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# Particle dynamics of branes

Ploegh, André René

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# Chapter 1

# Introduction

A long standing problem in physics is the unification of all known forces in nature. These are gravity and the strong, the weak and the electromagnetic force. The last three forces are combined in what is called the standard model of particle physics. This model has been developed by Salam, Glashow and Weinberg between 1970 and 1973. Combining the standard model with gravity is however a complicated issue. String theory is one of the most promising attempts to achieve this. The full mathematical structure of string theory is complicated and unfinished, only perturbatively the description of a string is by now well understood. A non-perturbatively description is still lacking.

As the name suggest, in string theory the fundamental objects are strings. These can be thought of as one-dimensional objects that trace out two-dimensional surfaces in time. Different vibrational modes of the string are associated to different particles and forces. One of these modes is the graviton. For this reason string theory is a candidate for unifying the known four forces in nature. The graviton belongs to the massless sector. The first massive states have masses around the Planck mass $\sim 10^{-5}$ grams or energy $\sim 10^{19}$GeV. This corresponds to lengths of about $10^{-33}$ centimeters. Since this is out of reach of today's accelerators, the main focus is on the massless sector.

One of the surprising things about string theory is that, although we start from a string, the massless spectrum gives rise to higher-dimensional objects. For example, think of a two-dimensional membrane. A second intriguing aspect of string theory is that it requires a ten-dimensional space-time instead of the four-dimensional universe we live in. It is therefore not surprising that we have even higher-dimensional objects than membranes, these are called branes. Branes will play a central role in this thesis. These objects tell us about non-perturbative aspects of string theory.

One way to learn more about string theory, is by looking at its low energy limit.

There is also a good physical reason to do this. Namely, the aim of string theory is to unify the standard model with gravity. However the results of experiments done so far in accelerators can be explained by the standard model. A new milestone in accelerator physics is the Large Hadron Collider (LHC) which will come online this summer at CERN. The energy scale of this accelerator is of the order of ten tera-electron-Volt ($10^{12}$ eV). This corresponds to lengths of about $10^{-19}$ meters. Any deviation from the standard model possibly observed at LHC should then be explainable by the low energy limit of string theory.

The low energy limit of string theory leads to so-called supergravities. A supergravity is a classical theory which extends Einstein's theory of general relativity to fermions in such a way that bosonic degrees of freedom are related to fermionic degrees of freedom, the number of bosons equals the number of fermions. The low energy limit of string theory leads to five different supergravities, but these theories are related through a whole web of dualities. This suggests that each of these five theories are different limits of a single theory called M-theory. There is very little known about this theory.

We mentioned that string theory requires a ten-dimensional space-time. Our observable universe is only four-dimensional including time. Somehow we have to rationalize away six dimensions. The standard way is via dimensional reduction. With this we mean that we assume that the ten-dimensional universe can be considered as a direct-product space of our four-dimensional universe and a compact six-dimensional space which is of small size. The extra dimensions need to be smaller than $10^{-18}$ meters else we would have observed them by now[1].

Let us get back to the branes. In this thesis we are going to study branes that are solutions of the two so-called type II supergravities that follow from considering the low energy limit of string theory. The dimensions of the extended object form the worldvolume of the brane. For example, in case of the string we would have a two-dimensional worldvolume consisting out of time and one spatial direction. The other space-time dimensions form the transverse space. The main focus will be on two types of branes. If time is part of the worldvolume the brane is called a (timelike) $p$-brane. The $p$ stands for the number of spatial directions of the worldvolume. In total we have $p+1$ dimensions. If time is not part of the worldvolume it is called an (spacelike) S$p$-brane. Here $p$ stands for the number of spatial worldvolume directions minus one. In this way $p$ refers in both cases to a $(p+1)$-dimensional worldvolume.

To investigate these brane solutions we could try to solve the equations of motion that follow from the action of the supergravity directly. This is however not the way we are going to proceed. As the title of the thesis suggest, we are going to look at

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[1] There is however a different string theory scenario where not all extra dimensions have to be small. This is the so-called brane world or Randall-Sundrum scenario [1, 2]. In this model all interactions except gravity are restricted to a four dimensional hyperplane, which represents our universe. This model has the advantage that, since gravity is spread over the whole space-time, it gives an explanation as to why the gravitational force is weak compared to the other three forces.

branes from a particle point of view. With this we mean that we are going to look at brane solutions whose dynamics depends only on one parameter, just like particles do. This parameter will be related to a coordinate of the transverse space. Because of this the worldvolume coordinates do not appear explicitly in the solutions. In this way we see that the worldvolume directions do not really matter. For this reason we first reduce over the worldvolume via dimensional reduction and then try to solve the remaining lower-dimensional equations of motion. This is the first step in reducing the problem of finding brane solutions. Since we have reduced over the worldvolume the theory is a ($p$=$-1$)-brane, such that we indeed have a zero-dimensional worldvolume. If time is part of the reduced worldvolume, the lower-dimensional theory lives in a Euclidean space-time. Such a solution is called a ($-1$)-brane or instanton. If time is not part of the reduced worldvolume, is it called an S($-1$)-brane.

Alternatively, we will show that we can reduce over the transverse directions that are not related to the parameter describing the dynamics of the solutions. As it turns out, this way we can generate a potential in the lower-dimensional action, which we then call a massive theory. A theory is called massless if there is no potential. If the lower-dimensional theory requires a Minkowski space-time we have two different solutions. We call it a cosmology if the solution depends explicitly on time only. If it does not depend on time explicitly, we call it a domain-wall. This can be considered as the stationary version of a cosmology[2].

It is important to mention that we will only consider consistent reductions. With this we mean that we can always undo the steps of the reduction in such a way that we are guaranteed that we also have a solution of the action we started with. In this way we construct a solution of the higher-dimensional theory. This procedure is called uplifting or oxidation.

To solve the lower-dimensional equations of motion we have to make a difference between the massless and massive theories.

The massless case is the easiest. Due to the dimensional reduction, the lower-dimensional action will have a much bigger symmetry group than the action we started with (not including diffeomorphisms). This we can use to simplify our quest for brane solutions further. In solving the lower-dimensional equations of motion we will first see that we can decouple the scalar sector from the gravity sector. As as result, we solve the metric independently from the scalar fields. We will not need to look for the most general scalar field solutions. Instead we will look for much easier solutions, namely generating solutions. With this we mean that if we act with the symmetry group on this solution, we automatically find the most general solution possible.

In case we have a massive theory, a general solution is difficult to give due to the presence of the potential. There is no decoupling from the gravity sector. Instead we will show when we can write the second order equations in terms of first order

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[2] In chapter 6 and 7 we consider a few examples where we have a Euclidean theory *with* a potential. We call these solutions instantons as well.

equations.

In this thesis we want to achieve the following. We are first going to show that $p$- and S$p$-branes can be linked to lower-dimensional actions whose solutions are respectively given by instantons or S$(-1)$-branes if we reduce over the worldvolume of the brane. And similarly, if we reduce a $p$-brane or S$p$-brane over all but one of its transverse directions we find a domain-wall or a cosmology. The main goal of this thesis is: *Derive the solutions that correspond to the lower-dimensional action.* In case the lower-dimensional theory is massless we look for the generating solution. For a massive theory the focus will be on re-writing the second order differential equations as first order equations. We will further see that a specific class of massive solutions behave as if there is no potential at all.

If we would uplift our solution back to the original theory we have constructed a general brane solution of the original theory. Along the way we will see that there are all kind of links between the lower- and higher-dimensional solutions.

The plan of the thesis is as follows. In the next chapter we begin with giving a short introduction to string theory. The focus will be on introducing the relevant concepts. In particular we will spend some time on the $p$- and S$p$-branes.

In chapter 3 we are going to explain how one does a dimensional reduction. We will restrict to two different types of reductions relevant for branes. At the end of the chapter we explain how dimensional reduction can be applied to branes.

In chapter 4 we look for time-dependent S$p$-branes via reducing over its worldvolume. This way we will obtain a massless theory. With the help of the generating solution and the symmetry group we will be able to construct the most general S$p$-brane with deformed worldvolume.

In chapter 5 we are going to look at massive theories, i.e. cosmologies and domain-walls. Solving a theory with a potential is more complicated, but we will show that often the equations of motion can be written as first order equations. Furthermore we will show that under certain conditions the problem is basically the same as looking for solutions of a model without a potential.

There is a link between these two solutions, which is called the domain-wall / cosmology correspondence. The correspondence can be summarized roughly as stating that for every cosmology there exists also a domain-wall and *vice-versa.* In chapter 6 chapter we are going to see what happens if we put this correspondence in a supergravity setting. We will see that this leads to a new type of brane, the so-called E$p$-brane. We point out a relation to S$p$-branes.

All the solutions considered so far live in a Minkowski space-time. In chapter 7 we are going to consider solutions that require a Euclidean space-time i.e. instantons. In chapter 3 we show how such theories can be obtained from dimensional reduction of ordinary Lorentzian supergravities. The main focus will be on finding the generating Euclidean solutions.

The last chapter will be about conclusions and possible future research.

There are four appendices. Appendix A contains all the necessary conventions that we use for general relativity and differential geometry. Appendix B is concerned with spinors in arbitrary dimensions. This is used in chapter 6. In appendix C we give an overview of Lie groups and Lie algebras. In the last appendix we give the published papers.