$K^+ \to \pi^+\pi^0$ decays at next-to-leading order in the chiral expansion on finite volumes

Lin, C.-J.D.; Martinelli, G.; Pallante, E.; Sachrajda, C.T.; Villadoro, G.

Published in:
Nuclear Physics B

Publication date:
2003

ELSEVIER

Nuclear Physics B (Proc. Suppl.) 119 (2003) 383-385

NUCLEAR PHYSICS B
PROCEEDINGS
SUPPLEMENTS

www.elsevier.com/locate/npe

$K^+ \rightarrow \pi^+\pi^0$ decays at next-to-leading order in the chiral expansion on finite volumes *

C.-J.D. Lin[a†], G. Martinelli[b], E. Pallante[c], C.T. Sachrajda[a,d] and G. Villadoro[b]

[a] Dept. of Physics and Astronomy, Univ. of Southampton, Southampton SO17 1BJ, England
[b] Dip. di Fisica, Universitá di Roma "La Sapienza", Piazzale A. Moro 2, I-00185 Roma. Italy
[c] SISSA, Via Beirut 2-4, 34013, Trieste, Italy
[d] Theory Division, CERN, CH-1211 Geneva 23, Switzerland

We present the ingredients for determining $K^+ \rightarrow \pi^+\pi^0$ matrix elements via the combination of lattice QCD and chiral perturbation theory (χPT). By simulating these matrix elements at unphysical kinematics, it is possible to determine all the low-energy constants (LECs) for constructing the physical $K^+ \rightarrow \pi^+\pi^0$ amplitudes at next-to-leading order (NLO) in the chiral expansion. In this work, the one-loop chiral corrections are calculated for arbitrary meson four-momenta, in both χPT and quenched χPT (qχPT), and the finite-volume effects are studied.

1. INTRODUCTION

The need for a high-precision prediction for $K \rightarrow \pi\pi$ amplitudes is underlined by the recent experimental measurement of $\mathrm{Re}(\epsilon'/\epsilon)$ and the long-standing puzzle, the AI = 1/2 rule. Although the finite-volume (FV) techniques developed in Refs. [3–5] can ultimately enable an accurate calculation of $K \rightarrow \pi\pi$ decay rates, the most practical approach to the numerical calculation of these decay rates remains the combination of lattice QCD and (quenched and partially quenched) χPT. Apart from a calculation for the CP-conserving, AI = 3/2, $K \rightarrow \pi\pi$ decay in Ref. [1], all the numerical studies hitherto follow a strategy proposed in Ref. [2], which only allows the determination of these amplitudes at leading-order (LO) in the chiral expansion[3]. Because of the large kaon mass and the presence of final state interactions, non-LO corrections in this expansion are significant. In a recent work [6,7], we have proposed to perform lattice simulations at unphysical kinematics over a range of meson masses and momenta, from which we can determine all the necessary LECs for constructing the physical matrix element $\langle\pi^+\pi^0|\mathcal{O}^{\Delta S=1}|K^+\rangle$ at NLO in the chiral expansion. We have suggested a specific unphysical kinematics[4], the SPQR kinematics as explained in detail in Refs. [6,7], which enables such a procedure, and have studied the following AS = 1 operators (α, β are colour indices)

$$
\begin{aligned}
Q_4 &= (\bar{s}_\alpha d_\alpha)_L(\bar{u}_\beta u_\beta - \bar{d}_\beta d_\beta)_L \\
&\quad +(\bar{s}_\alpha u_\alpha)_L(\bar{u}_\beta d_\beta)_L, \\
Q_7 &= \frac{3}{2}(\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_R, \qquad (1)\\
Q_8 &= \frac{3}{2}(\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_R,
\end{aligned}
$$

where e_q is the electric charge of q and $(\bar{\psi}_1\psi_2)_{L,R}$ means $\bar{\psi}_1\gamma_\mu(1 \mp \gamma_5)\psi_2$.

2. FINITE-VOLUME EFFECTS

In Ref. [6], we investigate the FV corrections, power-like in $1/L$, which arise from replacing sums by integrals in the one-loop calculation that involves the diagrams in Fig. 1 [5]. It can be shown

*SHEP 02-18.

†Presenter at the conference.

[3]In Ref. [1], the decay amplitude is also obtained at the precision of leading-order in the chiral expansion.

[4]Another choice is considered in Ref. [8].

[5]Such a calculation for $\langle\pi^+\pi^0|Q_4|K^+\rangle$ at two particular kinematics, $M_K = M_\pi$ and $M_K = 2M_\pi$ with all mesons

0920-5632/03/$ see front matter
doi:10.1016/S0920-5632(03)01561-5

that a diagram which does not have an imaginary part in' Minkowski space will only have FV corrections exponential in L, therefore only diagram (c) contributes to the $1/L^n$ corrections. Because the two-pion final state has $I = 2$, this diagram only contains four-quark intermediate states and there are no disconnected quark-loops in the quark-flow picture. For the same reason, it does not receive contributions from the η' propagator. Hence the $1/L^n$ effects are identical in χPT and quenched χPT (qχPT) for $K^+ \to \pi^+\pi^0$ at this order, and only at this order. This is not true for AI = $1/2$ decay amplitudes [9,10].

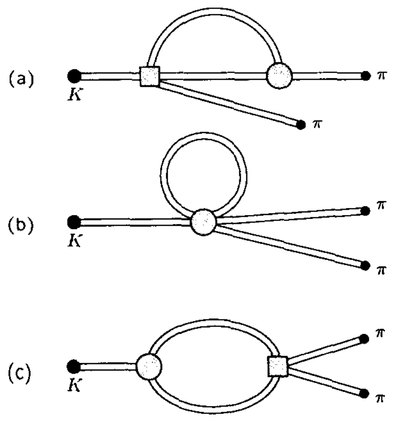

Figure 1. *One-loop* diagrams *for* $K^+ \to \pi^+\pi^0$ amplitudes. *The grey circles (squares) are weak* (strong) *vertices. The* diagrams *for* wavefunction renormalisation are *not shown here.*

We have found that in the center-of-mass frame, for all the $K^+ \to \pi^+\pi^0$ amplitudes, these one-loop $1/L^n$ corrections are independent of the weak operators and can be removed by a universal factor derived first by Lellouch and Lüscher in Ref. [3]. This modifies the conclusion of Refs. [11,12], in which a FV effect resulting in the shift of the two-pion total energy in the argument of the tree-level amplitudes is interpreted as a genuine $1/L^n$ correction to the matrix elements, and therefore the FV effects in $\langle\pi^+\pi^0|Q_4|K^+\rangle$ are found to depend on M_K.

at rest, have been performed in Refs. [11,12].

We are currently investigating the FV effects of these amplitudes in a moving frame. The Lellouch-Lüscher factor has not yet been 'derived for this, while the modification of Lüscher's quantisation condition [13,14] relating the infinite-volume $\pi\pi$ scattering phase to the FV two-pion energy spectrum, due to the moving frame was obtained in Ref. [15]. As a by-product of our work, we verify that the energy shift obtained in one-loop perturbation theory in a moving frame agrees with the expansion of the quantization condition in Ref. [15] to the same order.

3. ONE-LOOP CHIRAL CORRECTIONS IN INFINITE VOLUME

We evaluate the one-loop correction by using dimensional regularisation and subtracting $\log(4\pi) - \gamma_E + 1 + 2/(4 - d)$. The lowest-order amplitudes are all proportional to $1/f^3$, where f is the light pseudoscalar meson decay constant in the chiral limit. At NLO, we choose to express $1/f^3$ in terms of $1/(f_\pi^2 f_K)$. This factor fully absorbs the dependence upon the Gasser-Leutwyler LECs L_4 and L_5 introduced via wavefunction renormalisation.

In Ref. [6], the one-loop diagrams have been calculated for arbitrary external meson four-momenta in both χPT and qχPT. The results are lengthy and are presented on a web site [16]. In Fig. 2, we show an example of these results for $\langle\pi^+\pi^0|\mathcal{O}_{7,8}|K^+\rangle$. These plots are the ratios between the one-loop corrections and the lowest-order matrix elements with both final-state pions at rest. Fig. 2a is the result in χPT and Fig. 2b is that in qχPT. This figure suggests that in a quenched numerical calculation of these matrix elements, it is not appropriate in general to perform chiral extrapolation using unquenched χPT results [17]. This is confirmed by numerical data [18].

4. CONCLUSIONS

We have made theoretical progress towards the calculation of $K \to \pi\pi$ decay amplitudes via the combination of lattice QCD and χPT. We find

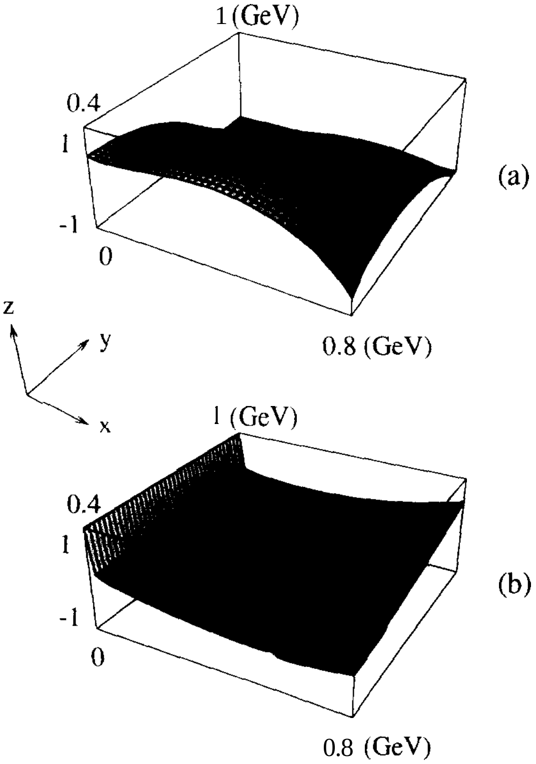

Figure 2. Ratio *between* the *one-loop correction, at the* renormalisation scale *0.7 GeV, and the lowest-order result for* $\langle\pi^+\pi^0|\mathcal{O}_{7,8}|K^+\rangle$ *in* (a) *χPT and (b) $q\chi PT$. The* x axis is M_π and *the y* axis is M_K. In *(b), the* coupling *accompanying the kinetic term of the η'* propagator is *set to zero, and the η' mass is taken to be M_0 = 0.5 GeV. The one-loop results are not very sensitive to these parameters. The singular behaviour* along *the line* $M_\pi = \sqrt{2}M_K$ *in (b) is due to the fact* that *when performing the* $q\chi PT$ calculation, we *use a* basis *in which the pseudo-Goldstone states are $\bar{q}q'$ mesons, where q and q' are u, d and* s, and *the $\bar{s}s$ meson becomes tachyonic when* $M_\pi > \sqrt{2}M_K$.

it feasible to determine all the LECs for constructing ${}_{I=2}\langle\pi\pi|\mathcal{O}^{\Delta S=1}|K\rangle$ at NLO in the chiral expansion. A quenched numerical study is in progress [18]. As for the *AI* = 1/2 channel, we find the situation to be considerably more complicated [9].

REFERENCES

1. S. Aoki *et* al. [JLQCD Collaboration], Phys. Rev. D 58 (1998) 054503 [arXiv:hep-lat/9711046].
2. C. W. Bernard *et al.,* Phys. Rev. D 32 (1985) 2343.
3. L. Lellouch and M. Liischer, Commun. Math. Phys. 219 (2001) 31 [arXiv:hep-lat/0003023].
4. C.-J. D. Lin *et al.,* Nucl. Phys. B 619 (2001) 467 [arXiv:hep-lat/0104006].
5. C.-J. D. Lin *et* al., Nucl. Phys. Proc. Suppl. 109 (2002) 218 [arXiv:hep-lat/0111033].
6. C.-J. D. Lin *et* al., arXiv:hep-lat/0208007.
7. P. Boucaud *et* al., Nucl. Phys. Proc. Suppl. 106 (2002) 329 [arXiv:hep-lat/0110206].
8. J. Laiho and A. Soni, Phys. Rev. D 65 (2002) 114020 [arXiv:hep-ph/0203106].
9. G. Villadoro, these proceedings.
10. M. Golterman and E. Pallante, Nucl. Phys. Proc. Suppl. 83 (2000) 250 [arXiv:hep-lat/9909069].
11. M. F. Golterman and K. C. Leung, Phys. Rev. D 56 (1997) 2950 [arXiv:hep-lat/9702015].
12. M. F. Golterman and K. C. Leung, Phys. Rev. D 58 (1998) 097503 [arXiv:hep-lat/9805032].
13. M. Liischer, Commun. Math. Phys. 105 (1986) 153.
14. M. Liischer, Nucl. Phys. B 354 (1991) 531.
15. K. Rummukainen and S. Gottlieb, Nucl. Phys. B 450 (1995) 397 [arXiv:hep-lat/9503028].
16. C.-J.D. Lin *et al.,* http://www.hep.phys.soton.ac.uk/kpipi/.
17. C. Bernard *et al., Panel discussion on chiral extrapolation,* these proceedings.
18. M. Papinutto, these proceedings.