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## Event-based simulation of quantum phenomena

Zhao, Shuang

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*Document Version*

Publisher's PDF, also known as Version of record

*Publication date:*

2009

[Link to publication in University of Groningen/UMCG research database](#)

*Citation for published version (APA):*

Zhao, S. (2009). *Event-based simulation of quantum phenomena: application to quantum interference, Einstein-Podolsky-Rosen experiments, quantum computation and quantum cryptography*. s.n.

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# Event-based Simulation of Quantum Phenomena

*Application to Quantum Interference, Einstein-Podolsky-Rosen Experiments,  
Quantum Computation and Quantum Cryptography*

**Shuang Zhao**

**2009**



University of Groningen  
**Zernike Institute**  
**for Advanced Materials**

***To my parents and my husband***

Zernike Institute PhD thesis series 2009-05  
ISSN 1570-1530

The work described in this thesis was performed at the Department of Applied Physics of the Rijksuniversiteit Groningen, the Netherlands.

ISBN electronic version: 978-90-367-3745-6  
ISBN printed version: 978-90-367-3746-3  
Printed by Ipskamp Drukkers, the Netherlands

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RIJKSUNIVERSITEIT GRONINGEN

# Event-based Simulation of Quantum Phenomena

*Application to Quantum Interference, Einstein-Podolsky-Rosen Experiments,  
Quantum Computation and Quantum Cryptography*

## Proefschrift

ter verkrijging van het doctoraat in de  
Wiskunde en Natuurwetenschappen  
aan de Rijksuniversiteit Groningen  
op gezag van de  
Rector Magnificus, dr. F. Zwarts,  
in het openbaar te verdedigen op  
vrijdag 13 maart 2009  
om 14:45 uur

door

**Shuang Zhao**

geboren op 21 mei 1979  
te Harbin China

**Promotor:** Prof.dr. H.A. De Raedt

**Beoordelingscommissie:** Prof.dr. S. Miyashita  
Prof.dr. J.Th.M. De Hossen  
Prof.dr. P. Rudolf

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