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Abstract

The spectroscopic factors for the low-lying quasi-hole states observed in the ^{16}O$(e,e'p)^{15}$N reaction are reinvestigated with a variational Monte Carlo calculation for the structure of the initial and final nucleus. A computational error in a previous report is rectified. It is shown that a proper treatment of center-of-mass motion does not lead to a reduction of the spectroscopic factor for p-shell quasi-hole states, but rather to a 7% enhancement. This is in agreement with analytical results obtained in the harmonic oscillator model. The center-of-mass effect worsens the discrepancy between present theoretical models and the experimentally observed single-particle strength. We discuss the present status of this problem, including some other mechanisms that may be relevant in this respect.

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I. INTRODUCTION

In present-day $(e, e'p)$ experiments [1] one can determine quite accurately single-particle (s.p.) overlap functions between an A-particle nuclear target in its ground state and low-lying (quasi-hole) states of the $(A-1)$ residual system. The spectroscopic factor, defined as the norm of the overlap function, is equal to unity only in the fixed-center mean-field approximation. Experimentally one finds [2] that spectroscopic factors of about $S \approx 0.6-0.7$ are needed to explain the data. Such deviations of S from unity are normally ascribed to nucleon-nucleon (NN) correlations. The observed values indicate the sensitivity of the quasi-elastic $(e, e'p)$ cross section to correlation effects in the initial and final state. Several nuclear many-body calculations have been made in attempts to explain the reduced spectroscopic factors of the $p_{1/2}$ and $p_{3/2}$ quasi-hole states seen in the $^{16}\mathrm{O}(e, e'p)$ reaction [3–6].

Fixed-center mean-field wave functions are not realistic for light systems like ^{16}O, since they are not translationally invariant, i.e. they contain spurious center-of-mass (c.m.) motion. Variational Monte Carlo (VMC) calculations of the ^{16}O ground state [7] and the $^{15}\mathrm{N}(p_{3/2})$ quasi-hole state [3] , in which the nuclear wave functions are explicitly translationally invariant, have been made to address this problem. It was reported in [3] that c.m. correlations reduced the p-shell spectroscopic factor by $\sim 12\%$ even in absence of dynamical NN correlations. However, as was noted in [8], this reduction contradicts the results obtained in the harmonic oscillator (h.o.) model, which predicts an enhancement by 7% for the p-shell spectroscopic factor in ^{16}O [9] due to c.m. correlations. The discrepancy with the h.o. model values indicated an error in the computer program of [3]. It has now been identified, and the corrected results are presented in the third Section of this paper. The correction simply involves a rescaling of the s.p. overlap function reported in [3] by the factor $(\frac{16}{15})^{3/2}$. This brings the results without dynamical NN correlations into perfect agreement with the h.o. results, but the full calculation now predicts a p-shell spectroscopic factor $S_p = 0.98$, considerably different from the experimental values.

The remainder of the paper is organized as follows. In Section II we examine the concept of overlap functions with translational invariance of the nuclear wave functions taken into account, and point out some consequences for the description of knock-out reactions. We also mention general results of the h.o. model and apply it to the case of the ^{16}O nucleus. The theoretical description of the quasi-hole states observed in the $^{16}\mathrm{O}(e, e'p)$ reaction is treated in Section III, where we correct the results of [3]. It is pointed out that a proper treatment of c.m. motion in other calculations would also worsen the discrepancy with experimental data. In Section IV we look at the order of magnitude of two simple corrections to the $(e, e'p)$ cross section. The present status of the problem is discussed in Section V.

II. CENTER-OF-MASS MOTION AND KNOCK-OUT REACTIONS

A. Overlap functions in self-bound systems

For systems which are localized around a fixed force center (e.g. the electrons of an atom), one defines a s.p. overlap function ϕ between normalized A and $(A-1)$ particle systems as

$$\phi(x_A) = \sqrt{A} \int dx_1 \dots dx_{A-1} \Phi^{\dagger}_{(A-1)}(x_1, \dots, x_{A-1}) \Phi_{(A)}(x_1, \dots, x_A). \tag{1}$$

(The notation x_i includes the spatial coordinate, $\mathbf{r}_i$, and the appropriate spin and isospin degrees of freedom.) In the case of Fermi systems, the $\Phi_{(A)}$ and $\Phi_{(A-1)}$ are antisymmetric wave functions, and as a consequence, the normalization of the overlap function (or spectroscopic factor S) has the property

$$S = \int dx_A |\phi(x_A)|^2 \leq 1. \tag{2}$$

In nuclei there is no fixed external force center, but the nucleons are localized around their c.m. due to their mutual interactions. The eigenstates of such a self-bound system can be factored as

$$\Phi_{(A)}(x_1, \ldots, x_A) = \exp(i\mathbf{K} \cdot \mathbf{R}_A)\Psi_{(A)}(x_1, \ldots, x_A), \tag{3}$$

with $\Psi_{(A)}$ the *intrinsic*, translationally invariant, wave function of the system, and the plane-wave factor describing the c.m. motion; $\mathbf{K}$ is the total momentum and $\mathbf{R}_A$ is the position of the c.m. The properties of the overlap functions are somewhat different in this case. A detailed analysis of many s.p. quantities in self-bound systems (the one-body density matrix, natural orbitals, spectral function etc.) will be given in a future publication [10].

For the present discussion it will be sufficient to note that, if the intrinsic wave functions are normalized according to

$$\int dx_1 \ldots dx_A \delta(\mathbf{R_A}) |\Psi_{(A)}(x_1, \ldots, x_A)|^2 = 1, \tag{4}$$

then the overlap function ψ must be defined as [11]

$$\psi(x_A) = \sqrt{A} \int dx_1 \ldots dx_{A-1} \delta(\mathbf{R}_{A-1}) \Psi^\dagger_{(A-1)}(x_1, \ldots, x_{A-1}) \Psi_{(A)}(x_1, \ldots, x_A). \tag{5}$$

As shown in Section II.B, the overlap function defined according to Eqs.(4-5) is indeed the counterpart for self-bound systems of Eq.(1), and it is the natural quantity appearing in the description of knock-out reactions when some standard approximations are made.

It should also be kept in mind that the spectroscopic factors in self-bound Fermi systems,

$$S = \int dx_A |\psi(x_A)|^2, \tag{6}$$

can be larger than one [with deviations of the order $\mathcal{O}(1/A)$]. Although this was already known before [9,12], the fact does not seem to be widely appreciated. The most extreme–though somewhat forced–example is that of two identical fermions (e.g. two spin-up neutrons). The intrinsic wave function can be written as $\Psi_{(2)} = f(\mathbf{r}_1 - \mathbf{r}_2)$, with $\int d\mathbf{r} |f(\mathbf{r})|^2 = 1$. Then the overlap function with the (only) intrinsic one-particle state, $\Psi_{(1)}(\mathbf{r}_1) = 1$, is given by $\psi(\mathbf{r}_2) = \sqrt{2} f(-\mathbf{r}_2)$, and the corresponding spectroscopic factor is $S = 2$, regardless of the Fermi or Bose nature of the particles. A more realistic illustration for three particles is considered in the Appendix.

B. Intrinsic transition matrix elements for knock-out reactions

In this Section we extend the standard derivation of the plane-wave impulse approximation (PWIA) amplitude in fixed-center systems to the case of knock-out reactions on self-bound systems, in order to justify the definition Eq.(5) for the overlap function. We consider for simplicity the transition matrix elements of a scalar one-particle external probe, e.g. the operator $\rho(\mathbf{q}) = \sum_{j=1}^{A} \exp(i\mathbf{q}\cdot\mathbf{r}_j)$, between translationally invariant initial and final states of the form (3). The matrix element is readily separated into a momentum conserving δ-function and the intrinsic transition matrix element,

$$\langle \Phi_{(A)f}|\rho(\mathbf{q})|\Phi_{(A)i}\rangle = (2\pi)^3\delta(\mathbf{K}_i + \mathbf{q} - \mathbf{K}_f)\langle \Psi_{(A)f}|\rho(\mathbf{q})|\Psi_{(A)i}\rangle, \tag{7}$$

given by

$$\langle \Psi_{(A)f}|\rho(\mathbf{q})|\Psi_{(A)i}\rangle = \int dx_1 \ldots dx_A \delta(\mathbf{R}_A)\Psi^\dagger_{(A)f}(x_1,\ldots,x_A)\sum_{j=1}^{A}\exp(i\mathbf{q}\cdot\mathbf{r}_j)\Psi_{(A)i}(x_1,\ldots,x_A). \tag{8}$$

In knock-out reactions where the residual $(A-1)$ nucleus is left in a bound state $\Psi_{(A-1)}$, the intrinsic final state can be specified by its asymptotic behavior,

$$\lim_{|\mathbf{r}_A - \mathbf{R}_{A-1}|\to\infty} \Psi_{(A)\mathbf{p}}(x_1,\ldots,x_A) = C\exp[i\mathbf{p}\cdot(\mathbf{r}_A - \mathbf{R}_{A-1})]\Psi_{(A-1)}(x_1,\ldots,x_{A-1}), \tag{9}$$

with $\mathbf{p}$ the relative momentum between the knocked-out nucleon and the c.m. of the remaining ones.

In the plane-wave approximation, distortion effects on the escaping particle are neglected, and we simply approximate

$$\Psi_{(A)\mathbf{p}}(x_1,\ldots,x_A) \approx \frac{(2\pi)^{-3/2}}{\sqrt{A!(A-1)!}}\mathcal{A}\left\{\exp[i\mathbf{p}\cdot(\mathbf{r}_A - \mathbf{R}_{A-1})]\Psi_{(A-1)}(x_1,\ldots,x_{A-1})\right\}, \tag{10}$$

with $\mathcal{A}$ the antisymmetrized sum over all coordinate permutations. The wave functions $\Psi_{(A)\mathbf{p}}$ are not orthogonal to the initial state $\Psi_{(A)i}$, and also not mutually orthogonal, since

$$\langle \Psi_{(A)\mathbf{p}}|\Psi_{(A)\mathbf{p}'}\rangle = \int dx_1 \ldots dx_A \delta(\mathbf{R}_A)\Psi^\dagger_{(A)\mathbf{p}}(x_1,\ldots,x_A)\Psi_{(A)\mathbf{p}'}(x_1,\ldots,x_A) \tag{11}$$

$$= \delta(\mathbf{p}-\mathbf{p}') - C(\mathbf{p},\mathbf{p}'), \tag{12}$$

with the non-orthogonality correction $C(\mathbf{p},\mathbf{p}')$ given by

$$C(\mathbf{p},\mathbf{p}') = (2\pi)^{-3}\int dx dx'\,\rho(x,x')\exp[i\mathbf{r}\cdot(\mathbf{p}' + \frac{1}{A-1}\mathbf{p})]\exp[-i\mathbf{r}'\cdot(\mathbf{p} + \frac{1}{A-1}\mathbf{p}')]. \tag{13}$$

The quantity $\rho(x,x')$ is the one-body density matrix of the intrinsic $(A-1)$-particle state $\Psi_{(A-1)}$ [10,11],

$$\rho(x,x') = (A-1)\int dx_1 \ldots dx_{A-2}\delta(\mathbf{R}_{A-2})\Psi^\dagger_{(A-1)}(x_1,\ldots,x_{A-2},x)\Psi_{(A-1)}(x_1,\ldots,x_{A-2},x'). \tag{14}$$

Similarly as for fixed-center systems, the nonorthogonality correction $C(\mathbf{p},\mathbf{p}')$ is suppressed by the momentum dependence of the Fourier transform of $\rho(x,x')$ for momenta p or p' larger than typical momenta contained in the bound-state $\Psi_{(A-1)}$, and can be neglected under the usual $(e,e'p)$ kinematical conditions.

The amplitude itself can now be written as

$$\langle\Psi_{(A)\mathbf{p}}|\rho(\mathbf{q})|\Psi_{(A)i}\rangle = \sqrt{A}\sum_{j=1}^{A}\int dx_1\ldots dx_A\delta(\mathbf{R}_A)\ \Psi^{\dagger}_{(A-1)}(x_1,\ldots,x_{A-1}) \exp[-i\mathbf{p}\cdot(\mathbf{r}_A-\mathbf{R}_{A-1})]\exp[i\mathbf{q}\cdot\mathbf{r}_j]\ \Psi_{(A)}(x_1,\ldots,x_A), \tag{15}$$

The impulse approximation implies that the nucleon which has absorbed the momentum of the probe gets ejected, and corresponds to retaining only the term with $j=A$ in Eq.(15). The result is

$$\langle\Psi_{(A)\mathbf{p}}|\rho(\mathbf{q})|\Psi_{(A)i}\rangle \approx A_{\rm PWIA}(\mathbf{p},\mathbf{q}) = (2\pi)^{-3/2}\int dx_A \exp[-i\mathbf{r}_A\cdot(\mathbf{p}-\frac{A-1}{A}\mathbf{q})]\psi(x_A), \tag{16}$$

where $\psi(x_A)$ given by Eq.(5), i.e. the PWIA amplitude still scales with the Fourier transform of the overlap function, at missing momentum $\mathbf{p}-\frac{A-1}{A}\mathbf{q}$.

The antisymmetrization correction terms (with $j\neq A$) in Eq.(15) can be recombined into

$$A_{\rm A.S.}(\mathbf{p},\mathbf{q}) = (2\pi)^{-3/2}\int dxdx' F(x,x')\exp[i\mathbf{r}\cdot(\frac{A-1}{A}\mathbf{q}+\frac{1}{A-1}\mathbf{p})]\exp[-i\mathbf{r}'\cdot(\mathbf{p}+\frac{\mathbf{q}}{A})], \tag{17}$$

with

$$F(x,x') = \sqrt{A}(A-1)\int dx_1\ldots dx_{A-2}\delta(\mathbf{R}_{A-2})\Psi^{\dagger}_{(A-1)}(x_1,\ldots,x_{A-2},x)\Psi_{(A)}(x_1,\cdots,x_{A-2},x,x'). \tag{18}$$

This correction is again suppressed by the momentum dependence of the Fourier transform of $F(x,x')$ when p and q are large. The magnitude of the nonorthogonality and antisymmetrization corrections will be estimated in Section IV.B for the kinematical conditions used in the $(e,e'p)$ experiment of [13].

C. Intrinsic Slater determinants

Descriptions of all but the lightest nuclei usually involve fixed-center (shell-model) wave functions of the Slater determinant (SD) type,

$$\Phi_{(A)}(x_1,\ldots,x_A) = \frac{1}{\sqrt{A!}}{\rm Det}[\phi_{h_j}(x_i)]_{i,j=1,\ldots,A}. \tag{19}$$

These wave functions contain spurious c.m. motion which does not correspond to the intrinsic degrees of freedom in self-bound systems. The correct intrinsic wave function describing the uncorrelated motion of A particles in A orbits around their c.m. is given by

$$\Psi_{(A)}(x_1,\ldots,x_A) = \frac{1}{\sqrt{A!}}\mathrm{Det}[\phi_{h_j}(\mathbf{r}_i - \mathbf{R}_A, \alpha_i)]_{i,j=1,\ldots,A}, \tag{20}$$

where α_i represents spin and isospin degrees of freedom. These intrinsic Slater determinants (ISD) are obviously translationally invariant, and should be used in combination with Eqs.(4-5), to calculate s.p. overlap functions and spectroscopic factors.

The effect of the spurious c.m. motion on overlap functions and spectroscopic factors is non-trivial, even when considering transitions between a single ISD $\Psi_{(A)}$ like (20) and a one-hole state $\Psi_{(A-1)h_n}$ obtained by removing the orbital h_n,

$$\Psi_{(A-1)h_n}(x_1,\ldots,x_{A-1}) = \frac{1}{\sqrt{(A-1)!}}\mathrm{Det}[\phi_{h_j}(\mathbf{r}_i - \mathbf{R}_{A-1}, \alpha_i)]_{i,j=1,\ldots,A;i\neq A;j\neq n}. \tag{21}$$

In contrast to the situation in fixed-center systems, the presence of the c.m. δ-function in Eqs.(4-5) makes the calculation of the normalization and overlap integrals quite complicated, even for ISD wave functions. Only if the orbitals that build up the ISD are chosen to be h.o. wave functions can the overlap function and spectroscopic factor be evaluated analytically. The simplest non-trivial illustration is for a h.o. well containing three neutrons, one spin-down and two spin-up; this is worked out in the Appendix.

The general h.o. model has been studied in [9], where overlaps were considered between an A-nucleon ground-state configuration in the h.o. shell model, and the $(A-1)$-nucleon one-hole states. It was found that, in each spin-isospin space $\sigma\tau$, the spectroscopic factor for the valence hole state (corresponding to a hole in the occupied shell with the largest oscillator quantum number N_v) is larger than one and given by

$$S_v^{(\sigma\tau)} = \left(\frac{A}{A-1}\right)^{N_v}. \tag{22}$$

Moreover, for h.o. orbitals the spectroscopic sum rule remains satisfied in the intrinsic frame[1]: in each spin-isospin space the sum of the spectroscopic factors $S_h^{(\sigma\tau)}$ of all one-hole ISD's yields the number of particles $A^{(\sigma\tau)}$

$$\sum_h n_h^{(\sigma\tau)} S_h^{(\sigma\tau)} = A^{(\sigma\tau)}, \tag{23}$$

with $n_h^{(\sigma\tau)}$ the number of particles occupying the main h.o. shell h. Since the sum rule is satisfied and the spectroscopic factor for the valence hole state is larger than one, the deeper hole states are partly spurious and have $S \leq 1$.

For the case of ^{16}O we have $n_s = 1$, $n_p = 3$, $N_v = 1$; the h.o. model predicts for a hole in the p-shell $S_p = \frac{16}{15} \approx 1.07$, and, through the sum rule (23), $S_s = 4 - n_p \times \frac{16}{15} = \frac{4}{5}$ for a hole in the s-shell. More realistic choices of the s.p. wave functions, such as Woods-Saxon (W.S.), preclude an analytical treatment of c.m. effects. Nevertheless one can expect results close to the h.o. values for a light nucleus like ^{16}O, where h.o. and W.S. wave functions are

[1]In general, the spectroscopic sum rule is not satisfied by considering just one-hole states of ISD's made of non-h.o. orbitals; this is also illustrated for a three-neutron example in the Appendix.

rather similar. We checked this by a direct computation of the overlap functions in ^{16}O with W.S. wave functions in the ISD's, exploiting the fact that for ISD the many-body integrals in Eqs.(4-5) can be reduced to a sequence of one-body integrals. e.g. we have (apart from normalization factors)

$$\psi_{h_n}(x_A) = \sum_{m=1}^{A} \phi_{h_m}(\frac{A-1}{A}\mathbf{r}_A, \alpha_A) \times \int d\mathbf{k} \mathrm{Det}\left[\int dx \exp[i\mathbf{k}\cdot\mathbf{r}]\phi^*_{h_i}(x)\phi_{h_j}(\mathbf{r}-\frac{1}{A}\mathbf{r}_A,\alpha)\right]_{i,j=1,\ldots,A;i\neq n;j\neq m}. \quad (24)$$

As expected the spectroscopic factors obtained with the W.S. wave functions practically coincide with the h.o. results.

Note that the correlated many-body wave functions considered in the next section are more complicated than single ISD's, and Monte Carlo quadrature was used there to calculate the s.p. overlap functions and spectroscopic factors. We verified that, for the case of single ISD's, the Monte Carlo quadrature and a direct evaluation of Eq.(24) lead to the same result.

III. SPECTROSCOPIC FACTORS FOR QUASI-HOLE STATES IN ^{16}O

In the variational Monte Carlo framework of [3,7], the intrinsic wave function of the ^{16}O ground state has the form

$$\Psi_{(A)}(x_1,\ldots,x_A) = \hat{F}(x_1,\ldots,x_A)\frac{1}{\sqrt{A!}}\mathrm{Det}\left[\phi_{h_j}(\mathbf{r}_i-\mathbf{R}_A,\alpha_i)\right]_{i,j=1,\ldots,A} \quad (25)$$

The intrinsic Slater determinant is built up with the s.p. wave functions ϕ_h, and incorporates the mean-field aspects. As explained in Section II.C, it is explicitly translationally invariant. The correlation factor $\hat{F}$ contains two- and three-particle correlations of central, spin, isospin, tensor and spin-orbit type, and is translationally invariant by itself.

The correlation factor and the s.p. wave functions were determined by a minimization of the expectation value of the many-body Hamiltonian

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}, \quad (26)$$

where the Argonne v_{14} model of the two-nucleon interaction [14] and the Urbana model VII of the three-nucleon interaction [15] were used. A detailed description of the variational procedure can be found in [7].

The intrinsic wave functions $\Psi_{(A-1)h_n}$ of the quasi-hole states in ^{15}N are approximated as

$$\Psi_{(A-1)h_n}(x_1,\ldots,x_{A-1}) = \hat{F}(x_1,\ldots,x_{A-1})\frac{1}{\sqrt{A!}}\mathrm{Det}\left[\phi_{h_j}(\mathbf{r}_i-\mathbf{R}_{A-1},\alpha_i)\right]_{i,j=1,\ldots,A;i\neq A;j\neq n}, \quad (27)$$

i.e. by retaining the correlation factor and s.p. orbitals of ^{16}O and omitting in the determinant the column with the corresponding ϕ_{h_j}.

The overlap function for the $p_{3/2}$ quasi-hole state was then calculated according to Eqs.(4-5). When dynamical NN correlations were neglected (by putting the correlation factor $\hat{F} = 1$), a spectroscopic factor $S_{p_{3/2}} = 0.88$ was found. This case corresponds to the s.p. overlap between single intrinsic Slater determinants discussed in Section II.C, for which the harmonic oscillator model predicts $S_p = 1.07$. The large discrepancy indicates an error in the computer program used in [3], which has now been identified and corrected.

To correct the error in [3], the s.p. overlap functions calculated with translationally invariant wave functions simply have to be rescaled by $(\frac{16}{15})^{\frac{3}{2}}$. The corrected spectroscopic factor with only c.m. correlations is thus given by $S_{p_{3/2}} = 0.88 \times (\frac{16}{15})^3 = 1.07$, and is now in perfect agreement with the h.o. model. The results including dynamical NN correlations in [3] must be changed likewise, and we get $S_{p_{3/2}} = 1.06$ when central NN correlations are added, and $S_{p_{3/2}} = 0.98$ for the complete calculation. The value $S_{p_{3/2}} = 0.90$, obtained by including dynamical NN correlations, but no c.m. correlations, does not change.

We note that the proper treatment of c.m. motion makes the discrepancy with the experimentally obtained spectroscopic factors severely worse. In [13] a spectroscopic factor $S_{p_{1/2}}(g.s.) = 0.61$ is reported for the $1/2^-$ ground state in ^{15}N, whereas the lowest $3/2^-$ state at 6.32 MeV has $S_{p_{3/2}}(6.32) = 0.53$. The experimental low-lying $p_{3/2}$ strength is fragmented over three states at 6.32, 9.93 and 10.70 MeV, of which the 6.32 MeV state is the dominant one, with 87% of the total strength. Since we did not include fragmentation due to low-energy configuration mixing, our variational result $S_{p_{3/2}} = 0.98$ should be compared to the total experimental value $S_{p_{3/2}} = 0.53/0.87 = 0.61$. The c.m. correction is not limited to variational calculations, but affects other theoretical models as well, such as the Green's function calculations in [4–6], for which c.m. motion was neglected. Thus, the result $S_{p_{3/2}} = 0.91$ in [5] would (in first order) be changed by c.m. correlations to $S_{p_{3/2}} = 0.91 \times \frac{16}{15} = 0.97$, a value similar to ours, while that of [6] (where low-energy fragmentation is taken into account) is increased from $S_{p_{3/2}}(6.32) = 0.76$ to $S_{p_{3/2}}(6.32) = 0.81$.

IV. OTHER CORRECTIONS TO THE $(E, E'P)$ CROSS SECTION

A correct treatment of c.m. motion enhances the spectroscopic factor of valence hole states at the mean-field level, leading e.g. to a 7% enhancement for the p-shell spectroscopic factors in ^{16}O. This results in present calculations [3–6] of the ^{15}N spectroscopic factors giving values that are much larger than the experimental values. In this Section we discuss two simple mechanisms, not taken into account in [3–6], that could possibly further reduce the theoretical cross section for knock-out of a p-shell nucleon. Firstly, the spectroscopic factor itself could change at the mean-field level if we allow for different mean fields in the target and residual nucleus. Secondly, corrections to the PWIA reaction amplitude due to nonorthogonality and antisymmetrization may become more important when c.m. motion is taken into account.

A. Different s.p. orbitals in target and residual nucleus

The assumption [implicit in Eq.(27)], of having the same set of s.p. wave functions describing the target and residual nucleus, is likely to become less adequate as A decreases.

In principle one could check this by making a separate variational calculation for the ^{15}N states, but the sensitivity of the results on the shape of the s.p. wave functions is not large enough to do this reliably. In order to have an idea about the magnitude of this effect we calculated p-shell spectroscopic factors in a fixed-center mean-field model, using slightly different W.S. well shapes for the protons in ^{15}N and ^{16}O, and keeping the neutron wells identical. In this case it is easy to show that $S_p = Q_s^4 Q_p^{10}$, with Q_s (Q_p) the overlap between the (slightly) different proton s-orbitals (p-orbitals) in ^{16}O and ^{15}N. If we fitted the W.S. parameters to the experimental charge densities available in the literature [16], we find $S_p = 0.946$, i.e. a substantial 5.4% reduction. However, a minimal change in the ^{16}O geometry (just reducing the W.S. radius to reproduce the rms radius of ^{15}N) gives only a 1.2% reduction. We tend to regard the latter value as the more realistic. The rms radius of ^{15}N is better established than the whole charge density. The ^{15}N charge density in [16] has the peculiarity of overshooting the ^{16}O charge density in the interior, and this feature is responsible for the more substantial effect. Also note that, lacking information, the neutron wells were taken identical in ^{15}N and ^{16}O. Relaxing this assumption would also contribute to the reduction of the proton p-shell spectroscopic factor. We estimate therefore the reduction of $S_{p_{3/2}}$ due to changes in the s.p. wave functions to be at most about 5% and probably less.

B. Nonorthogonality and antisymmetrization corrections

Here we look at the effects of nonorthogonality of the final scattering states $\Psi_{(A)\mathbf{p}}$ in Eq.(10) and the antisymmetrization correction (17) of the amplitude. With fixed-center wave functions these effects are known to be small under normal $(e, e'p)$ kinematical conditions, but it is not inconceivable that c.m. effects change this for intrinsic wave functions. Note e.g. that in the fixed-center frame the orthogonality corrections vanish if the final state is a SD with one of the bound hole states replaced with a continuum state of the same mean field, whereas this is no longer the case when ISD are used.

In order to handle the nonorthogonality corrections on the amplitude, we consider the Löwdin transformation on the set of final scattering states $\Psi_{(A)\mathbf{p}}$ [17]. The orthonormal set of Löwdin transformed states can be expressed as

$$|\tilde{\Psi}_{(A)\mathbf{p}}\rangle = \int dp' [N^{-1/2}]_{\mathbf{p}\mathbf{p}'} |\Psi_{(A)\mathbf{p}'}\rangle, \tag{28}$$

in terms of the overlap matrix $[N]$,

$$[N]_{\mathbf{p}\mathbf{p}'} = \langle \Psi_{(A)\mathbf{p}'} | \Psi_{(A)\mathbf{p}} \rangle = \delta(\mathbf{p} - \mathbf{p}') - C(\mathbf{p}, \mathbf{p}'), \tag{29}$$

given by Eq.(11). It can be shown that this transformation does not change the correct asymptotic behavior (9) of the final scattering states.

The amplitude can now be expanded in powers of the small correction C, and up to first order we get

$$\langle \tilde{\Psi}_{(A)\mathbf{p}} | \rho(\mathbf{q}) | \Psi_{(A)i} \rangle = \langle \Psi_{(A)\mathbf{p}} | \rho(\mathbf{q}) | \Psi_{(A)i} \rangle + \frac{1}{2} \int d\mathbf{p}' C(\mathbf{p}, \mathbf{p}') \langle \Psi_{(A)\mathbf{p}'} | \rho(\mathbf{q}) | \Psi_{(A)i} \rangle. \tag{30}$$

Using Eqs.(16-17) and neglecting antisymmetrization corrections in the (small) second term of Eq.(30), the amplitude becomes

$$\langle\tilde{\Psi}_{(A)\mathbf{p}}|\rho(\mathbf{q})|\Psi_{(A)i}\rangle = A_{\rm PWIA}(\mathbf{p},\mathbf{q}) + A_{\rm A.S.}(\mathbf{p},\mathbf{q}) + A_{\rm Löw}(\mathbf{p},\mathbf{q}), \tag{31}$$

with the first order Löwdin correction given by

$$A_{\rm Löw}(\mathbf{p},\mathbf{q}) = -\frac{1}{2}\int d\mathbf{p}' C(\mathbf{p},\mathbf{p}') A_{\rm PWIA}(\mathbf{p}',\mathbf{q}). \tag{32}$$

In the harmonic oscillator model the magnitude of the different terms in Eq.(31) can be easily estimated by concentrating on the Gaussian part of the momentum dependence. As an example, the exponent in the Gaussian part of the left-hand-side of Eq.(5) contains

$$\sum_{i=1}^{A}(\mathbf{r}_i - \mathbf{R}_A)^2 + \sum_{i=1}^{A-1}(\mathbf{r}_i - \mathbf{R}_{A-1})^2 = \frac{A-1}{A}(\mathbf{r}_A - \mathbf{R}_A)^2 + 2\sum_{i=1}^{A-1}(\mathbf{r}_i - \mathbf{R}_{A-1})^2, \tag{33}$$

and it follows that

$$\psi(x) = \exp[-\frac{1}{2b^2}\frac{A-1}{A}r^2] \times [\text{Polynomial in } \mathbf{r}]. \tag{34}$$

As a consequence, the Gaussian part of the momentum dependence of $A_{\rm PWIA}$ is

$$A_{\rm PWIA} \sim \exp[-\frac{b^2}{2}\frac{A}{A-1}(\mathbf{p} - \frac{A-1}{A}\mathbf{q})^2]. \tag{35}$$

Similarly we find

$$\rho(x,x') \sim \exp[-\frac{1}{2b^2}(r^2 + r'^2)], \tag{36}$$

$$C(\mathbf{p},\mathbf{p}') \sim \exp[-\frac{b^2}{2}\frac{A-1}{A-2}\{(\mathbf{p}' + \frac{1}{A-1}\mathbf{p})^2 + (\mathbf{p} + \frac{1}{A-1}\mathbf{p}')^2\}], \tag{37}$$

$$\begin{aligned} A_{\rm Löw}(\mathbf{p},\mathbf{q}) &\sim \int d\mathbf{p}' C(\mathbf{p},\mathbf{p}') \exp[-\frac{b^2}{2}\frac{A}{A-1}(\mathbf{p}' - \frac{A-1}{A}\mathbf{q})^2], \\ &\sim \exp[-\frac{b^2}{2}\frac{A}{A-1}\{(\mathbf{p} + \frac{1}{A}\mathbf{q})^2 + \frac{A-2}{2A}\mathbf{q}^2\}]. \end{aligned} \tag{38}$$

The last result also holds for the antisymmetrization correction $A_{\rm A.S.}(\mathbf{p},\mathbf{q})$.

The NIKHEF ^{16}O$(e,e'p)$ experiment [13] was performed under quasi-elastic parallel kinematics, with a roughly constant laboratory proton kinetic energy $T_{p_L} \approx 90$ MeV and missing momentum $p_m = p_L - q_L$ scanned in the region (-150 , 250) MeV/c. Under these kinematical conditions the magnitude[2] of the correction term (38) is very small compared to the magnitude (35) of the leading PWIA term in the amplitude, the ratio ranging from 4×10^{-6} at the most negative p_m, to 3×10^{-5} at p_m=0, and growing to 6×10^{-3} at the most positive p_m (corresponding to the smallest q_L). To get a 10% correction at $p_m = 0$, one would need protons ejected with only $T_{p_L} \approx 20$ MeV. We conclude that, under normal $(e,e'p)$ kinematics, the nonorthogonality[3] and antisymmetrization corrections are unimportant also for the translationally invariant wave functions considered here, just as in the fixed-center case.

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[2]Note that in the laboratory frame the relative momentum p in Section II.B is given by $p = p_L - \frac{1}{A}q_L$

[3]There is also the effect of nonorthogonality between the scattering states $\Psi_{(A)\mathbf{p}}$ and the initial

V. DISCUSSION

The observed magnitude of the experimental cross sections of $(e, e'p)$ reactions at small missing energies is not satisfactorily explained by present theoretical models. For ^{16}O we pointed out that c.m. correlations enhance the cross section leading to p-shell quasi-hole states by about 7%. As a result theoretical predictions [3–5] that consider effects primarily due to short-range and tensor NN correlations give $S_{p_{3/2}} \approx 0.97$ for ^{15}N. Including also low-energy configuration mixing in the target and residual nucleus [6] provides additional depletion and fragmentation of strength and lowers this to $S_{p_{3/2}} \approx 0.81$, which is still far above the experimental value $S_{p_{3/2}}(6.32) = 0.53 \pm 0.05$. At present this discrepancy does not seem to be understood.

More generally, the large difference between the experimental value of the p-shell quasi-hole strength and the present variational result indicates that an important ingredient is missing in the variational wave function. This is also signaled by recent calculations for $A \leq 7$ nuclei [18], in which it is found that the quality of the VMC wave function (used as input in subsequent Green's function Monte Carlo calculations) deteriorates with increasing A, when compared to the final GFMC result. The possibility of α-cluster components in the surface part of the wave function should be looked at.

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APPENDIX A: HARMONIC OSCILLATOR MODEL FOR THREE PARTICLES IN A S^2P CONFIGURATION

Consider one spin-down and two spin-up neutrons in a harmonic oscillator well. This is the simplest non-trivial illustration of the h.o. model. The ground-state configuration has a spin-up and a spin-down neutron in the s-shell and one spin-up neutron in the p-shell. Its intrinsic wave function is

$$\Psi_{(3)}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; \text{spins}) = \exp[-\frac{b^2}{2}\sum_{j=1}^{3}(\mathbf{r}_j - \mathbf{R}_3)^2]\,\{(z_1 - Z_3)(|uud\rangle - |udu\rangle) + (z_2 - Z_3)(|duu\rangle - |uud\rangle) + (z_3 - Z_3)(|udu\rangle - |duu\rangle)\}\,, \qquad \text{(A1)}$$

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state $\Psi_{(A)i}$, which is straightforward to include in this analysis. The correction term to the amplitude has a momentum dependence $\exp[-\frac{b^2}{2}(\frac{A}{A-1}p^2 + \frac{A-1}{2A}q^2)]$ and is of the same order of magnitude as the other correction terms.

where the spin states are differentiated by u, d, the occupied p-orbital is chosen in the z-direction, and the h.o. length parameter is b.

The three possible one-hole states are

$$\Psi^{(u)}_{(2)p}(\mathbf{r}_1, \mathbf{r}_2, \text{spins}) = \exp[-\frac{b^2}{2}\sum_{j=1}^{2}(\mathbf{r}_j - \mathbf{R}_2)^2](|ud\rangle - |du\rangle),$$
$$\Psi^{(u)}_{(2)s}(\mathbf{r}_1, \mathbf{r}_2, \text{spins}) = \exp[-\frac{b^2}{2}\sum_{j=1}^{2}(\mathbf{r}_j - \mathbf{R}_2)^2]((z_1 - Z_2)|ud\rangle - (z_2 - Z_2)|du\rangle),$$
$$\Psi^{(d)}_{(2)s}(\mathbf{r}_1, \mathbf{r}_2, \text{spins}) = \exp[-\frac{b^2}{2}\sum_{j=1}^{2}(\mathbf{r}_j - \mathbf{R}_2)^2](z_1 - z_2)|uu\rangle. \tag{A2}$$

In the notation of Eq.(22-23) we have $n^{(d)}_s = n^{(u)}_s = n^{(u)}_p = 1$, $A^{(u)} = 2$ and $A^{(d)} = 1$. The spectroscopic factors for the valence $p^{(u)}$ and $s^{(d)}$ hole states are, according to Eq.(22), $S^{(u)}_p = \frac{3}{2}$ and $S^{(d)}_s = (\frac{3}{2})^0 = 1$. The spectroscopic factor for the $s^{(u)}$ hole state can be found from the sum rule (23), $S^{(u)}_s = 2 - \frac{3}{2} = \frac{1}{2}$.

The same results are easily found by applying Eqs.(4-5) with the intrinsic wave functions (A1-A2). The normalizations of the wave functions are

$$\langle\Psi_{(3)}|\Psi_{(3)}\rangle = \frac{\pi^3 3^{5/2}}{b^8}, \tag{A3}$$
$$\langle\Psi^{(u)}_{(2)p}|\Psi^{(u)}_{(2)p}\rangle = \frac{\pi^{3/2}2^{5/2}}{b^3}, \tag{A4}$$
$$\langle\Psi^{(u)}_{(2)s}|\Psi^{(u)}_{(2)s}\rangle = \frac{\pi^{3/2}2^{1/2}}{b^5}, \tag{A5}$$
$$\langle\Psi^{(d)}_{(2)s}|\Psi^{(d)}_{(2)s}\rangle = \frac{\pi^{3/2}2^{3/2}}{b^5}. \tag{A6}$$

The overlap functions are then given by

$$\psi^{(u)}_p(\mathbf{r_3}) = \frac{b^{5/2}2^{5/4}}{\pi^{3/4}3^{3/4}} z_3 exp[-\frac{b^2}{3}r_3^2], \tag{A7}$$
$$\psi^{(u)}_s(\mathbf{r_3}) = \frac{b^{3/2}2^{1/4}}{\pi^{3/4}3^{3/4}} exp[-\frac{b^2}{3}r_3^2], \tag{A8}$$
$$\psi^{(d)}_s(\mathbf{r_3}) = \frac{b^{3/2}2^{3/4}}{\pi^{3/4}3^{3/4}} exp[-\frac{b^2}{3}r_3^2], \tag{A9}$$

and their normalization agrees with the values for the spectroscopic factors mentioned above.

The spectroscopic sum rule (23) in terms of the intrinsic one-hole states is a consequence of the special nature of a harmonic oscillator mean-field; in contrast to fixed-center systems it does not hold for general mean-field s.p. wave functions. As an example we can distort the h.o. mean field by taking different h.o. length parameters b and b' for the s and p orbitals in the present s^2p model. The result for $S^{(d)}_s$ now becomes

$$S^{(d)}_s = \frac{3(x^2+14x+9)^{-7/2}[(x-1)^2(x+3)^2 + 16(x+1)(x^2+14x+9)]}{(x+1)[(2(2x+1))^{-5/2} + 16(2(x+1)(x+5))^{-5/2}]}, \tag{A10}$$

with $x = (\frac{b'}{b})^2$. For $x = 1$ we recover the result $S_s^{(d)} = 1$, i.e. in a pure h.o. mean field the spectroscopic strength of the spin-down neutron is fully contained in the intrinsic $s^{(d)}$ one-hole state. For $x \neq 1$ we find $S_s^{(d)} < 1$; the remainder of the strength is contained in more complicated configurations.

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