mathcal{S}_2^j(E^j,a_1^j)$. If this person marries then there is a probability $\pi_j^{-j}(0|E^j)$ of being matched to an uneducated spouse and a probability $\pi_j^{-j}(1|E^j)$ of finding an educated partner. Note that the value function is conditional on the choices made by all other individuals in the same cohort. In particular, $\mathcal{P} = \{\pi^f(1),\pi^m(1)\}$ summarizes the education frequencies and $\mathcal{A}^{-j} = \{a_1^{-j}(0), a_1^{-j}(1)\}$ gives the level of savings of potential spouses. The financial asset constraint in (7) shows that it is only possible to borrow against own human capital net of subsistence costs and not the income of a future spouse.

For each gender and education level, write $\hat{\mathcal{S}}_1^j(E^j|\mathcal{P},\mathcal{A}^{-j})$ for the the level of life-time utility that is obtained after substituting out for the optimal financial assets choice $a_1^j(E^j)$ and corresponding consumption level in (7) whilst ignoring the utility cost of education. Then:

$$\mathcal{S}_1^j(\theta^j|\mathcal{P},\mathcal{A}^{-j}) = \max_{E^j} \Big[\hat{\mathcal{S}}_1^j(E^j|\mathcal{P},\mathcal{A}^{-j}) - \theta^j E^j\Big]. \tag{8}$$

Since the cost of education is monotonically increasing in θ^j while the benefit is independent of it, the optimal choice of education is characterized by a threshold rule:

$$\bar{\theta}^j \equiv \hat{\mathcal{S}}_1^j(1|\mathcal{P}, \mathcal{A}^{-j}) - \hat{\mathcal{S}}_1^j(0|\mathcal{P}, \mathcal{A}^{-j}), \quad E^j = \begin{cases} 1 & \text{if } \theta^j \leq \bar{\theta}^j \\ 0 & \text{if } \theta^j > \bar{\theta}^j \end{cases} \tag{9}$$

It follows that the fraction of individuals of gender j with a college education is $\pi^j(1) = F^j(\bar{\theta}^j)$.

Definition 1. A marriage market equilibrium for a given cohort of individuals is a set of education frequencies $\mathcal{P} \equiv \{\pi^f(1), \pi^m(1)\}$ and a set of financial asset choices $\mathcal{A} \equiv \{\mathcal{A}^f, \mathcal{A}^m\} = \{a_1^f(0), a_1^f(1), a_1^m(0), a_1^m(1)\}$ such that for each gender $j \in \{f, m\}$ and each realization of the utility cost $\theta^j \sim F^j$ the choice of education E^j and the corresponding level of assets $a_1^j(E^j)$ jointly maximize expected life-time utility (7).

The maintained assumption in the remainder of this paper is that the equilibrium is such that there is a (financial) benefit of being married to an educated spouse. That is, $(1+r)a_1^j(1) + H_2^j(1) > (1+r)a_1^j(0) + H_2^j(0)$. This rules out the possibility that educated individuals overspend to such an extent in period 1 that their net worth at the start of period 2 is less than that of someone who did not go to college.

2.4 Decomposition

In order to gain insight into the considerations that drive the individual's education choice, the threshold level $\bar{\theta}^j$ can be decomposed into two parts:

$$\bar{\theta}^j = LMB^j + MMB^j. \tag{10}$$

The first term is the *labour market benefit.* It is defined as the benefit of education for a (hypothetical) person who knows for certain that he or she will remain single for his or her entire life (a 'lifelong single'):

$$LMB^j \equiv \hat{\mathcal{L}}_1^j(1) - \hat{\mathcal{L}}_1^j(0), \tag{11}$$

where $\hat{\mathcal{L}}_1^j(E^j)$ has been introduced in equation (2) above. The remainder consists of the *marriage market benefit.* This term is itself comprised of several elements:

$$MMB^j \equiv \Delta SB^j + \frac{q}{1+\rho}\left[\Delta UG^j + \Delta MP^j\right]. \tag{12}$$

First there is the part due to differences in savings behaviour:

$$\begin{aligned} \Delta SB^j \equiv & \left[u(w_1^j(1)[1-\epsilon] - \bar{c} - \upsilon - a_1^j(1), 0) + \frac{1}{1+\rho}\mathcal{S}_2^j(1, a_1^j(1)) - \hat{\mathcal{L}}_1^j(1)\right] \\ & - \left[u(w_1^j(0) - \bar{c} - a_1^j(0), 0) + \frac{1}{1+\rho}\mathcal{S}_2^j(0, a_1^j(0)) - \hat{\mathcal{L}}_1^j(0)\right]. \end{aligned} \tag{13}$$

The possibility of marriage affects the pre-marital savings decision of an individual. For each level of education, the financial asset level in equilibrium will therefore differ from the optimal choice made by a lifelong single. Conditional on remaining single $\hat{\mathcal{L}}_1^j(E^j)$ is the maximum attainable utility, such that both terms in square brackets are negative. The overall sign will depend on the relative magnitude of each.

The second part is attributable to differences in the utility gain of being married:

$$\begin{aligned}\Delta UG^j \equiv & \Big[\pi_j^{-j}(1|0)\mathcal{M}_2^j(1,1,a_1^j(1),a_1^{-j}(1)) + \pi_j^{-j}(0|0)\mathcal{M}_2^j(1,0,a_1^j(1),a_1^{-j}(0)) - \mathcal{S}_2^j(1,a_1^j(1))\Big] \\ & - \Big[\pi_j^{-j}(1|0)\mathcal{M}_2^j(0,1,a_1^j(0),a_1^{-j}(1)) + \pi_j^{-j}(0|0)\mathcal{M}_2^j(0,0,a_1^j(0),a_1^{-j}(0)) - \mathcal{S}_2^j(0,a_1^j(0))\Big]. \quad (14)\end{aligned}$$

Keeping the matching probabilities constant at those for an uneducated person, the expected increase in welfare from being married relative to being single differs by education type. If a person without education has more to gain by sharing a household with a spouse then this term is negative.

The final part can be ascribed to differences in matching probabilities:

$$\Delta MP^j \equiv \Big[\pi_j^{-j}(0|0) + \pi_j^{-j}(1|1) - 1\Big]\Big[\mathcal{M}_2^j(1,1,a_1^j(1),a_1^{-j}(1)) - \mathcal{M}_2^j(1,0,a_1^j(1),a_1^{-j}(0))\Big]. \quad (15)$$

If there is positive assortative matching then being educated has the advantage of increasing the probability of marrying an educated spouse, assuming that this is desirable. On the other hand, if the matching process is completely random then this term disappears because the conditional probabilities in the first set of brackets coincide with the unconditional ones and sum to unity.

3 A fully specified example

In this section I will develop a fully specified version of the general model outlined above. It will serve to illustrate gender differences in the benefits and costs of education (Section 4) and thereby help to explain the college gender gap reversal (Section 5).

3.1 Assumptions

Assume that the individual's felicity function is given by:

$$u(c,b) = \begin{cases} \dfrac{\Big[c^{\phi}(1+b)^{1-\phi}\Big]^{1-1/\sigma} - 1}{1-1/\sigma} & \text{if } \sigma > 0, \sigma \neq 1 \\ \phi \ln c + (1-\phi)\ln(1+b) & \text{if } \sigma = 1 \end{cases} \quad (16)$$

The felicity function features a constant intertemporal substitution elasticity σ with respect to a Cobb-Douglas composite of consumption and the number of children, with $0 < \phi \leq 1$ representing the weight of consumption. These preferences are quasi-homothetic because (i) there is a fixed consumption cost, which drives a wedge between total consumption expenditures $\bar{c} + c$ and utility-generating consumption c, and (ii) children are not a necessary 'good', as $1+b$ enters the felicity function and not b itself. For future reference I define the intertemporal substitution elasticity of consumption as:

$$\sigma^* \equiv -\frac{u_c(c,b)}{u_{cc}(c,b)c} = \frac{1}{1-\phi(1-1/\sigma)}; \quad \sigma^* \gtreqless 1 \quad \Leftrightarrow \quad \sigma \gtreqless 1. \tag{17}$$

The value function of a single at the start of period 2, as defined in (3), can then be written as:

$$\mathcal{S}_2^j(E^j, a_1^j) = \frac{\phi}{\Gamma_2(1)} \frac{\Gamma_2(1)\Gamma_2(\sigma)^{-1/\sigma^*} W_2^j(E^j, a_1^j)^{1-1/\sigma^*} - 1}{1-1/\sigma^*}, \tag{18}$$

where $W_2^j(E^j, a_1^j)$ is the individual's wealth net of fixed costs from the perspective of period 2:

$$W_2^j(E^j, a_1^j) = (1+r)a_1^j + H_2^j(E^j) - \frac{2+r}{1+r}\bar{c}. \tag{19}$$

The parameter $\Gamma_2(\sigma)$ captures the marginal propensity to consume out of wealth in period 2 and is defined as:

$$\Gamma_2(\sigma) = \left[1 + \frac{1}{1+r}\left(\frac{1+r}{1+\rho}\right)^{\sigma^*}\right]^{-1}. \tag{20}$$

To derive the value function of a married individual I need to specify the process of decision-making within a couple. In line with a large part of the literature I postulate that the resulting allocation choices are Pareto efficient.[4] This implies that the couple acts *as if* it maximizes a weighted average of the individual utility functions of the husband and the wife (see for example Chiappori (1992)). The couple's periodic welfare function can be written as:

$$U(c_t^f, c_t^m, b) = \alpha u(c_t^f, b) + (1-\alpha)u(c_t^m, b), \tag{21}$$

where $0 < \alpha < 1$ is the Pareto weight of the female. The larger is α, the more the allocation on the Pareto frontier that is chosen by the couple tends to favour the wife. By taking this weight as exogenously given and constant I assume that the couple acts as a 'unitary' household and is fully committed to the marriage. In Appendix B I relax this assumption and allow for bargaining within the family.

If a married couple decides to have b children then all of these are born at the start of period 2 and they remain in the household for exactly 1 period. The cost of having children is three-fold. First, they increase the fixed consumption cost that the household has to incur. Second, parents are required to spend a minimum amount of time with their children. Child care is created according to a production function with a constant elasticity of substitution between father and mother time $\xi > 1$. The child-care constraint is then:

$$\Omega(n_2^f, n_2^m) = \left[(n_2^f)^{1-1/\xi} + (n_2^m)^{1-1/\xi}\right]^{\frac{1}{1-1/\xi}} \geq N_b b, \tag{22}$$

[4] Although reasonable for repeated day-to-day decisions such as the division of consumption expenditure, it is arguably less realistic for 'big' choices like how many children to raise.

where n_2^j is the time input of parent j and N_b is the minimum time requirement of a single child. Finally, the mother has to incur an additional time cost related to child birth of T_b per child.

Under the assumption that there are no bequests to children, the consolidated budget constraint for the household is given by:

$$\sum_i \Big[(1+r)a_1^i + H_2^i(E^i) - w_2^i(E^i)n_2^i\Big] - w_2^f(E^f)T_b b = \sum_i \left[c_2^i + \frac{c_3^i}{1+r}\right] + \left[\frac{2+r}{1+r}Q_a + Q_b b\right]\bar{c}, \quad (23)$$

where $1 < Q_a \leq 2$ is the equivalence scale for two adults and $0 < Q_b \leq 1$ is the adult equivalent of a child. With $Q_a < 2$ and $Q_b < 1$ there are economies of scale for a multi-person household in providing the fixed consumption cost (think of sharing a house, washing clothes, etcetera). Labour supply in period 2 is $1 - n_2^m$ for the male and $1 - n_2^f - T_b b$ for the female and can either be interpreted as hours worked or the fraction of the period worked full-time.

The problem of the household is to maximize the sum of discounted felicity (21) for period 2 and 3, subject to the budget constraint (23) and the minimum child care requirement (22). Assuming an interior solution, the most efficient allocation of child care between the parents for a given number of children is essentially one of minimizing the total associated time cost in terms of foregone wages. The unit cost function can be defined as:

$$\omega(E^f, E^m) = \min_{n_2^f, n_2^m} \Big[w_2^f(E^f)n_2^f + w_2^m(E^m)n_2^m\Big] \quad \text{s.t.} \quad \Omega(n_2^f, n_2^m) = 1. \quad (24)$$

The total cost of a child is then given by:

$$C_b(E^f, E^m) = Q_b\bar{c} + N_b\omega(E^f, E^m) + T_b w_2^f(E^f), \quad (25)$$

which depends positively on the education levels (or wages) of the parents. The optimal intra-family sharing rule is such that in every period a fraction $\beta(\alpha, \sigma) \in (0, 1)$ of total spending on private consumption goods goes to the female while the remainder is dedicated to the male, where:

$$\beta(\alpha, \sigma) = \left[1 + \left(\frac{1-\alpha}{\alpha}\right)^{\sigma^*}\right]^{-1}. \quad (26)$$

It follows that the share of a woman is increasing in her Pareto weight α. The value function for a married female can then be written as:

$$\mathcal{M}_2^f(E^f, E^m, a_1^f, a_1^m) = \frac{1}{\Gamma_2(1)} \frac{\Gamma_2(1)\Gamma_2(\sigma)^{-1/\sigma^*}\Phi(\sigma)\left[\dfrac{\beta(\alpha,\sigma)^{\phi}W_2(E^f, E^m, a_1^f + a_1^m)}{C_b(E^f, E^m)^{1-\phi}}\right]^{1-1/\sigma} - 1}{1 - 1/\sigma}, \quad (27)$$

where $\Phi(\sigma)$ is a constant[5] and $W_2(E^f, E^m, a_1)$ is household wealth net of subsistence costs:

$$W_2(E^f, E^m, a_1) = (1+r)a_1 + H_2^f(E^f) + H_2^m(E^m) + C_b(E^f, E^m) - \frac{2+r}{1+r}Q_a\bar{c}, \tag{28}$$

with $a_1 = a_1^f + a_1^m$ denoting aggregate savings. The household allocation only depends on total financial and human wealth and not its distribution over the spouses. This is known as income pooling and it is a consequence of the assumption of a unitary household with fixed Pareto weights. Joint wealth is higher than the sum of individual wealth as given in (19) due to (i) economies of scale, provided that $Q_a < 2$, and (ii) the possibility to produce children, in combination with a negative 'subsistence level'.[6]

Below I will sometimes refer to the case without fertility, by which I mean that $\phi = 1$ such that $b = 0$ (a corner solution) and $C_b(E^f, E^m)$ drops out of (28). The 'value' of a spouse can then be summarized as the sum of financial and human wealth $(1+r)a_1^j(E^j) + H_2^j(E^j)$. This is not possible when there are children involved, as then the education level by itself also matters in determining the opportunity cost of child care.

3.2 Equilibrium

For a woman with education E^f the optimal choice of savings in period 1, $a_1^f(E^f)$, has to satisfy the following first-order condition:

$$\begin{aligned} 0 = & -\frac{\phi}{\left[w_1^f(E^f)[1-\epsilon E^f] - \bar{c} - \upsilon E^f - a_1^f(E^f)\right]^{1/\sigma^*}} \\ & + \frac{1+r}{1+\rho}\Gamma_2(\sigma)^{-1/\sigma^*}\Bigg\{(1-q)\frac{\phi}{W_2^f(E^f, a_1^f(E^f))^{1/\sigma^*}} + q\Phi(\sigma)\beta(\alpha,\sigma)^{\phi(1-1/\sigma)} \\ & \times \left[\pi_f^m(1|E^f)\frac{C_b(E^f,1)^{-(1-\phi)(1-1/\sigma)}}{W_2(E^f,1,a_1^f(E^f)+a_1^m(1))^{1/\sigma}} + \pi_f^m(0|E^f)\frac{C_b(E^f,0)^{-(1-\phi)(1-1/\sigma)}}{W_2(E^f,0,a_1^f(E^f)+a_1^m(0))^{1/\sigma}}\right]\Bigg\}. \end{aligned} \tag{29}$$

The first term captures the direct negative utility effect of increased savings through a lower level of current consumption. The remaining two terms show the expected positive consequence for future consumption, which depends on expectations regarding marriage. By the second-order condition the solution to this equation depends negatively on the savings of educated and uneducated men, which means that financial assets are strategic substitutes in the marriage game.

In general it is not possible to solve for the equilibrium amounts of financial assets analytically. As a special case, consider a person of gender j who is single for certain such that $q = 0$. Then

[5] The parameter $\Phi(\sigma)$ is defined as:

$$\Phi(\sigma) = [\phi^\phi(1-\phi)^{1-\phi}]^{1-1/\sigma}$$

[6] That is, b enters the utility function through the term $1+b$. This can be seen as a subsistence level of -1 for children.

the optimal assets choice is independent of those of all other individuals and given by:

$$a_1^j(E^j) = w_1^j(E^j)[1 - \epsilon E^j] - \bar{c} - \upsilon E^j - \Gamma_1(\sigma) W_1^j(E^j), \tag{30}$$

where $W_1^j(E^j)$ is net total wealth from the perspective of period 1:

$$W_1^j(E^j) = H_1^j(E^j) - \delta\bar{c} - \upsilon E^j, \tag{31}$$

and $\Gamma_1(\sigma)$ is the corresponding propensity to consume:

$$\Gamma_1(\sigma) \equiv \left[1 + \frac{1}{1+r}\left(\frac{1+r}{1+\rho}\right)^{\sigma^*} + \frac{1}{(1+r)^2}\left(\frac{1+r}{1+\rho}\right)^{2\sigma^*}\right]^{-1}. \tag{32}$$

Note that this is the solution to the problem expressed in (2) given the functional form of the felicity function. The value function of a lifelong single in period 1 can therefore be written as:

$$\hat{\mathcal{L}}_1^j(E^j) = \frac{\phi}{\Gamma_1(1)} \frac{\Gamma_1(1)\Gamma_1(\sigma)^{-1/\sigma^*} W_1(E^j)^{1-1/\sigma^*} - 1}{1 - 1/\sigma^*}. \tag{33}$$

4 Gender differences in the benefit of education

The next two sections describe potential gender differences in the labour market benefit of education (Section 4.1) and the marriage market benefit of education (Section 4.2). The implications for the marriage market equilibrium are derived numerically in Section 4.3.

4.1 The labour market benefit

Recall that the labour market benefit of education is the difference in life-time utility with and without education for a person who remains single for certain. Using (33) I obtain:

$$LMB^j = \phi\Gamma_1(\sigma)^{-1/\sigma^*} \frac{W_1^j(1)^{1-1/\sigma^*} - W_1^j(0)^{1-1/\sigma^*}}{1 - 1/\sigma^*}. \tag{34}$$

where total wealth $W_1^j(E^j)$ consists of human capital $H_1^j(E^j)$ net of tuition fees υE^j and fixed costs $\delta\bar{c}$. Without loss of generality, I set $w_1^j(E^j) = w^j(E^j)$ and define $\eta_t^j(E^j)$ to be the net growth rate of wages from period $t-1$ to period t. This growth rate could in general depend on both gender and education level and captures factors such as experience build-up or human capital depreciation. Human wealth can then be written as:

$$H_1^j(E^j) = w^j(E^j)\left[1 - \epsilon E^j + \frac{1 + \eta_2^j(E^j)}{1+r} + \frac{(1 + \eta_3^j(E^j))(1 + \eta_2^j(E^j))[1 - R]}{(1+r)^2}\right]. \tag{35}$$

Gender differences in the labour market benefit of education (34) can stem from several sources. First of all, there is ample evidence for the existence of a 'gender wage gap': after accounting for measurable skill levels, women earn less than men (see Jarrell and Stanley (2004) for a meta-analysis for the US). This implies that men and women might not receive the same college wage premium, which is a measure of how much more a college graduate earns compared to a person without such a degree. Usually it is defined as the log difference in wages $\ln(w^j(1)/w^j(0))$, which corresponds to the coefficient on a college dummy in a Mincerian-style semi-log wage regression. Several authors have argued that the college wage premium is higher for women as obtaining education gives them a double dividend: not only does it increase their productivity, but it also reduces the gender gap in wages. Dougherty (2005) attributes this gap to 'discrimination, tastes, and circumstances', all of which might be inversely related to a woman's educational attainment. Similarly, Chiappori et al. (2009) postulate that discrimination is weaker against educated women because they are expected to show more labour market commitment and to invest more on the job. Hubbard (2011) on the other hand, claims that this gender imbalance is actually a statistical fluke which is the result of censoring of the highest wages in the data. As these top wages are disproportionally earned by men, ignoring the fact that they are only recorded up to a maximum tends to depress the male college wage premium. After correcting for this 'topcoding bias' he finds that there has not been a significant gender difference in the college wage premium for at least a decade.

In the context of the model, with $\sigma = 1$ (log felicity) and $\bar{c} = \upsilon = 0$ (no fixed costs or tuition fees) the labour market benefit is linearly increasing in the college wage premium. In this case only *relative* wages of educated and uneducated workers matter for the education threshold $\bar{\theta}^j$ and a higher college wage premium for one of the sexes immediately translates into a higher labour market benefit (given equal wage growth). However, with $\sigma \neq 1$ or positive fixed costs and tuition fees also the *absolute* wage levels matter, as the following proposition illustrates.

Proposition 1. *Assume there are no fixed costs, no tuition fees, and equal wage growth for both sexes over time and for each level of education. If the college wage premium is the same for both sexes but females earn less then equally qualified males, then:*

$$LMB^f \gtreqless LMB^m \quad \text{if} \quad \sigma \lesseqgtr 1$$

Differences in the labour market benefit between the sexes depend positively on the common college wage premium.

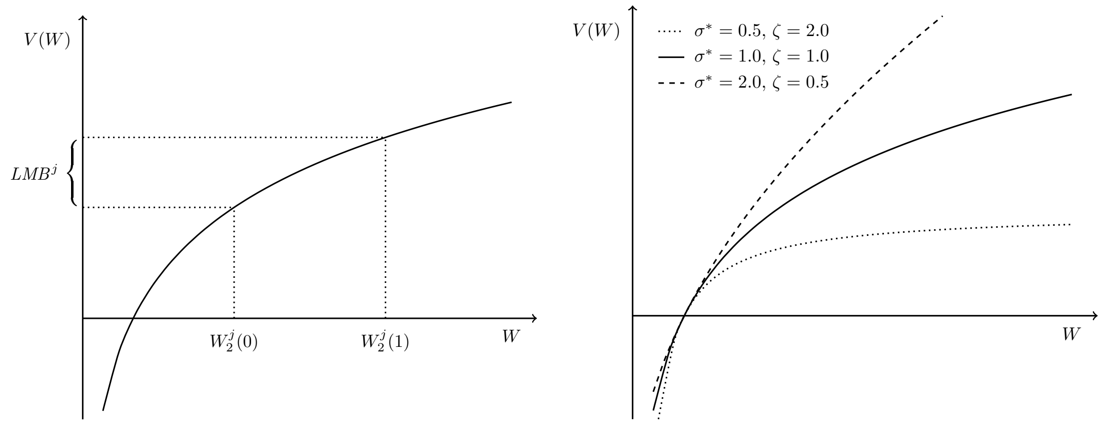
Proof. See Appendix A. ☐

Hence, even with a common college wage premium there might be a gender difference in the labour market benefit of education which is solely attributable to the curvature of the utility function. In the microeconometric literature the case with $0 < \sigma < 1$ is deemed more realistic, which would imply that women have a greater labour market benefit than men in the presence of a gender wage gap. To understand this, note that under the assumption of perfect capital markets the utility level of a lifelong single is an increasing function of individual wealth, say $\hat{\mathcal{L}}_1^j(E^j) = V(W_1^j(E^j))$. The labour market benefit is defined as the difference in life-time utility with and without education for a lifelong single, see Figure 5(i). By a first-order approximation it can be written as:

$$
\begin{aligned}
LMB^j \equiv V(W_1^j(1)) - V(W_1^j(0)) &\approx V'(W_1^j(0))[W_1^j(1) - W_1^j(0)] \\
&= V'(H_1^j(0) - \delta\bar{c})H_1^j(0)\left[\frac{H_1^j(1)}{H_1^j(0)} - \frac{\upsilon}{H_1^j(0)} - 1\right].
\end{aligned} \tag{36}
$$

Figure 5: Curvature of the utility function

(i) Labour market benefit (ii) Different elasticities



Assuming equal wage growth for both sexes, human capital is proportional to the wage by a factor that is gender independent, see (35). Therefore, if the college wage premium is the same for men and women then so is the ratio of human wealth with and without education. By taking the derivative of (36) with respect to $H_1^j(0)$ while keeping $H_1^j(1)/H_1^j(0)$ fixed I obtain:

$$
\frac{\partial LMB^j}{\partial H_1^j(0)} = V'(W_1^j(0))\left\{\left[1 - \zeta(W_1^j(0))\frac{H_1^j(0)}{H_1^j(0) - \delta\bar{c}}\right]\frac{H_1^j(1) - \upsilon - H_1^j(0)}{H_1^j(0)} + \frac{\upsilon}{H_1^j(0)}\right\}, \tag{37}
$$

where ζ is the elasticity of the marginal utility of wealth:

$$
\zeta(W) = -\frac{V''(W)W}{V'(W)}. \tag{38}
$$

Under the assumption that the utility function features a constant intertemporal substitution elasticity it follows that $\zeta = 1/\sigma^*$ and therefore independent of wealth. Figure 5 depicts the utility function for several values of ζ in order to illustrate the difference in curvature. The higher is ζ, the faster marginal utility declines with wealth. Proposition 1 states that if $\bar{c} = \upsilon = 0$ the derivative expressed in (37) is negative if and only if $\sigma < 1$ (or equivalently $\sigma^* < 1$, see (17)) such that $\zeta > 1$. Women, for whom $w^j(0)$ and thereby $H_1^j(0)$ is lower, will then have a higher labour market benefit of education than men. This difference is greater the larger is the common college wage premium. If $\bar{c} > 0$ then the result also holds with $\sigma = 1$, see Proposition 2.

Proposition 2. *Let $0 < \sigma \leq 1$. Assume there are positive fixed costs, no tuition fees, and equal wage growth for both sexes over time and for each level of education. If the college wage premium is the same for both sexes but females earn less then equally qualified males, then $LMB^f > LMB^m$.*

Proof. See Appendix A. ☐

This proposition underlines the importance of fixed costs in the model. As these costs are the same for every individual, irrespective of gender or education level, they weigh heaviest on those that receive the lowest wages. These are likely to be uneducated women, and for them education offers a possibility to escape poverty (in line with the insights from the empirical work of DiPrete and Buchmann (2006)). With $\sigma > 1$ the above result might still be valid but not necessarily for all parameter values. The presence of a tuition fee mitigates the result, as it is also the same for both sexes but does not have to be paid by people who choose to remain uneducated.

Taken together propositions 1 and 2 show that, in the context of this model, the labour market benefit of education can be greater for women than for men even when they do not have a higher college wage premium (the evidence for which is mixed).

A final potential source of gender difference in the labour market benefit is the relative wage growth of educated versus uneducated workers. However, to the best of my knowledge evidence for this is absent and it is also not obvious which gender would be favoured. For example, on the one hand it could be argued that educated women start at a lower level of wages but have a greater wage growth than men after they show labour market commitment, but conversely it might be the case that they run into 'glass ceilings' preventing them from attaining the highest-paid jobs. It gets even more complicated if wage growth captures returns to experience in proportion to hours worked, because then the child care allocation at home matters for the relative experience build-up of husband and wife and wage growth becomes endogenous. In the remainder of this paper I will simplify matters by assuming that wages are constant over the life-cycle.

4.2 The marriage market benefit

Taking wages as given, looking for potential gender differences in the labour market benefit of education is relatively straightfoward. The marriage market benefit is much more complex to analyze, as it depends on the matching probabilities which are themselves determined in the overall marriage market equilibrium. By keeping the probabilities constant across the sexes I derive some partial equilibrium insights.

Consider the simplest case in with log felicity, no children, no fixed costs and no savings in period 1. Then the level of utility during marriage is increasing in the log of household wealth. The difference in utility gain can be written as:

$$\Delta UG^j = \frac{1}{\Gamma_2(1)} \Bigg\{ \pi_j^{-j}(1|0) \ln\left(1 + \frac{H_2^j(1) - H_2^j(0)}{H_2^j(0) + H_2^{-j}(1)}\right) + \pi_j^{-j}(0|0) \ln\left(1 + \frac{H_2^j(1) - H_2^j(0)}{H_2^j(0) + H_2^{-j}(0)}\right) - \ln\left(1 + \frac{H_2^j(1) - H_2^j(0)}{H_2^j(0)}\right) \Bigg\}. \tag{39}$$

The first line captures the expected return to education when married, which is compared to the corresponding return in the single state. Note that the married individual's allocated share of total wealth drops out of the expression with log felicity as it is independent of the education level of the spouse. It is clear that under these assumptions $\Delta UG^j < 0$. The *absolute* increase in human wealth resulting from education $H_2^j(1) - H_2^j(0)$ does not depend on whether a person ends up being married or single from period 2 onward, yet if there is already some positive spousal wealth $H_2^{-j}(E^{-j})$ then this yields *relatively* less additional utility. Under the assumption that the college wage premium is the same for both sexes but females earn less then equally qualified males I find $\Delta UG^f < \Delta UG^m$ because (i) a woman has more to gain from an education in relative terms than a man if she remains single and (ii) a woman can expect to marry a richer spouse, which provides her with fewer incentives to accumulate wealth.[7] Concerning the part of the marriage market benefit attributable to matching probabilities it is exactly the other way around:

$$\Delta MP^j = \left[\pi_j^{-j}(0|0) + \pi_j^{-j}(1|1) - 1\right] \frac{1}{\Gamma_2(1)} \ln\left(1 + \frac{H_2^{-j}(1) - H_2^{-j}(0)}{H_2^j(1) + H_2^{-j}(0)}\right). \tag{40}$$

Since wages for women are lower they have more to gain than men by marrying an educated spouse compared to an uneducated one.

These insights are not straightfoward to generalize to the case with $\sigma \neq 1$, as then the sharing rule for wealth also plays a role. With endogenous fertility matters get even more complicated, because a college education not only implies a greater amount of human wealth but also a higher opportunity cost of time spent on child care. Biological differences between men and women then play an important role. The change in the cost of a child if a woman decides to become educated can be written as:

$$C_b(1, E^m) - C_b(0, E^m) = N_b[\omega(1, E^m) - \omega(0, E^m)] + T_b[w_3^f(1) - w_3^f(0)]. \tag{41}$$

For men the expression is similar but the increase in the child-bearing cost component is not present. This implies that in this model, all other gender differences aside, educated women are less desirable partners than educated men.

Overall, the marriage market return to education tends to be lower for women than for men. There is one notable exception. Under the restrictive assumptions that felicity is linear in consumption, fertility is exogenous and there are no savings in period 1, the marriage market benefit of education reduces to $MMB^j = \Delta MP^j$ which might be higher for women.[8]

[7] Write:

$$H_2^j(1) - H_2^j(0) = w^j(0)\left[\frac{w^j(1)}{w^j(0)} - 1\right]\left[1 + \frac{1-R}{1+r}\right]$$

Then if $w^f(1)/w^f(0) = w^m(1)/w^m(0)$ but $w^f(0) < w^m(0)$ it follows that:

$$H_2^f(1) - H_2^f(0) < H_2^m(1) - H_2^m(0) \quad \text{and} \quad H_2^f(1) + H_2^m(0) < H_2^f(0) + H_2^m(1)$$

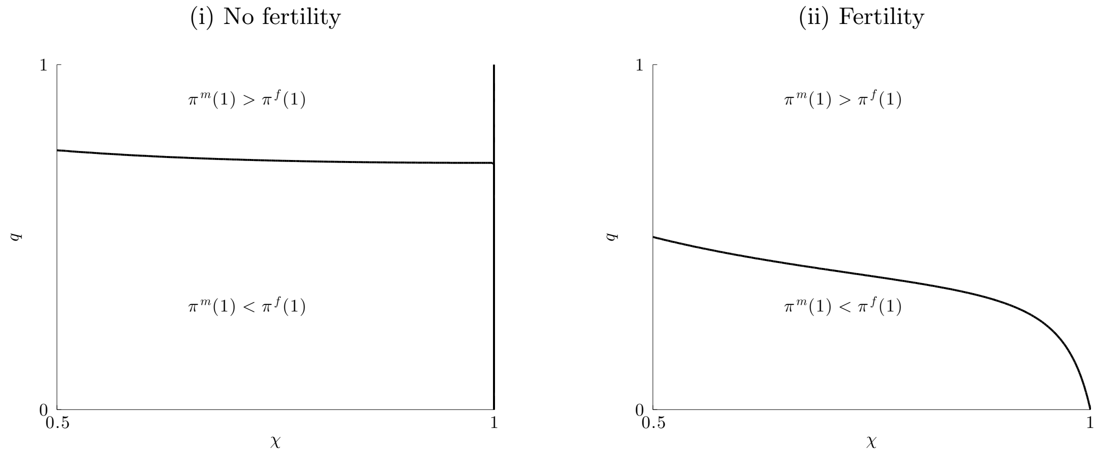
[8] Incidentally, these are the type of assumptions often made in models with endogenous matching.

4.3 Numerical results

In order to see how gender differences in the benefits and of a college education translate into differences in educational choices of men and women I solve for the marriage market equilibrium numerically (computational details are given in Appendix D). The focus here is on the sign of $\pi^m(1) - \pi^f(1)$, the actual levels are not of much interest until the next section.[9]

I restrict attention to the case that both the utility cost distribution and the college wage premium are the same for men and women. Wage levels might differ and the female to male wage ratio is denoted by $\chi \leq 1$. Figure 6 shows the interaction between this gender wage gap χ and the probability of marriage q in determining relative education frequencies. In both panels the solid line depicts combinations of q and χ that yield a symmetric equilibrium with $\pi^m(1) = \pi^f(1)$. Consider first panel (i), where there are no children and no fixed costs. If there is no gender wage gap then men and women are exactly the same. Regardless of the probability of marriage the education frequencies will be equal, which is why there is a vertical line at $\chi = 1$. Under the assumption that $\sigma < 1$ the labour market benefit of education for women is greater than that for men as long as $\chi < 1$ by Proposition 1. This explains why $\pi^f(1) > \pi^m(1)$ if the probability of marriage is low. On the other hand, if the probability of marriage is high (above the solid horizontal line) then educated men outnumber educated women. In this case a man knows that he is likely to end up marrying a wife who earns less than he does, which gives him an incentive to invest in his own education in order to generate a higher level of household income. For a woman it is the other way around, she is almost certain to marry a richer husband.

Figure 6: Relative education frequencies with equal cost distributions



Notes: The solid line shows the combinations of the probability of marriage q and the female to male wage ratio χ for which the equilibrium is symmetric ($\pi^m(1) = \pi^f(1)$). In panel (i) felicity depends only on consumption and there are no fixed costs. In panel (ii) felicity depends on both consumption and children, fixed costs are positive and there is a time cost of child birth for the mother.

The horizontal line of demarkation between the two types of equilibria is downward sloping because a higher value of χ means that the labour market benefit of education for women is closer to that of men. The relative education frequencies will then switch sign for a lower value of q. The line

[9]The parameter values used for this illustrative exercise are similar to the ones described in Section 5.2 below.

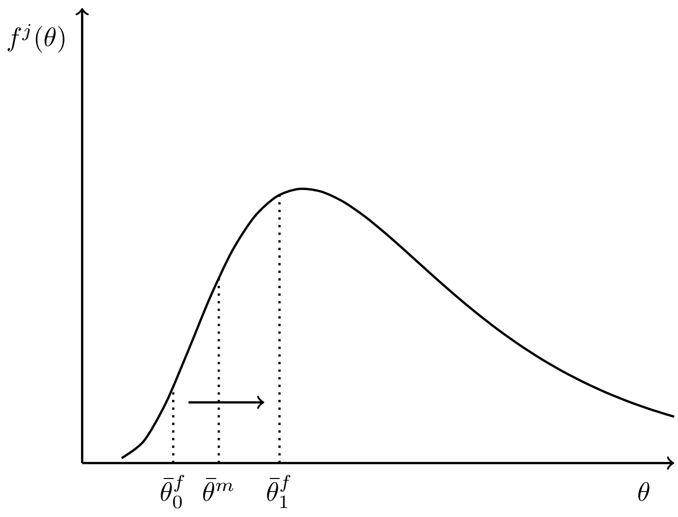
shifts if one of the model parameters other than q or χ changes. For example, if the intertemporal substitution elasticity σ decreases or the fixed cost $\bar{c}$ increases then the return to education for women tends to rise relative to that of men (in line with Proposition 1 and 2) and the line shifts up. An increase in sorting λ has a similar effect, as it implies that the part of the marriage market benefit due to matching probabilities goes up for women. If the tuition fee υ is positive instead of zero then the line shifts down but its slope increases. Since $\sigma < 1$ an increase in the common college wage premium favours women more than men and the line goes up.

Consider now panel (b), which includes fertility. Even in absence of a gender wage gap men and women are not the same as there is a positive time cost of child birth for mothers. With $\chi = 1$ there will only be a symmetric equilibrium if $q = 0$, as in absence of marriage children do not play a role. If the preference for consumption relative to children ϕ goes down then the line rotates in a counter-clockwise direction around the point $(1, 0)$.

5 The college gender gap reversal

In this section I will use the model in order to understand why women have overtaken men in terms of educational attainment. The main argument is as follows. Suppose that the distribution of utility costs of education is the same among men as it is among women. For illustrative purposes I have drawn a unimodal curve in Figure 7. Recall that the fraction of individuals of a given gender that obtain education is represented by the area to the left of the threshold level $\bar{\theta}^j$. Initially the benefits are lower for women, that is $\bar{\theta}^f$ is to the left of $\bar{\theta}^m$. Over time the threshold levels shift in such a way that $\bar{\theta}^f > \bar{\theta}^m$. For this explanation to be valid there must be (i) gender differences in the benefit of education and (ii) a change in the relative benefits for men and women over time. The aim here is to show that there exists a reasonable set of parameter values under which the model indeed generates a college gender gap reversal.

Figure 7: College gender gap reversal with equal cost distributions



5.1 Data

I use data from the Integrated Public Use Microdata Series (IPUMS) for the US (Ruggles et al. (2010)), see Appendix E for a description. I pick two cohorts, one born in 1950 and one born in 1970. When they are 40 years of age (in 1990 and 2010, respectively) I obtain some key statistics. The first two lines of Table 1 give the proportion of women and men who have obtained at least a Bachelor's degree for each cohort. While for the 1950 cohort the fraction of educated men exceeded that of educated women, by 1970 this inequality had been reversed. For men the graduation rate even decreased a little.

Table 1: Graduation rates and matching parameters

	Cohort 1950	Cohort 1970
$\pi^f(1)$	0.252	0.337
$\pi^m(1)$	0.304	0.291
q	0.905	0.830
λ	0.563	0.564

Source: Integrated Public Use Microdata Series (IPUMS) for 1990 and 2010.

Table 2 shows how men and women are matched in the data. Unfortunately it is not possible to retrieve information about characteristics of the former spouse for individuals who are separated or divorced, so the matching pattern is solely based on those who are (still) married at age 40 or are cohabiting.[10] The second row and column do not add up exactly to $\pi^m(1)$ and $\pi^f(1)$, respectively, because in reality educated people are more likely to get and stay married. From these cross tables I compute the index of marital sorting λ, see Table 1. It is greater than zero, indicating that there is positive assortative matching in education. Over time it has remained virtually constant. This is in contrast to studies that claim that marital sorting has become stronger, see for example Fernández et al. (2005). However, this conclusion is often based on the correlation coefficient between male and female education which indeed increased from 0.472 for the 1950 cohort to 0.523 for individuals born in 1970.[11] This rise can be entirely explained by the increased supply educated women without any change in the underlying preferences for assortative matching (a conclusion which is also reached by Chiappori et al. (2011) in the context of a different model). I calculate the probability of marriage q as one minus the fraction of people who are classified as 'never married/single' and are not cohabiting. Table 1 shows that this probability has declined over time.

As the year 1990 is the last one for which there are records of completed fertility per woman in the database, I can calculate the average number of children for each type of household only for the cohort born in 1950.[12] These are given in square brackets in Table 2. Completed fertility

[10] I define a cohabiting couple as one in which a 'household head' lives together with an 'unmarried partner' with a maximum age difference of 10 years. See Appendix E.

[11] The correlation coefficient is given by:

$$cor = \lambda \frac{\min\{\pi^f(1), \pi^m(1)\} - \pi^f(1)\pi^m(1)}{\sqrt{\pi^f(1)[1-\pi^f(1)]}\sqrt{\pi^m(1)[1-\pi^m(1)]}}$$

[12] After 1990 there is a variable that gives the number of children currently living in the household, but this one

Table 2: Matching patterns

(a) Cohort 1950

		E^f 0	E^f 1	
E^m	0	0.596 [2.310]	0.075 [1.779]	0.671
	1	0.148 [2.082]	0.181 [1.888]	0.329
		0.744	0.256	1

(b) Cohort 1970

		E^f 0	E^f 1	
E^m	0	0.519	0.129	0.648
	1	0.094	0.258	0.352
		0.613	0.387	1

Source: Integrated Public Use Microdata Series (IPUMS) for 1990 and 2010.
Notes: Average number of children in square brackets for 1950.

is highest for uneducated women married to uneducated men and lowest for educated women married to uneducated men.

5.2 Parameterization

I assume that the length of each period is 18 years. Obtaining a college degree requires a fraction $\epsilon = 0.25$ of period 1, while the retirement phase is a share $R = 0.3$ of the final period. The interest rate is set at 4% and the impatience discount factor at 2.5% per annum which translates into $r = (1.04)^{18} - 1$ and $\rho = (1.025)^{18} - 1$. Wages remain constant over the life-cycle so that $\eta_t^j(E^j) = 0$. Spouses have equal Pareto weights $\alpha = 1 - \alpha = 0.5$ in the household welfare function which implies that the share of wealth allocated to the wife is $\beta(\alpha, \sigma) = 0.5$ irrespective of the value of σ. I assume that child care requires $N_b = 0.25$ per child. If performed by one parent this would correspond to the loss of a quarter of each working day on average during 18 years. With a substitution elasticity between parents of $\xi = 4$ the actual time burden will be less.[13] The cost of child birth for the mother is set at $T_b = 0.02$ or about 4 months with no wages. The equivalence scale of two adults is $Q_a = 1.7$ while each additional child requires $Q_b = 0.5$ (in line with the so-called 'Oxford scale').

Wages for uneducated males are normalized to $w^m(0) = 1$ while uneducated females earn $w^f(0) = 0.75$. The male to female wage ratio is then $1/\chi \approx 1.33$, which is in line with the average findings for the gender wage gap in the US as reported in the meta-analysis of Jarrell and Stanely (2004). The college wage premium is equal for both sexes and set to a value of $\ln(w^j(1)/w^j(0)) = 0.47$, within the range estimated in Hubbard (2011). I abstract from tuition fees ($\upsilon = 0$) and assume that fixed costs are constant at $\bar{c} = 0.1$ which corresponds to 10% of the wage of an uneducated male. With an intertemporal substitution elasticity of $\sigma = 0.5$ this configuration satisfies the

is much less useful for my purposes.

[13] As parents are imperfect substitutes the sum of father time and mother time will be less than $N_b b$ as long as the care burden is allocated between them.

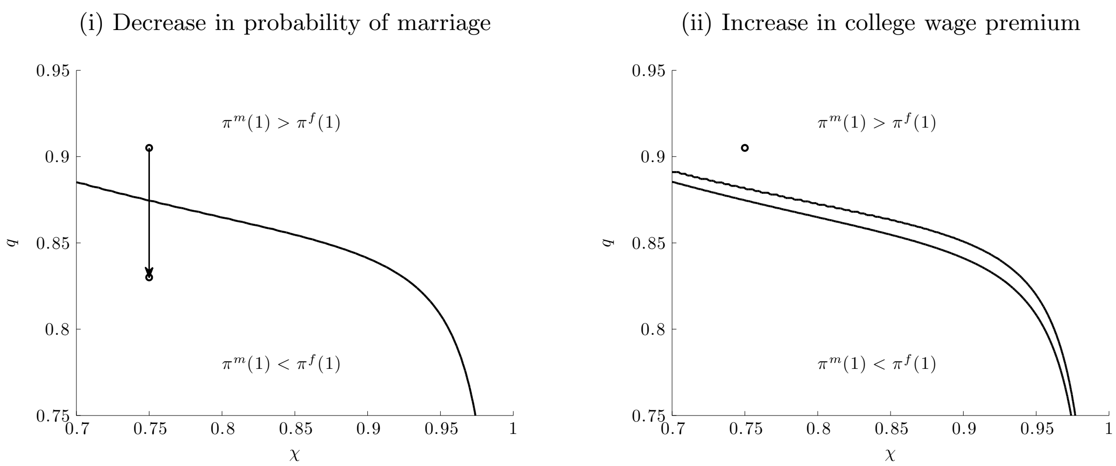
premises of Proposition 2 and the labour market benefit of education is greater for women than for men.

The remaining parameters are set in such a way as to match some of the key statistics for the 1950 cohort. The marital sorting index is $\lambda = 0.563$ and the probability of marriage $q = 0.905$ as in the data. The value of the preference parameter $\phi = 0.671$ ensures that the number of children born to parents who are both uneducated equals the average of 2.310 reported in Table 2. The intertemporal substitution elasticity for consumption then equals $\sigma^* = 0.598$. After calculating the threshold values for education (with $\bar{\theta}^f < \bar{\theta}^m$) I choose the parameters of a lognormal distribution to ensure that the cumulative density equals $\pi^f(1) = 0.252$ at $\bar{\theta}^f$ and $\pi^m(1) = 0.304$ at $\bar{\theta}^m$.

5.3 Numerical results

In Figure 8 the initial equilibrium which represents the 1950 cohort is indicated with a dot at the point where $\chi = 0.75$ and $q = 0.905$. The model will generate a college gender gap reversal if the equilibrium for the 1970 cohort is below the solid line in Figure 8 instead of above it. There are two conditions under which this may occur, which need not exclude each other. First, the dot may shift following a change in the probability of marriage q and/or the female to male wage ratio χ. From Table 1 it is clear that q has decreased from 0.905 for the 1950 cohort to 0.830 for the generation born in 1970. Given the current parameterization this would be sufficient to let women overtake men in educational attainment, see the arrow in Figure 8(i). The reversal would still occur if there is a moderate decrease in the gender wage gap (an increase in χ). The second possibility is that the line itself shifts up. This would happen, for example, if the common college wage premium increases over time or the level of subsistence consumption $\bar{c}$ rises (see Section 4.3). However, even an increase in the college wage premium to 1.8 does not result in a sufficient upward shift of the line to obtain a reversal, see Figure 8(ii). At the same time, the education frequencies for both men and women become too large relative to the data.

Figure 8: College gender gap reversal



For the case that there is only a decrease in q the numerical decomposition of the education threshold is given in Table 3. The bottom line gives the education frequencies $\pi^j(1)$ for each gender and cohort. The row above reports the corresponding threshold level $\bar{\theta}^j$. This number by itself is not very informative as it depends on the scaling of wages.[14] As only the relative magnitudes matter I have chosen to rescale the numbers in such a way that $\bar{\theta}^f = 100$ for the 1950 cohort. The threshold can be decomposed into the labour market benefit of education LMB^j and the marriage market benefit MMB^j (the two lines above). The latter is itself made up of three separate components, namely the parts due to differences in the utility gain ΔUG^j, matching probabilities ΔMP^j and savings behaviour ΔSB^j. Wages remain constant across cohorts, which means that the labour market benefit is also unchanged. It is significantly higher for females than for males, but initially it is outweighed by a much lower marriage market return. As women have lower wages and expect to marry a more wealthy spouse they have less incentive to add to household income by obtaining an education (given diminishing marginal utility of wealth) as is evidenced by the large negative number reported for ΔUG^j. On the other hand the part of the marriage market benefit attributable to matching probabilities is greater for women, in line with the insights from Section 4.2. As q decreases over time, the marriage market benefit of women goes up relative to that of men and the female advantage in the labour market benefit now dominates. The percentage of women that obtain education increases from 25.2% to 31.3% while for men it decreases from 30.4% to 28.1%. As a consequence, there is a reversal of the college gender gap. If the drop in the marriage probability is accompanied by a small increase in the common college wage premium to 1.61 then both education frequencies go up (to 33.6% and 30.1%, respectively), which brings them close to the values for the 1970 cohort reported in Table 1.

5.3.1 Robustness

There are three comments to be made with respect to the robustness of the results derived above.

First of all, the ability of the model to generate a college gender gap reversal depends critically on the choice of parameters in the 1950 benchmark. For example, with a higher intertemporal substitution elasticity σ or a lower level of subsistence consumption $\bar{c}$ the difference in labour market benefit between men and women is smaller and the drop in q alone might not be sufficient to induce women to catch up. In addition, the choice of parameters for the lognormal distribution affects the extent to which education choices respond to changes in benefits. If the threshold levels are close together in the 1950 benchmark, then the standard deviation will have to be low in order to be consistent with a gap of 0.052 in college rates. Consequently, the change in education frequencies as a result of threshold shifts will be greater.

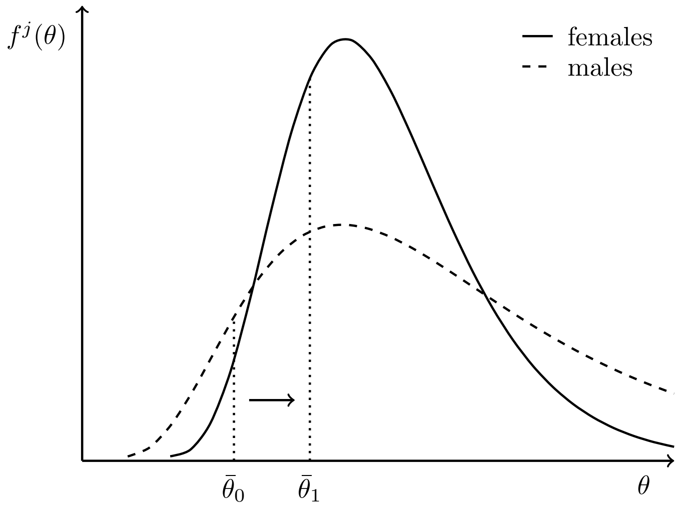
Secondly, many factors are taken as given in the model that need not be independent. For example, an increase in female wages might make them more 'picky' in choosing a partner and therefore less likely to marry. In addition they are more likely to 'choose love over money' which would decrease the degree of sorting (as in Fernández et al. (2005)). If wages depend on work experience then the increase in female labour supply following a drop in marriage rates might lower the gender wage gap.

[14] For example, if all wages are multiplied by 100 then consumption levels change accordingly. Marginal utility is then lower and the threshold level, which is defined as the difference between utility levels of educated and uneducated individuals, will be smaller.

Table 3: Decomposition of the benefit of education

		Cohort 1950		Cohort 1970	
		female	male	female	male
	$\frac{q}{1+\rho}\Delta UG^j$	-75.434	-8.474	-59.848	-10.894
$+$	$\frac{q}{1+\rho}\Delta MP^j$	16.043	3.574	13.246	3.899
$+$	ΔSB^j	19.972	0.563	12.181	0.764
$=$	MMB^j	-39.419	-4.337	-34.421	-6.231
$+$	LMB^j	139.419	108.625	139.419	108.625
$=$	$\bar{\theta}^j$	100.000	104.288	104.998	102.394
	$\pi^j(1)$	0.252	0.304	0.313	0.281

Figure 9: College gender gap reversal with unequal cost distributions



Finally, the fully specified example used here makes a few strong assumptions about the behaviour of married couples that can be relaxed. In Appendix B the 'unitary' household is replaced by one in which spouses bargain over allocations, while Appendix C shows what happens if individuals care for welfare of their partner. In both cases it is possible to find reasonable parameter values under which the model generates a college gender gap reversal if there is a decrease in the probability of marriage.

5.3.2 The role of costs

So far I have ignored possible gender differences in the utility cost of education. This cost is probably inversely related to a person's level of cognitive skills (such as IQ) and non-cognitive skills (for example self-motivation and discipline). Both Becker et al. (2010) and Jacob (2002) provide evidence to support the claim that whereas there are only minor gender differences in cognitive skills, women have on average better non-cognitive skills than men and among them there is less variability.[15] If so, then there is an alternative explanation for the college gender gap reversal which relies exclusively on cost differences, see Becker et al. (2010). Suppose that men and women have similar benefits from a college education but that the distribution of psychic costs is gender-specific as in Figure 9. In particular, men have both a higher mean and a greater standard deviation. Initially the threshold level is such that the mass of men to the left of it is greater than the share of women. Over time the benefits of a college education increase and the threshold shifts to the right. The number of college educated females grows faster than males and the college gender gap is reversed.

Becker et al. (2010) focus on gender differences in costs as they argue that nowadays the benefits of a college education are the same for men and women, if not still higher for men. Part of this

[15]This leaves open the question, however, of whether this is an innate biological difference between the sexes, the result of conscientious investment decisions, or the by-product of a culture that rewards and condones different types of behaviour in men and women.

argument is based on the observation that the college wage premium has been similar in recent years, as found by Hubbard (2011). However, I have shown that despite a common college wage premium it is still possible for women to have a higher (labour market) return to education. Hence there is a role for gender differences in the benefit of education in explaining the college gender gap reversal, which could be complemented by differences in costs.

6 Conclusion and discussion

In this paper I have shown under which conditions a basic theoretical life-cycle model with optimal education choices and exogenous marriage probabilities can generate a reversal in college graduation rates of men and women. In particular, I derive that the labour market benefit of education for women can be higher than for men even if they have the same college wage premium if the elasticity of the marginal utility of wealth is greater than unity or there are fixed costs. Initially this might be dominated by a lower marriage market return, but a decrease in the probability of marriage can induce women to overtake men in educational attainment.

There are some potential contributions to the college gender gap reversal which have been noted in the literature but do not fit into the framework presented here. The first is the role of uncertainty in income and marital status. DiPrete and Buchmann (2006) observe that higher education provides women with insurance against poverty through three channels: higher wages, lower risks of divorce, and less out-of-marriage childbearing. Divorce tends to be more costly for women than for men as custodial arrangements are usually such that the children reside with their mother. The rise in divorce rates over the last decades might therefore have incentivized women to invest in their financial independence.

Secondly, I have ignored the effect that education seems to have on the probability of getting and staying married. In an endogenous matching framework, educated individuals are more desirable spouses and therefore more likely to be married. Whereas in the current model there is no economic rationale for positive assortative matching, it might be the case that education levels are complements instead of substitutes in generating marital surplus (as in Chiappori et al. (2009)). It is not a priori clear, however, why this would affect the marriage market benefit of education for men and women in an asymmetric way.

Finally, as mentioned in the introduction there is a separate strand of literature which models the education decision as one made by parents for their offspring (see for example Echevarria and Merlo (1999)). In this context, parental preferences and expectations about the future wellfare of children of both sexes play a role in educational investments.

Given the near universal pattern of the college gender gap reversal in developed countries, any valid theory of its occurence should be consistent with international evidence. In future work I hope to take up the empirical challenge to disentangle various potential explanations by exploiting both time series and cross-country variation. Secondly, I wish to embed the household model into a general equilibrium framework in order to study the role of child care subsidies in the choice of education and fertility.

References

Becker, G. S., W. H. Hubbard, and K. M. Murphy (2010). Explaining the worldwide boom in higher education of women. *Journal of Human Capital 4*(3), 203–241.

Chiappori, P. A. (1992). Collective labor supply and welfare. *Journal of Political Economy 100*(3), 437–467.

Chiappori, P. A., M. Iyigun, and Y. Weiss (2009). Investment in schooling and the marriage market. *American Economic Review 99*(5), 1689–1713.

Chiappori, P. A., B. Salanié, and Y. Weiss (2011). Partner choice and the marital college premium. Discussion paper no. 1011-04, Columbia University.

DiPrete, T. A. and C. Buchmann (2006). Gender-specific trends in the value of education and the emerging gender gap in college completion. *Demography 43*(1), 1–24.

Dougherty, C. (2005). Why are the returns to schooling higher for women than for men? *The Journal of Human Resources 40*(4), 969–988.

Echevarria, C. and A. Merlo (1999). Gender differences in education in a dynamic household bargaining model. *International Economic Review 40*(2), 265–286.

Fernández, R., N. Guner, and J. Knowles (2005). Love and money: A theoretical and empirical analysis of household sorting and inequality. *The Quarterly Journal of Economics 120*(1), 273–344.

Fernández, R. and R. Rogerson (2001). Sorting and long-run inequality. *The Quarterly Journal of Economics 116*(4), 1305–1341.

Goldin, C., L. F. Katz, and I. Kuziemko (2006). The homecoming of American college women: The reversal of the college gender gap. *Journal of Economics Perspectives 20*(4), 133–156.

Hubbard, W. H. (2011). The phantom gender difference in the college wage premium. *The Journal of Human Resources 46*(3), 568–586.

Jacob, B. A. (2002). Where the boys aren't: Non-cognitive skills, returns to school and the gender gap in higher education. *Economics of Education Review 21*, 589–598.

Jarrell, S. B. and T. Stanley (2004). Declining bias and gender wage discrimination? A meta-regression analysis. *The Journal of Human Resources 39*(3), 828–838.

Ruggles, S., J. T. Alexander, K. Genadek, R. Goeken, M. B. Schroeder, and M. Sobek (2010). Integrated Public Use Microdata Series: Version 5.0. machine-readable database, University of Minnesota, Minneapolis.

A Proofs

Note from (34) that the labour market benefit of education can be written as:

$$LMB^j = \phi\Gamma_1(\sigma)^{-1/\sigma^*} \frac{W_1^j(0)^{1-1/\sigma^*}}{1-1/\sigma^*}\left[\left(\frac{W_1^j(1)}{W_1^j(0)}\right)^{1-1/\sigma^*} - 1\right], \tag{A.1}$$

where $1 - 1/\sigma^* = \phi(1 - 1/\sigma)$. Total wealth $W_1^j(E^j)$ consists of human capital $H_1^j(E^j)$ net of tuition fees υE^j and fixed costs $\delta\bar{c}$, where:

$$H_1^j(E^j) = w^j(E^j)\left[1 - \epsilon E^j + \frac{1+\eta_2^j(E^j)}{1+r} + \frac{(1+\eta_3^j(E^j))(1+\eta_2^j(E^j))[1-R]}{(1+r)^2}\right]. \tag{A.2}$$

Proposition 1. *Assume there are no fixed costs, no tuition fees, and equal wage growth for both sexes over time and for each level of education. If the college wage premium is the same for both sexes but females earn less then equally qualified males, then:*

$$LMB^f \gtreqless LMB^m \quad \text{if} \quad \sigma \lesseqgtr 1$$

Differences in the labour market benefit between the sexes depend positively on the common college wage premium.

Proof. According to the premises of the proposition:

$$\bar{c} = 0; \quad \upsilon = 0; \quad \eta_t^f(1) = \eta_t^m(1) \text{ and } \eta_t^f(0) = \eta_t^m(0) \quad \forall t \in \{1,2,3\}; \quad \frac{w^f(1)}{w^f(0)} = \frac{w^m(1)}{w^m(0)}$$

Under these assumptions:

$$W_1^j(E^j) = H_1^j(E^j).$$

If $w^j(1)/w^j(0)$ is fixed then so is $H_1^j(1)/H_1^j(0)$. Taking the derivative of (A.1) with respect to $H_1^j(0)$ while keeping $H_1^j(1)/H_1^j(0)$ constant gives:

$$\frac{\partial LMB^j}{\partial H_1^j(0)} = \phi\Gamma_1(\sigma)^{-1/\sigma^*} H_1^j(0)^{-1/\sigma^*}\left[\left(\frac{H_1^j(1)}{H_1^j(0)}\right)^{1-1/\sigma^*} - 1\right] \lesseqgtr 0 \quad \Leftrightarrow \quad \sigma^* \lesseqgtr 1 \quad \Leftrightarrow \quad \sigma \lesseqgtr 1,$$

since $H_1^j(1)/H_1^j(0) > 1$. The result follows because $H_1^f(0) < H_1^m(0)$ if $w^f(0) < w^m(0)$. Note that:

$$\frac{\partial^2 LMB^j}{\partial H_1^j(0)\partial\left(\frac{H_1^j(1)}{H_1^j(0)}\right)} = (1 - 1/\sigma^*)\phi\Gamma_1(\sigma)^{-1/\sigma^*} H_1^j(1)^{-1/\sigma^*} \lesseqgtr 0 \quad \Leftrightarrow \quad \sigma \lesseqgtr 1.$$

$\square$

Proposition 2. *Let $0 < \sigma \leq 1$. Assume there are positive fixed costs, no tuition fees, and equal wage growth for both sexes over time and for each level of education. If the college wage premium is the same for both sexes but females earn less then equally qualified males, then $LMB^f > LMB^m$.*

Proof. According to the premises of the proposition:

$$\bar{c} > 0; \quad \upsilon = 0; \quad \eta_t^f(1) = \eta_t^m(1) \text{ and } \eta_t^f(0) = \eta_t^m(0) \quad \forall t \in \{1,2,3\}; \quad \frac{w^f(1)}{w^f(0)} = \frac{w^m(1)}{w^m(0)}$$

Under these assumptions:

$$W_1^j(E^j) = H_1^j(E^j) - \delta\bar{c}.$$

If $w^j(1)/w^j(0)$ is fixed then so is $H_1^j(1)/H_1^j(0)$. Taking the derivative of LMB^j with respect to $H_1^j(0)$ while keeping $H_1^j(1)/H_1^j(0)$ constant gives:

$$\frac{\partial LMB^j}{\partial H_1^j(0)} = \phi\Gamma_1(\sigma)^{-1/\sigma^*}\left[H_1^j(0) - \delta\bar{c}\right]^{-1/\sigma^*}\left\{\left[\left(\frac{H_1^j(1) - \delta\bar{c}}{H_1^j(0) - \delta\bar{c}}\right)^{1-1/\sigma^*} - 1\right]\right.$$
$$\left. - \left(\frac{H_1^j(1) - \delta\bar{c}}{H_1^j(0) - \delta\bar{c}}\right)^{1-1/\sigma^*} \frac{\delta\bar{c}\left[\frac{H_1^j(1)}{H_1^j(0)} - 1\right]}{H_1^j(0) - \delta\bar{c}}\right\} < 0,$$

since $H_1^j(1)/H_1^j(0) > 1$ and $\sigma \leq 1$ such that $\sigma^* \leq 1$. The result follows because $H_1^f(0) < H_1^m(0)$ if $w^f(0) < w^m(0)$.

□

B Household bargaining

Suppose that a married couple does not act as a unitary whole or representative agent. Instead future husbands and wives bargain cooperatively over household allocations just before they get married but fully commit to them afterwards. Provided that the bargaining outcome is Pareto efficient the household still acts as though it maximizes a weighted average of individual utility functions as in (21) but now the weight attached to the female α is endogenous. Let $\tilde{\mathcal{M}}_2^j(E^j, E^{-j}, a_1|\alpha)$ denote the value function of a married individual conditional on α, which is the same as (27) above.

Previously it was possible to ignore any utility gain from being married other than that derived from the sharing of resources and children because it would cancel out in the decomposition of the education threshold (assuming it is independent of the education level of the spouse). In the context of bargaining, however, it influences how sensitive allocations are to changes in education and assets. From now on ν denotes the discounted flow of 'conjugal bliss' at the start of period 2.

A disadvantage of the bargaining approach is that the solutions will in large part be driven by the specification of threat points, the choice of which may not be obvious. Here I take the utility when single $\mathcal{S}_2^j(E^j, a_1^j)$ as specified in (18) as the outside option of each individual.[16] I focus on Nash bargaining, in which case α is given by:

$$\begin{aligned} \alpha(E^f, E^m, a_1^f, a_1^m) = \underset{\alpha}{\operatorname{argmax}} \Big\{ & \Big[\tilde{\mathcal{M}}_2^f(E^f, E^m, a_1^f + a_1^m|\alpha) + \nu - \mathcal{S}_2^f(E^f, a_1^f)\Big]^{\psi} \\ & \times \Big[\tilde{\mathcal{M}}_2^m(E^m, E^f, a_1^m + a_1^f|\alpha) + \nu - \mathcal{S}_2^m(E^m, a_1^m)\Big]^{1-\psi} \Big\}, \end{aligned} \tag{B.1}$$

where the parameter $\psi \in [0, 1]$ captures the relative bargaining strength of the female. I will restrict attention here to the classical case with $\psi = 0.5$ so that the overall bargaining position of each individual is determined by his or her outside option. Note that α is a function of individual education levels and savings.

The value of being married is now defined as:

$$\mathcal{M}_2^j(E^j, E^{-j}, a_1^j, a_1^{-j}) = \tilde{\mathcal{M}}_2^j(E^j, E^{-j}, a_1^j + a_1^{-j}|\alpha(E^f, E^m, a_1^f, a_1^m)) + \nu. \tag{B.2}$$

In this context, individuals have a strategic reason for obtaining education and accumulating financial assets. A college degree or a greater stock of savings enhances their position within the household and thereby increases the share of wealth that they will extract. This allows a further decomposition of the benefit of education. In particular, the part of the marriage market benefit attributable to differences in utility gain can now be written as:

$$\Delta UG^j = \Delta MU^j + \Delta PW^j. \tag{B.3}$$

The first term ΔMU^j is related to the marginal utility of wealth and is obtained by keeping the Pareto weights fixed when comparing one marital state with another. The second one ΔPW^j captures the strategic effect of changing Pareto weights. The remainder of the decomposition re-

[16] Other possibilities include the non-cooperative household allocation or the utility derived from remaining single with the possibility of marriage to a different person (but this would require a different model).

mains unchanged. An important difference, however, is that the part due to matching probabilities ΔMP^j might be negative: Having an educated spouse is now less desirable as this will deteriorate the own Pareto weight.

In order to get a consistent parameterization with $\bar{\theta}^f < \bar{\theta}^m$ I increase the intertemporal substitution elasticity to $\sigma = 0.65$ and set $\nu = 3$. The model still generates a college gender gap reversal following a drop in the probability of marriage, see Table B.1 for a decomposition of the threshold levels in the two equilibria.

Table B.1: Decomposition with household bargaining

		Cohort 1950		Cohort 1970	
		female	male	female	male
	$\frac{q}{1+\rho}\Delta MU^j$	−49.986	−11.379	−43.144	−11.986
+	$\frac{q}{1+\rho}\Delta PW^j$	6.483	3.980	5.743	3.691
=	$\frac{q}{1+\rho}\Delta UG^j$	−43.503	−7.399	−37.401	−8.295
+	$\frac{q}{1+\rho}\Delta MP^j$	13.260	1.418	10.731	1.720
+	ΔSB^j	5.629	0.389	4.166	0.401
=	MMB^j	−24.614	−5.592	−22.504	−6.174
+	LMB^j	124.614	107.067	124.615	107.067
=	$\bar{\theta}^j$	100.000	101.475	102.111	100.893
	$\pi^j(1)$	0.252	0.304	0.328	0.283

C Caring preferences

As a second extension I revert to the unitary model but now endogenize the level of conjugal bliss ν introduced in Appendix B. Up to this point I have assumed that each individual's preferences are completely egotistic: he or she only cares about his or her private consumption and the own utility derived from the presence of children. It is easy to generalize this to the case of 'caring preferences' by assuming that after marriage the felicity function of spouse j becomes:

$$\bar{u}^j(c_t^j, c_t^{-j}, b) = u(c_t^j, b) + \nu^j u(c_t^{-j}, b), \quad 0 \leq \nu^j < 1. \tag{C.1}$$

In order to be able to meaningfully compare the welfare of a married person with that of a single (for whom $\nu^j = 0$) I need to ensure that $u(c_t^{-j}, b) \geq 0$. This is not necessarily the case for the chosen felicity function if $\sigma \leq 1$ but can be achieved by a suitable scaling of endowments, for example by multiplying all wages and the fixed cost by a factor 100.

The periodic household welfare function (21) can now be written as:

$$\begin{aligned} U(c_t^f, c_t^m, b) &= \alpha \bar{u}^f(c_t^f, c_t^m, b) + (1-\alpha)\bar{u}^m(c_t^m, c_t^f, b) \\ &= \big[\alpha + (1-\alpha)\nu^m\big] u(c_t^f, b) + \big[(1-\alpha) + \alpha\nu^f\big] u(c_t^m, b). \end{aligned} \tag{C.2}$$

From the structure of the welfare function it follows that the optimal allocations are similar to the ones with egotistic preferences, only the female Pareto weight α is replaced by:

$$\bar{\alpha} = \frac{\alpha + (1-\alpha)\nu^m}{1 + (1-\alpha)\nu^m + \alpha\nu^f}. \tag{C.3}$$

The more a person is cared for, the higher is his or her 'new' Pareto weight. For a female the corresponding value function is given by:

$$\begin{aligned} \mathcal{M}_2^f(E^f, E^m, a_1^f, a_1^m) = \frac{1}{\Gamma_2(1)} \Bigg\{ & \Gamma_2(1)\Gamma_2(\sigma)^{-1/\sigma^*}\Phi(\sigma)\left[\frac{W_2(E^f, E^m, a_1)}{C_b(E^f, E^m)^{1-\phi}}\right]^{1-1/\sigma} \\ & \times \frac{\beta(\bar{\alpha},\sigma)^{\phi(1-1/\sigma)} + \nu^f[1-\beta(\bar{\alpha},\sigma)]^{\phi(1-1/\sigma)}}{1-1/\sigma} - \frac{(1+\nu^f)}{1-1/\sigma} \Bigg\}. \end{aligned} \tag{C.4}$$

The wife does not derive utility only from her share of wealth $\beta(\bar{\alpha}, \sigma)$ but to a lesser degree also from that of her husband. The anticipated positive effect on the welfare of the spouse provides additional incentives for education and saving in the first period. Again this allows for a further decomposition of the benefit of education. I write:

$$\Delta UG^j = \Delta MU^j + \nu^j \Delta CP^j. \tag{C.5}$$

where ΔMU^j is the difference in the own utility from consumption and children, while ΔCP^j captures that of the future spouse. The overall marriage market benefit of education is now more likely to be positive (in constrast to the benchmark case and the extension with household bargaining).

I repeat the parameterization procedure with $\nu^f = \nu^m = 0.1$. The results are reported in Table C.1. The marriage market benefit of education is positive for males in the 1950 cohort but negative for females. The model still generates a college gender gap reversal following a drop in the probability of marriage but the education frequencies are somewhat closer together for the 1970 cohort.

Table C.1: Decomposition with caring preferences

		Cohort 1950		Cohort 1970	
		female	male	female	male
	$\frac{q}{1+\rho}\Delta MU^j$	-71.812	-7.972	-57.201	-9.930
$+$	$\frac{q}{1+\rho}\nu^j \Delta CP^j$	1.870	4.098	1.740	3.515
$=$	$\frac{q}{1+\rho}\Delta UG^j$	-69.942	-3.874	-55.461	-6.415
$+$	$\frac{q}{1+\rho}\Delta MP^j$	16.794	3.945	14.617	4.282
$+$	ΔSB^j	17.927	0.177	10.794	0.362
$=$	MMB^j	-35.221	0.248	-30.050	-1.771
$+$	LMB^j	135.221	105.091	135.221	105.091
$=$	$\bar{\theta}^j$	100.000	105.339	105.171	103.320
	$\pi^j(1)$	0.252	0.304	0.302	0.284

D Computational details

The equilibrium of the model can be found by solving the following subproblems.

- *The value function of a single at the start of period* 2

  I assume that the solution to the optimization problem of a single at the start of period 2 is interior and verify this ex-post. This implies that I have an analytical expression for all allocation choices and the corresponding value function for both sexes $j \in \{f, m\}$ and any set of state variables $\{E^j, a_1^j\}$.

- *The value function of a married individual at the start of period* 2

  I assume that the solution to the optimization problem of a couple at the start of period 2 is interior and verify this ex-post. Given the Pareto weight α I have an analytical expression for all allocation choices and the corresponding value function of each spouse for any set of state variables $\{E^f, E^m, a_1^f, a_1^m\}$. If the weight is endogenous then I have to numerically find the value of α that maximizes the Nash bargaining objective function (B.1).

- *The equilibrium in savings at the start of period* 2 *conditional on education*

  For any given set of education frequencies $\mathcal{P} = \{\pi^f(1), \pi^m(1)\}$ I calculate the equilibrium choices of savings $\mathcal{A} = \{a_1^f(0), a_1^f(1), a_1^m(0), a_1^m(1)\}$ numerically. Under the assumption of fixed Pareto weights this amounts to solving 4 first-order conditions (similar to (29)) in 4 endogenous variables using rootfinding techniques.

  In the case of Nash bargaining it is more complicated. I set up a grid of feasible values for $a_1^j(E^j)$ for any combination of gender and education level. Then I find the best response for a person of gender j with education level E^j for any set of choices $\mathcal{A}^{-j} = \{a_1^{-j}(0), a_1^{-j}(1)\}$ made by individuals of the opposite gender with numerical optimization. I use spline interpolation and rootfinding techniques to obtain a consistent equilibrium.

- *The marriage market equilibrium*

  Obtaining the marriage market equilibrium amounts to finding a fixed point. For any guess regarding the education frequencies $\mathcal{P} = \{\pi^f(1), \pi^m(1)\}$ I calculate the equilibrium in savings and the corresponding values functions. This yields an education threshold $\bar{\theta}^j$, which provides a new guess $\pi^j(1) = F^j(\bar{\theta}^j)$. I iterate over $\mathcal{P}$ until the solutions converge.

E Data

I use data from the Integrated Public Use Microdata Series (IPUMS) for the US (Ruggles et al. (2010)).

To create Figure 1(i) I take the default US sample for every available year from 1970 up to and including 2011. I only select individuals who are 40 years of age. I create a dummy for college education which takes the value of 1 if a person has 4 years of college or more and 0 otherwise. (From 1990 onwards a more detailed education variable is available which explicitly includes the highest degree earned.) Then I calculate the proportion of college-educated individuals of each sex using the person weights present in the data.

In order to obtain the matching probabilities and marriage patterns reported in Table 1 and 2 I take the 1% sample for 1990 and 2010. I start by selecting individuals from age 30 up to and including age 50. I create a cohabitation dummy that takes the value of 1 if a household head lives together with an unmarried partner and 0 otherwise. For all married and cohabiting couples I make college dummies for the male and the female in the household. I then restrict the sample to those individuals who are 40 years of age (but whose partner might have any age between 30 and 50). Individuals of 40 years old that are not included in the final sample are those that live with a partner of the same sex and those that have a partner with whom the difference in age is more than 10 years. I calculate the probability of marriage q as one minus the proportion of individuals who have never been married and are not currently cohabiting. The degree of sorting λ can be obtained from:

$$\lambda = \frac{\pi(1,1) - \pi^f(1)\pi^m(1)}{\min\{\pi^f(1), \pi^m(1)\} - \pi^f(1)\pi^m(1)}. \tag{E.1}$$

List of research reports

12001-HRM&OB: Veltrop, D.B., C.L.M. Hermes, T.J.B.M. Postma and J. de Haan, A Tale of Two Factions: Exploring the Relationship between Factional Faultlines and Conflict Management in Pension Fund Boards

12002-EEF: Angelini, V. and J.O. Mierau, Social and Economic Aspects of Childhood Health: Evidence from Western-Europe

12003-Other: Valkenhoef, G.H.M. van, T. Tervonen, E.O. de Brock and H. Hillege, Clinical trials information in drug development and regulation: existing systems and standards

12004-EEF: Toolsema, L.A. and M.A. Allers, Welfare financing: Grant allocation and efficiency

12005-EEF: Boonman, T.M., J.P.A.M. Jacobs and G.H. Kuper, The Global Financial Crisis and currency crises in Latin America

12006-EEF: Kuper, G.H. and E. Sterken, Participation and Performance at the London 2012 Olympics

12007-Other: Zhao, J., G.H.M. van Valkenhoef, E.O. de Brock and H. Hillege, ADDIS: an automated way to do network meta-analysis

12008-GEM: Hoorn, A.A.J. van, Individualism and the cultural roots of management practices

12009-EEF: Dungey, M., J.P.A.M. Jacobs, J. Tian and S. van Norden, On trend-cycle decomposition and data revision

12010-EEF: Jong-A-Pin, R., J-E. Sturm and J. de Haan, Using real-time data to test for political budget cycles

12011-EEF: Samarina, A., Monetary targeting and financial system characteristics: An empirical analysis

12012-EEF: Alessie, R., V. Angelini and P. van Santen, Pension wealth and household savings in Europe: Evidence from SHARELIFE

13001-EEF: Kuper, G.H. and M. Mulder, Cross-border infrastructure constraints, regulatory measures and economic integration of the Dutch – German gas market

13002-EEF: Klein Goldewijk, G.M. and J.P.A.M. Jacobs, The relation between stature and long bone length in the Roman Empire

13003-EEF: Mulder, M. and L. Schoonbeek, Decomposing changes in competition in the Dutch electricity market through the Residual Supply Index

13004-EEF: Kuper, G.H. and M. Mulder, Cross-border constraints, institutional changes and integration of the Dutch – German gas market

13005-EEF: Wiese, R., Do political or economic factors drive healthcare financing privatisations? Empirical evidence from OECD countries

13006-EEF: Elhorst, J.P., P. Heijnen, A. Samarina and J.P.A.M. Jacobs, State transfers at different moments in time: A spatial probit approach

13007-EEF: Mierau, J.O., The activity and lethality of militant groups: Ideology, capacity, and environment

13008-EEF: Dijkstra, P.T., M.A. Haan and M. Mulder, The effect of industry structure and yardstick design on strategic behavior with yardstick competition: an experimental study

13009-GEM: Hoorn, A.A.J. van, Values of financial services professionals and the global financial crisis as a crisis of ethics

13010-EEF: Boonman, T.M., Sovereign defaults, business cycles and economic growth in Latin America, 1870-2012

13011-EEF: He, X., J.P.A.M Jacobs, G.H. Kuper and J.E. Ligthart, On the impact of the global financial crisis on the euro area

13012-GEM: Hoorn, A.A.J. van, Generational shifts in managerial values and the coming of a global business culture

13013-EEF: Samarina, A. and J.E. Sturm, Factors leading to inflation targeting – The impact of adoption

13014-EEF: Allers, M.A. and E. Merkus, Soft budget constraint but no moral hazard? The Dutch local government bailout puzzle

13015-GEM: Hoorn, A.A.J. van, Trust and management: Explaining cross-national differences in work autonomy

13016-EEF: Boonman, T.M., J.P.A.M. Jacobs and G.H. Kuper, Sovereign debt crises in Latin America: A market pressure approach

13017-GEM: Oosterhaven, J., M.C. Bouwmeester and M. Nozaki, The impact of production and infrastructure shocks: A non-linear input-output programming approach, tested on an hypothetical economy

13018-EEF: Cavapozzi, D., W. Han and R. Miniaci, Alternative weighting structures for multidimensional poverty assessment

14001-OPERA: Germs, R. and N.D. van Foreest, Optimal control of production-inventory systems with constant and compound poisson demand

14002-EEF: Bao, T. and J. Duffy, Adaptive vs. eductive learning: Theory and evidence

14003-OPERA: Syntetos, A.A. and R.H. Teunter, On the calculation of safety stocks

14004-EEF: Bouwmeester, M.C., J. Oosterhaven and J.M. Rueda-Cantuche, Measuring the EU value added embodied in EU foreign exports by consolidating 27 national supply and use tables for 2000-2007

14005-OPERA: Prak, D.R.J., R.H. Teunter and J. Riezebos, Periodic review and continuous ordering

14006-EEF: Reijnders, L.S.M., The college gender gap reversal: Insights from a life-cycle perspective www.rug.nl/feb