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Stabilizing control for power converters connected to transmission lines

Marius Zainea, Arjan van der Schaft and Jean Buisson

Abstract— This paper proposes a switching control strategy for the set-point stabilization of a power converter connected via a transmission line to a resistive load. The strategy employs a Lyapunov function that is directly based on energy considerations of the power converter, as well as of the transmission line described by the telegraph equations. The proposed stabilizing switching control still allows a certain freedom in the choice of the control law, a comparison between a maximum descent strategy and a minimum commutation strategy being discussed on a simple example.

I. INTRODUCTION

Devices like power converters (Boost, Buck, Čuk, multilevel converters) are widespread industrial devices. They are used in many applications such as variable speed DC motor drives, computer power supply, cell phone and cameras. Those devices are electrical circuits controlled by switches (transistors, diodes). Aiming at reducing switching losses and EMI (Electromagnetic Interference) of power converters, a lot of soft switching techniques are developed so that high efficiency, small size and low weight can be achieved. When they are operating in normal conditions, those circuits have been designed such that the commutation of the switches does not produce discontinuities. In this context, they can be modelled by switched systems (without jump). For this class of systems, multiple approaches for control have been developed, mainly based either on continuous time approaches (i.e. sliding mode [1], passivity based control [2], stabilizing control [3],...), or on hybrid approaches (i.e. model predictive control [4], supervisory control [5],...).

The goal of this paper is to show how the hybrid stabilizing control scheme presented in [6] can be extended to the case where the power converter and load are not situated at the same location, in which case the transmission line between the power converter and the load needs to be taken into account. The advantages of the proposed method are that it uses a simple Lyapunov function deduced on energy considerations and that the control variable is directly boolean. The difficulty of the studied problem resides in the fact that the transmission line model is a distributed parameter model described by PDEs (the telegraph equations), to which the original switching stabilizing control method of [6] cannot be directly applied. To solve this problem, the power converter part and the line and the load part are analyzed separately, where a candidate Lyapunov function is proposed for each

part on the same energy criteria as in the original method. Then it can be shown that the sum of the two candidate Lyapunov functions represents a Lyapunov function for the entire system.

Section II introduces the models used for the power converter subsystem as well as for the line and the load subsystem. Section III recalls first how the stabilizing control can be applied when the power converter and the load are directly connected together and, second, how this method can be extended to the power converter – line – load system. The case of the "boost" converter is discussed in section IV, where two strategies for control are analyzed and the paper ends with the conclusions section.

II. MODELS OF THE SYSTEMS WITH SWITCHING POWER CONVERTERS

The systems under consideration are electrical power converters, which are used to adapt the energy supplied by a power source to a load. Those systems include power sources, energy storage elements (inductances or capacitors), dissipative elements (resistances), transformers, gyrators and switching components. In the following, the storage and dissipative elements are supposed to be linear and the transformers and gyrators are supposed to be constant. The physical switches are considered to be ideal: in the state *on*, their voltage is null and in the state *off*, their current is null. In most of those systems, physical switches are associated by pairs. In each pair, one physical switch is controlled (e.g. transistor) while the other one may be not (e.g. diode). In a normal operating mode both physical switches commutate at the same time, which is equivalent to assuming that only the continuous conduction modes are considered. This association between two switches constitutes a commutation cell, which is simply called switch in the following.

In order to derive models for physical systems, different energy based approaches, such as circuit theory, bond graphs [7], Euler Lagrange, Hamiltonian approach [8] can be used. For switching systems, extensions have been proposed in [9] for the Hamiltonian approach or in [10], [11] among many other references for the bond graph approach.

When connecting a resistive load to a power converter, taking into consideration or not the transmission line, the most common cases are those presented in figure 1. The situation depicted by figure 1(a) correspond, for example, to the populars "boost", "buck" and "buckboost" converters, while the situation depicted by figure 1(b) correspond, for example, to the multicellular converter.

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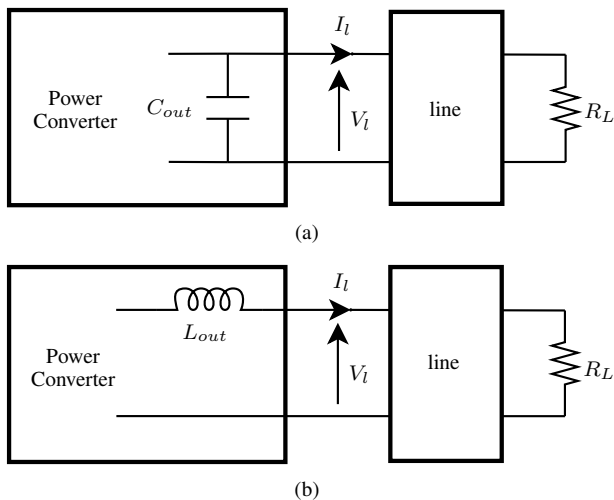
(a)

(b)

Fig. 1. The two most common cases when connecting the load to the power converter through a transmission line

A. The Power Converter Model

If storage elements are independent, all previously cited approaches can lead, for one mode (or switch configuration), to model (1), which is called "port-Hamiltonian systems" (with dissipation) [8], [9], [12]. [1]

$$\begin{cases} \dot{x} = (J - R) z + gu + g_l v \\ w = -g_l^T z \end{cases} . \qquad (1)$$

The vector $u \in \mathbb{R}^m$ corresponds to the energy sources which are generally either constant in DC-DC or DC-AC converters or sinusoidal in AC-DC or AC-AC converters. This vector is supposed constant in the following. The couple (v, w) is represented either by (I_l, V_l) for the case depicted by figure 1(a) or by (V_l, I_l) for the case depicted by figure 1(b). Vector $x \in \mathbb{R}^n$ is the state vector and n is the number of energy storage elements. State variables are the energy variables (fluxes linkage in the inductors, charges in the capacitors), $z \in \mathbb{R}^n$ is the co-state vector. Co-state variables are the corresponding co-energy variables (currents, voltages). In the case where the components are linear, the relation between those two vectors is given by:

$$z = Fx \qquad (2)$$

where $F = F^T \succ 0$. In simple cases, F is a diagonal matrix the elements of which are the inverse of the values of capacitances or inductances. The quantity $\dot{x}^T z$ represents the power entering the storage elements. The energy, which is stored in the system, can be expressed as:

$$E(x) = \frac{1}{2} x^T F x \qquad (3)$$

Both $n \times n$ matrices J and R are called structure matrices. The matrix J is skew-symmetric, $J = -J^T$; it corresponds to a power continuous interconnection in the network model.

[1] In [8], [9] this was originally called a "port-controlled Hamiltonian system".

The matrix R is nonnegative; it corresponds to the energy dissipating part of the circuit.

When the switches change their configuration, the continuous conduction hypothesis is assumed, which is equivalent to consider that physical switches commutate by pairs, implying further that storage elements are still independent and the state and co-state keep the same components. It also results that, for those systems, there is no jump on state variable when switching [10]. Those hybrid systems can be considered from the hybrid point of view as switching systems. As J, R and g may depend upon the mode, the model can be expressed as:

$$\begin{cases} \dot{x} = (J(\rho) - R(\rho)) z + g(\rho) u + g_l v \\ w = -g_l^T z \end{cases} , \qquad (4)$$

where $\rho \in \{0, 1\}^p$ is a boolean vector describing the configuration or mode of the system, p is the number of switches (or pairs of physical switches). Due to the assumption made on how the power converter and the line are connected together, w is a component of z and, thus, g_l does not depend on ρ. Matrices $J(\rho)$ and $R(\rho)$ have the same properties than J and R.

For this class of physical systems with pairs of physical switches, it is assumed in the following that the three matrices in (4) can be expressed using an affine relationship:

$$J(\rho) = J_0 + \sum_1^p \rho_i J_i, \qquad (5a)$$

$$R(\rho) = R_0 + \sum_1^p \rho_i R_i, \qquad (5b)$$

$$g(\rho) = g_0 + \sum_1^p \rho_i g_i, \qquad (5c)$$

where ρ_i are the components of ρ. This property which has been verified on many usual devices (Buck, Boost, Čuk) [13], [9], and it has also been formally proved for multicellular serial converters [14].

B. Ideal Line and Load Model

Consider the ideal lossless transmission line [12], where the spatial variable, q, belongs to the interval $[0, 1]$. The energy variables associated to the line are the charge density $Q = Q(t, q) \, dq$, and the flux density $\varphi = \varphi(t, q) \, dq$. The total energy stored at time t in the transmission line is given as:

$$E_l(Q, \varphi) = \int_0^1 \frac{1}{2} \left(\frac{Q^2(t, q)}{C_l(q)} + \frac{\varphi^2(t, q)}{L_l(q)} \right) dq \qquad (6)$$

where $C_l(q)$ and $L_l(q)$ are respectively the distributed capacitance and distributed inductance of the line. Moreover, the voltage and the current are given by:

$$V\left(t,q\right) = \frac{Q\left(t,q\right)}{C_l\left(q\right)}$$
$$I\left(t,q\right) = \frac{\varphi\left(t,q\right)}{L_l\left(q\right)} \quad (7)$$

satisfying the telegraph equations:

$$\frac{\partial Q}{\partial t} = -\frac{\partial I}{\partial q}$$
$$\frac{\partial \varphi}{\partial t} = -\frac{\partial V}{\partial q} \quad (8)$$

Additionally, for the system that consists of the transmission line and the resistive load, R_L, the following constraints hold:

$$V\left(t,0\right) = V_l$$
$$I\left(t,0\right) = I_l \quad (9a)$$
$$V\left(t,1\right) = R_L I\left(t,1\right) \quad (9b)$$

where $V\left(t,0\right)$ and $V\left(t,1\right)$, and, respectively, $I\left(t,0\right)$ and $I\left(t,1\right)$ are the voltages, respectively the currents, at the beginning and at the end of the line.

III. LYAPUNOV FUNCTION

The control approach which is proposed in this paper is based on a common Lyapunov function for the different modes, with its derivative depending on the control variable ρ. In [6], for the case where the power converters is directly connected to the load, it has been shown how ρ can be chosen such that the derivative of the Lyapunov function can be kept always negative. In the following, in III-B, it is shown how this result can be extended to the case where a transmission line is used to connect the power converter and the load, but, first, the initial method (without the line) is recalled.

A. The Power Converter Directly Connected to the Load

If the case where the power converter is directly connected to the load is analyzed, then the model expressed by (4) becomes:

$$\dot{x} = \left(J\left(\rho\right) - \tilde{R}\left(\rho\right)\right) z + g\left(\rho\right) u, \quad (10)$$

where

$$\tilde{R}\left(\rho\right) = R\left(\rho\right) + g_l \tilde{R}_L g_l^T, \text{ with} \quad (11a)$$

$$\tilde{R}_L = \begin{cases} R_L, \text{ for the figure 1(b) case} \\ 1/R_L, \text{for the figure 1(a) case} \end{cases}, \quad (11b)$$

and $\tilde{R}\left(\rho\right)$ has the same properties as $R\left(\rho\right)$.

1) Admissible Reference: The objective is to design a switching control law such that the output of the system take some specified value. Using the same approach as with an average model where the control ρ is considered continuous but constrained, the following definition of an admissible reference is proposed.

Definition 1: $z_0 = F x_0$ is called an admissible reference for system (10) and (2) where u is constant, if there exists $\rho_0 \in \mathbb{R}^p$, $0 \le \rho_{0i} \le 1$ such that constraint (12):

$$0 = \left(J\left(\rho_0\right) - \tilde{R}\left(\rho_0\right)\right) z_0 + g\left(\rho_0\right) u, \quad (12)$$

is satisfied.

Remark 2: If $p < n$ and $J\left(\rho_0\right) - R\left(\rho_0\right)$ is structurally invertible, then the admissible reference belongs to a subspace of $\mathbb{R}^p$. p state variables that are considered as the output of the model will be specified. The other state variables, as well as ρ_0, are fixed by constraint (12). In other cases, (12) is still satisfied, but $\left(J\left(\rho_0\right) - \tilde{R}\left(\rho_0\right)\right)$ may be singular, so x_0 is not necessarily unique and some state variables can be chosen arbitrarily.

2) Lyapunov Function:
Definition 3: A function H is a Lyapunov function for the system represented by (1) or (10) and (2) in x_0 if:

- $H\left(x, x_0\right) > 0$ anywhere excepted in x_0 where it holds $H\left(x_0, x_0\right) = 0$,
- H is radially unbounded,
- for any x, a control ρ can be chosen such that $\dot{H}\left(x, x_0\right) < 0$.

If such a control law is applied, then x will converge asymptotically toward x_0. The following results states how a Lyapunov function can be determined for the case where the power converter is directly connected to the load.

Theorem 4: Considering the system represented by (10) and (2), it is always possible to find a boolean state feedback $\rho\left(x\right)$ such that the function defined by $\tilde{H}\left(x, x_0\right) = E\left(x - x_0\right) = \frac{1}{2}\left(x - x_0\right)^T F\left(x - x_0\right)$, where x_0 is an admissible reference according to definition 1, is a Lyapunov function for the resulting closed–loop system.

Proof: Since there is no jump, $\tilde{H}$ is positive, continuous and null only for $x = x_0$. The time derivative of $\tilde{H}$ depends on the value of the control ρ and will be denoted by $\dot{\tilde{H}}_\rho$.

$$\begin{aligned} \dot{\tilde{H}}_\rho &= \left(x - x_0\right)^T F \dot{x} \\ &= \left(z - z_0\right)^T \left(\left(J\left(\rho\right) - \tilde{R}\left(\rho\right)\right) z + g\left(\rho\right) u\right) \end{aligned} \quad (13)$$

Using the skew symmetry property of $J\left(\rho\right)$ and the property of the admissible reference, this expression becomes:

$$\begin{aligned} \dot{\tilde{H}}_\rho &= -\left(z - z_0\right)^T \tilde{R}\left(\rho\right)\left(z - z_0\right) \\ &\quad + \left(z - z_0\right)^T \left(J\left(\rho\right) - J\left(\rho_0\right)\right) z_0 \\ &\quad - \left(z - z_0\right)^T \left(\tilde{R}\left(\rho\right) - \tilde{R}\left(\rho_0\right)\right) z_0 \\ &\quad + \left(z - z_0\right)^T \left(g\left(\rho\right) - g\left(\rho_0\right)\right) u \end{aligned} \quad (14)$$

And finally, replacing $\tilde{R}$, J, g using (5) and (11)

$$\begin{aligned} \dot{\tilde{H}}_\rho &= -\left(z - z_0\right)^T \tilde{R}(\rho)\left(z - z_0\right) \\ &\quad + \sum_{1}^{p}\left(z - z_0\right)^T \left(\left(J_i - R_i\right) z_0 + g_i u\right)\left(\rho_i - \rho_{0i}\right) \end{aligned} \quad (15)$$

Since $\tilde{R}(\rho)$ is a nonnegative matrix, the first term of this expression is never positive, and since $0 \le \rho_{0i} \le 1$, every

term of the sum can be made negative by choosing each ρ_i according to the sign of $(z - z_0)^T ((J_i - R_i) z_0 + g_i u)$. ■

Remark 5: In the engineering practice, the control variable ρ has to be considered in a context where the application specifications demand either a limited or a constant switching frequency. The inclusion of these specification related to Lyapunov based control has been discussed in [15]. However, for the sake of simplicity, throughout this paper, there is not any such limitation, the switching frequency being variable and unbounded.

The solution provided by the proof of the theorem 4 for the choice of ρ is referred next as the *maximum descent strategy*. However, this choice is very conservative and, in general, it is not the only one. Thus, one can relax the commutation conditions and still have stability and the most relaxed choice one can have is that of commutating when $\dot{\tilde{H}}_\rho = 0$. Additionally, if the minimum number of switches commutate then the resulting strategy is referred next as the *minimum switching strategy*.

B. The Power Converter Connected to the Load Using a Transmission Line

1) Admissible Reference: Similar to the case without the line, first some admissible reference has to be defined. An equilibrium point for the line is defined by:

$$\frac{\partial Q}{\partial t} = \frac{\partial \varphi}{\partial t} = 0, \tag{16}$$

which, due to (8), implies that

$$I_0 = \frac{\varphi_0(q)}{L(q)} \tag{17a}$$

$$V_0 = \frac{Q_0(q)}{C(q)} \tag{17b}$$

are constant as functions of both time and spatial variable q. Moreover, when the load resistance is considered, the following constraint holds:

$$V_0 = R_L I_0. \tag{18}$$

Then, the admissible reference for the case when the power converter is connected to the resistive load through a transmission line is formulated like in the case without line:

Theorem 6: Every admissible reference for the system formed by a power converter connected directly to a resistive load is an admissible reference for the system where also a transmission line is present.

Proof: At equilibrium, (18) holds which implies that $v_0 = \tilde{R}_L w_0$ holds too. Thus, using also that $w_0 = -g_l^T z_0$, equation (12) is recovered. ■

2) Lyapunov Function: Like in section III-A.2, a suitable Lyapunov function can be formulated for the entire system based on energy considerations.

Theorem 7: For the system including a power converter, a transmission line and a resistive load, it is always possible to find a boolean state feedback $\rho(x)$ such that the function defined by $H = E(x - x_0) + E_l(Q - Q_0, \varphi - \varphi_0)$ is a Lyapunov function for the resulting closed–loop system,

where x_0 is an admissible reference according to definition 1 and (Q_0, φ_0) is the corresponding equilibrium of the line.

Proof: Consider first the term $E(x - x_0) = \frac{1}{2}(x - x_0)^T F(x - x_0)$. Then, from (4), the computation of the time derivative of this term leads to (19).

$$\begin{aligned}
\dot{E} &= (x - x_0)^T F \dot{x} \\
&= (z - z_0)^T [(J(\rho) - R(\rho)) z + g(\rho) u + g_l v] \\
&= -(z - z_0)^T R(\rho)(z - z_0) + (z - z_0)^T g_l (v - v_0) \\
&\quad + (z - z_0)^T \sum_1^p [(J_i - R_i) z_0 + g_i u](\rho_i - \rho_{0i}) \\
&= \dot{\tilde{H}}_\rho - (w - w_0)(v - v_0)
\end{aligned} \tag{19}$$

Consider now the line energy function $E_l(Q - Q_0, \varphi - \varphi_0)$, evaluated in the shifted state variable. Then, using also (6) – (9), the expression of the time derivative of E_l is given by:

$$\begin{aligned}
\dot{E}_l &= \int_0^1 \frac{1}{C_l(q)} (Q(t, q) - Q_0(q)) \frac{\partial Q}{\partial t} dq \\
&\quad + \int_0^1 \frac{1}{L_l(q)} (\varphi(t, q) - \varphi_0(q)) \frac{\partial \varphi}{\partial t} dq \\
&= (V(t, 0) - V_0)(I(t, 0) - I_0) \\
&\quad - (V(t, 1) - V_0)(I(t, 1) - I_0) \\
&= (w - w_0)(v - v_0) - (V(t, 1) - V_0)^2 / R_L
\end{aligned} \tag{20}$$

Thus, the global time derivative is given by:

$$\dot{H} = \dot{\tilde{H}}_\rho - (V(t, 1) - V_0)^2 / R_L \tag{21}$$

Similar to theorem 4, ρ can be chosen such that $\dot{\tilde{H}}_\rho < 0$, and, thus, the same choice for ρ can be used to make $\dot{H}$ negative. ■

Remark 8: By developing further (15), knowing that (11) holds, the following is obtained:

$$\begin{aligned}
\dot{\tilde{H}}_\rho &= -(z - z_0)^T R(\rho)(z - z_0) \\
&\quad - (z - z_0)^T g_l \tilde{R}_L g_l^T (z - z_0) \\
&\quad + \sum_1^p (z - z_0)^T ((J_i - R_i) z_0 + g_i u)(\rho_i - \rho_{0i}) \\
&= \dot{H}_\rho - (w - w_0)^2 \tilde{R}_L.
\end{aligned} \tag{22}$$

Using also (21), the following relation can be derived between $\dot{H}$ and $\dot{\tilde{H}}_\rho$:

$$\dot{H} = \dot{\tilde{H}}_\rho + (w - w_0)^2 \tilde{R}_L - (V(t, 1) - V_0)^2 / R_L \tag{23}$$

Hence, in general, the choice of ρ such that $\dot{\tilde{H}}_\rho \leq 0$ is neither necessary, nor sufficient for $\dot{H}$ to be negative! Nevertheless, it can be noticed that the maximum descent strategy is robust regarding the presence of the line.
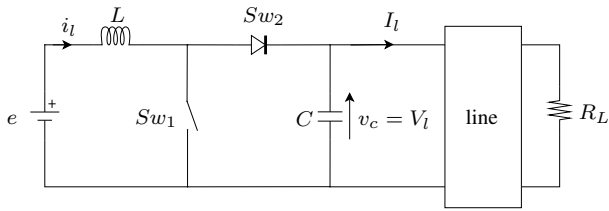
Fig. 2. The Boost converter with a transmission line

IV. EXAMPLE

Figure 2 represents a simplified circuit of a well known power converter called (ideal) Boost converter. Under the hypothesis previously formulated, only two operating modes are considered: one, the diode is conducting when the controlled physical switch is open ($\rho = 1$) and second, blocked when the controlled physical switch is closed ($\rho = 0$).

The state vector $x = (x_l, x_c)^T$ is composed of the flux linkage in the inductance and the charge in the capacitor. The co-state vector $z = (i_l, v_c)^T$ is composed of the current in the inductance and voltage on the capacitor. The matrices corresponding to (4), (5) and (10) are:

$$J(\rho) = \begin{pmatrix} 0 & -\rho \\ \rho & 0 \end{pmatrix} \quad R(\rho) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{(24a)}$$

$$g(\rho) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad g_l = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \quad \text{(24b)}$$

$$F = \begin{pmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{C} \end{pmatrix} \quad \tilde{R}(\rho) = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{1}{R_L} \end{pmatrix} \quad \text{(24c)}$$

The state equation is:

$$\begin{pmatrix} \dot{x}_l \\ \dot{x}_c \end{pmatrix} = \begin{pmatrix} 0 & -\frac{\rho}{C} \\ \frac{\rho}{L} & 0 \end{pmatrix} \begin{pmatrix} x_l \\ x_c \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e + \begin{pmatrix} 0 \\ -1 \end{pmatrix} I_l \quad \text{(25)}$$

The set point is defined by:

$$(v_{c0}, i_{l0}) = \left(\frac{e}{\rho_0}, \frac{e}{\rho_0^2 R_L} \right), \quad \text{(26)}$$

while the equilibrium point of the transmission line is given by:

$$(V_0, I_0) = \left(v_{c0}, \frac{v_{c0}}{R_L} \right). \quad \text{(27)}$$

The proposed Lyapunov function is:

$$H = \frac{1}{2} \frac{(x_l - x_{l0})^2}{L} + \frac{1}{2} \frac{(x_c - x_{c0})^2}{C} + \frac{1}{2} \int_0^1 \left[\frac{(Q - Q_0)^2}{C_l} + \frac{(\varphi - \varphi_0)^2}{L_l} \right] dq \quad \text{(28)}$$

And its derivative:

$$\dot{H} = [(v_c - v_{c0}) i_{l0} - (i_l - i_{l0}) v_{c0}] (\rho - \rho_0) - (V(t,1) - V_0)^2 / R_L \quad \text{(29)}$$

In the simulation, normalized values have been used ($e = 1$V, $R_L = 1\Omega$, $L = 1$H, $C = 1$F). First the output voltage
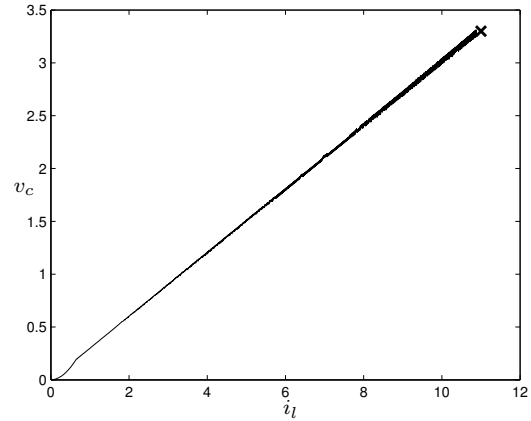

Fig. 3. The state evolution, from the origin to the equilibrium point, using a maximum descent strategy, where the reference point ($i_{l0} = 11.11$A, $v_{c0} = 3.33$V) is indicated by the **x**
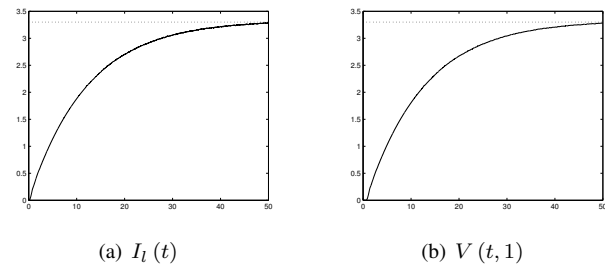

(a) $I_l(t)$ (b) $V(t,1)$

Fig. 4. The time evolution of the line input current, $I_l(t) = I(t,0)$, and of the load voltage drop, $V(t,1)$, when a maximum descent strategy is employed. The dotted lines represent the corresponding admissible reference points

is specified $v_{c0} = 3.33$V. Then $\rho_0 = 0.3$ and $i_{l0} = 11.11$A. The line has been modeled using a ladder representation with five cells, where the numerical values of the storage elements used in the cell model are 0.1H for the inductance and 0.1F for the capacitor.

The simulations were realized using the two control strategies: the maximum descent and the minimum switching, with the origin used each time as the initial value for the state vector. In figure 3 the (co-)state evolution is presented for the maximum descent control strategy. This strategy ensures that the derivative of the Lyapunov function is always negative by keeping negative the term $[(v_c - v_{c0}) i_{l0} - (i_l - i_{l0}) v_{c0}] (\rho - \rho_0)$, which leads to a sliding mode. In figure 4 are presented the time evolutions of the line input current and of the load voltage drop when such a strategy is applied.

In figure 5 the (co-)state evolution is presented for a minimum switching control strategy. This strategy takes the decision of changing mode only when the Lyapunov function derivative is becoming zero. In figure 6 are presented the time evolutions of the line input current and of the load voltage drop when such a strategy is applied. It can be noticed that, even though there is overshoot, the system converges faster than when the maximum descent strategy is used.
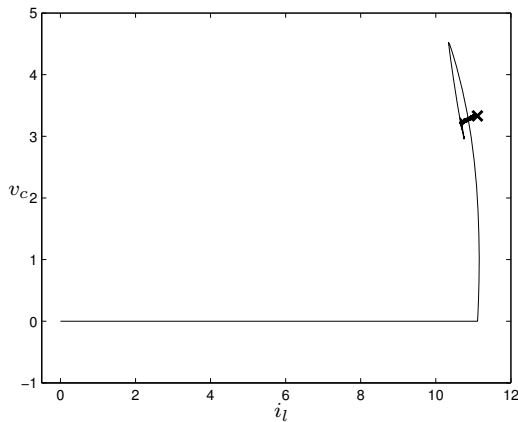
Fig. 5. The state evolution, from the origin to the equilibrium point, using a minimum switching strategy, where the reference point $(i_{l0} = 11.11A, v_{c0} = 3.33V)$ is indicated by the **x**
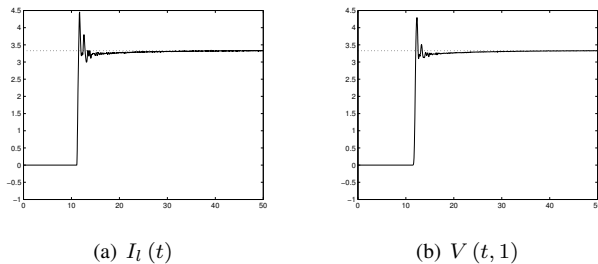

(a) $I_l(t)$ (b) $V(t, 1)$

Fig. 6. The time evolution of the line input current, $I_l(t) = I(t, 0)$, and of the load voltage drop, $V(t, 1)$, when a minimum switching strategy is employed. The dotted lines represent the corresponding admissible reference points

V. CONCLUSIONS

This paper presented a hybrid stabilizing control law that brings the system to an admissible reference point. To achieve this, a Lyapunov function has been proposed for the case when the load is connected to the power converter through a transmission line. This Lyapunov function has been deduced to be, using energy considerations, the sum of the candidate Lyapunov functions for the power converter part and for the transmission line. Moreover, the admissible reference point is the same for the power converter part either the line is present or not. Such a control law has then been applied to the "boost" converter, with two strategies outlined: the maximum descent, where the derivative of the Lyapunov function has been minimized, and the minimum commutation, where the commutation decision has been taken only when the derivative of the Lyapunov function became equal to zero. The second strategy proved to produce faster tracking performances, but exhibits overshoot. Future works will be concerned with the lossy line case, where the admissible reference point changes and the equilibrium voltage and current of the line are no longer constant with the spatial variable. Further more extensions to the case of nonlinear storage elements are planned as well as including

performances constraints that should lead to intermediate switching strategies regarding the maximum descent and the minimum switching strategies.

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