Atmospheric variability and the Atlantic multidecadal oscillation

Sterk, Alef Edou

Chapter 1

Introduction

1.1 Motivation and setting of the problem

In this work we study low-frequency variability of the North Atlantic climate system with particular emphasis on the Atlantic Multidecadal Oscillation and recurrent atmospheric flow patterns. We study suitable low-order models for the oceanic and atmospheric circulation using concepts of dynamical systems theory.

Models for climatological phenomena typically involve a Hopf bifurcation. A particular example is El Niño (Dijkstra, 2005, Chapter 7). In the models of the present work a Hopf bifurcation leads to periodic attractors representing multidecadal temperature swings in the ocean and planetary waves in the atmosphere. Along several routes to chaos the periodic attractors bifurcate into strange attractors. Key notions are unpredictability and low-dimensional chaos (Broer and Takens, 2010), which have also been associated with the onset of turbulence (Ruelle and Takens, 1971). In the ocean model we find (quasi-periodic) period doublings leading to (quasi-periodic) Hénon-like strange attractors. In the atmosphere model we find period doubling cascades, Hopf-Neĭmark-Sacker bifurcations followed by the breakdown of a quasi-periodic attractor, and intermittency.

The present study is a typical example of *experimental mathematics*. Even in the simplified setting of this work rigorous proofs are out of reach so that mathematical theorems are replaced by educated guesses.

Motivation and aims. Observational and model based studies provide evidence for variability in the North Atlantic sea surface temperature with a time scale of several decades. This phenomenon is called the Atlantic Multidecadal Oscillation (AMO). Investigations with ocean-only models show that the spatio-temporal properties of the AMO are explained by an internal oscillatory mode of these models (referred to as the AMO mode). Many studies, however, indicate that the growth rate of the AMO mode is negative due to atmospheric damping of sea surface temperature anomalies. On the other hand, the atmosphere itself shows variability on a multitude of time and spatial scales, which may excite the AMO mode. In the present work we study:

1. the dynamics of atmospheric low-frequency variability as observed at midlatitudes in the northern hemisphere;

2. the effect of atmospheric forcing on the AMO mode.

We apply concepts of dynamical systems theory to suitable low-order models.

Low-order models. In the geophysical literature, models for the atmospheric and oceanic circulation are typically derived from first principles, such as conservation laws, global balances, etc. This approach leads to systems of partial differential equations that govern the evolution of geophysical fields, such as velocity, pressure, or temperature.

In this work we derive *low-order models* from the equations of motion by means of Galerkin projections. The idea is to expand the unknown fields of the equations of motion in terms of a chosen basis, determining the spatial structure, with unknown time-dependent coefficients. In this expansion only a finite number of terms is retained. Then an orthogonal projection onto the basis gives a set of finitely many ordinary differential equations for the expansion coefficients.

Unfortunately, there is no theory which suggests how a basis must be chosen so that the dynamics of the low-order model qualitatively represent the dynamics of the original equations of motion. In the geophysical literature different bases have been used, such as Empirical Orthogonal Functions and their variants (Selten, 1995; Kwasniok, 1996), or eigenvectors computed from a linear stability problem of a particular steady state (Van der Vaart et al., 2002). Since these bases are computed numerically from a discretised model, they have the disadvantage that physical parameters have to be fixed in advance. For a bifurcation analysis, artificial parameters have to be re-introduced.

In the present study we use analytical basis functions, which are solutions of appropriate boundary value problems in order to satisfy certain boundary conditions. The advantage is that physical parameters are preserved in the projection. Hence, we can perform a bifurcation analysis where the bifurcation parameters have a straightforward physical interpretation. We select the retained basis functions based on physical considerations. We only retain those basis functions so that the truncated expansions can represent patterns on the relevant spatial scales.

1.2 Atmospheric low-frequency variability

The observed planetary-scale atmospheric circulation exhibits persistent and irregularly recurring patterns on time scales beyond 10 days during northern hemisphere winters. One way of characterising low-frequency variability is by means of spectral analysis of observed atmospheric fields. Fraedrich and Böttger (1978) studied spatio-temporal spectra of the variance of the 500 mb geopotential heights and found that the low-frequency component is characterised by temporal periods larger than 10 days and zonal wave numbers less than 5. Benzi and Speranza (1989) re-examined previous studies of amplification of waves with wave number 3 and of onset of Pacific anomalies. In addition, they summarise the main physical features of atmospheric low-frequency variability: on average planetary waves have a fixed geographic position, anomalies are vertically coherent, and low-frequency variability seems to be related to ultralong wave amplification through a non-standard form of baroclinic instability in which orography plays an essential role.

The central question of Chapter 2 is:

> Does the atmospheric variability characterising the northern hemisphere midlatitude circulation result from dynamical processes specific to the interaction of zonal flow and planetary waves with orography, and what are these processes?

To that end, we study a low-order model derived from the 2-layer shallow-water equations on a β-plane channel. The main ingredients of the low-order model are a zonal flow, a planetary scale wave (wave number 3), orography, and a baroclinic-like forcing.

We use orography height (h_0) and magnitude of zonal wind forcing (U_0) as control parameters to study the bifurcations of equilibria and periodic orbits. Along two curves of Hopf bifurcations an equilibrium loses stability ($U_0 \geq 12.5$ m/s) and gives birth to two distinct families of periodic orbits. These periodic orbits bifurcate into strange attractors along three routes to chaos: period doubling cascades, breakdown of 2-tori by homo- and heteroclinic bifurcations, or intermittency ($U_0 \geq 14.5$ m/s and $h_0 \geq 800$ m).

The observed attractors exhibit spatial and temporal low-frequency patterns comparing well with those observed in the atmosphere. For $h_0 \leq 800$ m the periodic orbits have a period of about 10 days and patterns in the vorticity field propagate eastward. For $h_0 \geq 800$ m, the period is longer (30-60 days) and patterns in the vorticity field are non-propagating. The dynamics on the strange attractors are associated with low-frequency variability: the vorticity fields show weakening and strengthening of

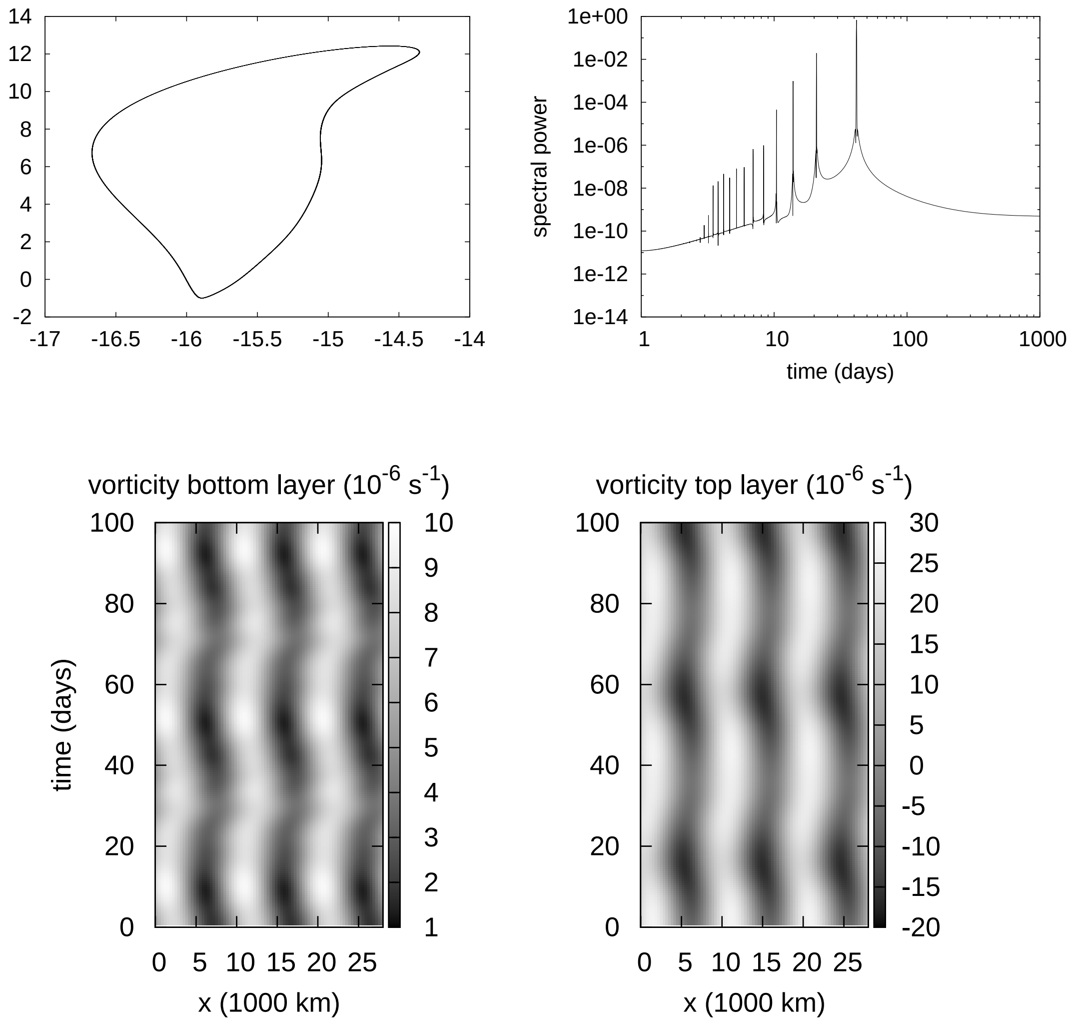

Figure 1.1. Top: Periodic orbit born at a Hopf bifurcation ($U_0 = 14.64$ m/s, $h_0 = 1400$ m) and its power spectrum. The period is approximately 60 days. Bottom: Hovmöller diagram computed from the periodic orbit in the top panel. The magnitude of the vorticity field is plotted as a function of time and longitude while keeping the latitude fixed at $y = 1250$ km. Observe that this wave is non-propagating in both layers.

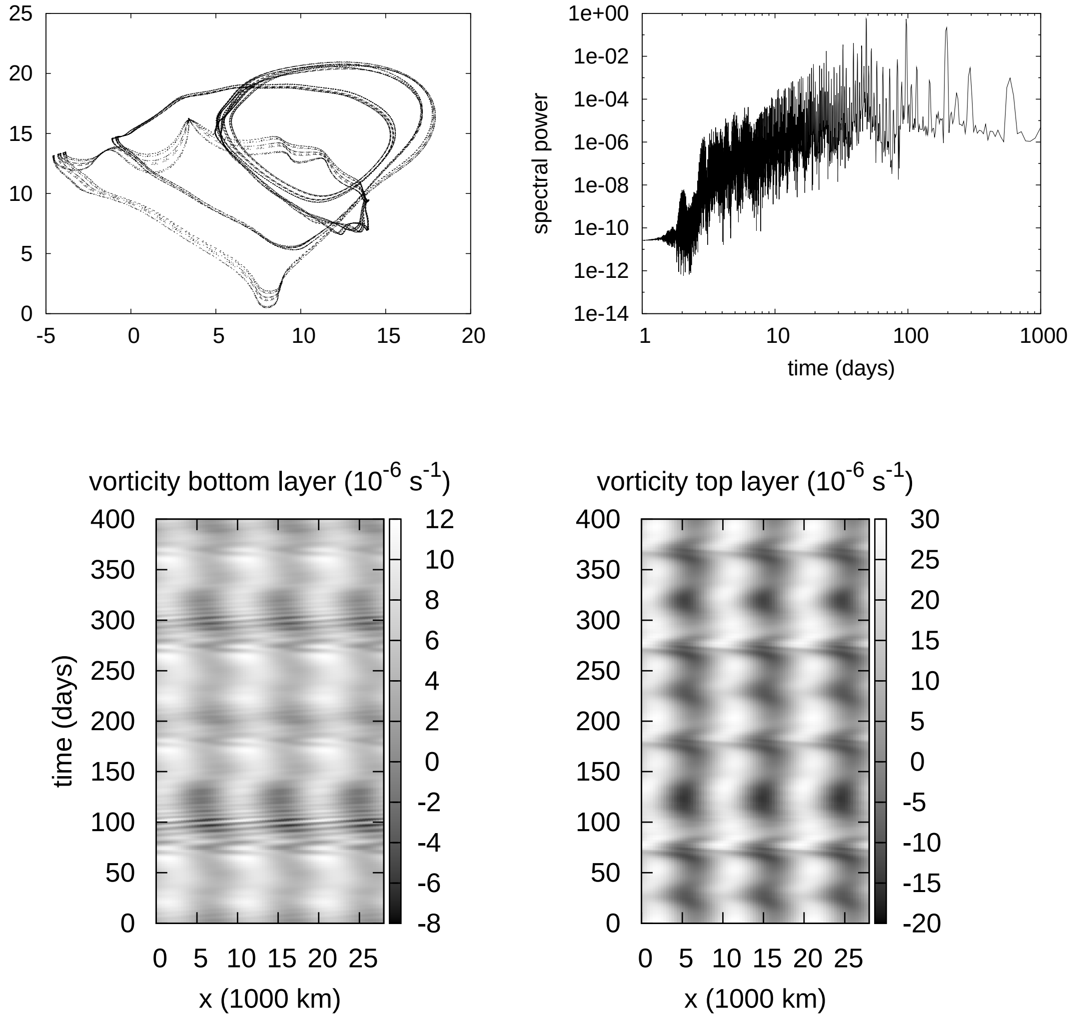

Figure 1.2. Same as Figure 1.1, but for $U_0 = 15$ m/s and $h_0 = 1400$ m. The periodic orbit has bifurcated into a strange attractor through a cascade of period doubling bifurcations. The non-propagating nature is 'inherited' from the bifurcating periodic orbit. Observe the irregular variability in the bottom layer, which is due to the harmonics induced by the period doubling bifurcations.

non-propagating planetary waves on time scales of 10-200 days. The spatio-temporal characteristics are 'inherited' (by intermittency) from the two families of periodic orbits and are detected in a relatively large region of the parameter plane. An example of this scenario for the period doubling case is shown in Figures 1.1 and 1.2.

The scenario presented in Chapter 2 is different from scenarios involving 'multiple equilibrium theories,' which associate atmospheric flow patterns with equilibria of the governing equations (Charney and DeVore, 1979). Recently, this idea has been adopted by Crommelin et al. (2004) to explain transitions between flow patterns by means of Shil′nikov-like strange attractors appearing in the Hopf-saddle-node scenario (Broer and Vegter, 1984). Such attractors give rise to intermittency near two saddle equilibria representing different flow patterns. See Broer and Vitolo (2008) for an overview of low-order atmosphere models where Shil′nikov-like strange attractors play an essential role.

1.3 The Atlantic Multidecadal Oscillation

Analyses of sea surface temperature observations provide evidence for variability in the North Atlantic Ocean with a time scale of several decades and a well-defined spatial pattern. This variability is called the Atlantic Multidecadal Oscillation (AMO). Figure 1.3 shows a specific pattern characterising the difference between the warm period 1950–1964 and the cold period 1970–1984.

The starting point for Chapter 3 is the paper of Te Raa and Dijkstra (2002) in which they study a minimal model for thermally driven flows in 3-dimensional ocean basin. Their bifurcation analysis shows the existence of an oscillatory mode, hereafter referred to as the AMO mode, which is characterised by a multidecadal time scale, westward propagation of temperature anomalies, and a phase difference between the anomalous meridional and zonal overturning circulations (Figure 1.4). The main goals of Chapter 3 are:

1. to develop a low-order model which captures the spatio-temporal signature of the AMO mode;
2. to study the stability of the AMO mode upon variation of parameters;
3. to study the effect of annual atmospheric forcing on the AMO mode.

From the model of Te Raa and Dijkstra we derive a low-order model, which consists of a system of 27 ordinary differential equations.

In models for thermally driven ocean flows different forcing mechanisms can be used. Two typical choices are restoring and prescribed heat flux. Restoring heat flux

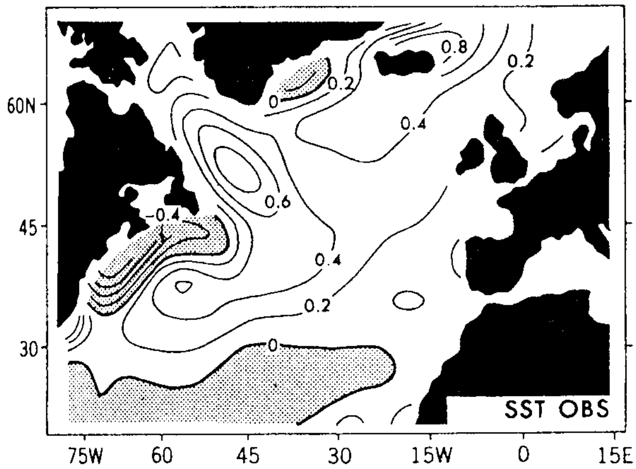

Figure 1.3. Pattern of the sea surface temperature anomaly determined by Kushnir (1994): difference between the averages taken over the warm years 1950–1964 and the cold years 1970–1984. The picture is taken from Latif (1998).

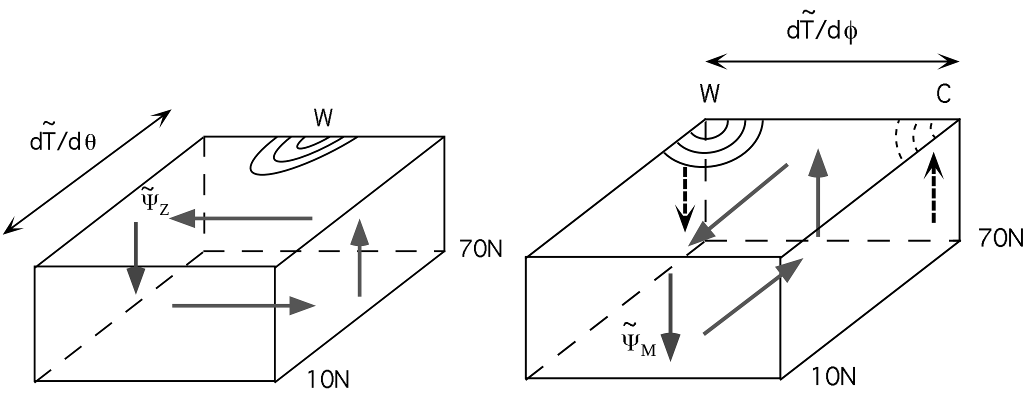

Figure 1.4. Two phases of the AMO mode as detected by Te Raa and Dijkstra (2002). A warm anomaly in the north central part of the basin induces a meridional temperature gradient, which induces an anomalous zonal overturning circulation. The latter causes the warm anomaly to travel westward, which leads to a zonal temperature gradient. The latter induces an anomalous meridional overturning circulation, and the second half of the oscillation starts.

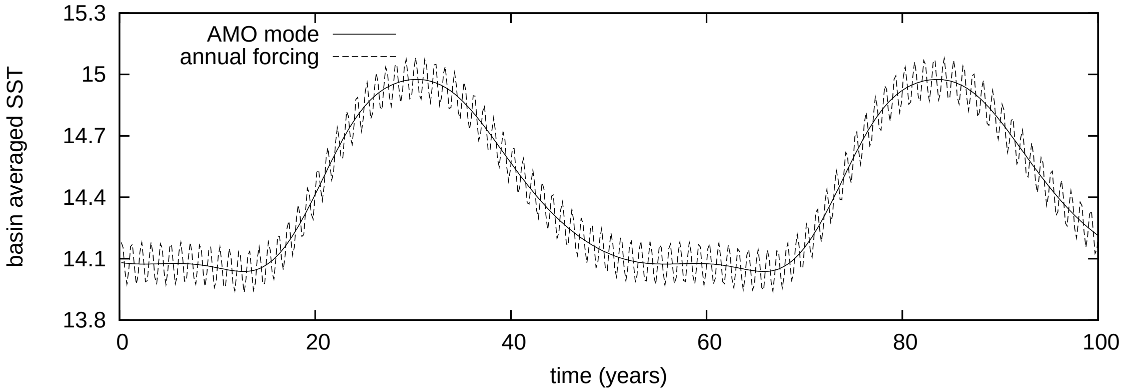

Figure 1.5. The basin averaged sea surface temperature plotted as a function of time for the AMO mode (solid) and the 2-torus attractor in the annually forced system (dashed).

is proportional to the temperature difference between the atmosphere and the sea surface, which results in atmospheric damping on sea surface temperature anomalies. Prescribed heat flux is a constant term, which results in net zero atmospheric damping.

Initially we use restoring heat flux as forcing, where a parameter Δ controls the equator-to-pole atmospheric temperature gradient. For the standard value $\Delta = 20°C$ the attractor of the low-order model is a stable equilibrium, which corresponds to a steady ocean flow. From this equilibrium we compute the corresponding heat flux which we then use as prescribed heat flux. Moreover, we introduce a parameter γ which interpolates between restoring ($\gamma = 0$) and prescribed heat flux ($\gamma = 1$). Hence, we study the stability of steady ocean flows as a function of atmospheric damping on the sea surface temperature. By increasing γ from 0 to 1 a supercritical Hopf bifurcation at $\gamma_H \approx 0.951$ gives birth to a periodic attractor, which has the spatio-temporal characteristics of the AMO mode as found by Te Raa and Dijkstra.

By means of a Poincaré return map we study the stability of the periodic orbit upon variation of the parameters Δ and γ. For $\Delta = 20°C$, the periodic orbit remains stable up to $\gamma = 1$, but for $\Delta > 20°C$ period doubling bifurcations occur. For $\Delta = 22°C$ the periodic orbit undergoes two period doublings, and for $\Delta = 24°C$ a complete cascade takes place. Strange attractors appearing after the period doubling cascade are Hénon-like: they are the closure of the unstable manifold of a saddle fixed point. See Figure 1.6 (left panel) for an example.

Finally, we impose periodic forcing to model annual variations in the surface heat flux. The Hopf bifurcation of the autonomous model turns into a Hopf-Neĭmark-

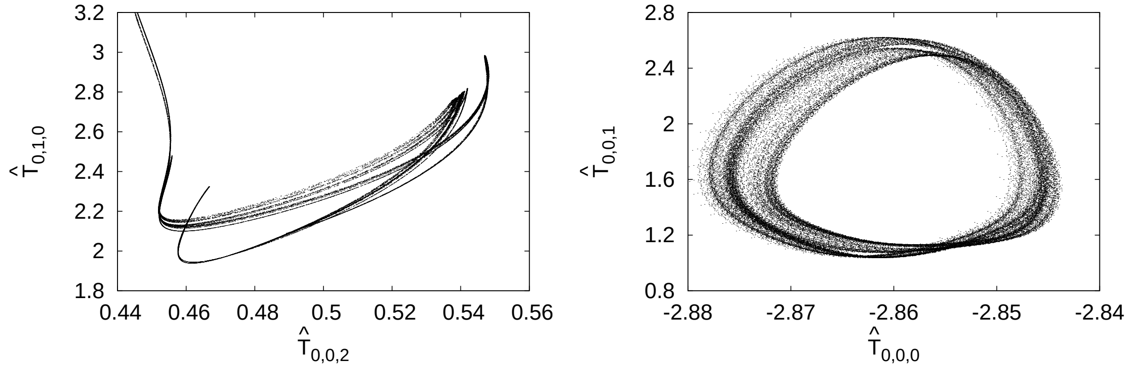

Figure 1.6. Left: A Hénon-like strange attractor which appears after a period doubling cascade of the AMO mode ($\Delta = 24°$C, $\gamma = 0.998$). Right: A quasi-periodic Hénon-like strange attractor which appears after a sequence of doublings of an invariant circle ($\Delta = 24°$C, $\gamma = 0.997185$).

Sacker bifurcation, which gives birth to a 2-torus attractor. The dynamics on the 2-torus attractor corresponds to the annual cycle imposed on the original AMO signal, see Figure 1.5. Clearly, the peak-to-peak amplitude of the quasi-periodic signal is larger than that of the original periodic signal, which shows that periodic forcing amplifies the AMO. The AMO mode is, however, not excited since the HNS bifurcation occurs for almost the same value of γ for which the Hopf bifurcation occurs in the autonomous system.

We study the dynamics of the nonautonomous system by means of a Poincaré stroboscopic map. In particular, the period doubling bifurcations of the autonomous system become doublings of an invariant circle of the Poincaré map. For $\Delta = 24°$C we detected at least 11 doublings, and we conjecture that a full cascade takes place. After these doublings we find quasi-periodic Hénon-like strange attractors, see Figure 1.6 (right panel). These attractors are the closure of the unstable manifold of a saddle invariant circle. Similar attractors were detected in the Lorenz-84 atmospheric model with seasonal forcing studied by Broer, Simó and Vitolo (2002, 2005).

1.4 Excitation of the AMO

Prescribed heat flux in ocean models is a strong idealisation since it amounts to net zero atmospheric damping. In reality sea surface temperature anomalies are substantially damped by the atmosphere. Dijkstra et al. (2008) estimate that realistic values of γ are less than $\gamma_H \approx 0.951$, which implies that AMO mode is damped. The central question of Chapter 4 is:

> Can atmospheric low-frequency variability excite a weakly damped AMO mode?

Here, weakly damped refers to the parameter range $\gamma < \gamma_H$, but for values of γ not too far away from the Hopf bifurcation. We speak of excitation when multidecadal variability related to the AMO mode occurs in this parameter range.

The results of Chapter 3 show that annual atmospheric forcing increases the amplitude of the AMO in supercritical conditions. But since there is no decrease

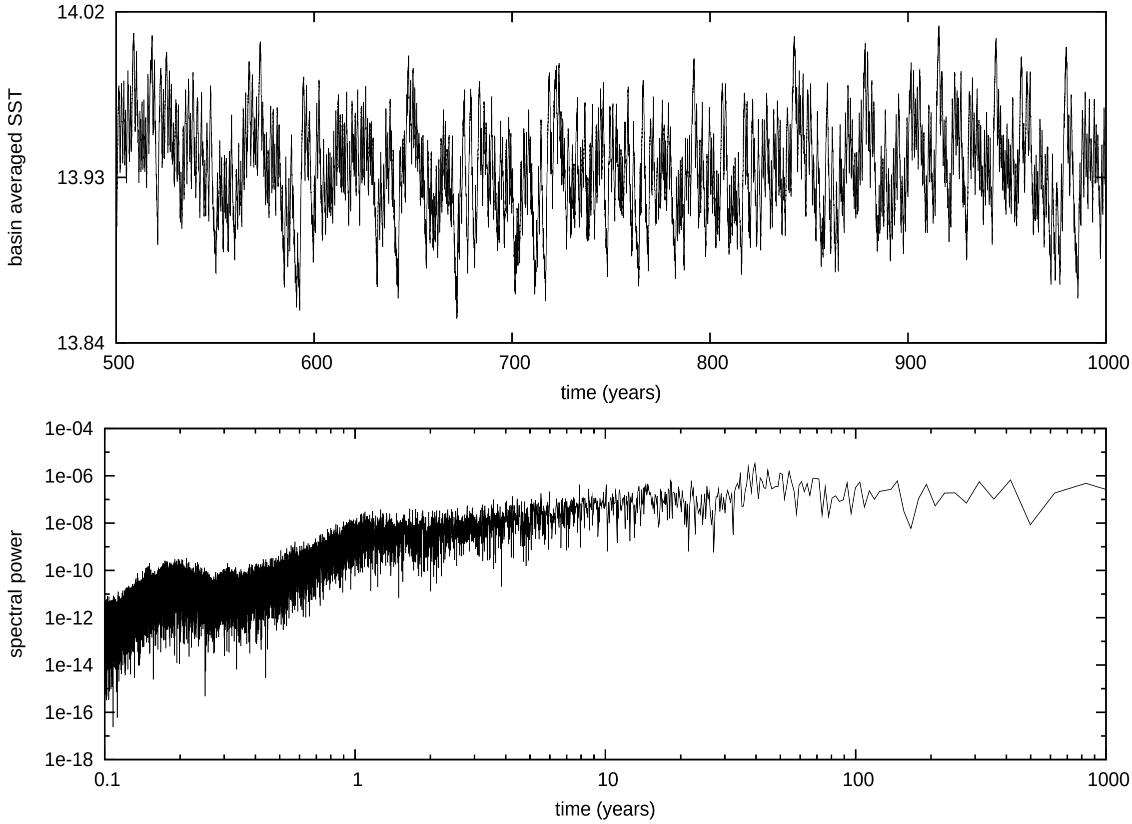

Figure 1.7. Induced multidecadal variability for $\Delta = 20°C$ and $\gamma = 0.90$. For these parameter values the autonomous ocean model of Chapter 3 has a stable equilibrium representing a steady ocean flow. With additional irregular forcing obtained from the atmosphere model in Chapter 2 multidecadal variability does occur. Top: basin averaged sea surface temperature as a function of time. Bottom: power spectrum.

of the critical value of γ, we cannot speak of excitation. On the other hand, we know that the atmosphere itself exhibits variability on a multitude of time scales. In Chapter 4 we study the low-order ocean model of Chapter 3 with additional forcing from the low-order atmosphere model of Chapter 2. The resulting model should not be considered as a coupled ocean-atmosphere model, but rather as an ocean model with additional atmospheric forcing.

With chaotic atmospheric forcing, the AMO is indeed excited. Figure 1.7 shows the basin averaged sea surface temperature when $\Delta = 20°\text{C}$ and $\gamma = 0.9$. The time series shows high-frequency fluctuations which are due to the fast variability of the atmosphere, but one can also observe a slower time scale. This is confirmed by the power spectrum, which shows a maximum for multidecadal time scales.

Invariant objects of the autonomous ocean model (equilibria, periodic orbits) no longer exist when chaotic atmospheric forcing is applied. Instead, the chaotic atmospheric forcing causes high-frequency, irregular fluctuations in the state variables of the ocean model. Nevertheless, the 'ghost' of the formerly existing nearby Hopf bifurcation still influences the dynamics. Such behaviour is typical for scenarios involving intermittency. In Chapter 4 we give a preliminary interpretation of this phenomenon. A more precise explanation is the subject of future research.

1.5 Discussion

The results presented in this work provide ample motivation for a further investigation of both mathematical and physical topics.

Persistence of dynamical phenomena? Reduction of infinite-dimensional systems to finite-dimensional systems is a challenging problem. On the one hand there are computational procedures such as discretisation by means of finite-differences or Galerkin-like projections. On the other hand there exist conceptual reductions to lower-dimensional models such as restrictions to invariant manifolds containing attractors. However, often the available theorems are not constructive. The challenge lies in reconciling the computational methods with the conceptual methods. The present study is only a first step in the coherent analysis of the infinite-dimensional models under consideration. There are two important open questions:

1. Which dynamical features of the low-order model persist as the number of retained basis functions is increased in the Galerkin projection?

2. Which dynamical features of the low-order models persist in the infinite-dimensional systems?

For the former question, one can think of the approach used for a Rayleigh-Bénard convection problem in Puigjaner et al. (2004, 2006, 2008).

The persistence question is also relevant from a physical point of view. Higher-dimensional Galerkin projections can describe phenomena with a smaller spatial scale, and the interaction between phenomena with different spatial scales likely affects the global dynamics. Therefore, an important question is:

> How can one objectively compare the dynamics of a low-dimensional model with the dynamics of higher-dimensional models?

Related to this question is how to compare the dynamics of low-order models with observations.

Existence of global attractors or inertial manifolds? Apart from the computational approach, a rigorous mathematical investigation of the infinite-dimensional systems should be undertaken. Indeed, in the present study the notion 'infinite-dimensional dynamical system' has been used in a rather loose sense. An important open question is:

> What is the state space of the infinite-dimensional model generated by the partial differential equations of Chapters 2 and 3?

Answering this question requires proving the existence of (weak) solutions. The idea would be to follow the methods used for the 2-dimensional Navier-Stokes equations and certain reaction-diffusion equations, see Temam (1997) and Robinson (2001). For these equations the Galerkin method is used to construct a sequence of successive approximations which converge to a solution of the weak form of the equations in a suitable Hilbert space. This Hilbert space then serves as a suitable state space on which an evolution operator can be defined. When this has been achieved one can try to prove the existence of finite-dimensional global attractors inside inertial manifolds.

Bibliography for Chapter 1

Benzi, R. and Speranza, A. (1989), Statistical properties of low frequency variability in the Northern Hemisphere, *Journal of Climate* **2**, pp. 367–379.

Broer, H.W., Simó, C. and Vitolo, R. (2002), Bifurcations and strange attractors in the Lorenz-84 climate model with seasonal forcing, *Nonlinearity* **15**, pp. 1205–1267.

Broer, H.W., Simó, C. and Vitolo, R. (2005), Quasi-periodic Hénon-like attractors in the Lorenz-84 climate model with seasonal forcing, *in* F. Dumortier, H.W. Broer, J. Mahwin, A. Vanderbauwhede and S.M. Verduyn-Lunel (eds), *Equadiff 2003, Proceedings International Conference on Differential Equations, Hasselt 2003*, World Scientific, pp. 714–719.

Broer, H.W. and Takens, F. (2010), *Dynamical Systems and Chaos*, Vol. 172 of *Applied Mathematical Sciences*, Springer.

Broer, H.W. and Vegter, G. (1984), Subordinate Šil'nikov bifurcations near some singularities of vector fields having low codimension, *Ergodic Theory and Dynamical Systems* **4**, pp. 509–525.

Broer, H.W. and Vitolo, R. (2008), Dynamical systems modelling of low-frequency variability in low-order atmospheric models, *Discrete and Continuous Dynamical Systems B* **10**, pp. 401–419.

Charney, J.G. and DeVore, J.G. (1979), Multiple Flow Equilibria in the Atmosphere and Blocking, *Journal of the Atmospheric Sciences* **36**, pp. 1205–1216.

Crommelin, D.T., Opsteegh, J.D. and Verhulst, F. (2004), A Mechanism for Atmospheric Regime Behavior, *Journal of the Atmospheric Sciences* **61**, pp. 1406–1419.

Dijkstra, H.A. (2005), *Nonlinear Physical Oceanography: A Dynamical Systems Approach to the Large Scale Ocean Circulation and El Niño*, second edn, Springer.

Dijkstra, H.A., Frankcombe, L.M. and von der Heydt, A.S. (2008), A Stochastic Dynamical Systems View of the Atlantic Multidecadal Oscillation, *Philosophical Transactions of the Royal Society A* **366**, pp. 2545–2560.

Fraedrich, K. and Böttger, H. (1978), A Wavenumber-Frequency Analysis of the 500 mb Geopotential at 50°N, *Journal of the Atmospheric Sciences* **35**, pp. 745–750.

Kushnir, Y. (1994), Interdecadal Variations in North Atlantic Sea Surface Temperature and Associated Atmospheric Conditions, *Journal of Climate* **7**, pp. 141–157.

Kwasniok, F. (1996), The reduction of complex dynamical systems using principal interaction patterns, *Physica D* **92**, pp. 28–60.

Latif, M. (1998), Dynamics of Interdecadal Variability in Coupled Ocean–Atmosphere Models, *Journal of Climate* **11**, pp. 602–624.

Puigjaner, D., Herrero, J., Giralt, F. and Simó, C. (2004), Stability analysis of the flow in a cubical cavity heated from below, *Physics of Fluids* **16**, pp. 3639–3655.

Puigjaner, D., Herrero, J., Giralt, F. and Simó, C. (2006), Bifurcation analysis of multiple steady flow patterns for Rayleigh-Bénard convection in a cubical cavity at $Pr = 130$, *Physical Review E* **73**. 046304.

Puigjaner, D., Herrero, J., Simó, C. and Giralt, F. (2008), Bifurcation analysis of steady Rayleigh–Bénard convection in a cubical cavity with conducting sidewalls, *Journal of Fluid Mechanics* **598**, pp. 393–427.

Robinson, J.C. (2001), *Infinite-Dimensional Dynamical Systems*, Cambridge University Press.

Ruelle, D. and Takens, F. (1971), On the nature of turbulence, *Communications in Mathematical Physics* **20**, pp. 167–192.

Selten, F.M. (1995), An efficient description of the dynamics of barotropic flow, *Journal of the Atmospheric Sciences* **52**, pp. 915–936.

Te Raa, L.A. and Dijkstra, H.A. (2002), Instability of the Thermohaline Ocean Circulation on Interdecadal Timescales, *Journal of Physical Oceanography* **32**, pp. 138–160.

Temam, R. (1997), *Infinite-Dimensional Dynamical Systems in Mechanics and Physics*, Vol. 68 of *Applied Mathematical Sciences*, second edn, Springer.

van der Vaart, P.C.F., Schuttelaars, H.M., Calvete, D. and Dijkstra, H.A. (2002), Instability of time-dependent wind-driven ocean gyres, *Physics of Fluids* **14**, pp. 3601–3615.