Endurance and age

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Endurance and Age: Evidence from Long-Distance Running Data

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Abstract

This paper analyzes the impact of aging on long-run endurance. We analyze the determinants of running speed on distances from 5K to the marathon. We model running speed as a function of distance, age, and sex. We find evidence for interaction between age and sex, but not between distance and sex. Our model shows that official track and field age grading overestimates human performance at high ages, although athletes are still able to perform at a high level at old ages.

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1 Introduction

This paper analyzes human capability to run at high speed on various distances at different ages.[1] To that purpose we analyze top performances by age of a large population (US citizens) across various long-run distances. Our goal is to analyze the joint impact of age, sex, and distance on running speed, while the main focus is on the impact of aging. How fast do old runners slow down? Is there any relation between the statistical estimates of the aging elasticity of running speed and biological results? Are there substantial differences between men and women? Do elderly women perform better on the long distances? These and more questions will be addressed.

There are some raw notions on the impact of aging on running performance. For instance the *World Association of Veteran Athletics* (WAVA) publishes age-group dependent correction factors. These correction factors are used in grading results in track and field events. One of the serious flaws of the WAVA data is the rather arbitrary subdivision of ages into age classes. Our goal is to give more precise cross-event cross-sex estimates of correction factors. Moreover, there is some suspicion against the WAVA-data in the sense that the WAVA-tables overestimate the capabilities of very old runners (see Fair, 1994). We present estimates of pooled models that use information on age and sex of the runner and the distance covered to explain performance. The final results give "normal" values speed depending on age and sex on various distances varying from the 5K up to the marathon. These estimates give a precise indication of human capabilities in endurance efforts across the life span.

Why do we use running data if we want to analyze the impact of aging on endurance? There is on main reason: running data are of a high quality and publicly available. Moreover, running data present a precise record of age-dependent results. The use of long-distance events implies that we focus on a particular aspect of the human body: physical endurance. This implies that we do not focus on muscular strength, "souplesse", or elegance. In other words: we include aerobic events and exclude the track and field sprint events. We include data on distances between 5 kilometer and the marathon (sometimes even 50 miles) for ages between 3 and 95. We use both data for men and women. Our methodology is based on smoothing out measurement errors in the

[1] Comments and suggestions made by Karel-Jan Alsem, Ray Fair, Lambrecht Kok, Ruud Koning, Gerard Kuper, Bert Scholtens, and Joost Wessels are gratefully acknowledged.

data via ordinary least squares estimation[2]. Section 2 presents the basic theoretical notions on modelling running speed. We do not derive a formal model, but present some plausible physiological assumptions in explaining running speed. We introduce the data in Section 3, give some basic statistics, and analyze the data in Section 4. We give a model that predicts the optimal velocity based on age, distance, and sex. We present various robustness checks on the results and compare our results with others. We sum with some conclusions in Section 5.

2 **Theory**

In this paper we analyze endurance of the human body by the key variable speed on long distance running events. Below we present some insights found in the literature. What is the lifetime pattern of capabilities of the human body with respect to endurance? This is a problem that is usually addressed by physiologists. Running speed has two main dimensions. First, one can analyze the so-called anaerobic running, say sprinting. Here the muscular structure of the body is one of the key determinants of speed. Also the reaction time at the start is important. In anaerobic events the physiologic differences between women and men determine to a large extent the differences in results between the sexes. It is known that in sprint events women *e.g.* need a longer distance to reach their maximum speed (67 meter for women versus 50 meter for men, see Grubb, 1998). It is also obvious that in sprint events there are large differences in speed during the event. This fact makes it troublesome to use average speed during an event as an indicator. Secondly, one can analyze the aerobic events, where endurance of the body is the most prominent physical characteristic. Here it is the ability of the body to transport oxygen to the muscles (the so-called VO2max) that plays a key role. Again the muscular structure differences between women and men make that men are able to run faster in absolute sense. But there is less variation between men and women in speed during the events. We find in analyzing velocities across distances the differences between men and women to be rather small (see hereafter). The aerobic events are also characterized by the fact that a constant velocity is optimal during the race for distances longer than 800 meter (see Keller 1974). In this paper we only analyze the aerobic events. So we refrain from all sprints and start even with the long-distance of 5K.

As statisticians we are interested in the running speed of all members of the entire world population. This leads to a measurement problem, since it is most likely that not all people will

[2] An alternative would be to estimate a human production frontier that fits the maximum efforts.

participate in sports events. It is evident that there is a serious selection bias in measuring the finishers in track and field and road racing events. This is a problem that we cannot solve, but some attention to this issue is necessary before we can turn to the top performance of the human body. Let's assume that we can measure physical endurance by the analysis of the total time used in a certain road race event. Let q denote the log of a runner's time in the race. For all runners of a given age we could draw a theoretical frequency distribution for q. Figure 1 presents a density function of this type. It includes data from a student race, organized in May 2000, for 358 female runners. Being a student race, one could reasonably assume that the data apply at least to women of the same age class. The density plot gives the log of the time in seconds on 5K. We can see that the lower bound is around 7, while the upper bound is around 7.8. A Jarque-Bera of 1.39 indicates that the distribution is normal[3]. The fastest time is a little more than 19 minutes, which is still far away from the international world record (14:28.09) though. Still we could argue that in this Dutch student population the log of 7 can be seen as the lower bound. If this density applies to Dutch students, probably a similar distribution applies to other population groups (although with a different mean and variance). The argument we make here is that our main concern is for the lower bound (or in terms of velocity upper bound), while there is always a density function linked with this lower bound. From the results we showed one might conclude that the density function is normal.

In this paper we focus on the lower bounds of running time (see Blest, 1996). By doing so, we are interested in the human production frontier. This implies that we analyze running data given perfect race conditions, the use of the best training methods and equipment possible, and perhaps more important, the mental ability of the athlete to compete. As shown above, we cannot analyze behavior worse than the frontier, apart from saying that the outcomes are probably normally distributed for a certain age. It would be interesting to use individual specific historical data to analyze the problem, because the sports-age of the individual can be important for current performance. A lack of public available data prevents analyses of this type though. We follow Fair (1994) in his analysis of the problem how fast do old men slow down (but note that we analyze old women as well). Fair postulates a relationship between the lower bound and age as depicted in Figure 2. The lower bound is infinite for small babies, falls down to a certain age, stays perhaps rather constant (between age h and j) and then, unfortunately, starts to rise. Fair is interested in the latter part and postulates a rather modest linear incline from age j up to a certain

[3] Grubb (1999) argues that the density plot of the racing time is likely to be more skewed than the density plot of the speed. We take the lognormal distribution of the racing time.

age *k,* where after the increase becomes quadratic. Fair estimates this age *k* for men only using age-specific data for both running and field events. For various models Fair observes that the age of 47-48 seems to be the critical age. Moreover Fair concludes that the rate of physical deterioration of the human body is really slow, but higher than the veteran athletic organizations make us do believe. This implies that official age-correction factors, like the WAVA of the *World Association of Veteran Athletics* or the the *Masters Age-Graded Tables* (MAGT), really disfavor the old runners.

Without going into detail into physiology: why would we expect a pattern like in Figure 2? It is sure that babies and the dead don't run. All in between is rather mysterious though. It is the growth of the body, muscles and hormones that increases performance. For endurance the so-called VO2max is relevant. This is the capacity of the body to absorb oxygen and transport it to the muscles within a certain time period. The VO2max increases with age while the human body grows. Through training, athletes are able to increase their natural level of VO2max. It is also experience with certain training habits that allows athletes to be able to use the maximum capacity to absorb oxygen for a certain period of time. After a certain age the VO2max decreases. Fair refers to some studies that report results for ages between 40 and 70 years. Although the estimates vary, a yearly decline of 0.5 to 0.9 per cent per year, seems to be a meta-outcome (see Dehn and Bruce, 1972, Heath *et al.*, 1981, Rogers *et al.*, 1990). For ages close to 40 the lower figure applies, while the decline is more serious for the older people. For ages higher than 70, to our knowledge, there is no comparable medical study available.

Up to now we discussed one dimension of the problem: how does velocity relate to age. We are also interested in the impact of distance on velocity. It is clear that anaerobic events can be run at higher speeds. Since we are not able to present theory in this respect, we concentrate on statistical evidence on the results. Riegel (1981) and Blest (1996) use a loglinear model $log(t) = a+b\ log(dist)$ to fit world records, where t is the time in seconds and $dist$ the distance in meters. Normal values for a are around –2.7 and for b around 1.1. This model can also be used in the terms of speed: $log(v)=-a+(1-b)log(dist),$ where v is the velocity in meters per second. (In the appendix we give estimates of this model based on world records from 100 meter up to 50 miles for men and women. We will use the findings later on). Francis (1943) proposed the following model: $v = C+A/(log(dist)-B),$ where C is the speed at very long distances, exp(B) is the asymptote of the distance where the maximum speed can be observed and A is a measure of the decrease of speed. In the literature (see Grubb, 1998) it is found that female runners have a lower long-distance speed *C*, but also a lower rate of slowing down *A*. Mosteller and Tuckey (1977) use a shifted power transform $v=A(dist-B)^x+C$ to linearize the Francis-model, where *x* is a negative power. In the literature there are estimates of *B*. *B* is the distance at which the race changes from an anaerobic to an aerobic event (say from sprint to long-distance race). Keller (1974) estimates *B* to be 291 meter. Women tend to have lower speed at long distances, but are found to have lower rates of slowing down.

A next aspect of the model is the relation between age and distance in its impact on running speed. Do older runners perform relatively better in long-distance events? In professional athletics one can observe that older runners switch from 1500 meter to 3K or 5K events, and 10K runners to the half marathon and marathon. The ability to run at real high speed seems to decline with age, while endurance can be stimulated and intensified by training. Fair finds evidence for this hypothesis for men between 35 and 70 years. After 70 years the performances on long distances seem to slow down relatively stronger. This fact implies that a simple analysis of distance correction must be age dependent. Simple analyses of theoretical running times between differences like Robinson and Tawn (1995) for the 3K and 1500 meters (with a factor 2.16) must be interpreted with care.

We follow Fair with respect to the modeling strategy to a large extent. We use his postulated relation between running effort and age. Our study has a different angle though. We analyze only long-distance running data. Fair analyzes also field events like the long jump and pole vault. Since these events focus more on muscular ability we refrain from those. Moreover we analyze men and women (Fair only studies men). As such we can use more data and test for similarities between men and women. Another difference with the Fair-study is that we analyze the whole lifetime development, while Fair concentrates on the ages over 35. Since Fair is interested in the mysterious age where serious decline of the male body starts he uses a special frontier-function method to estimate the curve of Figure 2. He considers the estimated residuals to be nonnegative to proxy the frontier. We take another approach and consider the empirics of Figure 2 to contain measurement error. We therefore use a higher-order polynomial to estimate the whole curve. To that extent we pool data from the various events for both women and men, since we expect the curves to have identical shapes across distances and sex. We do not estimate the frontier, but the average shape. Experimenting with frontier estimation did not change the conclusion with respect of the shape of the fitted patterns (it was relevant to the precise position of the frontier of course).

3Data

Athletics and statistics go hand in hand. Due to the richness of data sources various problems are addressed in the literature[4]. In athletics there is some tradition in measuring the effects of aging. Various athletic organizations use age-graded records (see *Mundle et al.* 1991) to be used by race officials for age-graded events. The most prominent sources are the official *World Association of Veteran Athletics* (WAVA) records, and the *Masters Age-Graded Tables* (MAGT). The WAVA published the first set of data in 1989 and updates this set every 5 years. The data contain data for both outdoor and indoor records for men and women for various events ranging from the 100 meter to one hour runs and for various field events. The WAVA data give records per 5-year class. The MAGT data presents similar information for track events. One of the disadvantages of both sets is that it is not transparent how the tables are compiled. Moreover, there is some suspicion that the theoretical tables are biased against older runners. Therefore we start our analysis by observing actual records per age and distance.

We use data from the Long-Distance Running Association[5]. This association keeps records of road racing events on various distances in the US. For each age the fastest time ever ran by an American citizen is recorded with the name, sex, and date of birth of the runner, as well as the name and date of the race. Moreover, the files contain information on special conditions of the event (downhill track, wind, etc). The following distances are recorded: 5K, 5M, 10K, 12K, 15K, 10M, 20K, Half Marathon, 25K, 30K, Marathon and 50M (K stands for kilometers and M for miles). For the last event not so many data are available. Nevertheless, this data set provides a unique set to analyze age-dependent abilities. The recorded ages vary from 3 to 95 years. In this section we give a preliminary statistical analysis of the data. In the next section we present a model that estimates the velocity of human beings across distances, age, and sex.

Figure 3 gives a first glance at the data. We plot the actual velocity of women on the marathon. This figure shows two lines: MARV for the raw data and HMARATV for the so-called filtered series. By filtering the series we try to get rid of measurement error. This simple plot shows a pattern that is familiar to all the data. Up to the age of 30 there is a serious progress in velocity of women. After that we observe an almost linear decline. The figure also shows another

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[4] Famous problems are the comparison of events (Who holds the best world record?, see Grubb, 1998), the analysis of the progress of world records (Henry, 1955), world records for 200 and 400 meters per lane, and optimal velocity in the race (Keller, 1974).

feature of the data: measurement errors. There are various reasons to believe that measurement errors are present in the data. Individual strong performance can trouble the general pattern. Moreover, for very young and very old ages there are probably a few participants in the events (say below 10 and above 75 years old). Since we are interested in the general development of human endurance we are interested in series without the measurement errors. We solve this problem in two ways. In the analysis of Figure 3 we correct the data for each event by filtering. Filtering the data has the advantage of being a rather simple procedure (we use the Hodrick-Prescott[6] filter). Through the filtering we are able to compare the theoretical velocity of different ages per event. We compute from the filtered series so-called age-correction factors. For each event the age with the maximum speed gets the value 1. The other figures represent the relative maximum speed for the corresponding age and can be used to correct results for a specific distance. For both men and women, we give these factors for all distances (except the 50M, since there are not enough data), as well as correction factors for the arithmetic average of the speeds over events (last column). Tables 1 and 2 contain the data. These data can be used for intra-event comparisons.

The series show similar patterns for both men and women. There are some preliminary conclusions:

- The top performance of both men and women is in the ages 27 to 30 years
- At the age of 4 people have about half of the final capacity and at the age of 10 about 75 per cent of their final ability. Young females develop quicker than young males. An annual increase in performance at young age of about 8 per cent is possible.
- After the age of 30 a gradual decline starts. Both male and female runners have about 90 per cent of their maximum performance at the age of 45. At the age of 60 years male runners have about 80 per cent of their original capacity, while female runners have only 75 per cent.
- Male runners show a serious decline in performance after 70 years old. This implies that our simple statistical analysis does not confirm Fair's results. Fair estimates a turning point of 47-48 years. A difference between our analysis and Fair's is that we use all available age observations, while Fair skips the ages above 70 years. Female runners start to decline after 55 years but at a slower rate than male after 70.

[5] See the website http://www.usaldr.org

[6] The Hodrick-Prescott-filter is a two-sided filter that minimizes the variance of the raw series around the filtered series.

- Overall there is a rather optimistic picture for the human body: Even at high ages it is still possible to perform at a high rate of endurance.

The rates of decline are modest and in the range of the estimates of the loss of VO2max. A preliminary conclusion from this descriptive statistical analysis is rather optimistic. Human beings are able to perform at a relative high percentage of the maximum capacity up to a high age. We should not forget that we estimate the “frontier”. There is no evidence that the normal distribution depicted in Figure 1, remains constant over the life span, i.e. the mean and variance may be age dependent. There is also no proof of the fact that anyone is willing to the utmost best to perform at the highest level. But one can turn the argument by saying that there is also no evidence that it is impossible to perform at a high level at higher ages. Another conclusion is that it might be dangerous to demand a constant performance of workers after 30 years old.

In the next section we turn to a model of velocity, distance and sex and combine all the information in more precise comparisons of performances. Since the shape of the curves seem to be similar across distances and sexes we can exploit the whole information set and analyze the performances across distances, ages and sex. This allows for a more precise estimation of the human age-endurance curve.

4 Modeling velocity

Before we start to estimate a model that combines information on distance, age, and sex to explain velocity, we first do some preliminary analysis on the relation between velocity and distance. We use the Mosteller and Tukey (1977) model, as described in Section 2: $v(i)=A(dist(i)-B)^x+C(i)$. As Grubb (1998) argues the estimates of *B* are pretty standard and we follow his suggestion to fix *B* at 250 meters. Moreover we use Grubb’s assumption on the power transformation and set x=-0.267. Next we estimate *A* and *C(i)* for both men and women. By doing so we give some indication of the effect of differences in distance on velocity. We use a pooled model with fixed effects (for the distances) on a balanced panel for men (7 to 78 years) and women (6 to 70 years). For men we find for *A*: 17.39 (*t*-value 49.64). This estimate is close to Grubb’s result for absolute world records. For women we get for *A*: 17.35 (*t*-value 40.19). Grubb finds for women 16.15. So our cross-section variation suggests that including all the age-dependent records increases the loss women show as opposed to using the world records alone.

Moreover, our estimate for women is now close to the men's parameter. Figure 4 gives for the ages 6 to 78 the long-distance speed *C(i)* for men and women. Grubb finds 4.69 for men, since he uses the absolute world records, while we get 3.53 on average and 4.43 as a maximum. For women Grubb gets 4.12, while we find on average 2.98 (and 3.71 as a maximum). The difference between the men's and women's average long-run velocities is rather similar. The graph shows that the pattern over the life period of long-run velocity is somewhat different for women. Girls show relatively high long-run speed ability. Between 30 and 45 women do relatively a bit better, after 45 there is a steady increase in the relative long-distance performance of men (up to 70). From this initial partial analysis of the relation between velocity and distance we conclude that there is no big difference in the slowing down over distance between men and women. There are differences in age effects though, since the long-run implied velocities of men and women differ over the life span. This finding calls for a simultaneous modeling of velocity, age, distance, and sex.

From the postulated relation between maximum velocity and age and the initial data-analysis in the previous section it be seen that the relation between velocity and age is nonlinear. Unlike Fair, who only analyses the curves for ages over 35, we want to estimate the whole curve. One can use the data in two respects. First, we might estimate a kind of frontier, assuming that the high extreme values of velocity are on the true production frontier. Second, we might consider again the extreme values to be outliers due to measurement errors. The latter approach seems reasonable, because we deal with road racing data (as opposed to track data). Moreover it seems to be the case that for extreme ages (young and old) there are less competitors. Finally global inspection of the data in Section 3 shows that some of the events have less competition. The 12K and 30K events seem to show lower velocities. Especially the 50M is an outlier, since there is low data availability. Despite the fact that there might be differences in levels between the various distances there are no obvious dissimilarities in the shapes of the curves. Figure 5 gives the contour plot of all the raw data for the men's events. One can see the rough surface of the velocity contour. In this section we will smooth this surface.

We therefore estimate an n^{th} order polynomial for all the events at once. First we distinguish between men and women. After that we test for pooling men and women. In the regressions we interact age and distance. The methodology used is rather straightforward. We postulate a similar pattern for each distance and stack the data in one matrix. The dependent variable is *y(ijt)* denoting velocity for age *t* on event *i* for sex *j.* The independent variables are age *t*, its higher order terms, distance *i*, and its higher order terms. Moreover we include cross terms.

Experimenting with the order of the polynomial yielded a 5th order polynomial in age and distance. Table 3 gives the results for men and women in separate regressions. The second and third columns denote the estimated coefficients and corresponding t-values for men; columns 4 and 5 give the similar data for women. It appears that the outcomes are quite similar for men and women. There is no difference in the shape of the spline. Therefore we decided to use a pooled model for both men and women and use fixed effects for the sex. Table 4 presents the results. The F-test on pooling the model equals 0.40, which justifies pooling. This set of results is our base model.

Next we explore our pooled set for the interaction between age, distance, and sex. Table 5 gives our best model. We do find interaction between age and sex, distance and sex, and age and distance. Looking at the coefficients we find that up to 54 there is a benefit in running at longer distances. After 54 it seems more profitable again to switch to shorter distances. We do not find any statistical evidence for interaction among the three directly. The results of this table can be used to give a theoretical prediction of the speed of men and women across distances and age. Figure 6 gives the smoothed contour plot for the male velocity. This Figure corresponds to Figure 5 that contains the raw data. The female contour looks at first sight quite identical, but is slightly different (see Figure 7). As is clear from the results of smoothing per distance aging is somewhat different for females. While men show a slow linear decline up to the age of 70 to 72 and a steep decline after that, women have a stronger but constant decline after 42. For both we observe relatively strong performances on the marathon as compared to the 30K. This might be due to less competition on the 30K.

The contour plots are rather difficult to use in day-to-day practice. Therefore we transform the plots into age-correction factors. We normalize the maximum performance for both men and women to 1. Table 6 gives the implied correction factors for men and Table 7 for women. We again exclude the 50M. We are able to compare Table 6 with Table 1, the table constructed on the smoothing per distance. The main advantage of Table 6 is that in this table we can compare performance between distances. One can see that the absolute top performance is for the 5K event at 27-28 years old for men. In the longer distances there is a bit higher optimal age to perform. Table 7 gives the same results for women.

Next we turn to the implied differences between females and males. Figure 7 gives the contour plot of the differences in speed between men and women. The plot clearly shows that girls are faster than boys up to the age of 6 years. After that, men get an advantage up to the age of 30. Women improve relatively to men to the age of 42. After that men gain relatively up to 72. After that age women definitely get the stronger results. It seems that the distance is not a crucial determinant of differences between women and men (see also below in the robustness check of the model).

4.1 Robustness

The results up to know show two things that demand a detailed robustness analysis. First, we found in our preliminary analysis of the correlation between speed and distance no serious difference in slopes between men and women. In our pooled model we did find significance of the cross products of distance and sex. The second observation concerns the weighting of events in the estimation. From eyeballing the data it is clear that some of the distances are less dense in competition than others. Here we observe that the 5M, 12K, 30K, and of course 50M below average predicted speed, while the half marathon shows more competition and higher speeds therefore. To that extent we re-estimate the model for the events 5K, 10K, 15K, 10M, 20K, half marathon, 25K, and marathon. The results are in Table 8. The Table shows that the interaction between distance and sex disappears. This is in line with the simple correlation between velocity and distance for men and women. Moreover, one can observe that the order of the polynomial decreases. For distance we find only a third-order effect, while for age a fourth-order adjustment.

We use the predictions from the restricted model to forecast the excluded events and compare the actual and fitted values. It appears that our model overestimates the realized male speed on the 5M by 0.23, on the 12K by 1.10, and on the 30K again by 0.22 on average. One could interpret these findings by the relative poor performance on these distances. On the 50M our model predicts far too high speeds. So the out of sample performance of the restricted model is poor. For women we get 0.21 on the 5M, 1.07 on the 12K, and 0.36 on the 30K. Also the 50M predictions are too high. We conclude to state that we prefer to use all available information, but to be aware of the overestimation of performances on the 5M, 12K and 30K.

4.2 Comparison with other age-grading results

How do our results compare to the results found by others? In order to validate the results and to show the usefulness we need to give an indication of the relative performance of our model. First we highlight the innovations of the model:

- We analyze the full lifetime of a runner instead of an analysis of veteran records only;
- We analyze endurance events simultaneously for men and women across distance and age;
- We fully focus on endurance (and do not analyze sprints or field events);
- We use regular estimation and consider the residuals to measurement error instead of a production frontier method.

How do these innovations lead to other insights than others have found before? Here we can list a set of possible comparative analyses:

- The analysis of absolute world records across events. Is the slope of the velocity-distance curve similar to ours? Here we compare our findings with Grubb (1998);
- Can we use our model, adjusted for the frontier-effect to predict world records? Since we use U.S. records we need to adjust our curves for the world-records.
- After 40, do our results compare to the WAVA, MAGT, and Fair (1994) results?

First we compare our own single event correction factors with the model-correction factors. We take the results for a 10-year old a 70-year old as a benchmark. First we look at the men's results. A 10-year old boy on the 5K needs to be 2.2 per cent faster according to the estimated model. On the half marathon the difference is 3.3 per cent, while for the marathon the difference is almost 1.7 per cent. Although the model predicts lower velocities for other distances, like the 10K or the 10M, we could argue that the model estimates are a bit higher for 10-year old boys. For a 40-year old man we observe that the model predicts about 1 to 2 percent higher speeds on all distances. For a 70-year old man the model predicts almost the same correction factor on the 5K as in the filtered single event series. For the half marathon the model predicts about 2 per cent higher speed, while for the marathon we again get about a 2 per cent lower speed in the model. These results imply that the model adds information that is relevant to the outcomes. For women we observe that the model predicts the speed of a 10-year old girl lower than the single event estimate (about 3.7 per cent). For a 40-year old women we find a slight overestimation of the model of 1 to 2 per cent on the distances. For a 70-year old women the model predicts 2 to 5 per cent faster speeds (5 per cent for the marathon). In general there are differences up to 4 per cent between 10 and 70-year old people and even higher differences for the extreme ages.

Next we take a look at the relationship between velocity and distance. To what extent does our model imply results found by others for the absolute world records? To that extent we compare our results for the age-independent top-performance per distance with the world records. We predict the marathon-time from the 5K record. Here we should keep in mind that the 5K record is a track record, while the marathon is a road event. As shown above we do not find a significant difference between the slope of the speed-distance curves for men and women in our data. Does this hold for the absolute world records? To that extent we replicate Grubb's results by using world records for distances from 100 meter dash up to 50M (21 events in total). See the appendix for the results for the model *log(v)=--A+(1-B) log(dist)*. The results in the appendix show that there are differences in the rate of slowing down, but that unlike Grubb's results, we can confirm that also for the world records there is a common slope. The pooling experiment is the basis for this conclusion. The individual estimation by sex shows that speed of slowing down is a bit bigger for women than for men. This should be kept in mind in the analysis of the world records. If we find a proper estimation for the curves of women, it is most likely that we predict too fast marathon times for men. We need to note further that the US population is surely not representative for the average world athletic population. There is more heterogeneity in the world record holders probably. Our prediction from the world record on the 5K (speed is 6.58 meters/second) is that the marathon could be run at a speed of 87.7 per cent. This implies that the men's marathon could be ran at a speed of 5.77 m/s, which implies a record time of 2:01.52. This is far below the current record time of 2:05.42. For women this would imply 87.4 per cent, which implies that the current 5K (with a speed of 5.76 m/s) would lead to a marathon time of 2:19.42, one minute below the current record time. This implies that our model, although tough for the men, is suited to fit absolute world records.

Finally we compare our results with those found by Fair and the WAVA norms. Note that Fair only analyses men's results. Fair gives general scores for the 5K-21.0975K from 35 to 90, and separate scores for 100 meter, 200 meter, 400mter-21.0975K, 30K-20M, and the marathon. Fair concludes that his scores are less tough for older runners. For instance for a 70-year old runner the MAGT predicts a loss of 39 per cent, while Fair estimates a loss of 43%. For the same distances we find an average correction factor of about 0.68, implying a loss of even 47 per cent. For the marathon Fair finds for a 70-year old a loss of 35 per cent. We find a loss of 43 per cent.

So both Fair and our results indicate rather strong performances on the marathon, but Fair is by fare more optimistic than our results. This might be due to the fact that Fair does not use the high-age results in his estimation.

The WAVA records are for 5-year age groups after 40 years. Table 9 gives the relevant scores for men and Table 10 for women. Again we compare the results for 40 year old and 70 year old men and women. For 40-year old there is hardly any difference in scores. It seems that the WAVA-scores assume that athletes are able to maintain their high velocity at a higher age (mostly 33 to 35 for the long distances instead of our 30 years at the maximum). This assumption mainly affects the differences at 40. At 70 years old we find for women a higher loss rate in our model. For the 5K this is about 5 per cent, for the marathon we find over 10 per cent. For men we also find a higher loss rate in our data, increasing from 2 per cent at the 5K to 7 per cent at the marathon. Again we can confirm Fair's conclusion that the age grading is not to the benefit of old runners can be confirmed by our analysis.

5 Summary and conclusion

In this paper we analyze the impact of aging on physical endurance of the human body. We use age-dependent running data and estimate a model for velocity of long-distance runners. We use information on age, distance, and sex to explain the velocity. Through that we are able to characterize human endurance over the life period. It comes to the fore that the rate of slowing down after 30 years old is amazingly low for both men and women. There is no real robust finding that man and woman slow down at a different rate across distances. Sex interacts with age though in explaining velocity. Age interacts with distance: up to 54 it seems profitable to run longer distances, after that longer distances are tougher for older women and men. Girls perform better than boys, and especially very old ladies do better than the age equivalent men (if they are still alive).

Our model fits the general views by other studies, but differs in detail. We agree with Fair (1994) on the exaggerated optimism of official age grading instruments as the official MAGT or the WAVA –norms. We are even a bit more pessimistic than Fair for very high ages. Our model gives a reasonable approximation for the world records, especially for the women's events.

Appendix – Correlations between velocity and distance for world records

In this appendix we demonstrate the relationship between speed and distance. We use the world record data provided by the IAAF for 100m, 200m, 400m, 800m, 1K, 1M, 2K, 3K, 5K, 10K, 20K, 25K, 30K. Moreover the IAAF presents unofficial records for the half marathon and the marathon. From the Ultra Long Distance Association we use the other data up to 2000K. In estimation we skip the four longest events in estimation.

Distance in km; *t* indicates the time in seconds; *v* the speed in meters/second

Distance	*t*-men	*t*-women	*v*-men	*v*-women
0.1	9.79	10.49	10.2145	9.5329
0.2	19.32	21.34	10.3520	9.3721
0.4	43.18	47.6	9.2635	8.4034
0.8	101.11	113.28	7.9122	7.0621
1	131.96	148.98	7.5781	6.7123
1.5	206	230.4	7.2816	6.5104
1.609	223.13	252.56	7.2110	6.3708
2	284.79	325.36	7.0227	6.1470
3	440.67	486.11	6.8078	6.1714
5	759.36	868.09	6.5845	5.7598
10	1582.75	1771.78	6.3181	5.6440
20	3415.6	4008.8	5.8555	4.9890
21.0975	3557	4003	5.9313	5.2704
25	4435.8	5369.2	5.6360	4.6562
30	5358.8	6425.6	5.5983	4.6688
42.195	7542	8443	5.5947	4.9976
48.27	9451	10876	5.1074	4.4382
50	9818	11319	5.0927	4.4174
64.36	12339	15613	5.2160	4.1222
80.45	17451	20418	4.6101	3.9402
100	22601	25248	4.4246	3.9607
160.9	43939	49661	3.6619	3.2400
200	59468	68431	3.3632	2.9227
1609	1109667	1089520	1.4500	1.4768
2000	1379280	1479840	1.4500	1.3515

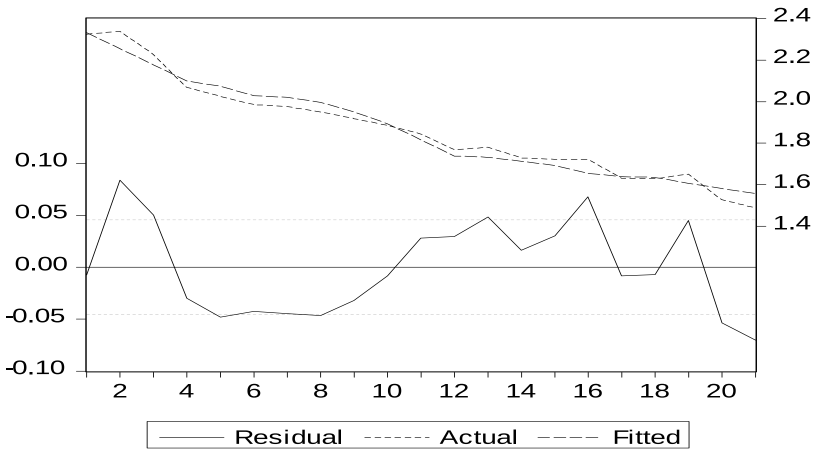
We estimate the model: *log(v)=-A+(1-B) log(dist)* for 100m up to 50M. We exclude the longer events, since it appears that the results point at outliers. First we estimate separately for men and women. Table A below gives the results.

Table A – World-record-curve estimation

	Men	Women
A	-2.85	-2.81
t-statistic	(65.46)	(59.56)
B	1.11	1.12
t-statistic	(23.17)	(23.47)
N	21	21
SSR	0.041	0.046

N= number of observations; SSR = sum of squared residuals

The figure below gives a residual plot for the men's regression. The horizontal numbering coincides with the world records for 100 meter up to 100K.



The plot shows that the model overestimates the velocities on 800 meter to 5K. The model indicates that the 200meter, half and whole marathon show good performances.

If we pool the data we find a common slope of *b*=-1.12 (32.36) and a *a-men* = -2.90 and a *a-women* = -2.76. The SSR of this pooled regression is 0.092, which justifies pooling form an *F*-value of only 0.30.

Finally we estimated the world record curves with the so-called stochastic frontier estimation method. This method is known in productivity analysis and assumes the following model:

$$log(v)=-A+(1-B)\ log(dist)\ -e+f$$

where *e* is a residual that cannot be negative and *f* is white noise. The parameter estimates of *A* and *B* lead to a frontier. We estimate the model such that we penalize positive *e*'s in the sum of squared residuals with a factor of 100 (see also Fair, 1994). We use data for the 100 meter up to the marathon and find for men and women and find for men *A*=2.903 and *B* = -1.110. For women we find *A*=2.800 and the same estimate for *B*=-1.110. So also for the frontier method we find a single slope. It appears that the 200 meter event is a stochastic outlier for both men and women.

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Table 1 – Single event age-correction factors for men

Age	5k	5m	10k	12k	15k	10m	20k	halfmar	25k	30k	mar	mean
1												
2												
3	0.562	0.448										0.487
4	0.593	0.503				0.553						0.526
5	0.624	0.556	0.634		0.582	0.591					0.459	0.565
6	0.655	0.607	0.663		0.615	0.628	0.657	0.597			0.508	0.603
7	0.685	0.653	0.692	0.626	0.647	0.664	0.679	0.628	0.580	0.668	0.557	0.639
8	0.714	0.695	0.720	0.655	0.680	0.699	0.700	0.659	0.621	0.684	0.604	0.674
9	0.741	0.733	0.749	0.685	0.712	0.733	0.722	0.690	0.660	0.700	0.649	0.707
10	0.766	0.767	0.776	0.715	0.743	0.764	0.743	0.720	0.698	0.718	0.692	0.738
11	0.790	0.796	0.801	0.743	0.773	0.794	0.765	0.749	0.734	0.736	0.732	0.767
12	0.813	0.822	0.826	0.770	0.800	0.821	0.787	0.778	0.768	0.755	0.768	0.795
13	0.835	0.845	0.849	0.796	0.826	0.847	0.809	0.805	0.800	0.775	0.802	0.820
14	0.855	0.866	0.870	0.819	0.851	0.871	0.831	0.831	0.829	0.796	0.833	0.844
15	0.874	0.885	0.890	0.841	0.873	0.893	0.853	0.856	0.856	0.819	0.861	0.866
16	0.892	0.902	0.909	0.861	0.894	0.913	0.874	0.878	0.880	0.842	0.886	0.887
17	0.908	0.918	0.925	0.880	0.913	0.930	0.895	0.899	0.901	0.865	0.908	0.906
18	0.923	0.933	0.940	0.897	0.930	0.946	0.914	0.919	0.919	0.887	0.928	0.923
19	0.937	0.946	0.953	0.914	0.945	0.959	0.932	0.936	0.935	0.907	0.945	0.939
20	0.950	0.958	0.964	0.930	0.959	0.970	0.948	0.951	0.949	0.925	0.959	0.952
21	0.961	0.968	0.974	0.945	0.970	0.979	0.962	0.965	0.961	0.942	0.971	0.964
22	0.971	0.976	0.982	0.959	0.980	0.986	0.974	0.976	0.971	0.956	0.980	0.975
23	0.980	0.983	0.989	0.970	0.987	0.992	0.983	0.985	0.980	0.968	0.988	0.983
24	0.987	0.988	0.994	0.980	0.993	0.996	0.991	0.991	0.987	0.979	0.993	0.990
25	0.992	0.992	0.998	0.988	0.997	0.999	0.996	0.996	0.992	0.987	0.997	0.995
26	0.996	0.995	0.999	0.994	0.999	1.000	0.999	0.999	0.996	0.994	0.999	0.998
27	0.999	0.997	1.000	0.997	1.000	1.000	1.000	1.000	0.999	0.998	1.000	1.000
28	1.000	0.999	0.999	0.999	0.999	0.998	1.000	1.000	1.000	1.000	1.000	1.000
29	1.000	1.000	0.997	1.000	0.998	0.996	0.998	0.998	1.000	1.000	0.998	0.999
30	0.998	1.000	0.995	0.999	0.995	0.992	0.996	0.995	0.998	0.998	0.996	0.997
31	0.996	1.000	0.992	0.998	0.991	0.987	0.992	0.991	0.995	0.994	0.993	0.994
32	0.992	0.999	0.988	0.995	0.987	0.981	0.988	0.987	0.990	0.988	0.989	0.990
33	0.987	0.998	0.984	0.992	0.982	0.975	0.984	0.981	0.984	0.982	0.984	0.986
34	0.981	0.995	0.980	0.988	0.977	0.968	0.979	0.976	0.977	0.975	0.979	0.980
35	0.975	0.991	0.975	0.983	0.971	0.960	0.972	0.970	0.969	0.967	0.974	0.974
36	0.967	0.985	0.969	0.977	0.965	0.953	0.965	0.964	0.961	0.960	0.968	0.968
37	0.960	0.979	0.964	0.970	0.958	0.946	0.958	0.958	0.953	0.953	0.962	0.961
38	0.952	0.972	0.957	0.962	0.951	0.939	0.950	0.951	0.944	0.946	0.955	0.953
39	0.944	0.965	0.950	0.955	0.943	0.932	0.942	0.945	0.936	0.939	0.948	0.946
40	0.936	0.958	0.944	0.947	0.936	0.925	0.935	0.938	0.929	0.932	0.940	0.939
41	0.929	0.950	0.937	0.939	0.928	0.919	0.928	0.931	0.922	0.926	0.933	0.932
42	0.921	0.942	0.929	0.931	0.920	0.913	0.921	0.923	0.916	0.920	0.925	0.924
43	0.913	0.933	0.922	0.922	0.912	0.907	0.915	0.915	0.911	0.913	0.917	0.917
44	0.905	0.924	0.916	0.914	0.905	0.902	0.910	0.907	0.906	0.906	0.909	0.910
45	0.898	0.916	0.909	0.905	0.898	0.896	0.905	0.899	0.902	0.899	0.901	0.903
46	0.890	0.908	0.903	0.897	0.891	0.891	0.900	0.892	0.898	0.892	0.894	0.897
47	0.883	0.901	0.897	0.889	0.885	0.886	0.895	0.886	0.893	0.884	0.888	0.890

48	0.876	0.895	0.891	0.881	0.879	0.881	0.890	0.880	0.888	0.876	0.882	0.884
49	0.870	0.890	0.885	0.874	0.873	0.876	0.886	0.875	0.883	0.868	0.876	0.878
50	0.863	0.885	0.880	0.867	0.868	0.871	0.880	0.871	0.876	0.860	0.871	0.873
51	0.857	0.879	0.874	0.859	0.863	0.866	0.874	0.867	0.869	0.853	0.865	0.867
52	0.851	0.873	0.868	0.852	0.858	0.860	0.867	0.863	0.861	0.846	0.860	0.861
53	0.845	0.867	0.861	0.844	0.853	0.854	0.859	0.859	0.852	0.839	0.853	0.854
54	0.839	0.858	0.853	0.836	0.847	0.847	0.850	0.854	0.842	0.833	0.846	0.847
55	0.832	0.848	0.845	0.828	0.841	0.839	0.841	0.848	0.832	0.826	0.838	0.839
56	0.825	0.838	0.837	0.819	0.834	0.831	0.831	0.841	0.822	0.819	0.829	0.830
57	0.818	0.827	0.828	0.810	0.826	0.821	0.820	0.834	0.813	0.811	0.821	0.821
58	0.810	0.817	0.818	0.800	0.818	0.811	0.809	0.825	0.803	0.802	0.812	0.812
59	0.802	0.806	0.809	0.790	0.810	0.800	0.797	0.815	0.794	0.792	0.804	0.802
60	0.793	0.795	0.800	0.779	0.802	0.790	0.786	0.805	0.784	0.781	0.797	0.793
61	0.785	0.785	0.791	0.768	0.794	0.780	0.775	0.794	0.775	0.770	0.790	0.783
62	0.776	0.776	0.782	0.756	0.786	0.770	0.764	0.783	0.765	0.759	0.784	0.773
63	0.768	0.767	0.775	0.745	0.778	0.761	0.753	0.772	0.756	0.747	0.777	0.764
64	0.760	0.759	0.768	0.734	0.770	0.752	0.743	0.760	0.747	0.735	0.771	0.755
65	0.753	0.752	0.761	0.722	0.762	0.745	0.734	0.748	0.738	0.723	0.764	0.746
66	0.746	0.745	0.755	0.711	0.753	0.739	0.725	0.737	0.730	0.710	0.756	0.737
67	0.740	0.738	0.749	0.700	0.743	0.733	0.717	0.725	0.721	0.696	0.747	0.729
68	0.734	0.732	0.743	0.688	0.732	0.729	0.709	0.714	0.713	0.682	0.737	0.720
69	0.729	0.725	0.736	0.677	0.721	0.725	0.702	0.704	0.705	0.668	0.725	0.711
70	0.722	0.718	0.728	0.668	0.709	0.720	0.694	0.694	0.696	0.654	0.712	0.702
71	0.715	0.711	0.720	0.658	0.697	0.716	0.686	0.685	0.686	0.640	0.698	0.693
72	0.707	0.703	0.711	0.648	0.686	0.712	0.677	0.676	0.674	0.625	0.683	0.684
73	0.698	0.695	0.702	0.636	0.675	0.707	0.666	0.668	0.660	0.611	0.667	0.673
74	0.686	0.686	0.692	0.620	0.665	0.702	0.655	0.660	0.645	0.597	0.650	0.663
75	0.673	0.675	0.681	0.601	0.655	0.695	0.643	0.652	0.628	0.583	0.633	0.651
76	0.658	0.663	0.669	0.578	0.645	0.688	0.629	0.644	0.609	0.571	0.614	0.638
77	0.641	0.649	0.656	0.552	0.635	0.678	0.615	0.634	0.588	0.561	0.595	0.625
78	0.622	0.633	0.641	0.526	0.625	0.666	0.599	0.622	0.566	0.551	0.574	0.610
79	0.602	0.616	0.626		0.615	0.652	0.582	0.609	0.541		0.552	0.594
80	0.581	0.598	0.609		0.604	0.633	0.565	0.594	0.516		0.528	0.576
81	0.559	0.578	0.591		0.592	0.609	0.546	0.578	0.491		0.504	0.557
82	0.536	0.556	0.570		0.577	0.579	0.526	0.559	0.467		0.479	0.536
83	0.513	0.532	0.547		0.561	0.545	0.506	0.540	0.442		0.454	0.514
84	0.490	0.507	0.522		0.542	0.507		0.519			0.429	0.490
85	0.466	0.481	0.495		0.521	0.466		0.498			0.404	0.464
86	0.441	0.454	0.466		0.500	0.424					0.378	0.437
87	0.415	0.429	0.436			0.381					0.353	0.410
88	0.388	0.405	0.404								0.327	0.382
89	0.359	0.383	0.372								0.302	0.354
90	0.331	0.362	0.338								0.277	0.325
91	0.302	0.342	0.304									0.295
92	0.273		0.267									0.265
93	0.244		0.230									0.234

Based on the Hodrick-Prescott filter of raw data per distance

Table 2 - Single event age-correction factors for women

Age	5k	5m	10k	12k	15k	10m	20k	halfmar	25k	30k	marat	mean
1												
2												
3												
4	0.675	0.655	0.690		0.584		0.498					0.615
5	0.703	0.685	0.719	0.701	0.627	0.615	0.549	0.637			0.577	0.646
6	0.731	0.714	0.749	0.715	0.670	0.657	0.600	0.668	0.626	0.645	0.619	0.678
7	0.757	0.743	0.777	0.727	0.710	0.699	0.649	0.698	0.653	0.668	0.660	0.708
8	0.782	0.770	0.803	0.739	0.747	0.739	0.695	0.728	0.678	0.690	0.699	0.738
9	0.806	0.795	0.828	0.750	0.780	0.777	0.737	0.757	0.703	0.711	0.735	0.765
10	0.827	0.817	0.851	0.761	0.810	0.810	0.775	0.784	0.726	0.731	0.769	0.790
11	0.846	0.836	0.871	0.773	0.835	0.840	0.808	0.808	0.749	0.752	0.799	0.813
12	0.862	0.853	0.889	0.786	0.857	0.866	0.836	0.830	0.771	0.772	0.826	0.834
13	0.876	0.868	0.905	0.800	0.876	0.889	0.860	0.850	0.793	0.791	0.851	0.853
14	0.889	0.882	0.919	0.814	0.893	0.908	0.880	0.869	0.815	0.810	0.873	0.870
15	0.901	0.894	0.932	0.829	0.909	0.925	0.897	0.887	0.837	0.829	0.892	0.886
16	0.912	0.906	0.942	0.844	0.923	0.940	0.911	0.904	0.859	0.848	0.909	0.901
17	0.922	0.917	0.952	0.859	0.936	0.953	0.923	0.920	0.881	0.868	0.924	0.915
18	0.933	0.929	0.960	0.874	0.948	0.965	0.934	0.935	0.901	0.887	0.938	0.929
19	0.943	0.940	0.968	0.889	0.959	0.975	0.945	0.949	0.920	0.905	0.950	0.941
20	0.954	0.951	0.975	0.904	0.968	0.983	0.954	0.961	0.936	0.922	0.961	0.953
21	0.964	0.961	0.981	0.921	0.977	0.990	0.963	0.971	0.950	0.938	0.970	0.963
22	0.973	0.970	0.986	0.937	0.984	0.995	0.972	0.980	0.962	0.953	0.978	0.973
23	0.980	0.977	0.991	0.953	0.989	0.998	0.979	0.987	0.973	0.966	0.985	0.981
24	0.987	0.984	0.994	0.967	0.994	0.999	0.985	0.993	0.982	0.978	0.990	0.988
25	0.992	0.990	0.997	0.978	0.997	1.000	0.990	0.997	0.989	0.987	0.995	0.993
26	0.995	0.994	0.999	0.987	0.999	1.000	0.994	0.999	0.995	0.994	0.998	0.997
27	0.998	0.997	1.000	0.993	1.000	0.998	0.997	1.000	0.998	0.998	0.999	0.999
28	0.999	0.999	1.000	0.997	1.000	0.997	0.999	0.999	1.000	1.000	1.000	1.000
29	1.000	1.000	1.000	0.999	0.999	0.994	1.000	0.997	1.000	1.000	0.999	1.000
30	1.000	0.999	0.999	1.000	0.998	0.991	0.999	0.993	0.999	0.998	0.998	0.999
31	0.999	0.997	0.997	1.000	0.996	0.988	0.997	0.989	0.996	0.995	0.996	0.996
32	0.998	0.993	0.995	0.998	0.994	0.985	0.994	0.985	0.993	0.990	0.993	0.993
33	0.996	0.988	0.992	0.995	0.991	0.981	0.990	0.980	0.989	0.985	0.990	0.990
34	0.993	0.981	0.988	0.992	0.987	0.977	0.984	0.975	0.985	0.979	0.986	0.985
35	0.988	0.974	0.984	0.987	0.982	0.972	0.978	0.969	0.981	0.972	0.981	0.980
36	0.983	0.965	0.978	0.982	0.976	0.966	0.971	0.964	0.976	0.965	0.976	0.974
37	0.977	0.957	0.972	0.976	0.969	0.960	0.963	0.958	0.972	0.958	0.971	0.967
38	0.970	0.948	0.964	0.969	0.961	0.953	0.955	0.952	0.966	0.950	0.964	0.960
39	0.961	0.940	0.956	0.961	0.951	0.945	0.947	0.945	0.960	0.943	0.956	0.952
40	0.952	0.932	0.946	0.952	0.942	0.938	0.939	0.937	0.952	0.935	0.947	0.944
41	0.942	0.924	0.937	0.943	0.932	0.930	0.932	0.930	0.943	0.927	0.938	0.935
42	0.932	0.917	0.927	0.932	0.922	0.922	0.924	0.922	0.932	0.919	0.928	0.926
43	0.922	0.910	0.917	0.921	0.913	0.913	0.916	0.914	0.921	0.911	0.918	0.917
44	0.911	0.903	0.908	0.910	0.904	0.904	0.909	0.906	0.908	0.903	0.908	0.907
45	0.901	0.897	0.900	0.898	0.895	0.894	0.901	0.897	0.896	0.895	0.897	0.898
46	0.891	0.890	0.892	0.886	0.887	0.885	0.892	0.889	0.883	0.888	0.887	0.889
47	0.882	0.884	0.884	0.874	0.879	0.875	0.884	0.882	0.872	0.882	0.877	0.880

48	0.873	0.878	0.877	0.862	0.872	0.865	0.875	0.875	0.861	0.876	0.868	0.872
49	0.865	0.872	0.870	0.851	0.865	0.856	0.865	0.869	0.851	0.869	0.860	0.864
50	0.857	0.865	0.863	0.841	0.858	0.848	0.856	0.864	0.843	0.863	0.852	0.856
51	0.849	0.858	0.856	0.830	0.850	0.840	0.848	0.858	0.835	0.856	0.844	0.848
52	0.840	0.849	0.848	0.819	0.841	0.833	0.840	0.851	0.826	0.848	0.835	0.840
53	0.831	0.840	0.839	0.807	0.830	0.826	0.831	0.843	0.818	0.841	0.826	0.831
54	0.822	0.830	0.830	0.793	0.819	0.818	0.823	0.833	0.808	0.832	0.816	0.821
55	0.812	0.819	0.821	0.779	0.807	0.810	0.813	0.822	0.798	0.824	0.804	0.811
56	0.802	0.807	0.810	0.764	0.795	0.800	0.803	0.810	0.787	0.814	0.791	0.799
57	0.791	0.794	0.799	0.749	0.783	0.790	0.791	0.797	0.774	0.803	0.777	0.787
58	0.780	0.779	0.788	0.733	0.771	0.779	0.778	0.783	0.760	0.791	0.763	0.774
59	0.769	0.765	0.775	0.719	0.758	0.767	0.764	0.768	0.745	0.776	0.749	0.760
60	0.757	0.751	0.762	0.705	0.746	0.754	0.750	0.754	0.731	0.760	0.735	0.746
61	0.745	0.737	0.748	0.691	0.734	0.741	0.736	0.740	0.716	0.742	0.721	0.732
62	0.732	0.724	0.734	0.679	0.722	0.728	0.721	0.726	0.701	0.723	0.707	0.719
63	0.719	0.711	0.720	0.668	0.711	0.716	0.707	0.713	0.687	0.703	0.692	0.705
64	0.706	0.699	0.706	0.657	0.700	0.704	0.692	0.699	0.674	0.681	0.678	0.691
65	0.693	0.688	0.693	0.648	0.690	0.692	0.679	0.686	0.661	0.659	0.663	0.678
66	0.681	0.678	0.681	0.639	0.680	0.681	0.665	0.671	0.649	0.636	0.648	0.665
67	0.668	0.668	0.670	0.631	0.669	0.671	0.652	0.656	0.637	0.613	0.632	0.653
68	0.655	0.658	0.659	0.625	0.656	0.661	0.639	0.640	0.625	0.589	0.616	0.640
69	0.642	0.649	0.649	0.619	0.642	0.653	0.628	0.623	0.614	0.567	0.600	0.628
70	0.629	0.640	0.640	0.613	0.626	0.646	0.618	0.607	0.602	0.545	0.583	0.616
71	0.616	0.631	0.631		0.610	0.641	0.609	0.590	0.590	0.522	0.567	0.605
72	0.603	0.621	0.622		0.595	0.636	0.602	0.575	0.579	0.500	0.551	0.594
73	0.591	0.610	0.613		0.581	0.630	0.596	0.559	0.567	0.477	0.536	0.584
74	0.579	0.598	0.604		0.569	0.624	0.592	0.544	0.555	0.456	0.522	0.574
75	0.567	0.584	0.595		0.559	0.616	0.589	0.529	0.543		0.509	0.564
76	0.555	0.568	0.584		0.549	0.605	0.586	0.515	0.531		0.497	0.553
77	0.543	0.550	0.573		0.540	0.592	0.584	0.501	0.520		0.484	0.541
78	0.530	0.532	0.561		0.529	0.576		0.489	0.508		0.471	0.529
79	0.518	0.513	0.547		0.518	0.558		0.477			0.458	0.515
80	0.505	0.493	0.532		0.504	0.538		0.465			0.443	0.500
81	0.491	0.474	0.515		0.490	0.515					0.428	0.484
82	0.476	0.455	0.496		0.475	0.492					0.411	0.467
83	0.460	0.435	0.477		0.460	0.468					0.395	0.449
84	0.443	0.416	0.457								0.378	0.431
85	0.426	0.398	0.437								0.362	0.412
86	0.408	0.380	0.417								0.345	0.393
87	0.389	0.364	0.397								0.328	0.374
88	0.369	0.348	0.377								0.312	0.355
89	0.349	0.334	0.357								0.295	0.336
90	0.329										0.278	0.316

Based on the Hodrick-Prescott filter of raw data per distance

Table 3 – Determinants of velocity by sex

Dependent variable: velocity (km/hour)

Determinant	Men-Coefficient	Men t-Statistic	Women Coefficient	Women t-Statistic
AGE	1.306261	14.79284	0.836727	10.17093
AGE2	-0.039844	-8.209404	-0.023082	-5.028293
AGE3	0.000442	3.722053	0.000201	1.774895
AGE4	-1.21E-06	-0.915005	-1.87E-07	-0.146970
AGE5	-6.32E-09	-1.148809	-4.36E-09	-0.823512
DIST	-0.777827	-6.573045	-0.697869	-6.349411
DIST2	0.063799	5.586313	0.058116	5.392797
DIST3	-0.002764	-5.899633	-0.002646	-5.941430
DIST4	5.13E-05	6.251469	5.09E-05	6.497409
DIST5	-3.18E-07	-6.510774	-3.23E-07	-6.897610
AGEDIST	0.002126	5.105083	0.002178	3.655274
AGE2DIST	-2.06E-05	-4.963723	-1.94E-05	-3.530442
C	10.24510	16.44872	12.02039	18.14745
N	939		895	
SSR	491		453	

AGE = age; AGEi = AGE to the power i; DIST=distance, DISTi= distance to the power i, AGEDIST=age*distance.

The standard errors are White heteroskedasticity corrected. N = number of observations, SSR= sum of squared residuals.

Table 4 – Pooled model

Dependent variable: velocity

Determinant	Pooled coefficient	Pooled t-statistic
AGE	1.009961	13.51307
AGE2	-0.027144	-6.699851
AGE3	0.000196	1.999959
AGE4	8.95E-07	0.824747
AGE5	-1.27E-08	-2.840426
DIST	-0.735576	-8.197983
DIST2	0.061087	7.031849
DIST3	-0.002713	-7.649653
DIST4	5.12E-05	8.280870
DIST5	-3.21E-07	-8.737353
AGEDIST	0.002017	4.593177
AGE2DIST	-1.83E-05	-4.495986
SEX	2.497457	66.60256
C	10.14690	18.68413
N	1834	
SSR	1187	

SEX= 1 if male; 0 if female. For the other symbols see Table 3.

The standard errors are White heteroskedasticity corrected. N = number of observations, SSR= sum of squared residuals.

Table 5 – Pooled model with interaction between age, distance and sex

Determinant	Pooled coefficient	Pooled t-statistic
AGE	0.824657	12.88426
AGE2	-0.022305	-6.456528
AGE3	0.000181	2.151617
AGE4	5.06E-08	0.053614
AGE5	-5.39E-09	-1.362133
DIST	-0.687333	-8.160965
DIST2	0.057415	7.193231
DIST3	-0.002620	-8.077685
DIST4	5.05E-05	8.904949
DIST5	-3.21E-07	-9.476466
AGEDIST	0.002170	6.046590
AGE2DIST	-2.03E-05	-5.933829
SEX	-1.739029	-3.323421
AGESEX	0.492449	8.949931
AGE2SEX	-0.018266	-8.640562
AGE3SEX	0.000280	8.753267
AGE4SEX	-1.48E-06	-8.948754
DISTSEX	-0.100495	-1.811543
DIST2SEX	0.007054	2.023087
DIST3SEX	-0.000170	-2.194547
DIST4SEX	1.21E-06	2.310592
C	12.00374	23.97868
N	1834	
SSR	945	

See for an explanation of the symbols Tables 3 and 4.
The standard errors are White heteroskedasticity corrected. N = number of observations, SSR= sum of squared residuals.

Table 6 – Predicted age-correction factors for men

Age	5k	5m	10k	12k	15k	10m	20k	halfmar	25k	30k	mar
1	0.404	0.370	0.355	0.344	0.328	0.322	0.297	0.289	0.258	0.216	0.208
2	0.459	0.425	0.410	0.399	0.384	0.378	0.354	0.346	0.314	0.274	0.266
3	0.510	0.476	0.462	0.451	0.436	0.430	0.406	0.399	0.368	0.327	0.321
4	0.558	0.525	0.511	0.500	0.485	0.479	0.456	0.448	0.418	0.378	0.373
5	0.603	0.570	0.556	0.545	0.531	0.525	0.502	0.494	0.464	0.425	0.421
6	0.645	0.612	0.598	0.588	0.573	0.568	0.545	0.538	0.508	0.469	0.466
7	0.684	0.651	0.638	0.627	0.613	0.608	0.585	0.578	0.548	0.510	0.508
8	0.720	0.687	0.674	0.664	0.650	0.644	0.622	0.615	0.586	0.548	0.547
9	0.753	0.721	0.708	0.697	0.684	0.679	0.657	0.650	0.621	0.583	0.583
10	0.783	0.751	0.739	0.728	0.715	0.710	0.689	0.681	0.653	0.616	0.617
11	0.811	0.780	0.767	0.757	0.744	0.739	0.718	0.711	0.683	0.646	0.648
12	0.837	0.806	0.793	0.783	0.770	0.766	0.745	0.738	0.710	0.673	0.676
13	0.860	0.829	0.817	0.807	0.795	0.790	0.769	0.762	0.735	0.698	0.703
14	0.882	0.851	0.838	0.829	0.816	0.812	0.791	0.785	0.757	0.721	0.727
15	0.901	0.870	0.858	0.848	0.836	0.832	0.812	0.805	0.778	0.742	0.748
16	0.918	0.887	0.875	0.866	0.854	0.849	0.830	0.823	0.796	0.761	0.768
17	0.933	0.902	0.891	0.882	0.870	0.865	0.846	0.839	0.813	0.778	0.786
18	0.946	0.916	0.904	0.895	0.884	0.879	0.860	0.854	0.827	0.793	0.801
19	0.958	0.928	0.916	0.907	0.896	0.892	0.873	0.866	0.840	0.806	0.815
20	0.968	0.938	0.927	0.918	0.907	0.902	0.884	0.877	0.852	0.818	0.828
21	0.976	0.947	0.935	0.927	0.916	0.912	0.893	0.887	0.861	0.828	0.839
22	0.983	0.954	0.943	0.934	0.924	0.919	0.901	0.895	0.870	0.836	0.848
23	0.989	0.960	0.949	0.940	0.930	0.926	0.908	0.901	0.876	0.843	0.856
24	0.994	0.964	0.953	0.945	0.935	0.931	0.913	0.907	0.882	0.849	0.862
25	0.997	0.968	0.957	0.949	0.938	0.934	0.917	0.911	0.886	0.854	0.867
26	0.999	0.970	0.959	0.951	0.941	0.937	0.920	0.914	0.889	0.857	0.871
27	1.000	0.971	0.961	0.953	0.943	0.939	0.922	0.916	0.891	0.859	0.874
28	1.000	0.972	0.961	0.953	0.943	0.939	0.922	0.916	0.892	0.861	0.876
29	0.999	0.971	0.960	0.953	0.943	0.939	0.922	0.916	0.893	0.861	0.877
30	0.998	0.969	0.959	0.951	0.942	0.938	0.921	0.916	0.892	0.861	0.877
31	0.995	0.967	0.957	0.949	0.940	0.936	0.920	0.914	0.890	0.859	0.876
32	0.992	0.964	0.954	0.946	0.937	0.933	0.917	0.911	0.888	0.857	0.875
33	0.988	0.961	0.950	0.943	0.934	0.930	0.914	0.908	0.885	0.854	0.872
34	0.984	0.956	0.946	0.939	0.930	0.926	0.910	0.905	0.881	0.851	0.869
35	0.979	0.952	0.942	0.934	0.925	0.922	0.906	0.900	0.877	0.847	0.866
36	0.974	0.947	0.937	0.929	0.920	0.917	0.901	0.896	0.873	0.843	0.862
37	0.968	0.941	0.931	0.924	0.915	0.912	0.896	0.890	0.868	0.838	0.857
38	0.962	0.935	0.925	0.918	0.909	0.906	0.890	0.885	0.862	0.833	0.852
39	0.956	0.929	0.919	0.912	0.903	0.900	0.884	0.879	0.857	0.827	0.847
40	0.949	0.922	0.912	0.906	0.897	0.894	0.878	0.873	0.850	0.821	0.841
41	0.943	0.915	0.906	0.899	0.890	0.887	0.872	0.866	0.844	0.815	0.835
42	0.936	0.908	0.899	0.892	0.883	0.880	0.865	0.860	0.837	0.808	0.829
43	0.928	0.901	0.892	0.885	0.876	0.873	0.858	0.853	0.831	0.801	0.823
44	0.921	0.894	0.884	0.878	0.869	0.866	0.851	0.846	0.824	0.795	0.816
45	0.914	0.887	0.877	0.870	0.862	0.859	0.844	0.839	0.817	0.788	0.809
46	0.906	0.879	0.870	0.863	0.855	0.852	0.837	0.831	0.809	0.781	0.802
47	0.899	0.872	0.862	0.856	0.847	0.844	0.829	0.824	0.802	0.773	0.795
48	0.891	0.864	0.855	0.848	0.840	0.837	0.822	0.817	0.795	0.766	0.788

49	0.884	0.857	0.847	0.841	0.833	0.829	0.815	0.809	0.787	0.759	0.781
50	0.876	0.849	0.840	0.833	0.825	0.822	0.807	0.802	0.780	0.751	0.774
51	0.868	0.842	0.832	0.826	0.818	0.814	0.800	0.794	0.773	0.744	0.766
52	0.861	0.834	0.825	0.818	0.810	0.807	0.792	0.787	0.765	0.737	0.759
53	0.854	0.827	0.817	0.811	0.803	0.800	0.785	0.780	0.758	0.729	0.752
54	0.846	0.820	0.810	0.803	0.795	0.792	0.778	0.772	0.751	0.722	0.744
55	0.839	0.812	0.803	0.796	0.788	0.785	0.770	0.765	0.743	0.714	0.737
56	0.832	0.805	0.795	0.789	0.781	0.778	0.763	0.758	0.736	0.707	0.730
57	0.824	0.798	0.788	0.782	0.773	0.770	0.755	0.750	0.728	0.700	0.722
58	0.817	0.790	0.781	0.774	0.766	0.763	0.748	0.743	0.721	0.692	0.715
59	0.810	0.783	0.774	0.767	0.759	0.756	0.741	0.736	0.714	0.685	0.707
60	0.803	0.776	0.766	0.760	0.751	0.748	0.733	0.728	0.706	0.677	0.699
61	0.795	0.769	0.759	0.752	0.744	0.741	0.726	0.721	0.699	0.670	0.692
62	0.788	0.761	0.752	0.745	0.737	0.733	0.718	0.713	0.691	0.662	0.684
63	0.781	0.754	0.744	0.738	0.729	0.726	0.711	0.706	0.683	0.654	0.676
64	0.773	0.746	0.737	0.730	0.722	0.718	0.703	0.698	0.676	0.646	0.668
65	0.766	0.739	0.729	0.722	0.714	0.710	0.695	0.690	0.668	0.638	0.659
66	0.758	0.731	0.721	0.714	0.706	0.702	0.687	0.682	0.659	0.630	0.651
67	0.750	0.723	0.713	0.706	0.698	0.694	0.679	0.673	0.651	0.622	0.642
68	0.742	0.715	0.705	0.698	0.689	0.686	0.670	0.665	0.642	0.613	0.633
69	0.733	0.706	0.696	0.689	0.680	0.677	0.661	0.656	0.633	0.604	0.623
70	0.725	0.697	0.687	0.680	0.671	0.668	0.652	0.647	0.624	0.594	0.613
71	0.715	0.688	0.678	0.671	0.662	0.658	0.642	0.637	0.614	0.584	0.603
72	0.706	0.678	0.668	0.661	0.652	0.648	0.632	0.627	0.604	0.573	0.592
73	0.696	0.668	0.658	0.650	0.641	0.638	0.622	0.616	0.593	0.562	0.581
74	0.685	0.657	0.647	0.639	0.630	0.626	0.610	0.605	0.581	0.551	0.568
75	0.673	0.645	0.635	0.628	0.618	0.615	0.598	0.593	0.569	0.538	0.556
76	0.661	0.633	0.623	0.615	0.606	0.602	0.585	0.580	0.556	0.525	0.542
77	0.648	0.620	0.610	0.602	0.592	0.589	0.572	0.566	0.542	0.511	0.527
78	0.634	0.606	0.595	0.588	0.578	0.574	0.557	0.552	0.528	0.496	0.512
79	0.620	0.591	0.580	0.573	0.563	0.559	0.542	0.536	0.512	0.480	0.495
80	0.604	0.575	0.564	0.557	0.547	0.543	0.525	0.519	0.495	0.463	0.478
81	0.587	0.558	0.547	0.539	0.529	0.525	0.508	0.502	0.477	0.445	0.459
82	0.569	0.540	0.529	0.521	0.510	0.506	0.489	0.483	0.458	0.426	0.439
83	0.549	0.520	0.509	0.501	0.490	0.486	0.468	0.462	0.437	0.405	0.417
84	0.528	0.499	0.488	0.479	0.469	0.465	0.447	0.440	0.415	0.382	0.394
85	0.506	0.476	0.465	0.457	0.446	0.442	0.423	0.417	0.392	0.359	0.370
86	0.482	0.452	0.441	0.432	0.421	0.417	0.398	0.392	0.366	0.333	0.344
87	0.456	0.426	0.414	0.406	0.395	0.390	0.371	0.365	0.339	0.305	0.315
88	0.428	0.398	0.386	0.377	0.366	0.362	0.343	0.336	0.310	0.276	0.285
89	0.398	0.368	0.356	0.347	0.336	0.331	0.312	0.305	0.279	0.245	0.253
90	0.366	0.336	0.324	0.315	0.303	0.299	0.279	0.272	0.246	0.211	0.219
91	0.332	0.301	0.289	0.280	0.268	0.264	0.244	0.237	0.210	0.175	0.182
92	0.295	0.265	0.252	0.243	0.231	0.226	0.206	0.199	0.172	0.137	0.143
93	0.256	0.225	0.213	0.203	0.191	0.186	0.166	0.159	0.132	0.096	0.101
94	0.214	0.183	0.171	0.161	0.149	0.144	0.123	0.116	0.089	0.052	0.056
95	0.170	0.138	0.126	0.116	0.103	0.098	0.077	0.070	0.042	0.006	0.009

Table 7– Predicted age-correction factors for women

Age	5k	5m	10k	12k	15k	10m	20k	halfmar	25k	30k	mar
1	0.541	0.507	0.491	0.478	0.459	0.451	0.420	0.410	0.372	0.324	0.332
2	0.581	0.546	0.531	0.518	0.500	0.492	0.461	0.451	0.413	0.366	0.375
3	0.618	0.584	0.569	0.557	0.538	0.531	0.500	0.491	0.453	0.406	0.417
4	0.653	0.620	0.605	0.593	0.574	0.567	0.537	0.527	0.490	0.444	0.456
5	0.686	0.653	0.639	0.626	0.609	0.601	0.572	0.562	0.525	0.480	0.493
6	0.718	0.685	0.670	0.658	0.641	0.634	0.604	0.595	0.558	0.514	0.528
7	0.747	0.714	0.700	0.688	0.671	0.664	0.635	0.626	0.590	0.545	0.560
8	0.774	0.742	0.728	0.716	0.699	0.692	0.664	0.655	0.619	0.575	0.591
9	0.799	0.767	0.754	0.742	0.725	0.719	0.691	0.681	0.646	0.603	0.620
10	0.823	0.791	0.778	0.766	0.750	0.743	0.716	0.707	0.672	0.628	0.647
11	0.845	0.813	0.800	0.789	0.773	0.766	0.739	0.730	0.695	0.653	0.672
12	0.865	0.834	0.821	0.810	0.794	0.787	0.760	0.751	0.717	0.675	0.696
13	0.884	0.853	0.840	0.829	0.813	0.807	0.780	0.771	0.737	0.696	0.717
14	0.900	0.870	0.857	0.846	0.831	0.825	0.798	0.790	0.756	0.715	0.737
15	0.916	0.885	0.873	0.862	0.847	0.841	0.815	0.806	0.773	0.732	0.756
16	0.930	0.900	0.887	0.877	0.862	0.856	0.830	0.821	0.788	0.748	0.773
17	0.942	0.912	0.900	0.890	0.875	0.869	0.844	0.835	0.802	0.762	0.788
18	0.953	0.924	0.911	0.901	0.887	0.881	0.856	0.847	0.815	0.775	0.802
19	0.963	0.934	0.922	0.912	0.897	0.892	0.867	0.858	0.826	0.787	0.815
20	0.972	0.942	0.930	0.921	0.907	0.901	0.876	0.868	0.836	0.797	0.826
21	0.979	0.950	0.938	0.928	0.914	0.909	0.885	0.876	0.845	0.806	0.836
22	0.985	0.956	0.944	0.935	0.921	0.916	0.892	0.884	0.852	0.814	0.844
23	0.990	0.961	0.950	0.940	0.927	0.921	0.898	0.890	0.859	0.821	0.852
24	0.994	0.965	0.954	0.945	0.931	0.926	0.903	0.895	0.864	0.826	0.858
25	0.997	0.969	0.957	0.948	0.935	0.930	0.906	0.899	0.868	0.830	0.863
26	0.999	0.971	0.959	0.950	0.938	0.932	0.909	0.901	0.871	0.834	0.867
27	1.000	0.972	0.961	0.952	0.939	0.934	0.911	0.903	0.873	0.836	0.870
28	1.000	0.972	0.961	0.952	0.940	0.935	0.912	0.904	0.874	0.838	0.872
29	0.999	0.972	0.961	0.952	0.940	0.934	0.912	0.904	0.875	0.838	0.874
30	0.998	0.970	0.959	0.951	0.939	0.934	0.911	0.904	0.874	0.838	0.874
31	0.996	0.968	0.957	0.949	0.937	0.932	0.910	0.902	0.873	0.837	0.873
32	0.993	0.965	0.955	0.946	0.934	0.929	0.907	0.900	0.871	0.835	0.872
33	0.989	0.962	0.951	0.943	0.931	0.926	0.904	0.897	0.868	0.833	0.870
34	0.985	0.958	0.947	0.939	0.927	0.922	0.901	0.893	0.865	0.829	0.867
35	0.980	0.953	0.943	0.934	0.923	0.918	0.897	0.889	0.861	0.826	0.864
36	0.974	0.948	0.937	0.929	0.918	0.913	0.892	0.884	0.856	0.821	0.860
37	0.968	0.942	0.932	0.924	0.912	0.907	0.886	0.879	0.851	0.816	0.855
38	0.962	0.936	0.925	0.917	0.906	0.901	0.880	0.873	0.845	0.810	0.850
39	0.955	0.929	0.919	0.911	0.900	0.895	0.874	0.867	0.839	0.804	0.845
40	0.948	0.922	0.912	0.904	0.893	0.888	0.867	0.860	0.832	0.798	0.838
41	0.940	0.914	0.904	0.896	0.885	0.881	0.860	0.853	0.825	0.791	0.832
42	0.932	0.906	0.896	0.889	0.878	0.873	0.852	0.845	0.818	0.784	0.825
43	0.924	0.898	0.888	0.880	0.869	0.865	0.844	0.837	0.810	0.776	0.817
44	0.916	0.889	0.880	0.872	0.861	0.857	0.836	0.829	0.802	0.768	0.809
45	0.907	0.881	0.871	0.863	0.852	0.848	0.828	0.821	0.793	0.760	0.801
46	0.898	0.872	0.862	0.854	0.844	0.839	0.819	0.812	0.784	0.751	0.793
47	0.888	0.862	0.853	0.845	0.834	0.830	0.810	0.803	0.775	0.742	0.784
48	0.879	0.853	0.843	0.836	0.825	0.821	0.801	0.794	0.766	0.733	0.775

49	0.869	0.843	0.834	0.826	0.816	0.811	0.791	0.784	0.757	0.723	0.766
50	0.859	0.834	0.824	0.817	0.806	0.801	0.781	0.775	0.747	0.714	0.756
51	0.850	0.824	0.814	0.807	0.796	0.792	0.772	0.765	0.737	0.704	0.747
52	0.840	0.814	0.804	0.797	0.786	0.782	0.762	0.755	0.727	0.694	0.737
53	0.829	0.804	0.794	0.787	0.776	0.772	0.752	0.745	0.717	0.684	0.727
54	0.819	0.794	0.784	0.776	0.766	0.761	0.741	0.735	0.707	0.674	0.717
55	0.809	0.783	0.774	0.766	0.756	0.751	0.731	0.724	0.697	0.664	0.706
56	0.799	0.773	0.763	0.756	0.745	0.741	0.721	0.714	0.686	0.653	0.696
57	0.788	0.763	0.753	0.745	0.735	0.730	0.710	0.703	0.676	0.643	0.685
58	0.778	0.752	0.742	0.735	0.724	0.720	0.700	0.693	0.665	0.632	0.675
59	0.767	0.742	0.732	0.724	0.714	0.709	0.689	0.682	0.655	0.621	0.664
60	0.757	0.731	0.721	0.714	0.703	0.699	0.678	0.671	0.644	0.610	0.653
61	0.746	0.720	0.711	0.703	0.692	0.688	0.668	0.661	0.633	0.600	0.642
62	0.736	0.710	0.700	0.692	0.682	0.677	0.657	0.650	0.622	0.589	0.630
63	0.725	0.699	0.689	0.682	0.671	0.666	0.646	0.639	0.611	0.577	0.619
64	0.715	0.688	0.678	0.671	0.660	0.655	0.635	0.628	0.600	0.566	0.608
65	0.704	0.678	0.668	0.660	0.649	0.644	0.624	0.617	0.589	0.555	0.596
66	0.693	0.667	0.657	0.649	0.638	0.633	0.613	0.606	0.578	0.544	0.584
67	0.682	0.656	0.646	0.638	0.627	0.622	0.601	0.594	0.566	0.532	0.572
68	0.671	0.645	0.635	0.627	0.615	0.611	0.590	0.583	0.555	0.520	0.560
69	0.660	0.634	0.623	0.615	0.604	0.599	0.578	0.571	0.543	0.508	0.548
70	0.649	0.622	0.612	0.604	0.593	0.588	0.567	0.559	0.531	0.496	0.536
71	0.638	0.611	0.600	0.592	0.581	0.576	0.555	0.548	0.519	0.484	0.523
72	0.626	0.599	0.589	0.581	0.569	0.564	0.543	0.535	0.507	0.472	0.510
73	0.614	0.587	0.577	0.569	0.557	0.552	0.530	0.523	0.494	0.459	0.497
74	0.602	0.575	0.565	0.556	0.544	0.540	0.518	0.511	0.481	0.446	0.483
75	0.590	0.563	0.552	0.544	0.532	0.527	0.505	0.498	0.468	0.433	0.470
76	0.578	0.550	0.540	0.531	0.519	0.514	0.492	0.484	0.455	0.419	0.455
77	0.565	0.537	0.527	0.518	0.506	0.501	0.478	0.471	0.441	0.405	0.441
78	0.552	0.524	0.513	0.505	0.492	0.487	0.465	0.457	0.427	0.391	0.426
79	0.538	0.511	0.500	0.491	0.478	0.473	0.450	0.443	0.413	0.376	0.410
80	0.524	0.496	0.485	0.476	0.464	0.458	0.436	0.428	0.398	0.361	0.395
81	0.510	0.482	0.471	0.462	0.449	0.443	0.420	0.413	0.382	0.345	0.378
82	0.495	0.467	0.455	0.446	0.433	0.428	0.404	0.397	0.366	0.329	0.361
83	0.480	0.451	0.440	0.430	0.417	0.412	0.388	0.380	0.349	0.311	0.343
84	0.463	0.435	0.423	0.414	0.400	0.395	0.371	0.363	0.332	0.294	0.325
85	0.447	0.418	0.406	0.397	0.383	0.377	0.353	0.345	0.314	0.275	0.305
86	0.429	0.400	0.388	0.378	0.365	0.359	0.335	0.327	0.295	0.256	0.285
87	0.411	0.381	0.369	0.360	0.346	0.340	0.315	0.307	0.275	0.236	0.265
88	0.392	0.362	0.350	0.340	0.326	0.320	0.295	0.287	0.255	0.215	0.243
89	0.371	0.342	0.329	0.319	0.305	0.299	0.274	0.265	0.233	0.193	0.220
90	0.350	0.320	0.308	0.298	0.283	0.277	0.251	0.243	0.210	0.170	0.196
91	0.328	0.298	0.285	0.275	0.260	0.254	0.228	0.220	0.187	0.146	0.171
92	0.305	0.274	0.262	0.251	0.236	0.230	0.204	0.195	0.162	0.121	0.145
93	0.280	0.249	0.237	0.226	0.210	0.204	0.178	0.169	0.135	0.094	0.117
94	0.254	0.223	0.210	0.199	0.184	0.177	0.151	0.142	0.108	0.066	0.088
95	0.227	0.196	0.183	0.172	0.156	0.149	0.122	0.113	0.079	0.037	0.058

Table 8 – Robustness check on pooled regression

Variable	Coefficient	t-Statistic
AGE	0.948574	22.95836
AGE2	-0.029061	-18.55534
AGE3	0.000331	14.03722
AGE4	-1.43E-06	-11.75441
DIST2	-0.010091	-25.12365
DIST3	0.000180	26.67442
AGEDIST	0.002234	7.336064
AGE2DIST	-2.05E-05	-6.573997
SEX	-1.957528	-4.375522
AGESEX	0.467546	8.689487
AGE2SEX	-0.017642	-8.386783
AGE3SEX	0.000277	8.522789
AGE4SEX	-1.49E-06	-8.724601
C	9.281462	26.21734
N	882	
SSR	479	

Table 9 – World Athletic Veteran Association correction factors for men

age	5K	8K	10K	12K	15K	10mile	20K	hmar	25K	30K	MAR
30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
31	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
32	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
33	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
34	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
35	0.996	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
36	0.990	0.993	0.995	0.997	0.999	1.000	1.000	1.000	1.000	1.000	1.000
37	0.983	0.987	0.988	0.990	0.992	0.993	0.995	0.996	0.997	1.000	1.000
38	0.976	0.980	0.982	0.983	0.985	0.986	0.988	0.989	0.991	0.993	0.997
39	0.969	0.973	0.975	0.976	0.978	0.979	0.981	0.982	0.984	0.986	0.990
40	0.962	0.966	0.968	0.970	0.972	0.972	0.975	0.975	0.977	0.979	0.984
45	0.928	0.932	0.933	0.935	0.937	0.938	0.940	0.940	0.942	0.944	0.949
50	0.893	0.896	0.898	0.900	0.901	0.902	0.904	0.905	0.907	0.909	0.913
55	0.857	0.860	0.862	0.863	0.865	0.866	0.868	0.869	0.870	0.872	0.876
60	0.819	0.822	0.824	0.825	0.827	0.828	0.830	0.830	0.832	0.834	0.838
65	0.779	0.782	0.783	0.785	0.787	0.787	0.790	0.790	0.791	0.794	0.798
70	0.737	0.739	0.740	0.742	0.743	0.744	0.746	0.747	0.748	0.750	0.754
75	0.690	0.692	0.694	0.695	0.696	0.697	0.699	0.700	0.701	0.703	0.707
80	0.639	0.641	0.642	0.644	0.645	0.645	0.648	0.648	0.649	0.651	0.655
85	0.581	0.583	0.584	0.585	0.586	0.587	0.589	0.590	0.591	0.593	0.596
90	0.511	0.513	0.514	0.515	0.516	0.517	0.519	0.520	0.521	0.523	0.526
95	0.417	0.419	0.419	0.421	0.422	0.423	0.425	0.426	0.426	0.428	0.432
100	0.262	0.263	0.264	0.266	0.266	0.267	0.269	0.270	0.270	0.273	0.276

Source: www.wava.org.

Table 10 – World Athletic Veteran Association correction factors for women

age	5K	8K	10K	12K	15K	10mile	20K	hmar	25K	30K	mar
30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
31	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
32	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
33	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
34	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
35	0.991	0.995	0.997	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
36	0.984	0.988	0.990	0.991	0.993	0.994	0.996	0.997	0.999	1.000	1.000
37	0.976	0.980	0.982	0.984	0.986	0.986	0.989	0.989	0.991	0.993	0.998
38	0.968	0.972	0.974	0.976	0.978	0.979	0.981	0.981	0.983	0.985	0.990
39	0.960	0.964	0.966	0.968	0.970	0.971	0.973	0.974	0.975	0.978	0.982
40	0.953	0.957	0.959	0.960	0.962	0.963	0.965	0.966	0.968	0.970	0.975
45	0.913	0.917	0.919	0.921	0.923	0.924	0.926	0.926	0.928	0.930	0.935
50	0.873	0.877	0.879	0.881	0.883	0.884	0.886	0.886	0.888	0.890	0.895
55	0.832	0.836	0.838	0.840	0.842	0.843	0.844	0.845	0.847	0.849	0.854
60	0.790	0.793	0.795	0.797	0.799	0.800	0.802	0.802	0.804	0.806	0.811
65	0.745	0.749	0.751	0.752	0.754	0.755	0.757	0.757	0.759	0.761	0.766
70	0.697	0.701	0.703	0.704	0.707	0.707	0.709	0.709	0.711	0.713	0.718
75	0.646	0.649	0.651	0.653	0.655	0.656	0.657	0.658	0.660	0.662	0.667
80	0.590	0.593	0.595	0.597	0.599	0.599	0.601	0.601	0.603	0.605	0.610
85	0.527	0.530	0.532	0.534	0.536	0.536	0.538	0.538	0.540	0.542	0.547
90	0.452	0.455	0.457	0.459	0.461	0.462	0.463	0.463	0.465	0.468	0.472
95	0.353	0.356	0.358	0.360	0.362	0.363	0.364	0.364	0.366	0.369	0.373
100	0.193	0.196	0.198	0.200	0.202	0.203	0.203	0.204	0.206	0.208	0.213

Source: www.wava.org

Figure 1 – Density function of the log of the 5K running time by female students

Source: results of a student-race Lauwersloop, held on May 27, 2000. Data for 358 female runners on distances of 3.8, 4.2, 4.8, 5.5, 6.6, and 8K, all transformed into 5K equivalents.

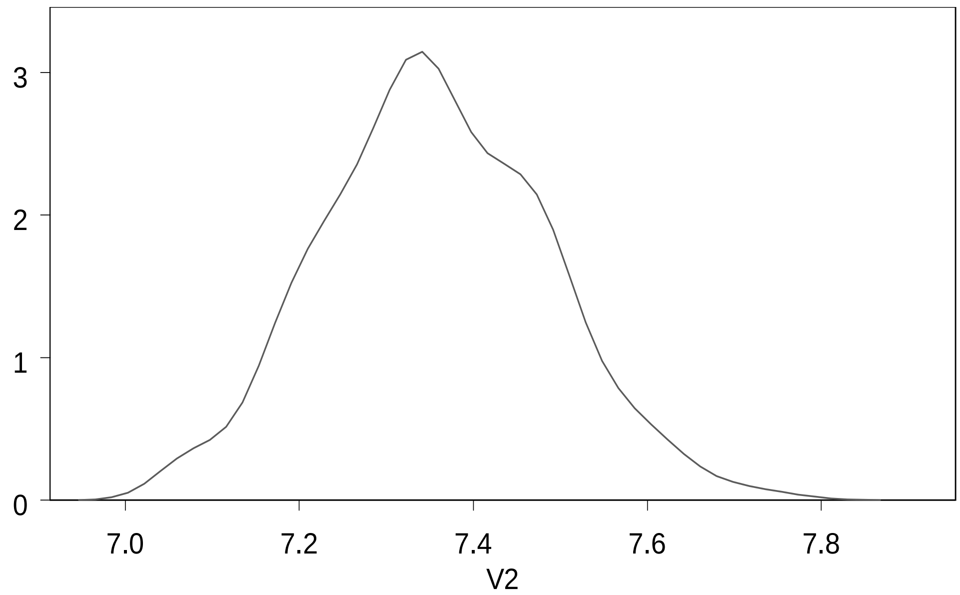

Figure 2 – Postulated relation between the lower bound of a density plot of running times and age

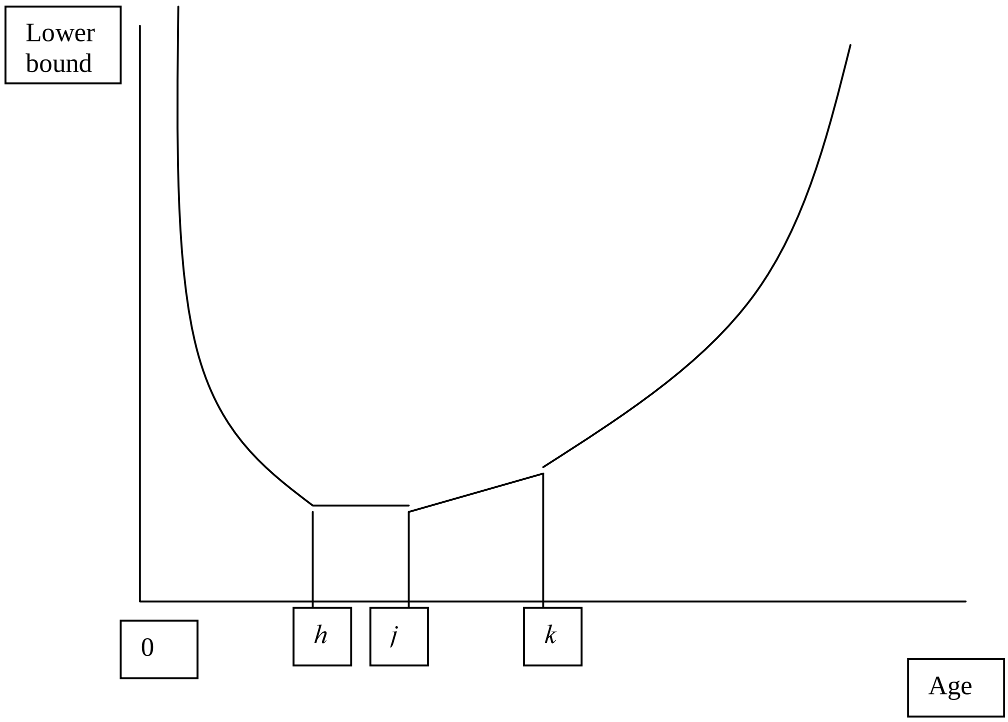

Figure 3 – Velocity of women on the marathon

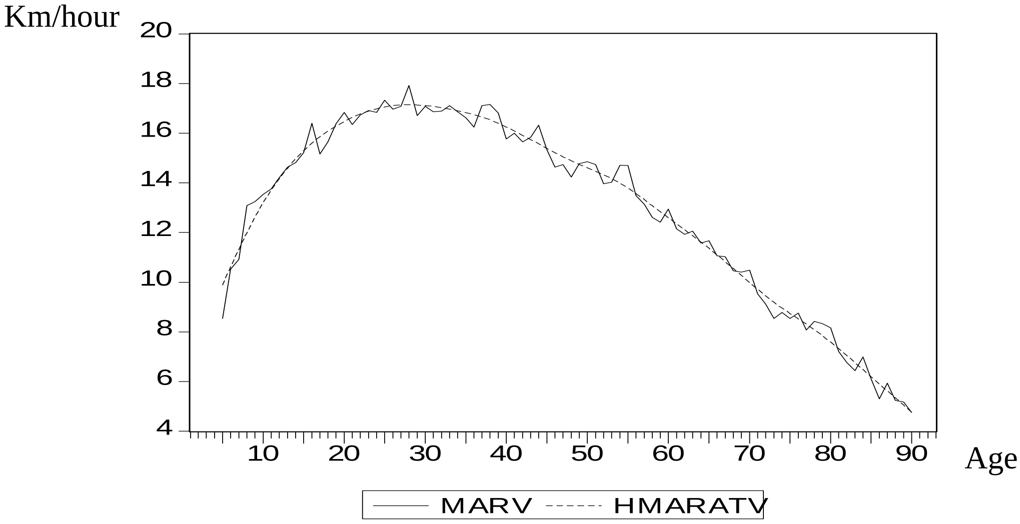

Figure 4 – Estimated age-dependent long distance velocity of men and women

Velocity in meters per second

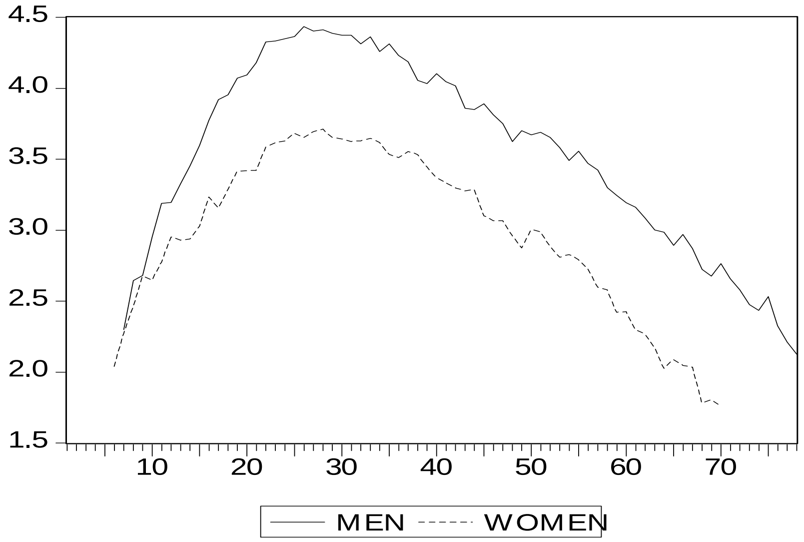

Figure 5 – Contour plot of men's velocity

On the axes:
x-axis: age (from 1 to 95)
y-axis: distance (from 50 miles to 5 K)
z-axis: velocity of men (km/hour)

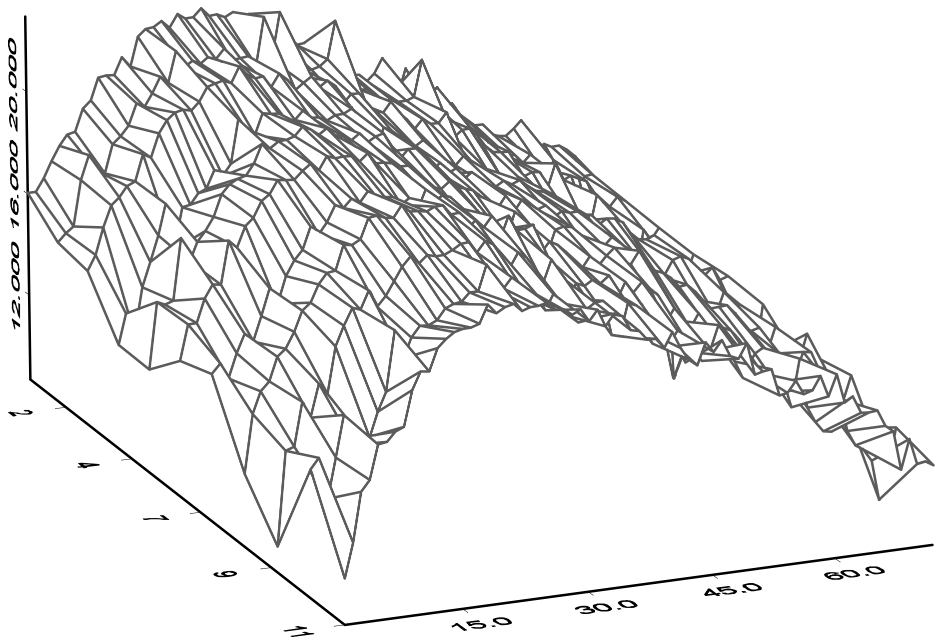

Figure 6 – Contour plot of theoretical velocity of men

On the axes:
x-axis: age (from 1 to 95)
y-axis: distance (from Marathon to 5 K)
z-axis: velocity of men (km/hour)

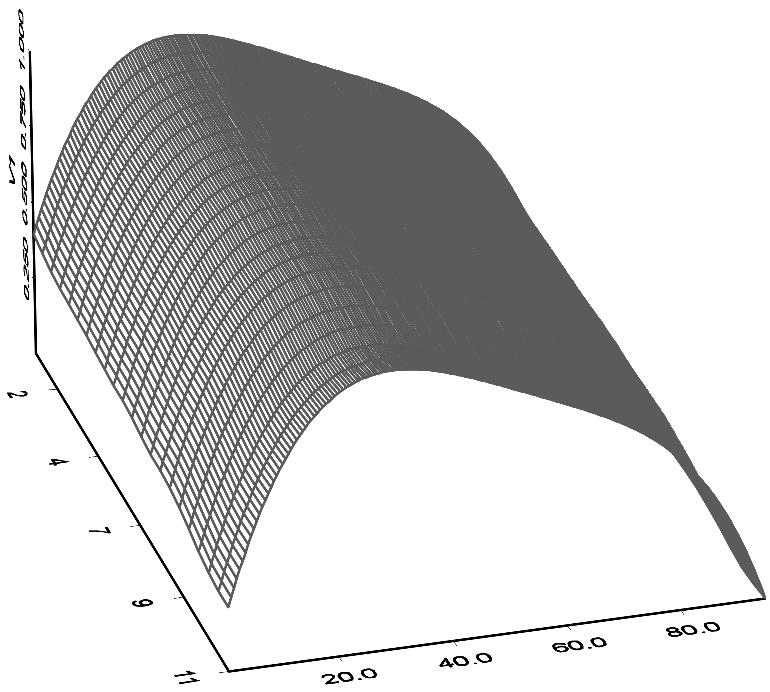

Figure 7 – Contour plot of theoretical velocity of women

On the axes:
x-axis: age (from 1 to 95)
y-axis: distance (from Marathon to 5 K)
z-axis: velocity of men (km/hour)

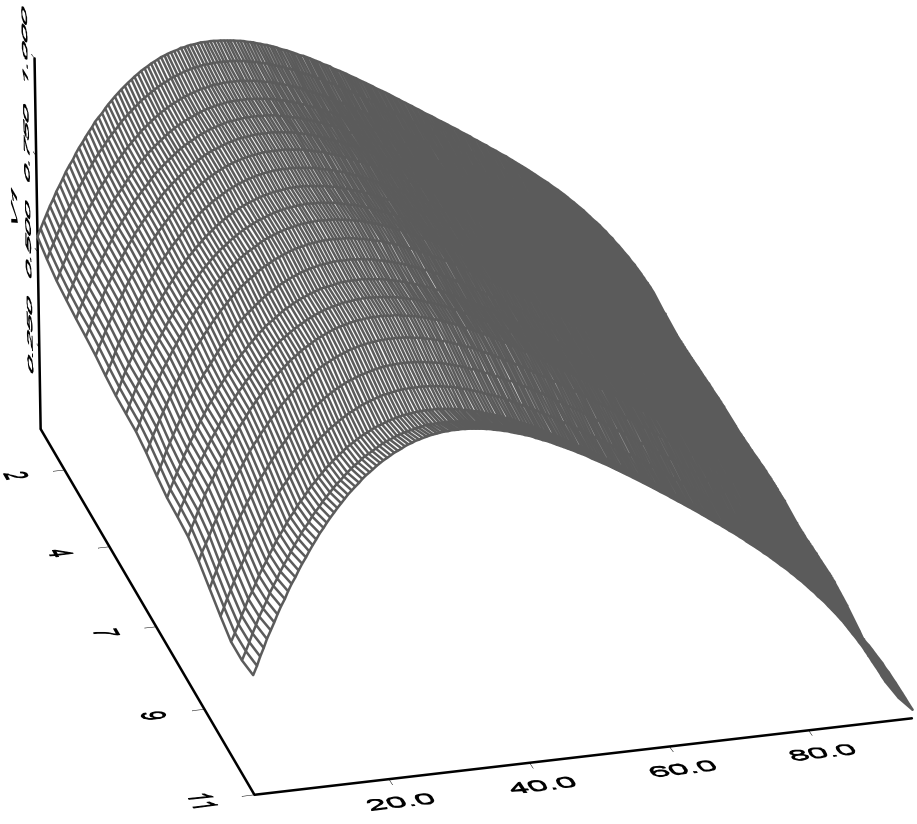

Figure 8 – Contour plot for difference in velocity between men and women

On the axes:
x-axis: age (from 1 to 95)
y-axis: distance (from 50 miles to 5 K)
z-axis: velocity differential between men and women (km/hour)