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A Saturated Output Feedback Controller for the Three Phase Voltage Sourced Reversible Boost Type Rectifier

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Abstract – In this article, we present a saturating controller to regulate the output voltage load in a three phase Boost type rectifier. The controller only needs the output voltage signal to be implemented. Moreover, by forcing the inductance currents to track desired suitable sinusoidal signals in phase with the input source voltages, the controller can ensure a near the unity power factor functioning of the system. The saturation feature in this controller is motivated by the fact that the allowed combinations of switches in the rectifier circuit define a finite set of *allowed control vectors*, moreover, the convex combinations of these vectors raise up a convex hull in the space of control inputs, an hexagon in this example. Thus, based upon the assumption of fast switching, we can ensure that any control vector lying inside this hexagon, and much more, inside its inscrit circle, can be physically implementable by means of a PWM strategy or a convex combination of the *allowed control vectors*.

I. Introduction

In many processes the controllers are restricted to live in finite sets. This is the case in most of the circuits in the field of power electronics, where the control input is normally a binary signal to be introduced in the gate of a thyristor acting as a switch. Classical examples of these systems are the DC–DC converters, AC–DC converters better known as rectifiers, DC–AC converters more frequently referred as inverters, among others.

In this paper we present a saturated controlled that preserves stability of the three phase boost type rectifier. The finite number of allowed switch positions in the circuit, yields a finite set of *allowed control vectors*, which define a convex hull in the space of the control inputs. Thus, assuming the switches can conmute fast enough, we can say that the only implementable control vectors are those who are bounded by the boundaries of this convex hull, for instance they can be implemented via a PWM strategy or by means of a convex combination of the allowed control vectors. Better to consider the convex hull boundaries to propose a bound for a saturated control, we consider the

inscrit circle to the convex hull, thus yielding an easier condition to analize since only its radium is considered. Furthermore, by defining a reference signal tracking problem on the input currents of the converter, the power factor can be made very close to unity as long as the tracked signal is in phase with the rectified input voltages. The load voltage DC component is selected acordind to a given condition on the parameters, which in fact, implies that at least in the steady state, the control vector lies inside the inscrit circle. The required reference signal amplitude for the input currents is determined via a steady state analysis and a partial system inversion.

This work was motivated by the seminal paper [4] where the problem of output voltage regulation in the three phase rectifier is formulated and some guidelines are given to implement a controller based on the definition of subspaces in the control input space. Later in [3] they presented an approach based on the concept of sliding modes and the computation of the equivalent control.

The rest of the paper is organized as follows: In section 2 the formulation of the problem is given. The model of the rectifier is studied in detail, the $\alpha\beta$-transformation is used in order to express the model in a reduced frame, and finally the control objective is presented. Section 3 is devoted to the controller design. In section 4 the main result of the paper is presented. The proposed controller is analyzed and stability proofs are given. In section 5 simulations results are provided for assessing the performance of the proposed saturated control law. Finally, in section 6, we give some conclusions and further research.

II. Problem formulation

A. Mathematical model

Consider the three phase Boost type rectifier shown in Figure 1. This circuit is composed in its main part by a bridge of three legs, where each leg contains two switches connected in cascade. We will refer to this switches as the upper switch and the lower switch. In the special case of the rectifier, the switches in each leg work in a complementary way, that is, when one of them is connected, i.e., in the ON position, the other is disconnected, i.e., in the OFF position. For this reason, we assign to each leg only one control input, namely δ_1, δ_2 and δ_3 taking only two values each one. We

will assign the value -1 in the case the upper switch in the OFF position and the lower switch in the ON position, and +1 in the other case ($\delta_i = -\bar{\delta}_i$, $i = 1, 2, 3$).

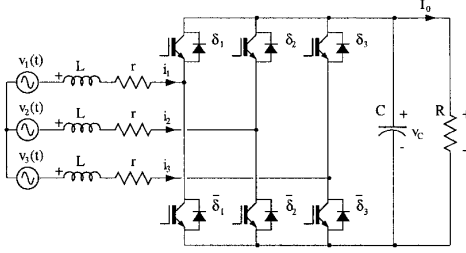The permissible combinations of the switches connections are given in table 1, all they form eight possible choices.

Fig. 1: Circuit schematic of a three phase Boost type rectifier

Vector	δ_1	δ_2	δ_3
U_0	-1	-1	-1
U_1	1	-1	-1
U_2	1	1	-1
U_3	-1	1	-1
U_4	-1	1	1
U_5	-1	-1	1
U_6	-1	-1	1
U_7	1	1	1

Table 1: Permitted switch positions

A state space model of the system is given by,

$$\begin{bmatrix} L\frac{d}{dt}i_1 \\ L\frac{d}{dt}i_2 \\ L\frac{d}{dt}i_3 \end{bmatrix} = -\begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$- \frac{1}{2}v_C \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \end{bmatrix}\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} \quad (2.1)$$

$$C\frac{d}{dt}v_C = \frac{1}{2}\begin{bmatrix} \delta_1 & \delta_2 & \delta_3 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} - I_0 \quad (2.2)$$

where $i = [i_1, i_2, i_3]^T$ is the vector of line currents, or the inductance currents; v_C is the output capacitor voltage; $[v_1, v_2, v_3]^T$ is the vector of the source line voltages; I_0 is the output load current; $\delta = [\delta_1, \delta_2, delta_3]^T$ is the vector of control inputs which represents the switch position and takes values in the discrete set -1, 1; L is the inductance filter at the line source inputs, r its parasitic inductance resistance and C is the output capacitor.

Assume that the system is balanced, i.e., the voltage and current source signals have the same amplitudes but are displaced $\frac{2}{3}\pi$ rad one with respect to the other and the passive elements on each line have the same values. So, voltages and currents fulfill the following equations,

$$v_1 + v_2 + v_3 = 0 \quad (2.3)$$
$$i_1 + i_2 + i_3 = 0 \quad (2.4)$$

Also assume that the vector of source line voltages is composed by purely sinusoidal signals, i.e.,

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} V\cos(wt) \\ V\cos(wt - \frac{2}{3}\pi) \\ V\cos(wt - \frac{4}{3}\pi) \end{bmatrix}$$

where V is the amplitude of the voltage signal in [Volts] and w its frequency in [rad/sec].

For our purpose we will consider that the output current load is due only to a purely resistive element, i.e., $I_0 = v_C/R$, where R represents the load resistance.

B. Coordinate Transformation. $\alpha\beta$-Model

Consider the commonly called *Blondel-Park*-transformation or 3/2-transformation given by,

$$\mathcal{P} = \sqrt{\frac{2}{3}}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \quad (2.5)$$

We apply the above transformation to the model (2.2) in the following way,

$$x = \mathcal{P}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}, \quad v = \mathcal{P}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}, \quad u = \mathcal{P}\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix}$$

$x = \begin{bmatrix} x_1, & x_2 \end{bmatrix}^T$, $v = \begin{bmatrix} v_1, & v_2 \end{bmatrix}^T$ and $u = \begin{bmatrix} u_1, & u_2 \end{bmatrix}$.

This transformation toguether with the variable change $y = v_C$, yields the following model which is now expressed in a coordinate set referred to a fixed frame,

$$\begin{bmatrix} L\frac{d}{dt}x_1 \\ L\frac{d}{dt}x_2 \end{bmatrix} = -\begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - \frac{y}{2}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$C\frac{d}{dt}y = \frac{1}{2}\begin{bmatrix} u_1 & u_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - I_0 \quad (2.6)$$

Notice that in order to arrive to this model the following fact has been used,

$$\delta^T i = u^T x$$

which stems from the fact that transformation (2.5) is power conserving.

For our purpose we will consider that the output current load is due only to a purely resistive element, i.e., $I_0 = \frac{y}{R}$, where R represents the load resistance. This yields the model,

$$\begin{bmatrix} L\frac{d}{dt}x_1 \\ L\frac{d}{dt}x_2 \end{bmatrix} = -\begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - \frac{y}{2}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$
$$(2.7)$$

$$C\frac{d}{dt}y = \frac{1}{2}\begin{bmatrix} u_1 & u_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \frac{y}{R} \quad (2.8)$$

Line source voltages $[v_1, v_2]^T$ take now the values,

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = E\begin{bmatrix} \cos(wt) \\ \sin(wt) \end{bmatrix}$$

or in matrix form,

$$v = e^{\mathcal{J}wt} \begin{bmatrix} E \\ 0 \end{bmatrix}, \quad e^{\mathcal{J}wt} \triangleq \begin{bmatrix} \cos(wt) & -\sin(wt) \\ \sin(wt) & \cos(wt) \end{bmatrix}$$

where we have defined $E = \sqrt{\frac{3}{2}}V$ by simplicity.

Table 2 resumes the transformation of the control input vectors. The vectors in the new frame, i.e., $u = [u_1, \ u_2]^T$, form the commonly called *input space* which in this specific problem can be drawn on a plane as shown in figure 2. We will refer to this vectors as the *allowed control vectors*, they form a convex hull, an hexagon in this special case. Moreover, inscrit to this hexagon, there is an inscrit circle with radium equal to $\sqrt{2}$.

No.	δ_1	δ_2	δ_3	u_1	u_2
U_0	-1	-1	-1	0	0
U_1	1	-1	-1	$2\sqrt{\frac{2}{3}}$	0
U_2	1	1	-1	$\sqrt{\frac{2}{3}}$	$\sqrt{2}$
U_3	-1	1	-1	$-\sqrt{\frac{2}{3}}$	$\sqrt{2}$
U_4	-1	1	1	$-2\sqrt{\frac{2}{3}}$	0
U_5	-1	-1	1	$-\sqrt{\frac{2}{3}}$	$-\sqrt{2}$
U_6	-1	-1	1	$\sqrt{\frac{2}{3}}$	$-\sqrt{2}$
U_7	1	1	1	0	0

Table 2: Permitted switch positions in the $\alpha\beta$-model

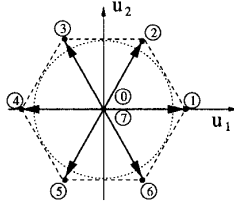Fig. 2: Input space

C. Control Objective

The control objective is to propose a control vector $u(t)$ of the system modeled by equations (2.7,2.8), whose goal is twofold. First, to drive the output capacitor voltage to a desired constant value, i.e., $y(t) \to V_d$ as $t \to \infty$, where V_d is a constant desired output voltage. Second, the control input vector $u(t)$, should be designed in such a way that the inductance currents $x(t) \to x_d(t)$ as $t \to \infty$ where $x_d(t) \in \mathcal{C}^1$ is a time varying vector of desired currents with constant amplitude I_d yet to be defined. This vector is turning counterclockwise at a frequency w and in phase with the source line voltage vector $v = [v_1, \ v_2]$, in order to ensure a near the unity power factor functioning.

The desired currents vector $x_d = [x_{1d}, \ x_{2d}]^T$ is described by,

$$x_d = e^{\mathcal{J}wt} \begin{bmatrix} I_d \\ 0 \end{bmatrix} \qquad (2.9)$$

or more explicitely,

$$x_{1d} = I_d \cos(wt), \quad x_{2d} = I_d \sin(wt) \qquad (2.10)$$

where I_d is a constant value yet to be determined.

Moreover, due to the fact that the only implementable control vectors are those contained in the hexagon (see Fig. 2), then the amplitude of the computed control vector should be restricted, and for easy of analysis we will consider the radium of the inscrit circle, i.e., $||u(t)|| \leq \sqrt{2}$.

III. Controller design

Consider that the desired currents given by (2.9) fulfill the currents equation (2.7), that is,

$$L\dot{x}_d = -rx_d + v - \frac{y}{2}u^* \qquad (3.1)$$

where according to eq. (2.9),

$$\dot{x}_d = w\mathcal{J}e^{\mathcal{J}wt} \begin{bmatrix} I_d \\ 0 \end{bmatrix} = w\mathcal{J}x_d \qquad (3.2)$$

Solving from the last expression for u, we obtain,

$$u^* = \frac{2}{y}(v - rx_d - wL\mathcal{J}x_d) \qquad (3.3)$$

being u^* a control signal which needs still to be saturated.

Notice that this expression is divided by y. This will make our result local since we should restrict the output voltage to be $y > c$, being c a constant strictly bigger than zero.

Notice that u^* can be written as a rotating vector in the following way,

$$u^* = \frac{2}{y}(v - rx_d - wL\mathcal{J}x_d) \qquad (3.4)$$

$$= \frac{2}{y}e^{\mathcal{J}wt} \begin{bmatrix} E - rI_d \\ wLI_d \end{bmatrix} \qquad (3.5)$$

and finally,

$$u^* = \frac{2}{y}e^{\mathcal{J}(wt-\phi)} \begin{bmatrix} a \\ 0 \end{bmatrix} \qquad (3.6)$$

where,

$$a = \sqrt{(E - rI_d)^2 + w^2L^2I_d^2}, \quad \phi = \operatorname{atan}(\frac{wLI_d}{E - rI_d}) \quad (3.7)$$

$$\cos(\phi) = \frac{E - rI_d}{a}, \quad \sin(\frac{wLI_d}{a}) \qquad (3.8)$$

In order to ensure that the controller is contained in the inscrit circle, we propose to saturate only the amplitude of the control vector (3.6) and to maintain the angle. Thus, the resulting controller to be applied takes the form,

$$u = \begin{cases} \frac{2a}{y}e^{\mathcal{J}(wt-\phi)}\epsilon_1, & ||u^*|| \leq \sqrt{2} \\ \sqrt{2}e^{\mathcal{J}(wt-\phi)}\epsilon_1, & ||u^*|| > \sqrt{2} \end{cases} \qquad (3.9)$$

where $\epsilon_1 = [1, \ 0]^T$ is the unitary canonical vector.

Now we analize the the steady state response of the system in order to compute I_d, that is, eq. (2.8) is evaluated for

$x = \overline{x}_= x_d$, $y = \overline{y}_= V_d$ and $u(x,y,t) = u(\overline{x}, \overline{y}, t) = u(x_d, V_d, t)$. This yields,

$$
\begin{align}
0 &= \frac{1}{2}\frac{2}{V_d}[1, \ 0]e^{-\mathcal{J}(wt-\phi)}e^{\mathcal{J}wt}\begin{bmatrix} I_d \\ 0 \end{bmatrix} - \frac{V_d}{R} \quad (3.10) \\
&= \frac{I_d(E - rI_d)}{V_d} - \frac{V_d}{R} \quad (3.11)
\end{align}
$$

The last equation has two solutions for I_d, nevertheless, only one of them is physically meaningfull, and is given by,

$$
I_d = \frac{E}{2r} - \sqrt{\frac{E^2}{4r^2} - \frac{V_d^2}{rR}} \quad (3.12)
$$

With the results above we can state our main result.

IV. Main result

Proposition IV.1 *Consider the system described by eqs. (2.7,2.8) in closed loop with the saturated controller (3.9). Then the output voltage y locally asymptotically reaches the desired constant value V_d which is chosen such that the following condition is satisfied,*

$$
V_d^2 > 2\left(1 + \frac{r^2 + 0.5r}{w^2 L^2}\right)(E^2 + w^2 L^2 I_d^2) \quad (4.1)
$$

where I_d is computed according to (3.12)

Moreover, the inductance currents vector $x = [x_1, \ x_2]^T$ locally asymptotically tracks the rotating desired currents vector $x_d = [x_{d1}, \ x_{d2}]$ given by (2.9), thus ensuring a near the unity power factor functioning.

A. Proof:

The proof is divided into two parts, first we consider the case when the control vector $||u^*|| \leq \sqrt{2}$, and thus we chose $u = u^*$. Second, we analize the case $||u^*|| > \sqrt{2}$, so u is chosen as the vector in the same direction as vector u^* but with amplitude $\sqrt{2}$.

Case 1:

In this case, the controller is given by (3.3),

$$
u = u^* = \frac{2}{y}(v - rx_d - wL\mathcal{J}x_d)
$$

This control in closed loop with the system (2.7,2.8), yields the following error dynamics,

$$
\begin{align}
\dot{\tilde{x}} &= -\frac{r}{L}\tilde{x} \quad (4.2) \\
C\dot{y} &= \frac{1}{y}(v^T - rx_d^T + wLx_d^T J)x - \frac{y}{R} \quad (4.3)
\end{align}
$$

where $\tilde{x} \triangleq x - x_d$.

Let's define $z = \frac{1}{2}y^2$, then $\dot{z} = y\dot{y}$, thus the second eq. in the error dynamics become,

$$
C\dot{z} = v^T\tilde{x} + v^Tx_d - rx_d^T\tilde{x} - rI_d^2 + wLx_d^TJ\tilde{x} - \frac{2z}{R} \quad (4.4)
$$

where we used the fact that $v^Tx_d = EI_d$.

Analizing the equilibrium point in the error dynamics,

$$
\begin{align}
0 &= \overline{\tilde{x}} \quad (4.5) \\
0 &= EI_d - rI_d^2 - \frac{2\overline{z}}{R} \quad (4.6)
\end{align}
$$

From the first equation, $\overline{\tilde{x}} = 0$, and from the second equation with $\overline{z} = \frac{V_d^2}{2}$,

$$
0 = EI_d - rI_d^2 - \frac{V_d^2}{R} \quad (4.7)
$$

which coincides with (3.11).

Thus, by defining $\tilde{z} \triangleq z - \overline{z}$, the error dynamics can be written finally as,

$$
\begin{align}
\dot{\tilde{x}} &= -\frac{r}{L}\tilde{x} \quad (4.8) \\
C\dot{\tilde{z}} &= v^T\tilde{x} - rx_d^T\tilde{x} + wLx_d^TJ\tilde{x} - \frac{2\tilde{z}}{R} \quad (4.9)
\end{align}
$$

whose equilibrium point is the origin.

Consider now the following change of coordinates,

$$
\begin{align}
\psi_1 &= x_d^T\tilde{x} \quad (4.10) \\
\psi_2 &= x_d^TJ\tilde{x} \quad (4.11)
\end{align}
$$

The error dynamics is then transformed to,

$$
\begin{align}
\dot{\psi}_1 &= -\frac{r}{L}\psi_1 - w\psi_2 \quad (4.12) \\
\dot{\psi}_2 &= -\frac{r}{L}\psi_2 + w\psi_1 \quad (4.13) \\
\dot{\tilde{z}} &= \frac{E}{CI_d}\psi_1 - \frac{r}{C}\psi_1 + \frac{wL}{C}\psi_2 - \frac{2}{RC}\tilde{z} \quad (4.14)
\end{align}
$$

which can be written in matrix form as follows,

$$
\begin{bmatrix} \dot{\psi}_1 \\ \dot{\psi}_2 \\ \dot{\tilde{z}} \end{bmatrix} = \begin{bmatrix} -\frac{r}{L} & -w & 0 \\ w & -\frac{r}{L} & 0 \\ \frac{E - rI_d}{CI_d} & \frac{wL}{C} & -\frac{2}{RC} \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \\ \tilde{z} \end{bmatrix} \quad (4.15)
$$

which is a stable system.

Case 2:

In this case $||u^*|| > \sqrt{2}$, then the controller is selected according to (3.9) to be,

$$
u = \sqrt{2}e^{\mathcal{J}(wt-\phi)}\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \phi = \text{atan}\left(\frac{wLI_d}{E - rI_d}\right) \quad (4.16)
$$

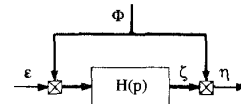For this part of the proof we will require the following lemma.

Fig. 3: Block diagram of system considered in the lemma

Lemma IV.2 *Consider the system (see Fig. 3) described by the following dynamics,*

$$\zeta = H(p)\Phi\varepsilon \tag{4.17}$$

$$\eta = \Phi^T\zeta \tag{4.18}$$

where ε, $\eta \in I\!\!R$; ζ, $\Phi \in I\!\!R^2$; $H(p) \in I\!\!R^{2\times2}(p)$.
  Assume,

$$\Phi = \left[\begin{array}{c} \gamma\sin(wt - \theta) \\ \gamma\cos(wt - \theta) \end{array} \right] \tag{4.19}$$

Then the system can be expressed as the linear system,

$$\eta = \Sigma(p)\varepsilon \tag{4.20}$$

where the new tranfer function $\Sigma(s)$ is defined as,

$$\Sigma(s) = \frac{\gamma^2}{2}\left[H(s - jw) + H(s + jw)\right] \tag{4.21}$$

□□□

The system (2.7,2.8) can be represented by the block diagram of Fig. 4 after some simple manipulations
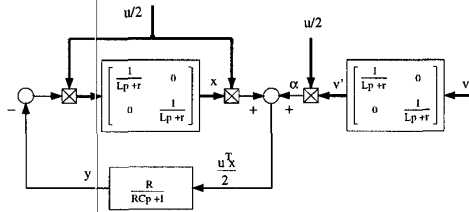

Fig. 4: Modified block diagram of the system

In the block diagram in Fig. 4 we can identify an structure similar to that of the lemma IV.2, that is, a linear block being pre and post multiplied by a rotating vector. Thus, by applying the result in this lemma, the whole system can be now expressed as an interconnection of two linear systems as shown in Fig. 5, having by input a signal α yet to be determined.
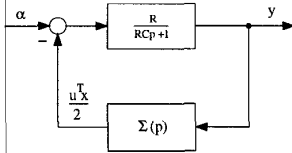

Fig. 5: Reduction of the original block diagram to a feedback linear system

In this case,

$$\Phi = \frac{u}{2}, \quad \gamma = \frac{1}{\sqrt{2}}, \quad H(s) = \text{diag}\left\{\frac{1}{Ls + r}, \frac{1}{Ls + r}\right\} \tag{4.22}$$

Applying the result in lemma IV.2 we obtain,

$$\Sigma(s) = \frac{1}{2}\left(\frac{Ls + r}{L^2s^2 + 2Lrs + w^2L^2 + r^2}\right) \tag{4.23}$$

The closed loop transfer function of the linear system shown in Fig. 5 is given by,

$$G(s) = \frac{L^2s^2 + 2Lrs + w^2L^2 + r^2}{\left(Cs + \frac{1}{R}\right)\left(L^2s^2 + 2Lrs + \frac{L}{2}s + w^2L^2 + r^2 + \frac{r}{2}\right)} \tag{4.24}$$

which is a stable system.

In order to compute the steady state value of the output of this linear system, we should compute before the steady state value of the signal α, which is actually a bounded signal that converges to a constant value as we will see in what follows.

From block diagram in Fig. 4, we see that vector v' is obtained from the low pass filtering of the rotating vector v, i.e.,

$$v' = \frac{1}{Lp + r}e^{\mathcal{J}wt}\left[\begin{array}{c} E \\ 0 \end{array} \right] \tag{4.25}$$

$$\tag{4.26}$$

This gives the following vector, which has been obtained by using the phasor concepts.

$$v'_{ss} = \frac{E}{b_1}e^{\mathcal{J}(wt - \beta_1)} \tag{4.27}$$

where

$$b_1 = \sqrt{r^2 + w^2L^2}, \quad \beta_1 = \text{atan}(\frac{wL}{r}) \tag{4.28}$$

$$\cos(\beta_1) = \frac{r}{b_1}, \quad \sin(\beta_1) = \frac{wL}{b_1} \tag{4.29}$$

Thus, α in the steady state is, given by,

$$\alpha_{ss} = \frac{u^Tv'_{ss}}{2} = \frac{E}{\sqrt{2}b_1}\cos(\phi - \beta_1) \tag{4.30}$$

where $\cos(\phi - \beta_1) = \frac{(E - rId)r + w^2L^2I_d}{ab_1}$. So α_{ss} can be written also as,

$$\alpha_{ss} = \frac{E}{\sqrt{2}ab_1^2}\left[(E - rId)r + w^2L^2I_d\right] \tag{4.31}$$

We can now compute the steady state value for the system output y as follows,

$$y_{ss} = \lim_{s\to0} sG(s)\frac{\alpha_{ss}}{s} \tag{4.32}$$

$$= \frac{RE}{\sqrt{2}a(b_1^2 + \frac{r}{2})}\left[(E - rId)r + w^2L^2I_d\right] \tag{4.33}$$

which can be also written as,

$$y_{ss} = \frac{V_d^2w^2L^2}{\sqrt{2}a(b_1^2 + \frac{r}{2})} + \frac{REr}{\sqrt{2}a(b_1^2 + \frac{r}{2})}\left(E - rId + \frac{w^2L^2I_d^2}{E}\right) \tag{4.34}$$

As shown in eq. (3.6)

$$u^* = \frac{2}{y}e^{\mathcal{J}(wt - \phi)}\left[\begin{array}{c} a \\ 0 \end{array} \right] \tag{4.35}$$

A condition for this control vector to enter into the circle of radium $\sqrt{2}$ is that in the steady state,

$$||u^*_{ss}|| < \sqrt{2} \tag{4.36}$$

That is,

$$y_{ss} > \sqrt{2}a \tag{4.37}$$

An upper bound for a from (3.7) is,

$$a < \sqrt{E^2 + w^2 L^2 I_d^2} \tag{4.38}$$

while a lower bound for y_{ss} from (4.34) is,

$$y_{ss} > \frac{V_d^2 w^2 L^2}{\sqrt{2}\sqrt{E^2 + w^2 L^2 I_d^2}(b_1^2 + \frac{r}{2})} \tag{4.39}$$

Thus, the condition (4.37) is implied by the following condition,

$$\frac{V_d^2 w^2 L^2}{\sqrt{2}\sqrt{E^2 + w^2 L^2 I_d^2}(b_1^2 + \frac{r}{2})} > \sqrt{2}\sqrt{E^2 + w^2 L^2 I_d^2} \tag{4.40}$$

From where (4.1) is easily obtained.

□□□

Remark: Condition (4.1) reveals the amplification characteristic of the Boost rectifier.

V. Simulation results

The control strategy discused before was applied to the rectifier system. Moreover, we propose a set of initial conditions that exhibit the saturating feature of the proposed controller.

Using the software package SIMULINK -and the numerical values shown below-, digital computer simulations were performed for evaluating the performance of the nonlinear output feedback controller strategy. The simulations were carried out using the actual nonlinear system model (2.7, 2.8) with initial conditions for the currents vector $x(0) = [0, 0]$ and for the output capacitor voltage $y(0) = 150 Volts$. This initial conditions put in evidence the saturating feature of the controller proposed.

$$L = 10\mu H, \quad \omega_0 = 2\pi 60 \text{rad/sec}, \quad C = 1mF, \quad r = .25\Omega$$
$$V = 110V, \quad R = 25\Omega, \quad V_d = 325V$$

The first diagram in Fig. 6 shows the phase plot of the inductance currents $x_2(x_1)$, we can observe that the currents vector reaches the desired currents vector asymptotically. Second diagram in the same figure shows the phase plot of the controls $u_2(u_1)$, notice that in the transient period the control vector trajectory u^* in dotted line, can take values that go outside the inscrit circle, thus enabling the saturating feature of the controller proposed. Finally, in the third diagram, the time response of the output capacitor voltage y is presented. This signal reaches the desired final value $V_d = 325$ Volts in the steady state. Moreover, the non-minimum phase behaviour of the system response can be observed as well.
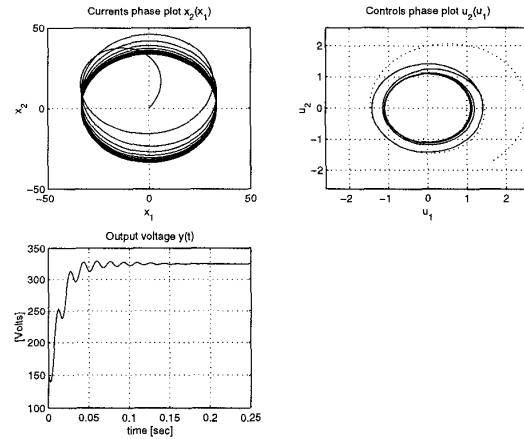

Fig. 6: Response of the system under the proposed controller

VI. Conclusions

In this article we have proposed an output feedback saturated controller for the three phase Boost type rectifier which ensures a near the unity power factor functioning. The satutarion feature of the controller is motivated by the fact that the convex combinations of the *allowed control vectors* raise up a convex hull in the space of control inputs, an hexagon in this example. Thus, based upon the assumption of fast switching, we can ensure that any control vector lying inside the inscrit circle to this convex hull, can be physically implementable by means of a PWM strategy or a convex combination of the *allowed control vectors* which is commonly called *baricentric method*. For the stability proof, we have shown that whenever the control vector u^* lies outside the inscrit circle, by applying the saturated one u, the amplitude of the control vector u^* in steady state is reduced so it enters into the inscrit circle, provided a condition among the parameters and the desired values is guaranteed.

REFERENCES

[1] ELBULUK M. E., G. C. VERGHESE AND D. E. CAMERON, "Nonlinear Control of Switching Power Converters" *Cont. Theo. and Adv. Tech.*, Vol. 5, No. 4, pp. 601-617. 1989

[2] H. NIJMEIJER AND A.J. VAN DER SCHAFT **Nonlinear Dynamical Control Systems**, Springer-Verlag, NY, 1990.

[3] SABANOVIC-BEHLILOVIC N., A. SABANOVIC AND T. NINOMIYA, "PWM in Three-Phase Switching Converters - Sliding Mode Solution. *Proc. of the IEEE PESC'94*, pp. 560-565, 1994.

[4] A. SABANOVIC, N. SABANOVIC AND K. OHNISHI "Sliding modes in power converters and motion control systems", *Int. J. Control*, Vol. 57, No. 5 pp. 1237-1259, 1993.

[5] SIRA-RAMÍREZ, H., G. ESCOBAR AND R. ORTEGA, "On passivity-based sliding mode control of switched dc–to–dc power converters", *35th IEEE CDC*, Kobe, Japan, Dec. 13-15 1996.