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ORGANISATIONAL NICHE BOUNDARIES IN THE *N*-SPACE

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SOM theme G Cross-contextual comparison of institutions and organizations

Abstract

The paper investigates organizational boundary spanning from the point of view of neighborhood relations. Neighborhood is defined with the closeness of organizations' resource utilization patterns. The key resource is the clientele's demand for organizational outputs (products, party programs, membership, etc.). Demand is characterized qualitatively by *n* taste descriptors that span an *n*-dimensional resource space. Organizational niche boundaries may take different forms and size. To avoid niche overlap over boundaries, organizations can configure in the resource space in different clusterings. Which are the densest arrangements that allow for the coexistence of maximal number of organizations? How can these coexisting neighborhoods build up? How do competition, new entry and the number of immediate neighbor's change around the niche boundary with space dimension? The paper applies results of the sphere packing problem in *n*-dimensional geometry to answer these questions.

1. NEIGHBOURHOOD IN THE RESOURCE ENVIRONMENT

The way of organizational boundary spanning depends on internal settings and also on what lies beyond the boundaries. This paper concentrates on the external environment with a special focus on the neighborhood relations around the organizational surface. How many bystanders can cluster around an organization? How do they position themselves relative to the focal organization and to each other? How do environmental characteristics, like the number of clients' taste descriptor, affect grouping density? Is rivalry unavoidable or the configuration allows for coexistence without crossing the neighbors' domain? Looking for answers to these questions, the paper applies the armoire of n-dimensional geometry.

1.1 Resource spaces

The paper takes the view of organizational ecology on organizational neighborhood (Carroll and Hannan 2000). Two organizations are neighbors if their resource utilization patterns are similar, that is, if they operate in nearby domains of the resource environment. These organizations can be, for example, firms that cater and compete for the same customers or parties that aim their programs at electors with similar political preferences.

The resource environment is modeled as a segment of the n-space where spatial axes stand for different resource characteristics, and points in this space stand for resource configurations (Hannan and Freeman 1989). Organizations can survive by absorbing environmental resource. But resources only play a role if they are scarce. Physical resource scarcity (like lack of raw materials) is rare and transient in market economies. It may still occur, for example if a technological breakthrough leads to the emergence of a brand new industry with a sudden demand for a new type

of physical resource. Human resources like skilled personnel can be also scarce for a while, but training programs can ease the shortage.

However, there is one sort of "human resource" for organizations that is almost always scarce in market economies: their clients' need for services. Speaking about firms, this key resource is the customers' purchasing power (Tirole 1988); in case of non-profit organizations, it is the clientele's membership and contribution (McPherson 1983); for political parties, it is the support of the electors (Downs 1957). The paper considers the people to whom the organization offers its services the key resource. The resource space displays customer tastes with respect the organization's product (product characteristic space, Lancaster 1966) or people's social status that influence their chance of affiliation with a certain non-profit organization (sociodemographic space, McPherson 1983). The n-space may also display the political preferences of a constituency along n political issues that span the issue-space (Enelow and Hinich 1990).

1.2 Niches

Organizations, be they economic firms, non-profit organizations or political parties, address a certain range of people with their output (products, services, political programs). For the sake of brevity, I refer to these people as customers, whatever is the concrete form of the organizational offer. The domain in the n-dimensional resource space from where the organization draws resource (attract customers) is called the organization's catchment area or *niche*.

The notion of niche comes from bioecology: the niche is the subset of the resource space that represents the environmental resource conditions with which in place the given population sustains (Levins 1968). The resource space offers qualitatively different resources that the individual (specimen, organization) can or can not absorb. The quantitative aspects of the resource supply play a crucial role as

well: the availability of resource can be good or bad. The distinction between qualitative and quantitative resource aspects brings to the distinction between the *fundamental-* and the *realized* niche (Levins 1968, Hannan and Freeman 1989).

The fundamental niche contains those resource points (customer tastes) in space that the organization can absorb; thus, the fundamental niche reflects the organization's resource utilization pattern. In case of the realized niche, interaction effects like competition are also taken into account. Resource competition intensity between two organizations is measured with the overlap between their fundamental niches. The stronger organization may outforce the other from certain resource domains, so the realized niche is typically smaller than the fundamental niche.[1] The distinction between fundamental niche (niche "on its own") and realized niche (niche shaped by external conditions and strategy) plays an important role in this paper.

Niche spanning. Organizations try to minimize competition pressure, that is, their catchment area overlap with others. They can do this by modifying the size or the location of their niches.

Size modification may mean a defensive contraction of the niche, i.e., organizations stop to offer products to certain customer groups, parties downsize their political programs. The price they pay for this change is addressing fewer people. However, such actions might also have positive aspects on niche utilization efficiency. According to ecological theories, there is a trade-off between niche breadth and the organization's fitness to customers: the broader the niche, the lower this fitness. This relation is called the principle of allocation (Levins 1968, Freeman and Hannan 1983). The underlying idea is that organizational forms (or species) can distribute a given amount of "adaptive capacity" above two tasks. They either invest

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[1] Symbiotic relations like cooperations can make additional resource space segments accessible for the organizations; then, their realized niches can be even larger then their fundamental niches.

in maintaining a broad niche that allows them to persist under various environmental conditions. Alternatively, they can invest in adapting only to a narrow range of conditions in which they will really excel. This brings to the generalism-specialism problem of organization science. Generalist organizations have a broad niche; they address a broad range of customers with their products. But, this can lead to an overspanning of the niche boundaries: making an appeal to very different tastes does not make good to the quality of the organization's output. Generalist products typically cannot be tuned to specific customer demands because of the heterogeneity of these demands. Consequently, generalists' attraction on the addressed people is not as high as that of specialists on their own specific customer taste groups to whom they dedicate their output. Under a broad range of conditions, there is an optimum point along the niche width vs. fitness trade-off; organizations with this optimal niche breadth maximize their sales, voter support, etc.

Assume that consumer tastes are characterized by n distinct aspects. The customer's taste preference with respect to a product is represented with a point in the n-space where axes display the possible ranges of the n taste aspects. This preference point is called the ideal point of the customers. The bigger the distance between the customer's ideal point and the point that characterizes the product, the lower the product's attraction on the customer (keeping the prices constant). The product's point in the n-space is the center of the organization's niche. For analytical reasons, the paper considers single product organizations. In case of multi-product firms, the organization's niche would be the union of the product niches.

Niche profile. The shape of the niche influences how the organization interacts with its task environment. How many neighbors it can have and how do these organizations position around its boundary. The literature of spatial modeling discusses two kind of niche shapes: niches are either n-dimensional cubes

(hypercubes, Hutchinson 1978, McPherson 1983) or *n*-dimensional spheres (hyperspheres, Carroll 1985, Péli and Nooteboom 1999). The choice between the two options depends on how people "calculate" the distance between their ideal taste points and the organization's offer. Mismatches occur along all the *n* taste dimensions. The question is that how these imperfections aggregate in the customers' judgement.

First, people can evaluate their distance from the product's point along each aspect separately. The client may require that the mismatch between product and her/his preference should not exceed a certain threshold (*R*) along either of the taste dimensions (Figure 1a). For example, buyers take into account the length and the waist size of jeans independently. In case of good fit, the mismatch is usually within the 1-inch range along both dimensions. When errors between product features and customers' do not add up in the customer's evaluation, then the obtaining niches are *n*-dimensional rectangles. Sometimes weights are assigned to taste dimensions to represent their relative importance to the others. These weights can be set to unity and the scales along the *n* product dimensions can be standardized without affecting the coming arguments on organizational positioning in this paper. With these modifications in place, the rectangular niches transform into *n*-dimensional cubes.

Second, the clientele can be sensitive to the overall match between their ideal point and that of the product. They make cross-dimensional comparisons, measuring the good and bad matches against each other along the product dimensions. Errors along the *n* taste aspects do add up in the customers' evaluation. In this case, the *n*-dimensional Euclidean distance is a suitable measure of customer-product fit (Péli and Nooteboom 1999). Organizational niches are *n*-dimensional spheres around the product's ideal point (provided that the above mentioned scale transformations are applied).

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2. NICHE POSITIONING

Assume a consumer market with organizations that position their products in the n-space of customer tastes in a way that minimizes catchment area overlap. The market is crowded, so efficient niche arrangements are searched that allow for the co-existence of a high number of actors. The breadth of organizations' fundamental niches is similar. What is the best way of non-overlapping niche positioning in the n-space?

The economic literature on this topic starts with the paper of Harold Hotelling (1929) who addressed the question of best shop positioning along one spatial dimension (e.g., the main street or a beach). Customers face a certain inconvenience because of their distance from the nearest shop. This "transportation cost" can be counterbalanced by smaller prices, and the price-distance duo determines the size of the shop's catchment area. The model has been modified in several ways (Salop 1979, Darnell 1990), and has been extended to two spatial dimensions (Christaller 1966, Lösch 1967).

This paper assumes that prices have no remarkable influence on catchment area shape. So, the coming arguments are confined to markets where price competition (Bertrand competition, Tirole 1988) is modest, like in the newspaper industry (Carroll 1985). But fortunately, this assumption does not confine model validity in case of non-profit organizations and political parties, where only the output's quality and closeness to the client determines appeal. Let's consider the cases of cubic and spherical niches separately.

2.1 Niches as cubes

The n-space always can be tiled up with n-cubes without a residual and overlap. For example, the unity-edge cubes constitute the cells of an n-dimensional co-ordinate

system. Given the *n*-dimensional rod of organizational niche positioning, the distance between the nearest cube centers (*R*) still has to be decided. This *R* value may depend on model settings and also on how tightly the market is packed with organizations. For example, customers' indifference threshold may determine where the niche boundaries lie. If this threshold is *R*, then the best distance between products is 2*R*.

The space packing problem has a simple solution for cubic niches. Good spatial arrangements with spherically symmetric niches is certainly more complicated. However, their investigation leads to some unexpected conclusions.

2.2 Spherical niches

Spherically symmetric niches obtain if the distance between people's ideal tastes and organizational offer is measured it with their Euclidean distance. It is useful to make a distinction between spherical niches with and without a finite boundary.

Finite fundamental niche boundary. Recall that indifference thresholds drew the boundaries of cubic fundamental niches. In the spherical case, a *tolerance* threshold can determine the edge of the fundamental niche: beyond a limit radius, no customer chooses the product (Figure 2a). Thus, the size of the fundamental niche can be given by customers' sensitivity to the product's distance from their ideal points. (Within the tolerance limits, the actual distance from the ideal point also counts: the closer the product, the bigger the chance of a transaction.)

The size of the realized niche can follow from the organization's own optimizing considerations. The optimal, sales- or vote maximizing, catchment area breadth is usually smaller than customers' maximal tolerance distance that draws the borderline of the fundamental niche. The existence of such an optimum means that it does not pay off to span the catchment area to all people who can be a client of the

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organization. For example in politics, an overspanned realized niche means an inconsistent and vague party program that alienates voters at the center of the niche.

Infinite fundamental niche boundary. The fundamental niche can also be spherically symmetric without having any boundaries; for example, if the organization's attraction on the clients decreases asymptotically with distance (Figure 2b). Then, the fundamental niche is "infinite" (of course, the resource space size is always finite). This niche interpretation shows a strong analogy with modern physic's notion on particle locations. According to quantum mechanics, an electron's position is not a point or discrete domain in space, but rather a cloud around a central point. The density of the electron cloud measures the electron's "intensity of presence" that thins out with radius in infinity.

Note that the realized niche is always finite, and not always because of the resource space limits. The core part of the catchment area can be better defended from competitors than remote peripheries. Competition with neighbors cuts off the "infinite" tails of the fundamental niche, leaving a confined central domain for the realized niche (Figure 3).

2.3 Sphere packing

What is the best way of niche positioning? How can the maximal number of organizations be placed into the space in a way that avoids or minimizes boundary overlap between neighbors? Non-overlapping n-spheres cannot fill up the n-space without a residual (except for one-dimensional spaces where "cubes" and "spheres" both degenerate to line segments). There are better and worse methods to fill up the space with spherical niches. The paper applies results on the *sphere-packing problem* in geometry (Conway and Sloane 1998). The central question in this field is how can we pack up the n-space with non-overlapping spheres of unity radius in the densest

way. The efficiency of a given space filling up is measured with packing density (Δ), the ratio of total sphere content to the volume of the populated space segment. We assume that the spherical catchment areas are of the same size, for example because the "tolerance" distance of people beyond which they do not engage in transaction is not organization specific. This is clearly a simplification. However, going along with the assumption of unique niche radius allows to focus on basic niche positioning effects, and later research can add variations to this baseline model.

Studying dense space packings with non-overlapping (but touching or "kissing") spherical niches helps to determine dense center arrangements when organizational catchment areas overlap. The niche configurations that maximize the space covering ratio (Δ) of touching but non-competing organizations are also good at minimizing overlap between neighbors.

2.4 Spheres in square-lattice center arrangement

A feasible way of spherical niche packing is to arrange the sphere centers into a rod of squares, just as we did in the cubic case (Figure 4). This arrangement is called *square-lattice packing* (Conway and Sloane 1998). If niche boundaries are finite and overlap is excluded and $n > 1$, then cavities are left in the space populated by customers without a product that matches their taste or by voter groups without being addressed by any parties. The center of a cavity is called the *deep hole* (Figure 4). It is the farthest location from any neighboring sphere center; the biggest the distance between hole and sphere center, the deeper the hole. When niches are allowed to overlap, the space may be filled up completely. Then, the deep holes of the previous setting become zones of inferior organizational offer; attraction of any organization on the customers at these locations is very low. These weak points can attract new entries, and possibly new competition. The square-lattice sphere center arrangement is not ideal in this respect, because it is certainly not the densest sphere

packing. But its advantage is that it builds up relatively easily in any dimensions, according to the following procedure.

Assume a new market segment that becomes gradually populated. New organizations look for niche locations at the *n*-space with a strategy that mixes imitation and differentiation. Imitation of incumbent firms can be useful in new, unexplored situations. Organizations with adjacent niches have similar resource utilization patterns. Therefore, the new niche should be close to that of some incumbent organization. But because the neighbors are also potential competitors, the new niche location should also be somewhat different (product differentiation, Tirole 1988). So, organizations with definite fundamental niche boundaries may adopt the following two-step a niche allocation procedure:

(i) Identify an incumbent niche that is not yet completely surrounded by neighbors.

(ii) Discriminate one of your niche co-ordinates from theirs with $2R$ in the direction of a free location, where R is niche radius.

These two requirements define a gradual filling up the *n*-space segment that gives a square-lattice packing.

In case of infinite spherically symmetric fundamental niches, the search procedure is similar. Now having no closed fundamental niche boundaries, certain optimum conditions may determine the best distance from neighbors and so the breadth of the realized niche. This optimal center distance is determined by two trade-offs: one is between generalism and specialism, the other is between imitation advantages and thicker niche overlap that comes with niche center closeness.

2.5 Dense niche arrangements

We are now looking for niche groupings that maximize packing density. The focus is on niches with finite boundaries (spheres) that touch without intersecting. Again,

these dense arrangements will also define reasonably good center configurations for infinite fundamental niches.

One-dimensional spheres are line segments that can completely cover the line. Packing density is maximal, $\Delta = 1$. In two dimensions, the best is the hexagonal arrangement, also known from beehives, $\Delta = 0.91$ (Figure 5a). The packing density of the densest "cannon ball" packing in 3D is 0.74 (Figure 5b). For the sake of comparison: the density of the two- and three-dimensional square-lattice packings is 0.79 and 0,52 respectively.

High packing density allows to approach the carrying capacity of the given resource environment (market, political community, etc). However, it is difficult to find and to build up a really dense packing with conscious positioning, in lack of the same simple building-up algorithm that was specified for the square-lattice arrangement. Now, trials and errors can build up dense packings. New entrants can invade loosely packed spots in the resource space, and selection processes can filter out the organizational surplus from overcrowded domains (Hannan and Freeman 1989, Carroll and Hannan 2000).

3. ADDING NEW DIMENSIONS TO SPACE

This part adds dynamics to the boundary spanning configuration. A shift in spatial dimension reflects that customers consider an additional product descriptor at purchasing (e.g., the degree of environment friendliness). In politics, resource space expansion may mean the introduction of a new political issue that penetrates political discourse. The coming sections demonstrate how the emergence of new spatial dimensions affects the niche packing structure, igniting waves of reconfigurations. This implies a change in niche size and location and blurs up neighborhood relations.

New cavities open in space; earlier friendly neighbors might cross the others' borders, starting a new round of competition and selection.

Space dimensionality may also contract. For example, the number of prevalent political issues decreases to a few or even to one in case of shock situations like wars, revolutions and acts of terror. Technical change may also make once crucial product characteristics obsolete, and so, irrelevant for the customer. In the long run, dimensional contraction induces the inverse of the expansion effects we specify in this part. But in the short run, both increasing and decreasing space dimension incurs a period of reconfiguration with volatile conditions.

Changing space dimension affects packing densities, niche border arrangements (the number and configuration of neighbors), and also the optimal niche size. These factors influence the chances of new organizational entries that may or may not involve the crossing incumbents' boundaries. The consequences on square-lattice packings and dense sphere packings are qualitatively similar. However, the effects differ sharply in magnitude.

3.1 Packing density change

The previous part has already compared the densities of square-lattice based sphere arrangements and optimal sphere packings up to 3 dimensions. The space-covering ratio decreased with n in both cases. Beyond three dimensions, the densest packings are not known yet.[2] Fortunately, an upper bound on the possible best packing is given for each n, and the already discovered densest packings well approximate these bounds up till about 10 dimensions. As spatial dimensions do not exceed ten in most applications, we can rely on the already found densest packings when speaking about close-to-optimal spatial niche arrangements.

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[2] Actually, the Gauss-conjecture, that the cannon ball packing is the densest in 3-D, was proven only a couple of years ago.

Table 1 displays that the best packing density dramatically rapidly converges to zero with the number of spatial dimensions. For example, $\Delta = 0.47$ at five dimensions. So, organizations with finite spherical niches can only reach less than the half of the customer tastes typified by five aspects, if niches are similar in size and boundaries do not intersect. Having ten dimensions, the 90% of the space is empty, with the same conditions in place. The density fall of the "airy" square-lattice packings is even more dramatic. Then, the spheres cover about 16% in 5-D and 0,25% in 10-D of total space (this is 1/3 and 1/40 of the respective best packing densities). The resource space becomes virtually empty in higher dimensions if niche overlap is excluded.

Having fundamental niches without a finite border, the space is fully covered even by a single niche; then, each organization has some positive attraction on any of the taste groups. However, these positive attractions are minuscule beyond a limited core zone of the niche (Figure 2b). So, new entrants may invade the soft points at the deep hole locations. Entry probability increases with the size and number of these soft points. The coming two sections address these aspects.

3.2 Deep holes get deeper

Square-lattice packings. The depth of a hole in a sphere packing is the distance of the farthest point of the cavity from the closest sphere center. Setting radius R to unity and applying the Pythagoras theorem, we get $\sqrt{2}$ for this distance in two dimensions (Figure 4). Applying again the Pythagoras theorem along the third axis yields a $\sqrt{3}$ deep hole value in 3-D. In general, the deep hole size (D) is:

$$D = R\sqrt{n}$$

The deepness of the hole goes to infinity with the number of spatial dimensions in cubic sphere center arrangements. This provides an abundant possibility for new entrants to locate their niches if n is high enough. Moreover, some remarkable change occurs at the organizational boundaries in square-lattice packings as space dimension reaches four. The deep hole distance for $n = 4$ is:

$$D = R\sqrt{4} = 2R.$$

From this D value, the sphere radius takes R, therefore the distance between the niche boundary and the hole center is also R. That is, another touching sphere of the same R radius can be inserted into every deep hole! The obtaining *chessboard lattice* packing doubles the density of the original setting (Conway and Sloane 1998). Assume a three dimensional space fully packed with organizations in the square-lattice arrangement of spherical niches. If a new commodity aspect expands the market into a fourth dimension, then this opens the gate for the entry of a new wave of organizations with the same niche breadth. The number of organizations, and so the market concentration, doubles without even igniting competition. This sort of space expansion may seem to be an "economic perpetuum mobile" that provides infinite opportunities for organization founding. But it does not. A new taste dimension may well induce some more spending, but the total demand usually does not double in the market. The same or somewhat higher purchasing power is distributed over a much higher number of taste configurations in four dimensions than in three. The amount of resource per space volume unit becomes less and less in higher dimensions: the resource space thins out with n.

Beyond four dimensions, the biggest touching sphere radius that fits into the deep hole exceeds R. For example, the maximal non-overlapping radius in the hole exceeds the incumbent organizations' niche radius by 24% and 45% in 5 and 6

dimensions, respectively. This gives a size advantage to newcomers, even if the incumbent niches were at their optimal size the before the dimension shift. This is because the optimal catchment area size tends to increase with the n under a broad range of conditions, and therefore the sales or vote maximizing organizations tend to be bigger in higher dimensional spaces (Péli 2002). The increasing resource scarcity caused by the space thinning out makes optimum size even more important. The incumbent organizations can only adjust their niches to the higher optimum by invading their neighbors' domains and so changing the status quo; the newcomers can take the right size simply by filling up the holes between incumbents. This argument gives a theoretical prediction for empirical research:

> *When resource space dimensionality exceeds four, a robust wave of new generalist entrants can outperform incumbent generalist organizations.*

Dense sphere center arrangements. Close-to-optimal niche packings pose barriers to the invasion of powerful new entrants. Though the deep hole tends to deepen with n at dense sphere arrangements as well, the hole size has an upper bound at $2R$. Were a deep hole deeper than $2R$, another sphere of radius R could be inserted into the hole; so the packing were not dense by definition.

The space covering can further improve even in dense sphere packings, if the assumption on uniform niche radius is released. Think of soap foam, where the big bubbles are surrounded by a big amount of miniature balls in the interstices. In societies, the big and small bubbles are generalist and specialist organizational niches. Specialist organizations may persist with a small niche breadth that is far beyond the optimum. Some of them stabilize its position by maintaining a strong individual reputation. Specialists can usually also go along with a cheap and lean structure because of their small size. But smaller size also means vulnerability; bully

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generalists can easily outforce them in face-to-face resource competition. Therefore, specialists are often margin dwellers. They absorb the peripheral spots of the taste space, weakly populated by customers and often abandoned by generalists (Carroll 1985). Increasing space dimensionality increases the number and also the aggregated volume of unexploited resource pockets, around generalists. Small players may fit to the holes even in the densest arrangements, scavenging on residual resource. This argument helps to explain the observation that specialist numbers can soar in sophisticated markets with a high number of taste aspects (Péli and Nooteboom 1999).

3.3 Higher dimensions, more kisses

The kissing number (τ) of a sphere packings informs about the maximal number of touching neighbors around a sphere. In square-lattice center arrangements, a sphere can have two neighbors along each dimension. Therefore

$$\tau = 2n.$$

The number of immediate neighbors increases proportionally to spatial dimension in square-lattice packings. The kissing number is even higher in dense packings, and its change with n is also faster than linear. For example, the maximal number of neighbors is 6 in two dimensions, 24 in four dimensions and 240 in eight dimensions (Table 1). The more neighbors you have that bid for similar customers, from the more directions you can expect an attack. Higher dimensional spaces are quite uncomfortable places if the number of potential foes is concerned. In this respect, the square-lattice arrangements with their lower kissing numbers are not safer than the dense packings, because their deep holes are the breeding places of powerful new generalist organizations if $n \geq 5$. Also counting the "deadly kisses" of these big guys

attracted into space by loose niche arrangements, neighbors in the entailing grouping can be even outnumber those in the densest packings.

4. SUMMARY

The paper analyzed the consequences of different niche positionings in *n*-dimensional resource spaces. Two types of niche geometry were studied. Customers' indifference thresholds that were independently given for each taste dimension defined the borders of cubic (rectangular) niches. An analogy from bioecology: a species can sustain in an environment if variations in vital physical conditions like heat, humidity, luminosity are kept within limits (Levins 1968).

Spherically symmetric niches obtain if consumers make an overall judgement about the organization's offer, measuring the good and bad matches along taste aspects against each other. Customers at the same distance from the niche center in any directions are the same satisfied with the organization's output. The niches are *n*-spheres with a definite border if the organization's attraction becomes zero beyond a limit distance from the customer. Then, the customer's tolerance limit defines the border of the fundamental niche, organization's operation zone in lack of competition and interaction effects. The *realized niche* can also be a finite sphere, for example if organizations adjust their niche size to an optimum (maximum sales, votes, membership, etc.). Resource competition with other organizations in the form of boundary overlap can also draw finite (but usually non-spherical) realized niches.

The paper compared square-lattice based sphere arrangements with the known densest sphere packings. An advantage of the former configuration is that it builds up relatively simply, if organizations combine imitation with product differentiation. Dense packings usually do not build up by following a simple stepwise search strategy. Rather, selection forces may shape the niche distribution,

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eliminating agents from densely overlapping domains with higher probabilities. Selection processes may take more time than step-by-step niche filling by newcomer organizations. Rapid environmental changes may necessitate reconfigurations that sweep away selection's work to build up dense niche structures. So, one might rather expect simpler niche packings like the square-lattice center arrangement in organizational environments. However, these loose configurations can be unstable; in higher dimensional spaces they behave like time bombs activated by increasing space dimension.

The proportion of empty space increases much faster in the squared center arrangement than in the densest packings. For example, the latter are about two times denser in 4-Ds and three times denser in 5-D. A joint consequence of loose packing and dwindling packing density is that a massive stream of new organizations can enter the market, doubling the population size without niche overlap (in the finite boundary case) at space dimension four. From five dimensions, the niche size of new entrants can be even bigger than that of incumbents'.

The maximal number of neighbors (τ) increases much faster in dense arrangements than in the square-lattice packing. However, if we add the potential new entrants lured by the big deep holes if $n > 4$, then the number of neighbors can become really high. Paradoxically, dense packings can be still safer places for incumbent organizations than the looser and simpler square-lattice arrangements.

4.1 Further research

The findings on n-dimensional boundary effects suggest an agenda for empirical research. Tightly populated product characteristic spaces should be investigated when a new taste aspect appears. From $n = 4$, an invasion of newcomer generalists becomes more and more possible, especially in loose spatial arrangements. The sphere packing analysis might give account for some of the sudden and radical

reconfiguration waves that we can see in certain market histories. Can one identify the emergence of a new product aspect or taste dimension just before the period of turbulence?

Theoretical research in this domain could release some model assumptions. One topic is the systematic assessment of the consequences of size variances on sphere packings. Organizations may compensate the customer with a lower price for the low appeal of a generalist product, and they can reach high prices with well-tuned specialist products. Specialist organizations may also perform quite well in aspects that are hard to be displayed as resource space axes, like product scarcity or the prestige of a specific low-scale production method (Carroll and Swaminathan 2000).

Another research line may release the assumption that resource distribution is even in the niche neighborhood. Markets typically feature unimodal, or sometimes polimodal taste distributions, where mainstream customer flavors form market centers with abundant demand, surrounded by peripheries of thin resource. Demand inhomogeneities may substantially influence the size and also the contours of the fundamental niche. The mathematical results say little about space packings with unequal bodies. However, computer simulations might be able to model organizational interactions over boundaries, taking into account the effects of resource-, price- and strategy differences in spatial positioning. The sphere packings studied in this paper may provide the initial frame along which the modeling can proceed.

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Table 1. The Known Densest Packings and Kissing Numbers

| Dimension n	Packing Density Δ for the best known packing	Packing Density Δ cubic center arrangement	Kissing Number τ
1	1	1	2
2	0.90690	0.7854	6
3	0.74048	0.5236	12
4	0.61685	0.3084	24
5	0.46526	0.1645	40
6	0.37295	0.0807	72
7	0.29530	0.0369	126
8	0.25367	0,0159	240
9	0.14577	0,0064	272
10	0.09962	0,0025	372
11	0.06624		519.78
12	0.04945		756
13	0.03201		1060.67
14	0.02162		1422
15	0.01686		2340
16	0.01471		4320
17	0.008811		5346
18	0.005958		7398
19	0.004121		10668
20	0.003226		17400

Source: Conway and Sloane (1998, p. 15). Note that τ is not always integer. If hyperspheres can have different number of neighbors in a packing, then τ is calculated as an average.

Figure 1. Cubic and Spherical Niches in Two Dimensions

(a)

(b)
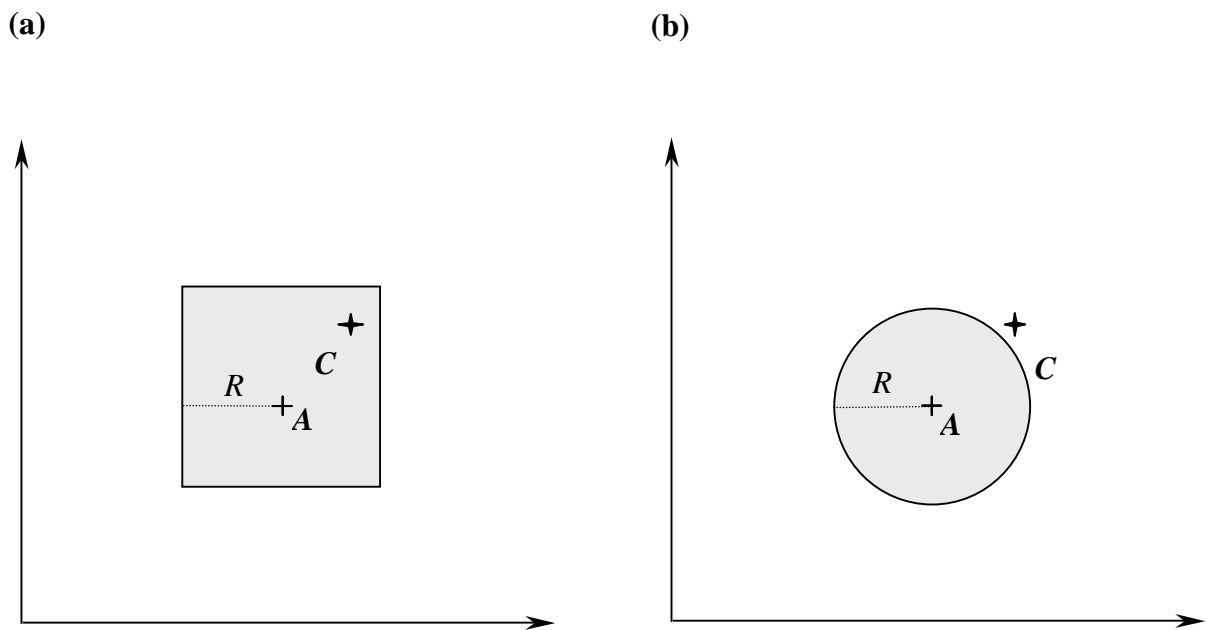
The organization's ideal point is A. The range of address is $2R$ in both cases. The same customer with aste point C falls into the niche in (a) but not in (b).

Figure 2. Fundamental Niches with and without a Definite Boundary

(a)   **(b)**

Attraction

C_1

C_2

Customers' tolerance range
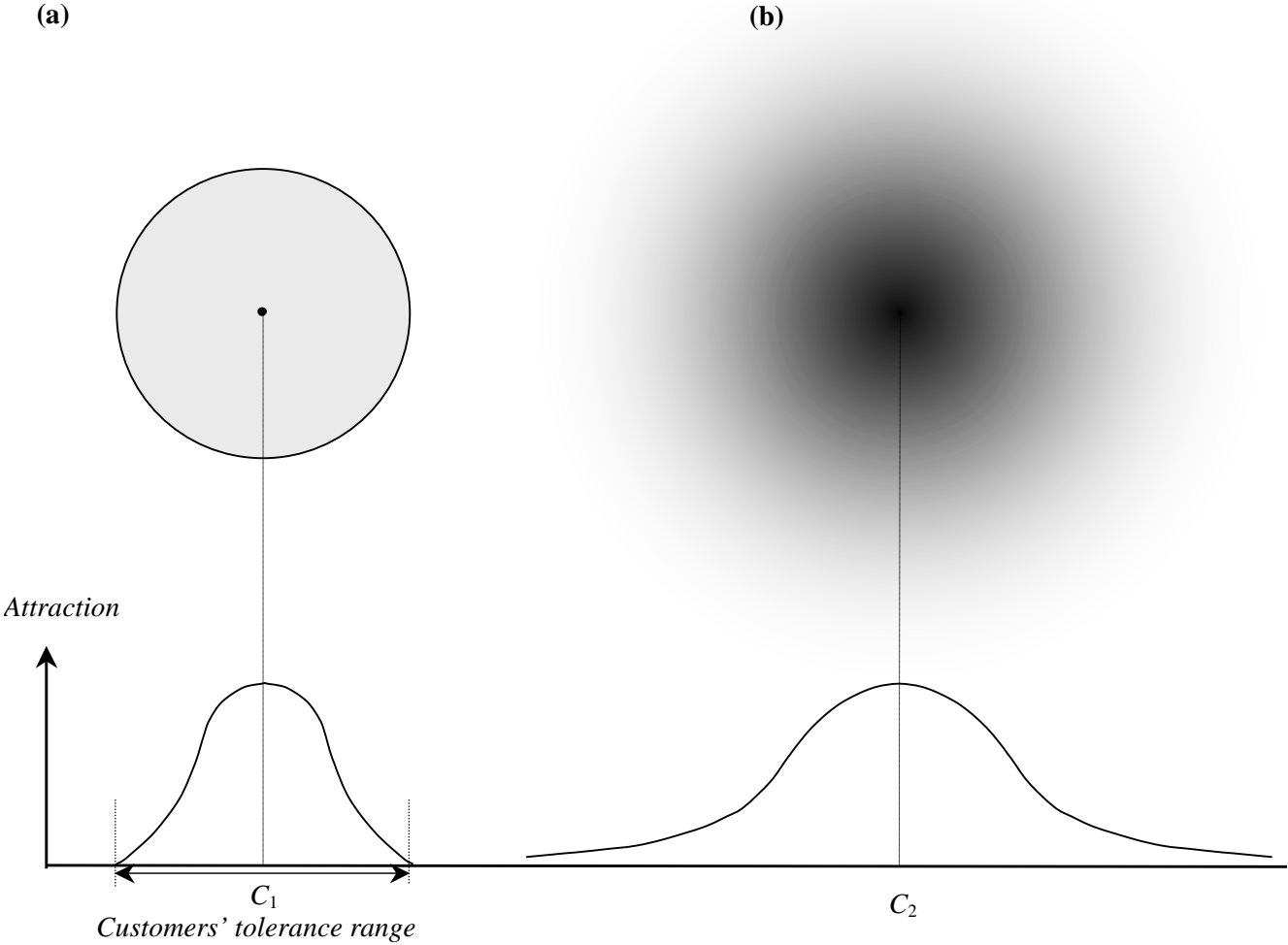
The organizations' attraction on clients decreases with distance from niche centers C_1 and C_2. Attraction loss is asymptotic in the second case, yielding a spherically symmetric unbounded fundamental niche.

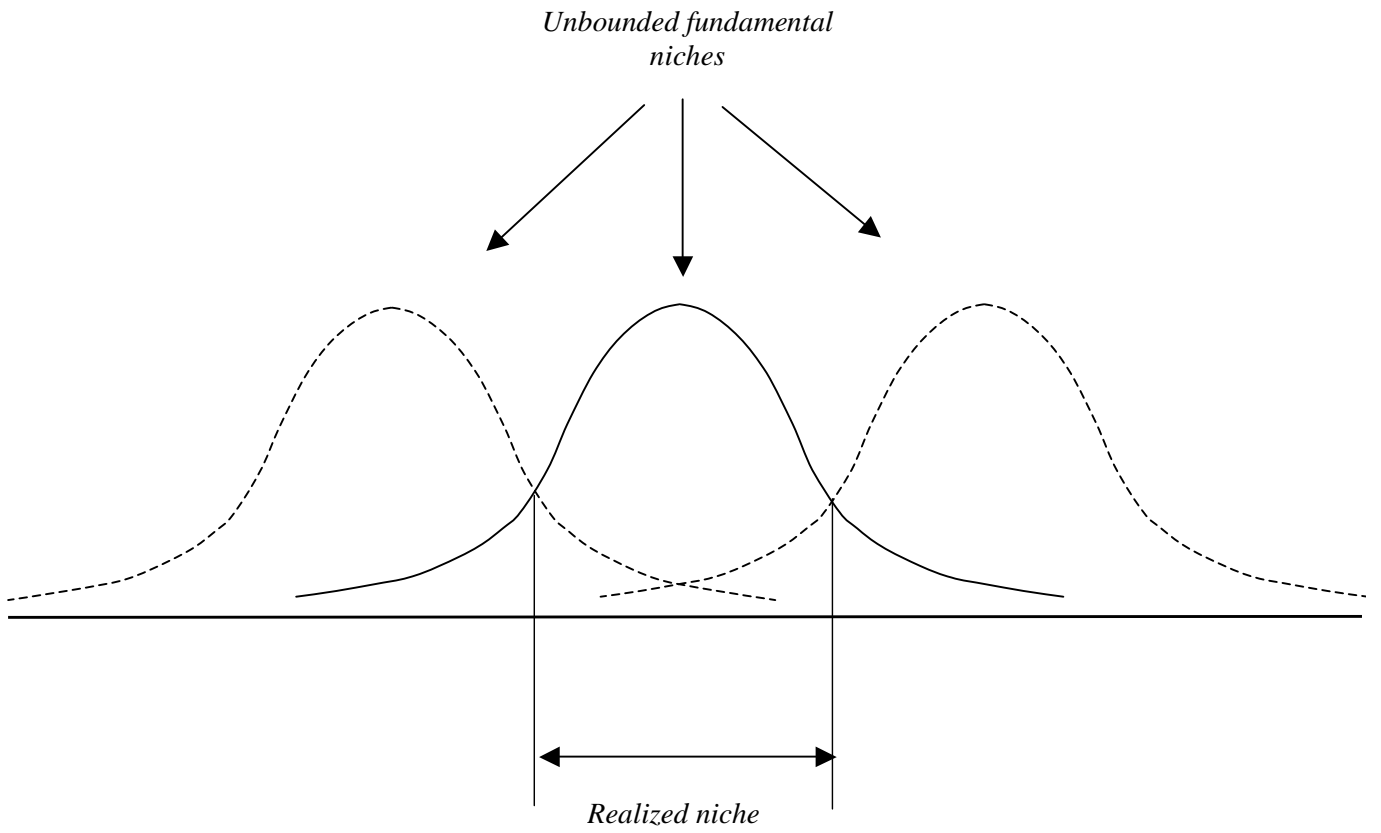**Figure 3. Finite Realized Niche Boundary Set by Competition**

Unbounded fundamental niches

Realized niche

Figure 4. Sphere Centers in Square-lattice Arrangement
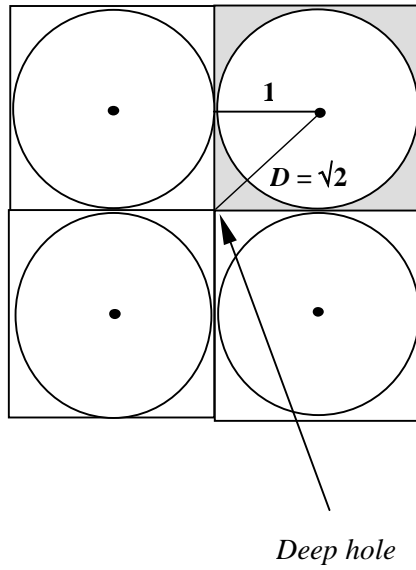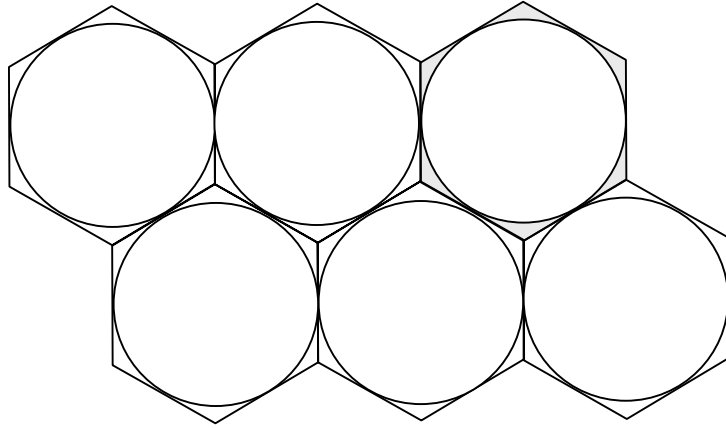

Deep hole

Figure 5. The Densest Sphere Packings in Two and Three Dimensions

(a) Hexagonal packing

(b) Cannon ball packing