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# Risk, Cohabitation and Marriage.

Padma Rao Sahib\* and Xinhua Gu\*\*

SOM-theme D Demographic and geographic environment.

## Abstract

This paper introduces imperfect information, learning, and risk aversion in a two sided matching model. The model provides a theoretical framework for the commonly occurring phenomenon of cohabitation followed by marriage, and is consistent with empirical findings on these institutions. The paper has three major results. First, individuals set higher standards for marriage than for cohabitation. When the true worth of a cohabiting partner is revealed, some cohabiting unions are converted into marriage while others are not. Second, individuals cohabit within classes. Third, the premium that compensates individuals for the higher risk involved in marriage over a cohabiting partnership is derived. This premium can be decomposed into two parts. The first part is a function of the individual's level of risk aversion, while the second part is a function of the difference in risk between marriage and cohabitation.

*Keywords:* imperfect information, risk, class partition, stochastic dominance.

*JEL classification:* D83; D84; J12.

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## 1. Introduction

There are not many among the married who can claim that their spouses have remained the same as they were on their wedding day. Individuals marry and subsequently discover new characteristics in their spouses. Some of these characteristics may come as pleasant surprises; finding out that your spouse is an excellent cook is one example. However, it is evident from the high rate of divorce in many countries that many surprises are not as pleasant.

Because divorce can be psychologically painful and costly, some researchers hypothesize that couples may choose to cohabit instead of marrying. Whether cohabitation is a result of increased divorce rates is unclear; Waters and Ressler (1999) study state level data on cohabitation and divorce and conclude that the causality between divorce and cohabitation runs in both directions. What is clear however, is that cohabitation as a life style has gained in popularity in recent decades. This is particularly the case in North America and Europe. For example, between the mid 1970s and the late 1980s the percentage of women aged 20-24 who were in cohabiting unions rose from 11% to 49% in France (Kiernan 1996).

Although prevalent, cohabitation is short-lived. It tends to precede marriage rather than replace it. Bumpass and Sweet (1989) use data from the United States and find that 40% of all cohabiting couples either marry or stop living together within a year, and only a third of cohabiting couples are still cohabiting after two years. They also find that 60% of those who marry after living in a cohabiting union, marry their cohabiting partners. This is corroborated by more recent data from the United Kingdom; Ermisch and Franscesconi (1999) also find that more than half of first cohabiting unions are converted into marriage. The most frequently cited reason for unmarried individuals living together is to assess compatibility before marriage (Bumpass et al. 1991). This indicates that for many couples, cohabitation serves as a probationary period.

The goal of this paper is to develop a theoretical model consistent with the observed facts on cohabitation. As is evident from empirical findings, cohabitation is often transitory and is often viewed as a kind of trial marriage. It appears to involve less commitment as cohabiting unions are more likely to dissolve than marriages.

The model developed in this paper adopts the view that couples cohabit because they want to evaluate one another as potential spouses. They find out each others suitability as spouses while living together. Upon finding this out, they either marry or separate. In addition, this study introduces imperfect information, learning and relaxes the assumption of risk neutrality; a new feature in the growing literature on two-sided matching models (see Burdett and Coles 1999 for a survey). It extends Rao Sahib and Gu (2000), in which premarital cohabitation and subsequent marriage are investigated in a risk neutral setting.

The remainder of the paper is organized as follows. Section 2 discusses the model. Section 3 describes the decision making of couples. The role of risk in cohabiting and marital unions is discussed in section 4. Section 5 shows that cohabiting unions occur within classes and section 6 concludes the paper.

## 2. Search in the marriage market

### 2.1. The modelling framework

This subsection describes the modelling framework. It is a steady state matching model with nontransferable utility and heterogeneous agents as developed in Burdett and Coles (1997), hereafter simply BC. As our model is an extension of this model and we retain some of the terminology used, we begin by first describing the BC model. We then describe how our model differs from the BC model.

A model of a decentralized marriage market in which positive assortative mating arises as an equilibrium outcome is developed in BC. The positive assortative mating found in BC is in terms of a comprehensive index. This index, termed *pizazz*, captures the worth of an individual as a marriage partner. If a couple marry, they each receive as utility the *pizazz* of the other. Equilibrium is characterized by a class partition. Men and women form classes with marriages occurring only among members of the same class. There exists an ordering among classes; men and women with high *pizazz* marry one another while men and women with low *pizazz* marry one another. The type of assortative mating found in BC is somewhat different from the usual notion of assortative mating found in sociological and biosocial studies. In the latter, positive assortative mating refers to a positive correlation between the traits of couples, for example, a positive correlation in age, education or height. Not all two-sided search models yield positive assortative mating as an equilibrium outcome; see for example, Shimer and Smith (2000).

Narcissism is ruled out in the BC model. Having high *pizazz* is useful only because it allows one to attract opposite sex individuals who also have high *pizazz*, and increases the utility from marriage. The optimal policy for a single individual to follow is to marry the first single encountered of the opposite sex whose *pizazz* is above a certain reservation level. The steady state is characterized by a constant distribution of *pizazz* among the pool of single individuals and a balanced flow of market exits and entries.

In this paper, we deviate from the BC framework in four ways. First, BC assume that an individual's *pizazz* can be observed on contact. In their model, individuals who find a suitable spouse, marry and leave the marriage market forever. In contrast, we assume that single individuals can observe the true *pizazz* of a potential partner only imperfectly (we retain however, the term *pizazz* to mean the worth of an individual as a marriage partner). This provides a motivation for

cohabiting unions which are initiated by single men and women in an attempt to learn each other's true worth as marriage partners. Of course, cohabitation is just one interpretation of this information collecting period; it can also be called courtship or dating. Couples continue to cohabit until true pizazz is revealed, and at that point, they either marry or separate<sup>1</sup>.

Second, in BC, individuals leave the marriage market through marriage or death, and there is an exogenous in-flow of new singles into the market. In our model, however, cohabiting unions sometimes dissolve resulting in singles re-entering the marriage market.

Third, it is assumed in BC that individuals are risk neutral. We relax the assumption of risk neutrality and consider the case when individuals are risk averse. This allows us to derive a general expression for the risk premium for marriage over a cohabiting union.

Fourth, although it is not explicitly stated in BC, the distribution of offers of marriage from opposite-sex singles faced by an agent first-order stochastically dominates the distribution faced by an agent from a lower pizazz class. This continues to hold for the offer distributions involved in our model. However, the introduction of uncertainty and learning allows us to use the concept of second-order stochastic dominance to analyze the role of risk in cohabiting and marital unions.

We retain however, several of the assumptions made in BC. We assume that a large and equal number of male and female singles participate in a marriage market. Although couples first cohabit and then marry, we continue to refer to this pool of singles as the marriage market. Clearly, as men and women either cohabit or marry and leave the singles market, the distribution of pizazz will change over time. However, we assume as in BC, that singles are only partially rational and believe that the environment is stationary. This implies that they believe that the distribution of pizazz in the singles market for both men and women is constant over time. This is an important assumption as we assume that individuals follow stationary strategies.

Individuals are assumed to adopt utility maximizing strategies given the behaviour of other singles in the market. In particular, it is assumed that participants in the marriage market seek to maximize the expected discounted lifetime utility by searching for the best possible matches, discounted at a rate  $r$  to the present. The optimal policy to be followed by individuals consists of a certain set of opposite sex singles to whom they will make offers of cohabitation if they meet. Individual lifetimes follow an exponential distribution, and the probability that an individual dies in time interval of duration  $h$  is  $\delta h$ .

Pairs of market distributions of pizazz for men and women which are con-

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<sup>1</sup>Chade (1999) also develops a model in which individuals observe one another's type imperfectly. However, in his model, individuals never learn one others' true type. After matching on imperfectly observed type, they leave the market forever.

sistent with flow-in and flow-out distributions that are equal, make up the set of steady state equilibria. Our focus however, is on the decision problem faced by couples rather than the calculation of steady state equilibria. In our model, the calculation of the steady state is complicated by the fact that in addition to outflows of singles due to marriage and death, cohabiting unions and marriages may dissolve resulting in individuals re-entering the singles market.

We next describe our model in which couples are assumed to first cohabit and then marry.

## 2.2. The model with cohabitation

We assume as it is often the case, that singles have difficulty contacting members of the opposite sex and meet bilaterally (this is in contrast to Bloch and Ryder 2000 in which couples are matched via a centralized matching procedure). In addition, a single who proposes to cohabit with a member of the opposite sex may be rejected. To capture this uncertainty in initiating a cohabiting relationship, we assume that for any single in the market, the rate at which offers of cohabitation arrive follows a Poisson process.

In the rest of the paper, we simplify exposition by considering the decision maker to be a single woman who follows an optimal search strategy to find a potential marriage partner (the arguments are identical for the case of a single man, and are dealt with by symmetry). We assume that when a woman meets a man, she only observes his pizazz imperfectly. She observes  $y$ , though his true pizazz is  $x$ . Symmetrically, the man also obtains a noisy observation  $y$  on the woman's pizazz  $x$ . We assume that singles believe that the distributions of both true pizazz and observed pizazz are time-invariant. We refer to  $x$  as true pizazz and  $y$  as observed pizazz.

The distribution of true pizazz among men is denoted  $G_M(x)$ . That is, if a woman meets a man, then  $G_M(x)$  is the probability that the man's pizazz is no greater than  $x$ , or alternatively,  $G_M(x)$  may be used to denote the proportion of men in the market whose pizazz is lower than  $x$ . Similarly, one can define women's pizazz distribution, denoted  $G_W(x)$ . For simplicity, we assume  $G_M(x) = G_W(x) = G(x) = P(X \leq x)$  with support  $[\underline{x}, \bar{x}]$ . In this sense, the search-matching process is random.

We assume that observed pizazz is the sum of true pizazz and random noise. That is, the woman observes  $y$ , a realization of  $Y$ , the random variable used to denote observed pizazz. We assume that  $Y = X + \varepsilon$  and  $\varepsilon$  is independent of  $X$ . Also, it is assumed that  $E(\varepsilon) = 0$  and  $Var(\varepsilon)$  is small.

Upon observing  $y$ , the woman forms an expectation of the true pizazz of the man,  $E(X | Y = y)$  denoted  $m(y)$  or simply  $m$  (see the Appendix for an example when there are two types of husbands). Although  $m$  is expected true pizazz conditional on observed pizazz, for the sake of brevity, we will refer to  $m$

simply as expected pizazz. She then faces two choices. The first is to enter into a cohabiting union with him. The second is to remain single in the hope of a better draw from the same distribution in the next period.

Here,  $m$  can be interpreted as the Bayes estimate of  $x$ , denoted  $\hat{x}$ . This estimate is obtained by minimizing a risk function:  $\min_{\hat{x}} E_{x|y} \frac{1}{2} (\hat{x} - x)^2$ . Notice that this is the expectation of a quadratic loss function with respect to the posterior distribution. For example, consider the case when  $X \sim N(\mu, \sigma^2)$ ,  $\varepsilon \sim N(0, \sigma_\varepsilon^2)$  and  $Y = X + \varepsilon$  with  $\varepsilon \perp X$ . Then,  $\hat{x} = m(y) = w\mu + (1 - w)y$  where  $w = \sigma_\varepsilon^2 / (\sigma^2 + \sigma_\varepsilon^2)$ .

We make two assumptions. The first (A1) is that  $m'(y) > 0$ . This implies that  $m$  is increasing in  $y$ . This captures the common view that first impressions matter. If a man looks like he would be a good husband, then the woman will believe that he is likely to be a good husband. This assumption is satisfied in the normal distribution case mentioned above, as  $m'(y) = 1 - w > 0$ .

Moreover, the assumption that  $m$  is increasing in  $y$  is useful because  $m$  is then invertible. The support of  $M$  denoted  $[\underline{m}, \overline{m}] = [m(\underline{y}), m(\overline{y})]$ . Denoting the distribution of  $M$  by  $F(m)$  we can write

$$\begin{aligned} F(m) &= P(M \leq m) = P(Y \leq y(m)) = F_Y(y(m)) \\ \text{where } y &= y(m) \text{ is the inverse function of } m = m(y). \end{aligned}$$

Note that in this model, a single woman deciding whether or not to enter a cohabiting relationship has two pieces of information available to her. One is the observed pizazz of a potential partner  $y$ . The other is her knowledge of the distribution of true pizazz conditional on observed pizazz (more details about this distribution may be found in note 2 of the Appendix). This distribution is denoted distribution is  $Q_{X|Y}(x | y) = P(X \leq x | Y = y)$ .

Using these two pieces of information, she calculates  $m(y)$ . All decisions regarding whether or not to enter a cohabiting relationship or stay in one are based on  $m$ . By basing inference on  $m$  instead of  $y$ , the woman increases the precision of her inference about true pizazz. For the case of the normal distribution mentioned above,  $M \sim N(\mu, \sigma_m^2)$  where  $\sigma_m^2 = \sigma^4 / (\sigma^2 + \sigma_\varepsilon^2)$ . Notice that

$$\sigma_m^2 < \sigma^2 < \sigma_y^2$$

as  $\sigma_y^2 = \sigma^2 + \sigma_\varepsilon^2$ . Also, notice that  $\sigma_{m|x}^2$  which is  $(1 - w)^2 \sigma_\varepsilon^2$  is less than  $\sigma_{y|x}^2 (= \sigma_\varepsilon^2)$ .

In addition, a woman's own attractiveness, captured by her pizazz, determines the pizazz and the number of men that she can attract. That is, women with high pizazz are able to attract men who also have high pizazz. However, a woman who has high worth as marriage partner may not appear to have high worth as a marriage partner. This depends on the distribution of true pizazz conditional on observed pizazz. A woman may have a 'bad' day, and appear less attractive than

she really is. That is, her observed pizazz,  $y$  is equal to the sum of  $x$  and a large negative draw from the noise distribution. Alternatively, a woman who has low pizazz may be lucky and appear more attractive than she really is. Given that  $y$  is an imperfect indicator of true pizazz, one could ask if a possible strategy may be for the woman to simply state her true pizazz, when she meets a man. However, men who encounter a woman are more likely to trust their own judgment than the pizazz stated by the woman. They trust their own estimate of her true pizazz based on observed pizazz. As a result, as true pizazz is revealed only at some moment in cohabitation, what affects a woman's cohabitation prospects are estimates of her true pizazz based on observed pizazz,  $m = E(X | Y = y)$ .

Notice however, that a woman's  $m$  is random from her own point of view. She does not know the realization of observed pizazz that will occur when she encounters a man. However, the expected discounted utility of rejecting an offer of cohabitation and continuing to be single denoted  $R$ , is constant in search models of this type. Therefore, as  $m$  is stochastic, we parameterize  $R$  by  $\tilde{m}_e$  instead of parameterizing it with  $m$ . Here,  $\tilde{m}_e$  is the woman's expectation of her  $m$  conditional on her private knowledge of her own true pizazz,  $x$ . We refer to this woman as an  $\tilde{m}_e$ -type woman. That is,  $\tilde{m}_e = E(m(y) | x) = E_\varepsilon(m(x + \varepsilon))$ . In the case of the normal distribution,  $\tilde{m}_e = w\mu + (1 - w)x$ . Notice that this is a weighted average of two quantities. The first quantity is her true type,  $x$ . The second quantity is her estimate of the estimate made by the man she meets, which is  $\mu$ . In this expression,  $\mu = E(M) = E(E(X | Y)) = E(X)$ .

There exists a set of men who are willing to cohabit with a  $\tilde{m}_e$ -type woman. We denote the distribution of  $m$  among such men by  $F(m | \tilde{m}_e)$ . As  $m$  is invertible, the conditional distribution  $Q_{X|Y}(x | y)$  may be expressed in terms of  $m$ , and can be written as  $Q(x, m)$  or more precisely as  $Q(x, m | \tilde{m}_e)$ . This is the distribution of true pizazz  $x$  conditional on observed pizazz  $y = y(m)$  among men who are willing to cohabit with a woman with conditional expected pizazz  $\tilde{m}_e$ .

We denote by  $\alpha$  the total arrival rate of offers of cohabitation from all men faced by a particular woman. As men and women face different opportunities based on their attractiveness, a fraction of these offers, denoted  $\alpha(\tilde{m}_e)$  are received by a woman with  $\tilde{m}_e$ , and  $\alpha(\tilde{m}_e) \leq \alpha$ .

As cohabiting unions are initiated based on a noisy signal of an individual's worth as a marriage partner, some cohabiting unions progress to marriage when couples discover each other's true worth as marriage partners while others dissolve. We assume that the rate at which cohabiting unions evolve into marriage also follows a Poisson process with rate  $\lambda$ . This transition to marriage is also related to a woman's type and the rate faced by a  $\tilde{m}_e$ -type woman is written as  $\lambda(\tilde{m}_e)$ . Those women who are more attractive are more likely to transit to marriage. Those who are less attractive are more likely to have their cohabiting unions dissolve so that they must return to the singles pool.

The second assumption (A2) we make concerns  $Q(x, m | \tilde{m}_e)$ . We assume that



$\partial Q(x, m | \widetilde{m}_e) / \partial m < 0$ . This assumption implies that for any given  $m_B < m_A$  we have  $Q(x, m_A | \widetilde{m}_e) < Q(x, m_B | \widetilde{m}_e)$  for all  $x \in S_X$ . That is,  $Q(x, m_A | \widetilde{m}_e)$  first order stochastically dominates  $Q(x, m_B | \widetilde{m}_e)$ . Recall that a distribution function:

$F(x)$  first order stochastically dominates  $G(x)$  if

$$F(x) \leq G(x) \text{ for all } x. \quad (1)$$

In addition, if  $F(x)$  first order stochastically dominates  $G(x)$ , then for a non-decreasing function  $u(x)$ , we have

$$E_{G(x)}u(x) < E_{F(x)}u(x) \quad (2)$$

For example, suppose that a single woman encounters two single men, denoted  $A$  and  $B$  with observed pizazz  $y_A$  and  $y_B$  respectively. Assume that man  $A$  appears to be the better partner than man  $B$ . That is,  $y_B < y_A$ . Since we have assumed that  $m'(y) > 0$ , this implies that  $m_B < m_A$ . The assumption of first order stochastic dominance implies the following: for any value of true pizazz  $x$ , the probability that the true pizazz of man  $A$  is more than  $x$  is greater than the probability that the true pizazz of man  $B$  is more than  $x$ .

This holds in the case of the normal distribution mentioned above. In this case,

$$\partial Q(x, m | \widetilde{m}_e) / \partial m = -\frac{1}{\sigma_1} \phi\left(\frac{x - m}{\sigma_1}\right) < 0$$

where  $\phi(\cdot)$  is the standard normal density. Recall that  $(X | y) \sim N(m, \sigma_1^2)$  where  $\sigma_1^2 = w\sigma^2$  and  $w = \sigma_\varepsilon^2 / (\sigma^2 + \sigma_\varepsilon^2)$ .

We can consider the matching process leading to marriage as occurring in three stages. In the first stage, which can be regarded as a “pre-draw” stage, a single individual is in the marriage market. In this stage, she only knows the distribution of expected true pizazz conditional on observed pizazz,  $F(m)$ .

In the second stage, the woman draws  $x$  (a draw from the distribution of true pizazz) but observes  $y$  (a draw from the distribution of observed pizazz). She chooses to cohabit with him if his value of  $m$  (her inference of his true pizazz based on  $y$ ) exceeds a certain threshold level, denoted  $m_r$ . This threshold level,  $m_r$  is her reservation level of expected true pizazz conditional on observed pizazz. If she enters a cohabiting union with a man with expected pizazz  $m$ , she receives utility equal to  $u(m)$  during that period. If not, she remains single and the process repeats itself in the next period. It is of course possible that she herself is rejected by the man she has encountered, in which case she has no choice but to remain single until the next period.

Sometime during the cohabitation period, the true pizazz of each partner is revealed to the other. This marks the third stage. We assume for simplicity that learning occurs only once and only during cohabitation. At this point, each party

must decide whether to maintain or quit the current match. If the couple choose to maintain the match, they are assumed to establish a formal marriage and leave the marriage forever. If the cohabiting couple do not to marry, they are assumed to separate.

This distinction is made only to aid understanding. Events in fact occur at random intervals, because of the Poisson assumption that underlies the processes of cohabitation and marriage. The next section develops the formal model further in terms of these three stages. The optimal policy to be followed in each stage is derived and for ease of exposition, we continue to assume that the decision maker is a woman.

### 3. The decision making process

#### 3.1. Stage 1: being single

This is the “pre-draw” stage, when a  $\tilde{m}_e$ -type woman is in the singles market. At this stage, she observes neither  $x$  nor  $y$ . She only knows  $F(m)$ , the distribution of expected true pizazz conditional on observed pizazz. Standard dynamic programming arguments yield,

$$R(\tilde{m}_e) = \frac{1}{1+r h} \left[ u_{\tilde{m}_e}(0) h + \alpha(\tilde{m}_e) h E_m V_{\tilde{m}_e}(m) + (1 - \alpha(\tilde{m}_e) h) R(\tilde{m}_e) \right] + o(h). \quad (3)$$

In the above,  $R(\tilde{m}_e)$  is the expected discounted utility of being single in the current period and following an optimal policy from the next period onward. The term  $u_{\tilde{m}_e}(0)$  is the utility received in the current period. This term is in fact,  $u_{\tilde{m}_e}(m)$  where  $m$  is the expected pizazz of her cohabiting partner. However, she has no partner at this stage and therefore the argument of  $u_{\tilde{m}_e}(\cdot)$  is zero. The term  $\alpha(\tilde{m}_e) h$  is the probability that the woman receives an offer of cohabitation during the short time interval  $h$ . The expected discounted value of having an offer from an  $m$ -type man is denoted  $E_m V_{\tilde{m}_e}(m)$ . An  $m$ -type man is one whose expected true pizazz conditional on observed pizazz is  $m$ . The term,  $1 - \alpha(\tilde{m}_e) h$  is the probability that no offer of cohabitation is received in the time period  $h$ . In this case, the woman has no choice but to remain single till the next period.

We assume for now that  $u_{\tilde{m}_e}(m) = m$ . Letting  $h \rightarrow 0$ , we have  $o(h)/h \rightarrow 0$  and we can rewrite (3) to obtain

$$R(\tilde{m}_e) = \frac{\alpha(\tilde{m}_e)}{r + \alpha(\tilde{m}_e)} E_m V_{\tilde{m}_e}(m). \quad (4)$$

A reservation pizazz policy is the optimal policy to be followed in this stage; this is demonstrated in section 3.4.

### 3.2. Stage 2: should we live together?

At this stage, the woman encounters a man and based on his observed pizazz, estimates his true pizazz. That is, she calculates  $E(X | Y = y)$  or simply  $m$ . The woman is confronted with two alternatives: continue to be single or enter into a cohabiting union with the man she has encountered. The value function at this stage is:

$$V_{\tilde{m}_e}(m) = \max \left\{ \varphi_{\tilde{m}_e}(m), R(\tilde{m}_e) \right\}. \quad (5)$$

In the above expression,  $\varphi_{\tilde{m}_e}(m)$  is the expected discounted utility of accepting an offer of cohabitation from an  $m$ -type man and behaving optimally from the next period onwards. This term can be written as follows:

$$\varphi_{\tilde{m}_e}(m) = \frac{1}{1+rh} \left[ u(m)h + \lambda(\tilde{m}_e)h E_{x|m} J_{\tilde{m}_e}(x) + (1 - \lambda(\tilde{m}_e)h) \varphi_{\tilde{m}_e}(m) \right] + o(h). \quad (6)$$

The first term  $u(m)h$  is the utility received in the short time interval  $h$  of being in a cohabiting union with a man with expected pizazz equal to  $m$ . The term  $\lambda(\tilde{m}_e)h$  is the probability that the woman receiving a proposal for marriage from her cohabiting partner in time interval  $h$ . The value of receiving an offer of marriage and behaving optimally next period onwards is denoted  $J_{\tilde{m}_e}(x)$ . Notice that the expectation of this value function is taken with respect to  $x$  conditional on  $m$ . In the event that no offer of marriage is received during time interval  $h$ , and this occurs with probability  $(1 - \lambda(\tilde{m}_e)h)$ , the woman remains in a cohabiting relationship. We continue to assume that  $u_{\tilde{m}_e}(m) = m$ . Then, as  $h \rightarrow 0$  (??) reduces to

$$\varphi_{\tilde{m}_e}(m) = \frac{m + \lambda(\tilde{m}_e) E_{x|m} J_{\tilde{m}_e}(x)}{r + \lambda(\tilde{m}_e)}. \quad (7)$$

It should be pointed out that there is no uncertainty about true pizazz being revealed at some moment during cohabitation. When this occurs, couples either marry or separate. However, the optimal policy to be followed is determined ex-ante, before any offers of marriage are made. Therefore, we need to introduce the Poisson parameter  $\lambda(\tilde{m}_e)$ , which links the cohabitation stage and the marriage stage of the model. The optimal policy to be followed in this stage is also derived in section 3.4.

### 3.3. Stage 3: should we marry?

Stage 3 is when the woman finds out the true pizazz,  $x$ , of her cohabiting partner. She then chooses to either separate from him or marry him. Assuming that the woman receives as utility, the true pizazz of her husband in each time period following marriage, that is  $u_{\tilde{m}_e}(x) = x$ , then the utility from marriage is  $\psi_{\tilde{m}_e}(x) = x/r = \psi(x)$ . This is based on the assumption that once a couple marry, they stay married forever and never return to the singles market. If the

woman does not marry, she returns to the singles market and the expected discounted utility she receives is then  $R(\tilde{m}_e)$ , as discussed in stage 1. Therefore, the value function at this stage can be written as

$$J_{\tilde{m}_e}^{\sim}(x) = \max \left\{ \psi_{\tilde{m}_e}^{\sim}(x), R(\tilde{m}_e) \right\} \quad (8)$$

The optimal strategy to be followed at this stage is a reservation pizazz policy. The woman marries her cohabiting partner if his true pizazz,  $x \geq x_r$  where

$$x_r = rR(\tilde{m}_e) = \psi^{-1}(R(\tilde{m}_e)). \quad (9)$$

In the above equation,  $x_r$  is the woman's minimum acceptable level of pizazz. This result depends on the constancy of  $R(\tilde{m}_e)$  w.r.t.  $x$  which is shown next.

### 3.4. Derivation of optimal strategies.

We first show that the optimal strategy to be followed in stage 2 is a reservation pizazz strategy. To show this, we first demonstrate that  $\varphi_{\tilde{m}_e}^{\sim}(m)$  is strictly increasing in  $m$ . To this end, we first examine the expectation of value function in stage 3. This is:

$$EJ_{x|m}^{\sim}(x) = E_{x|m} \max \left\{ \psi_{\tilde{m}_e}^{\sim}(x), R(\tilde{m}_e) \right\}. \quad (10)$$

Evaluating the above expression (see the Appendix for details) yields:

$$E_{x|m}J_{\tilde{m}_e}^{\sim}(x) = \frac{1}{r} \int_{\psi^{-1}(R(\tilde{m}_e))}^{\bar{x}} [1 - Q(x, m | \tilde{m}_e)] dx + R(\tilde{m}_e) \quad (11)$$

which is a function of  $(m, \tilde{m}_e)$ . Differentiating w.r.t.  $m$  we obtain,

$$\frac{\partial E_{x|m}J_{\tilde{m}_e}^{\sim}(x)}{\partial m} = \frac{1}{r} \int_{\psi^{-1}(R(\tilde{m}_e))}^{\bar{x}} \left[ -\frac{\partial Q(x, m | \tilde{m}_e)}{\partial m} \right] dx > 0 \quad (12)$$

because of assumption A2.

This allows for an additional interpretation of assumption A2. Notice from (10) that  $J_{\tilde{m}_e}^{\sim}(x)$  is nondecreasing in  $x$ . Using  $E_Q(\cdot)$  to denote the expectation taken using distribution  $Q$ , we can write

$$E_{Q(x, m_B | \tilde{m}_e)}J_{\tilde{m}_e}^{\sim}(x) < E_{Q(x, m_A | \tilde{m}_e)}J_{\tilde{m}_e}^{\sim}(x) \quad (13)$$

as  $Q(x, m_A | \tilde{m}_e)$  first order stochastically dominates  $Q(x, m_B | \tilde{m}_e)$ . The inequality in (13) can clearly be seen to hold by examining (11). This may be

explained as follows: Suppose that a woman cohabits with a man with conditional mean pizazz  $m_A$  as opposed to cohabiting with a man with conditional mean pizazz  $m_B$  and that  $m_B < m_A$ . Then the woman's expected discounted utility of following an optimal strategy when the true pizazz of her cohabiting partner is revealed is higher in the case of man  $A$  than it is in the case of man  $B$ .

The assumption that  $\partial Q(x, m | \tilde{m}_e) / \partial m < 0$  is important in terms of the mathematics of the model. Next, it is used in proving that  $\varphi_{\tilde{m}_e}(m)$  is increasing in  $m$ . Recall that  $\varphi_{\tilde{m}_e}(m)$  is the expected discounted utility of being in a cohabiting union in the current period and following an optimal strategy from the next period onward. Differentiating (7):

$$\frac{\partial \varphi_{\tilde{m}_e}(m)}{\partial m} = \frac{1}{r + \lambda(\tilde{m}_e)} \left[ 1 + \lambda(\tilde{m}_e) \frac{\partial E_{x|m} J_{\tilde{m}_e}(x)}{\partial m} \right]. \quad (14)$$

Substituting (12) in the above yields:

$$\frac{\partial \varphi_{\tilde{m}_e}(m)}{\partial m} = \frac{1}{r + \lambda(\tilde{m}_e)} \left[ 1 + \frac{\lambda(\tilde{m}_e)}{r} \int_{\psi^{-1}(R(\tilde{m}_e))}^{\bar{x}} \left[ -\frac{\partial Q(x, m | \tilde{m}_e)}{\partial m} \right] dx \right] > 0, \quad (15)$$

because of assumption A2. This ensures that a unique solution to the stage-2 problem exists. The optimal strategy to be followed in stage 2 is also a reservation pizazz policy. An  $\tilde{m}_e$ -type woman cohabits with a man if his estimated pizazz  $m \geq m_r$  where  $m_r = \varphi_{\tilde{m}_e}^{-1}(R(\tilde{m}_e))$ .

To solve for the rejection utility for stage 1, we first evaluate the expectation of (5), which can be written as follows

$$EV_{\tilde{m}_e}(m) = E_M \max \{ \varphi_{\tilde{m}_e}(m), R(\tilde{m}_e) \}. \quad (16)$$

Evaluating the above (see the Appendix for the derivation of the following equation), yields the rejection utility for this  $\tilde{m}_e$ -type woman:

$$\begin{aligned} R(\tilde{m}_e) &= \frac{\alpha(\tilde{m}_e)}{r(r + \lambda(\tilde{m}_e))} \int_{\varphi_{\tilde{m}_e}^{-1}(R(\tilde{m}_e))}^{\bar{m}} [1 - F(m | \tilde{m}_e)] \\ &\quad \times \left[ 1 - \frac{\lambda(\tilde{m}_e)}{r} \int_{\psi^{-1}(R(\tilde{m}_e))}^{\bar{x}} \frac{\partial Q(x, m | \tilde{m}_e)}{\partial m} dx \right] dm \end{aligned} \quad (17)$$

Notice that from the above equation, it is clear that  $R(\tilde{m}_e)$  does not depend on particular values of  $x$  or  $m$ . This constancy is important in establishing that a reservation strategy is optimal in stages 2 and 3.

Blackwell's theorem (see Sargent 1987) ensures that there exists a unique solution to this functional equation in  $R(\widetilde{m}_e)$  which is continuous in  $[\varphi_{\widetilde{m}_e}^{-1}(R(\widetilde{m}_e)), \overline{m}]$ . By symmetry, we will have another functional equation in  $R(m_e)$  for an  $m_e$ -type man as the decision-maker, which is identical to (17) except that  $m_e$  and  $\widetilde{m}_e$  are exchanged. The assumption that men and women face the same underlying distribution, that is  $G_M(x) = G_W(x)$  allows us to use the symmetry argument. These two equations are used to derive the results on class partitioning which will be discussed in the next section.

Since the rest of the paper deals with matching and class partitions, we simply denote the type of the decision maker as  $\widetilde{m}_e$ . The type of the potential partners of this decision maker is denoted  $m_e$ . As long as a sorting result is attained for one sex, the same can be derived for the other sex by symmetry. This symmetric equilibrium induces sorting of individuals at the aggregate level.

#### 4. Stochastic dominance and the class partition

In this section, we show that cohabiting unions occur only between couples in the same class and that there is an ordering among classes. The number of classes is finite. We prove these results by establishing a series of lemmas.

**Lemma 1.** *During the search for a cohabiting partner, the distribution of  $m$  among opposite-sex singles faced by a  $\widetilde{m}_e$ -type single first-order stochastically dominates the distribution of  $m$  among opposite sex singles faced by an  $\widetilde{\widetilde{m}}_e$ -type single if  $\widetilde{\widetilde{m}}_e < \widetilde{m}_e$ . That is,  $F(m | \widetilde{m}_e) < F(m | \widetilde{\widetilde{m}}_e)$ .*

Proof: In what follows,  $m'$  denotes the upper bound of the distribution of  $\widetilde{M}_e$ .

$$\begin{aligned} F(m | \widetilde{m}_e) &= P(M \leq m | \widetilde{M}_e \leq m') = P(M \leq m | M \leq m') \\ &= F(m) / F(m'), \quad \text{for } \underline{m} \leq m \leq m', \text{ and } \widetilde{m}_e \leq m' \end{aligned}$$

In the above, we substitute  $m'$  in place of the conditioning variable  $\widetilde{m}_e$ ;  $m'$  will in fact be determined later in the matching process. We can replace  $\widetilde{M}_e$  by  $M$  because we assume that beliefs are rational and because the distribution of pizazz is the same for both men and women. Notice that  $F(m | \widetilde{m}_e)$  is a truncated distribution with truncated support  $[\underline{m}, m']$ . As  $\widetilde{m}_e$  decreases, its upper bound may or may not fall and the resultant support will shrink if the bound does fall, thus generating a series of truncated distributions. Because  $F(m | \widetilde{m}_e) < F(m | \widetilde{\widetilde{m}}_e)$  for  $\widetilde{\widetilde{m}}_e \leq m''$ ,  $\widetilde{m}_e \leq m'$  and  $m'' < m'$ , we have the result that  $F(m | \widetilde{m}_e)$  first-order stochastically dominates  $F(m | \widetilde{\widetilde{m}}_e)$ .

**Lemma 2.** *A cohabiting individual with expected pizazz ( $\widetilde{m}_e$ ) of either sex is more likely to attract a spouse of higher true pizazz than a cohabiting individual with lower expected pizazz  $\widetilde{\widetilde{m}}_e$ .*

Proof: No truncation is required in this case, instead we simply specify the following:

$$Q(x, m | \widetilde{m}_e) = Q(x, m) \quad \text{for } m \leq m' \text{ and } \forall x \in [\underline{x}, \bar{x}], \text{ where } \widetilde{m}_e \leq m'$$

In this case, the support for the conditional distribution does not shrink. However, it is the upper bound  $m'$  of the parameterizing variable  $\widetilde{m}_e$  that defines the range of  $m$ . The cohabitor's  $m$  has the same upper bound as the decision-maker. The stochastic dominance property still holds, as desired. For example, if  $\widetilde{m}_e \leq m''$ ,  $\widetilde{m}_e \leq m'$ ,  $m'' < m'$ , and if  $\widetilde{m}_e < \widetilde{m}_e$ , then it follows from assumption A2 that the family of distributions  $Q(x, m | \widetilde{m}_e)$  first-order stochastically dominates that of distributions  $Q(x, m | \widetilde{m}_e)$ .

**Lemma 3.** *A higher-type individual of either sex faces a higher arrival rate both of offers of cohabitation and of offers of marriage than his or her competitors of lower types.*

Proof:  $\alpha(\widetilde{m}_e) = \alpha F(m')$  for  $\widetilde{m}_e \leq m'$ , which says that  $\alpha(\widetilde{m}_e)$  is calculated by discounting the overall arrival rate of cohabitation offers  $\alpha$  at a rate of  $[1 - F(m')]$  because with such a probability no opposite-sex singles with type  $m$  greater than  $m'$  are willing to propose to a  $\widetilde{m}_e$ -type person for cohabitation. By the same logic, the arrival rate of marriage offers faced by either sex must undergo the discounting procedure such as:  $\lambda(\widetilde{m}_e) = \lambda F(m')$  for  $\widetilde{m}_e \leq m'$ .

Notice that these lemmas already contain some of the elements related to class partitioning. Singles are better off proposing to opposite sex singles of either the same type as themselves or of a lower type. However, the lower bound of preferences of singles is not yet clear. These lemmas are concerned only with the upper bound of compatible partners' pizazz levels. Further results based on reservation pizazz policies are needed to identify lower bounds of pizazz intervals within which couples can be matched.

Lemma 1 may be interpreted as defining the opportunity set for a person of type  $\widetilde{m}_e$ ,  $[\underline{m}, m'] \equiv M_{opp}$ . Any opposite-sex single with  $m$  higher than  $m'$  is unattainable. The reservation pizazz policy derived from the search model determines the acceptance set  $[m_r, \bar{m}] \equiv M_{acc}$  for this person. Any opposite-sex below the lower bound  $m_r$  is considered unacceptable. Then, the intersection of the two sets,  $M_{opp} \cap M_{acc}$ , identifies the set of opposite-sex partners that are both attainable and acceptable to this person. This is a set of mutually agreeable pizazz types. Note that the reservation demand of opposite-sex singles determines the upper bound of this set while the single's own reservation demand decides its lower bound.

**Lemma 4.** *The best single of either sex accepts any opposite sex single in the top class. That is, the single whose  $\widetilde{m}_e = \bar{m}$  will cohabit with a member of the opposite sex with  $m \in (m_1, \bar{m}]$  and marry a member of the opposite sex with  $x \in (x_1, \bar{x}]$ .*

Proof: Consider the case of the woman whose  $\widetilde{m}_e = \overline{m}$  whom we refer to as the best woman. Using lemmas 1, 2 and 3, we can show that  $F(m | \overline{m}) = F(m)$ ,  $Q(x, m | \overline{m}) = Q(x, m)$ , for  $m \in [\underline{m}, \overline{m}]$ ,  $x \in [\underline{x}, \overline{x}]$ , and  $\alpha(\overline{m}) = \alpha$ ,  $\lambda(\overline{m}) = \lambda$ . From this and from (17), we find that the expected discounted utility of being single for the best woman satisfies the following equation:

$$\overline{R} \equiv R(\overline{m}) = \frac{\alpha}{r(r + \lambda)} \int_{\varphi_{\overline{m}}^{-1}(\overline{R})}^{\overline{m}} [1 - F(m)] \left[ 1 - \frac{\lambda}{r} \int_{\psi^{-1}(\overline{R})}^{\overline{x}} \frac{\partial Q(x, m)}{\partial m} dx \right] dm \quad (18)$$

Solving this equation for  $\overline{R}$  determines the best single's reservation demand for the pizazz types of his or her cohabiters and spouses. We define:  $m_r(\overline{m}) = \varphi_{\overline{m}}^{-1}(\overline{R}) \equiv m_1$  and  $x_r(\overline{m}) = \psi^{-1}(\overline{R}) \equiv x_1$ .

We have now established that the best woman will cohabit with men whose  $m \in (m_1, \overline{m}]$  and marry men with  $x \in (x_1, \overline{x}]$ . We refer to men whose  $m \in (m_1, \overline{m}]$  as making up the top class for cohabiting unions and men whose  $x \in (x_1, \overline{x}]$  as making up the top class for marital unions. By symmetry, the top-class of women can also be defined. Notice that the best man will not necessarily cohabit or marry the best woman. This is the outcome of a positive discount rate and the difficulty in encountering singles of the opposite sex.

**Lemma 5.** *A top-class single of either sex cohabits and marries any top-class single of the opposite sex.*

Proof: This is a natural corollary of lemma 4, can be proved rigorously as follows. As we have shown that the best man will accept any woman in the top-class as a partner, all men must be willing to accept women in the top class as partners. This implies that any woman with  $\widetilde{m}_e \in (m_1, \overline{m})$  faces the same prospects as the best woman. She faces the entire distribution of pizazz among men and the total arrival rate of offers from all men. Therefore, for such women,  $F(m | \widetilde{m}_e) = F(m)$ ,  $Q(x, m | \widetilde{m}_e) = Q(x, m)$ , for  $m \in [\underline{m}, \overline{m}]$ ,  $x \in [\underline{x}, \overline{x}]$ , and  $\alpha(\widetilde{m}_e) = \alpha$ ,  $\lambda(\widetilde{m}_e) = \lambda$ . Substituting these quantities into (17) yields the expected discounted utility of being single for such a woman:

$$R(\widetilde{m}_e) = \frac{\alpha}{r(r + \lambda)} \int_{\varphi_{\widetilde{m}_e}^{-1}(R(\widetilde{m}_e))}^{\overline{m}} [1 - F(m)] \left[ 1 - \frac{\lambda}{r} \int_{\psi^{-1}(R(\widetilde{m}_e))}^{\overline{x}} \frac{\partial Q(x, m)}{\partial m} dx \right] dm \quad (19)$$

Equations 7 and 11 and lemmas 2 and 3 imply  $\varphi_{\widetilde{m}_e}^{-1}(\cdot) = \varphi_{\overline{m}}^{-1}(\cdot)$ . Notice that (19) and (18) are identical except that the term  $R(\overline{m})$  appears in (18) while the term  $R(\widetilde{m}_e)$  appears in (19). Therefore,  $R(\widetilde{m}_e) = \overline{R}$  for  $\widetilde{m}_e \in (m_1, \overline{m}]$ . Therefore, the reservation values of women in the top class other than the best woman,  $m_r(\widetilde{m}_e)$  and  $x_r(\widetilde{m}_e)$ , are identical to the reservation values of the best woman:  $m_1$  and  $x_1$ , respectively.



**Lemma 6.** *A second-class single of either sex accepts any second-class single of the opposite sex. That is singles with  $m \in (m_2, m_1]$  cohabit with each other while couples with whose  $x \in (x_2, x_1]$  marry each other.*

Proof: Consider a woman who is not in the top class but she is the best among all women who are not in the top class. That is, her  $\widetilde{m}_e = m_1$ . This woman will be rejected by all men in the top class, but she is the most attractive among women not in the top class. From lemma 1, 2 and 3, she faces  $F(m | m_1) = F(m) / F(m_1)$ ,  $Q(x, m | m_1) = Q(x, m)$ , for  $m \in [\underline{m}, m_1]$ ,  $x \in [\underline{x}, \bar{x}]$ , and  $\alpha(m_1) = \alpha F(m_1)$ ,  $\lambda(m_1) = \lambda F(m_1)$ . Using these results, and applying (17) to the shrunk support  $[\underline{m}, m_1]$  results in the expected discounted utility of being single for this woman:

$$R_1 \equiv R(m_1) = \frac{\alpha}{r(r + \lambda F(m_1))} \int_{\varphi_{m_1}^{-1}(R_1)}^{m_1} [F(m_1) - F(m)] \left[ 1 - \frac{\lambda F(m_1)}{r} \int_{\psi^{-1}(R_1)}^{\bar{x}} \frac{\partial Q(x, m)}{\partial m} dx \right] dm \quad (20)$$

Solving the above for  $R_1$  identifies the men that form the second-class:  $m \in (m_2, m_1]$ ,  $x \in (x_2, x_1]$ , where  $m_2 = \varphi_{m_1}^{-1}(R_1)$ ,  $x_2 = \psi^{-1}(R_1)$ . By symmetry, one can also obtain the second-class of women. By the same reasoning as in the proof of lemma 5, we have  $R(\widetilde{m}_e) = R_1$  for  $\widetilde{m}_e \in (m_2, m_1]$ , and lemma 6 then follows.

**Proposition 1** *Singles in class- $i$  only accept opposite-sex singles in the same class,  $1 \leq i \leq n$ , where  $n$  is a finite number.*

Proof: Repeating the procedures in lemma 4, 5, 6, and applying mathematical induction, one can conclude that  $R(\widetilde{m}_e)$  is a non-decreasing step function so that  $R_{n-1} < \dots < R_1 < \bar{R}$ . Class partitioning is such that  $(\underline{m} <) m_n < \dots < m_1 < \bar{m}$  and  $(\underline{x} <) x_n < \dots < x_1 < \bar{x}$ . It can be proved by contradiction that  $n$  is finite. The proof is identical to the one in BC in which the number of classes that make up the class partition for marriage is shown to be finite.

The implications of this theorem are as follows: (1) singles can be split up into  $n$  distinct classes. In equilibrium, matches, either cohabiting or marital, take place only between the two sexes in the same class; (2) the class partition for marriage is consistent with that of cohabitation; (3) the aggregate matching induced by class partitioning is not Pareto optimal due to imperfect information. That is, a low-type single can increase her welfare through mismatch in cohabitation and this can be achieved by mis-signalling at the partner's expense. A cohabitor of higher type will suffer from a utility loss in comparison to what she could get when matched with a partner of the same class under perfect information. What is more, she faces the prospect of re-entering the marriage market after

the relationship dissolves. The next section addresses the issue of risk in greater detail.

## 5. Risk premia, cohabitation and marriage

This section addresses issues related to the risk of mis-match and the risk aversion of the individual. For simplicity, we omit the conditioning variable  $\tilde{m}_e$  in what follows. The results are obtained by linking the agent's optimal policies in stages 2 and 3. We begin with the following proposition:

**Proposition 2** *Individuals are more discriminating when they marry than when they cohabit. That is, reservation values for potential spouses are higher than reservation values for potential cohabiting partners.*

Proof: If we consider a woman in the top class, then her  $\tilde{m}_e \in [m_1, \bar{m}]$  and  $R(\tilde{m}_e) = \bar{R}$ . If we write  $m_r$  in place  $m_1$  and  $x_r$  in place of  $x_1$  to emphasize that these are reservation cut-off levels, then  $\varphi_{\tilde{m}}(m_r) = \bar{R}$  and  $\psi(x_r) = \bar{R}$ . Therefore, from (7) we have,

$$\bar{R} = \frac{m_r + \lambda E_{x|m=m_r} J(x)}{r + \lambda} \quad (21)$$

Using  $\bar{R}r = x_r$  and rearranging terms yields,

$$\lambda [E_{x|m=m_r} J(x) - \bar{R}] = x_r - m_r.$$

Using (34) from the Appendix we have,

$$RP \equiv x_r - m_r = \left(\frac{\lambda}{r}\right) \int_{x_r}^{\bar{x}} (x - x_r) dQ(x, m_r) > 0. \quad (22)$$

The above equation holds not just for the top class, but for any class<sup>2</sup>. Notice that (22) has two interpretations. The first is an interpretation similar to the one found in the search literature on the costs and benefits of search. The left hand side represents the opportunity cost of searching one more time with a marriage proposal of  $x_r$  at hand. The right hand side represents the prospective benefit of this search which is the expected discounted utility associated with a future draw of  $x > x_r$ . Individuals set lower reservation levels for cohabitation than for

<sup>2</sup>The general relation related to (22) for class  $i$  or  $\tilde{m}_e \in (m_i, m_{i-1}]$  should take the form of

$$x_i - m_i = \frac{\lambda}{r} \int_{x_i}^{x_{i-1}} (x - x_i) d_x Q(x, m_i)$$

Since  $Q(x, m | \tilde{m}_e) = Q(x, m)$  for  $\forall m \leq m_{i-1}$  according to lemma 2, evaluating this at  $m = m_i$  yields  $Q(x, m_i | \tilde{m}_e) = Q(x, m_i)$ .

marriage because observed pizazz is a noisy signal of true pizazz. As cohabitation unions precede marriage, individuals may miss out on the right husband or wife by setting reservation levels for cohabitation too high.

A second interpretation is based on a comparison of the distributions  $F(m)$  and  $G(x)$  and noting that  $F(m)$  second-order stochastically dominates  $G(x)$ . That is,  $G(x)$  is riskier than  $F(m)$ . Recall that  $G(x)$  is the distribution of true pizazz while  $F(m)$  is the distribution of expected pizazz. Before a single individual receives any offers, there are only two pieces of information available: the individual's prior distributions of  $G(x)$  and  $F(m)$ . Based on this, the individual has to make an ex-ante decision on the optimal policy to be followed when cohabiting or marrying. For example, in the normal distribution case, it can be shown that  $G(x)$  is just a mean-preserving spread of  $F(m)$  since  $\mu_m = \mu_x$  and  $\sigma_m^2 < \sigma_x^2$ . Since the distributions have equal means, the individual's response when facing the riskier distribution  $G(x)$ , is to become more choosy. The term  $RP$  can be interpreted as a kind of risk premium that compensates the individual for the higher risk in marriage versus cohabitation.

**Proposition 3** *Higher reservation values for cohabitation translate into higher reservation values for marriage. Consider two individuals indexed  $i$ ,  $i = k$  and  $k + 1$ . Then,*

$$m_r^k > m_r^{k+1} \Rightarrow x_r^k > x_r^{k+1}. \quad (23)$$

Proof: Combining the optimal search policies in stage 2 and 3 we have,  $x_r = rR = r\varphi(m_r)$ . Using (14), we have  $dx_r/dm_r = r\varphi'(m_r) > 0$ .

**Proposition 4** *Individuals who set higher reservation values for cohabitation are more risk averse than others. That is,*

$$RP'(m_r) \geq 0 \Leftrightarrow \int_{x(m_r)}^{\bar{x}} \left[ -\frac{\partial Q(x, m_r)}{\partial m_r} \right] dx \geq P(x \geq x_r \mid m = m_r)$$

Proof: This follows from (34) and (35), from which we know that

$$\int_{x_r}^{\bar{x}} (x - x_r) dQ(x, m_r) = \int_{x_r}^{\bar{x}} [1 - Q(x, m_r)] dx.$$

The above derivation uses (22) as an implicit function:  $x = x(m)$ . This proposition asserts that the risk premium rises with the level of reservation cohabitation pizazz, provided that the probability that the partner satisfies the individual's reservation demand is lower than the degree of stochastic dominance.

**Proposition 5** *For any individual in any class, the risk premium to compensate for the higher risk of marriage over cohabitation is equal to the quantity of risk multiplied by its price.*

Proof: Re-writing (22) to its equivalent:

$$x_r - m_r = P(x < x_r | m = m_r) E[\xi(x_r - x) | x < x_r, m = m_r] \quad (24)$$

In the above, the quantity of risk<sup>3</sup>,  $P(x < x_r | m = m_r)$ , is the probability that the true pizzazz of the cohabiting partner turns out to be below the woman's reservation cut-off for marriage even though he was acceptable as a cohabiting partner. The 'price' of this risk is the expectation of  $\xi(x_r - x)$ , conditional on this disappointing revelation. The term  $\xi = (1 + r/\lambda)^{-1} < 1$  is the multiplier in the measurement of the risk. The higher the discount rate  $r$ , and the lower the transition rate  $\lambda$  of cohabitation to marriage, the smaller is the effect of this multiplier on the price of risk. The term,  $(x_r - m_r)$ , is the loss in utility compared with what the woman would have gained had she chosen a partner with  $x > x_r$  as well as  $m \geq m_r$  (that is, if the woman had chosen a man who was both a good cohabiting partner as well as a good husband).

So far, individuals are assumed to be risk neutral. We now relax this assumption. Note however, that relaxing this assumption does not alter the results obtained in the last section.

**Proposition 6:** *For a risk averse individual, the total risk premium can be decomposed into two parts. One is  $(x_r - m_r)$ , this compensates the individual for bearing the risk due to the second-order stochastic dominance of  $F(m)$  over  $G(x)$ ; the other part is  $\xi RP_{pa}$  which is a function of the individual's risk aversion.*

Proof: We assume that the individual's utility function  $u(x)$ , possesses the usual properties of being strictly increasing and concave. Therefore, (22) can then be

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<sup>3</sup>Stringently, the quantity of risk for class  $i$  should be measured as

$$P(x < x_i | m_i \leq m < m_{i-1}) = \int_{-\infty}^{x_i} f(x | y_i \leq y < y_{i-1}) dx$$

using  $y'(m) > 0$ . In addition,

$$f(x | y_i \leq y < y_{i-1}) = \int_{y_i}^{y_{i-1}} f(x, y) dy / \int_{y_i}^{y_{i-1}} f_Y(y) dy$$

and the price of risk for this class is  $\xi$  multiplied by

$$E(x_i - x | x < x_i, m_i \leq m < m_{i-1}) = \int_{-\infty}^{x_i} \int_{y_i}^{y_{i-1}} (x_i - x) f(x, y) dx dy / \int_{-\infty}^{\infty} \int_{y_i}^{y_{i-1}} f(x, y) dx dy.$$

However less stringently, one can condition on  $m = m_r$  in (24) in addressing the issue of mismatch risk. This is the risk that the current partner is only good enough as a cohabitor since  $m = m_r$  but not good enough to be a spouse since  $x < x_r$ .

written as:

$$u(x_r) - u(m_r) = \frac{\lambda}{r} \int_{x_r}^{\bar{x}} [u(x) - u(x_r)] d_x Q(x, m_r) > 0 \quad (25)$$

Since the utility function is assumed to be increasing, we have  $u(x_r) > u(m_r)$ . The application of Jensen's inequality to the above equation yields

$$u(x_r) - u(m_r) < P(x < x_r | m = m_r) E[\xi(u(x_r) - u(x)) | x < x_r, m = m_r]$$

We can consider  $x$  as  $m_r$  plus an actuarially neutral gamble of  $z (= x - m_r)$  in the sense that  $E_{x|m_r} z = 0$ . Taking the second order Taylor series expansion of  $u(x)$  around  $m_r$ , the first-order expansion of  $u(x_r)$  around  $m_r$ , and using the Arrow-Pratt measure of local risk premium (see chapter 4 of Copeland and Weston 1979 for a derivation) yields:

$$RP_{pa} = - \left( \frac{\sigma^2(z | m_r)}{2} \right) \frac{u''(m_r)}{u'(m_r)} > 0$$

Here  $RP_{pa}$  captures the difference between the quantities on both sides of Jensen's inequality or the degree of the agent's risk aversion if the risk in question is not too large. Mathematical manipulation yields

$$\begin{aligned} x_r - m_r + \xi RP_{pa} &= P(x < x_r | m = m_r) E[\xi(x_r - x) | x < x_r, m = m_r] \\ &\quad - \frac{\xi u''(m_r)}{2 u'(m_r)} \int_{\underline{x}}^{x_r} (x - m_r)^2 d_x Q(x, m_r) \end{aligned}$$

Introducing the modified Pratt-Arrow local risk premium results in:

$$RP_{pa}^M = - \frac{\sigma^2(z | x > x_r, m = m_r)}{2} \frac{u''(m_r)}{u'(m_r)}$$

Taking the Taylor series expansions once again gives

$$x_r - m_r + \xi RP_{pa}^M = P(x < x_r | m = m_r) E[\xi(x_r - x) | x < x_r, m = m_r] \quad (26)$$

which reduces to (24) if  $u'' = 0$ . This result however depends on the assumption that  $\sigma_\varepsilon^2$  is small enough so that the quantity of risk is not very large, and the Arrow-Pratt approximation may be used.

## 6. Conclusion

Marriages today are commonly preceded by a period of cohabitation. For many couples, this period of cohabitation serves as a trial marriage. It is a period in which they can decide if their cohabiting partner is the right choice as a spouse.

Despite a vast empirical literature, mainly in demography, there are few theoretical models that can be used to study the modern phenomenon of cohabitation followed by marriage. This paper develops such a model. The framework used is a two sided search-matching model. To this framework, the paper adds imperfect information, learning and risk; new concepts in the search-matching literature. This extended model is consistent with empirical findings from demographic studies on cohabitation and marriage.

## Appendix

### Note 1: The calculation of $m(y)$ when there are two types of husbands.

This note gives an example of the case when husbands are of two types: good and bad. Denote the type of the husband by the random variable  $X$ . Assume that  $X$  takes on two possible values  $x_G$  and  $x_B$  with probability  $p$  and  $1 - p$  respectively. Good husbands are men who have pizazz  $x_G$ , while bad husbands are men who have pizazz  $x_B$ .

In addition, when a woman meets a man, he can either appear to be a good husband or appear to be a bad husband. That is, his observed pizazz,  $y$  is a realization of the random variable  $Y$ , and can take on one of two values,  $y_g$  or  $y_b$ . Those that seem like good husbands have observed pizazz  $y_g$  while those that seem like bad husbands have observed pizazz have  $y_b$ . However, appearances are not always to be trusted, as some bad husbands may appear to be good husbands and vice versa.

The discrete (two-type) counterpart of  $m(y)$  is  $x_G P(x_G | y) + x_B P(x_B | y)$ . A woman then infers her partner's true type  $x$  from his signal  $y$  according to Bayes' law:

$$P(x_G | y) = \frac{P(y | x_G) p}{P(y | x_G) p + P(y | x_B) (1 - p)}$$

Notice that  $P(x_B | y) = 1 - P(x_G | y)$ .

Let  $\alpha = P(y_g | x_G)$ . Note that  $\alpha$  is the probability that a good husband looks like a good husband (he correctly signals his type). Then, the posterior probability that a man is a good husband given that he looks like a good husband, is

$$P(x_G | y_g) = \frac{\alpha p}{\alpha p + (1 - \alpha) (1 - p)}$$

which is positively related to  $\alpha$  and  $p$ .

For comparison, the continuous counterpart of  $[P(x_G | y), P(x_B | y)]$  is  $f(x | y) = f(y | x) f_X(x) / f_Y(y)$ . The continuous counterpart to  $(\alpha, 1 - \alpha)$  is  $f(y | x)$ .

### Note 2: The joint distribution of observed and true pizazz.

Suppose that the joint density of true pizazz,  $X$  and observed pizazz,  $Y$  is denoted  $f_{X,Y}(x, y)$ . The distribution of true pizazz conditional on observed pizazz is:

$$Q_{X|Y}(x | y) = \Pr(X \leq x | y) = \frac{\int_{\underline{x}}^x f_{X,Y}(x, y) dx}{\int_{\underline{x}}^{\bar{x}} f_{X,Y}(x, y) dx}. \quad (27)$$

The expectation of true pizazz conditional on observed pizazz denoted  $m(y)$  can be written as

$$m(y) = E(X | Y = y) = \int_{\underline{x}}^{\bar{x}} x Q_{X|Y}(x | y) dx = \frac{\int_{\underline{x}}^{\bar{x}} x f_{X,Y}(x, y) dx}{\int_{\underline{x}}^{\bar{x}} f_{X,Y}(x, y) dx}. \quad (28)$$

Since  $m$  is increasing in  $y$ , the support of  $m$  is  $[\underline{m}, \bar{m}] = [m(\underline{y}), m(\bar{y})]$ . It is then legitimate to invert the function  $m$ , denoted  $y(m)$ . The distribution of the random variable  $M = E(X | Y)$  is given by

$$F(m) = \Pr(M \leq m) = \Pr[Y(m) \leq y(m)] = \int_{\underline{y}}^{y(m)} \int_{\underline{x}}^{\bar{x}} f_{X,Y}(x, y) dx dy \quad (29)$$

Therefore,

$$Q_{X|Y}(x | y) = Q_{X|Y}(x | y(m)) \equiv Q(x, m), \quad (30)$$

with domain  $S_X \times S_M$  where  $S_X = [\underline{x}, \bar{x}]$  and  $S_M = [\underline{m}, \bar{m}]$ .

### Derivation of (11).

The expectation of the value function in stage 3 ((10)) is

$$E_{x|m}J = E_{x|m} \max [\psi_{\tilde{m}_e}(x), R(\tilde{m}_e)].$$

Recall that in the above, the decision maker is a woman. Her expected true pizazz conditional on observed pizazz (based on her private knowledge of her own  $x$ ) is denoted  $\tilde{m}_e$ . The true pizazz of the potential husband is denoted  $x$  and his expected true pizazz conditional on his observed pizazz  $y$ , is denoted  $m$ . Expanding the above, we obtain,

$$\begin{aligned} E_{x|m}J &= \Pr(\psi_{\tilde{m}_e}(x) \geq R(\tilde{m}_e)) E_{x|m} [\psi_{\tilde{m}_e}(x) | \psi_{\tilde{m}_e}(x) \geq R(\tilde{m}_e)] \\ &\quad + [\Pr(\psi_{\tilde{m}_e}(x) < R(\tilde{m}_e))] R(\tilde{m}_e). \end{aligned} \quad (31)$$

The policy to be followed by the woman is to marry her cohabiting partner if his true pizazz,  $x$  is greater than a threshold level,  $x_r$ . That is,  $\Pr(\psi_{\tilde{m}_e}(x) \geq R(\tilde{m}_e)) = \Pr(x \geq x_r)$  and  $\Pr(\psi_{\tilde{m}_e}(x) < R(\tilde{m}_e)) = \Pr(x < x_r)$ . Using  $x_r = rR(\tilde{m}_e)$ , (31), may be simplified as follows

$$EJ = \frac{1}{r} \{ \Pr(x \geq x_r | y) [E(x | x \geq x_r)] + \Pr(x < x_r | y) x_r \} \quad (32)$$

Adding and subtracting the term  $\Pr(x > x_r | y) x_r$  to RHS of eq32) and re-arranging terms yields,

$$EJ = \frac{1}{r} \{ \Pr(x \geq x_r | y) [E(x | x \geq x_r) - x_r] + x_r \} \quad (33)$$

Recall that in the above,  $\Pr(x \geq x_r | y)$  is in fact conditional on the woman's own expected pizazz based on her private knowledge of her true pizazz. Therefore, we write  $\int_{x_r}^{\bar{x}} dQ(x, m | \tilde{m}_e)$  in place of  $\Pr(x \geq x_r | y)$ . Using  $x_r = rR(\tilde{m}_e)$ , (33) may be written as

$$EJ = \frac{1}{r} \left\{ \int_{x_r}^{\bar{x}} (x - x_r) dQ(x, m | \tilde{m}_e) \right\} + R(\tilde{m}_e). \quad (34)$$



The term  $\int_{x_r}^{\bar{x}} (x - x_r) dQ(x, m | \tilde{m}_e)$  can be integrated by parts (this is done using a change of variable technique; denoting  $Q(x, m | \tilde{m}_e)$  by  $G(x)$ , and using  $dG(x) = -d[1 - G(x)]$ ). In addition, using  $\psi_{\tilde{m}_e}(x_r) = R(\tilde{m}_e)$  yields (11) in the text of the paper:

$$EJ = \left(\frac{1}{r}\right) \int_{\psi_{\tilde{m}_e}^{-1}(R(\tilde{m}_e))}^{\bar{x}} [1 - Q(x, m | \tilde{m}_e)] dx + R(\tilde{m}_e). \quad (35)$$

### Derivation of (17):

Recall that the expectation of the value function (from (5) in stage 2 is

$$EV_{\tilde{m}_e}(m) = E_M \max \left\{ \varphi_{\tilde{m}_e}(m), R(\tilde{m}_e) \right\}. \quad (36)$$

We begin by re-writing (4):

$$(\alpha(\tilde{m}_e) + r) R(\tilde{m}_e) = \alpha(\tilde{m}_e) E_M \max \left\{ \varphi_{\tilde{m}_e}(m), R(\tilde{m}_e) \right\} \quad (37)$$

The RHS of the above equation can be shown to equal

$$\begin{aligned} & \alpha(\tilde{m}_e) \left\{ P(\varphi_{\tilde{m}_e}(m) < R(\tilde{m}_e)) R(\tilde{m}_e) \right\} + \\ & \alpha(\tilde{m}_e) \left\{ P(\varphi_{\tilde{m}_e}(m) \geq R(\tilde{m}_e)) E \left[ \varphi_{\tilde{m}_e}(m) | \varphi_{\tilde{m}_e}(m) \geq R(\tilde{m}_e) \right] \right\} \end{aligned}$$

Therefore, eq(37) can be written as:

$$rR(\tilde{m}_e) = \alpha(\tilde{m}_e) P(\varphi_{\tilde{m}_e}(m) \geq R(\tilde{m}_e)) \left\{ E \left[ \varphi_{\tilde{m}_e}(m) | \varphi_{\tilde{m}_e}(m) \geq R(\tilde{m}_e) \right] - R(\tilde{m}_e) \right\}.$$

Using  $\varphi_{\tilde{m}_e}(m_r) = R(\tilde{m}_e)$ , the above equation can be re-written as

$$\begin{aligned} R(\tilde{m}_e) &= \frac{\alpha(\tilde{m}_e)}{r} \int_{m_r}^{\bar{m}} [\varphi_{\tilde{m}_e}(m) - \varphi_{\tilde{m}_e}(m_r)] dF(m | \tilde{m}_e) \\ &= -\frac{\alpha(\tilde{m}_e)}{r} \int_{m_r}^{\bar{m}} [\varphi_{\tilde{m}_e}(m) - \varphi_{\tilde{m}_e}(m_r)] d(1 - F(m | \tilde{m}_e)). \end{aligned}$$

Integrating by parts and noting that  $F(\bar{m} | \tilde{m}_e) = 1$  yields

$$R(\tilde{m}_e) = \frac{\alpha(\tilde{m}_e)}{r} \int_{m_r}^{\bar{m}} [1 - F(m | \tilde{m}_e)] \varphi'_{\tilde{m}_e}(m) dm. \quad (38)$$

Substituting (15) in the above yields

$$\begin{aligned}
R(\tilde{m}_e) &= \frac{\alpha(\tilde{m}_e)}{r(r + \lambda(\tilde{m}_e))} \int_{\varphi_{\tilde{m}_e}^{-1}(R(\tilde{m}_e))}^{\bar{m}} [1 - F(m | \tilde{m}_e)] \\
&\quad \times \left[ 1 - \frac{\lambda(\tilde{m}_e)}{r} \int_{\psi^{-1}(R(\tilde{m}_e))}^{\bar{x}} \frac{\partial Q(x, m | \tilde{m}_e)}{\partial m} dx \right] dm
\end{aligned} \tag{39}$$

which is (17) in the text of the paper.

## References

- Bloch, F and H. Ryder (2000), "Two-Sided Search, Marriages and Matchmakers." *International Economic Review* February, 41(1), 93-115.
- Bumpass, L. and J. Sweet (1989), "National Estimates of Cohabitation." *Demography* 26, 615-625.
- Bumpass, L., J.A. Sweet and A. Cherlin (1991), "The role of cohabitation in declining rates of marriage." *Journal of Marriage and the Family* 53(4), 913-927.
- Burdett, K. and M. Coles (1997), "Marriage and Class." *The Quarterly Journal of Economics* CXII, February, 141-168.
- Burdett, K. and M. Coles (1999), "Long-term Partnership Formation : Marriage and Employment." *The Economic Journal* 109, June, 307-334.
- Chade, H (1999) Two-Sided Search, Heterogeneity, and Noisy Observable Types, mimeo, Arizona State University.
- Copeland, E.T. and J.F. Weston (1983), *Financial Theory and Corporate Policy*, second edition, Addison-Wesley Publishing Company.
- Ermisch, J. and M. Francesconi (1999), "Cohabitation in Great Britain: Not for Long, but Here to Stay." Institute for Social and Economic Research, University of Essex working paper.
- Kiernan, K.E. (1996), "Partnership Behaviour in Europe: Recent Trends and Issues." in *Europe's Population in the 1990s* edited by D. Coleman. Oxford: Oxford University Press.
- Rao Sahib P and X Gu (2000) "Living in Sin" and Marriage: a Matching Model, *Journal of Population Economics*, forthcoming.
- Sargent, T.J. (1987), *Dynamic Macroeconomic Theory*, Harvard University Press.
- Shimer R. and L. Smith (2000), Assortative Matching and Search 68(2), *Econometrica*, March, 343-69.
- Waters, M.S. and R. Ressler (1999), "An economic model of cohabitation and divorce" *Journal of Economic Behaviour and Organization* Vol 40, 195-206.