Behavioural Cournot competition

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To Ton

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This dissertation is the final outcome of my research, first at the Department of Quantitative Economics at The University of Maastricht, and later at the Department of International Economics and Business at the University of Groningen. It was in Maastricht that the genesis of this work began, when I worked there as a mathematician.

Arjen van Witteloostuijn, who later was to become my supervisor for this dissertation, asked me whether a competition between two companies could also give rise to complex, dynamic phenomena, and I set off exploring chaos theory. A joint publication resulted from our findings.

At that time, however, I did not yet intend to take my doctoral degree, since my heart was committed to teaching. But in the background ideas for research kept bubbling up: outlines of concepts together with a few elaborations were neatly stored in a red portfolio. In the early beginnings of the year 2002 the concepts in the “red portfolio” demanded for a thorough investigation, and after years of teaching I now felt mature enough to devote all my time to research.

Arjen van Witteloostuijn was enthusiastic from the start and his enthusiasm and expert knowledge continually stimulated me in successfully bringing seemingly endless calculations to a close. My supervisor also offered me the opportunity to continue my work on this thesis in Durham in the early months of 2003, where our discussions, together with the beautiful and inspiring surroundings, enriched me and deepened my insight.

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Heerlen, 2003

CONTENTS

LIST OF VARIABLES AND NOTATION

Notation

This dissertation contains many mathematical expressions and equations. Parameters and variables are mostly denoted by a lower case character in italics, such as d. Exceptions are matrices, profits, consumer surplus, welfare, reaction functions and utility functions, which are denoted by upper case characters. Changes or differences are denoted by Δ. In general, subscripts are used to distinguish time periods or the firm to which a parameter or variable refers. However, superscripts are used i) to distinguish the firm in case of profits, and also ii) to denote equilibrium outputs (by the use of *).

List of variables

a	weight of market share in the objective function in Chapters 4, 5 and 6
A	matrix corresponding to a linearized system of difference or differential equations
b	slope of the inverse demand function
c	coefficient of the linear part of the production cost function
CS, CS_{cl}	consumer surplus; consumer surplus of the classical Cournot textbook benchmark case
d	coefficient of the quadratic part of the production cost function (Chapters 4, 5 and 6)
f	function
F	relative total welfare over a whole business cycle in Chapter 4
F_d	relative total welfare over a period with decreased economic activity in Chapter 4
F_e	relative total welfare over a period with increased economic activity in Chapter 4
G	expression for the reaction function if Cardan's method is involved
h	habit formation
i, j	index indicating a firm (such as in "... firm i ...") also often used as a subscript index to indicate the firm to which a specific parameter or variable corresponds
l	adjustment costs per unit production, corresponding to a decrease of production (see also u)
L	Lyapunov exponent
m	market size (in the normalized model)
n	number of firms in oligopoly
p, p^*	market price; equilibrium market price
q, q^*	total market supply; equilibrium market supply
R	reaction function (used with superscripts ^{1}, ^{2}, ^{i} or ^{j})
R^c	compound reaction function
R	relative total profit (used without superscripts) over a whole business cycle in Chapter 4

R_d	relative profit over a period with decreased economic activity in Chapter 4
R_e	relative profit over a period with increased economic activity in Chapter 4
s	status function, non-profit part of the utility function
t	index to indicate a time period (such as in "... time t ...") also often used as a subscript index
u	adjustment costs per unit production, corresponding to an increase of production (see also l)
U	utility function (pay-off function)
W, W_{cl}	welfare; welfare corresponding to the classical Cournot case
x, x^*	output of a firm; equilibrium output of a firm (often with a subscript index)
α	parameter of the reaction function in Chapter 2 weight of (production) size in the objective function
γ	depreciation factor corresponding to habit formation
δ	disturbance term
Δm	the change of the market size
μ	adjustment coefficient
$\Delta\Pi$, ΔCS	difference in profit; difference in consumer surplus
ΔW	difference in welfare
Π, Π_{cl}	profit (often with superscript index); profit corresponding to the classical Cournot profit in equilibrium
$\aleph$	chaotic set

CHAPTER 1

INTRODUCTION

Cournot's assumptions concerning behaviour.

The title of this Ph.D thesis - Behavioral Cournot Competition - indicates that this study is closely related to human behaviour. If Augustin Cournot were to look from a heavenly point of view to the extensions and modifications of his original model, he would probably be highly astonished or paradisiacally self-satisfied by what he has stirred up. It is fitting to pay him the honour he deserves, by reflecting on Cournot models, because he was in fact an early game theorist who attempted to model firms' behaviour. In his "Récherches sur les principes mathématiques de la théorie de la richesse" (1838) Cournot treated the case of oligopolic competition and assumes myopic behaviour, i.e. each incumbent firm reacts on the total market supply of all rivals in the previous period. Since each rival tries to maximize his own profit, such a competitor is able to calculate its actual production level, referred to as the "Best Reply" or mathematically stated as the reaction function. The basic game theoretical model of Cournot already contains two assumptions regarding human behaviour, namely naïve or myopic expectations, concerning rivals' production levels and the assumption of managers' profit-maximizing behaviour as well. Clearly both assumptions may be subject to discussion as we will reflect on in this introduction. Furthermore Cournot assumed that consecutive reactions of two competitors, the "tatônnement proces", would lead to a steady state, nowadays called Cournot-Nash equilibrium, though the equilibrium's stability is by no means certain (a Nash equilibrium is a profile of strategies such that each player's strategy is an optimal response to the other players' strategies, Fudenberg and Tirole (1991)) .

What about stability?

Under the assumption of naïve expectations, the resulting linear downward sloping reaction function does not imply (local) stability automatically if more than two competitors are involved. Theocharis (1960) examined the (Cournot) solution of the oligopoly problem and states that: "If there are two sellers the Cournot solution is always dynamically stable; if their number is three, we shall get finite oscillations about the equilibrium position and if their number is greater than three there will always be instability". If we realize that Theocharis used constant marginal production costs, a decreasing linear (inverse) demand function, and myopic expectations, we may understand that stability issues still receive significant attention in recent research. Modifications of the basic assumptions, such as other (than naïve) expectations, more general demand functions and production cost functions, multimarket and multi-product competition and learning behaviour of players lead to a wide variety of outcomes. Fisher (1961) examined the implications on stability of speeds of adjustment and increasing marginal costs, whereas Okugushi (1976) investigated equilibrium's existence, stability and uniqueness under a more general formation of expectations. Other contributions are Furth (1986), Okugushi and Szidarovsky (1990) (multi-product firms), Szidarovsky and Yen (1995) (quadratical adjustment costs around the Cournot-Nash equilibrium) and Zhang and Zhang (1996) (multi-product and multi-market case). Kohlstad and Mathiesen (1987) and Gaudet and Salant (1991) provide conditions for uniqueness (and use two different methods of proof) and, in their paper, Long and Soubeyran (2000) provide a proof of equilibrium's existence and uniqueness, using yet another technique (the contraction mapping approach). Dastidar (2000) proves that under broad conditions uniqueness implies local stability. One cannot say that the stability (and existence) issue receives less attention today.

Reaction curves and chaos.

Another branch of research sprouts from the Cournot tree, because the classical and familiar downward slope of the reaction curve may not always be taken for granted. This may have significant implications for the stability of the Cournot-Nash equilibrium, even in duopoly models; if the adjustment process fails to converge to a Cournot-Nash equilibrium, firms' supply paths may also exhibit cyclical and chaotic patterns. Rand (1978) proved, in a rather technical and beautiful paper, that a duopoly game characterized by unimodal reaction functions - monotone increasing for rival's outputs to the left of the maximum and (familiar) monotone decreasing for competitor's supplies to the right - may imply the inherent chaotic nature of the dynamical system. The famous paper of Li and Yorke (1975) made researchers aware of the fact that simple non-linear equations in a dynamical model may lead to chaotic time paths of variables under study and gave a significant impetus to the further development of chaos theory. Although applications of the mathematical findings concerning chaos theory became quite fashionable, naturally researchers tried to provide microeconomic foundations for the occurrence of non-monotonic reaction curves concerning Cournot competition.

Van Witteloostuijn and van Lier (1990, see also Chapter 2) improve upon Rand (1978) and Dana and Montrucchio (1986) by providing a nonlinear model of Cournot duopoly competition with (realistic) positive monopoly output. They prove the occurrence of chaotic regimes and use simulation experiments to illustrate the properties of chaos. They also use the term "dualist" concerning a firm with an asymmetric reaction pattern: if the rival's supply is below a certain level a firm's "Best Reply" to an increase of the rival's supply is also an increase of production level (so-called aggressive behaviour). However, if the competitor's output exceeds a certain level, a firm's response to more aggressive play by its competitor is less aggressive play. Although van Witteloostuijn and van Lier's reaction curves may describe firms' behaviour adequately, the functional form is based on empirical reflections (such as entry deterrence), and a possible microeconomic foundation for the occurrence of hill-shaped reaction curves is still missing. By considering a general expression for the slope of a reaction curve, concerning arbitrary demand and cost functions, one gets indications for choices of these functions which may lead to a positive slope or a switch in slope's sign. Since then two contributions (which indeed use other specifications for the demand and cost functions) to the microeconomic foundation for the existence of non-monotonic reaction curves are worth mentioning. First Puu (1991,1998) shows that the assumptions of constant unit production costs and an iso-elastic demand function result in unimodal reaction curves (and periodic or chaotic supply patterns concerning cases with two or three competitors). So in Puu's models firms (or managers) still show pure profit-maximizing behaviour, whereas the essence of their models is determined by the choice of economists' second-favourite demand curve (constant elasticity demand).

Second Kopel (1996) proves that introducing cost functions with an interfirm externality - marginal production costs not only depend on firm's own quantities, but also on the supply offered by the rival - leads to unimodal (quadratic) reaction functions and very complicated dynamics. However the well thought-out models of both Puu and Kopel show one salient shortcoming: firms' monopoly output equals

zero. Realizing that a positive monopoly output (and the use of an aggressive strategy against an entrant) may be used as an instrument to deter entry, a zero monopoly supply, as an implication of a microeconomic assumption, seems not very realistic.
If we reflect on the models of Puu and Kopel, we observe that the models' modifications - in comparison to Cournot's original model - take into account "environmental" factors such as the form of the demand curve and the influence of the rival's supply level on their own production costs. The essence of their modifications is determined by extra exogenous factors (besides the self-evident assumption of direct competition), whereas the classical profit-maximizing behaviour of the firm (represented by its managers or run by its owner) remains unchanged. Of course this "endogenous", profit-maximizing, behaviour of the firm is completely determined by human beings, i.e. the (top) management team and the firm's culture. Assuming that every human being possesses some degree of selfish behaviour, management targets do not have to coincide with the firm's interests and profitability. Almost every human being is motivated by love, sex, status and money, not necessarily ranked in this specific order. Therefore, if, for instance, salaries and bonuses of top managers are not only determined by a firm's profits, but may be influenced by a concern's market share or size (sales volumes) as well, it is questionable whether the target of managers is really pure profit maximization. In their pathbreaking paper "Equilibrium Incentives in Oligopoly" Fershtman and Judd (1987) consider the separation of ownership and management of firms. They demonstrate that competing firms' owners will often distort their managers' objectives away from strict profit maximization for strategic reasons.

Do firms really maximize profits?

Concerning principal-agent models, such as the models of Fershtman and Judd (1987), Sklivias (1987) and Vickers (1985), managers' behaviour is influenced by incentive contracts written by the firms' owners. Owners may commit their (top) managers to nonprofit maximizing behaviour. But apart from the fact that managers' objectives can be directed by owners - by using salaries not only based on firms' profits but also based on sales volumes or revenues as well - firms' cultures and ingrained habits of top managers must not be underestimated. First we consider empirical findings concerning managers' nonprofit maximizing objectives. Then we also briefly reflect on the principal-agent models, which provide a possible explanation for managers' behaviour as well.

Each textbook on organizational behaviour (Robbins (2001)) treats the well-established phenomenon of the resistance to change on a firm's work floor; managers dislike retrenchment whereas they prefer expansion of their departments. The psychological fact that managers prefer to be the head of large departments provides an example of human behaviour, apart from the explanation of managerial behaviour by incentive contracts written by owners.
We have to realize that a firm's behaviour as a whole is fully determined by the behaviour and habits of its (top) management team. A firm's culture and behaviour may be looked upon as a weighted sum of the behaviour and habits of its chief executives. If research points at nonprofit maximizing motives of managers, this justifies the introduction of firms' nonprofit maximizing objectives in competition models as well. From an empirical perspective cumulative evidence supports that firms are not pure profit-maximizers, but that sales or market share also form some of their objectives. A study of Niskanen (1971) indicates that bureaucrats maximize

budgets. Note that this finding may imply that the size of departments is part of the goal of managers (naturally firm's size depends on the size of its departments). Deneffe and Masson (2002) test the hypothesis of profit maximization of hospitals and come to the conclusion (resulting from regression analysis) that hospitals maximize a combination of profits and size (number of patients). Van Witteloostuijn (1998) notes that: "The key assumption in managerial economics is that managers may fail to maximize profits, since they may be driven by other motives … the common denominator of such nonprofit motives is that managers are assumed to favour size: managers prefer to be the head of large rather than small organizations."

Van Witteloostuijn (1998) uses a data set concerning the sales adjustment behaviour of 10 global chemical companies in years of profitability growth versus decline in the period 1967-1992 and shows that "an increase in profitability goes hand in hand with an upward adjustment of sales, whereas a decline of profitability is not associated with a downward adjustment of sales volume." This tentative evidence supports managerial "love for size". It may not come as a surprise that a firm's market share may also be a crucial part of its objective in strategic competition. Because a firm's behaviour is determined by the behaviour and preferences of its management, empirical research concerning their preferences is crucial. Peck (1988) reports the results of a survey into corporate objectives among 1,000 American and 1,031 Japanese top managers. Increasing market share ranks third in the American and second in the Japanese subsample, whereas return on investment is first among American and third among Japanese top managers.

Besides psychological reasons concerning the "love for size or market share", clearly managers' behaviour will be influenced by managerial compensation and top-management incentives (as the recent case of the Ahold-concern shows, managers may even use fraudulent accounting to ensure their bonuses). Empirical studies in managerial compensation reveal that executive bonuses and salaries are associated with both firm size and profit level, with the size correlations being the stronger of the two (Jensen and Murphy (1990) and Lambert, Larcker and Weigelt (1991)). If the salaries of executives are more closely correlated with the scale of operation (size) than with a firm's profitability, this association may be disadvantageous for the firm. Influenced by bonuses, a manager may select projects that increase his firm's size (or market share) and his level of compensation, but may have a negative impact on his firm's market performance and social welfare. Realizing that the previous examples of empirical research on managerial objectives and compensation schemes are far from complete, they do justify extensions of the (classical) Cournot competition model.

As already noted principal-agent models may provide an explanation for managers' nonprofit maximizing behaviour. In the models of Fershtman and Judd (1987), Sklivias (1987) and Vickers (1985), a two-stage sequential game is considered, because owners (principals) and managers (agents) are separated. In the first stage the owners write the incentive contract for their managers. In the second stage this incentive contract forces the managers to maximize the objective function "$\alpha\Pi + (1-\alpha)S$", where Π and S respectively equal profits and revenues (we note that the objective can be rewritten as "$\Pi + (1-\alpha)\cdot c\cdot x$", where c equals marginal production costs and x equals production level; Vickers considers an objective function with a combination of profits and production levels). In stage two the managers' function is to observe the (till now uncertain) demand and cost functions and to maximize their objective function by manipulating quantities (Cournot) or prices (Bertrand). Knowing the Nash equilibrium in this second stage, (by backward induction) owners can choose the weights α in the incentive contract in the first stage in such a way that profits are maximized, given the rival's weight (so-called subgame perfect equilibrium). Note that the owner's target is to maximize profits; if an owner doesn't hire a manager the objective function is Π and the weight α equals 1 in these models. These two-stage sequential games lead to incentive contracts with weights $\alpha < 1$ corresponding to quantity competition and $\alpha > 1$ concerning price competition. In both cases managers behave as nonprofit maximizers. Fershtman and Judd (1987) refer to the resulting equilibrium as the "incentive equilibrium" and prove that "… incentive equilibria in the quantity game generates greater output, lower rents, lower prices, and a more efficient allocation of production than the usual Cournot equilibria." In case of Bertrand competition incentive equilibria imply smaller output (and higher market prices) than the usual Bertrand equilibria. These principal-agent models explain why managers' objective function may be a (linear) combination of profits and revenues (or equivalently profits and production levels)(1).

However, we note that these principal-agent models assume highly rational behaviour of firms' owners and managers and also assume that the incentive contract of the competitor is known without delay. In this thesis we will refer to this rational behaviour as "strategic consciousness". In this introduction we will also pay detailed attention to the viewpoint of Organizational Ecology and we will discuss the "inertia hypothesis" of Hannan and Freeman (1984). From this standpoint the weights α, attributed to sales volumes or revenues, are part of the "blueprint" of a firm (a firm's culture) and therefore these weights are rather fixed relative to environmental changes and disturbances or in other words: they represent a form of a firm's inertia. The considerations in this thesis emphasize the rather fixed character of managerial behaviour and focus on the consequences concerning firms' profits, survival chances and social welfare for several weight combinations of rivals in a duopoly setting.

We consider the introduction of the models of this thesis justified, because, whatever the explanation of managers' behaviour will be (owner-manager relation, psychological or blueprint of the firm), empirical research points at managers' nonprofit maximizing objectives.

The Behavioral Cournot model.

The behavioral model also takes into account firms' productions adjustment costs around a fixed level of supply. Both this latter extension and the introduction of the concept of inertia need further clarification. In the framed part on the next page, in the outline of the general model, we recognize the "classical" Cournot model in which firms (or managers) maximize their actual profits (revenues - production costs), using the rivals' supplies in the previous period as a forecast for their actual production levels (myopic expectations).

Concerning adjustment costs, production expansion and shrinking obviously lead to additional expenses of the firm, on top of the standard unit production costs. Van Witteloostuijn, Boone and van Lier (2003) note that: "A prominent long-run cost of production flexibility follows from the investment or devestment of capital. This is straightforward investment economics. Additionally, a major short-run cost of production flexibility originates in human resources". Because adjustment costs are

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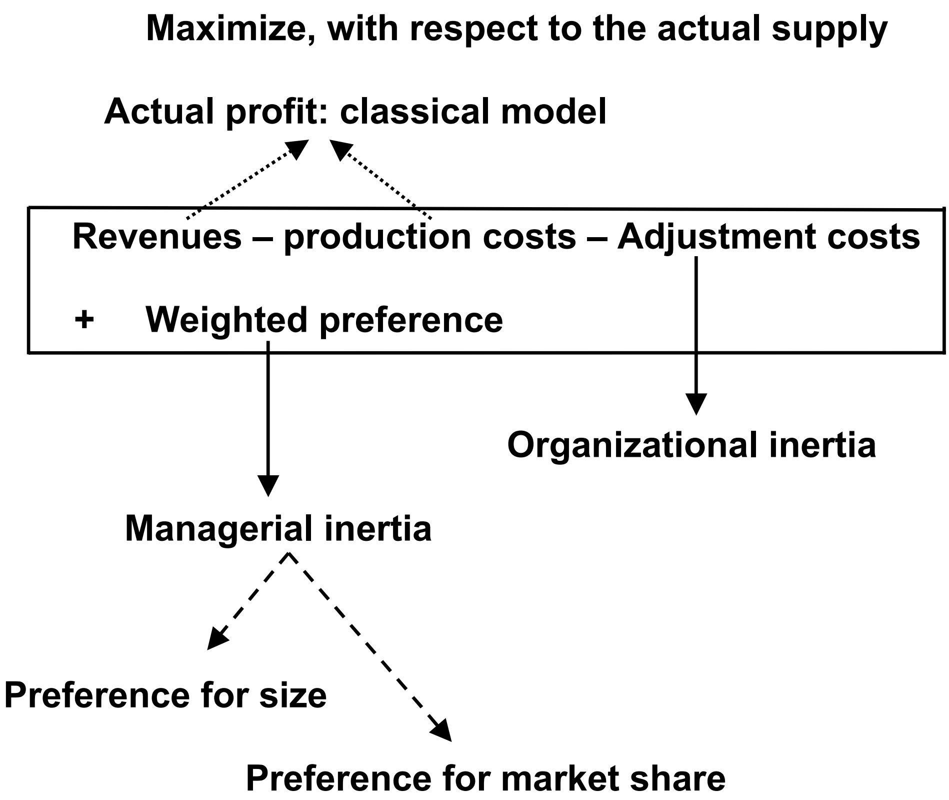
(1) Yet another explanation is provided by a dynamic optimization model of Glaeser and Schleifer (2001). They demonstrate that, if consumers are prepared to pay a higher price for quality, the not-for-profit status may be more advantageous for firms (operating in sectors such as child care, long term care for the aged, hospitals and schools). "When consumers care deeply about non-verifiable quality, entrepeneurs prefer non-profit status, because it softens incentives [concerning pure profit] and brings higher prices ex-ante."

strongly related to human resource management practices, studies concerning these practices are of importance and illustrative. Huselid (1995) reveals, in his single-country (US) study, that firms differ in their choice for high- or low-commitment human resource management practices. Naturally high commitment human resource practices are relatively expensive, because of high salaries and long-term contracts and therefore possibly lead to higher adjustment costs. Furthermore Gooderham, Nordhaug and Ringdale (1999) report significant and systematic cross-country heterogeneity concerning human resource management practices. One may expect that firms are more or less slow in changing their production level over a downward or upward phase of a business cycle or stated differently: firms show inertia concerning environmental turbulence.

The general model



In the (extended) Behavioral Cournot model apparently two forms of firm's inertia can be distinguished, organizational and managerial inertia. We now discuss this important concept (from Organizational Ecology (OE)).

Inertia.

Hannan and Freeman (1984) reflect on the question how quickly an organization can be reorganized in relation to environmental opportunities and threats. They argue that the concept of structural inertia must be defined in relative and dynamic terms: "In particular, structures of organizations have high inertia when the speed of reorganization is much lower than the rate at which environmental conditions change. Thus the concept of inertia, like fitness, refers to a correspondence between the behavioral capabilities of a class of organizations and their environments". We can look upon inertia as an organizational "blueprint" built up in the history of a firm. For instance, if a firm invests in extra capacity, possibly to deter entry (Bulow, Geanakoplos and Klemperer (1985) , Dixit (1980)), this capacity can be considered as a form of (relative) inertia, because it takes effort, costs and time to reduce capacity. Also a firm's advertising policy, formed in a concern's past, may be seen as an example of an organizational "blueprint", subject to resistance if the environment changes.

Van Witteloostuijn, Boone and van Lier (2003) distinguish three forms of inertia, namely population (system), organizational and managerial inertia. Population inertia deals with a whole population of organizations (and is therefore related to a higher level of analysis) and may be the result of the existence of path dependencies in the competitive process. They state that: "…switching costs at the demand side, which result from network economies, may impede firms from introducing superior technologies that impose a high cost on those who deviate from the market standard". This phenomenon may result in the overall use of more inferior technologies despite of the availability of better technologies. Other examples of population inertia are industry-level exit and entry barriers. Our "Behavioral Cournot" model does not take into account system inertia, but focuses on a market with two incumbent firms which possess organizational or managerial inertia.

Organizational inertia.

Adjustment costs around a certain production level are clearly an example of inertia at the organizational level. As already mentioned earlier, firm's adjustment costs are strongly related to human resource management practices and there exist inter-firm differences and cross-country heterogeneities between these practices as well. On the one hand these human resource practices are related to countries' laws and may be the result of trade union negotiation processes. On the other hand, in a society with a high speed of technological development, firms naturally attach importance to the maintenance or improvement of their level of knowledge, thus implying favourable labour contracts (permanent appointments, advantageous dismissal procedures and golden handshakes) concerning high-educated personnel. Concerning the form of firms' production adjustment cost function, Hamermesh and Pfann (1996) discuss a large number of empirical studies that have revealed evidence for different shapes of these functions. Production adjustment cost functions may be symmetric or asymmetric, linear or non-linear, depending upon the country, industry and period under study. In Chapter 4 the consequences of a linear, asymmetric adjustment cost function will be examined on competitors' profits, market supply and social welfare. One of the model's outcomes is that firms do not instantaneously react on environmental turbulence, such as a business cycle, but stay inert concerning their production levels if the market size starts to change. The issue of inter-firm heterogeneity, concerning adjustment costs, turns out to be an interesting subject of research.

Managerial inertia.

As may be clear from previous considerations, empirical evidence supports the presence of nonprofit maximizing behaviour, in OE terminology: managerial inertia, on an organization's work floor. In the "Behavioral Cournot" competition models two forms of managerial inertia are included, namely preference for size (or sales volume) and preference for market share. We note that in the principal-agent models of Fershtman and Judd (1987), Sklivias (1987), Vickers (1985) and Basu (1995) - Basu considers a three-stage sequential game, where in the first stage owners decide to hire a manager or not - in essence always preference for size is considered. Concerning the model's schematic presentation (p.6), the term "weighted preference" is used. In this thesis a model parameter (α or a) is introduced to tune the managerial level (weight) of preference for size or market share (in the models α or a equal to 1 indicates equal weights for profit and size or market share in firm's objective function).Therefore the implications of firms' heterogeneity, with respect to their levels of preference, can be studied by allowing different values of these model parameters between rivals. And if we choose this specific parameter equal to zero, we deal with managers' (classical) profit-maximizing behaviour (concerning the principal-agent models profit-maximizing behaviour corresponds with an owner who hires no managers). This profit-maximizing case provides a natural point of reference. Clearly a difference between managers' preference for size or market share strikes the eye: whereas the first form of managerial inertia deals with the absolute supply level of the firm, the second form takes into account a firm's production levels in relation to market supply. In this thesis also the implications of larger levels of preference are examined (for instance the level of managerial inertia may increase in a declining market). Chapter 3 deals with preference for size, whereas Chapters 5 and 6 deal with managerial preference for market share. We note that the analysis of the consequences of managers' preference for market share is mathematically much more complicated than cases which consider preference for size.

Research topics in this thesis.

Before we present a schematic summary of the Chapters and their topics of research we briefly mention and motivate these research issues.

- **Demand turbulence**. In empirical studies in the field of Organizational Ecology (OE), the market size is referred to as the carrying capacity of a population of organizations. Obviously carrying capacity highly influences the survival chances of firms in a population; a declining market size intensifies competition, because all incumbent rivals depend on market demand. In our models (in Chapters 3 and 4) demand turbulence is introduced by modeling declining, increasing and cyclical demand (using the "market size" parameter m).

- **Strategic competition**. Because in the "Behavioral Cournot" models the output levels of each firm crucially depend on the rival's supply level, competition is per definition strategic (we use naïve expectations, i.e. a competitor uses rival's previous output in its decision making). A crucial question in this thesis is whether firms may use their (level of) structural inertia, such as adjustment costs and managerial preferences, as a strategic instrument. If firms are able to adapt their levels of inertia rationally (as in the principal-agent two-stage sequential games), they (for instance the owners) may choose an optimal level of inertia. If firms cannot change their managerial levels of preference or adjustment costs quickly enough, from the standpoint of Organizational Ecology, selection will favour those firms with the most advantageous levels of inertia (concerning profitability) in a Darwinian selection process. Research focusses on this intriguing issue in Chapters 3, 4, 5 and 6.

- **Structural inertia**. We already motivated in detail the modeling of forms of structural inertia, supported by results from empirical research.

- **Cost- and product heterogeneity**. Differences between competitors may include a heterogeneity in the efficiency of their production technologies. In Chapter 3 this difference is reflected by different unit production costs, whereas in the other Chapters only equally efficient rivals are studied. We also focus on homogeneous products, although future research will include product heterogeneity (and Bertrand competition) as well. In Chapter 3 an interesting issue is whether a (relative) inefficiency in the production process may be compensated by managerial preference for size in a competitive setting (now we hope your level of curiosity has increased further).

- **Social welfare**. The examination of the influence of structural inertia on social welfare - defined as the sum of both rivals' profits (producer surplus) and consumer surplus - receives significant attention in Chapters 4 and 5. If managers' bonuses are strongly correlated with (growth of) size or market share, executives might be willing to sacrifice profitability, to enhance sales and hence their own compensation. To anticipate one of the outcomes (of Chapter 5): as an implication of preference for market share, indeed firms' equilibrium profits are sacrificed. The question is which level of managerial inertia is still "healthy", concerning social welfare.

- **Complicated dynamics**. Baumol and Benhabib (1989) discuss the implications of chaos for economic modeling and state that: "Yet all that may be involved, as we will see, is the phenomenon referred to as chaos, a case that is emphatically not pathological, but in which a dynamic mechanism that is very simple and deterministic yields a time path so complicated that it will pass most standard tests of randomness." In Chapter 2 hill-shaped reaction curves (without a microeconomic foundation) result in chaotic supply paths. In Chapter 5 it will be proved that managerial preference for market share may result in non-monotonic reaction curves, thus implying the possibility of chaos. In our model firms' naïve expectations may lead to an error between the predicted and actual output. If firms were to detect some patterns in their forecasting errors, they would possibly revise their expectation formation. Hommes (1998) examines elaborately the autocorrelation between these forecasting errors concerning a price adjustment (Cobweb) model, using various expectation schemes of the agents. His concepts and use of techniques may be interesting for future research, concerning competing firms.

The following matrix summarizes the topics which receive attention in this thesis. This schematic overview reveals that heterogeneity between firms, reflected in their production technologies is only considered in Chapter 3, mainly due to the fact that reaction functions corresponding with managerial preference for market share possess a rather complicated mathematical form. If different levels of preference for market share and different production costs are involved in the model, Cournot-Nash equilibria resulting from the duopoly case can only be obtained by using numerical methods.

Research framework summarized (+ means this topic receives attention, whereas -- indicates no current attention)

	Ch. 2	Ch. 3	Ch. 4	Ch. 5	Ch. 6
Demand turbulence	--	+	+	--	--
Strategic competition	+	+	+	+	+
Structural inertia	--	+	+	+	+
Cost/Product Heterogeneity	--	+	--	--	--
Welfare	--	--	+	+	--
Complicated dynamics	+	--	--	--	+

The models' outcomes from two different perspectives.

We may reflect on the analytic results of this thesis' game theoretic modeling from two perspectives, namely from an Organizational Ecology (OE) and an Industrial Organization (IO) point of view. Because both economic disciplines use competition as a central concept, game theoretic models may contribute to the insights of both disciplines. OE primarily involves empirical work (which does not mean that no mathematical modelling is used in this field) and has mainly focused on diffuse or indirect competition, i.e. rivals influence each other because they compete for the same limited resources (such as the market demand for their products). IO, however, describes and models many different types of competition, varying from perfect (diffuse) competition to oligopolistic (direct) competition; firms may use strategic actions to deter entry or to outcompete rivals. The two-stage sequential games of

Fershtman and Judd (1987), Sklivias (1987) and Vickers (1985) also assume strategic use of nonprofit maximizing objectives (determined by firms' owners). Because structural inertia plays a crucial role in this thesis, we first consider the inertia hypothesis of OE and then we will reflect on the IO standpoint, i.e. firms' use of strategic moves.

The inertia hypothesis.

Hannan and Freeman (1984) argue that (assumption 1): "Selection in populations of organizations in modern societies favors forms with high reliability of performance and high levels of accountability". Briefly speaking, a firm's reliability and accountability requires that structures of roles, authority and communication must be reproducible from day to day. So firms create and maintain standardized routines ("blueprints") and this naturally implies (relative) inertia; the speed at which a firm may reorganize its procedures and "blueprints" is subject to resistance and therefore may be lower than the rate at which the environment changes. These arguments lead to Hannan and Freeman's (1984) famous inertia hypothesis:

> "Selection within populations of organizations in modern societies favors organizations whose structures have high inertia"

However, others argue that firms should be highly flexible concerning their bureaucratic blueprint (Volberda (1998)) and another study on organizational decline suggests that "strategic paralysis" - a firm's inert strategic behaviour - foreshadows a firm's bankruptcy (D'Aveni (1989)). The analytic outcomes of our game theoretical models contribute to the inertia debate. For instance one of the results is that (van Witteloostuijn, Boone and van Lier (2003):

> "An organizationally flexible (i.e., without production adjustment costs) firm will be outcompeted by an organizationally inert (i.e., with production adjustment costs) rival in a declining market. The opposite holds true in a booming market."

This result (of Chapter 4) supports Hannan and Freeman's inertia hypothesis in a declining market. The results of the analysis of Chapters 3, 4, 5 and 6 may be looked upon from the OE standpoint. This means that the weights attributed to size (α in the models of Chapter 3) or market share (a in the models of Chapters 5 and 6) in the objective function, or the adjustment cost parameters (l and u in Chapter 4) are fixed (inert). So all sorts of heterogeneities concerning these levels of inertia between two rivals can be considered.

"Strategic consciousness".

As already mentioned earlier, firms may hold idle capacity to deter a possible entrant (Bulow, Geanakoplos and Klemperer (1985)) and owners may use their managers' incentive contracts as a strategic weapon in direct competition (Fershtman and Judd (1987) and others). This strategic action rests on three assumptions. First a firm (owner) has to be aware of the strategic consequences of its action - we use the term "strategic consciousness" - and second a firm must have the possibility to change the relevant characteristic, thereby using it as a strategic instrument. Third the firm must have adequate information concerning its rival's relevant parameters (full information about rival's incentive contracts). For instance, what will be the consequence of delayed information, concerning rival's "incentive parameters"?

One of the outcomes of Chapter 4's analysis is that production adjustment costs are advantageous in a declining market. However in the light of a firm's structural inertia, firms may not be able to heighten these costs - for instance by changing human resource management practices - in the short run. And firms may not be aware of the implications of production adjustment costs. On the other hand, if competitors are able to adapt their production adjustment costs, because enough time is available and they behave rationally, this may result in a two-stage sequential game; firms first "choose" their level of organizational inertia and then both firms manipulate their output levels. The same reflections, concerning rational and strategic behaviour, hold for managerial inertia.

One of the interesting analytic results is that there exists an optimal level of structural inertia. Concerning managerial inertia we mention a result of Chapter 5.

> "If a firm attributes weight to its market share, whereas its competitor is a (classical) profit-maximizer (for instance an owner who hires no managers), there exists a level of preference for market share, which maximizes the "market share loving" firm's profit."

From an OE standpoint selection favors firms with a certain level of managerial inertia in a Darwinian selection process. However, from a IO point of view a firm may change its level of managerial inertia to enhance its profitability and strengthen its strategic position in the market. We conclude this introduction with advices for the reader and some minor notes.

To the reader.

- You may wish to read and study the chapters of this thesis separately. Chapters 2, 3, 4 and 5 can be studied as independent units. The study of Chapter 6, however, requires some previous analytic results of Chapter 5. Beside these prerequisites, first reading Chapter 2 is recommended, because both Chapters 2 and 6 deal with complicated dynamics, including chaos.

- The production cost function considered in the analysis of Chapters 4, 5 and partly Chapter 6 is quadratic. This allows us to study also the implications of production technologies with increasing and decreasing returns to scale. You may wish to reflect on the implications of constant marginal production costs: just choose the parameter d in the expressions equal to zero.

- Chapter 6 concludes with a reflection on Stackelberg leadership. This deviation from the "Behavioral Cournot" model, is motivated by the fact that, as an implication of preference for market share, the size of the "market share loving" firm may amply exceed the competitor's supply volume.

I hope that in reading this thesis, you may experience the same level of satisfaction as I did in delivering a report of my research.

CHAPTER 2

CHAOTIC PATTERNS IN COURNOT COMPETITION

Apart from a few adjustments, this Chapter is based on a publication in 1990: Van Witteloostuijn A. and Van Lier A.: “Chaotic patterns in Cournot competition”, *Metroeconomica*, 41, 161-185, 1990.

1. Chaotic patterns in economics

Chaos theory offers a new mode of analyzing the complexity of nonlinear (economic) dynamics. A growing list of applications is mainly focused on modeling macroeconomic (growth and business) cycles and dynamic (consumer's and firms') choice. This Chapter provides a nonlinear dynamic model of Cournot competition. The model improves upon Rand (1978) and Dana and Montrucchio (1986) by permitting monopoly output to be positive. The existence of chaotic regimes is proven and simulation experiments illustrate the implications.

In the 1970s and 1980s chaos theory broke and still “breaks across the lines that separate scientific disciplines. Because it is a science of the global nature of systems it has brought together thinkers from fields that had been widely separated” (Gleick (1987, p. 5)). Gleick (1987, Chapter 2) does not hesitate to characterize the rise of chaos theory as a revolution, since it “has become not just a canon of belief but also a way of doing science. ... Some carry out their work explicitly denying that it is a revolution; others deliberately use Kuhn’s language of paradigm shifts to describe the changes they witness” (pp. 38-39).

The essential notion of chaos theory is that (even simple) dynamic systems may generate seemingly random and chaotic patterns. Irregular and unpredictable time paths result from deterministic sources. Baumol and Quandt (1985) offer the illustrative, if imprecise, description that “chaos is defined as a fully deterministic behaviour pattern which is, in at least some respects, undistinguishable from a random process or, rather, a process perturbed by substantial random elements. It displays extreme sensitivity to changes in parameter values, and is characterized by an infinite number of equilibria each approached by (superimposed) cycles of different periodicities, and whose simultaneous presence is what gives the appearance of randomness to a time series generated by a deteministic process” (p. 3). Chaos theory reaches an analytical apparatus which has found application in many scientific disciplines.

This Chapter loosely defines chaos as to three features of dynamic trajectories: (i) sensitive dependence on initial conditions; (ii) existence of periodic orbits of all periods; and (iii) existence of an uncountable set of initial conditions that each give rise to (asymptotically) aperiodic time paths (Kelsey (1988, p. 9)). The point of departure is a first-order difference equation (with a continuous function f),

$$x_{t+1} = f(x_t)\,, \tag{2.1}$$

which can be associated with chaotic trajectories if nonlinearity gives a hill-shaped function. The key point is that “it cannot be too strongly emphasised that the process is generic to most functions with a *hump of tunable steepness*" (May (1976, p. 461, italics added)). Particular specifications of equation (2.1) can give a sequence of bifurcations such that "the pattern never repeats" (May (1976, p. 461)). For the moment, this intuition suffices. An excellent, general review of the merits of nonlinear dynamics is May (1976), whereas Kelsey (1988) and Baumol and Benhabib (1989) offer nice introductions of chaos theory in economics. A mathematical introduction to chaotic one- and higher- dynamical systems is provided by Devaney (1989). For more detailed mathematical information of maps on the interval (after all the function f of equation (2.1) is such a map) we refer to De Melo and Van Strien (1993).

In the late 1970s and early 1980s the methodology of nonlinear dynamics also entered the economic scenery. The most widespread use of chaos theory lies in the field of macroeconomic (business and growth) cycles (Stutzer (1980), Benhabib and Day (1980,1982), Day (1982,1983), Dana and Malgrange (1984), Day and Shafer (1985,1987), Grandmont (1985,1986), Boldrin and Montrucchio (1986), Deneckere and Pelikan (1986) and Julien (1988)). These models induce "the profession's growing awareness of the fact that, even in the absence of extraneous shocks, the internal (nonlinear) dynamics of an economy may generate quite complex periodic orbits or even nonexplosive 'chaotic' deterministic trajectories, that may be hard to distinguish from 'truly random' time series ... . Indeed, the recent approach to endogenous business cycles relies often on advances made lately in the mathematical theory of nonlinear dynamical systems, in particular the analysis of sudden qualitative changes displayed by their trajectories ('bifurcations')" (Grandmont and Malgrange (1986)) ([1]).

The second class of applications of chaos theory to economic frameworks are nonlinear models of (consumers' and firms') dynamic choice (Rand (1978), Benhabib and Day (1981), Baumol and Quandt (1985), Dana and Montrucchio (1986), Granovetter and Soong (1986), Rasmussen and Mosekilde (1988), and Iannaccone (1989)). These contributions "show that rational choice in a stationary environment can lead to erratic behaviour ... . We mean by erratic behaviour choice sequences that do not converge to a long-run stationary value or to any periodic pattern" (Benhabib and Day (1981, p. 459)). A particular type of nonlinear models of dynamic choice focuses on Cournot competition (Rand (1979), and Dana and Montrucchio (1986)). This Chapter offers a constructive critique of the two existing nonlinear models of Cournot (duopoly) competition.

The Chapter is organized as follows. Section 2.2 describes the essential features of the two existing nonlinear models of Cournot (duopoly) competition. A basic flaw of these models is the (implicit) assumption that monopoly output is zero. Section 2.3 presents a model of Cournot duopoly competition which permits monopoly output to be positive. Section 2.4 illustrates the model's features with the help of the results of simulation experiments. Section 2.5 briefly indicates the applicability of the analytical apparatus of nonlinear dynamics to topics of theory of competition in industrial organization.

([1]) Of course, business and growth cycles with chaotic patterns may follow endogenously from agents' behavior, where the agents' decision making induces a chaotic sequence of choices (for instance, *via* agents' hill-shape offer curves in Grandmont (1985))

2. Nonlinear models of Cournot competition

Examples of the introduction of chaos theory in industrial organization are scarce. Dana and Montrucchio (1986) argue that “the only exception is the seminal paper of Rand ..., which shows, in a very abstract manner, that the Cournot tâtonnement in a duopoly model may display a complicated dynamical structure” (p. 41). The current state of the art is not much different. Only Dana and Montucchio’s (1986) treatment of Cournot duopoly models provides a further contribution to the application of nonlinear dynamics to topics in industrial organization ([2]). The nonlinear models of Cournot competition indicate that rivalry in a market can be associated with turbulent movements of the firms’ quantities if the competitors’ reaction functions are hill-shaped.

Recall that the Cournot (1838) duopoly model implies that a firm i chooses a supply quantity (x_i) so as to maximize a profit (Π^i) function, conditional upon the quantity offered by the rival j (equal to x_j),

$$\underset{x_i}{\text{Max}}\, \Pi^i = p(x_i + x_j)\cdot x_i - c_i(x_i) \tag{2.2}$$

where p denotes the inverse demand function and so price and c the cost of production. Solving the maximand (2.2) gives the first-order condition

$$p(x_i + x_j) + x_i \cdot \mathrm{d}p/\mathrm{d}(x_i + x_j)\cdot(1 + \mathrm{d}x_j/\mathrm{d}x_i) - \mathrm{d}c_i/\mathrm{d}x_i = 0 \tag{2.3}$$

From condition (2.3) the firm’s reaction function follows:

$$x_i = R^i(x_j)\,, \tag{2.4}$$

where $i, j = 1,2$ and $i \neq j$ with the Nash assumption that a firm expects a passive reaction of the rival upon its quantity strategy $\mathrm{d}x_j/\mathrm{d}x_i = 0$. In the standard Cournot duopoly models (Tirole (1988)) chaotic patterns cannot emerge, since the rivals’ reaction functions (2.4) are assumed to be linear or insufficiently nonlinear (*i.e.*, without a hill-shape).

The introduction of nonlinear dynamics in a Cournot duopoly model requires that at least one of the rivals’ reaction functions is hill-shaped. The reason is straightforward. Suppose that neither of the reacton functions takes a hill-shaped form; that is $\mathrm{d}R^i/\mathrm{d}x_j \leq (\text{or} \geq)\, 0$ for $x_j \geq 0$, where $i, j = 1,2$ and $i \neq j$ .Assume that rival 1 and 2 react according to the time sequence ..., t - 1, t, t + 1, ... For example, if rival 1 offers a quantity $x_{1,t-1}$ at time t - 1, then rival 2 reacts by supplying $x_{2,t}$ at t which provokes rival 1’s reaction $x_{1,t+1}$ at $t+1$, *etcetera*. Then, $x_{i,t+1} = R^i(x_{j,t})$ so that $x_{i,t+1} = R^i(R^j(x_{i,t-1})) = k^i(x_{i,t-1})$. Now, $\mathrm{d}k^i/\mathrm{d}x_{i,t-1} = \mathrm{d}R^i/\mathrm{d}x_{j,t}\cdot \mathrm{d}R^j/\mathrm{d}x_{i,t-1} \leq (\text{or} \geq)\, 0$. Hence, the absence of hill-shaped reaction functions implies that the second-order

([2]) A further exception is perhaps Baumol and Quandt’s (1985) nonlinear model of advertizing.

difference equation of a rival's quantities shows not even a single hump because the sign of the derivative of the (composed) function k^i is unchanging.

Rand's (1978) approach to the Cournot duopoly model is very abstract indeed by directly postulating unspecified reaction functions with sufficient nonlinearity. Rand treats an example of an analytical hill-shaped function and a nonanalytical tent map. Analytical hill-shaped first-order difference equations can have chaotic regimes (Section 2.1). Besides, nonanalytical first-order difference equations can also give chaotic patterns for particular ranges of parameter values (May (1976, p. 465) and Devaney (1989, p. 52)). Dana and Montrucchio (1986) supplement Rand's treatment of chaotic behaviour in Cournot duopoly models by, among other things, providing five specified examples. Given the desired hill-shaped specification of the reaction function(s), they derive (an) associated specification of profit function(s).

Both Rand's and Dana and Montrucchio's reaction curves have the shape which is depicted in Figure 2.1 (for the analytical case). We will comment on the concepts "strategic complements and substitutes" in Section 2.3.

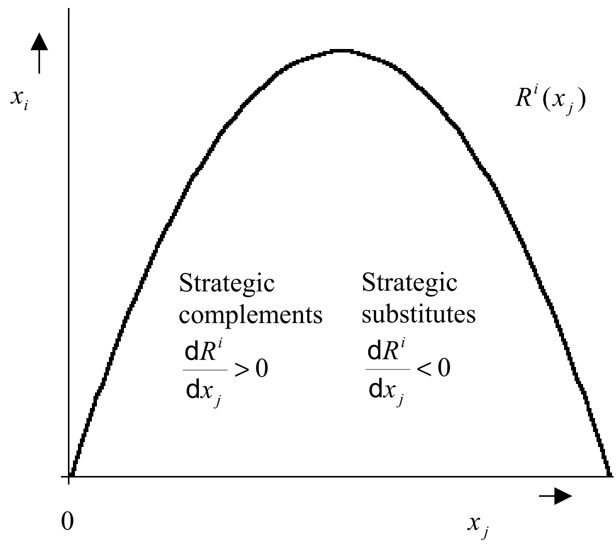


Fig. 2.1 Hill-shaped Cournot reaction curve with zero monopoly output.

The *ad hoc* assumption of hill-shaped reaction functions in Rand's and Dana and Montrucchio's analyses leaves an essential question unanswered: Can an economic rationale be provided for the (very complicated) nonlinear shape of the profit functions? Kelsey (1988) points out that the "shapes of the reaction functions [Rand] assumes are very extreme indeed. It does not look like they could be generated by plausible demand and cost functions" (p. 19). However we mention three microeconomic foundations for the occurrence of hill-shaped reaction curves. Puu (1991) proves that under the assumption that the quantity demanded is reciprocal to price (and with constant unit production costs), reaction curves are unimodal. And Kopel (1996, pp.2036-2038) demonstrates that cost functions incorporating an interfirm externality lead to quadratic and hill-shaped reaction curves as well. That these nonlinear shapes of the reaction curves can also be generated under the assumption of a non-profit maximizing objective function is the main subject of Chapter 5. If equation (2.2) is modified by adding a non-profit part, unimodal reaction curves are quite possible. General conditions and a specification will be provided in Chapter 5 whereas the interesting and complicated dynamics will be examined in Chapter 6.

But another crucial observation immediately strikes the eye. The usual illustration of a hill-shaped curve implies that $R^i(0) = x_i = 0$. The reaction curves in Rand (1978), Dana and Montrucchio (1986), Puu (1991) and Kopel (1996) all show this feature. This means that the (implicit) assumption is imposed that firm i offers a *zero* output in response to firm j's zero production: that is, monopoly output is taken to be zero! However, this assumption is not very realistic. This extreme case can be bypassed by introducing a $x_i^M = R^i(0) > 0$, where x_i^M represents firm i's monopoly output (which, for example, can follow from the standard maximization procedure of a monopolist). Figure 2.2 illustrates the shape of this reaction function.

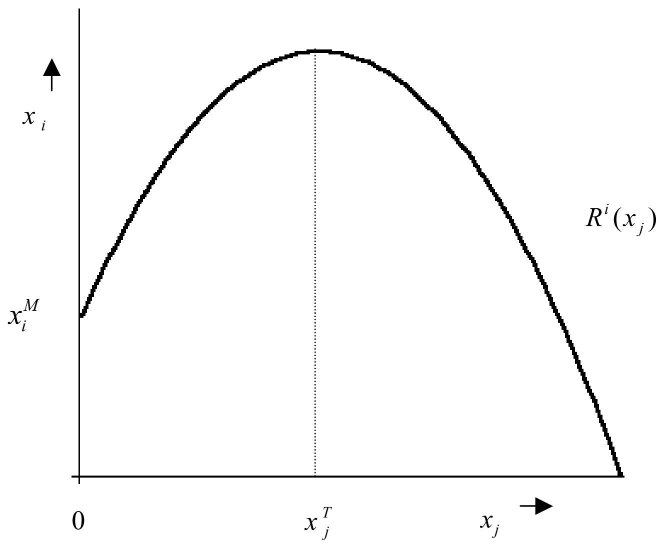


Fig. 2.2 Hill-shaped Cournot reaction curve with positive monopoly output.

The hill-shaped Cournot reaction curve with positive monopoly output induces a further question: Can a proof of the existence of chaotic regimes still be provided? Section 2.3 goes on to examine both questions of economic interpretation and proof of existence.

3. Cournot reaction curves with positive monopoly output

Concerning the economic interpretation we discuss three concepts namely the asymmetric reaction pattern, strategic substitutes and complements and idle capacity. Then a general formulation of Li and Yorke's Theorem will be given and this condition will be applied to three different duopoly scenarios: a "dualist" against an "imitator", a "dualist" against an "accommodator" and finally a "dualist" against a "dualist". The specification of a reaction function with positive monopoly output, which contains a parameter α, enables us to derive a condition for these parameter values corresponding with chaotic regimes.

Asymmetric reaction pattern.

The critical implication of the hill-shape of a reaction function is that a firm shows an *asymmetric* reaction pattern. For $x_j < x_j^T$ (Fig. 2.2) firm i and j's supplies are *positively* correlated, whereas $x_j > x_j^T$ is associated with a *negative* relationship between x_i and x_j. Hence, for $0 \le x_j < x_j^T$ firm i acts as a *follower* or *imitator*. If firm j expands output, so does firm i. Whenever firm j contracts supply, firm i too introduces a decrease of the quantity offered. However, if firm j expands its output beyond x_j^T, then firm i starts to act as a *fringe competitor* of *accommodator*. On the one hand, whenever firm j expands output, firm i simply adapts to reduced residual demand. On the other hand, if firm j contracts output, then firm i adopts an aggressive strategy by expanding its supply.

The asymmetric reaction pattern follows from the switch in sign of the first-order derivative of the reaction curve. A firm with an asymmetric reaction pattern can be called a *dualist*: that is, the firm's reply can be to imitate as well as to accommodate, depending on the scale of the rival's output.The reaction curve of a dualist is hill-shaped.

Strategic substitutes and complements.

Types of reaction patterns can be distinguished as to the features of the cross-partial derivatives of the firm's profit with respect to its opponents' action (Bulow *et al*. (1986, pp. 491-497), and Tirole (1988, p. 208)). Here it suffices to note that "with strategic substitutes *B*'s optimal response to more aggressive play by *A* is to be *less* aggressive ... . With strategic complements *B* responds to more aggressive play with *more* aggressive play" (Bulow *et al*. (1986, pp. 494)). In terms of Cournot competition this means that strategic substitutes predict $\mathrm{d}R^i/\mathrm{d}x_j < 0$, whereas strategic complements indicate $\mathrm{d}R^i/\mathrm{d}x_j > 0$. Hence, the substitute or complement nature of the firm's reaction pattern is reflected in the sign of the reaction curve's slope. Both cases are depicted in Figures 2.3a and b (Tirole (1988, p. 208)). The reaction function which follows from strategic *complements* (curves I), describes the reaction pattern of an *imitator*, whereas an *accommodator's* responses are reflected in the reaction curve with strategic *substitutes* (curves II).

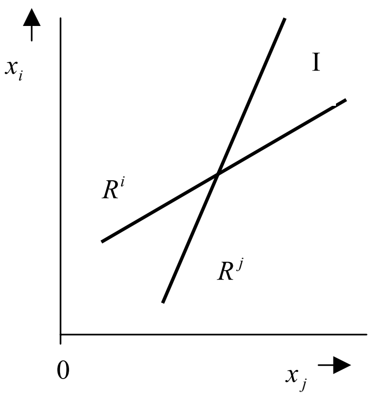


Fig. 2.3a Strategic complements.

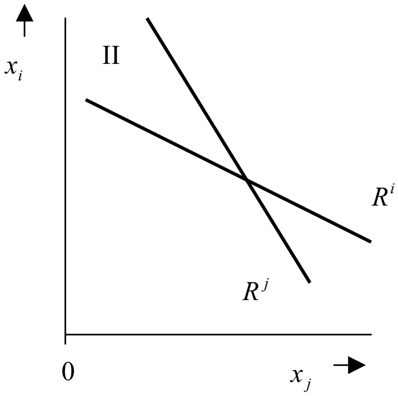


Fig. 2.3b Strategic substitutes.

Idle capacity

Bulow *et al.* (1985, p. 180, see also Fig. 2.2) provide an answer to the first question by offering an economic interpretation of hill-shaped reaction curves with positive monopoly output in terms of strategic substitutes and complements. Here the following brief intuition suffices. Starting from monopoly output a firm is willing to *increase* supply in reaction upon entry, which contradicts the downward slope of standard Cournot reaction curves: that is, starting from monopoly output the firm regards outputs as strategic complements. This assumption follows from the literature on entry deterrence. Two arguments offer a case in point. First, the aggressive strategy after entry is described in the literature on idle capacity as an entry-deterring instrument (Spence (1977, 1979), and Ware (1985)). Second, the post-entry expansion policy can be grounded in long-run reputation arguments, even if this strategy is not profit-maximizing from a short-run perspective (Milgrom and Roberts (1982, 1987), and Arvan (1986)).

However, the expansion policy does not pay if the rival's scale moves beyond a particular point. After a certain scale of expansion (x^T) the benefit of accommodation starts to dominate over the advantage of the aggressive strategy, which implies that the standard downward slope of the Cournot reaction curve sets in: that is, the firms consider outputs to be strategic substitutes. The hill-shape of Cournot reaction curves can follow from demand specifics. The key point is that the "assumption that each firm's marginal revenue is always decreasing in the other's output ... is quite a restrictive assumption. For example it is *never* satisfied in the relevant range for economists'second-favourite demand curve - constant elasticity demand" (Bulow *et al.* (1985, p. 178)).

Proof of existence by application of Li and Yorke's Theorem.

Granovetter and Soong (1985, pp. 92-93) provide a graphical intuition which suggests that chaotic regimes can occur in hill-shaped functions with a positive intercept (in a model of nonlinear consumers' choice) without, however, offering a proof. Li and Yorke (1975) provide however a theorem which can be used to *prove* the existence of chaotic regimes in general. This chapter employs an abbreviated version of this theorem.

Theorem of Li and Yorke
The first-order difference equation (2.1), where f is a continuous map of an interval J into itself, gives chaotic regimes if there exists a value of $x_t \in J$ such that

$$x_{t+3} \le x_t < x_{t+1} < x_{t+2} \text{ (}^3\text{)}. \tag{2.5}$$

Following Day (1982, 1983) Li and Yorke's Theorem can be re-expressed as

$$f(x_m) \le x_c < x^* < x_m\,, \tag{2.6}$$

where

$$x_m = f(x^*) = \max_{x>0} f(x) \text{ and } f(x_c) = x^*. \tag{2.7}$$

If the function $f(x)$ is hill-shaped, the inequality $x^* < x_m$ is equivalent to $x_c < x^*$ and the inequality $f(x_m) \le x_c$ can be formulated as $f^2(x_m) \le x^*$, where $f^2(x) = f(f(x))$, the second iteration of f. Then, the theorem of Li and Yorke can be re-expressed as

$$f^2(x_m) \le x^* < x_m\,. \tag{2.8}$$

Form (2.8) will be used to prove that there exists an uncountable set $\aleph$ of initial conditions that give rise to chaotic time paths for a significant class of hill-shaped reaction functions with positive monopoly output. The remainder of this Section contains the analysis of the three cases mentioned in the introduction: (1) one firm (re)acts as a dualist, whereas the rival is an imitator; (2) one firm is a dualist, while the rival responds as an accommodator; and (3) both rivals behave as dualists. It appears that all three cases can be associated with chaotic reaction patterns.

Case 1: Dualist against imitator.
The reaction function of firm i, $R^i(x_j)$, is assumed to be hill-shaped (with positive monopoly output), whereas the reaction function of firm j is supposed to resemble $R^j(x_i) = x_i$. This scenario describes competition between a dualist and a perfect imitator. Rival i and j react according to the time sequence *..., t, t*+1*, t*+2*, ...* . this means

$$x_{i,t+2} = R^i(x_{j,t+1}) = R^i[R^j(x_{i,t})] = R^i(x_{i,t})\,. \tag{2.9}$$

With doubled lengths of the time intervals equation (2.9) has the same form as the first-order nonlinear difference equation (2.1). Figure 2.4 illustrates the applicability of Li and Yorke's Theorem to a first-order difference equation which resembles Figure 2.2's hill-shaped Cournot reaction curve with positive monopoly output (the dualist, curve I) and 45°-line (the perfect imitator, curve II).

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(3) Hence, if there exists a cycle with period 3, then there are chaotic regimes as well. This result is related to Sarkovskii's (1964) theorem (Kelsey (1988, p. 5)).

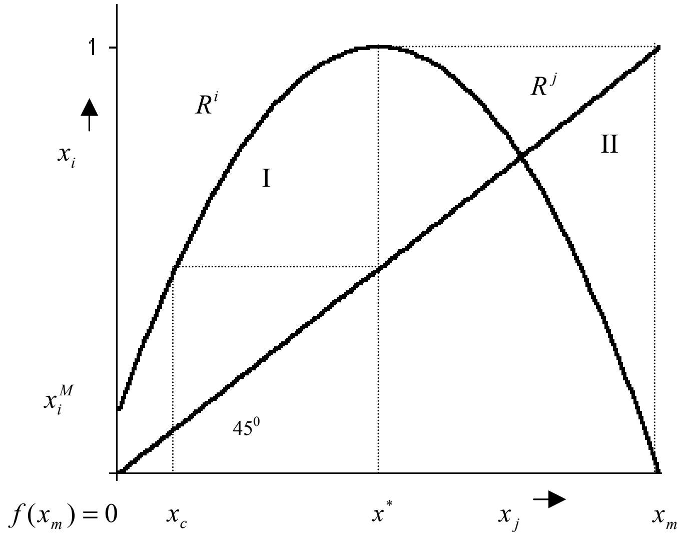


Fig. 2.4 Case 1: Dualist against imitator.

This Chapter uses the specification

$$x_i = R^i(x_j) = 1 - \alpha \cdot (x_j - 1 + 1/\sqrt{\alpha})^2,\ \alpha > 1 \tag{2.10}$$

for the dualist's hill-shaped reaction curve. Note that this function possesses a maximum for $x_j = 1 - 1/\sqrt{\alpha}$ with a value of $x_i = 1$, whereas the monopoly output equals $x_i^M = R^i(0) = 2\sqrt{\alpha} - \alpha > 0$ for $\alpha < 4$.

For a significant class of second-degree reaction functions R^i with positive monopoly output and assuming that firm j acts as a perfect imitator, the existence of chaotic regimes can be proven algebraically with the use of condition (2.8). Following Rand (1978) and Dana and Montrucchio (1986) output is scaled to $x_i, x_j \in [0,1]$. If $R^i(1) = 0$ and the maximum of R^i is 1, then condition (2.8) indicates that a necessary condition for proving Li and Yorke's chaos is $R^i(0) \le x^*$. That is, the monopoly output x_i^M is restricted by the upperbound x^* (*i.e.*, the location of the maximum).

Proposition 2.1 (chaotic regimes concerning a dualist and an imitator).

If $x_i = R^i(x_j) = 1 - \alpha \cdot (x_j - 1 + 1/\sqrt{\alpha})^2$ and $x_j = R^j(x_i) = x_i$, and for the parameter α it holds that $3.0795... \le \alpha < 4$, there exists an uncountable set $\aleph$ of initial conditions with chaotic (asymptotically aperiodic) time paths and for *every* natural number k there exists a time path with period k.

The proof of Proposition 2.1 is offered in Appendix 2.1. Proposition 2.1 indicates that Cournot duopoly competition can be associated with chaotic trajectories if a

dualist (that is, a firm with a hill-shaped reaction function) competes against a perfect imitator, even when monopoly output is assumed to be positive.
Proposition 2.1 is robust as regards to modifications of the assumption that rival j (re)acts as a perfect imitator. First, take the case where competitor j only imitates imperfectly.

Proposition 2.2 (concerning a dualist and an imperfect imitator).

If the reaction function of firm i has the parabolic form as indicated in Proposition 2.1 but with $3.0795... \leq \alpha < 4$, then the case where the reaction function of rival j reflects imperfect imitation also gives rise to chaotic time paths.

The proof of Proposition 2.2 is given in Appendix 2.2. The key point is that the reaction function of firm j is turned into $R^j(x_i) = x_i + \delta(x_i)$, where $\delta(x_i)$ indicates a small disturbance. The composed reaction function $R^i(R^j)$ then possesses the same shape as the one in the proof of Proposition 2.1, except for a small disturbance term, so that Li and Yorke's condition can still be satisfied if the disturbance is small enough.

Case 2: Dualist Against Accommodator.

The assumption that rival j (re)acts as an imitator, can be dropped in favor of the well-established case which assumes a downward sloping Cournot reaction curve. That is, the dualist i (reaction curve I) faces an accommodator j (reaction function II). This scenario is illustrated in Figure 2.5a for perfect accommodation. This means that the accommodator j (re)acts according to $x_j = R^j(x_i) = 1 - x_i$.

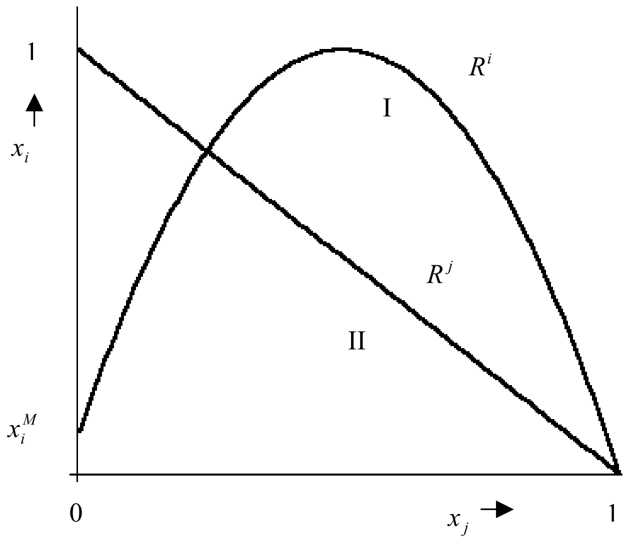


Fig. 2.5a Case 2: Dualist against accommodator.

Figure 2.5b shows graphically that Li and Yorke's theorem can be applied. The composed reaction function R^c (Fig. 2.5b) - with $x_{i,t+2} = R^c(x_{i,t}) = R^i(1 - x_{i,t})$ - can be used to illustrate that condition (2.6) does hold. By again making use of the re-expressed Theorem of Li and Yorke (2.8) this result can be proven algebraically.

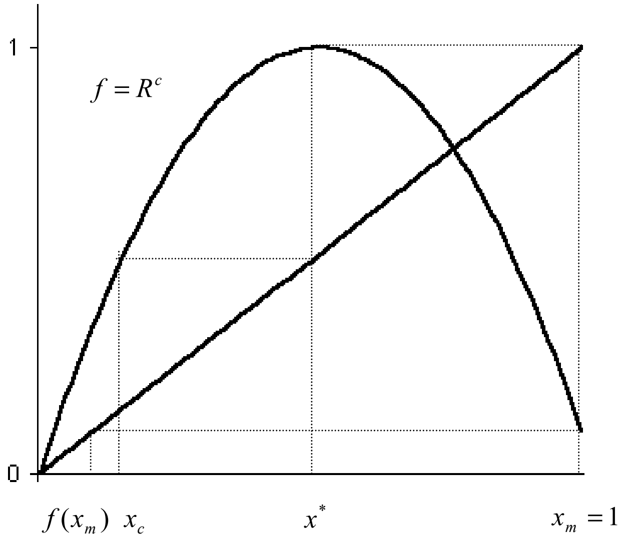


Fig. 2.5b Composed reaction function of a dualist against an accommodator.

Proposition 2.3 (chaotic regimes concerning a dualist and an accommodator).

If the reaction function of firm i has the parabolic form as indicated in Proposition 2.1 and for the parameter α it holds that $3.6708... \leq \alpha < 4$, then the case where the reaction function of rival j reflects perfect accommodation is also associated with chaotic time paths.

Proposition 2.3's proof is presented in Appendix 2.3. Proposition 2.3 indicates that Cournot accommodation can also give chaotic time patterns if one of the two firms decides on the basis of a hill-shaped reaction function with a positive intercept (*i.e.*, if one of the rivals is a dualist with positive monopoly output).

Case 3: Dualist Against Dualist.

The third case describes the scenario where both firms have the same hill-shaped reaction function with positive monopoly output. That is, the two rivals behave as dualists. This case is depicted in Figure 2.6a.

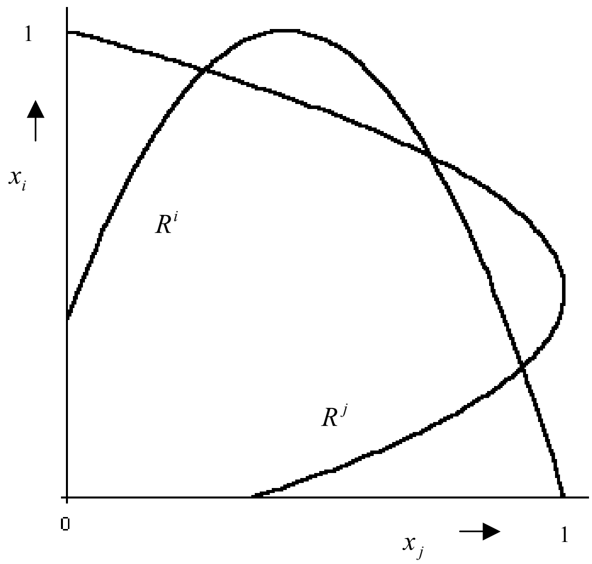


Fig. 2.6a Case 3: Dualist against dualist.

The analytical proof follows from Proposition 2.1 and because it is not very technical we give it here.

Proposition 2.4 (chaotic regimes concerning two dualists).

If the reaction functions of firm i and j have identical hill-shaped forms, chaotic regimes can be derived.

Proof

With reference to Appendix 2.1 this proof can be brief. Appendix 2.1 proves that the function $f_\alpha(x)$ in the difference equation gives rise to a chaotic set $\aleph$, which generates the chaotic time paths. Two dualists firm i and j react according to the same reaction function with doubled length of the time intervals. So, now the difference equation equals

$$x_{i,t+1} = f_\alpha(f_\alpha(x_{i,t}))$$

and with x_i replaced by x

$$x_{t+1} = f_\alpha(f_\alpha(x_t)) = f_\alpha^2(x_t)$$

The function f_α^2 - the second iteration of f_α - gives rise to (asymptotically aperiodic) time paths too, because a time path of f_α^2 can be derived by skipping the "odd terms" in a time path of f_α (in other words the chaotic sets of f_α^2 and f_α are equal).

Because f_α generates time paths with period k (Proposition 2.1), where k can be every natural number, the function f_α^2 gives the same result.

[End of proof]

Proposition 2.4 predicts that Cournot competition between two rivals which are making use of equivalent hill-shaped reaction functions with positive monopoly output, can be associated with chaotic trajectories of output.
Figure 2.6b shows the compound reaction curve ($= f_\alpha^2(x)$) of two dualists and can be used for a graphical proof of the existence of Li and Yorke's chaos.

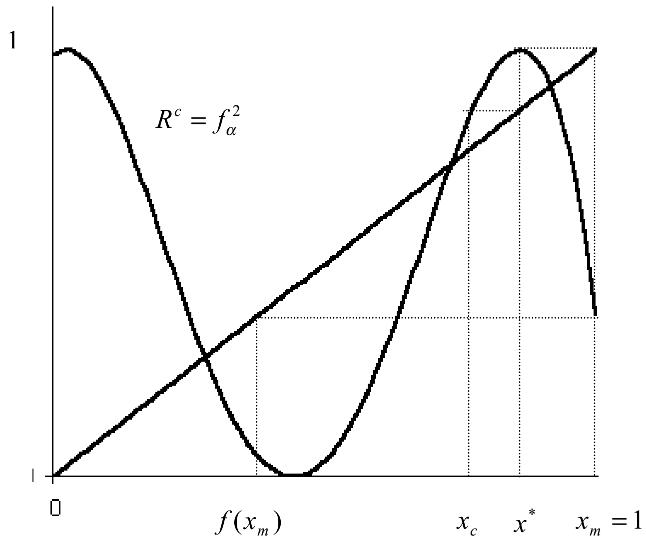


Fig. 2.6b Composed reaction function of a dualist against a dualist.

4. Simulation examples

Functional specifications.

The implications of hill-shaped Cournot reaction curves can be illustrated through simulation of competition for a series of (counter-)moves ([4]). The simulation experiments cover 120 moves (or periods $t = 0,...,119$): that is, both rivals act and react 60 times. Rival i sets supply in even-numbered periods ($t = 0, 2, 4,...,118$), whereas rival j's replies are effectuated at odd-numbered dates ($t = 1, 3, 5,...,119$). Two initializations dictate the simulation results. First, by varying the value of the parameter (α) the steepness of the hill-shaped reaction function (2.10) can be tuned. Second, variation of the first move ($x_{i,0}$) manipulates the initial competitive condition.

The experiments simulate the dualist against imitator rivalry (Section 2.3, case 1). Firm i is the dualist ($x_i = R^i(x_j) = 1 - \alpha \cdot (x_j - 1 + 1/\sqrt{\alpha})^2$) and firm j acts as a perfect imitator ($x_j = R^j(x_i) = x_i$). Since the results are similar for both rivals, this section presents only firm i's outputs (at even-numbered periods). Table 2.1 indicates the initial values of the simulation experiments.

Table 2.1 The initial values of the simulation experiments

Simulation experiment	Initial monopoly Output	Steepness parameter α	Figure
I	0.310	3.35	2.7
II	0.300	3.35	2.8
III	0.310	3.34	2.9 and 2.11
IV	0.998	3.35	2.10

The simulation experiments reveal three properties of complex dynamics: (i) chaotic regimes for particular parameter values; (ii) sensitive dependence on initial conditions; and (iii) sudden breaks in qualitative patterns. These features can pose serious problems to econometric estimation.

Property (i): Chaotic trajectories.

The first consequence of nonlinear dynamics can of course be the occurrence of chaotic trajectories. If rival firms are engaged in Cournot competition while at least one competitor is making supply decisions on the basis of a sufficiently steep, hill-shaped reaction function, the time pattern of both rivals' quantities mimics a random walk. The first simulation experiment (I) illustrates this point. Figure 2.7 depicts the series of supplies of firm i (the Lyapunov exponent L defined by $L = \lim_{n\to\infty} \frac{1}{n} \cdot \ln\left|(f^n)'(x)\right|$) is 0.48) ([5]).

([4]) For example, Baumol and Quandt (1985) and Baumol (1986) also offer interesting simulation examples, whereas Sterman (1989) presents the results of an experimental study.

([5]) The Lyapunov exponent indicates chaos for $L > 0$ (Lorenz (1989, pp. 186-191)).

In period $t = 0$ firm i starts to supply close to monopoly output ($x_{i,0} = 0.310$). The subsequent reactions of firm i reveal a chaotic trajectory. The series of firm i's supplies fails to show a systematic (periodic) pattern: history does not repeat.

For example, the pattern of quantities from period $t = 104$ (indicated by an arrow) to $t = 108$ differs qualitatively from the trajectories in both history and future. The time pattern of firm i's outputs mimics a random walk.

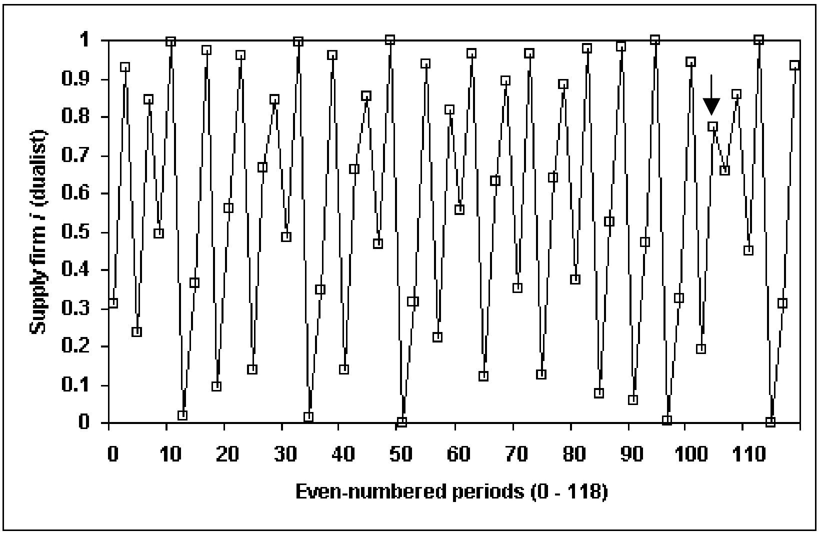


Fig. 2.7 A chaotic trajectory corresponding with $\alpha = 3.35$ and $x_{i,0} = 0.310$.

Property (ii): Sensitive dependencies.

The second property of complex dynamics can be illustrated by assuming a small change in the initial conditions. The second simulation (II) assumes monopoly output to be slightly below the first simulation's level. Figure 2.8 shows that the trajectory of rival i's quantities changes dramatically (L = 0.49). This means that history matters.

Period $t = 0$'s monopoly output is slightly below the first simulation's level ($x_{i,0}$ is decreased from 0.310 to 0.300). The trajectory of firm i's outputs in simulation experiment II is completely different from experiment I's pattern. For example, any resemblance between simulation I and II's output trajectory in period $t = 56$ (arrow) to $t = 66$ and $t = 78$ (arrow) to $t = 82$ is absent. This illustrates the observation that the pattern of quantities is extremely sensitive to minor changes (here a 0.01 reduction) in the level of initial monopoly output. Figure 2.9 shows that the same is true for small variations in the value of parameter α $(L = 0.49)$.

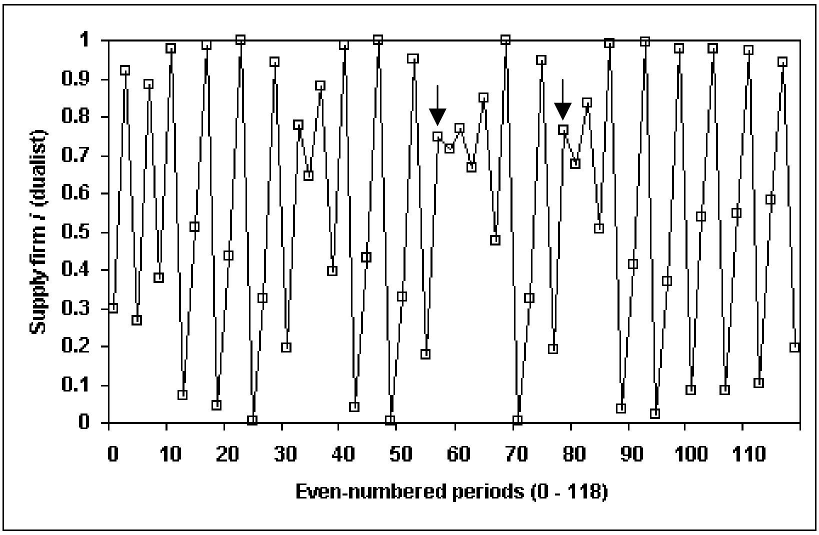


Fig. 2.8 Sensitive dependence on $x_{i,0}$ with $\alpha = 3.35$ and $x_{i,0} = 0.300$.

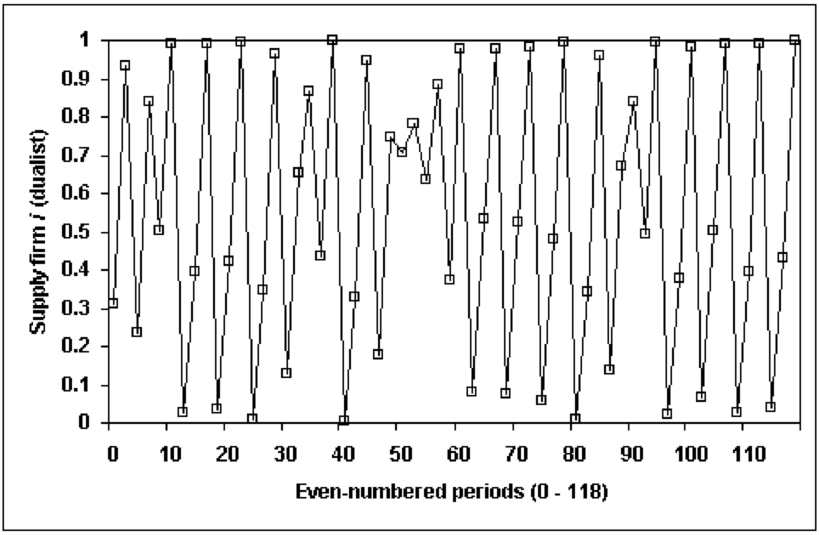


Fig. 2.9 Sensitive dependence on α with $\alpha = 3.34$ and $x_{i,0} = 0.310$.

The third simulation (III) retains monopoly output at experiment I's level, but introduces a minor reduction in α (α is reduced from 3.35 to 3.34). A sidelong glance at Figures 2.7 and 2.9 reveals that a 0.01 variation of parameter α induces a radical transformation of firm i's output pattern.

Property (iii): Qualitative breaks

A peculiar feature of complex dynamics is that a chaotic trajectory is associated with sudden breaks in the qualitative pattern. The fourth simulation experiment (IV) reveals this feature as supply suddenly shows a regularity for two significant time intervals. Figure 2.10 presents the result (L = 0.48). For two (short) periods of time the pattern suggests that history repeats and also suggest a stabilizing supply level. But the occurrence of these (fake) equilibria is just a temporary event.

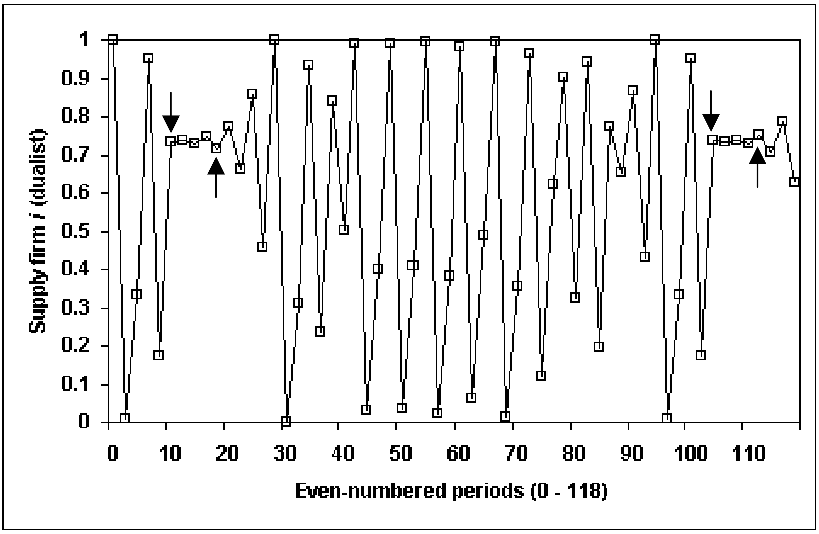


Fig. 2.10 Qualitative breaks with $\alpha = 3.35$ and $x_{i,0} = 0.998$.

Experiment IV retains α's value at simulation I and II's level, but assumes monopoly output to increase from $x_{i,0} = 0.300$ to $x_{i,0} = 0.998$. From period $t = 10$ to $t = 18$ and $t = 104$ to $t = 112$ (see arrows) firm i's supply remains almost constant, which suggests convergence to a single equilibrium point. However, after period $t = 20$ respectively $t = 114$ the pattern breaks down again.

Econometric Dilemma.

Deterministic chaos poses serious problems to econometric estimation (Baumol and Benhabib, 1989). On the one hand, a time trajectory which is extremely sensitive on initial conditions, is difficult to predict. On the other hand, it is problematic to distinguish deterministic chaos from stochastic randomness. This is even more relevant if one recognizes Kelsey's (1988, p. 12) observation that imposing a random error term on a hill-shaped function implies that chaos becomes more common. However, (at least) three arguments can be put forward to modify this claim.

First, chaotic trajectories are associated with (long) periods of regularity. This follows from the feature that sudden regularities characterize the qualitative pattern. Second, new econometric techniques have been (and are) developed to test whether deterministic chaos or stochastic randomness (predominantly) underlies a particular time series (Brock (1986)). Third, an additional argument follows from the specifics of this Chapter's application. The fact that *individual* firms can offer a chaotic series of quantities, does not necessarily mean that the trajectory of *market* supply is dictated by chaos as well.

First, if a dualist faces an imitator, the firms' chaotic output trajectories are replicated at the market level. To be precise, if market supply follows from the summation of two subsequent moves (*i.e.*, an output decision of both rivals), market output is determined by $2 \cdot x_i + \delta(x_i)$. Perfect imitation of a dualist (corresponding with $\delta(x_i) = 0$) implies that one firm's chaotic output trajectory is duplicated at the market level. Second, if a dualist and perfect accommodator are engaged in duopoly Cournot competition, the result is reversed. The firms' chaotic trajectories are not observed at the market level as market output follows from $x_i + (1 - x_i) = 1$. That is, perfect accommodation induces stationary market supply.

The implications for market output are not so obvious if two dualists compete over quantities. Summation of two subsequent output levels in the four simulation experiments mimics market supply (X) in a dynamic dualist against dualist game with doubled period lengths (T). Hence, $X_T = x_{i,2T} + x_{j,2T+1}$: that is, $X_0 = x_{i,0} + x_{j,1}$, $X_1 = x_{i,2} + x_{j,3}$ *etcetera*. The four simulation experiments give the same result: the chaotic output trajectories of both rivals seem to induce chaotic patterns of market supply. Bearing in mind that 120 moves give market supply for 60 periods ($T = 0, ..., 59$), Figure 2.11 illustrates this result for simulation III.

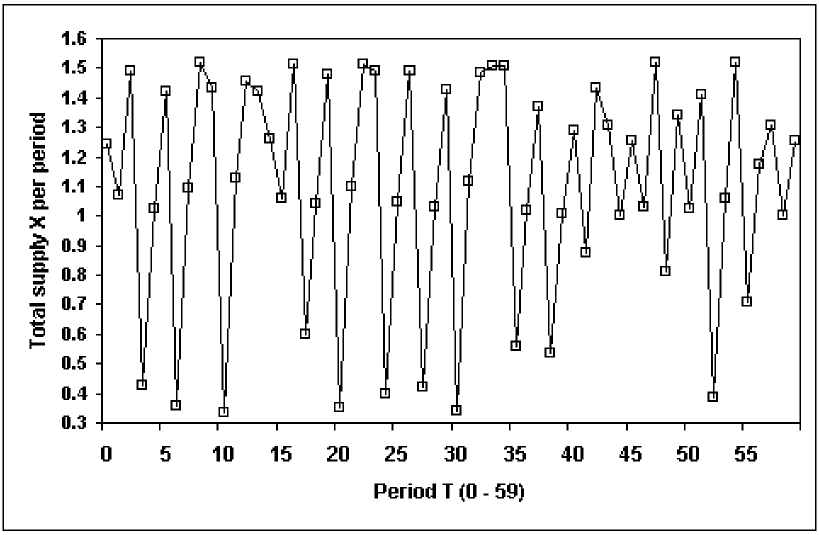


Fig. 2.11 Chaotic market supply with $\alpha = 3.34$ and $x_{i,0} = 0.310$.

5. Appraisal

This Chapter improves upon Rand's (1978) and Dana and Montrucchio's (1986) models of Cournot duopoly competition by permitting monopoly output to be positive. Bulow *et al.*'s (1985) argument indicates the economic plausibility of hill-shaped Cournot reaction curves with positive monopoly output. Specification of such a hill-shaped reaction function with positive monopoly output - the reaction function of the so-called dualist - leads to (individual) chaotic supply levels in three cases: (1) a dualist against an (imperfect) imitator, (2) a dualist against an accommodator and (3) a dualist against a dualist. And the cases (1) and (3) also imply a chaotic market supply and market price. Future research can be directed to several topics. First, this Chapter ignores the dilemma of the specification of cost and demand functions, as hill-shaped reaction curves (Figure 2.2 and Proposition 2.1) are postulated on the basis of *a priori* arguments. This raises the question whether there is a (large) class of economically plausible demand and/or cost functions which predicts such asymmetries. We already mentioned the contributions to this question of Puu (1991) - who uses the assumption that the quantity demanded is reciprocal to price - and Kopel (1996), who uses cost functions incorporating an interfirm externality. However their specifications lead to reaction functions with a zero monopoly output.

Second, other models of competition can be analyzed as to the (non)existence of complex dynamics, where the quest for chaotic regimes in models of competition is not to be restricted to one-shot Cournot games. Chapter 5 provides one answer to this second question. There we will prove that incorporating preference for market share as a non-profit part of the traditional objective function (eq. (2.2)) also leads to hill-shaped analytical and nonanalytical reaction curves.

Furthermore the influence of a difference in capacity of two rivals on the existence of chaotic regimes is an interesting topic for further research. And if the number of competing firms exceeds 2 - leading to complicated higher dimensional dynamics in the case of simultaneously reacting firms – we can ask ourselves under which conditions chaotic individual supplies and market supply still exist.

Appendix 2.1

Proof of Proposition 2.1; dualist against imitator.

Firm j acts as a perfect imitator, which gives (with doubled length of the time intervals) the following difference equation for x_i:

$$x_{i,t+1} = R^i(x_{i,t}) \tag{A1}$$

For the sake of convenience, x_i is replaced by x and R^i by f. Second-degree polynomials with the following properties are considered:

$$\max_{0 \le x \le 1} f(x) = f(x^*) = x_m = 1 \qquad \text{with } 0 < x^* < 1, \qquad \text{and} \tag{A2.i}$$

$$f(1) = 0\,. \tag{A2.ii}$$

The parabola

$$f_\alpha(x) = 1 - \alpha \cdot (x - 1 + 1/\sqrt{\alpha})^2 \qquad \text{with} \qquad \alpha > 1 \tag{A3}$$

satisfies the conditions (A2.i) and (A2.ii) (then $x^* = 1 - 1/\sqrt{\alpha}$). Applying Li and Yorke's (re-expressed) condition - $f^2(x_m)\ (= f(0)) \le x^* < x_m\ (=1)$ - with the further restriction $f(0) > 0$ (positive monopoly output) to the parabola (A3) leads to the inequalities

$$0 < 1 - \alpha \cdot (1/\sqrt{\alpha} - 1)^2 = 2\sqrt{\alpha} - \alpha \le 1 - 1/\sqrt{\alpha}\,. \tag{A4}$$

The inequality on the left hand side can be solved analytically and gives

$$0 < \alpha < 4\,. \tag{A5}$$

Numerically solving the inequality $2\sqrt{\alpha} - \alpha \le 1 - 1/\sqrt{\alpha}$ (which is equivalent to $\alpha \cdot \sqrt{\alpha} - 2 \cdot \alpha + \sqrt{\alpha} - 1 \ge 0$) imposes a second restriction on the parameter α :

$$\alpha \ge 3.0795\ \ldots\,. \tag{A6}$$

We note that the inequality is equivalent with $(\sqrt{\alpha})^3 - 2(\sqrt{\alpha})^2 + \sqrt{\alpha} - 1 \ge 0$ and also can be solved analytically using Cardan's Method leading to $\alpha \ge 3.0795957...$

Combining (A3), (A4), (A5) and (A6) now gives the result that the class of parabolas

$$\begin{aligned} &f_\alpha(x) = 1 - \alpha \cdot (x - 1 + 1/\sqrt{\alpha})^2 \qquad \text{with} \qquad 0 \le x \le 1 \\ &\text{and} \qquad 3.0795\ldots \le \alpha < 4 \end{aligned} \tag{A7}$$

has the properties (A2.i) and (A2.ii), $f(0) > 0$ and satisfies Li and Yorke's condition. Therefore, the difference equation (A1) gives rise to chaotic regimes.

[End of proof]

Appendix 2.2

Proof of Proposition 2.2; dualist against an imperfect imitator.

The reaction functions

$$R^i(x_j) = 1 - \alpha \cdot (x_j - 1 + 1/\sqrt{\alpha})^2 \quad \text{with} \quad 3.0795\ldots \le \alpha < 4, \quad \text{and} \tag{A8.i}$$

$$R^j(x_i) = x_i + \delta(x_i) \quad \text{with} \quad \delta(0) \ge 0 \tag{A8.ii}$$

are assumed. With doubled length of the time intervals the output of firm i at "time t + 1" is

$$\begin{aligned} x_{i,t+1} &= R^i(R^j(x_{i,t})) = 1 - \alpha \cdot [x_{i,t} + \delta(x_{i,t}) - 1 + 1/\sqrt{\alpha}]^2 = \\ &= 1 - \alpha \cdot (x_{i,t} - 1 + 1/\sqrt{\alpha})^2 - \alpha \cdot [\delta(x_{i,t})]^2 + 2 \cdot \alpha \cdot (1 - 1/\sqrt{\alpha} - x_{i,t}) \cdot \delta(x_{i,t}) \ . \end{aligned} \tag{A9}$$

If again x_i is replaced by x, substitution of the function $f_\alpha(x)$, as indicated by (A3) in Appendix 2.1, gives

$$x_{t+1} = f_\alpha(x_t) + \tau(x_t), \quad \text{and} \tag{A10.i}$$

$$\tau(x_t) = -\alpha \cdot [\delta(x_t)]^2 + 2 \cdot \alpha \cdot (1 - 1/\sqrt{\alpha} - x_t) \cdot \delta(x_t) \ . \tag{A10.ii}$$

where τ is a "disturbance term". If the function f_α satisfies Li and Yorke's condition, then this condition can still be satisfied when f_α is disturbed by a small (and continuous) τ. So, if δ (the disturbance of the linear reaction function R^j) is "small enough", the conditions for the existence of chaotic time paths can still be satisfied.

[End of proof]

Appendix 2.3

Proof of Proposition 2.3; dualist against (perfect) accommodator.

Firm j acts as a perfect accommodator, which gives (with doubled length of time intervals) the following difference equation for x_i:

$$x_{i,t+1} = R^i(1 - x_{i,t})\,. \tag{A11}$$

If $R^i(x_j) = 1 - \alpha \cdot (x_j - 1 + 1/\sqrt{\alpha})^2$ with $0 < \alpha < 4$ (reaction curve of a dualist with positive monopoly output), (A11) gives

$$x_{i,t+1} = 1 - \alpha \cdot (x_{i,t} - 1/\sqrt{\alpha})^2 \quad \text{with} \quad 0 < \alpha < 4\,. \tag{A12}$$

If again, for the sake of convenience, x_i is replaced by x, (A12) can be rewritten as

$$x_{t+1} = f(x_t) \quad \text{with} \quad f(x) = 1 - \alpha \cdot (x - 1/\sqrt{\alpha})^2 \quad \text{and} \quad 0 < \alpha < 4 \tag{A13}$$

The function $f(x)$ is a second-degree polynomial with maximum location $x^* = 1/\sqrt{\alpha}$ and maximum value $x_m = 1$. Applying Li and Yorke's condition - $f^2(x_m) \le x^* < x_m \ (=1)$ - to the function (A13) gives the inequalities

$$1 - \alpha \cdot (-\alpha + 2 \cdot \sqrt{\alpha} - 1/\sqrt{\alpha})^2 \le 1/\sqrt{\alpha} < 1\,. \tag{A14}$$

Combining the inequality on the right hand side with $0 < \alpha < 4$ reveals

$$1 < \alpha < 4 \tag{A15}$$

Numerically solving the inequality on the left hand side (which is equivalent to $\alpha^3 \cdot \sqrt{\alpha} - 4 \cdot \alpha^3 + 4 \cdot \alpha^2 \cdot \sqrt{\alpha} + 2 \cdot \alpha^2 - 4 \cdot \alpha \cdot \sqrt{\alpha} + 1 \ge 0$) imposes a second restriction on the parameter α:

$$\alpha \ge 3.6708\ldots\ . \tag{A16}$$

Combining (A13), (A15) and (A16) provides the result that the class of parabolas (composed of a hill-shaped reaction function with positive monopoly output and the reaction function of a perfect accommodator)

$$f_\alpha(x) = 1 - \alpha \cdot (x - 1/\sqrt{\alpha})^2 \quad \text{with} \quad 0 \le x \le 1 \quad \text{and} \quad 3.6708\ldots \le \alpha < 4\,. \tag{A17}$$

satisfies Li and Yorke's condition. Therefore, the difference equation (A11) gives rise to chaotic time paths.

[End of proof]

CHAPTER 3

BIG IS BEAUTIFUL
Cournot competition, habit formation and exit

This Chapter was previously published as a research memorandum (RM 93-016, Maastricht University of Limburg). Some minor changes have been made.

1. Introduction

Starting from the seminal contributions of Beaver (1966) and Altman (1968) many studies into the (*ex post*) prediction of bankruptcies have appeared in the literature on Accounting and Finance (AF). The key issue in this literature is the identification of financial ratios that can predict corporate failure a few years before the actual date of bankruptcy. The results indicate that bankrupt firms are associated with financial ratios that started to deteriorate several years (in general, 1 to 5) before the year of failure (Foster (1986: Chapter 15)). For example, Zmijewski (1983) reports that in a sample of 72 bankrupt and 3,573 nonbankrupt firms over the 1972-1978 period the former showed a net income/net worth ratio of -.591 and the latter of 0.091 one year prior to bankruptcy. Another example is Hambrick and D'Aveni (1989). Hambrick and D'Aveni (1988: 10) report the results of an investigation into 57 large corporate failures during the period 1972-1982. They point out that "the bankrupts showed signs of relative weakness as early as ten years before failing. ... That these bankruptcies were culminations of often ten-year declines is compelling testimony to organizational inertia" (1988: 13 and 20). For example, in the five years prior to the date of bankruptcy (t) the series of the mean net income/assets ratio of bankrupt firms is –4.56 (t–5), –21.79 (t–4), –21.30 (t–3), –85.11 (t–2) and –107.89 (t–1).

Moreover, a few exceptional studies take account of nonfinancial predictors. Noteworthy are Altman, Haldeman and Narayanan (1977), Ohlson (1980), Zmijewski (1983), Keasey and Watson (1987) and Storey, Keasy, Watson and Wynarczyk (1987), which include a measure of size as predictor of corporate bankruptcy. Their results indicate that "size appears as an important predictor" (Ohlson (1980: 122)) as bankrupt firms are, on average, significantly smaller than nonbankrupt corporations. This result is supported by empirical research on entry into and exit from industries. McDonald (1986), Dunne, Roberts and Samuelson (1988 and 1989), Baden-Fuller (1989), Lieberman (1990) and Baldwin and Gorecki (1991) reported that the exit rate is, by and large, significantly higher among small and young firms. For example, Dunne, Roberts and Samuelson (1989: 689) report that "in summary, failure rates are lower for older plants ... and for larger plants" on the basis of patterns of failure statistics for over 200,000 plants that entered manufacturing in the U.S. in the period 1967-1977.

The bottom line is that the key finding of the bankruptcy prediction models is that firms start to accumulate losses many years before the actual date of exit. However, apart from critique of the methods of empirical analysis and sample selection (Zavgren (1983)), Foster (1986: 559) notes that "economic theory has played a small role in the development of univariate or multivariate distress prediction models." That is, the model testing is not theory-guided, but based on *ad hoc* arguments. As Rees (1990: 406) observes: "Most of the empirical studies ... are wide-ranging searches for any statistically reliable relationships between failure and accounting variables without the benefit of theoretical backing. As such they have been occasionally characterised as 'brute empiricism'". Therefore, "if the literature on distress/failure prediction is to progress further, then more explicit and formal modelling of the economic interests and decision processes of the firm's major stakeholders will probably have to be undertaken" (Keasey and Watson (1991: 100)). The current paper takes up this challenge.

So far, the scarce theoretical AF-contributions have been of three types. First, statistical ruin theory is applied to the issue of the determination of the risk of bankruptcy. For example, Vinso (1979) introduces initial reserves, fixed costs and expected profit flows to calculate a firm's risk of failure. Second, a few theoretical AF-models focus on explaining bankruptcy by modeling creditors' confidence in terms of catastrophe theory. Notably Ho and Saunders (1980) and Scapens, Ryan and Fletcher (1981) argue that poor financial performance may induce creditors to suddenly withdraw credit. So, this type of model is concerned with explaining the breakdown of chronically failing firms by interference of outside stakeholders. Third, Wadhwani (1986) and Simmons (1989) derive reduced-form equations from a profit-maximizing framework in which firms engaged in price-taking (perfect) competition take account of determinants such as (future) product prices, debt ratios, interest rates, inflation levels, money wages, bankruptcy costs, share prices and bankruptcy probabilities.

The theoretical modeling of financial distress is, however, still in its infancy. Particularly the role of strategic competition in explaining financial distress is, as yet, largely ignored. The isue of strategic competition is central to Industrial Organization (IO). For a long time IO has relatively ignored the issue of organizational failure. In the 1980s a countable number of analytical papers started to model exit decisions of firms in a competitive environment. The notable contributions are Jovanovic (1982), Lippman and Rumelt (1982), Ghemawat and Nalebuff (1985 and 1990), Fudenberg and Tirole (1986), Frank (1988), Reynolds (1988), Whinston (1988), Baden-Fuller (1989), Dixit (1989,1992), Jovanovic and Lach (1989), Fishman (1990), Londregan(1990) and Dierickx, Matutes and Neven (1991). All models start from the assumption that firms seek to maximize the discounted profit flow. The key issue in this literature concerns the question which (group of) firm(s) decides to exit first, where firms' heterogeneity is, by and large, reflected in cost or size features.

Without exception, the models assume quantity (Cournot) competition, either among n atomistic firms or two duopolists (with endogenous or exogenous profit levels). Competition is strategic, as individual profit levels crucially depend upon the (exit and output) decisions of rivals. Cost differences may originate from many sources, for example, efficiency of closing (Baden-Fuller (1989)), learning-by-doing (Jovanovic and Lach (1989)), scale economies (Ghemawat and Nalebuff (1985)) and talent (Frank (1988)). The results of the models are diverse, and appear to depend crucially upon the underlying assumptions. Two results are worth noting. First, the well-documented natural selection argument is supported if inefficient firms decide to exit first (Jovanovic (1982), Lippman and Rumelt (1982), Fudenberg and Tirole (1986), Frank (1988), and Jovanovic and Lach (1989)). Second, the contrary result holds if small and inefficient firms survive at the detriment of large and efficient rivals (Ghemawat and Nalebuff (1985,1990), Baden-Fuller (1989), Fishman (1990), Londregan (1990), and Dierickx, Natutes and Neven (1991)). This counterintuitive result is supported by Baden-Fuller's (1989) investigation of the steel castings industry in the U.K., being the only empirical IO-study on exit that takes account of profit performance. His finding is that, although a significant number of closures suffered from negative cash flow/sales ratios prior to exit, “the least profitable plants ... did not close” (1989: 956).

Except for Dixit (1989, 1992), the consequence of organizational failure in IO-models is just-in-time exit. This result is driven by the assumption of long-run backward induction. That is, firms are perfectly forward-looking: calculating the (expected) future stream of profit, taking account of the end-game equilibrium that results from strategic competition, they decide to exit at the moment profitability falls below zero. Although Dixit’s (1989) model describes a case where firms accept temporary losses, this result also follows from long-run calculations. The point is that Dixit (1989: 629) argues that firms take account of favourable demand developments in the future: “The firm knows that by remaining active it can avoid incurring [costs] for reentry should future developments turn favourable; therefore, it is willing to incur some current loss to preserve this option”. The focus of the IO-models on just-in-time exit (or in Dixit’s case: temporary losses) makes the results unable to explain the empirical findings in the AF-literature that more often than not firms accumulate losses before actually exiting the market. This state of the art is the result of IO-models’ reliance on assumptions of extreme rationality. Whinston (1988: 584, note 27) argues that “in fact, the results are, if anything, overly favourable about the possibility of prediction, since they rely heavily on long chains of backward inductive reasoning that are likely to be quite sensitive to even small amounts of irrationality”. After observing the same flaw Fishman (1990: 71) concludes that “this consideration should lead economists to proceed with caution before taking such results at face value”. The assumptions of extreme rationality are, moreover, reflected in the fact that all IO-models take firms to seek maximization of the future profit flow.

So, the AF-studies on financial distress clearly reveal that firms’ bankruptcies are, by and large, preceded by many years in which losses are accumulated. However, the formal modeling of the economics behind this result is still in its infancy, and the recent exit games in the IO-tradition fail to give in to the request for theoretical explanations by their exclusive focus on extreme rationality and just-in-time exit. This Chapter is a preliminary attempt to fill the gap by focusing on two strategies in particular: just-in-time (zero-profit) exit versus chronic failure (ongoing operation whilst accumulating losses). Van Witteloostuijn (1998) distinguishes four possible outcomes of the process of a firm’s downturn (measured in terms of profitability): immediate exit, turnaround success, flight from losses and chronic failure. He presents a framework that summarizes arguments from varying economic (IO) and organizational (OS) perspectives that have, for the most part, developed independently. His framework provides an overview of the literature on organizational decline, related to (internal and external) causes, (financial and nonfinancial) conditions, (shape and size) courses and (profits or losses) consequences. In this Chapter answers to two questions are investigated: under what conditions of competition and demand does either strategy (exit or chronic failure) occur?; and what features, in terms of efficiency and scale, characterize the firms that either exit or stay? Both questions are scrutinized by extending Vickers’ (1985) model of managerial economics by introducing cost asymmetries and habit formation. Like Vickers, Sklivias (1987) also considers a two-stage sequential game with cost symmetries, whereas Fershtman and Judd (1987) and Basu (1995) introduce cost asymmetries between competing firms. Concerning all these models, owners write an incentive contract for their managers in stage one of the game and then, in the second stage, managers play the Cournot or Bertand game. By backward induction the owners determine the incentive contract such that profitability is maximized, given the rival's incentive contract. This implies that the values of the weights (α or θ in these models) of the nonprofit parts of managers' objective functions (size or revenues) are determined and fixed (Nash equilibrium) by the owners in this two-stage game. So these models assume a high degree of rationality. However, we will examine all weight combinations (of the nonprofit parts of the objective functions) between rivals and will not restrict ourselves to the fixed weigths resulting from a two-stage sequential game. We consider the nonprofit-maximizing behaviour of managers (their "love for sales volume") as a habit, developed in time and such a habit can be part of the "blueprint" of the firm (Hannan and Freeman (1984)). The concept of habit formation has been applied to the explanation of preference changes in models of decision making on consumption (Pollak (1970) and Alessie and Kapteyn (1991)) and labour supply (Phlips (1978) and Vendrik (1992)). This Chapter introduces the notion of habit formation in the theory of the firm. This is explained in Section 3.2. Section 3.3 goes on to present the results of the model for the case where firms are facing symmetric cost conditions: that is, firms control equally efficient production technologies. Section 3.4 deals with cost asymmetries: both efficient and inefficient firms (may) operate in the market. Section 3.5 concludes the Chapter by offering an appraisal and conclusion.

2. A model

In accordance with the exit games in the IO-literature the model is Cournot: competition is in quantities. The prime deviation from standard Cournot, and so IO's exit games, is the introduction of sales (production volume) in the objective function of Cournot oligopolists. To be precise, the point of departure is a combination of Vickers' (1985) analysis of sales maximization in Cournot-Nash competition, Becker and Murphy's (1988) explanation of rational addiction through habit formation, and Akerlof's (1991) arguments in favour of myopic behaviour. These elements - sales maximization, habit formation and myopic behaviour - imply a departure from extreme rationality. Below the key assumptions are discussed in turn.

Vickers (1985) presents a modern version of the old theory of managerial economics. The key assumption in managerial economics is that firms (or, to be precise, managers) fail to maximize profits, but are driven by other motives, well-known examples being sales (Baumol (1953)), growth (Marris (1964)) and staff expenditures (Williamson (1964)). The common denominator of such nonprofit motives is that managers are assumed to favour the promotion of a measure of size: managers prefer to be head of large budgets, organizations or staffs rather than small ones. This hypothesis is confirmed by studies in public choice (Mueller (1989)), which, for example, indicate that bureaucrats maximize budgets (Niskanen (1971)). Similarly, theories of economic organization (Milgrom and Roberts (1992)), particularly principal-agent models, emphasize the nonprofit motives of nonowning managers (Vickers (1985), Fershtman and Judd (1987), Sklivias (1987) and Basu (1995)). Note that size preference implies an asymmetry: managers like to grow, but dislike to retrench. Vickers (1985: 141) starts from the assumption that firms maximize in a period t

$$u_t = \Pi_t + \alpha x_t, \tag{3.1}$$

where α is a weight parameter, and u denotes utility, Π profit and x output or sales. So, equation (3.1) implies that "those who take the decisions in large firms are advanced by high sales rather than purely by profits" (Vickers (1985: 141)). The current model diverges form Vickers (1985) in three respects in order to fit more closely with the issues at hand: first, habit formation and myopic behaviour are included; second, cost asymmetries are introduced; and, third, firms are not subject to a zero-profit constraint. The bottom line is that these extensions permit an explicit focus on exit competition.

Becker and Murphy (1988) build upon the notion of rational habit formation (Stigler and Becker (1977) and Spinnewyn (1981)) in their explanation of addiction. As noted in the introduction, the concept of habit formation has been applied to the explanation of preference changes in models of decision making on consumption (Pollak (1970) and Alessie and Kapteyn (1991)) and labour supply (Phlips (1978) and Vendrik (1992)). The key argument is that people start to develop stronger preferences for consumption or working patterns over time if they get used to specific levels of consumption or numbers of working hours. A linear approximation of a habit formation function pertaining to Vickers' (1985) output or sales volume is

$$h_t = x_t + (1-\gamma)h_{t-1} \tag{3.2}$$

where γ is a depreciation factor and h denotes a habit parameter. If x is interpreted as consumption, equation (3.2) implies that "increases in past consumption raise current consumption" (Becker and Murphy (1988: 694)).

However, rational habit formation assumes that people in their current decisions take account of the future implications of developing a habit. As Akerlof (1991) notes, in practice "the standard assumption of rational, forward-looking, utility maximizing is violated" (1991: 1). This points to myopic habit formation. Recently, Akerlof (1991) lists arguments in favour of shortsightedness. For example, he notes that "the nonindependence of errors in decision making in the series of decisions can be explained with the concept from cognitive psychology of undue salience or vividness. For example, present benefits and costs may have undue salience relative to future costs and benefits" (1991: 1). One of his examples is organizational failure caused by escalating commitment (1991: 7-8), arguing that procrastination does not only occur in project initiation, but also in project termination. Note, moreover, that the standard assumption of Cournot-Nash conjectures implies a kind of myopic behaviour (to be precise, shortsighted expectations) as well. The three elements – sales (production) maximization, habit formation and myopic behaviour - give the essential assumption on firms' decision making in the current model. To be precise, firms are assumed to maximize

$$u_t = \Pi_t + \alpha h_t \,, \tag{3.3}$$

which implies that firms maximize a utility function that is composed of profit and output, while output is subject to habit formation (h_t follows from equation (3.2)). That is, after a while firms increasingly prefer to be large; to paraphrase Becker and Murphy (1988: 694), increases in past size raise preference for current size.

Note that the interpretation of equation (3.3) from the perspective of studies on organizations (OS) is straightforward (Cameron, Sutton and Whetten (1988)). Evidence from OS-contributions clearly indicates that managers prefer volume. Particularly, studies of firms' growth point out that "growth is frequently sought directly because it facilitates the internal management of an organization" (Whetten (1987: 30)). As far as habit formation is concerned, references to routinized behaviour abound in the OS-literature, notably the literature on forms of (managerial) inertia (Hannan and Freeman (1984) and Tushman, Newman and Romanelli (1986)). Note moreover that equation (3.3) implies that managerial inertia are asymmetric. Apart from the literature on corporate growth, Porter's (1976) arguments on managerial exit barriers, for example, support this view: "Managerial exit barriers are characteristics of the company's decision-making process which deter its management from making decisions to exit from a particular business even though they are justified on economic grounds" (1976: 23). Last but not least, the important role of myopic behaviour is stressed time and again in OS-contributions, an excellent example being Staw, Sandelands and Dutton (1981).

The model introduces strategic competition by assuming Cournot-Nash duopoly: two incumbent rivals, firm 1 and 2, compete over quantities by deciding on the output volume they bring to the market in period t. The model is completed by defining unit production cost c to be scale- and time-independent, and taking demand to be represented by the linear downward-sloping function

$$p_t = m_t - x_{1,t} - x_{2,t} \,, \tag{3.4}$$

where p denotes price, m is a size parameter, and x_1 and x_2 are the output volumes of firm 1 and firm 2, respectively. Noting that profit follows from $\Pi = (p-c)x$, substitution of the inverse demand function (3.4) and habit formation equation (3.2) into maximand (3.3) gives a firm's decision rule. That is, in a period $t + 1$ firm i decides to produce the output volume $x_{i,t+1}$ that maximizes

$$u_{t+1} = (m_{t+1} - x_{i,t+1} - x_{j,t} - c_i)x_{i,t+1} + \alpha_i[x_{i,t+1} + (1-\gamma_i)h_{i,t}], \tag{3.5}$$

where $i, j = 1, 2$ and $i \neq j$. Recall that the well-known Cournot-Nash assumption implies that firm i myopically expects firm j to sustain its period t's output volume in period $t + 1$: $x^e_{j,t+1} = x_{j,t}$, where e is an expectational operator. To focus on strategic competition only, assume for the time being that demand is stationary (m is time-independent) and cost is symmetric ($c_1 = c_2$).

Standard Cournot-Nash competition assumes pure profit maximizers, which follows from equation (3.1) or (3.5) by taking $\alpha_i = 0$. This gives the familiar Cournot-Nash duopoly equilibrium output (x^*) where $x^* = (m-c)/3$. Strict sales maximization without habit formation follows from $\alpha_i > 0$ and $\gamma_i = 1$, which reduces decision rule (3.5) to utility function (3.1). Then, Vickers (1985) shows that symmetric Nash equilibrium of the α-setting game (in a two-stage sequential game) gives output $x^* = 2(m-c)/5$. So, "compared with the case in which firms are managed by profit-maximizers, output per firm is higher, price is lower and profits are lower" (Vickers (1985: 142)). However, cases with negative profitability remain unexplored. On the one hand, if $m > c$, in both cases *neither* firm decides to exit, since *both* firms capture a positive profit in equilibrium. On the other hand, with $m < c$ *both* firms decide to undertake just-in-time exit. The outcome may be different if habit formation is introduced ($\alpha_i > 0$ and $0 < \gamma_i < 1$): that is, now α_i does not follow from highly rational decision making, but is the result of (fixed) managerial inertia. This allows us to consider a large set of (α_i, α_j) - combinations of the two rivals (satisfying nonnegative price restrictions) and reflect on the corresponding implications.

The basic model reflected in the equations (3.1)-(3.5) is developed below for two cases: Section 3.3 deals with cost symmetry ($c_1 = c_2$: Proposition 3.1), and Section 3.4 with cost asymmetry ($c_1 \neq c_2$: Proposition 3.5). In addition, spread over both sections attention is paid to five specific issues. Section 3.3 contains discussions of (i) disequilibrium and equilibrium profit (Proposition 3.2), (ii) speed of adjustment toward equilibrium (Proposition 3.2), and (iii) 'optimal' (that is: profit-maximizing) levels of habit formation (Proposition 3.4); Section 3.4 analyzes (iv) the n-firm case (Proposition 3.7); both sections include a discussion of (v) comparative statics of exit decisions (Propositions 3.3 and 3.6). When convenient, the results for both cases are compared in Section 3.4.

3. Cost symmetry

The first case assumes cost symmetry: $c_i = c_j$. By introducing $c_i = c_j = c$ in the basic model (3.1)-(3.5) stationary-state-equilibrium strategies can be calculated. The result is summarized in Proposition 3.1.

Proposition 3.1. A Cournot-Nash duopoly stationary-state-equilibrium with symmetric cost conditions ($c_1 = c_2$) and asymmetric habit formation ($\alpha_i \neq \alpha_j$) can be calculated, and is asymptotically stable for $0 < \gamma_i, \gamma_j < 1$.

Proof. With $c_i = c_j = c$ maximization of utility function (3.5) gives a system of four difference equations:

(i) $x_{1,t+1} = -x_{2,t}/2 + (m - c + \alpha_1)/2$,
(ii) $h_{1,t+1} = (1-\gamma_1)h_{1,t} - x_{2,t}/2 + (m - c + \alpha_1)/2$,
(iii) $x_{2,t+1} = -x_{1,t}/2 + (m - c + \alpha_2)/2$, and
(iv) $h_{2,t+1} = (1-\gamma_2)h_{2,t} - x_{1,t}/2 + (m - c + \alpha_2)/2$.

In matrix form this is $\underline{x}_{t+1} = A\underline{x}_t + \underline{b}$, where $\underline{x}_t = [x_{1,t}, h_{1,t}, x_{2,t}, h_{2,t}]^{\mathrm{T}}$, $\underline{b} = [(m - c + \alpha_1)/2, (m - c + \alpha_1)/2, (m - c + \alpha_2)/2, (m - c + \alpha_2)/2]^{\mathrm{T}}$ and $A = \begin{bmatrix} 0 & 0 & -1/2 & 0 \\ 0 & 1-\gamma_1 & -1/2 & 0 \\ -1/2 & 0 & 0 & 0 \\ -1/2 & 0 & 0 & 1-\gamma_2 \end{bmatrix}$. The eigenvalues of matrix A are found by solving the equation $\det(A - \lambda I) = (1-\gamma_1-\lambda)(1-\gamma_2-\lambda)(\lambda+1/2)(\lambda-1/2) = 0$ with the sufficient and necessary conditions $\vee_i [|\lambda_i| < 1]$ for asymptotic stability. Hence, asymptotic stability occurs for $0 < \gamma_i < 1$, where $i = 1, 2$. From the equilibrium condition $\underline{x}^* = A\underline{x}^* + \underline{b}$ and so $(I - A)\underline{x}^* = \underline{b}$ follow the stationary-state-equilibrium outcomes (3.6), (3.7) and (3.8) below.

[End of proof]

If habit asymmetry is introduced ($\alpha_i \neq \alpha_j$), Cournot-Nash duopoly competition with habit formation gives firm i's stationary-state-equilibrium output

$$x_i^* = (m - \alpha_j + 2\alpha_i - c)/3 \tag{3.6}$$

and stationary-state-equilibrium level of habit formation (h^*)

$$h_i^* = (m - \alpha_j + 2\alpha_i - c)/(3\gamma_i), \tag{3.7}$$

where $i, j = 1, 2$ and $i \neq j$. For brevity's sake henceforth the 'stationary-state-equilibrium' is, except in propositions and proofs, referred to as 'equilibrium'.

Equilibrium sales (3.6) support the intuition that firm i's production exceeds firm j's output if $\alpha_i > \alpha_j$. Firm i's equilibrium profit (Π) follows from

$$\Pi^{\,i} = (m - \alpha_i - \alpha_j - c)(m - \alpha_j + 2\alpha_i - c)/9 \tag{3.8}$$

where $i,\ j\ =\ 1,\ 2$ and $i \neq j$. So, firm i's profit falls below zero if $\alpha_i + \alpha_j > m - c$. Equation (3.6) confirms that symmetric pure profit maximization ($\alpha_i = \alpha_j = 0$) gives the symmetric Cournot-Nash equilibrium output $x^* = (m-c)/3$. Moreover the evident observation that firm i's equilibrium output increases in its own habit formation (α_i) and decreases in its rival's preference for size (α_j), is supported, where the first force is twice as influential as the second ($+2\alpha_i$ versus $-\alpha_j$). Note that γ_i and γ_j have no impact on equilibrium volumes (but they do influence the habits h_i^* in equilibrium). Furthermore, note that the concept of habit formation doesn't influence equilibrium's stability. Proposition 3.2 deals with the speed at which the (supply) equilibrium is reached.

Proposition 3.2. Output volumes converge rapidly toward stationary-state-equilibrium values. The case where both firms decide to exit is reached immediately, for example. Therefore, the disequilibrium profit captured during the time of adjustment is negligible.

Proof.

We use the difference equations, concerning the firms' supplies (Proposition 3.1):

$$x_{1,t+1} = (m - c + \alpha_1)/2 - x_{2,t}/2\,,$$
$$x_{2,t+1} = (m - c + \alpha_2)/2 - x_{1,t}/2\,,$$

Using the initial conditions ($x_{1,0}, x_{2,0}$) we obtain the following solutions

$$x_{1,t} = (m - c - \alpha_2 + 2\alpha_1)/3 + (x_{1,0} - x_{2,0} + \alpha_2 - \alpha_1)(1/2)^{t+1} + \\ (-x_{1,0} - x_{2,0} + 2m/3 + \alpha_1/3 + \alpha_2/3 - 2c/3)(-1/2)^{t+1}$$

and

$$x_{2,t} = (m - c - \alpha_1 + 2\alpha_2)/3 + (x_{2,0} - x_{1,0} + \alpha_1 - \alpha_2)(1/2)^{t+1} + \\ (-x_{1,0} - x_{2,0} + 2m/3 + \alpha_1/3 + \alpha_2/3 - 2c/3)(-1/2)^{t+1}\ .$$

So both $x_{1,t}$ and $x_{2,t}$ converge to their stationary-state-equilibrium values rapidly. Take, for example, the case where both firms decide to exit. Then: $\alpha_1 \le c - m$ and $\alpha_2 \le c - m$, which implies that

$x_{1,1} = (m - c - x_{2,0} + \alpha_1)/2 \leq -x_{2,0}/2 \leq 0$ and
$x_{2,1} = (m - c - x_{1,0} + \alpha_2)/2 \leq -x_{1,0}/2 \leq 0$.

Hence, both firms simultaneously leave the market in period $t = 1$. Over $T + 1$ periods firm i's average profit per period ($i = 1, 2$), $\Pi_t^{\,i}$, follows from

$$\Pi_t^{\,i} = [1/(T+1)]\sum_{t=0}^{T}[(m - c - x_{1,t} - x_{2,t})x_{i,t}]\,.$$

Summing the sequences after substitution of the above expressions for $x_{1,t}$ and $x_{2,t}$ gives $\Pi_t^{\,i} = a_i(T) + \Pi^{\,i}$, where $a_i(T)$ indicates the disequilibrium profit during the adjustment phase and $\Pi^{\,i}$ is the stationary-state-equilibrium profit expressed in equation (3.8). The adjustment profit can be expressed as

$$a_i(T) = \frac{1}{T+1}\sum_{t=0}^{T}\left[E_1(\tfrac{1}{2})^{t+1} + E_2(-\tfrac{1}{2})^{t+1} + E_3(-\tfrac{1}{4})^{t+1} + E_4(\tfrac{1}{4})^{t+1}\right],$$

where the constants E_1, E_2, E_3 and E_4 depend on $x_{1,0}$, $x_{2,0}$, m, c, α_1 and α_2. Using summation of geometric sequences, one can easily observe that the adjustment profit goes to zero with increasing T. Since stationary-state-equilibrium values are reached rapidly, total profit can be approximated by ignoring the payoff during the period of adjustment, which gives equation (3.8).

[End of proof]

So, the period of adjustment after setting an arbitrary pair of (positive) starting values of output volumes, $x_{1,0}$ and $x_{2,0}$, is short. For the case where both firms exit in equilibrium this state of affairs is reached in the subsequent period ($t = 1$). Therefore, the value of total profit can be approximated by ignoring the adjustment payoff, which gives equation (3.8).

Starting from Propositions 3.1 and 3.2 a set of equilibria can be characterized. The equilibrium features (and, for that matter, adjustment dynamics) crucially depend on the precise values of α_1, α_2, m and c. The fact that prices cannot be negative, imposes three restrictions on the set of feasible equilibria. Demand function (3.4) indicates that nonnegative prices result if

$$x_1^* + x_2^* \leq m \Leftrightarrow \alpha_1 + \alpha_2 \leq m + 2c\,, \tag{3.9}$$

$$x_1^* \leq m \Leftrightarrow 2\alpha_1 - \alpha_2 \leq 2m + c\,, \text{ and} \tag{3.10}$$

$$x_2^* \leq m \Leftrightarrow 2\alpha_2 - \alpha_1 \leq 2m + c\,. \tag{3.11}$$

Total exit implies that $x_i^* \leq 0$. So, firm i leaves the market if

$$\alpha_j - 2\alpha_i \geq m - c\,, \tag{3.12}$$

where $i, j = 1, 2$ and $i \neq j$.

With the help of the equations (3.6) to (3.12) environmental regimes and competitive equilibria can be identified. For the sake of argument, four distinct cost-demand regimes (each being composed of many market periods t) are discussed. First, the benchmark regime (Figure 3.1) is the case where $c = 0 < m$. Second, in the next regime (Figure 3.2) the market can be served profitably by two standard Cournot-Nash duopolists: $0 < c < m$. Third, a border regime (Figure 3.3) occurs for $c = m$. Fourth, in the subsequent regime (Figure 3.4) demand has fallen below the level where any output volume can be sold profitably, either by duopolists or by a monopolist: $c > m$. The shift from Figure 3.1 to Figure 3.4 can be interpreted as being the result of a dramatic decline in demand in the sense that environmental conditions change from favourable to unfavourable. Compare, for example, Figures 3.2 and 3.4 for the cases where $c = 12$ and m drops from 15 (Figure 3.2) to 10 (Figure 3.4).

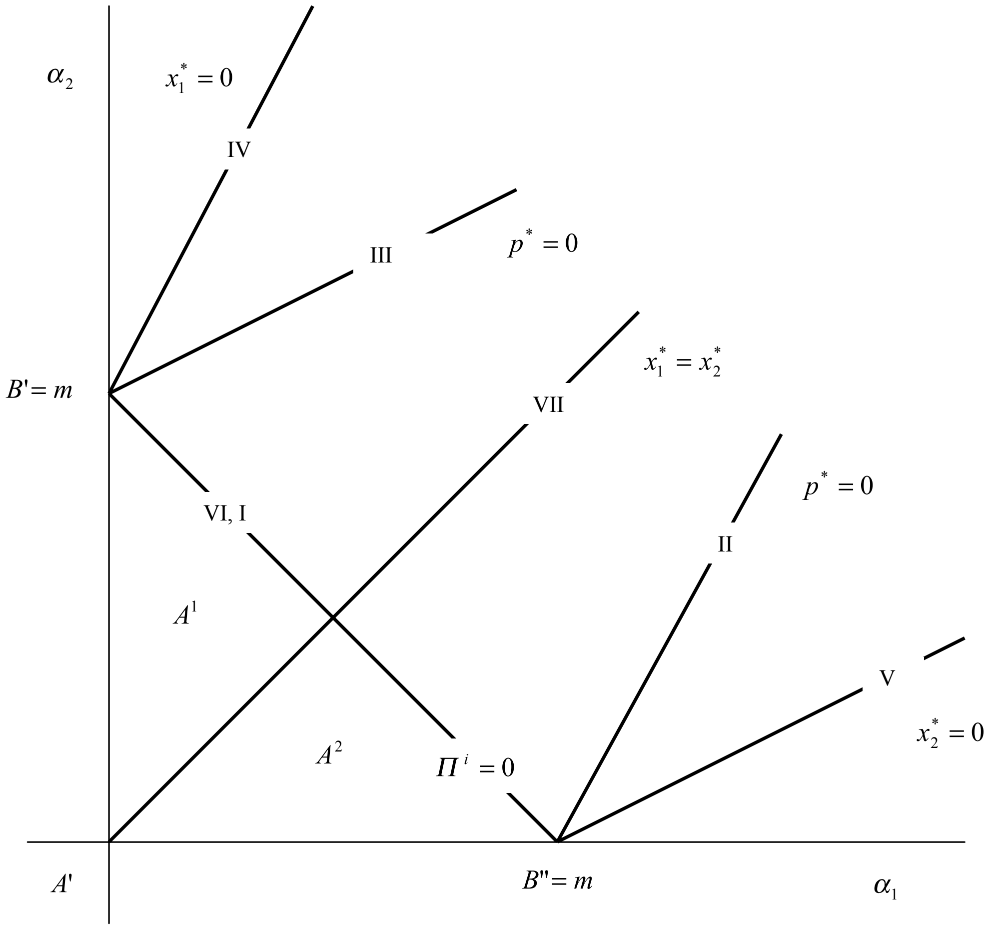


Fig. 3.1 Equilibrium outcomes for $c_1 = c_2 = 0 < m$

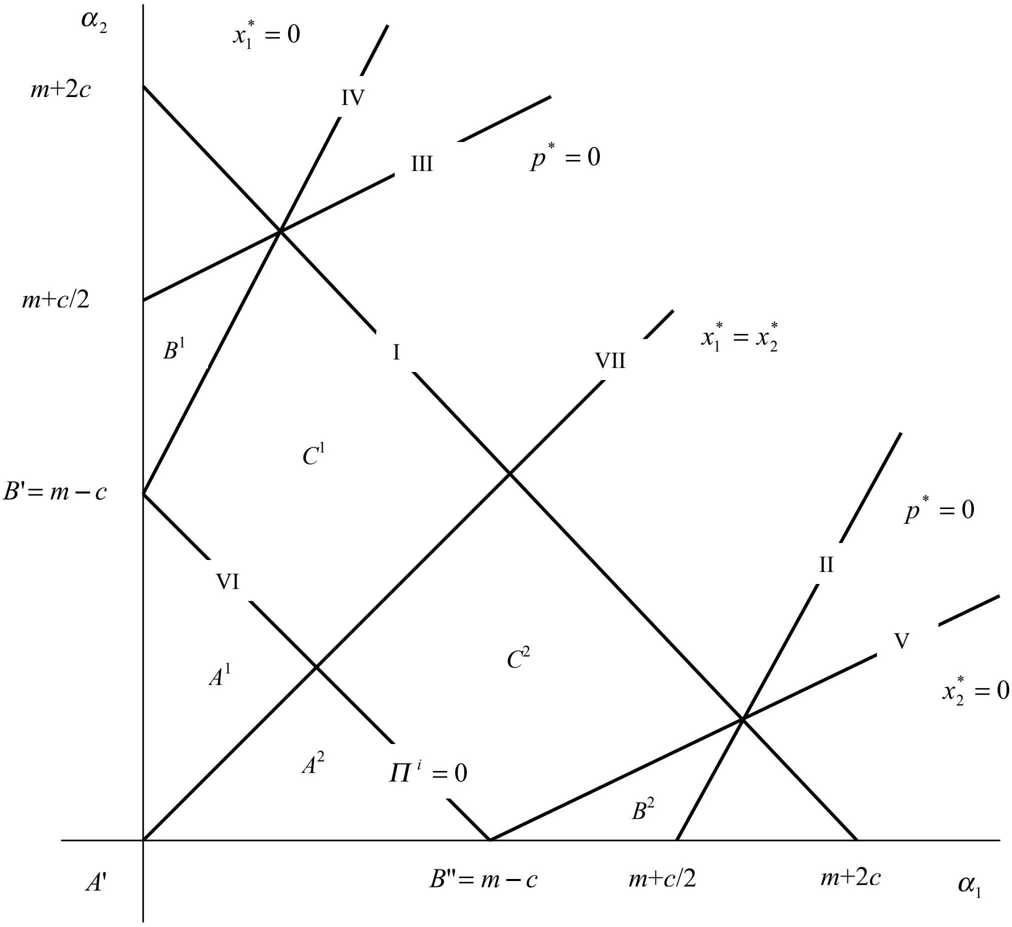


Fig. 3.2 Equilibrium outcomes for $0 < c_1 = c_2 = c < m$

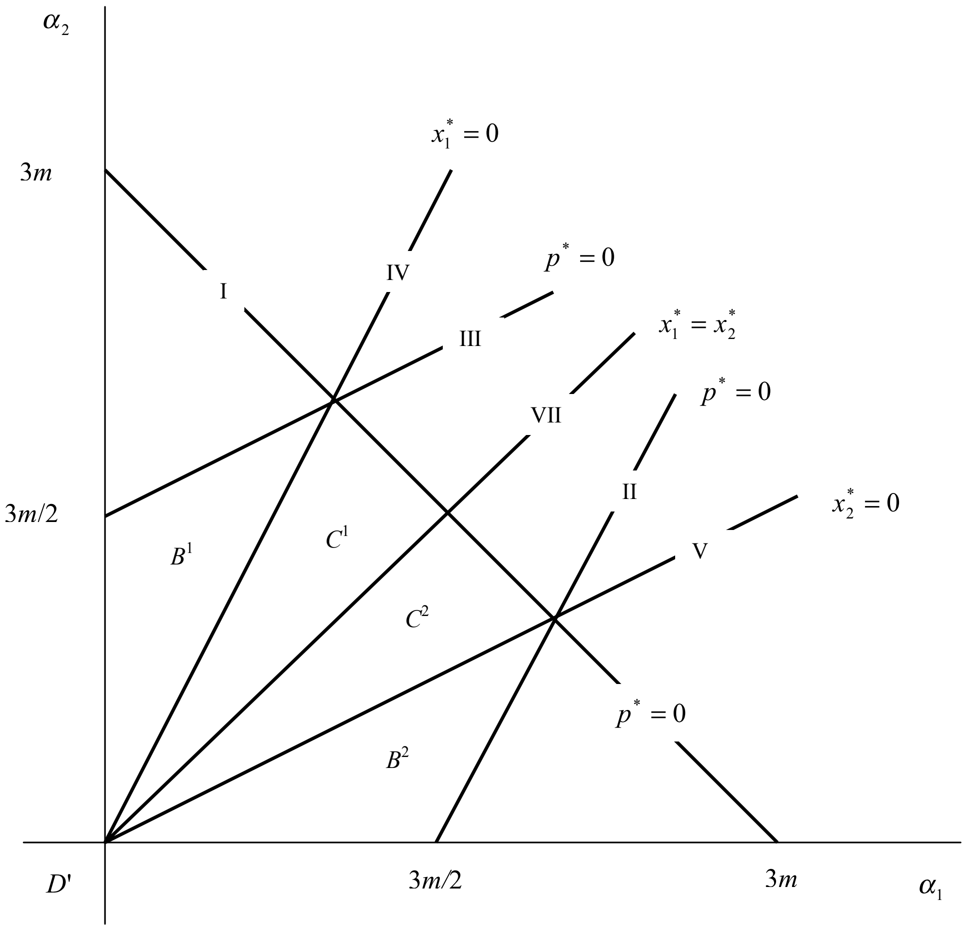


Fig. 3.3 Equilibrium outcomes for $c_1 = c_2 = c = m$

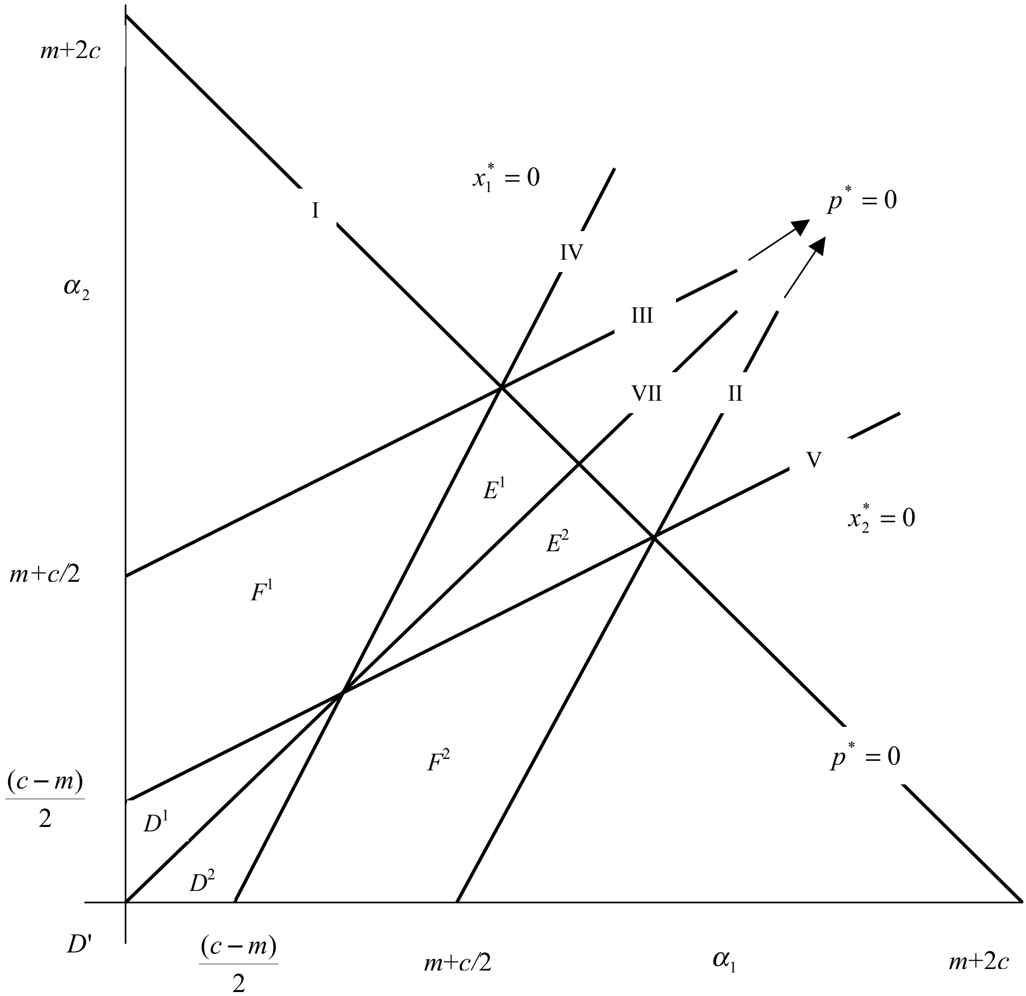


Fig. 3.4 Equilibrium outcomes for $c_1 = c_2 = c > m$

Beyond lines I, II and III price falls below zero (equations (3.9), (3.10) and (3.11), respectively). Lines IV and V depict firm 1's and firm 2's exit condition (equation (3.12): $\alpha_2 - 2\alpha_1 = m - c$ and $\alpha_1 - 2\alpha_2 = m - c$, respectively). Beyond lines VI profit starts dropping below zero (equation (3.8): $\alpha_1 + \alpha_2 = m - c$). At the right-hand side of lines VII firm 1's output exceeds firm 2's sales, whereas at the left-hand side of lines VII the opposite holds ($\alpha_1 > \alpha_2$ and $\alpha_2 > \alpha_1$, respectively). Figures 3.1-3.4 depict six specific equilibrium areas (*A*-*F*), apart from the border cases on the lines VI (zero profit) and VII (symmetric scale).

The six equilibrium outcomes can be briefly characterized as follows. In Figure 3.1 cost and demand conditions are favourable ($c = 0 < m$). Point *A'* is the standard Cournot-Nash equilibrium ($\alpha_1 = \alpha_2 = 0$). In equilibrium areas *A* both firms move away from standard Cournot-Nash by decreasing profit (though not below zero, given the condition that $c = 0 < m$) as a result of increasing habit formation ($\alpha_1, \alpha_2 > 0$), where firm j exceeds its rival i in size in equilibrium region A^i. On line VI the equilibrium takes the form of a zero-profit duopoly (with symmetric scale at the intersection point of the lines VI and VII, and with asymmetric size otherwise). In two limit cases the habit-motivated firm ($\alpha_i >> 0$) expels the profit-maximizing rival ($\alpha_j = 0$) from the market (points B' and B'') so that a zero-profit monopoly is reached. In Figure 3.2 cost conditions have deteriorated ($c > 0$), though not dramatically ($c < m$). Equilibrium areas A^i have decreased in size. In equilibrium areas B^i firm i is expelled from the market by its loss-making rival j, since firm j's managerial inertia dominates over firm i's preference for size ($\alpha_j >> \alpha_i$). Although market demand would enable two firms to produce profitably, in equilibrium areas C^i price falls below the level of average cost as a result of excessive market supply. Firm 1 is larger than firm 2 if $\alpha_1 > \alpha_2$ (C^2), firm 2 exceeds firm 1 in scale if $\alpha_1 < \alpha_2$ (C^1), and both firms are of equal size if $\alpha_1 = \alpha_2$ (line VII). In Figure 3.3 profit opportunities, even for a monopolist, are bound to disappear ($c = m$). Equilibrium areas A^i have finally disappeared. In equilibrium point D' both firms have decided to exit just in time. Neither firm is willing to accumulate losses for the sake of sustaining sales volume. Equilibrium point D' reflects a standard outcome of exit games in the IO-literature: $\alpha_1 = \alpha_2 = 0$ with simultaneous just-in-time exit. In Figure 3.4 the market is no longer viable, under whatever conditions ($c > m$), and equilibrium point D' has expanded to equilibrium area D: the profit motive dominates over habit formation. In equilibrium areas E^i, although the market is nonviable in terms of profitability, both firms stand the test of environmental decline (firm j being larger than rival i at either side of line VII). Given their preference for bigness, both firms are prepared to sacrifice profitability, even by accepting prices below the economic shut-down level ($p < c$). In equilibrium area F^i firm i gives in to deteriorating environmental circumstances, whilst firm j perseveres with operation. Contrary to firm i, firm j is willing to accept below average cost prices in order to be able to sustain sales volume.

So, from Figures 3.1 to 3.4 equilibrium areas appear and disappear, and grow and shrink. The results of this comparative statics can be summarized in Proposition 3.3.

Proposition 3.3. The (relative) size of the stationary-state-equilibrium areas where one or both firms decide to exit increases in c and decreases in m, with two notable exceptions: the size of the stationary-state-equilibrium area where only one firm leaves the market is independent from m for $0 \le c \le m$, and the stationary-state-equilibrium area where both firms stay in the market is independent from c for $c > m$.

Proof. The size of the admissible region is $\frac{1}{2}m^2 + \frac{1}{2}c^2 + 2mc$. Define three ratios of stationary-state-equilibrium areas: $R^1 = \dfrac{\text{One firm exits}}{\text{Both firms stay}}$; $R^2 = \dfrac{\text{Both firms exit}}{\text{Both firms stay}}$; and $R^3 = \dfrac{\text{Both firms exit}}{\text{One firm exits}}$. Note that $A = A^1 + A^2$, $B = B^1 + B^2$, $C = C^1 + C^2$, $E = E^1 + E^2$ and $F = F^1 + F^2$. By way of illustration, consider the following comparative statics. First, take the case where $0 \le c < m$. The size of the stationary-state-equilibrium areas A and C (where both firms stay in the market) is $\frac{1}{2}m^2 + 2mc - c^2$. Stationary-state-equilibrium area B (where one of both firms expels the rival from the market) is $\frac{3}{2}c^2$, which is independent from m. So, $R^1 = \frac{B}{A+C} = \frac{3c^2}{m^2 + 4mc - 2c^2}$, indicating that for a fixed value of m the size of the exit area B increases with c. Note that $R^2 = R^3 = 0$. Second, both stationary-state-equilibrium areas are of equal size ($A + C = B$) if $c = m$: then $R^1 = 1$, and $R^2 = R^3 = 0$. Third, take the case where $c > m$. Stationary-state-equilibrium area D (where both firms exit from the market) is $\frac{1}{2}(c-m)^2$. Stationary-state-equilibrium area E (where both firms stay in the market) is $\frac{3}{2}m^2$, which is independent from c. The size of stationary-state-equilibrium areas F (where only one firm leaves the market) is $\frac{3}{2}m(2c-m)$. So, with a fixed parameter m the ratios $R^1 = F/E$, $R^2 = D/E$ and $R^3 = D/F$ increase with c, implying an ((asymptotically) linear or quadratic) increase in the incidence of exit with increasing c: $R^1 = \frac{2c-m}{m}$, $R^2 = \frac{(c-m)^2}{3m^2}$; and $R^3 = \frac{(c-m)^2}{3m(2c-m)}$. Opposite results can be derived for the combination of variable m and fixed c.

[End of proof]

Of course, the result that the incidence of exit goes up if cost increases (c↑) (and, for that matter, if demand decreases (m↓)) is hardly surprising. Additionally, however, the model permits the calculation of (shifts in) absolute and relative sizes of the equilibrium areas. Figure 3.5 illustrates Proposition 3.3 for three cases: $\frac{\text{One firm exits}}{\text{Both firms stay}}$ ($R^1 = B/(A+C)$, or $R^1 = F/E$): Figure 3.5), $\frac{\text{Both firms exit}}{\text{Both firms stay}}$ ($R^2 = D/E$: Figure 3.6) and $\frac{\text{Both firms exit}}{\text{One firm exits}}$ ($R^3 = D/F$: Figure 3.7).

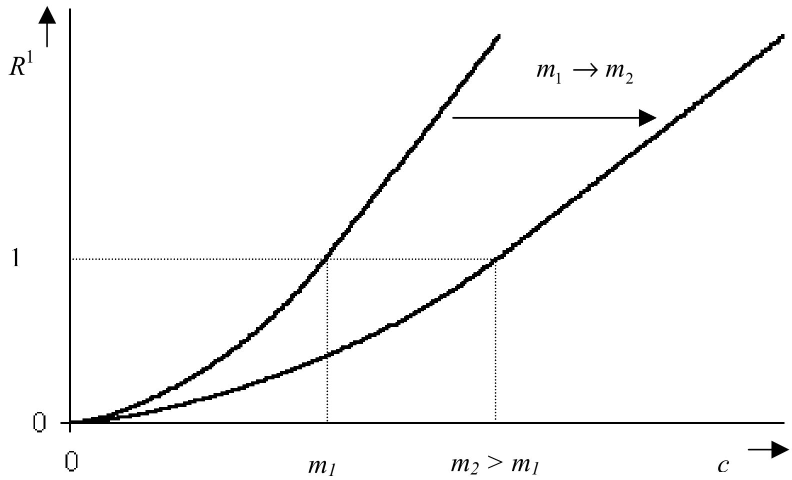


Fig. 3.5 Comparative statics for R^1: One firm exits / Both firms stay

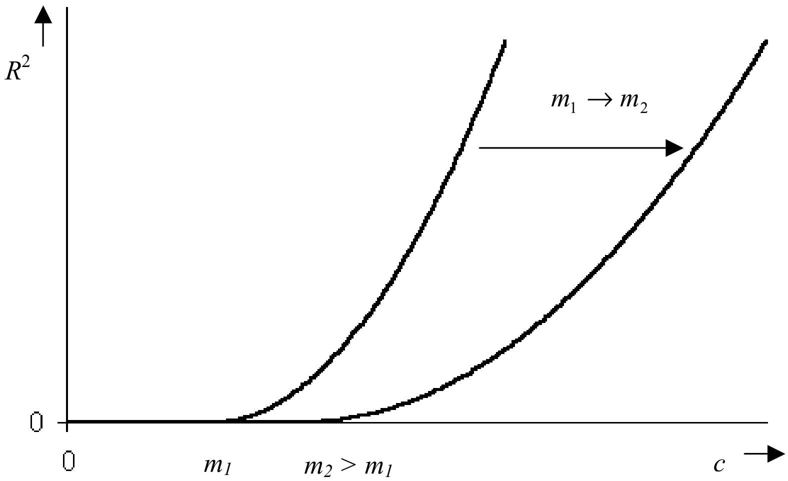


Fig. 3.6 Comparative statics for R^2: Both firms exit / Both firms stay

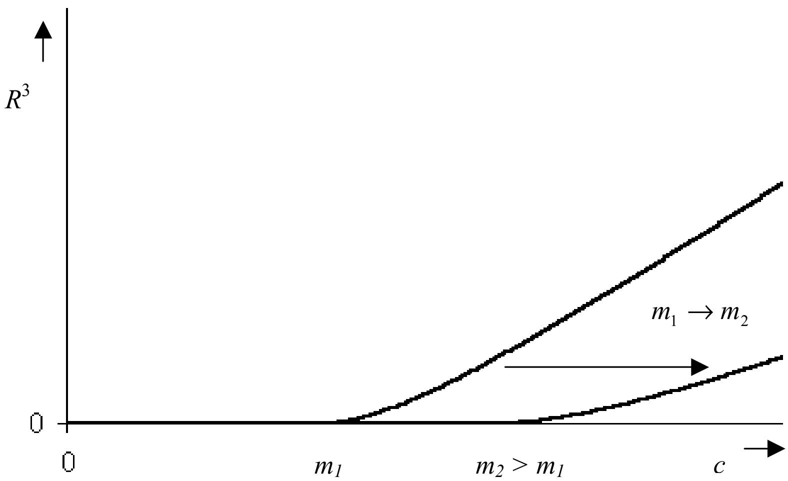


Fig. 3.7 Comparative statics for R^3: Both firms exit / One firm exits

Figures 3.5-3.7 depict the results for two cases of the demand parameter m: $m_1 < m_2$. Clearly, apart from the indicated exceptions, exit areas increase in c and decrease in m, either in a linear (R^1), quadratic (R^2) or asymptotically linear (R^3) way.

A final issue is related to Vickers' (1985) model, which assumes that firms *decide* on their preference for size (α) in an α-setting game. This paper relates to this issue by asking the question: what values of α_i maximize firm i's profit in equilibrium, given firm j's choice of α_j (we note that Fershtman and Judd (1987), Sklivias (1987) and

Basu (1995) also consider the choice of the weight attributed to revenues or, equivalently, sales volumes in a two-stage game; but we also consider the market size m , production costs c and habit formation symmetry). Proposition 3.4 indicates an answer to this question.

Proposition 3.4. With habit formation symmetry ($\alpha_i = \alpha_j = \alpha$) the profit-maximizing stationary-state-strategy for both firms is:
(i) negative habit formation ($\alpha < 0$) if $c < m$;
(ii) (zero-profit or standard, respectively) Cournot-Nash ($\alpha_i = \alpha_j = \alpha = 0$) if $c = m$ or $c < m$ and α restricted to be nonnegative; and
(iii) exit if $c > m$.
With habit formation asymmetry ($\alpha_i \neq \alpha_j$) the profit-maximizing stationary-state-reply of firm i to a fixed habit parameter α_j of firm j is:
(i) positive habit formation ($\frac{1}{4}(m - c - \alpha_j) > 0$) if $c < m$ and $\alpha_j < m - c$;
(ii) Cournot fringe follower ($\alpha_i = 0$) if $c < m$ and $\alpha_j = m - c$; and
(iii) exit otherwise.

Proof. First, take the case where $\alpha_i = \alpha_j = \alpha$. From equation (3.8) stationary-state-equilibrium profit $\Pi^i = \Pi^j = \Pi = \frac{1}{9}[-2\alpha^2 - (m-c)\alpha + (m-c)^2]$, which indicates a hill-shaped parabola of Π in α. Note that $\Pi = \frac{1}{9}(m-c)^2$ if $\alpha = 0$, and $\Pi = 0$ for $\alpha = \frac{1}{2}(m-c)$. Furthermore the fact that both firms stay in the market (positive production) leads to the condition $\alpha > -(m-c)$. The parabola shifts to the "South-East" with decreasing m. The parabola has a maximum at $\alpha = \frac{1}{4}(c-m)$, which is positive for $c > m$, zero for $m = c$ and negative for $c < m$. For $c > m$ stationary-state-equilibrium profit, Π , is negative, irrespective of the value of α.
Second, take the case where $\alpha_i \neq \alpha_j$. Assume that α_j is fixed and nonnegative. Equation (3.8) determines firm i's stationary-state-equilibium profit, which is a hill-shaped parabola in α_i with one maximum at $\alpha_i = \frac{1}{4}(m - c - \alpha_j)$. The parabola shifts to the "South-West" if α_j increases. With $c > m$ or $c < m$ but $\alpha_j > m - c$ profit is negative, whatever value of α_i is considered (see also Figures 3.4 and 3.2). If $c < m$ but $\alpha_j > m - c$, the maximum is at $\alpha_i = 0$. In the case where both $c < m$ and $\alpha_j < m - c$, firm i's profit is maximized at $\alpha_i = \frac{1}{4}(m - c - \alpha_j) > 0$.

[End of proof]

The intuition is illustrated in Figures 3.8 and 3.9 .

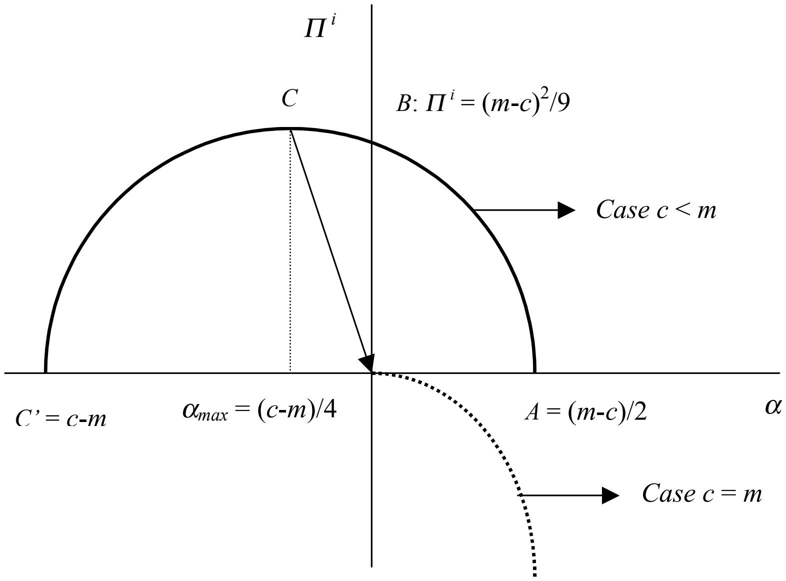


Fig. 3.8 The α-setting game for $\alpha_i = \alpha_j = \alpha$.

Figure 3.8 summarizes the implications for the case with habit symmetry. The curve *ABCC'* depicts symmetric profit ($\Pi^1 = \Pi^2 = \Pi$) for symmetric habit formation ($\alpha_1 = \alpha_2 = \alpha$). The *ABCC*'-curve shifts to the "South-East" with decreasing demand parameter m. If $c = m$, the *ABCC*'-curve intersects the Π-axis at the parabola's maximum in (0,0), which indicates zero-profit Cournot-Nash behaviour as the profit-maximizing strategy. Exit is the profit-maximizing strategy for $c > m$, because profits always lie below zero. The interesting case is $c < m$. If α is restricted to be nonnegative, both firms maximize profit in the standard symmetric Cournot-Nash duopoly outcome by setting $\alpha_1 = \alpha_2 = \alpha = 0$ (point B), as the admissible equilibria are restricted to the first quadrant. That is, at point B profit is maximized by having no preference for sales volume, which gives the standard profit objective function. Moving from point B to A along the *AB*-line, profit declines with increases in sales preference. At point A a zero profit is earned, which resembles the perfectly competitive outcome of a standard Bertrand-Nash duopoly game. At point A α equals $\frac{1}{2}(m-c)$. The line *AB* is the right half of a parabola, which intersects the Π-axis at $\Pi = \frac{1}{9}(m-c)^2$ $(\alpha = 0)$. If α is allowed to be negative, the "left half" of parabola *ABCC*', curve *BCC*', indicates a preference for smallness (that is, $\alpha < 0$), or: a negative utility of sales. At point C both firms maximize profit by restricting output, which resembles the collusive Cournot duopoly outcome ($\Pi^1 = \Pi^2 = \frac{1}{8}(m-c)^2$).

In Figure 3.9 the *EFGG*'-parabola depicts profit for α_i-choices, given a predetermined α_j. The *EFGG*'-curve shifts to the "South-West" if α_j increases, with its maximum at point H at (0,0) for $\alpha_j = m - c$. If $c < m$ and $\alpha_j < m - c$, the firm i benefits from positive habit formation up to a maximum at point F at $\alpha_i = \frac{1}{4}(m - c - \alpha_j)$. For $c < m$ and $\alpha_j = m - c$ this expression is maximized at point H

by setting $\alpha_i = 0$, which indicates Cournot fringe follower strategies by firm i in a leader-follower (Stackelberg) setting, where a small firm i ($\alpha_i = 0$) follows a large leader j ($\alpha_j >> 0$). If $c > m$ or $c < m$ but $\alpha_j > m - c$, exit is the profit-maximizing reply: the case with $c > m$ is trivial; with $c < m$ but $\alpha_j > m - c$ (see also Fig. 3.2) rival j has expanded such that firm i cannot operate profitably in the competitive fringe. Figure 3.9 supports, though in a different context, Dixit's (1992) observation that managerial inertia may well be optimal in the context of decision making in a dynamic environment.

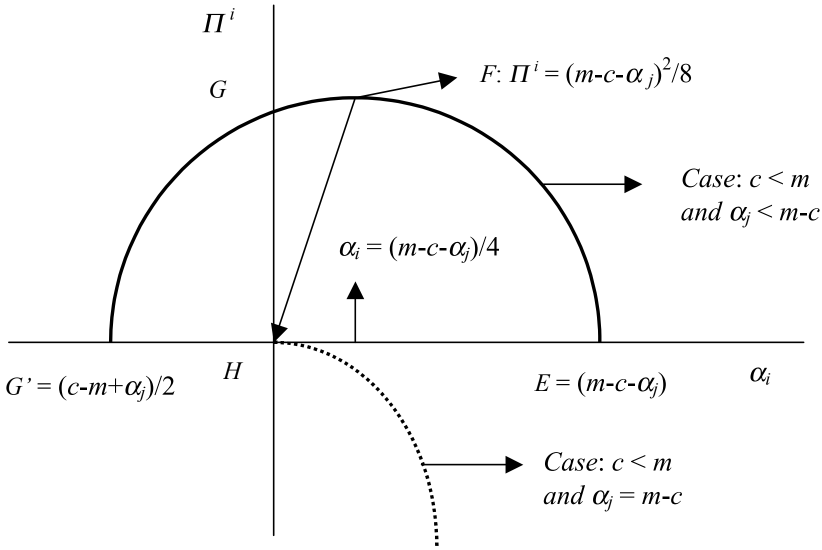


Fig. 3.9 The α-setting game for $\alpha_i \neq \alpha_j$.

4. Cost asymmetry

A popular explanation of organizational decline and exit is efficiency differentials: that is, $c_i \neq c_j$. The basic model (3.1)-(3.5) can now be re-analyzed, which gives Proposition 3.5.

Proposition 3.5. A Cournot-Nash duopoly stationary-state-equilibrium with asymmetric cost conditions ($c_i \neq c_j$) and asymmetric habit formation ($\alpha_i \neq \alpha_j$) can be calculated, and is asymptotically stable for $0 < \gamma_i, \gamma_j < 1$.

Proof. With $c_i \neq c_j$ maximizing equation (3.5) gives a set of four difference equations:

(i) $x_{1,t+1} = -\frac{1}{2}x_{2,t} + \frac{1}{2}(m - c_1 + \alpha_1)$,
(ii) $h_{1,t+1} = (1-\gamma_1)h_{1,t} - \frac{1}{2}x_{2,t} + \frac{1}{2}(m - c_1 + \alpha_1)$,
(iii) $x_{2,t+1} = -\frac{1}{2}x_{1,t} + \frac{1}{2}(m - c_2 + \alpha_2)$, and
(iv) $h_{2,t+1} = (1-\gamma_2)h_{2,t} - \frac{1}{2}x_{1,t} + \frac{1}{2}(m - c_2 + \alpha_2)$.

In matrix form this is $\underline{x}_{t+1} = A\underline{x}_t + \underline{b}$ with

$$\underline{b} = [\tfrac{1}{2}(m - c_1 + \alpha_1), \tfrac{1}{2}(m - c_1 + \alpha_1), \tfrac{1}{2}(m - c_2 + \alpha_2), \tfrac{1}{2}(m - c_2 + \alpha_2)]^{\mathrm{T}}$$

and A as in the proof of Proposition 3.1. Equilibrium conditions $\underline{x}^* = A\underline{x}^* + \underline{b} \iff (I - A)\underline{x}^* = \underline{b}$ determine the stationary-state-equilibrium values (3.13), (3.14) and (3.20), which, from condition $0 < \gamma_i < 1$, are asymptotically stable.

[End of proof]

If $c_i \neq c_j$, firm i's equilibrium sales (3.6) transform into

$$x_i^* = \tfrac{1}{3}(m - \alpha_j + 2\alpha_i - 2c_i + c_j), \tag{3.13}$$

and firm i's equilibrium habit formation (3.7) changes into

$$h_i^* = \tfrac{1}{3}\frac{m - \alpha_j + 2\alpha_i - 2c_i + c_j}{\gamma_i}, \tag{3.14}$$

where $i, j = 1, 2$ and $i \neq j$. Note that equation (3.13) confirms the intuition that firm i's equilibrium output increases in firm j's cost level. Equilibrium sales (3.13) indicate that firm i's output exceeds firm j's production if

$$\alpha_i > \alpha_j + (c_i - c_j) \ . \tag{3.15}$$

Condition (3.15) implies that lower habit formation can be compensated by an efficiency advantage.

The equivalence of the nonnegative price condition (3.9) to (3.11) is

$$x_1^* + x_2^* \leq m \Leftrightarrow \alpha_1 + \alpha_2 \leq m + (c_1 + c_2)\,, \tag{3.16}$$
$$x_1^* \leq m \Leftrightarrow 2\alpha_1 - \alpha_2 \leq 2m - c_2 + 2c_1\,, \text{ and} \tag{3.17}$$
$$x_2^* \leq m \Leftrightarrow 2\alpha_2 - \alpha_1 \leq 2m - c_1 + 2c_2\ . \tag{3.18}$$

Firm i's exit condition (3.12) shifts to

$$\alpha_j - 2\alpha_i \geq m - 2c_i + c_j\,, \tag{3.19}$$

where $i, j = 1, 2$ and $i \neq j$. Firm i's equilibrium profit (3.8) transforms into

$$\Pi^{\,i} = \tfrac{1}{9}(m - \alpha_i - \alpha_j - 2c_i + c_j)(m - \alpha_j + 2\alpha_i - 2c_i + c_j) \tag{3.20}$$

where $i, j = 1, 2$ and $i \neq j$. So, firm i's profit is negative for $\alpha_i + \alpha_j > m - 2c_i + c_j$.

By way of illustration, this section explores the implications of cost asymmetry for two cases: $0 < c_1$, $c_2 < m$ and $0 < c_1 < m < c_2$. For the sake of convenience, the assumption is that $c_1 < c_2$: so, firm 1 is the lowest-cost producer. If $0 < c_1$, $c_2 < m$, demand decline cannot explain exit, so that the focus is on the impact of strategic competition in combination with managerial inertia. Figures 3.1-3.4 are Figures 3.10-3.12's counterparts. The shift from Figures 3.1-3.4 to Figures 3.10-3.12 can, for example, be interpreted as being the result of a change in competitive conditions following an efficiency-enhancing innovation by firm 1. Two subcases can be discerned as to the size of firm 1's cost reduction (that is, $c_2 - c_1$): $m - 2c_2 + c_1 > 0$ (Figure 3.10) and $m - 2c_2 + c_1 < 0$ (Figure 3.11). For example, take the case where $m = 15$, $c_2 = 12$ and c_1 drops from 12 to 10 in the first subcase, and subsequently from 10 to 8 in the second subcase. The third case follows from $0 < c_1 < m < c_2$: firm 2, contrary to firm 1, cannot profitably operate in the market (Figure 3.12). This can be the result of, for example, a drop in demand from 15 to 10.

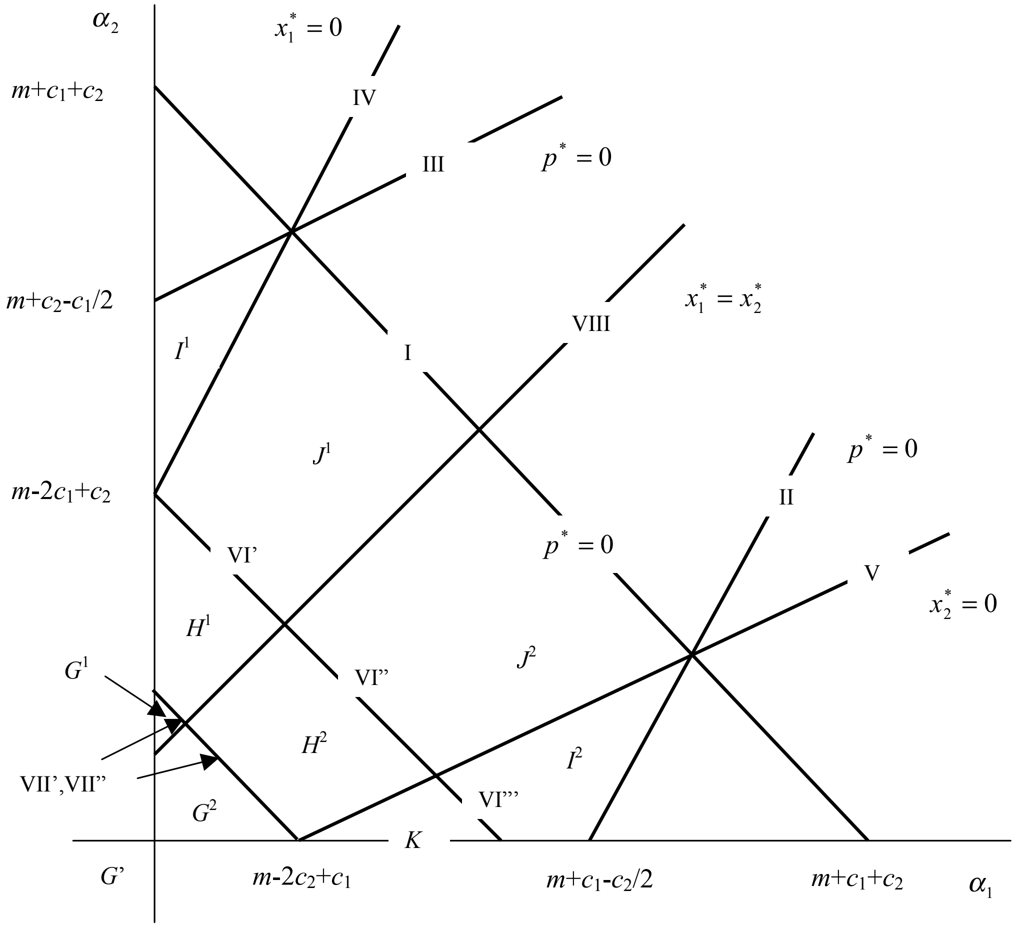


Fig. 3.10 Equilibrium outcomes for $0 < c_1 < c_2$ and $m - 2c_2 + c_1 > 0$

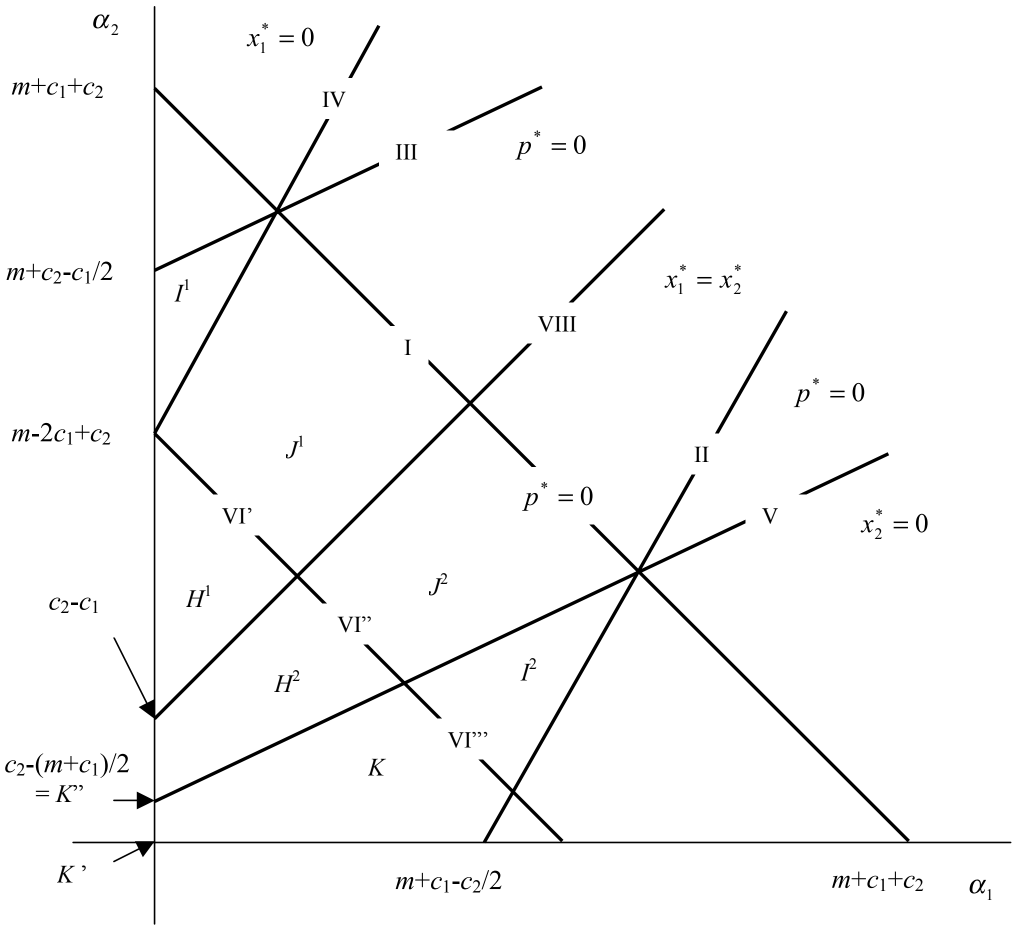


Fig. 3.11 Equilibrium outcomes for $0 < c_1 < c_2$ and $m - 2c_2 + c_1 < 0$

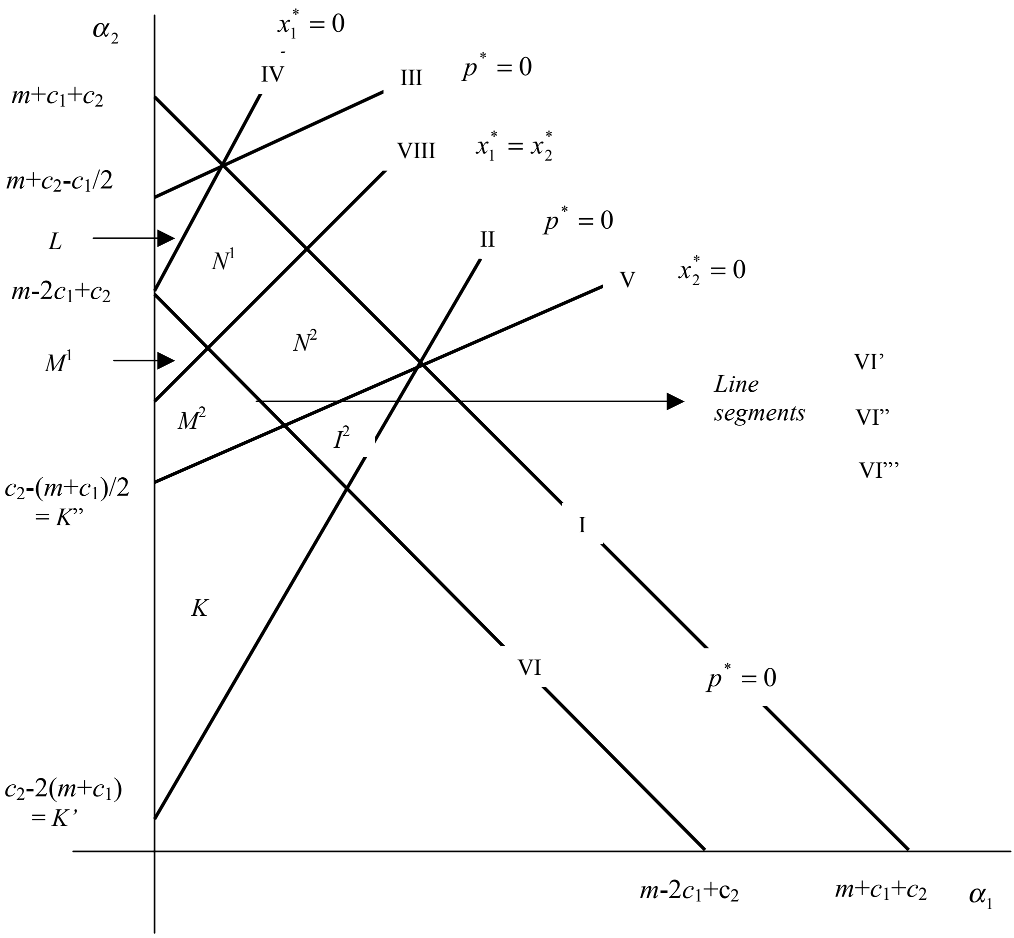


Fig. 3.12 Equilibrium outcomes for $c_1 < m < c_2$

The nonnegative price conditions determine the lines I, II and III (conditions (3.16) to (3.18), respectively). Lines IV and V represent firm 1's and firm 2's exit conditions, respectively (equation (3.19)). Lines VI'-VI''' and VII'-VII'' depict firm 1's and firm 2's zero-profit points (equation (3.20)). Finally, at the right-hand side of line VIII firm 1's output exceeds firm 2's sales, whereas at the left-hand side of line VIII the opposite holds (equation (3.15)). For the sake of brevity, this symmetric scale case is not discussed explicitly below.

Relative to the case with cost symmetry, the introduction of cost asymmetries triggers results that are different in emphasis or nature. As far as shifts in emphasis are concerned, the fact that the firms' exit and profit conditions have changed in favour of the lowest-cost firm is worth noting. For example, the intuition is confirmed that the exit areas of the efficient and inefficient firm are relatively reduced (I^1 and L) and enlarged (I^2 plus K), respectively. Differences in nature are more interesting, however. First, the interpretation of equilibria may have to be changed. Most importantly, equilibrium areas I^1 and L imply that the inefficient firm 2 survives at the detriment of the lowest-cost rival 1, and at the left-hand side of line VIII in the equilibrium regions G^1, H^1, J^1, M^1 and N^1 and on the quilibrium lines VI' and VII' the highest-cost firm 2's production volume exceeds the sales of its efficient rival 1. In both cases the explanation is that the preference for bigness overcompensates the efficiency disadvantage in the motive scheme of firm 2, whereas the opposite occurs in firm 1's motivation structure. Note that equilibrium areas L, M^1 and N^1 even occur if $c_2 > m$ (Figure 3.12), implying that the inefficient rival 2 dominates over the efficient firm 1 even in an environment that is only viable for an efficient supplier!

Second, if c_1 is reduced to such an extent that $m - 2c_2 + c_1 < 0$ (Figures 3.11 and 3.12), the standard Cournot-Nash duopoly equilibrium G' (pure profit maximizers: $\alpha_1 = \alpha_2 = 0$) vanishes. The reason is that in this case firm 1's cost advantage is such that standard Cournot-Nash duopoly competition drives the profit of the inefficient firm below the economic shut-down point ($p < c_2$): firm 1 can exploit its efficiency advantage, and is able to operate as a natural monopoly. Third, two new equilibrium areas have emerged. In equilibrium regions H^i and M^i both firms have decided to stay in the market (firm j exceeding rival i in size). Cournot-Nash duopoly competition gives a price $c_1 < p < c_2$ (or $p = c_1 < c_2$ on lines VI' and VI''). So, firm 1 earns a positive profit (or zero on lines VI' and VI''), whilst firm 2 accumulates losses as a result of its efficiency disadvantage. Firm 2's habit of liking bigness overcompensates its profit motive. Condition (3.15) determines the firms' relative sizes. In equilibrium area K the lowest-cost firm 1 survives at the detriment of the inefficient rival 2. Firm 1 operates as a natural monopoly which exploits its cost advantage so as to capture a positive profit (or zero on line VI'''). Note that equilibrium line K'-K'' implies that $\alpha_1 = 0$, firm 1 being a standard, efficiency-protected monopoly. Firm 2's preference for large sales cannot compensate the negative profit so as to trigger a decision to stay.

In comparison with the cost-symmetric case (Proposition 3.3) the comparative statics of the exit game with efficiency differentials introduces an additional finding worth reporting. This finding is summarized in Proposition 3.6.

Proposition 3.6. For the case where $c_1 < c_2$, the (relative) size of the stationary-state-equilibrium areas where the inefficient firm 2 decides to exit increases (in a nonlinear way) in c_2 with one notable exception: the size of the exit stationary-state-equilibrium region of firm 2 is independent from c_2 for $c_2 \geq 2(m + c_1)$.

Proof. Define an additional ratio of stationary-state-equilibrium areas: $R^4 = \dfrac{\text{Firm 2 exits}}{\text{Firm 1 exits}}$. Distinguish three cases (for $c_1 < c_2$). For all cases stationary-state-equilibrium areas I^1 and L (where firm 1 decides to leave the market) are $\frac{3}{4}(c_1)^2$. This is different for the sum of stationary-state-equilibrium areas I^2 and K (where firm 2 leaves the market), and so for R^4.

(i) $c_2 < \frac{1}{2}(m + c_1)$: $I^2 + K = \frac{3}{4}(c_2)^2$; and $R^4 = \dfrac{(c_2)^2}{(c_1)^2}$.

(ii) $\frac{1}{2}(m + c_1) \leq c_2 < 2(m + c_1)$: $I^2 + K = \frac{3}{4}(c_2)^2 - \frac{1}{4}(2c_2 - c_1 - m)^2$; and

$$R^4 = \frac{(c_2)^2}{(c_1)^2} - \frac{1}{3}\frac{(2c_2 - c_1 - m)^2}{(c_1)^2}.$$

(iii) $c_2 \geq 2(m+c_1)$: $I^2 + K = \frac{3}{4}(m+c_1)^2$; and $R^4 = \frac{(m+c_1)^2}{(c_1)^2}$, which is independent from c_2.

[End of proof]

The result is illustrated in Figure 3.13, where $R^4 = \frac{\text{Firm 2 exits}}{\text{Firm 1 exits}} = \frac{I^2+K}{I^1+L}$.

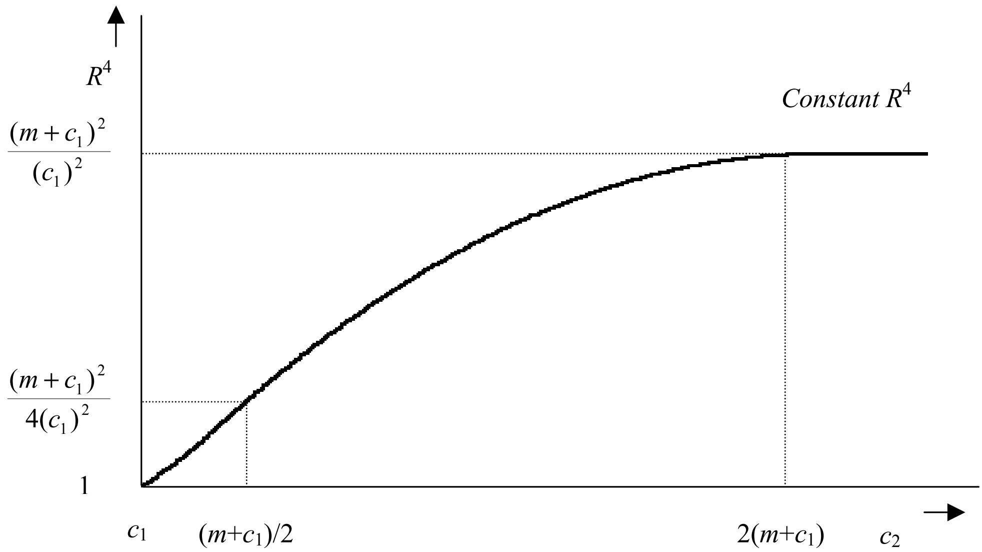


Fig. 3.13 Comparative statics for R^4: Firm 2 exits / Firm 1 exits

Figure 3.13 clearly depicts that R^4 remains constant beyond $c_2 \geq 2(m+c_1)$. For $c_2 < 2(m+c_1)$ the intuitive result is obtained that the ratio R^4, reflecting the relative incidence of firm 2's exit decision, increases (in a nonlinear way) in c_2.

The results for the duopoly model can easily be extended to the n-firm case. Proposition 3.7 summarizes the results of the n-firm Cournot-Nash competition game with habit formation and cost asymmetry.

Proposition 3.7. A Cournot-Nash oligopoly stationary-state-equilibrium with n firms, asymmetric cost conditions ($c_i \neq c_j$) and asymmetric habit formation ($\alpha_i \neq \alpha_j$) can be calculated, and is asymptotically instable for $n > 3$.

Proof. With n firms $p_t = m - \sum_{i=1}^{n} x_{i,t}$ (equation (3.4)). Maximizing the objective function (3.3) for period $t+1$, substituting $\Pi^i_{t+1} = x_{i,t+1}(m - x_{i,t+1} - c_i - \sum_{j \neq i} x_{j,t})$, and habit formation (3.2) for period $t+1$, gives the system of difference equations

$x_{i,t+1} = \frac{1}{2}(m - c_i + \alpha_i) - \frac{1}{2}\sum_{j \neq i} x_{j,t}$ and

$h_{i,t+1} = (1-\gamma_i)h_{i,t} + \frac{1}{2}(m - c_i + \alpha_i) - \frac{1}{2}\sum_{j \neq i} x_{j,t}$ for $i, j = 1,...,n$ and $i \neq j$.

In matrix form this is $\underline{x}_{t+1} = A\underline{x}_t + \underline{b}$, where

$\underline{x}_t = [x_{1,t}, h_{1,t}, x_{2,t}, h_{2,t}, ..., x_{n,t}, h_{n,t}]^{\mathrm{T}}$,

$\underline{b} = [\frac{1}{2}(m-c_1+\alpha_1), \frac{1}{2}(m-c_1+\alpha_1), \frac{1}{2}(m-c_2+\alpha_2), \frac{1}{2}(m-c_2+\alpha_2), ..., \frac{1}{2}(m-c_n+\alpha_n), \frac{1}{2}(m-c_n+\alpha_n)]^{\mathrm{T}}$,

and A the n-dimensional variant of matrix A in the proof of Proposition 3.1. Now the stationary-state-equilibrium volumes (21) and habit formations (22) can be calculated. The eigenvalues of matrix A are found by solving the equation

$\det(A - \lambda I) = 0 \Leftrightarrow \det(A - \lambda I) = -\prod_{i=1}^{n}[(1-\gamma_i) - \lambda](\frac{1}{2} - \lambda)^{n-1}[\lambda + \frac{1}{2}(n-1)] = 0$.

Hence, noting that $\lambda_n = -\frac{1}{2}(n-1)$, the stationary-state-equilibrium outcomes are asymptotically unstable for $n > 3$. For $n = 3$ the eigenvalues are $\lambda_1 = 1-\gamma_1$, $\lambda_2 = 1-\gamma_2$, $\lambda_3 = 1-\gamma_3$, $\lambda_4 = \lambda_5 = \frac{1}{2}$ and $\lambda_6 = -1$, which gives small upward and downward movements close to the stationary-state-equilibrium outcome. For $n > 3$ there exists one eigenvalue the absolute value of which exceeds 1. Now the fluctuations are limited by the nonnegative price restriction.

[End of proof]

Equilibrium volumes are

$$x_i^* = \frac{1}{(n+1)}(m - nc_i + \sum_{j \neq i} c_j + n\alpha_i - \sum_{j \neq i}\alpha_j) \tag{3.21}$$

and equilibrium habit formation is

$$h_i^* = \frac{1}{\gamma_i(n+1)}(m - nc_i + \sum_{j \neq i} c_j + n\alpha_i - \sum_{j \neq i}\alpha_j) \tag{3.22}$$

both for $i, j = 1,...,n$, $i \neq j$ and $n > 1$. Hence, if $\alpha_i = 0$ $(i = 1,...,n)$ and $c_i = c_j = ... = c_n$, the standard Cournot-Nash outcome with n firms is reached. The counterparts of the nonnegative price conditions (3.16) to (3.18) are

$$\sum_{i=1}^{n} x_i^* \leq m \Leftrightarrow \sum_{i=1}^{n}\alpha_i \leq m + \sum_{i=1}^{n} c_i \text{ , and} \tag{3.23}$$

$$x_i^* \leq m \Leftrightarrow n\alpha_i - \sum_{j \neq i}\alpha_j \leq nm + nc_i - \sum_{j \neq i} c_j \tag{3.24}$$

for $i, j = 1,...,n$ and $i \neq j$. Now equilibrium profit can be calculated.

So, the n-firm case is a straightforward extension of the duopoly game implying a multiplication of the number of equilibrium regions without changing the qualitative features of the equilibrium outcomes. Model (3.21) to (3.24) defines an n-dimensional area being constrained by $n+1$ hyperplanes in the first quadrant ($\alpha_i \geq 0$ for $i=1,...,n$). An important deviation from the duopoly case is however that for $n>3$ asymptotic instability occurs. The model can be analyzed through computer simulation by supplementing the model with the condition that sales are not allowed to fall below zero. That is, in period t firm i's sales volume follows from

$$x_{i,t}^* = \max\left\{0, \left[-\tfrac{1}{2}\sum_{j\neq i} x_{j,t-1}^* + \tfrac{1}{2}(m+\alpha_i - c_i)\right]\right\} \qquad (3.25)$$

for $i, j = 1,...,n$ and $i \neq j$.

5. Appraisal

The model reveals the precise conditions that underlie specific outcomes, ranging from standard Cournot-Nash duopoly competition over just-in-time exit to chronic failure, even by efficient firms. Essential in our model is that the weights attributed to size in the objective function, α_i, do (except from Proposition 3.4) not follow from highly rational decision making like in principal-agent models (such as Vickers (1985), Fershtman and Judd (1987), Sklivias (1987) and Basu (1995)), but are assumed to be the result of managerial inertia (habit formation). The analysis of all sorts of (α_1, α_2)-combinations leads to a variety of outcomes, such as an inefficient loss-making monopoly (region I^1 in Figures 3.10 and 3.11). On the one hand, as far as empirical regularities are concerned, the model outcomes support the two well-documented stylized facts referred to in the introduction. First, the observation that bankruptcy is negatively correlated to size is reflected in equilibrium areas B^i, F^i, I^i, K and L where the (larger) habit-motivated firm survives to the detriment of the (smaller) profit-motivated rival. Second, the empirical finding that, in many cases, firms accumulate losses before the actual date of bankruptcy appears in equilibrium areas B^i, C^i, E^i, F^i, H^i, I^i, J^i, L, M^i and N^i (ignoring the border cases on the line segments), where chronic failure is associated with pertaining negative profits for one or both firms.

On the other hand, from a theoretical angle the model reveals a broad set of outcomes and underlying causes through varying mixtures of cost (a)symmetries, demand turbulence, managerial inertia and strategic competition. Again, two remarks are worth making. First, IO-models concerning environmental decline, ignoring managerial inertia (so $\alpha_1 = \alpha_2 = 0$) are captured by the equilibrium points A' (cost-symmetric Cournot-Nash duopoly profit maximization), D' (duopoly just-in-time exit), G' (cost-asymmetric Cournot-Nash duopoly profit maximization) and K'-K'' (profit-maximizing efficient monopoly). Second, a key feature of this Chapter's model is the explanation of chronic failure by large, inefficient firms in equilibrium areas H^1, J^1, M^1 and N^1 (where the inefficient firm is larger than the lowest-cost rival) and, particularly, I^1 and L (where the inefficient firm has expelled the lowest-cost rival from the market). Table 3.1 summarizes the outcomes. For the sake of brevity, the cases with symmetric scale on the line segments VII (Figures 3.1-3.4, apart from the equilibrium point A') and VIII (Figure 3.10-3.12) are not included.

Table 3.1. *Equilibrium outcomes*

EQUILIBRIUM	INTERPRETATION	CONDITIONS
A'	Cost-symmetric profit-maximizing duopoly with symmetric scale facing favourable demand	$c_1 = c_2 = c$; $\alpha_1 = \alpha_2 = 0$; $\Pi^1 = \Pi^2 > 0$; $x_1^* = x_2^* > 0$; $0 \le c < m$
A^1,A^2	Cost-symmetric habit-motivated profit-making duopoly with asymmetric scale facing favourable demand	$c_1 = c_2 = c$; $0 < \alpha_i < \alpha_j$; $0 < \Pi^i < \Pi^j$; $0 < x_i^* < x_j^*$; $0 \le c < m$
VI	Cost-symmetric habit-motivated zero-profit duopoly with asymmetric scale facing favourable demand	$c_1 = c_2 = c$; $0 < \alpha_i < \alpha_j$; $\Pi^i = \Pi^j = 0$; $0 < x_i^* < x_j^*$; $0 \le c < m$

B', B''	Habit-motivated zero-profit monopoly facing favourable demand	$c_1 = c_2 = c$; $0 < \alpha_i < \alpha_j$; $\Pi^i = \Pi^j = 0$; $x_i^* = 0 < x_j^*$; $0 \leq c < m$
B^1, B^2	Habit-motivated loss-making monopoly facing favourable or neutral demand	$c_1 = c_2 = c$; $0 < \alpha_i < \alpha_j$; $\Pi^j < \Pi^i = 0$; $x_i^* = 0 < x_j^*$; $0 < c \leq m$
C^1, C^2	Cost-symmetric habit-motivated loss-making duopoly with asymmetric scale facing favourable or neutral demand	$c_1 = c_2 = c$; $0 < \alpha_i < \alpha_j$; $\Pi^j < \Pi^i < 0$; $0 < x_i^* < x_j^*$; $0 < c \leq m$
D'	Cost-symmetric profit-maximizing just-in-time exit with neutral or unfavourable demand	$c_1 = c_2 = c$; $\alpha_1 = \alpha_2 = 0$; $\Pi^1 = \Pi^2 = 0$; $x_1^* = x_2^* = 0$; $c \geq m$
D	Cost-symmetric habit-motivated just-in-time exit with unfavourable demand	$c_1 = c_2 = c$; $\alpha_1, \alpha_2 > 0$; $\Pi^1 = \Pi^2 = 0$; $x_1^* = x_2^* = 0$; $c > m$
E^1, E^2	Cost-symmetric habit-motivated loss-making duopoly with asymmetric scale facing unfavourable demand	$c_1 = c_2 = c$; $0 < \alpha_i < \alpha_j$; $\Pi^j < \Pi^i < 0$; $0 < x_i^* < x_j^*$; $c > m$
F^1, F^2	Habit-motivated loss-making monopoly facing unfavourable demand	$c_1 = c_2 = c$; $0 < \alpha_i \ll \alpha_j$; $\Pi^j < \Pi^i = 0$; $x_i^* = 0 < x_j^*$; $c > m$
G'	Cost-asymmetric profit-maximizing duopoly facing favourable demand with efficient leader	$m - 2c_2 + c_1 > 0$; $\alpha_1 = \alpha_2 = 0$; $0 < \Pi^2 < \Pi^1$; $0 < x_2^* < x_1^*$; $c_1 < c_2 < m$
G^1	Cost-asymmetric habit-motivated profit-making duopoly facing favourable demand with inefficient leader	$m - 2c_2 + c_1 > 0$; $0 < \alpha_1 + (c_2 - c_1) < \alpha_2$; $\Pi^1, \Pi^2 > 0$; $0 < x_1^* < x_2^*$; $c_1 < c_2 < m$
VII'	Cost-asymmetric habit-motivated duopoly facing favourable demand with inefficient zero-profit leader and efficient profit-making follower	$m - 2c_2 + c_1 > 0$; $0 < \alpha_1 + (c_2 - c_1) < \alpha_2$; $\Pi^2 = 0 < \Pi^1$; $0 < x_1^* < x_2^*$; $c_1 < c_2 < m$
G^2	Cost-asymmetric habit-motivated but profit-making duopoly facing favourable demand with efficient leader	$m - 2c_2 + c_1 > 0$; $0 < \alpha_2 < \alpha_1 + (c_2 - c_1)$; $0 < \Pi^2 < \Pi^1$; $0 < x_2^* < x_1^*$; $c_1 < c_2 < m$
VII''	Cost-asymmetric habit-motivated duopoly facing favourable demand with efficient profit-making leader and inefficient zero-profit follower	$m - 2c_2 + c_1 > 0$; $0 < \alpha_2 < \alpha_1 + (c_2 - c_1)$; $\Pi^2 = 0 < \Pi^1$; $0 < x_2^* < x_1^*$; $c_1 < c_2 < m$
H^1	Cost-asymmetric habit-motivated duopoly facing favourable demand with inefficient loss-making leader and efficient profit-making follower	$m - 2c_2 + c_1 \gtrless 0$; $0 < \alpha_1 + (c_2 - c_1) < \alpha_2$; $\Pi^2 < 0 < \Pi^1$; $0 < x_1^* < x_2^*$; $c_1 < c_2 < m$
VI'	Cost-asymmetric habit-motivated duopoly with inefficient loss-making leader facing (un)favourable demand and efficient zero-profit follower facing favourable demand	$m - 2c_2 + c_1 \gtrless 0$; $0 < \alpha_1 + (c_2 - c_1) < \alpha_2$; $\Pi^2 < \Pi^1 = 0$; $0 < x_1^* < x_2^*$; $c_1 < c_2 < m$ (or $c_1 < m < c_2$)
H^2	Cost-asymmetric habit-motivated duopoly facing favourable demand with efficient profit-making leader and inefficient loss-making follower	$m - 2c_2 + c_1 \gtrless 0$; $0 < \alpha_2 < \alpha_1 + (c_2 - c_1)$; $\Pi^2 < 0 < \Pi^1$; $0 < x_2^* < x_1^*$; $c_1 < c_2 < m$

VI’’	Cost-asymmetric habit-motivated duopoly with efficient zero-profit leader facing favourable demand and loss-making follower facing (un)favourable demand	$m - 2c_2 + c_1 \gtrless 0;\ 0 < \alpha_2 < \alpha_1 + (c_2 - c_1);$ $\Pi^2 < \Pi^1 = 0;\ 0 < x_2^* < x_1^*;\ c_1 < c_2 < m$ (or $c_1 < m < c_2$)
I^1	Inefficient habit-motivated loss-making monopoly facing (un)favourable demand	$m - 2c_2 + c_1 \gtrless 0;\ 0 < \alpha_1 + (c_2 - c_1) << \alpha_2;$ $\Pi^2 < \Pi^1 = 0;\ 0 = x_1^* < x_2^*;\ c_1 < c_2 < m$
I^2	Efficient habit-motivated loss-making monopoly facing favourable demand	$m - 2c_2 + c_1 \gtrless 0;\ 0 < \alpha_2 << \alpha_1 + (c_2 - c_1);$ $\Pi^1 < \Pi^2 = 0;\ 0 = x_2^* < x_1^*;\ c_1 < c_2 < m$
J^1	Cost-asymmetric habit-motivated loss-making duopoly facing favourable demand with inefficient leader	$m - 2c_2 + c_1 \gtrless 0;\ 0 < \alpha_1 + (c_2 - c_1) < \alpha_2;$ $\Pi^2 < \Pi^1 < 0;\ 0 < x_1^* < x_2^*;\ c_1 < c_2 < m$
J^2	Cost-asymmetric habit-motivated loss-making duopoly facing favourable demand with efficient leader	$m - 2c_2 + c_1 \gtrless 0;\ 0 < \alpha_2 < \alpha_1 + (c_2 - c_1);$ $\Pi^1,\ \Pi^2 < 0;\ 0 < x_2^* < x_1^*;\ c_1 < c_2 < m$
K’-K’’	Efficient profit-maximizing monopoly facing favourable demand	$m - 2c_2 + c_1 < 0;\ \alpha_1 = 0 \leq \alpha_2;$ $\Pi^2 = 0 < \Pi^1;\ x_2^* = 0 < x_1^*;\ c_1 < m < c_2$
K	Efficient habit-motivated profit-making monopoly facing favourable demand	$m - 2c_2 + c_1 \gtrless 0;\ 0 < \alpha_2 << \alpha_1 + (c_2 - c_1);$ $\Pi^2 = 0 < \Pi^1;\ x_2^* = 0 < x_1^*;\ c_1 < m < c_2$
VI’’’	Efficient habit-motivated zero-profit monopoly facing favourable demand	$m - 2c_2 + c_1 \gtrless 0;\ 0 < \alpha_2 << \alpha_1 + (c_2 - c_1);$ $\Pi^1 = \Pi^2 = 0;\ x_2^* = 0 < x_1^*;\ c_1 < m < c_2$
L	Inefficient habit-motivated loss-making monopoly facing unfavourable demand	$m - 2c_2 + c_1 < 0;\ 0 < \alpha_1 + (c_2 - c_1) << \alpha_2;$ $\Pi^2 < \Pi^1 = 0;\ x_1^* = 0 < x_2^*;\ c_1 < m < c_2$
M^1	Cost-asymmetric habit-motivated duopoly with inefficient loss-making leader facing unfavourable demand and efficient profit-making follower facing favourable demand	$m - 2c_2 + c_1 < 0;\ 0 < \alpha_1 + (c_2 - c_1) < \alpha_2;$ $\Pi^2 < 0 < \Pi^1;\ 0 < x_1^* < x_2^*;\ c_1 < m < c_2$
M^2	Cost-asymmetric habit-motivated duopoly with efficient profit-making leader facing favourable demand and inefficient loss-making follower facing unfavourable demand	$m - 2c_2 + c_1 < 0;\ 0 < \alpha_2 < \alpha_1 + (c_2 - c_1);$ $\Pi^2 < 0 < \Pi^1;\ 0 < x_2^* < x_1^*;\ c_1 < m < c_2$
N^1	Cost-asymmetric habit-motivated loss-making duopoly with inefficient leader facing unfavourable demand and efficient follower facing favourable demand	$m - 2c_2 + c_1 < 0;\ 0 < \alpha_1 + (c_2 - c_1) < \alpha_2;$ $\Pi^2 < \Pi^1 < 0;\ 0 < x_1^* < x_2^*;\ c_1 < m < c_2$
N^2	Cost-asymmetric habit-motivated loss-making duopoly with efficient leader facing favourable demand and inefficient follower facing unfavourable demand	$m - 2c_2 + c_1 < 0;\ 0 < \alpha_2 < \alpha_1 + (c_2 - c_1);$ $\Pi^1,\ \Pi^2 < 0;\ 0 < x_2^* < x_1^*;\ c_1 < m < c_2$

This Chapter’s model has, of course, its limitations. Table 3.1 reveals that the model does not cover two scenarios: temporary downturn, where a firm recovers after a temporary period of negative profit (Dixit (1989 and 1992)), and ultimate exit, where a firm leaves the market after a period of accumulated losses (Ho and Saunders (1980) and Scapens, Ryan and Fletcher (1981)). Van Witteloostuijn (1998) uses the terms “turnaround success” and “flight from losses” in his framework. Therefore, an immediate extension of the model could be in line with the theoretical exercitions from an AF-perspective (Ho and Saunders (1980) and Scapens, Ryan and Fletcher (1981)) by introducing the influence of institutional restrictions through modeling creditors’ confidence. This means that the firm’s profit constraint (or, to be precise, loss constraint) is endogenized rather than fixed to zero (IO-literature) of infinite (this Chapter).

Moreover, by way of illustration, three further extensions are worth mentioning. First, following Vickers (1985), the assumption of myopic behaviour may be relaxed. That is, habit formation becomes a strategic variable if managers (or other stakeholders, for that matter) are able to decide, at least to some extent, on routines formation by in advance taking account of the implications of specific decisional, organizational and ownership structures on the parameter α (in the models of Vickers (1985) and Fershtman and Judd (1987) owners write incentive contracts for their managers. Owners influence managers’ objective functions in such a way that profit is maximized, given the rival’s objective function). Proposition 3.4 reflects on the strategic choice of the weights α_i. Second, expansion (and shrinkage) may be assumed to be costly. In the well-established IO-tradition on decision making on capacity, expansion (or exit) requires investment (or loss) of sunk cost in building up (or breaking down) productive capacity (Tirole (1988)). Third, and related to the second, firms may be assumed to operate in a multimarket context, which has an impact on the nature and size of exit barriers (Van Wegberg and Van Witteloostuijn (1992) and Van Witteloostuijn and Van Wegberg (1992)). The key point is that the modeling framework in this paper permits the introduction of these and other extensions, and subsequently facilitates the analysis of the implications of newly introduced parameters.

CHAPTER 4

COURNOT COMPETITION WITH ASYMMETRICAL ADJUSTMENT COSTS SUBJECT TO A BUSINESS CYCLE

1. Introduction

In Chapter 3 we examined the consequences of managerial growth preferences and we used a game theoretical approach to model this form of firm's inertia. One of the intriguing results of this analysis is that management's preference for size (and its resistance to downsize) may be strategically beneficial in direct competition. For instance van Witteloostuijn, Boone and van Lier (2003) state that "A cost-efficient (i.e., low-cost) and managerial flexible (i.e., profit-maximizing) firm may well be outcompeted by a cost-inefficient (i.e., high-cost) and managerially inert (i.e., nonprofit-maximizing) rival. In the extreme, the latter firm may even survive at the expense of the former".

This outcome of mathematical modelling also supports the path-breaking inertia theorem in Organizational Ecology of Hannan and Freeman (1984). Briefly speaking, they argue that "Selection in populations of organizations in modern societies favors forms with high reliability of performance and high levels of accountability" This high levels of reliability and accountability require that organizational rules, procedures and structures are highly reproducible. Naturally the same factors that make a system or structure highly reproducible also cause firm's inertia (relative to environmental changes). These arguments, roughly speaking, lead to Hannan and Freeman's theorem (1984): "Selection within populations of organizations in modern societies favors organizations whose structures have high inertia".

Chapter 3 deals with the implications of a special form of inertia, namely managerial inertia under environmental turbulence. The behaviour of the firm's management – reflected in "love for size" and "resistance to retrenchment" is imprinted thoroughly in firm's structures and procedures and appararently this firm's blueprint is favoured in a selection process. Of course, under the assumption that firms are able to adapt rationally to environmental turbulence or competitive threats (rational adaptation theory versus the selection perspective), manipulation of the level of preference for size can serve as a strategic instrument. The α-setting game (which is closely related to the "delegation games" of Vickers (1985), Fershtman and Judd (1987) and others) and the optimal level of inertia in Chapter 3 are examples of rational and strategic action.

In this Chapter we focus on another form of inertia, reflected by firm's adjustment costs due to a change of production levels. Obviously the adjustment of production volumes is associated with all sorts of costs, because neither a decrease nor an increase of a firm's production level is likely to be costless. The form of inertia associated with adjustment costs could be called organizational inertia besides the other form - managerial inertia - considered in Chapter 3 (in Chapters 5 and 6 also managerial inertia reflected by preference for market share will be examined). Realizing that changes in firm's production volumes cause additional expenses, on top of the usual unit production costs, we briefly reflect on the main sources of these adjustment costs, inter-firm differences, and their functional form concerning the Cournot duopoly model in this Chapter. First prominent long-run cost of production change follows from investment or devestment of capital.

And second a major short-run cost of production change originates in human resources. Expansion of the usual supply, results in overwork and the recruitment of additional labour force. Both overwork pay and the attraction of new employees imply extra costs and may delay the firm's decision to enlarge production. On the other hand a decrease of production levels may imply the necessity to fire personnel and is associated with redundancy payments and golden handshakes. Besides extra costs also firm's specific skills and knowledge may be lost as a consequence of downsizing.

Concerning the shape of the firm-level employment adjustment cost function Hamermesh and Pfann (1996) discuss a large number of empirical studies and argue that the adjustment cost function may be symmetric or asymmetric and linear or non-linear, depending upon the country, industry and period under study. These adjustment costs may be very large. For example "Hamermesh (1989) suggests that lumpy costs of adjustment in the manufacturing plants he studies are so large that a shock must alter demand by 60 percent before employment is changed". One may expect that the implication of adjustment costs is that firms are more or less slow in changing their employment size over a downward or upward phase of the business cycle. Indeed Hamermesh and Pfann argue that the response to environmental shocks will not be instantaneous.

There also exist inter-firm differences in terms of adjustment as management literature reveals. Huselid (1995) reveals that firms differ in their choice for high- or low-commitment human resource management practices. Naturally high-commitment human resource practices are associated with high salaries, long- term contracts and permanent appointments, implying that such practices are relatively expensive in comparison to low-commitment human resource practices. Even between countries there exist differences between human resource management practices. Gooderham, Nordhaug and Ringdal (1999) report significant and systematic cross-country heterogeneity concerning these practices. On the one hand relatively expensive human resource practices dominate in Denmark, Germany, and Norway, whereas their relatively cheap counterparts are typical for France, Spain and the UK. Another source of differences between firms' organizational inertia may be the well-established phenomenon of resistance to change on an organization's work floor. A clear case of organizational inertia is caused by retrenchment costs and van Witteloostuijn (1998) presents a recent review of this issue, which has been studied heavily in the literature of organizational decline.

How do we use the conclusions of these empirical findings in our game theoretic approach? As a first step in modelling mathematically the complicated issue of organizational inertia, in this Chapter, we focus on a Cournot duopoly game, with asymmetrical adjustment costs of both competitors around the Cournot-Nash equilibrium. Production cost functions will be chosen both linear and quadratic. We note that, concerning most studies, unit production cost, i.e. marginal costs, are considered to be constant. The presence of a quadratic term in the production cost function enables us to examine the consequences of production technologies with decreasing and increasing returns to scale as well, albeit this choice sometimes complicates the expressions and formulas. Concerning the functional form of the adjustment costs, many choices can be made as empirical research reveals. For instance, Szidarovsky and Yen (1995) focus on conditions for the stability of dynamic oligopolies (with discrete time scale) under the assumption of quadratic adjustment costs concerning the production level of the previous period.

So concerning their model, adjustment costs take the functional form $C = k(x_t - x_{t\text{-}1})^2$, where k is a constant and the quantities x_t , $x_{t\text{-}1}$ represent the actual and previous supply levels respectively. However, we consider the quadratic functional specification rather nonrealistic in the context of hiring and firing employees; if adjustment costs are strongly related to human resource management practices, at least a linear term should have been added. In our duopoly model adjustment costs are constant per unit production change related to a fixed production level (corresponding to a reference market size m=1 in a normalized model). Then, the possible necessity to increase or decrease supply is induced by environmental turbulence, namely fluctuations in demand, i.e. the occurrence of business cycles.

The analysis of this Chapter focuses on the behavioral phases of two competitors, which may possess different adjustment costs per unit production change, as an implication of decreased or increased economic activity. The introduction of adjustment costs in the Cournot duopoly model naturally supports Hammermesh and Pfann's findings that the response to environmental shocks will not be instantaneous. Furthermore the forthcoming analysis reveals intriguing implications of inter-firm differences in adjustment costs, related to strategic benefits. Here we mention one of the outcomes:

An organizationally flexible (i.e., without production costs) firm may well be outcompeted by an organizationally inert (i.e., with production adjustment costs) rival. In the extreme, the latter may even survive at the expense of the former.

The "may" formulation of this statement has to do with a period of decreased or increased economic activity. It turns out that organizational inertia pays off in a declining market, whereas in a booming market flexibility pays off. The outcomes of the analysis of this Chapter can be looked upon from two different theoretical perspectives. First from the rational adaptation perspective and under the assumption that a change in the adjustment costs (per unit) is possible, these costs can serve as a firm's strategic instrument. By manipulating these adjustment costs (for instance by a change in human resource practices) a firm is able to expel its rival from the market or to make relatively higher profits during a period of economic upswing or downswing.

However the second, Organizational Ecology (OE), perspective, which focuses on Darwinian selection processes is also (partly) supported by the analytical outcomes, because selection favors firms with a high level of organizational inertia in a declining market.

How is this Chapter organized?

Section 4.2 deals with the formulation and normalization of the "classical" Cournot duopoly model, i.e. the model without adjustment costs,. A summary of both competitor's profits and social welfare is provided, subject to a business cycle. These "classical" outcomes will serve as a point of reference concerning the analytical results of the other Sections. Section 4.3 deals with the precise formulation of the Cournot model with asymmetrical adjustment costs around the original Cournot-Nash equilibrium. The Propositions derived in this Section describe the properties of reaction curves, as a consequence of adjustment costs and a change in market size. These findings play a central role in the analysis of the other Sections.

In Section 4.4 the behavioral phases of both rivals are considered in a declining market. We compare both firms' profits to the classical outcomes and reflect on several exit conditions concerning the competitor with the lowest adjustment cost parameter. Section 4.5 deals with a booming market and again classical profit will serve as a point of reference during three behavioral phases. Both competitors' profits will be compared. Computer experiments illustrate the analytical results. In Section 4.6 we use integral calculus to compute the total profits of both competitors during a downward, upward and a complete business cycle. This technique enables us to derive expressions concerning the relative profits of one firm in comparison to its rival over a period with decreased or increased economic activity. These outcomes lead to considerations whether a firm - with knowledge of its rival's adjustment cost parameters - can choose its own adjustment costs such that the rival's relative profits during a complete business cycle are minimized. These considerations refine the exit criteria suggested in Section 4.4.

The analysis of Section 4.7 focuses on the implications of adjustment costs concerning social welfare. Welfare is compared to the classical outcome of a Cournot duopoly game and also total welfare is considered over (parts of) a business cycle. Section 4.8 concludes with an appraisal.

2. Classical Cournot duopoly and a business cycle

The variables under manipulation in a classical Cournot model are the quantities both firms will produce. We assume a homogeneous market, i.e. each firm offers the same good for sale on the market. Furthermore we will assume that the inverse demand curve is linear and decreasing. If X_1+X_2 equals total output of both firms, firm 1 and 2, market price is determined by

$$p(X_1,X_2) = mA - b(X_1+X_2),\ A > 0 \text{ and } b > 0 \tag{4.1}$$

In this expression the variable m equals market size and $m = 1$ will be our point of departure, whereas $m < 1$ and $m > 1$ respectively refer to a decreased and an increased market size. We also assume that in every period the whole production is sold i.e. excess demand equals precisely zero. For both firms the functional form of the production costs is chosen to take the quadratic form $C(x_i) = CX_i + DX_i^2$, with $0 < C < A$. This will allow us to study cases with linear production costs ($D = 0$), decreasing marginal costs ($D < 0$) and increasing marginal costs ($D > 0$) i.e. we study production processes (functions) with constant, increasing and decreasing returns to scale. Marginal costs will always be positive on the interval $0 \leq X_i \leq (A/b)m_{max}$ for $D > -(C/2)(b/A)/m_{max}$. Here m_{max} equals maximal market size. Under the assumption of naïve (myopic) expectations of both firms ($X_{i,t}^e = X_{i,t\text{-}1},\ i = 1{,}2$) and the restriction of nonnegative prices we obtain the following optimization problem for firms 1 and 2:

$$\begin{aligned}
&\text{Max } \Pi^1(X_{1,t}\,; X_{2,t-1}) = X_{1,t}(mA - bX_{1,t} - bX_{2,t-1}) - CX_{1,t} - D(X_{1,t})^2,\\
&\text{subject to } 0 \leq X_{1,t} + X_{2,t-1} \leq m(\frac{A}{b})\\
&\text{Max } \Pi^2(X_{2,t}\,; X_{1,t-1}) = X_{2,t}(mA - bX_{2,t} - bX_{1,t-1}) - CX_{2,t} - D(X_{2,t})^2,\\
&\text{subject to } 0 \leq X_{2,t} + X_{1,t-1} \leq m(\frac{A}{b})
\end{aligned} \tag{4.2}$$

We can normalize these equations by the transformation $X_{1,t} = (A/b)x_{1,t}$ and $X_{2,t} = (A/b)x_{2,t}$. In the following we will restrict our study to the transformed equations of both competitors (we use c,d for the production cost parameters instead of C,D):

$$\begin{aligned}
&\text{Max } \Pi^1(x_{1,t}\,; x_{2,t-1}) = x_{1,t}(m - x_{1,t} - x_{2,t-1}) - cx_{1,t} - d(x_{1,t})^2,\\
&\text{with respect to } x_{1,t}\text{, and subject to the restriction } 0 \leq x_{1,t} + x_{2,t-1} \leq m\\
&\text{Max } \Pi^2(x_{2,t}\,; x_{1,t-1}) = x_{2,t}(m - x_{2,t} - x_{1,t-1}) - cx_{2,t} - d(x_{2,t})^2,\\
&\text{with respect to } x_{2,t}\text{, and subject to the restriction } 0 \leq x_{2,t} + x_{1,t-1} \leq m
\end{aligned} \tag{4.3}$$

Now it holds that $c = C/A$ and $d = D/b$, so for quadratic production costs the two transformed conditions equal (i) $0 < c < 1$ and (ii) $d > (-c/2)/m_{max} > -1/2$. Solving this optimization problem for both competitors we obtain a classical decreasing Cournot reaction curve (that never meets the restriction) with slope $-1/(2+2d)$. This leads to the following set of first order, linear difference equations:

$$x_{1,t} = \frac{m-c}{2+2d} - \frac{1}{2+2d}x_{2,t-1} \text{ and } x_{2,t} = \frac{m-c}{2+2d} - \frac{1}{2+2d}x_{1,t-1} \tag{4.4}$$

The intersection point of both reaction curves is the unique and symmetrical Cournot-Nash equilibrium $x_1^* = x_2^* = (m-c)/(3+2d)$ (under the usual textbook assumption that firms possess constant unit production costs C, this equilibrium, corresponding with the non-normalized model and market size m=1, equals $x_i^* = (A-C)/3b$).The sufficient conditions for the asymptotic stability of this equilibrium can be found in Fudenberg and Tirole (1991) or Devaney (1989) and are satisfied for d > -1/2, because the eigenvalues of this set of difference equations equal $\lambda_{1,2} = \pm\ 1/(2+2d)$. The basin of attraction of this stable Cournot-Nash equilibrium equals the whole set of feasible (i.e. nonnegative price p) initial values ($x_{1,0}$, $x_{2,0}$). Note that the eigenvalues inform us about the speed at which the equilibrium is approached; the smaller the absolute values of these eigenvalues are, the larger the speed will be at which the equilibrium will be approached. In this sense stability will be "weakened" if d decreases, for instance for production costs with decreasing marginal costs (i.e. concave cost functions, production processes with increasing returns to scale).

If we were to consider a three player model where each firm reacts on the aggregate production of the other competitors, the eigenvalues would be $\lambda_{1,2}$ = $1/(2+2d)$ and λ_3 = $-1/(1+d)$, implying that stability conditions no longer hold for $d \leq 0$. For linear cost functions Theocharis (1960) already investigated the classical case with 3 or more players, under the assumption of naïve expectations, and found cyclical solutions for 3 players and explosive solutions for more than 3 players. In case of instability, solutions are limited by the nonnegative price restriction. Concerning our duopoly model profits corresponding with the Nash equilibrium are equal for both firms:

$$\Pi_{cl}^{\ 1} = \Pi_{cl}^{\ 2} = (m-c)^2 \frac{(1+d)}{(3+2d)^2} \tag{4.5}$$

During a downward business cycle i.e. during a depression (m decreasing from m=1) both rivals will equally decrease their production levels and of course their profits will also decrease equally. Using calculus reveals that profits are decreasing functions with respect to the increasing parameters d, for d > -1/2, and c respectively. The more negative the parameter d is, the more concave the production cost function is; with constant c this parameter d reflects scale advantages of the production process. As one might expect the more efficient the production technologies, which these firms control are, the more profitable this is for both firms even in periods of depression. The expression for the profits also holds for m > 1, i.e. during an upswing of economic activity, and if we were to display the profit during a whole business cycle we would obtain a fluctuating pattern which keeps pace with the (graph of the) business cycle, but which is not symmetrical with respect to m=1 (the expression of $\Pi_{cl}^{\,i}$ contains the quadratic term $(m\text{-}c)^2$).

The consequences for the welfare W concerning two firms with adjustment costs will be the main issue of Section 4.7 so we now give the expression for the classical case. In the normalized model the consumer surplus CS equals $CS = \frac{1}{2}(q^*)^2$ with $q^* \equiv x_1^* + x_2^*$, the market supply of both competitors. Substituting firms' production levels corresponding with the Cournot-Nash equilibrium we obtain

$$CS_{cl} = \frac{2(m-c)^2}{(3+2d)^2} \tag{4.6}$$

Social welfare is defined as the sum of producer surplus and consumer surplus. Substitution of the expressions of firms' profits and consumer surplus leads to the following expression for social welfare pertaining to the classical case:

$$W_{cl} = (m-c)^2 \frac{(4+2d)}{(3+2d)^2} \tag{4.7}$$

Because W_{cl} increases as d decreases (or c decreases) we obtain the natural result that the more efficient a production technology is, the larger welfare will be. In the usual textbook benchmark case firms have constant unit production costs corresponding to $d=0$ (for the non-normalized model and market size $m=A$ this usual assumption leads to the expression $W_{cl} = 4(A\text{-}C)^2/(9b)$ for the welfare).

3. The model with asymmetrical linear adjustment costs

We now introduce linear adjustment costs pertaining to firm 1 around the production level $x_1 = x_1^*$ (for instance, x_1^* can be equal to the "classical" Cournot-Nash equilibrium $(1\text{-}c)/(3+2d)$ corresponding to market size $m=1$). We will distinguish adjustment costs equal to l_1 per unit of production decrease and equal to u_1 per unit of production increase where in general l_1 and u_1 are allowed to be different. Using normalized variables similar to Section 4.2, concerning firm 1, this model leads to the following optimization problem with restrictions:

$$\text{Max } \Pi^1(x_{1,t}\,;x_{2,t-1}), \text{with respect to } x_{1,t} \text{ and subject to } 0 \le x_{1,t} + x_{2,t-1} \le m, \text{ with}$$

$$\begin{aligned} \Pi^1 &= x_{1,t}(m - x_{1,t} - x_{2,t-1}) - cx_{1,t} - d(x_{1,t})^2 - l_1(x_1^* - x_{1,t}), \text{ for } x_{1,t} \le x_1^* \\ \Pi^1 &= x_{1,t}(m - x_{1,t} - x_{2,t-1}) - cx_{1,t} - d(x_{1,t})^2 - u_1(x_{1,t} - x_1^*), \text{ for } x_{1,t} > x_1^* \end{aligned} \tag{4.8}$$

This optimization problem is usually solved by setting the marginal profit equal to zero (first order condition), but here we have to deal with a discontinuity of the marginal profit at $x_{1,t} = x_1^*$.

$$\begin{aligned} \frac{\partial \Pi^1}{\partial x_{1,t}} &= m - c - (2+2d)x_{1,t} - x_{2,t-1} + l_1 \text{ for } x_{1,t} \le x_1^* \\ \frac{\partial \Pi^1}{\partial x_{1,t}} &= m - c - (2+2d)x_{1,t} - x_{2,t-1} - u_1 \text{ for } x_{1,t} > x_1^* \end{aligned} \tag{4.9}$$

Marginal profit displays a "jump" the size of l_1+u_1 and the second order condition for a maximum is satisfied ($\partial^2 \Pi^1 / \partial(x_{1,t})^2 = -2-2d < 0$). The optimal profit occurs for a specific $x_{1,t}^*$ satisfying $\partial \Pi^1/\partial x_{1,t} \ge 0$ for $x_{1,t} \le x_{1,t}^*$ and $\partial \Pi^1/\partial x_{1,t} \le 0$ for $x_{1,t} > x_{1,t}^*$. Note that the value $x_{1,t}^*$ is uniquely determined, because of the ever-decreasing marginal profit as $x_{1,t}$ increases. In the following analysis we use the expression "conditional reaction curve" indicating a reaction curve with adjustment costs around x_1^* and corresponding with market size m. We will use the notation $x_{1,t} = R^1(x_{2,t-1}, m| x_1^*)$ for player 1. The following Proposition reveals that there exists an interval of values $x_{2,t-1}$ of player 2, for which player 1 always reacts with $x_{1,t} = x_1^*$. In other words, as a consequence of adjustment costs, player 1 doesn't react immediately upon a change of its rival's production level (like in the classical Cournot model) but remains inert.

Proposition 4.1 (the inertia interval).

For $\max\{0, m\text{-}c\text{-}(2+2d)\,x_1^* - u_1\} \le x_{2,t-1} \le \min\{m\text{-}\,x_1^*, m\text{-}c\text{-}(2+2d)\,x_1^* + l_1\}$ it holds that $x_{1,t} = R^1(x_{2,t-1}, m| x_1^*) = x_1^*$.

Proof

First we use the expression of the marginal profit $\partial \Pi^1 / \partial x_{1,t}$ for $x_{1,t} \leq x_1^*$. It follows that for $x_{2,t\text{-}1} \leq m\text{-}c\text{-}(2\text{+}2d)\,x_1^* + l_1$ the marginal profit $\partial \Pi^1 / \partial x_{1,t} \geq 0$ at $x_{1,t} = x_1^*$. The marginal profit increases for lower values of $x_{1,t}$ ($\partial^2 \Pi^1 / \partial (x_{1,t})^2 = -2 - 2d < 0$) so $\partial \Pi^1 / \partial x_{1,t} \geq 0$ for $x_{1,t} \leq x_1^*$.

From the expression of the marginal profit $\partial \Pi^1 / \partial x_{1,t}$ for $x_{1,t} > x_1^*$, it follows that for $x_{2,t\text{-}1} \geq m\text{-}c\text{-}(2\text{+}2d)\,x_1^*$ - u_1 the marginal profit $\partial \Pi^1 / \partial x_{1,t} \leq 0$ at $x_{1,t} = x_1^*$. Because the marginal profit decreases for higher values of $x_{1,t}$ it holds that $\partial \Pi^1 / \partial x_{1,t} \leq 0$ for $x_{1,t} > x_1^*$. For $m\text{-}c\text{-}(2\text{+}2d)\,x_1^*$ - $u_1 \leq x_{2,t\text{-}1} \leq m\text{-}c\text{-}(2\text{+}2d)\,x_1^* + l_1$ the change of the sign of the marginal profit (from positive to negative) always takes place at $x_{1,t} = x_1^*$ and player 1 reaches the optimum location of his profit. The expressions "*max*" and "*min*" in the interval of inertia clearly follow from the restrictions $x_{2,t\text{-}1} \geq 0$ and $x_{1,t} + x_{2,t\text{-}1} \leq m$.

[End of proof]

Proposition 4.1 tells us that the lenght of the inertia interval equals l_1+u_1, the sum of the two adjustment costs per unit of decrease and increase of production respectively (of course under the condition that this interval does not meet the nonnegativity restrictions of production and price). Furthermore a point on the graph of this conditional reaction curve in the "midst of" the inertia interval is ($m\text{-}c$ -$(2\text{+}2d)\,x_1^*$, x_1^*) This point lies also on the graph of the nonconditional curve (without adjustment costs). The following Proposition reveals the functional form of firm 1's reaction curve concerning values of $x_{2,t\text{-}1}$ out of the inertia interval.

Proposition 4.2 (shifts of reaction curves, due to adjustment costs).

For $x_{2,t\text{-}1} < m\text{-}c\text{-}(2\text{+}2d)\,x_1^*$ - u_1 the functional form of the conditional reaction curve can be obtained from the functional form of the nonconditional curve (no adjustment costs) by

$$x_{1,t} = R^1(x_{2,t-1}, m | x_1^*) = R^1(x_{2,t-1} + u_1, m) = \frac{(m-c)}{(2+2d)} - \frac{(x_{2,t-1} + u_1)}{(2+2d)} \tag{4.10}$$

And for $x_{2,t\text{-}1} > m\text{-}c\text{-}(2\text{+}2d)\,x_1^* + l_1$ the following holds

$$x_{1,t} = R^1(x_{2,t-1}, m | x_1^*) = R^1(x_{2,t-1} - l_1, m) = \frac{(m-c)}{(2+2d)} - \frac{(x_{2,t-1} - l_1)}{(2+2d)} \tag{4.11}$$

(Note that the two parts of the graphs of the conditional reaction curve for $x_{2,t\text{-}1} < m\text{-}c\text{-}(2\text{+}2d)\,x_1^*$ - u_1 and for $x_{2,t\text{-}1} > m\text{-}c\text{-}(2\text{+}2d)\,x_1^* + l_1$ originate in the nonconditional curve by translation to the left and the right respectively.)

Proof

For $x_{2,t\text{-}1} < m\text{-}c\text{-}(2\text{+}2d)\,x_1^*$ - u_1 the marginal profit $\partial \Pi^1 / \partial x_{1,t} > 0$ at $x_{1,t} = x_1^*$. The marginal profit is decreasing with respect to the variable $x_{1,t}$ and will be equal to zero for the unique value of $x_{1,t} > x_1^*$ satisfying $m\text{-}c\text{-}(2\text{+}2d)x_{1,t} \text{-}x_{2,t\text{-}1}\text{-}u_1 = 0$.

Slightly rewriting this expression gives $m\text{-}c\text{-}(2\text{+}2d)x_{1,t}$ -$(x_{2,t\text{-}1}+u_1) = 0$.

Concerning the nonconditional reaction curve the variable $x_{1,t}$ equals the solution of m-c-$(2+2d)x_{1,t}$ -$x_{2,t-1} = 0$. This proves part one of the proposition.
For $x_{2,t-1} > m$-c-$(2+2d)\,x_1^* + l_1$ the marginal profit $\partial \Pi^1 / \partial x_{1,t} < 0$ at $x_{1,t} = x_1^*$. Now the unique value of $x_{1,t} < x_1^*$ is obtained from m-c-$(2+2d)x_{1,t}$ -$(x_{2,t-1}$-$l_1) = 0$.

[End of proof]

The form of the conditional reaction curve $R^1(x_{2,t-1},\ m|\,x_1^*)$ of firm 1 is fully determined by Propositions 4.1 and 4.2. Note that these Propositions also hold for arbitrary (non-linear) production cost functions $c(x_1)$ and that the linearity of $x_{2,t-1}$ in the formulas of the marginal profit leads to this rather simple expression for the inertia interval. An example of the graph of a conditional reaction curve together with the graph of the nonconditional curve serves as a visual illustration.

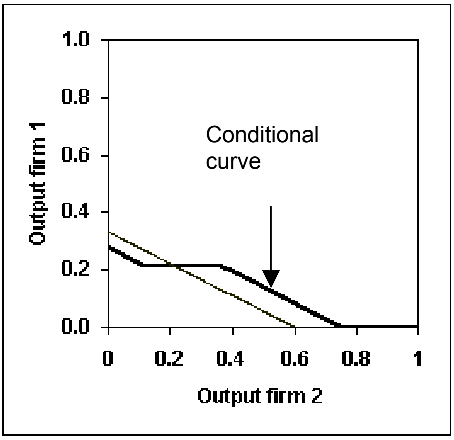


Fig. 4.1 Comparison of conditional and nonconditional reaction curves

The graph of the conditional reaction function is displayed in bold here. Here the production cost function equals $c(x_1) = 0.4x_1$-$0.1(x_1)^2$, $x_1^* = 0.2143$ (Cournot-Nash equilibrium for two identical players), and the adjustment costs around x_1^* per unit are l_1=0.15 (for $x_{1,t} < x_1^*$) and u_1=0.10 (for $x_{1,t} > x_1^*$). The inertia interval is determined by 0.2143-$0.10 \leq x_{2,t-1} \leq 0.2143+0.15$. The intersection point of the two curves equals the Cournot-Nash equilibrium ($(1$-$c)/(3+2d)$, $(1$-$c)/(3+2d)$) = (0.2143 , 0.2143).
In Sections 4.4 and 4.5 we will consider the behaviour of two competitors subject to environmental turbulence, namely fluctuations in the market size m. Therefore we have to consider the functional form of the conditional reaction function subject to such downswings and upswings of economic activity. The following Proposition shows that if the market size decreases (increases) with an amount Δm (starting from size m) the graph of the conditional reaction function of firm 1 shifts to the left (right) over a distance Δm.

Proposition 4.3 (shifts of reaction curves due to fluctuations in market size).
Using the notation $R^1(x_{2,t\text{-}1},\ m{+}\Delta m|\, x_1^*)$ for the conditional reaction function of firm 1 corresponding with market size $m{+}\Delta m$ it holds that

$$R^1(x_{2,t\text{-}1},\ m{+}\Delta m|\, x_1^*) = R^1(x_{2,t\text{-}1}\text{-}\Delta m,\ m|\, x_1^*) \tag{4.12}$$

Proof

This property can be easily understood by consideration of the manner the market size m appears in the formulas of Proposition 4.1 and 4.2.
For $x_{2,t\text{-}1} < m{+}\Delta m\text{-}c\text{-}(2{+}2d)\,x_1^* \text{ - } u_1$ and $x_{2,t\text{-}1} > m{+}\Delta m\text{-}c\text{-}(2{+}2d)\,x_1^* {+}l_1$ the functional form of the conditional reaction curve $R^1(x_{2,t\text{-}1},\ m{+}\Delta m|\, x_1^*)$ can be obtained by solving respectively the equations (see Proposition 4.2):

$$m{+}\Delta m\text{-}c\text{-}(2{+}2d)x_{1,t}\ \text{-}x_{2,t\text{-}1}\text{-}u_1 = 0 \text{ and } m{+}\Delta m\text{-}c\text{-}(2{+}2d)x_{1,t}\ \text{-}x_{2,t\text{-}1}{+}l_1 = 0$$

Rearranging the terms in both equations leads to

$$m\text{-}c\text{-}(2{+}2d)x_{1,t}\ \text{-}(x_{2,t\text{-}1}\text{-}\Delta m)\text{-}u_1 = 0 \text{ and } m\text{-}c\text{-}(2{+}2d)x_{1,t}\ \text{-}(x_{2,t\text{-}1}\text{-}\Delta m){+}l_1 = 0$$

In these equations $x_{1,t}$ simply equals $R^1(x_{2,t\text{-}1}\text{-}\Delta m,\ m|\, x_1^*)$.
Using Proposition 4.1 for market size $m{+}\Delta m$ it is clear that the inertia interval also shifts.

[End of proof]

Note that the fact that the expressions for the marginal profit depend linearly on the variable $x_{2,t\text{-}1}$ enables us to rewrite these expressions effectively. We again consider firm 1 with a production cost function equal to $c(x_1) = 0.4x_1\text{-}0.1(x_1)^2$, $x_1^* = 0.2143$, and adjustment costs around x_1^* per unit equal to $l_1{=}0.15$ (for $x_{1,t} < x_1^*$) and $u_1{=}0.10$ (for $x_{1,t} > x_1^*$). The following graphical presentation shows the shifting to the right of the conditional reaction curve of this firm subject to an increasing market size (departing from a full market size $m{=}1$, next $m{=}1.1$, and finally $m{=}1.2$).

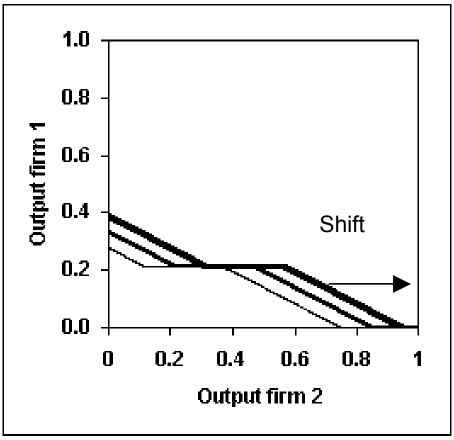


Fig. 4.2 The conditional reaction curve subject to an increasing market size

4. Phases of behaviour in a declining market

We have to be precise about what we mean by a declining market. During this following analysis our point of departure is a market size m=1 and we assume that this market size decreases till an all time low of the whole business cycle. In general four periods can be distinguished as parts of the business cycle. We can distinguish a period of expansion or upswing during which the economy recovers from the downswing (depression). The upper turning point, the crisis, is the (small) period in the economic development between (relative) growth and (relative) decline.The revival (period) corresponds to the lower turning point. So if the market size decreases, starting from m=1, the forthcoming period corresponds with the second half of the downswing moving to the lower turning point.

We now introduce a second competitor, firm 2, with the same (quadratic) production cost function $C(x_{2,t}) = c.x_{2,t}+d.(x_{2,t})^2$ implying that in this Chapter both firms possess an equally efficient production technology (in Chapter 3 we examined the case with cost asymmetry as well). However both firms can have different (linear) adjustment costs per unit production change around the Cournot-Nash equilibrium for m=1. Concerning firm 2 the optimization problem can be formulated as:

$$\text{Max } \Pi^2(x_{2,t}\,; x_{1,t-1}), \text{ with respect to } x_{2,t} \text{ and subject to } 0 \le x_{2,t} + x_{1,t-1} \le m,$$

$$\text{with } x_2^* = \frac{(1-c)}{(3+2d)}. \tag{4.13}$$

$$\Pi^2 = x_{2,t}(m - x_{2,t} - x_{1,t-1}) - cx_{2,t} - d(x_{2,t})^2 - l_2(x_2^* - x_{2,t}), \text{ for } x_{2,t} \le x_2^*$$

$$\Pi^2 = x_{2,t}(m - x_{2,t} - x_{1,t-1}) - cx_{2,t} - d(x_{2,t})^2 - u_2(x_{2,t} - x_2^*), \text{ for } x_{2,t} > x_2^*$$

Adjustment cost parameters u_1 (competitor 1) and u_2 are not of importance in this section because we analyse the behavioural consequences of a declining market. We assume that $l_1 \geq l_2$ i.e. the adjustment costs of firm 1 per unit production in a declining market (m<1) exceed those of its rival. As an example we choose the values l_1=0.15, l_2=0.05 and u_1=0.10, u_2=0.05 of the adjustment cost parameters around $x_1^* = x_2^*$=(1-c)/(3+2d). If the production cost function of both rivals equals $C(x_i) = 0.4x_i - 0.1(x_i)^2$, i=1,2 , then the Cournot-Nash equilibrium of both firms equals $x_1^* = x_2^* = 0.2143$. For m=1 the resulting graphs of the two reaction curves in Figure 4.3 clarify both firms' initial situation. The curve of firm 1 is printed bold. Note that these reaction curves still intersect at the (symmetric) Cournot-Nash equilibrium $x_1^* = x_2^* = 0.2143$. Due to both inertia intervals around (x_1^*, x_2^*) the two derivatives dx_1/dx_2 and dx_2/dx_1 are zero and the eigenvalues at this equilibrium equal $\lambda_{1,2} = 0$. The sufficient conditions for (global) asymptotic stability of this equilibrium are clearly satisfied and we could call this equilibrium "superstable" (from each initial situation the equilibrium is reached in at most 5 steps).

Stability of the equilibrium is crucial in the discussion of this section; because the Cournot-Nash equilibrium is reached very quickly we can compare supplies and profits of both firms with an easy mind. Comparative statics only makes sense in case of a stable equilibrium, otherwise we would have to study the consequence of the time path as well.

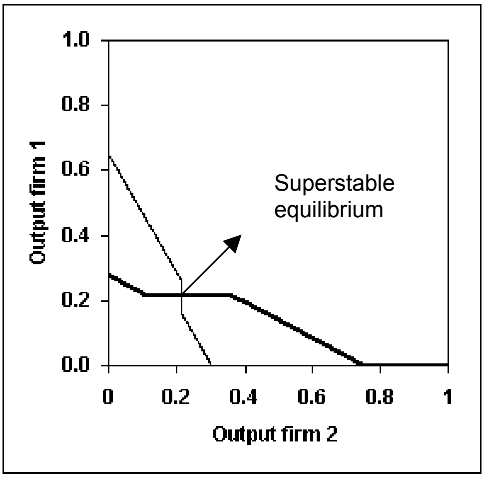


Fig. 4.3 The reaction curves of both firms at the start of phase 1 (m=1)

Due to a continuously decreasing market size (Proposition 4.3), the two reaction curves are shifted more and more (inwards) and three phases, each phase with its own characteristical behaviour of both firms, will occur. The following analysis focusses on the ("superstable" and stable) equilibria and their consequences for the profits of both rivals and not on the timepaths of the dynamical system (we leave out the t and t-1). During the analysis we will also apply general results on two benchmark cases. The first benchmark case is characterized by equal adjustment cost parameters $l_1=l_2$, i.e. we deal with completely symmetrical players and the second benchmark case corresponds with the choice $l_1=l > 0$ and $l_2=0$, i.e. firm 2 has no adjustment costs.

Phase 1: The "complete inertia" phase.

Using the "shifting" property we may conclude that both rivals maintain their original production levels for all market sizes m with $1 - l_2 \leq m \leq 1$, in spite of a declining market. Note that the market size for which phase 1 ends is fully determined by the firm with the smallest adjustment costs. As a consequence phase 1 doesn't exist if one of the competitors has no adjustment costs (benchmark case 2). In general it holds that during phase 1 the two firms meet no adjustment costs and their output levels both equal the original Cournot-Nash equilibrium for $m = 1$. If Δm_1 equals the decrease of the market size departing from $m = 1$, the end of the phase of complete inertia occurs at $\Delta m_1 = l_2$ corresponding with a market size of 1-l_2. Before we analyse both rivals' profits in detail some brief qualitative comments can be made. The classical production level would be $x^*_{cl} = (1-\Delta m_1-c)/(3+2d)$. However, due to adjustment costs, both rivals' productions are kept on a higher level. This upholding of original supply causes a lower price in comparison to the classical outcome and ,thus, consumers benefit from adjustment costs (the consumer surplus exceeds the surplus of the classical case; this is one of the subjects of Section 4.7). Also employees benefit from high commitment human resource practices reflected in beneficial labour contracts and permanent appointments. As we already mentioned in the introduction, adjustment costs are directly related to the costs of changing production levels and costs of changing labour force.

To support the line of thought the situation concerning both firms at the end of phase 1 (here $m = 0.95$) is displayed in Figure 4.4. As usual firm 1's reaction curve is printed in bold.

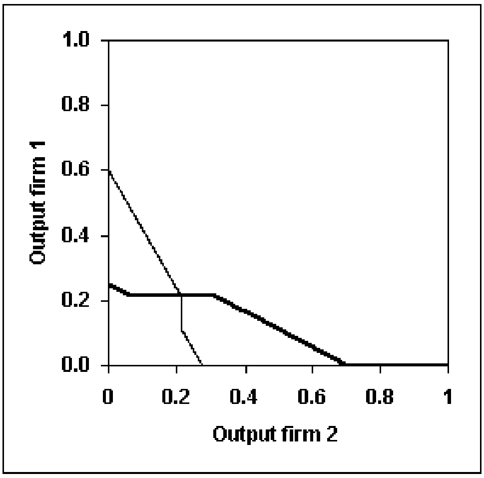


Fig. 4.4 The end of phase 1, m=1-l_2

Both firms' profits are equal during this phase and, using $m = 1 - \Delta m_1$, we obtain $\Pi^1 = x_1^*(1-\Delta m_1 - x_1^* - x_2^*) - cx_1^* - d(x_1^*)^2 = x_1^*(1 - x_1^* - x_2^*) - cx_1^* - d(x_1^*)^2 - \Delta m_1 x_1^*$. Note that the first part of this expression equals the profit corresponding to m=1. We summarize the results.

$$\begin{aligned}
&\text{During phase 1: } 0 \le \Delta m_1 \le l_2, m = 1 - \Delta m_1 \\
&\text{Production levels } x_1^* = x_2^* = \frac{(1-c)}{(3+2d)} \\
&\text{Profits } \Pi^1 = \Pi^2 = (1-c)^2 \frac{(1+d)}{(3+2d)^2} - \Delta m_1 \frac{(1-c)}{(3+2d)} \\
&\text{End of phase 1.} \\
&\text{Production levels } x_1^* = x_2^* = \frac{(1-c)}{(3+2d)} \\
&\text{Profits } \Pi^1 = \Pi^2 = (1-c)^2 \frac{(1+d)}{(3+2d)^2} - l_2 \frac{(1-c)}{(3+2d)}
\end{aligned} \tag{4.14}$$

During the "complete inertia" phase 1 the (linearly) decreasing profits of both firms are independent of l_1. In Section 4.2 we computed both competitors' profits concerning the classical situation without adjustment costs (eq. 4.5). It can be proved easily (by rewriting Π_{cl} in terms of Δm_1) that during phase 1 always $\Pi_{cl} > \Pi^1$ (or Π^2) and the difference is increasing with respect to the variable Δm_1.

$$\Pi^i - \Pi_{cl} = -\Delta m_1 \frac{(1-c)}{(3+2d)^2} - (\Delta m_1)^2 \frac{(1+d)}{(3+2d)^2} < 0, \text{for } i = 1,2 \tag{4.15}$$

Phase 2: The "inertia outperforms flexibility" phase

Let Δm_2 be equal to the further decrease of the market size, starting at the end of phase 1. During this phase market size equals $m = 1\text{-}l_2\text{-}\Delta m_2$. Firm 1 maintains its production level, whereas competitor 2 reduces its supply. The market size at which this phase ends can be computed by using the following graph:

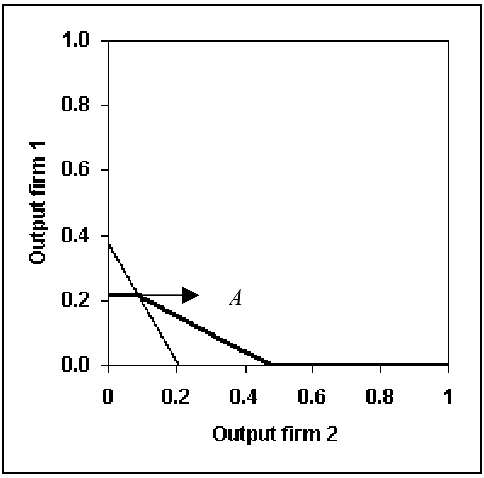


Fig. 4.5 The position of both reaction curves at the end of phase 2

We observe that, at the end of phase 2, the relevant part of the conditional reaction curve of player 2 just intersects the (shifted) inertia interval in point A of the conditional reaction curve of player 1. Concerning the point A on the inertia interval, it holds that $x_1 = x_1^*$ whereas the other co-ordinate equals $x_2 = x_2^* + l_1 - l_2 - \Delta m_2$ (at the start of phase 1 this co-ordinate equalled $x_2^* + l_1$, and market size has now decreased by $l_2 + \Delta m_2$). Using Propositions 4.2 and 4.3, the relevant part of the curve of firm 2 satisfies the equation (leaving out t and t-1) $1-\Delta m_2-c-(2+2d)x_2-x_1=0$.
Substituting the co-ordinates of point A in this equation, using $x_1^* = x_2^* = (1-c)/(3+2d)$ leads to the following property

Phase 2 ends for $\Delta m_2 = (l_1 - l_2)\dfrac{(2+2d)}{(1+2d)}$ so for $m = 1 - l_2 - (l_1 - l_2)\dfrac{(2+2d)}{(1+2d)}$ (4.16)

The expression for Δm_2 contains the difference $l_1 - l_2$ between both rivals' adjustment costs, so obviously by enlarging its adjustment costs l_1 firm 1 is able to enlarge the duration of this specific "inertia outperforms flexibility" phase.

We will now compute both firms' profits during this phase in such a way that the profits at the end of the previous phase will appear naturally in the formulas. Using the notation $x_1^{'}$ and $x_2^{'}$ for the equilibrium outputs of firms 1 and 2 respectively we have $x_1^{'} = x_1^*$ (= $(1-c)/(3+2d)$) and the supply $x_2^{'}$ is obtained by substituting $x_1^{'}$ in the expression of the relevant part of rival 2's conditional reaction curve.

$$1-\Delta m_2-c-(2+2d)x_2^{'}-x_1^{*}=0 \leftrightarrow x_2^{'}=x_1^{*}-\frac{1}{(2+2d)}\Delta m_2 \qquad (4.17)$$

Before putting into practice comparative statics, a brief comment on the stability of the Cournot-Nash equilibrium in this new phase is at place. Due to the inert reaction of player 1, during phase 2, the eigenvalues at the equilibrium $(x_1^{'},x_2^{'})$ still equal $\lambda_{1,2}=0$. Therefore this equilibrium maintains its "superstability", like in the "complete inertia" phase. Having expressed equilibrium supplies during this second phase in the equilibrium outputs of both competitors at the end of phase 1, we now substitute these quantities in the expressions for firm's profits. Note that the market size now equals $m=1-l_2-\Delta m_2$ and that we have to take into account adjustment costs in the expression of Π^2.

$$\Pi^1=x_1^{*}(1-l_2-\Delta m_2-x_1^{*}-x_2^{*}+\frac{\Delta m_2}{(2+2d)})-cx_1^{*}-d(x_1^{*})^2$$

$$\Pi^2=(x_2^{*}-\frac{\Delta m_2}{(2+2d)})(1-l_2-\Delta m_2-x_1^{*}-x_2^{*}+\frac{\Delta m_2}{(2+2d)}) \qquad (4.18)$$

$$-c(x_2^{*}-\frac{\Delta m_2}{(2+2d)})-d(x_2^{*}-\frac{\Delta m_2}{(2+2d)})^2-l_2\frac{\Delta m_2}{(2+2d)}$$

Evaluating these two expressions we summarize in (4.19) the results for the supplies and profits of both players during phase 2 and at the end of phase 2:

During phase 2, $0<\Delta m_2\le(l_1-l_2)\frac{(2+2d)}{(1+2d)}$, $m=1-l_2-\Delta m_2$

Production levels: $x_1^{'}=\frac{(1-c)}{(3+2d)}$, $x_2^{'}=\frac{(1-c)}{(3+2d)}-\Delta m_2\frac{1}{(2+2d)}$

Profits:

$$\Pi^1=(1-c)^2\frac{(1+d)}{(3+2d)^2}-l_2\frac{(1-c)}{(3+2d)}-\Delta m_2\frac{(1-c)(1+2d)}{(2+2d)(3+2d)}$$

$$\Pi^2=(1-c)^2\frac{(1+d)}{(3+2d)^2}-l_2\frac{(1-c)}{(3+2d)}-\Delta m_2\frac{(1-c)}{(3+2d)}+(\Delta m_2)^2\frac{1}{4(1+d)}$$

End of phase 2.

Production levels: $x_1^{'}=\frac{(1-c)}{(3+2d)}$, $x_2^{'}=\frac{(1-c)}{(3+2d)}-(l_1-l_2)\frac{1}{(1+2d)}$

Profits:

$$\Pi^1=(1-c)^2\frac{(1+d)}{(3+2d)^2}-l_1\frac{(1-c)}{(3+2d)}$$

$$\Pi^2=(1-c)^2\frac{(1+d)}{(3+2d)^2}-l_1\frac{(1-c)}{(3+2d)}-(l_1-l_2)\frac{(1-c)}{(1+2d)(3+2d)}+(l_1-l_2)^2\frac{(1+d)}{(1+2d)^2}$$

(4.19)

Of course for linear production costs, i.e. $d=0$, these expressions can be simplified a lot, but by allowing parameter d to be nonzero we can also study firms with decreasing or increasing marginal production costs.

First we return to the two benchmark cases. In case 1 ($l_1=l_2$) phase 2 doesn't occur and for benchmark case 2 ($l_1=l$, $l_2=0$) we can prove that the properties $\Pi^1 > \Pi_{cl}$ and $\Pi^2 < \Pi_{cl}$ hold in general during phase 2 (for case 2 phase 1 doesn't occur and the market size m equals $m = 1 - \Delta m_2$). The following proposition also contains a general result for firm 2 concerning the case $l_1>l_2$.

Proposition 4.4 (profits of both firms during phase 2).

Consider benchmark case 2, i.e. $l_1 = l, l_2 = 0$.

For firm 1 it holds that $\Pi^1 > \Pi_{cl}$ for $\Delta m_2 < \dfrac{(1-c)}{2(1+d)^2}$ during phase 2.

This result holds during the whole phase if $l < \dfrac{(1\text{-}c)(1+2d)}{4(1+d)^3}$.

For firm 2 it holds that $\Pi^2 < \Pi_{cl}$ for $\Delta m_2 < \dfrac{4(1-c)(1+d)}{(5+4d)}$ during phase 2.

This result holds during the whole phase if $l < \dfrac{2(1\text{-}c)(1+2d)}{(5+4d)}$.

Consider the case with $l_1 > l_2 > 0$.

Then it holds that $\Pi^2 < \Pi_{cl}$ for $\Delta m_2 < \dfrac{4(1-c)(1+d)}{(5+4d)} + l_2 \dfrac{8(1+d)^2}{(5+4d)}$ during phase 2.

This result holds during the whole phase if $l_1 - l_2 < \dfrac{2(1\text{-}c)(1+2d)}{(5+4d)} + l_2 \dfrac{4(1+d)(1+2d)}{(5+4d)}$.

Proof

First we consider benchmark case 2. Substituting l_2=0 in the expressions for the profits of the firms during phase 2 and rewriting $\Pi_{cl} = (1\text{-}\Delta m_2\text{-}c)^2(1+d)/(3+2d)^2$ we obtain

$$\begin{aligned}
\Pi^1 - \Pi_{cl} &= \Delta m_2 \frac{(1-c)}{(3+2d)^2(2+2d)} - (\Delta m_2)^2 \frac{(1+d)}{(3+2d)^2} \\
\Pi^2 - \Pi_{cl} &= -\Delta m_2 \frac{(1-c)}{(3+2d)^2} + (\Delta m_2)^2 \frac{(5+4d)}{4(1+d)(3+2d)^2}
\end{aligned} \tag{4.20}$$

The specific limiting values concerning the variable Δm_2 for which $\Pi^1>\Pi_{cl}$ and $\Pi^2<\Pi_{cl}$ hold can be derived easily. Imposing the condition that these specific values exceed the lenght of phase 2 (= $l(2+2d)/(1+2d)$) leads to the conditions for the adjustment cost parameter l (of firm 1).

For the case with $l_1 > l_2 >0$ we obtain (now $\Pi_{cl} = (1\text{-}l_2\text{-}\Delta m_2\text{-}c)^2(1+d)/(3+2d)^2$) for firm 2

$$\begin{aligned}
\Pi^2 - \Pi_{cl} &= -l_2 \frac{(1-c)}{(3+2d)^2} - (l_2)^2 \frac{(1+d)}{(3+2d)^2} - \Delta m_2 \frac{(1-c)}{(3+2d)^2} - \Delta m_2 l_2 \frac{2(1+d)}{(3+2d)^2} + \\
&+ (\Delta m_2)^2 \frac{(5+4d)}{4(1+d)(3+2d)^2} < -\Delta m_2 \frac{(1-c)}{(3+2d)^2} - \Delta m_2 l_2 \frac{2(1+d)}{(3+2d)^2} + \\
&(\Delta m_2)^2 \frac{(5+4d)}{4(1+d)(3+2d)^2} < 0 \text{ for } \Delta m_2 < \frac{4(1-c)(1+d)}{(5+4d)} + l_2 \frac{8(1+d)^2}{(5+4d)}
\end{aligned} \tag{4.21}$$

If $\Delta m_2 > (l_1\text{-}l_2)(2+2d)/(1+2d)$ then $\Pi^2 < \Pi_{cl}$ holds for the whole phase 2.

[End of proof]

Consider the case with $l_2=0$. Choosing the specific values $c=0.4$ and $d=-0.1$ for the production cost parameters leads to $\Pi^1 > \Pi_{cl}$ and $\Pi^2 < \Pi_{cl}$ for $l_1 < 0.165$ and $l_1 < 0.209$ (in 3 decimals) respectively. Proposition 4.4 reveals that the firm with adjustment costs in benchmark case 2 benefits in two ways from the asymmetry of the adjustment costs. First the profit of this firm exceeds the profit of the classical case and second, the profit of the firm with adjustment costs exceeds its rival's profit (without adjustment costs). And also in the general case with $l_1>l_2$ it holds that the profit of firm 2 is below the classical profit. We already demonstrated that firm 1 can enlarge the duration of phase 2 by enlarging its adjustment cost parameter and, as will become apparent from the analysis below (Proposition 4.5), this specific behavioral phase passes in favour of this firm, so that the adjustment cost parameter may act as a strategic instrument.

Then, of course, we assume that firm 1 actually is able to change its adjustment costs and acts rationally. However, from an Organizational Ecology point of view, apparently selection favors the competitor with the largest adjustment costs, whether this firm is aware of these benefits (and adapts rationally) or not. The results of our game theoretical model are in accordance with the conclusions of Hannan and Freeman (1984) who argue that: "…selection processes tend to favor organizations whose structures are difficult to change. That is, we claim that high levels of structural inertia in organizational populations can be explained as an outcome of an ecological-evolutionary process".

We have to realize that in the general expressions for both profits (during phase 2) firm 1's adjustment cost parameter l_1 is absent; profits are fully determined by the market size m, parameter l_2, and the production cost parameters. The question arises in what manner firm 1 would be able to use this parameter l_1 as a strategic weapon (under the assumption of rational adaptation). First we have to contemplate the underlying assumptions of the expressions for the profits. The supply levels (and so resulting profits) of both rivals are fully dictated by the intersection point of their reaction curves. And these reaction curves are already the consequence of a strategic decision; firm 1, for instance, maximizes its profit subject to the expected output of its rival. And because l_1 is absent in the expression for Π^1 firm 1 can't maximize its profit with respect to the parameter l_1.

However the analysis of benchmark case 2 leads one to suspect that enlarging the duration of phase 2 by enlarging the parameter l_1 does make sense. Although the profit of firm 1 also decreases in the declining market, the profit of its rival decreases more quickly. This result holds in general during phase 2 if $l_1>l_2$ and follows directly from the (negative) slopes of the profits with respect to Δm_2 during phase 2. From the formulas during the phases 1 and 2 it clearly follows that the slope with respect to Δm_2 of firm 1's profit is less negative than the slope with respect to the variable Δm_1 (after all it holds that $-\left\{{}^{(1-c)}\!/\!{}_{(3+2d)}\right\} * \left\{{}^{(1+2d)}\!/\!{}_{(2+2d)}\right\} > -{}^{(1-c)}\!/\!{}_{(3+2d)}$. However the slope of firm 2's profit at the start of phase 2 and with respect to Δm_2 equals $-{}^{(1-c)}\!/\!{}_{(3+2d)}$ (and increases somewhat because the expression of the profit contains a term with $(\Delta m_2)^2$).

Therefore firm 1's profit decreases less quickly than the profit of its rival indicating that during phase 2 the difference in profits clearly increases; the graphical display of Figure 4.6 confirms this fact. By enlarging its adjustment costs firm 1 would be able to enlarge the difference in profits, but note that this strategic action is at the expense of its own profit. However, enlarging the adjustment costs can be very useful if, during this "inertia outperforms flexibility" phase, the profit of firm 2 drops below zero, while firm 1 still enjoys positive profits. If firm 2 is forced to exit, firm 1 acquires a monopoly status with the corresponding larger monopoly profit thereafter. We have to analyse this case thoroughly. Using the expressions for Π^1 and Π^2, we can compute the difference in profits between both firms, and the condition for which competitor 1 keeps an advantage in profit over its rival (possibly during the remainder of phase 2) is easily derivable. We can also compute the unique Δm_2 (corresponding with market size m=1-l_2-Δm_2) at which firm 2 faces profit equal to zero, by solving the quadratic equation $\Pi^2 = 0$.

Combining the two results leads to a condition for the difference in adjustment costs between both competitors such that, during this phase, firm 2's profit drops below zero whereas firm 1 still possesses positive profits at that specific moment.The following proposition summarizes the analytical results.

Proposition 4.5 (exit and survival conditions concerning firm 2).

Let A be defined by $A = \dfrac{(1-c)(1+2d)}{(3+2d)(1+d)}$ and L be defined by

$L = \dfrac{(1-c)(1+2d)}{(3+2d)} - \sqrt{l_2 \dfrac{(1-c)(1+2d)^2}{(3+2d)(1+d)}}$.

If $d \le 0$ it holds that $L \le A$ for all adjustment costs l_2.

And if $d > 0$ it holds that $L \le A$ for $l_2 \ge \dfrac{(1-c)d^2}{(1+d)(3+2d)}$.

For $L \le l_1 - l_2 < A,\ \Pi^2 = 0$ while $\Pi^1 > 0$. For $0 < l_1 - l_2 < L,\ \Pi^1 > \Pi^2 > 0$ during phase 2. Firm 2 survives phase 2 but has fallen behind in profit.

Proof

Solving the quadratic equation (with respect to Δm_2) Π^2=0 leads to

$$\Pi^2 = (\Delta m_2)^2 \frac{1}{4(1+d)} - \Delta m_2 \frac{(1-c)}{(3+2d)} + (1-c)^2 \frac{(1+d)}{(3+2d)^2} - l_2 \frac{(1-c)}{(3+2d)} = 0 \leftrightarrow$$

$$\Delta m_2 = \frac{2(1-c)(1+d)}{(3+2d)} - \sqrt{4l_2 \frac{(1-c)(1+d)}{(3+2d)}}$$

Firm 2 faces profit equal to zero during phase 2 if

$$(l_1 - l_2)\frac{(2+2d)}{(1+2d)} \ge \Delta m_2 = \frac{2(1-c)(1+d)}{(3+2d)} - \sqrt{4l_2 \frac{(1-c)(1+d)}{(3+2d)}} \leftrightarrow l_1 - l_2 \ge L$$

From the expressions of the profits of both firms we can compute the difference during phase 2; Solving Π^1-$\Pi^2 > 0$ we obtain a condition for Δm_2:

$$\Pi^1 - \Pi^2 = \Delta m_2 \frac{(1-c)}{(2+2d)(3+2d)} - (\Delta m_2)^2 \frac{1}{4(1+d)} > 0 \leftrightarrow \Delta m_2 < \frac{2(1-c)}{(3+2d)}$$

This advantage in profit of firm 1 over its rival holds during phase 2 if l_1-$l_2 < A$.
The conditions for $L \leq A$ are easily derivable for $d \leq 0$ and $d > 0$.

[End of proof]

Note that the condition $L \leq A$ is always satisfied for linear production costs and quadratic production costs functions with decreasing marginal costs. For positive d, i.e. a convex production cost function, the condition $L \leq A$ is already satisfied for small adjustment costs of firm 2. Proposition 4.5 is an important one, because it reveals that, if firm 1 is capable of choosing its adjustment costs in the well defined interval (L,A), the rival can be expelled from the market during phase 2. We have to emphasize that the essence of firm 1's strategic action, provided that rational adaptation is possible, is to prolong this beneficial phase by using its adjustment costs. It should be mentioned that, although this rival faces smaller profits during this phase, for nonpositive profits it is required that the decline of the market is large enough. Moreover, even if a firm faces temporary negative profits, such a firm can remain active for some time awaiting a more prosperous period (Dixit (1989)). Van Witteloostuijn (1998) notes that: "Except for Dixit, the consequence of organizational failure in Industrial Organization models is immediate exit. This result is driven by the assumption that firms are perfectly forward-looking: calculating the expected future stream of profit, taking into account of the end-game equilibrium that results from strategic competition, they decide to exit at the moment profitability falls below zero. … Dixit argues that firms take notice of the chance that demand will develop favourably in the future…The outcome of Dixit's (1989) model resembles turnaround success or flight from losses, depending on the expected and actual development of demand."

In case of a small decline of the market size firm 2 keeps positive profits, but his profit continues to decrease not only absolutely but also relatively. And in case that the difference of the adjustment cost parameters isn't large enough, $0 < \Delta l = l_1 - l_2 < L$, firm 2 may survive phase 2. This surviving is relative because firm 2 has fallen behind in profit during that period of decreased economic activity in comparison with its competitor. Therefore in Section 4.6 we develope an analytical instrument to compute the relative profit of firm 2 (in comparison with firm 1) over a whole period of depression. To illustrate Proposition 4.5 we consider the example of both rivals we used before (adjustment cost parameters l_1=0.15, l_2=0.05 around $x_1^* = x_2^*$=(1-c)/(3+2d) and production costs $C(x_i) = 0.4x_i - 0.1(x_i)^2$, i=1,2). Applying Proposition 4.5 we can conclude that, if the difference in adjustment costs $\Delta l = (l_1 - l_2)$ lies between L=0.0841 and A=0.1905, firm 2 will face negative profits while firm 1 still has positive profits (in this case even during the remainder of phase 2). Because Δl=0.10 the condition $L \leq \Delta l < A$ is clearly satisfied. The profit of firm 2 becomes zero for m=0.7607. If l_1=0.15 phase 2 ends for $\Delta m_2 = \Delta l\ (2+2d)/(1+2d) = (0.1)(1.8)/(0.8) = 0.225$ so for a market size equal to m=1-0.05-0.225=0.725.

Both firms' profits during the consecutive phases 1 and 2 are graphically displayed in Figure 4.6 (as usual the graph of firm 1 is printed in bold). Note that at a market size of 0.76 the profit of player 2 becomes negative, but at that specific market size player 2 already has fallen behind in profit for a longer period (starting at the end of phase 1, m=0.95).

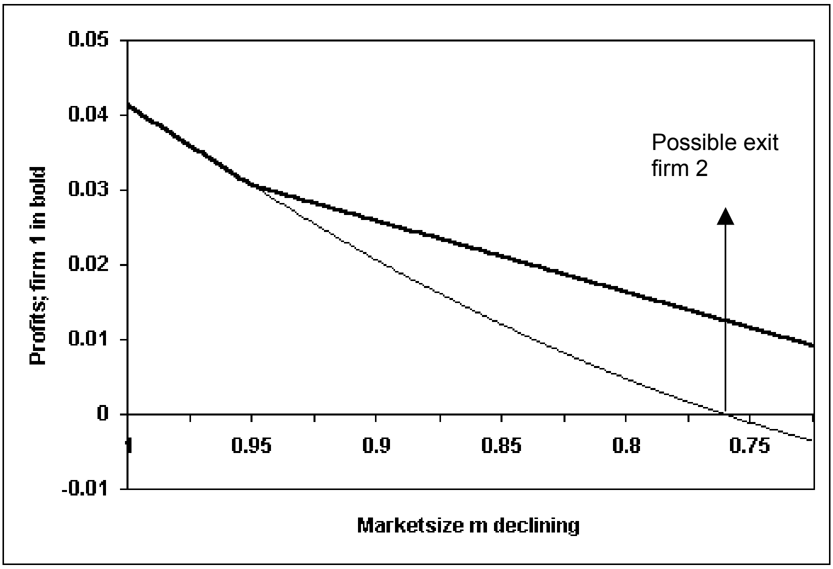


Fig. 4.6 The profits of the two firms during phases 1 and 2.

We will conclude the analysis of the "inertia outperforms flexibility" phase by displaying the consequence of Proposition 4.5 graphically. For every chosen production cost function ($C(x)=cx+dx^2$), Proposition 4.5 defines two specific regions in the Δl-l_2-plane (where $\Delta l=l_1-l_2$). Two important regions can be distinguished. The region D_2 consists of those $(\Delta l,l_2)$ combinations for which firm 2 faces nonpositive profits during phase 2 whereas firm 1 still possesses positive profits at that moment. And the "survival" region S consists of those $(\Delta l,l_2)$ combinations for which competitor 2 maintains positive profits but falls behind in profit in comparison with its rival during phase 2.

For linear production costs (c=0.4,d=0) the boundaries are printed as thin lines; region D_2 is bounded by the straight (horizontal) line and the curved line. The "survival" region S is bounded by the curved line and the nonnegative l_2-axis (and a part of the positive Δl-axis). The straight (bold) line and the curved (bold) line correspond with the boundaries of the regions D_2 and S for the concave cost function (c=0.4,d=-0.1). Note that the point $(\Delta l, l_2) = (0.10,0.05)$, corresponding with the previous example (Fig. 4.6), lies in the interior of region D_2. Computation shows that for the lineair cost case with c=0.4 the point (0.10,0.05) lies exactly on the curved thin line indicating that firm 2's profit becomes zero precisely at the end of phase 2. All benchmark cases 2 ($l_1 = l > 0$, $l_2 = 0$) correspond to points on the Δl-axis. If production cost parameters equal c=0.4 and $d = -0.1$, firm 1 has to possess adjustment costs with l at least 0.171, in order to ensure that its rival faces nonpositive profits.

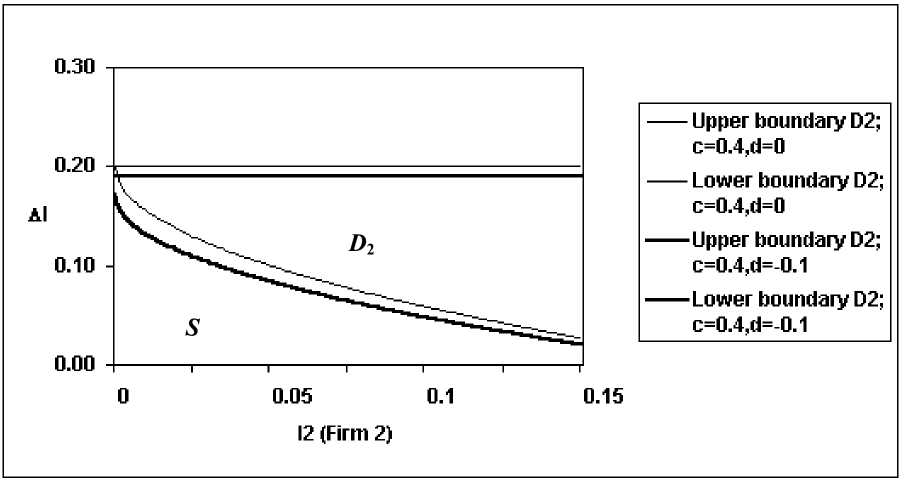


Fig. 4.7 The regions D_2 and S, for linear and concave production costs.

Phase 3: The "complete flexibility" phase.

The variable Δm_3 equals the further decrease of the market size, starting at the end of phase 2. During phase 3 the market size equals $m = 1\text{-}l_2\text{-}(l_1\text{-}l_2)(2+2d)/(1+2d)\text{-}\Delta m_3$. Both firms will decrease their production levels equally. To determine production levels and profits of both firms we will use techniques analogous to those we used for phase 2. It will turn out that, even if firm 2 survives the "inertia outperforms flexibility" phase, a condition for the difference in adjustment costs of both firms can be derived , such that during this "complete flexibility" phase firm 2 is forced to exit (because of accumulated losses).

We will now use the notations $x_1^{''}$ and $x_2^{''}$ for the production levels of firm 1 and its rival respectively. This Cournot-Nash equilibrium equals the intersection point of the relevant parts of both reaction curves. Taking into account market size the relevant parts of these curves for both competitors satisfy the equations (after rearranging terms):

$$\begin{aligned}
&1-(l_1-l_2)\frac{1}{(1+2d)}-\Delta m_3-c-(2+2d)x_1-x_2=0, \text{for player 1}\\
&1-(l_1-l_2)\frac{(2+2d)}{(1+2d)}-\Delta m_3-c-(2+2d)x_2-x_1=0, \text{for player 2}
\end{aligned} \tag{4.22}$$

Using some linear algebra the Cournot-Nash equilibrium concerning phase 3 can be obtained.

$$x_1^{''}=\frac{(1-c)}{(3+2d)}-\Delta m_3\frac{1}{(3+2d)},\quad x_2^{''}=\frac{(1-c)}{(3+2d)}-(l_1-l_2)\frac{1}{(1+2d)}-\Delta m_3\frac{1}{(3+2d)} \tag{4.23}$$

Note that the Cournot-Nash equilibrium corresponding to m=1 and also equilibrium supplies reached at the end of phase 2 appear in these expressions. The eigenvalues at this equilibrium equal $\lambda_{1,2} = \pm\, 1/(2+2d)$, like in a classical Cournot duopoly game (see Section 4.2), so stability is guaranteed during phase 3. We can interprete these production levels $x_1^{''}$ and $x_2^{''}$ as downward adjustments of the output levels $x_1^{'}$ and $x_2^{'}$ at the end of the previous phase 2. Making use of this interpretation, computation of both rivals' profits concerning this phase goes smoothly. We present a summary (4.24) of the results.

During phase 3, $\Delta m_3 \geq 0,\ m = 1 - l_2 - (l_1 - l_2)\dfrac{(2+2d)}{(1+2d)} - \Delta m_3$

Production levels:

$$x_1^{''} = \frac{(1-c)}{(3+2d)} - \Delta m_3 \frac{1}{(3+2d)},\ x_2^{''} = \frac{(1-c)}{(3+2d)} - (l_1 - l_2)\frac{1}{(1+2d)} - \Delta m_3 \frac{1}{(3+2d)}$$

Profits:

$$\Pi^1 = (1-c)^2 \frac{(1+d)}{(3+2d)^2} - l_1 \frac{(1-c)}{(3+2d)} - \Delta m_3 \frac{(1-c)(2+2d)}{(3+2d)^2} + (\Delta m_3)^2 \frac{(1+d)}{(3+2d)^2}$$

$$\Pi^2 = (1-c)^2 \frac{(1+d)}{(3+2d)^2} - l_1 \frac{(1-c)}{(3+2d)} - (l_1 - l_2)\frac{(1-c)}{(1+2d)(3+2d)} + (l_1 - l_2)^2 \frac{(1+d)}{(1+2d)^2}$$

$$- \Delta m_3 \frac{(1-c)(2+2d)}{(3+2d)^2} + \Delta m_3 (l_1 - l_2)\frac{(2+2d)}{(1+2d)(3+2d)} + (\Delta m_3)^2 \frac{(1+d)}{(3+2d)^2} \tag{4.24}$$

Naturally for $\Delta m_3 = 0$ these profits equal the profits at the end of phase 2. Furthermore note that four terms of the rather complicated expressions of both firms' profits are exactly equal. We first concentrate on the two benchmark cases and we discuss some results concerning the differences of these profits with the classical profit. For the symmetric benchmark case 1 ($l_1 = l_2 = l$) the results during phase 3 are straightforward; like in phase 1 it holds that $\Pi^i < \Pi_{cl}$, $i=1,2$. We already observed that the asymmetry of benchmark case 2 ($l_1 = l > 0$, $l_2 = 0$) leads to the asymmetric conclusion $\Pi^1 > \Pi_{cl}$, $\Pi^2 < \Pi_{cl}$, during the second phase. Naturally at the start of phase 3 these results still hold, but due to firm 1's adjustment costs (the production level of this firm now decreases too) the profit of this firm drops below classical profit in a further declining market. However, in most cases, firm 2's profit stays below the classical profit.

Proposition 4.6 (profits compared to classical Cournot-Nash profits).

Consider benchmark case 1, i.e. $l_1 = l_2 = l$.

$\Pi^i < \Pi_{cl}$ and this difference is increasing with respect to the variables Δm_3 (the decrease of the market size) and the adjustment cost parameter l, during phase 3.

Concerning the case $l_1 = l > 0,\ l_2 = 0$ the following holds:

If $\Pi^1 > \Pi_{cl}$ at the end of phase 2 the profit of this firm stays above the classical profit for $\Delta m_3 < \dfrac{(1-c)}{4(1+d)^2} - l\dfrac{(1+d)}{(1+2d)}$. If $\Pi^2 < \Pi_{cl}$ at the end of phase 2 the profit of this firm stays below the classical profit for $\Delta m_3 < (1-c) - l\dfrac{(5+4d)}{2(1+2d)}$.

Proof

Using the expressions for the profits during phase 3 for $l_1=l_2=l$ we obtain

$$\Pi^i - \Pi_{cl} = -l\,\frac{(1-c)}{(3+2d)^2} - l^2\,\frac{(1+d)}{(3+2d)^2} - (\Delta m_3)l\,\frac{2(1+d)}{(3+2d)^2} < 0, \text{ for } i = 1{,}2$$

Concerning the first benchmark case clearly this negative difference increases with respect to the variables Δm_3 and l. For the second benchmark case we again use the expressions for both profits Π^1 and Π^2 substituting $l_1=l, l_2=0$ and we rewrite classical profit as Π_{cl} = $[1-l(2+2d)/(1+2d)-\Delta m_3\ -c]^2(1+d)/(3+2d)^2$. Evaluation of these formulas leads to the required expressions for the differences Π^1-Π_{cl} and Π^2-Π_{cl} :

$$\Pi^1 - \Pi_{cl} = l\,\frac{(1-c)}{(1+2d)(3+2d)^2} - l^2\,\frac{4(1+d)^3}{(1+2d)^2(3+2d)^2} - \Delta m_3 l\,\frac{4(1+d)^2}{(1+2d)(3+2d)^2}$$

$$\Pi^2 - \Pi_{cl} = -l\,\frac{2(1-c)(1+d)}{(1+2d)(3+2d)^2} + l^2\,\frac{(1+d)(5+4d)}{(1+2d)^2(3+2d)^2} + \Delta m_3 l\,\frac{2(1+d)}{(1+2d)(3+2d)^2}$$

The proof is completed by solving the equations Π^1-$\Pi_{cl} > 0$ and Π^2-$\Pi_{cl} < 0$ with respect to the variable Δm_3.

[End of proof]

For the production cost parameters we make the usual choice c=0.4 and d=-0.1 and for the choice of the adjustment cost parameters l_1=0.1, l_2=0 the application of Proposition 4.4 clearly shows that $\Pi^1>\Pi_{cl}$ and $\Pi^2<\Pi_{cl}$ at the end of phase 2 (market size m=0.775). Now Proposition 4.6 reveals that firm 2's profit stays below the classical profit for $\Delta m_3 < 0.313$. Then the market size equals 0.463 corresponding with a deep depression. So we may conclude that competitor 2's profit even stays below the classical profit for a long period during the "complete flexibility" phase 3. We argued that firm 1's profits drop below classical profit because of its adjustment costs; the result of a computation indeed confirms that for m=0.702 the beneficial situation $\Pi^1>\Pi_{cl}$ no longer holds. At this point we have to realize that firm 1's adjustment costs not only can serve as a strategic instrument, but that there exists another incentive to enlarge adjustment costs as well. Because as a consequence of enlarging the parameter l_1 the profit of the firm stays above the classical profit during a long period of recession (at the expence of its rival). Combining the results of Propositions 4.4 and 4.6 it follows that $\Pi^1>\Pi_{cl}$ for all market sizes with m >1- $0.25(1-c)/(1+d)^2$ - $l_1(1+d)/(1+2d)$.

Using the expressions of the profits corresponding with phase 3 we now return to the analysis of the difference in profits between the two competitors.
It appears that the analysis leading to Proposition 4.5 can be expanded to phase 3.
The market size for which $\Pi^2 = 0$ can be determined solving a quadratic equation with respect to Δm_3. Combining this result with the condition $\Pi^1 > \Pi^2$ leads to the following proposition:

Proposition 4.7 (exit and survival regions concerning firm 2).

Let M be defined by $M = \frac{(1-c)(1+2d)^2}{(1+d)(3+2d)} - 2\sqrt{l_2 \frac{(1-c)(1+2d)^2}{(3+2d)(1+d)}}$.

If $d \le 0$ it holds that $M \le L$ (see Proposition 4.5) for all adjustment cost parameters l_2.

And if $d > 0$ it holds that $M \le L$ for $l_2 \ge \frac{(1-c)d^2}{(1+d)(3+2d)}$.

For $M < l_1 - l_2 < L,\ \Pi^2 = 0$ while $\Pi^1 > 0$ during phase 3.

Proof

Using the expressions for the profits of both firms the condition $\Delta\Pi = \Pi^1 - \Pi^2 > 0$ leads to

$$\Delta\Pi = (l_1 - l_2)\frac{(1-c)}{(1+2d)(3+2d)} - (l_1 - l_2)^2 \frac{(1+d)}{(1+2d)^2} - \Delta m_3 (l_1 - l_2)\frac{(2+2d)}{(1+2d)(3+2d)}$$

This expression is positive for $\Delta m_3 < (\Delta m)^A = \frac{(1-c)}{(2+2d)} - (l_1 - l_2)\frac{(3+2d)}{2(1+2d)}$

Solving the quadratic equation $\Pi^2=0$ with respect to Δm_3 gives

$$(\Delta m_3)^L = (1-c) - (l_1 - l_2)\frac{(3+2d)}{(1+2d)} - \sqrt{l_2 \frac{(1-c)(3+2d)}{(1+d)}}$$

Imposing the condition that $(\Delta m_3)^L < (\Delta m_3)^A$ completes the proof.

[End of proof]

Naturally the situation as decribed in Proposition 4.7 only occurs if the decline of the market is large enough.To illustrate Proposition 4.7 we consider the example of both rivals with the usual production cost parameters c=0.4 and d=-0.1. We now choose the adjustment cost parameters of both firms equal to l_1=0.10 and l_2=0.05 implying that the difference in adjustment costs per unit Δl equals 0.05. Note that the point $(\Delta l, l_2)$ = (0.05,0.05) no longer lies in the interior of region D_2. Proposition 4.5 ensures us that firm 2 survives phase 2 (in the sense of $\Pi^2>0$), because $\Delta l < L$=0.0841. Using c=0.4, d=-0.1 and l_2=0.05 application of Proposition 4.7 reveals that M is negative, namely M=-0.0222. This negative value of M has an important implication; it means that for $0 < \Delta l < 0.0841$ firm 2 always faces nonpositive profits during phase 3, whereas its rival still has positive profits at that specific moment.
The graph of both firms' profits, during three phases, can serve as an illustration (see Figure 4.8). In this graph kinks in the smooth curves of both profits, occurring at a market size m equal to 0.8375 correspond precisely with the start of behavioural phase 3. Note also that at that specific market size the slope of the curve of firm 1 decreases due to its adjustment costs whereas the slope of its rival increases somewhat (but stays negative).

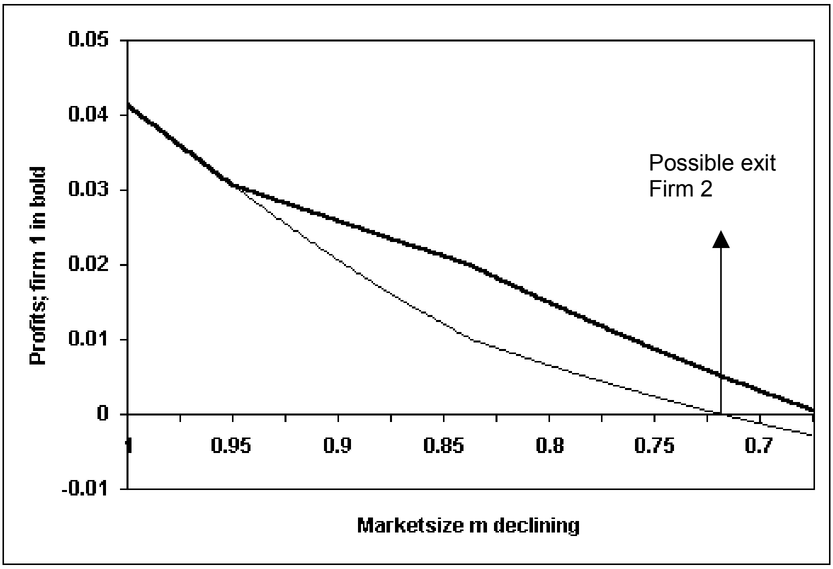


Fig. 4.8 Firm 2 faces nonpositive profits during phase 3.

Of course using the expression for $(\Delta m_3)^L$ the market size at which $\Pi^2=0$ can be computed exactly (here $m = 0.8375 - 0.1195 = 0.718$).
By Proposition 4.7 a new region D_3 can be added to the $(\Delta l, l_2)$-plane, which consists of those $(\Delta l, l_2)$ combinations for which $\Pi^2=0$ while $\Pi^1>0$, during phase 3.
Although firm 2 was able to survive phase 2 (in a way) yet its profits drop below zero during phase 3. The consequences of Propositions 4.5 and 4.7 can be displayed in one figure; choosing the concave production cost function $C(x)=0.4x-0.1x^2$ for both rivals leads to the following illustration:

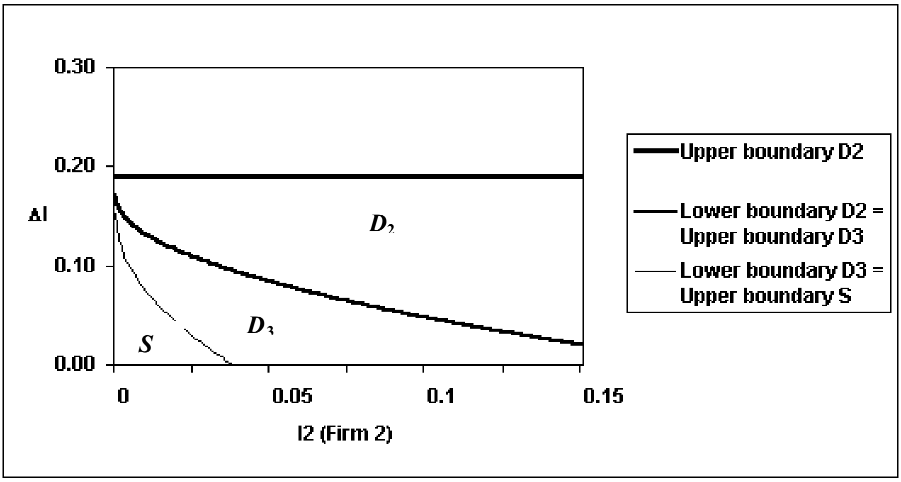


Fig. 4.9 The regions D_2 , D_3 and the "survival" region S.

As also indicated in the legenda the region D_2 is bounded by the straight line and the curve, both printed bold (and a very small part of the positive Δl-axis) whereas the region D_3 is bounded by the two curves printed bold and thin respectively, a part of the positive l_2-axis and a small part of the positive Δl-axis. The intersection point l_2^* of the "lower boundary" curve with the positive l_2-axis has an important interpretation: if firm 2's adjustment cost parameter l_2 exceeds l_2^*, then for all adjustment cost parameters l_1 of rival 1 with $l_1>l_2$ firm 2 faces nonpositive profits during phase 2 or 3. The value l_2^* can be easily solved by setting M (Proposition 4.7) equal to zero and leads to the following property.

$$\text{If } l_2 \geq \frac{(1-c)(1+2d)^2}{4(3+2d)(1+d)} \text{ and } l_1 > l_2 \text{ it holds that } \Pi^2 \leq 0,\ \Pi^1 > 0 \text{ during phase 2 or 3} \quad (4.25)$$

Applying this property to the case with production cost parameters c=0.4 and d=–0.1 we may conclude that, if firm 2's adjustment cost parameter exceeds the value 0.038, then for all $l_1>l_2$ the profit of this competitor will drop below zero, whereas firm 1 still possesses positive profits (naturally under the assumption that the decline of the market is large enough). The region S consists of those values for Δl and l_2 for which firm 1's profits drop to zero before the rival's profits fall below zero. The choice l_2=0, Δl=0.06 (so l_1=0.06) provides an example of this case. However a graphical display of both firms' profits shows clearly that, although Π^1=0 before Π^2=0, the competitor with the smallest adjustment costs has fallen behind in profit during a long period of economic recession.

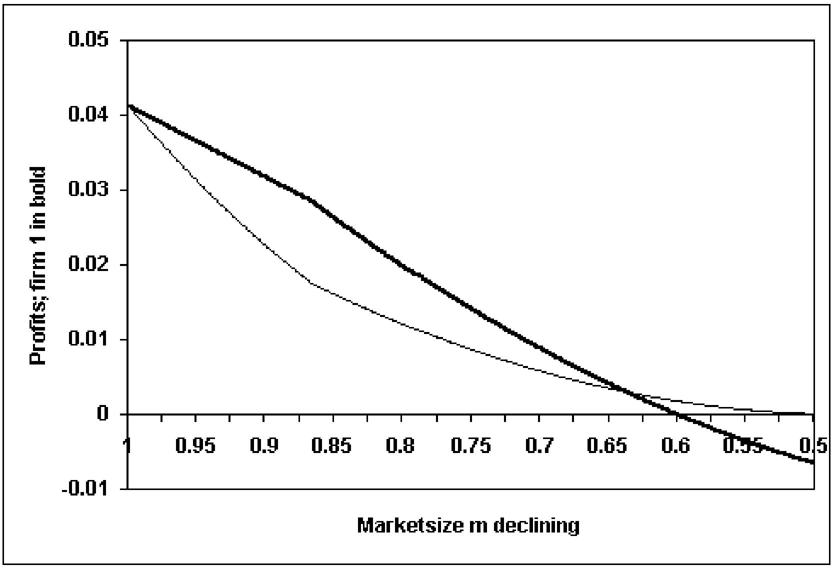


Fig. 4.10 The case "Π^1=0 before Π^2=0".

Phase 1 doesn't occur for this benchmark case and the "inertia outperforms flexibility" phase 2 ends at market size m=0.865, indicated by the kinks in both curves.

5. Phases of behaviour in an expanding market

In this section we again consider both rivals, but now subject to an expanding market. The adjustment costs (per unit production) for firms 1 and 2 for $x_i > x_i^*$ (i=1,2), where x_i^* equals the Cournot-Nash equilibrium for m=1, equal u_1 and u_2 respectively. We assume that $u_1 \geq u_2$ so firm 2 also possesses a smaller adjustment cost parameter corresponding with an increasing production. Due to a gradually increasing market size, both reaction curves are shifted (outwards) and as a consequence again three phases of production behaviour can be distinguished. Phase 1 is determined by the smallest adjustment cost parameter u_2 and holds for a market size m satisfying $1 \leq m \leq 1+u_2$ and during this specific phase, as a consequence of both inertia intervals, both firms maintain their supply levels at $x_1^* = x_2^*$=(1-c)/(3+2d). The positions of both reaction curves at the end of phase 1 and the end of phase 2 are displayed in the Figures 4.11a and 4.11b respectively. In this example production cost parameters are c=0.4, d=-0.1 for both firms and the adjustment cost parameters equal u_1=0.1 and u_2=0.05 for firm 1 and firm 2 respectively (l_1=0.15, l_2=0.05).

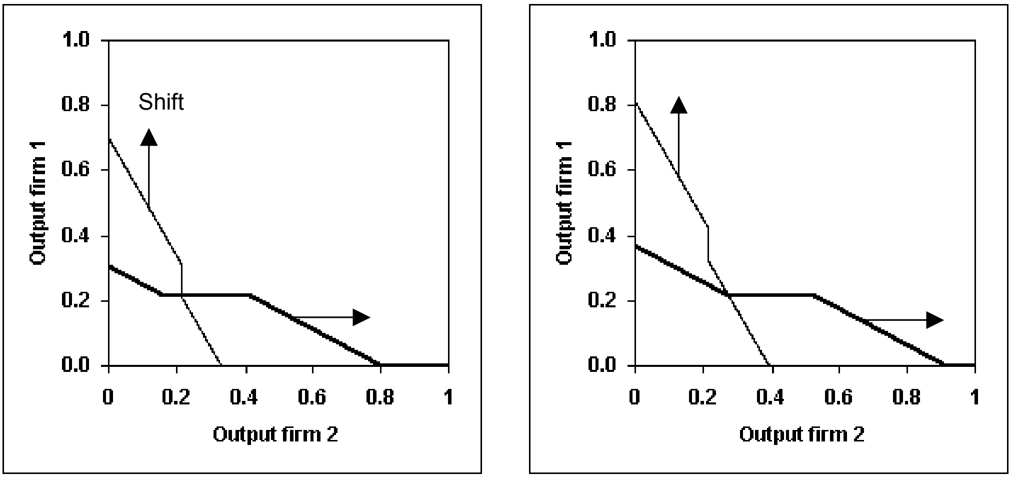


Fig. 4.11a The end of phase 1. **Fig. 4.11b The end of phase 2.**

The “complete inertia” phase 1 ends at a market size of 1.05 and during this phase both firms maintain their supply at level $x_1^* = x_2^*$= 0.214. During phase 2 the firm with the smallest adjustment costs, firm 2, loses its inertness and increases its production, whereas its rival, firm 1, maintains its supply level. Phase 2 ends at m = 1.1625. Finally during phase 3 both firms will increase their production levels equally (in comparison to their levels at the end of phase 2). During each phase production levels and profits can be determined by using similar computation techniques, so we will briefly summarize these results and lay the emphasis on interpretation. Like in Section 4.4 the difference between both competitors' profits besides the difference between the profits of each firm and classical profit (no adjustment costs at all) deserve our attention.

During the "complete inertia" phase 1 (4.26), $0 \le \Delta m_1 \le u_2,\ m = 1 + \Delta m_1$

Production levels $x_1^* = x_2^* = \dfrac{(1-c)}{(3+2d)}$

Profits $\Pi^1 = \Pi^2 = (1-c)^2 \dfrac{(1+d)}{(3+2d)^2} + \Delta m_1 \dfrac{(1-c)}{(3+2d)}$

End of phase 1. (4.26)

Production levels $x_1^* = x_2^* = \dfrac{(1-c)}{(3+2d)}$

Profits $\Pi^1 = \Pi^2 = (1-c)^2 \dfrac{(1+d)}{(3+2d)^2} + u_2 \dfrac{(1-c)}{(3+2d)}$

During phase 2 (4.27), $0 < \Delta m_2 \le (u_1 - u_2)\dfrac{(2+2d)}{(1+2d)},\ m = 1 + u_2 + \Delta m_2$

Production levels : $x_1^{'} = \dfrac{(1-c)}{(3+2d)},\ x_2^{'} = \dfrac{(1-c)}{(3+2d)} + \Delta m_2 \dfrac{1}{(2+2d)}$

Profits : (4.27)

$$\Pi^1 = (1-c)^2 \frac{(1+d)}{(3+2d)^2} + u_2 \frac{(1-c)}{(3+2d)} + \Delta m_2 \frac{(1-c)(1+2d)}{(2+2d)(3+2d)}$$

$$\Pi^2 = (1-c)^2 \frac{(1+d)}{(3+2d)^2} + u_2 \frac{(1-c)}{(3+2d)} + \Delta m_2 \frac{(1-c)}{(3+2d)} + (\Delta m_2)^2 \frac{1}{4(1+d)}$$

First we focus on both differences $\Pi^1 - \Pi_{cl}$ and $\Pi^2 - \Pi_{cl}$ during the first two behavioural phases and we analyse two benchmark cases. Like in Section 4.4 case 1 concerns symmetrical players ($u_1=u_2=u>0$) whereas case 2 deals with an asymmetrical situation ($u_1=u>0$, $u_2=0$). It appears that for symmetrical firms profits exceed classical profits during the “complete inertia” phase 1 (phase 2 doesn’t occur), whereas the difference Π^i-Π_{cl} (due to the adjustment costs) drops below zero during the “complete flexibility” phase 3. Concerning the asymmetrical benchmark case 2 phase 1 doesn’t occur and during phase 2 we obtain the interesting result that Π^1-$\Pi_{cl} < 0$ and Π^2-$\Pi_{cl} > 0$. So now, in contrast with the outcomes corresponding with a declining market (compare Proposition 4.4), asymmetry in adjustment costs benefits the competitor without adjustment costs. This result leads one to suspect that the essential strategic behaviour during a prosperous period corresponding to an increasing market size is flexibility, i.e. keeping the adjustment costs as small as possible. Proposition 4.8 summarizes the analytical results in detail.

Proposition 4.8 (profits of both firms during phases 1 and 2).

Consider benchmark case 1, i.e. $u_1 = u_2 = u$. For $u < \dfrac{1-c}{1+d}$, so in general, it holds that $\Pi^i > \Pi_{cl}$ during phase 1 and phase 2 does not occur.

Consider benchmark case 2, i.e. $u_1 = u > 0$ and $u_2 = 0$. Phase 1 does not occur and during phase 2 it always holds that $\Pi^2 > \Pi_{cl} > \Pi^1$.

Proof

Considering benchmark case 1 we use the expressions for Π^i during phase 1. Evaluating the expression for $\Pi_{cl} = (1+\Delta m_1 - c)^2(1+d)/(3+2d)^2$ leads to

$$\Pi^i - \Pi_{cl} = \Delta m_1 \frac{(1-c)}{(3+2d)^2} - (\Delta m_1)^2 \frac{(1+d)}{(3+2d)^2} > 0 \quad \text{for } \Delta m_1 < \frac{(1-c)}{(1+d)}.$$

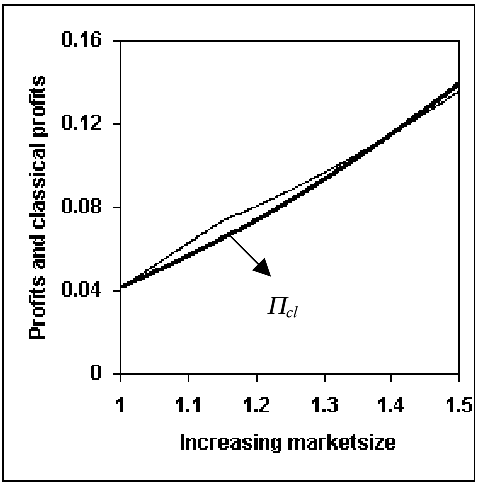
For benchmark case 2 we use the expressions for the profits during phase 2 and substitute $u_2=0$. Rewriting the classical profit as $\Pi_{cl} = (1+\Delta m_2 - c)^2(1+d)/(3+2d)^2$ gives

$$\Pi^1 - \Pi_{cl} = -\Delta m_2 \frac{(1-c)}{(2+2d)(3+2d)^2} - (\Delta m_2)^2 \frac{(1+d)}{(3+2d)^2} < 0 \text{ for all } \Delta m_2.$$

$$\Pi^2 - \Pi_{cl} = \Delta m_2 \frac{(1-c)}{(3+2d)^2} + (\Delta m_2)^2 \frac{(5+4d)}{4(1+d)(3+2d)^2} > 0 \text{ for all } \Delta m_2.$$

[End of proof]

To clarify the results of Proposition 4.8 graphically both firms' profits in relation to the classical profit, concerning benchmark cases 1 and 2, are displayed in Figures 4.12a and 4.12b respectively. For both cases we choose c=0.4, d=-0.1 as usual; the choices $u_1=u_2=u=0.15$ and $u_1=u=0.15, u_2=0$ correspond with the respective benchmark cases 1 and 2. The graph of the classical profit is printed in bold.



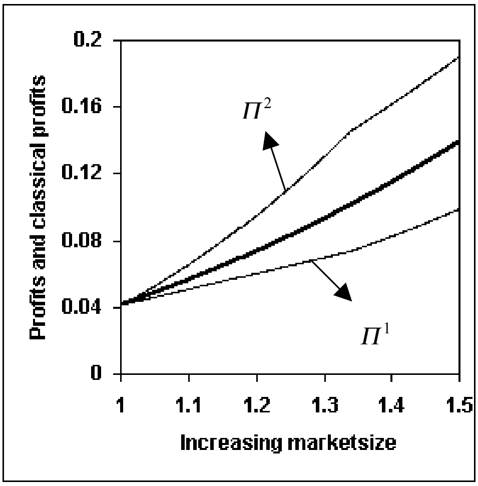


Fig. 4.12a Π^i and Π_{cl} for case 1. **Fig. 4.12b Π^1,Π^2 and Π_{cl} for case 2.**

Note that also phase 3 is included in the graphs. Corresponding to case 1 the transition from phase 1 to phase 3 occurs at m = 1.15 and concerning case 2 the behavioral change from phase 2 to phase 3 occurs at market size m = 1.3375. Apparently for case 1 profits drop below classical profits during phase 3 and for the asymmetrical case 2 the results for phase 2 also seem to hold during phase 3. Proofs concerning these phenomena will be given in the remainder of this section.

We now return to the difference in profits between both competing firms. By computing the difference in profits Π^2-Π^1 it can be proved easily that, during phase 2, firm 2 always enjoys an advantage in profit over its rival, assuming that $u_1>u_2$; this advantage is increasing with respect to the variable Δm_2 (see Proposition 4.9). At the end of phase 2 the profits and production levels of both firms become (4.28):

End of phase 2, $m = 1 + u_2 + (u_1 - u_2)\dfrac{(2+2d)}{(1+2d)}$.

Production levels $x_1^{'} = \dfrac{(1-c)}{(3+2d)}$, $x_2^{'} = \dfrac{(1-c)}{(3+2d)} + (u_1 - u_2)\dfrac{1}{(1+2d)}$

Profits :

$$\Pi^1 = (1-c)^2\frac{(1+d)}{(3+2d)^2} + u_1\frac{(1-c)}{(3+2d)}$$

$$\Pi^2 = (1-c)^2\frac{(1+d)}{(3+2d)^2} + u_1\frac{(1-c)}{(3+2d)} + (u_1-u_2)\frac{(1-c)}{(1+2d)(3+2d)} + (u_1-u_2)^2\frac{(1+d)}{(1+2d)^2} \tag{4.28}$$

Now it holds that the smaller firm 2's adjustment cost parameter u_2 is, the bigger the advantage in profits of firm 2 over its rival at the end of the "flexibility outperforms inertia" phase 2 will be. The flexibility of firm 2 in comparison to its competitor pays off. This advantage in profits increases even further in a continuously expanding market during the next and third "complete flexibility" phase (4.29).

During phase 3, $\Delta m_3 \geq 0, \quad m = 1 + u_2 + (u_1 - u_2)\dfrac{(2+2d)}{(1+2d)} + \Delta m_3$

Production levels : $x_1^{''} = \dfrac{(1-c)}{(3+2d)} + \Delta m_3\dfrac{1}{(3+2d)}$,

$$x_2^{''} = \frac{(1-c)}{(3+2d)} + (u_1-u_2)\frac{1}{(1+2d)} + \Delta m_3\frac{1}{(3+2d)}$$

Profits :

$$\Pi^1 = (1-c)^2\frac{(1+d)}{(3+2d)^2} + u_1\frac{(1-c)}{(3+2d)} + \Delta m_3\frac{(1-c)(2+2d)}{(3+2d)^2} + (\Delta m_3)^2\frac{(1+d)}{(3+2d)^2}$$

$$\begin{aligned}\Pi^2 &= (1-c)^2\frac{(1+d)}{(3+2d)^2} + u_1\frac{(1-c)}{(3+2d)} + (u_1-u_2)\frac{(1-c)}{(1+2d)(3+2d)} + (u_1-u_2)^2\frac{(1+d)}{(1+2d)^2}\\ &+ \Delta m_3\frac{(1-c)(2+2d)}{(3+2d)^2} + \Delta m_3(u_1-u_2)\frac{(2+2d)}{(1+2d)(3+2d)} + (\Delta m_3)^2\frac{(1+d)}{(3+2d)^2}\end{aligned} \tag{4.29}$$

Proposition 4.9 (profits of both firms subject to an increasing market size).

During phase 1 the profits Π^1 and Π^2 of both firms are equal and profits increase if the market size increases.

During the phases 2 and 3 it holds that, if $u_1 > u_2$, $\Pi^2 > \Pi^1$ and the difference $\Delta\Pi = \Pi^2 - \Pi^1$ increases if the market size m increases.

Proof

During phase 1 it holds that the derivatives of Π^1 and Π^2 with respect to Δm_1 both equal $(1\text{-}c)/(3+2d) > 0$. This proves the first part of the proposition.
During the phases 2 and 3 we obtain for the difference in profits

$$\text{Phase 2}: \Pi^2 - \Pi^1 = \Delta m_2 \frac{(1-c)}{(2+2d)(3+2d)} + (\Delta m_2)^2 \frac{1}{4(1+d)}$$

$$\text{Phase 3}: \Pi^2 - \Pi^1 = (u_1 - u_2)\frac{(1-c)}{(1+2d)(3+2d)} + (u_1 - u_2)^2 \frac{(1+d)}{(1+2d)^2} +$$

$$+ \Delta m_3 (u_1 - u_2)\frac{(2+2d)}{(1+2d)(3+2d)}$$

Clearly both differences are always positive and the derivatives of $\Delta\Pi$ with respect to Δm_2 and Δm_3 are also positive.

[End of proof]

Note that the difference in profits during phase 3 is also increasing with respect to the difference in adjustment costs (per unit production) Δu between both competitors. We illustrate Proposition 4.9 with a graph of both firms' profits. Again choosing the production cost function equal to $C(x)=0.4x\text{-}0.1x^2$ concerning both rivals and choosing the adjustment costs (per unit) for firms 1 and 2 equal to $u_1=0.10$ and $u_2=0.05$ respectively, leads to the following graph (the graph corresponding with firm 1 is printed in bold):

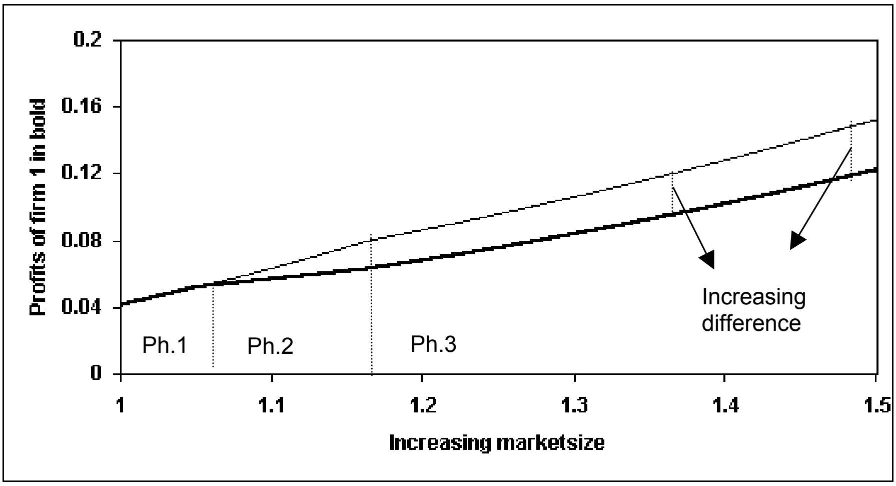


Fig. 4.13 Firms' profits subject to an increasing market size.

Note that phase 1 ends for $m=1.05$ and that phase 2 ends for $m=1.1625$.
The widening gap between both curves from the start of the second phase illustrates Proposition 4.9.

For the two benchmark cases we already showed some properties of both firms' profits in comparison to the classical profit (see Figures 4.12a and 4.12b). Using the expressions for the profits of both competitors during the "complete flexibility" phase

3 we now are able to give the proof we promised above of the phenomena occurring during this third behavioural phase.

Proposition 4.10 (profits compared to classical profits concerning phase 3).

Consider benchmark case 1, i.e. $u_1 = u_2 = u$.

During phase 3 it holds that $\Pi^i > \Pi_{cl}$ for $\Delta m_3 < \frac{(1-c)}{2(1+d)} - \frac{u}{2}$;

for $\Delta m_3 > \frac{(1-c)}{2(1+d)} - \frac{u}{2}$ the profits Π^i drop below the classical profit Π_{cl}.

Consider benchmark case 2, i.e. $u_1 = u > 0, u_2 = 0$.

Then it always holds that $\Pi^2 > \Pi_{cl}$ and the difference $\Pi^2 - \Pi_{cl}$ increases with respect to Δm_3. It also holds that $\Pi^1 < \Pi_{cl}$ and the (negative) difference $\Pi^1 - \Pi_{cl}$ decreases further with respect to Δm_3

Proof

Using the expressions for the profits during phase 3 for u_1=u_2=u and rewriting the classical profit as $\Pi_{cl} = (1+u+\Delta m_3-c)^2(1+d)/(3+2d)^2$ (please bear in mind that phase 2 doesn't occur) we obtain

$$\Pi^i - \Pi_{cl} = u\frac{(1-c)}{(3+2d)^2} - u^2\frac{(1+d)}{(3+2d)^2} - (\Delta m_3)u\frac{2(1+d)}{(3+2d)^2} > 0, \text{ for } i = 1,2$$

if and only if $\Delta m_3 < \frac{(1-c)}{2(1+d)} - \frac{u}{2}$.

For the second benchmark case we use the expressions for the profits Π^1 and Π^2 during phase 3. Substituting u_1=u, u_2=0 and rewriting the classical profit as (phase 1 doesn't occur) $\Pi_{cl} = [1+u(2+2d)/(1+2d)+\Delta m_3 -c]^2(1+d)/(3+2d)^2$ we obtain the required expressions for the differences Π^1-Π_{cl} and Π^2-Π_{cl} :

$$\Pi^1 - \Pi_{cl} = -u\frac{(1-c)}{(1+2d)(3+2d)^2} - u^2\frac{4(1+d)^3}{(1+2d)^2(3+2d)^2} - \Delta m_3 u\frac{4(1+d)^2}{(1+2d)(3+2d)^2}$$

$$\Pi^2 - \Pi_{cl} = u\frac{2(1-c)(1+d)}{(1+2d)(3+2d)^2} + u^2\frac{(1+d)(5+4d)}{(1+2d)^2(3+2d)^2} + \Delta m_3 u\frac{2(1+d)}{(1+2d)(3+2d)^2}$$

Clearly Π^1-$\Pi_{cl} < 0$ and Π^2-$\Pi_{cl} > 0$ both hold. The proof is completed by observing that the expression Π^1-Π_{cl} has a negative derivative and Π^2-Π_{cl} possesses a positive derivative with respect to the variable Δm_3.

[End of proof]

We conclude this section with some reflections. From Propositions 4.8 and 4.10 it follows that for "symmetrical" (u_1=u_2=u) rivals, adjustment costs - corresponding to an increasing production level - cause beneficial effects at least for some period. If both competitors, due to their adjustment costs, keep their production at a lower level in comparison with the classical situation, their profits exceed the classical profits. However in the long term, in case of a further increasing market size, this policy leads to relative losses. During the " complete flexibility" phase 3 profits of the symmetrical firms drop below classical profit implying that the earlier advantage changes into a disadvantage (combining the results of the Propositions 4.8 and 4.10 this occurs at a market size $m=1+u/2+(1-c)/(2+2d)$).

So if a firm is able to reduce its adjustment cost it should do this; sticking to adjustment costs is a short-sighted policy because this strategy, however inviting this may appear in the first period, only leads to benefits which are temporary.
Moreover the main result of Proposition 4.9 reveals that the rival which is able to lower its adjustment costs first, obtains a strategic benefit over its competitor. The analysis of benchmark case 2 shows that the profits of the flexible player (without adjustment costs) even exceed classical profit whereas the profits of the inert rival lie below classical profit. Summarizing: In the expanding market flexibility pays off at the expence of the rival. So starting from a completely symmetrical situation with adjustment costs the rival that is able to lower its adjustment costs first, obtains an advantage. In other words: the early bird catches the worm! Note that, from the standpoint of Organizational Ecology, the model's results, concerning an expanding market, do not support Hannan and Freeman's (1984) "Inertia hypothesis"; in a booming market Darwinian selection processes favor organizationally flexible firms.
This conclusion is totally opposed to the results corresponding to a declining market. There the firm with the highest level of organizational inertia, reflected by larger adjustment costs, has all the advantages and is even capable of forcing its rival to exit (nonpositive profits). Note that concerning completely symmetrical players ($l_1=l_2=l>0$), Proposition 4.6 reveals that both profits are always below classical profits during the consecutive phases 1 and 3. At first sight one would say that it is beneficial to lower adjustment costs (if possible), but appearances are deceptive. The results of Section 4.4 show clearly that, in the declining market, the less flexible player is strategically stronger.

6. Total profits and relative profits over a whole period

This section contains an analysis of both competitors' total profits during a period of decreased or increased economic activity. In our model the period of decreased activity is characterized by a market size $m \leq 1$. So starting at $m=1$ the market declines, reaches its lower turning point (point of revival) and then recovers again till its original level $m=1$. The period of increased activity, the prosperous period, corresponds to $m \geq 1$.To support our line of thought we first look back on the results of Section 4.4 corresponding to a period of decreased activity.

The Propositions 4.5 and 4.7 of that section revealed that firm 1 may use its adjustment cost parameter l_1 as a strategic instrument. By raising its adjustment costs (parameter l_1), the duration of the beneficial second behavioural phase is enlarged as well. As a consequence the rival's profits may drop below zero during phases 2 or 3, whereas firm 1's profits still are positive. But we also showed that there exist combinations $(\Delta l, l_2)$, for which firm 1's profits (Π^1) exceed its rival's profits (Π^2) during phase 2 and a part of phase 3, and yet for a certain market size Π^1 drops below zero, whereas Π^2 still remains positive for a short period. Such a case is displayed in Figure 4.10 of Section 4.4. The fact that firm 1 faces losses just before its competitor faces nonpositive profits should be put in perspective, because (as we already observed) firm 2 has fallen behind in profit over a long period during decreased economic activity. A leeway in total profits may, for instance, lead to less investment in comparison to the rival over a certain period. Therefore not only nonpositive profits, but also the total leeway in profits over a certain period of recession can serve as a, more refined, exit criterion.

We start our considerations by noting that the manner in which the market size decreases (or increases) during a period of decreased (or increased) economic activity doesn't need to be linear. Under the assumption of symmetry, a business cycle (or parts of such cycle) can be modeled using a sine function. Assuming that both competitors' reaction period is small in comparison with the whole period, integral calculus and areas play a crucial role in the analysis of this section. Before going more deeply into technical details, we want to clarify the basic idea visually using Figure 4.14. Assuming that the market size is a decreasing (sine) function this graph shows the development of the profits over time. Considering the time period $0 \leq t \leq 0.5$, during which the market size decreases from 1 till 0.7, the area of the region bounded by the bold curve, the lines $t=0$, $t=0.5$ and the t-axis can serve as a representation of firm 1's total profit. A similar observation holds for firm 2's total profit. As can be observed in the graphical display, firm 2's total profit over this period (Π_{tot}^2) equals a fraction R of its rival's total profit (Π_{tot}^1), i.e. $R = \Pi_{tot}^2/\Pi_{tot}^1$, and therefore equals the fraction of two areas. The further elaboration of this basic idea and the application to specific cases, subject to a period of decreased or increased economic activity, forms the essence of this section. First we specify our assumptions and conditions. Without loss of generality we consider a normalized time period, $0 \leq t \leq 2$, and we will perform the analysis under the assumption that the period of time between two reactions of both rivals equals $1/n$, where n (the number of reactions) is large enough.This assumption is crucial, because it allows us to replace finite sums by integrals. The goniometrical function with a market size equal to $m(t)=1-\alpha.sin(\pi t)$ can serve as a model for a (part of a) business cycle; For $0 \leq t \leq 1$ this specific function describes the market size during a whole period of decreased economic activity.

The all time low occurs at t=0.5, corresponding with a market size of m=1-α, and total recovery takes place at t=1 (for $1 \leq t \leq 2$ this function describes a complete period of increased activity with $m \geq 1$).The (output) behaviour of both rivals during a period with $m \leq 1$ depends on the value of the model parameter α which indicates the “depth” of the depression. If $\alpha \leq l_2$ both firms show a supply behaviour corresponding with the “complete inertia” phase 1, i.e. they maintain their production levels. With equal profits of both firms during the whole period this case isn’t very exiting analytically. But for a value of α which is big enough - the condition derived in Section 4.4 equals $\alpha > l_2+(l_1-l_2)(2+2d)/(1+2d)$ - both firms will pass through the respective behavioral phases 1,2,3 and 2,1 consecutively. Both profits over the time period $0 \leq t \leq 1$ then equal $\Pi^i(m(t))$, i=1,2 (of course the same considerations hold during a period with $m \geq 1$; replace l_i by u_i). To illustrate this development of those profits over time graphically during a period of depression, we choose two firms with the usual production cost function $C(x)=0.4x-0.1x^2$ and with adjustment costs per unit equal to l_1=0.06 and l_2=0 (also the benchmark case corresponding to Figure 4.10).The model for the market size $m(t)$ = 1-0.3$sin(\pi t)$ leads to goniometrical expressions for the two profits over time and the kinks in both graphs display transitions from one behavioural phase to another. Because firm 2 has no adjustment costs, phase 1 doesn’t occur. Note that the smallest market size occurs for t=0.5 (and equals 0.7) and that both competitors still don’t suffer losses at this all time low.The transition from phase 2 to phase 3 takes place at a market size m = 0.865; this corresponds to t = 0.149 (and later at t =0.851 the reverse transition takes place).

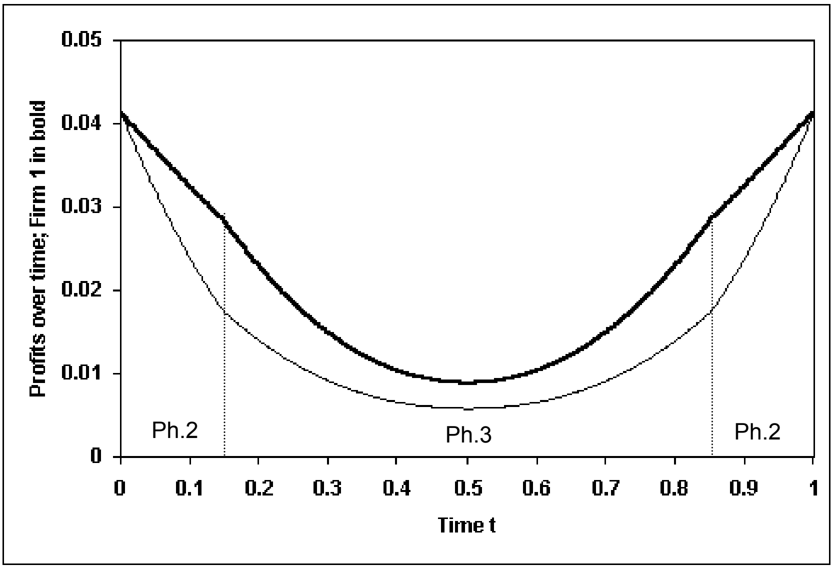


Fig. 4.14 Profits over time during a period of decreased economic activity, Business cycle model: $m(t)$ = 1- 0.3$sin(\pi t)$, $0 \leq t \leq 1$.

In general the following expressions can be used for the computation of both total profits and the fraction R_d (which expresses firm 2’s total profit as a fraction of the rival’s total profit during a period with $m \leq 1$, and $0 \leq t \leq 1$):

$$\Pi^1_{tot} = \sum_{k=1}^{n} \Pi^1(m(\tfrac{k}{n})) \text{ and } \Pi^2_{tot} = \sum_{k=1}^{n} \Pi^2(m(\tfrac{k}{n})).$$

$$R_d = \frac{\sum_{k=1}^{n} \Pi^2(m(\tfrac{k}{n}))}{\sum_{k=1}^{n} \Pi^1(m(\tfrac{k}{n}))}. \tag{4.30}$$

Such expressions, consisting of finite sums, can be computed easily using spreadsheet programmes. For n large enough the expressions for both total profits can be rewritten using integrals over the (corresponding) interval (here the interval $[0,1]$). We now refer to a property, concerning the approximation of integrals, which can be found in many books on numerical methods f.i. Kammer (1987).

$$\int_0^1 f(t)\mathrm{d}t \approx \frac{1}{n}\sum_{k=1}^{n} f(\tfrac{k}{n}) \text{ and for the error } E \text{ it holds that } E \le \frac{1}{2n}\max|f'| \text{ over } 0 \le t \le 1 \tag{4.31}$$

Concerning our application, a value of n=10 (indicating ten reactions of both competitors during the time period) can be large enough and concerning symmetrical functions (Figure 4.14) the integral's approximation by summation works even much more adequately (the maximum error E is less than C/n^2 with C a constant). Firm 2's total profit considered as a fraction of firm 1's total profit, during a period of decreased or increased activity ($m \le 1$, $m \ge 1$ respectively), can now be rewritten. We distinguish between the fractions R_d and R_e corresponding to $m \le 1$ and $m \ge 1$ respectively. Note that in the expressions for R_d and R_e it holds that $0 \le t \le 1$ and $1 \le t \le 2$ respectively.

$$R_d = \frac{\int_0^1 \Pi^2(m(t))\mathrm{d}t}{\int_0^1 \Pi^1(m(t))\mathrm{d}t} \text{ and } R_e = \frac{\int_1^2 \Pi^2(m(t))\mathrm{d}t}{\int_1^2 \Pi^1(m(t))\mathrm{d}t} \tag{4.32}$$

The expressions for R_d and R_e quickly become complicated in cases for which both firms' behaviour passes through several phases over time.The reason for this complication is straightforward: then several expressions for the profits have to be distinguished and the resulting formulas will not clarify the case under consideration. Of course these analytically complicated cases deserve our attention as well and can be studied by using computer simulations. We will apply this integral calculus to the second benchmark case (the first benchmark case, $l_1=l_2=l$, $u_1=u_2=u$, leads to equal profits and so $R_d = R_e = 1$). Concerning this second asymmetrical case, with $l_1=l>0$, $l_2=0$ and $u_1=u>0$, $u_2=0$ we impose a condition on the amplitude of the sine function used to model the market size m. The condition $\alpha \le Min[l(2+2d)/(1+2d), u(2+2d)/(1+2d)]$ ensures us that the behavioural phase corresponding to this benchmark case always equals phase 2 during both periods with $m \le 1$ and $m \ge 1$ and enables us study this case analytically. The condition for α needs some further explanation. It means that one firm is still inert, because of its larger adjustment costs, whereas its rival is flexible and changes its output level. Corresponding to the benchmark case with production cost parameters c=0.4,d=-0.1 and $l=u$=0.10 the condition for α equals $\alpha \le 0.225$ and concerning benchmark cases with linear production costs α has to be less than the minimum of $2l$ and $2u$.

The analytical results for the fractions R_d, R_e and the fraction R corresponding to a total business cycle are summarized in Proposition 4.11.

Proposition 4.11 (relative total profits of firm 2 compared to its rival).

Consider the second benchmark case with $l_1 = l > 0, l_2 = 0$ and $u_1 = u > 0, u_2 = 0$.

Assume that the amplitude of the sine function $\alpha \le Min\left\{ l\frac{(2+2d)}{(1+2d)}; u\frac{(2+2d)}{(1+2d)} \right\}$.

Over the whole period with decreased economic activity ($m \le 1$) it holds that

$$R_d = \frac{\Pi_{tot}^2}{\Pi_{tot}^1} = \frac{(1-c)^2(1+d)^2 - (2\alpha/\pi)(1-c)(1+d)(3+2d) + (\alpha^2/8)(3+2d)^2}{(1-c)^2(1+d)^2 - (\alpha/\pi)(1-c)(1+2d)(3+2d)}.$$

Over the whole period with increased economic activity ($m \ge 1$) it holds that

$$R_e = \frac{\Pi_{tot}^2}{\Pi_{tot}^1} = \frac{(1-c)^2(1+d)^2 + (2\alpha/\pi)(1-c)(1+d)(3+2d) + (\alpha^2/8)(3+2d)^2}{(1-c)^2(1+d)^2 + (\alpha/\pi)(1-c)(1+2d)(3+2d)}.$$

And over the whole cycle (with decreased and increased economic activity as well) it holds that $R = \frac{\Pi_{tot}^2}{\Pi_{tot}^1} = 1 + \alpha^2 \frac{(3+2d)^2}{8(1-c)^2(1+d)^2}$.

Proof

For the whole period with decreased economic activity ($m \le 1$) we substitute $\Delta m_2 = \alpha sin(\pi t)$ (Δm_2 is positive for $0 \le t \le 1$) and $l_2=0$ in the expressions for Π^1 and Π^2 during phase 2 (Section 4.4). The (goniometric) expressions for these profits equal:

$$\Pi^1 = (1-c)^2\frac{(1+d)}{(3+2d)^2} - \frac{(1-c)(1+2d)}{(2+2d)(3+2d)}\alpha sin(\pi t)$$

$$\Pi^2 = (1-c)^2\frac{(1+d)}{(3+2d)^2} - \frac{(1-c)}{(3+2d)}\alpha sin(\pi t) + \frac{1}{4(1+d)}\alpha^2 sin^2(\pi t).$$

Using $\int_0^1 sin(\pi t)\mathrm{d}t = 2/\pi$, $\int_0^1 sin^2(\pi t)\mathrm{d}t = 1/2$ we can compute the fraction R_d.

For the whole period with $m \ge 1$ we use $\Delta m_2 = -\alpha sin(\pi t)$ (Δm_2 is positive for $1 \le t \le 2$) and $u_2=0$ in the expressions for Π^1 and Π^2 during phase 2 (Section 4.5) and we obtain the same expressions for the profits, but now for $1 \le t \le 2$.

The fraction R_e can be computed using $\int_1^2 sin(\pi t)\mathrm{d}t = -2/\pi$, $\int_1^2 sin^2(\pi t)\mathrm{d}t = 1/2$.

And, concerning the complete business cycle, we can use the goniometric expressions for the profits as well, but now for $0 \le t \le 2$. Then the fraction R equals

$R = \frac{\int_0^2 \Pi^2(m(t))\mathrm{d}t}{\int_0^2 \Pi^1(m(t))\mathrm{d}t}$. Using $\int_0^2 sin(\pi t)\mathrm{d}t = 0$, $\int_0^2 sin^2(\pi t)\mathrm{d}t = 1$ we obtain R.

[End of proof]

These expressions for the relative total profits simplify for linear costs (d=0). To gain insight in the meaning of these fractions we will present some tables and graphs for various values of α and for various production cost functions. But first we draw several conclusions from the general expressions. One of the main results of Section 4.5 is that the most flexible competitor benefits from an expanding market. Concerning the second benchmark case firm 2 is the beneficial one during such a prosperous period and the first result states that

- for all $0 < c < 1$, $d > -1/2$ and $\alpha > 0$ it holds that R_e, firm 2's relative total profit in comparison with its rival's total profit during the whole period with $m \geq 1$, exceeds 1.

And during a period with a decreased economic activity we observed in Section 4.4 that the "flexible" firm, i.e. the firm with the smallest adjustment costs, has lower profits than its rival. The second result can be proven easily and states that

- the corresponding fraction R_d is less than 1 for $\alpha < {}^{8(1-c)}\!\big/_{\pi(3+2d)}$ and this condition holds for almost all reasonable cases (c=0.4,d=-0.1 gives $\alpha < 0.546$).

The third result, concerning the relative total profits of the firm without adjustment costs in comparison with its rival, is interesting and can be concluded directly from the expression for R:

- For all $\alpha > 0$, $0 < c < 1$ and $d > -1/2$ the fraction R, corresponding to the whole and completely symmetric business cycle, exceeds 1.

Apparently the advantages during the period of increased economic activity exceed the disadvantages during the whole period with $m \leq 1$. This interesting property can be illustrated with a graphical presentation of both rivals' profits subject to a complete business cycle. We consider an example of the second benchmark case with l_1=0.1, l_2=0 and u_1=0.1, u_2=0 (and the usual production cost function) and we choose the parameter α equal to 0.2 (the condition for α in Proposition 4.11 is satisfied). Figure 4.15 clearly shows that the volatility of the profits without adjustment costs exceeds the volatility of the profits with adjustment costs; adjustment costs damp the amplitude of the profit cycle (as we will illustrate this damping also holds for the total supply with adjustment costs). Furthermore the graph of the profits without adjustment costs also reveals an asymmetry around the (classical) profit for m=1; the amplitude of the profit corresponding to the period $m \geq 1$ is larger than the amplitude concerning the period with decreased economic activity. As already noted in Section 4.2 the expression for the classical profit Π^{i}_{cl} contains the quadratic term $(m\text{-}c)^2$ and this mathematical fact can serve as an explanation for the asymmetry. Sound intuition also leads to the same insight. Concerning linear production costs, the profit equals the product of two factors namely "output" and "market price minus costs" per unit product and during an expanding market both factors "output" (= $(m\text{-}c)/3$) and "price minus costs" per unit (= $(m\text{-}c)/3$) increase. The larger this market size m is, the larger these two factors become and also the larger profit's sensitivity to changes in output and price becomes.

This sensitivity property implies that decreasing the output (due to adjustment costs) by a fixed amount during the period of prosperity has a larger impact on the difference in profits, Π_{adj} - Π_{cl}, than increasing the supply by the same amount during the period of recession. This is exactly what happens in the case with adjustment costs. The formulas for the outputs during the behavioural phases 1, 2 and 3 corresponding to the declining and the expanding market reveal a beautiful symmetry. But the restriction of the output (compared with the classical output) during the prosperous period results in a larger difference in profits than supply's upholding during the period of recession. Therefore, over a complete business cycle, the totally flexible firm still keeps an advantage over its rival with adjustment costs. This argumentation clarifies the fact that concerning a completely symmetrical business cycle the factor R exceeds 1.

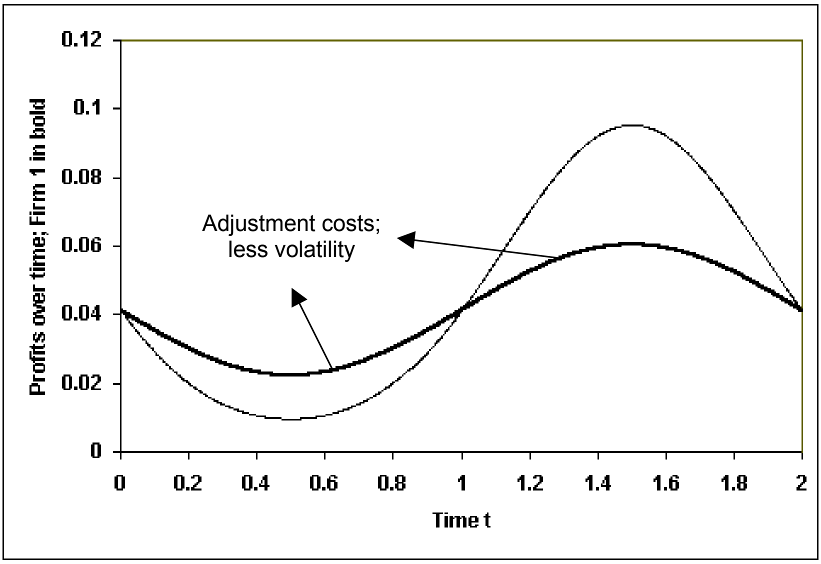


Fig. 4.15 Profits over time during a complete business cycle; firm 1 possesses adjustment costs whereas its rival has no adjustment costs.

In this example with $\alpha = 0.2$ (c=0.4, d=-0.1) the relative advantage of firm 2 over its rival is substantial; application of Proposition 4.11 gives R = 1.134 which indicates that firm 2's total profit is about 13% higher than its rival's total profit. So, in the end, flexibility outperforms inertia. Of course we have to realize that during the first period of decreased economic activity - this period corresponds to $0 \leq t \leq 1$ in our model of the whole cycle - R_d drops below 1. Therefore it is quite possible that firm 2 doesn't survive the period of recession and may never taste the benefits of the prosperous period. Application of Proposition 4.11 leads to a relative total profit of firm 2 compared with its rival of R_d= 0.671. The graphical display of the development of the fraction Π^2_{tot}/Π^1_{tot} during the complete business cycle illustrates this phenomenon (the fraction is obtained by computer simulation).

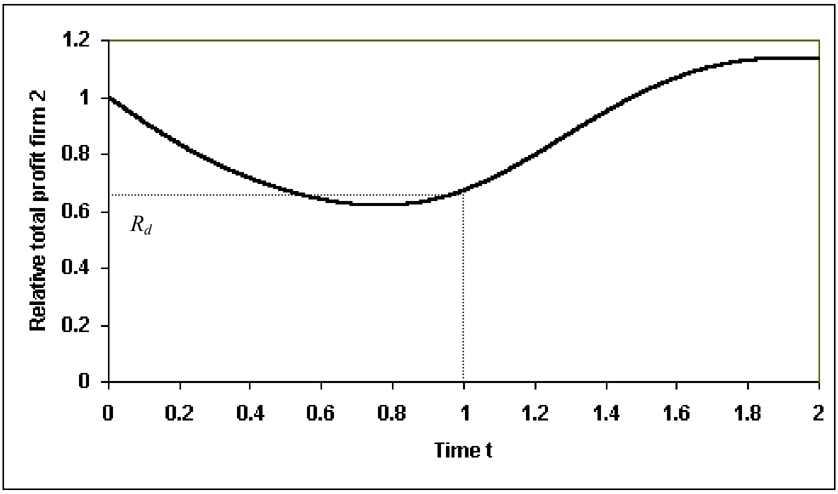


Fig. 4.16 The quotient of the total profits Π^2_{tot}/Π^1_{tot} over time during a complete business cycle with α=0.2 (l_1=0.1, l_2=0 and u_1=0.1, u_2=0 and c=0.4, d=-0.1).

We already stated that adjustment costs also influence market supply; the market supply's amplitude with adjustment costs is less than the amplitude corresponding to classical market supply (= $2(m-c)/(3+2d)$ in our normalized model). Figure 4.17 illustrates the effect of the adjustment costs on the market supply; we again make the choice corresponding with the second benchmark case namely l_1=0.1, l_2=0, u_1=0.1, u_2=0 (and c=0.4,d=-0.1) and α=0.2. The benchmark case's market supply (one firm possesses adjustment costs) is printed in bold. We argue that adjustment costs may serve as a policy instrument to bring peace to the economy.

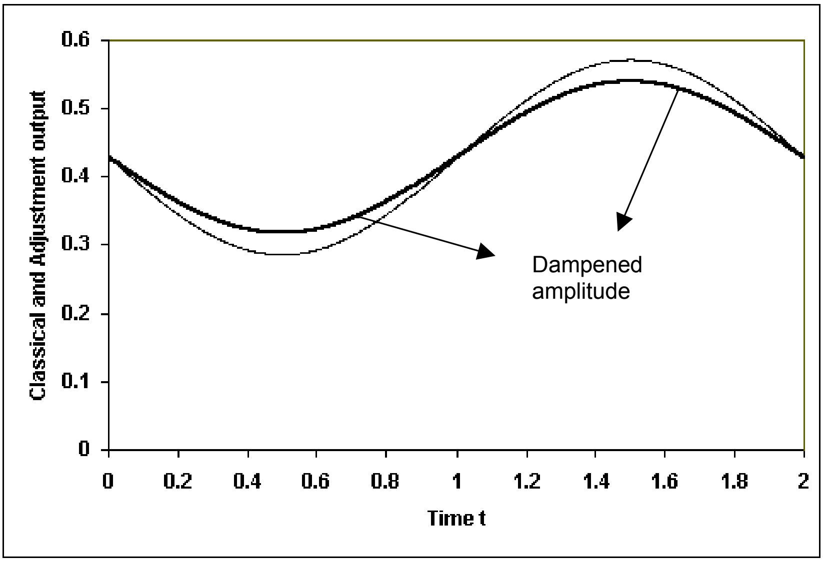


Fig. 4.17 Market supply corresponding to the second benchmark case (l_1=0.1, l_2=0, u_1=0.1, u_2=0) and classical market supply, with α=0.2.

In the introduction of this Section we referred to the benchmark case with c=0.4, d=-0.1, l_1=0.06 and l_2=0. We noted that the fact that Π^1 drops below zero whereas Π^2 still remains positive for a short period, should be put in perspective and that a total leeway in profits over a certain period can also serve as a refined exit criterion. The results of Proposition 4.11 can be applied to support this statement. For all α with $\alpha \leq 0.135$ the proposition's condition is satisfied and for $\alpha = 0.1$ computation leads to $R_d = 0.824$ as a measure for firm 2's relative total profit compared to its rival during the whole period with $0.9 \leq m \leq 1$. For $\alpha = 0.4$ (corresponding to a deep depression) we must resort to a spreadsheet programme to compute the value of the fraction R_d at the exact moment in time corresponding to $\Pi^1 = 0$ (then the market size equals m=0.6, see Fig. 4.10). In our model of the business cycle the market size equals m=0.6 at t=0.5 and the corresponding value of firm 2's relative total profit equals 0.76.

Such low values concerning the relative total profits of the firm without adjustment costs during a period with decreased economic activity aren't exceptional. To gain further insight in the values of the fractions R_d, R_e and R , concerning the second benchmark case (l_1>0,l_2=0; u_1>0,u_2=0), we present three tables with these values for several values of α and c. We choose linear production cost functions (d=0) and for the adjustment cost parameters we choose l_1=0.10 and u_1=0.10. Proposition 4.11 can be applied for all amplitudes $\alpha \leq 0.2$. And for all larger values of the business cycle's amplitude we use computer simulation to compute the quotients of the total profits Π^2_{tot}/Π^1_{tot} over a complete period with $m \leq 1$, $m \geq 1$ and a complete business cycle.The fractions are rounded off to two decimals.

Table 4.1, relative total profits $R_d = \Pi^2_{tot}/\Pi^1_{tot}$ over the whole period with $m \leq 1$.

	c=0.2	c=0.4	c=0.6
α=0.10	0.88	0.85	0.78
α=0.20	0.78	0.72	0.62
α=0.30	0.74	0.69	0.78

Table 4.2, relative total profits $R_e = \Pi^2_{tot}/\Pi^1_{tot}$ over the whole period with $m \geq 1$.

	c=0.2	c=0.4	c=0.6
α=0.10	1.12	1.16	1.25
α=0.20	1.25	1.34	1.51
α=0.30	1.31	1.41	1.62

Table 4.3, relative total profits $R = \Pi^2_{tot}/\Pi^1_{tot}$ over the whole cycle.

	c=0.2	c=0.4	c=0.6
α=0.10	1.02	1.03	1.07
α=0.20	1.07	1.13	1.28
α=0.30	1.13	1.23	1.52

The conclusions from Proposition 4.11 (which hold for a limited set of α's) still hold for larger amplitudes of the business cycle: in general $R_d < 1$, $R_e > 1$ and $R > 1$. Of course for large values of α, the advantages of the firm with adjustment costs, during a period with decreased economic activity, may change into disadvantages.
Obviously this is due to the adjustment costs which occur as extra costs during the behavioural "complete flexibility" phase 3.

Consideration of these three tables leads to the observation that in general the fraction R_d decreases if the "depth" of the recession (α) increases or the production costs per unit (c) increase. And the fractions R_e and R both increase if α or c increases. So in general relative benefits or losses become enlarged if the production process is less efficient or the amplitude of the business cycle is larger.

From the results presented in Table 4.3 we can draw another important conclusion. A totally flexible firm, without adjustment costs in both directions of production change, enjoys substantial benefits over a complete business cycle. Of course this statement only holds under the assumption that this flexible firm survives the period of recession (see Figure 4.16). However it should be noted that the strategically strongest firm has asymmetrical adjustment costs: the results concerning the fractions R_d and R_e clearly show that a strategically strong firm possesses sufficient adjustment costs corresponding to a decreasing output and is totally flexible - if possible the firm has no adjustment costs at all - if the output level has to be increased.

In Section 4.4 we reflected on the strategic behaviour of the firm with the largest adjustment costs during a period with a decreasing market size. By enlarging these adjustment costs this firm is able to enlarge the duration of the beneficial phase 2 and Propositions 4.5 and 4.7 provide some possible exit conditions for the rival (of course this rational behaviour assumes knowledge of the implications of adjustment costs, imformation concerning the rival and also the possibility to change the adjustment cost parameter l).

These reflections on exit criteria can be refined if we use the indicator R_d for firm 2's relative total profit in comparison with its competitor. Even if firm 2 maintains nonnegative profits over the whole period with decreased economic activity, total leeway in profits, expressed by R_d, can serve as another criterion for leaving the market. A value of $R_d = 0.69$ (Table 4.1, c=0.4,α=0.3) could mean for instance that firm 2's total investments, over the past period of recession, only add up to 69% of the competitor's total investments (assuming that investments are proportional to profit). If firm 1 is able to invest more over a period of recession, firm 2 can even be expelled from the market in future, because in the just recently recovered market firm 1 may react more adequately. In the following we reflect on the general case with $l_1 > l_2 \geq 0$.

Assuming that firm 1 has information about its rival's adjustment cost parameter l_2 and is also able to estimate the parameter α (the depth) of the downward business cycle, the parameter l_1 can also serve as a strategic instrument. Firm 1 may choose its adjustment cost parameter in such a way that the fraction R_d at the end of this recession (total recovery m=1 occurs at $t = 1$) is minimized. Because the behavioural "inertia outperforms flexibility" phase 2 is the most advantageous one for firm 1, in comparison with its rival, the optimal choice for l_1 satisfies the equation (assume $\alpha > l_2$)

$$l_2 + (l_1 - l_2)\frac{(2+2d)}{(1+2d)} \geq \alpha \tag{4.33}$$

This choice guarantees that the "complete flexible" behavioural phase 3 does not occur (If $\alpha \leq l_2$, $R_d = 1$ of course). We formulate Proposition 4.12:

Proposition 4.12 (minimization of firm 2's relative total profit due to recession).

Assume that the amplitude α of the (downward) business cycle exceeds the adjustment cost parameter l_2 of firm 2 (otherwise only phase 1 occurs). All choices of l_1 which satisfy the condition $l_1 \geq l_1^* = \alpha \frac{(1+2d)}{(2+2d)} + l_2 \frac{1}{(2+2d)}$ minimize the fraction R_d at the end of the recession.

Concerning benchmark case 2 the parameter l_2 equals zero and the minimized value of R_d then equals the expression in Proposition 4.11. As noted earlier the analytic expression for the fraction R_d is too complicated if $l_2 > 0$, but then Proposition 4.12 is also confirmed by computer experiments. These experiments show that if l_1 increases, the fraction R_d falls for $l_1 \leq l_1^*$ and for $l_1 > l_1^*$ R_d remains constant. Table 4.4 contains the values of l_1^* and the minimized value of R_d for several values of l_2 and α (with production cost parameters c=0.4 and d=-0.1).

Table 4.4, some values of l_1^*.

	$l_2 = 0$	$l_2 = 0.05$	$l_2 = 0.1$
α=0.10	l_1^*=0.044 ,R_d=0.82	l_1^*=0.072 ,R_d=0.92	**Only phase 1** ,l_1^*=0.1
α=0.20	l_1^*=0.089 ,R_d=0.67	l_1^*=0.117 ,R_d=0.71	l_1^*=0.144 ,R_d=0.78
α=0.30	l_1^*=0.133 ,R_d=0.56	l_1^*=0.161 ,R_d=0.47	l_1^*=0.189 ,R_d=0.39

Note that if firm 1 were to overestimate the amplitude parameter α a little bit the fraction R_d would still be minimized by the choice of l_1. Also note that, if rival 2 were to increase its adjustment cost parameter l_2 with an amount Δl_2, firm 1 only has to increase l_1 with $\Delta l_1 = \Delta l_2/(2+2d)$ to minimize R_d over the whole period. We note that the strategic choice of the adjustment cost parameter is also related to two-stage games, such as "delegation games" (Fershtman and Judd (1987), Vickers (1985) and others). Owners of the firms (who hire their managers) may change managers' salaries and contracts in such a way that adjustment costs increase, for instance in accordance with Proposition 4.12. The study of such game in relation to adjustment cost will be a topic for future research.

We conclude this section with an example of a firm 1 that is able to choose both its adjustment cost parameters l_1 and u_1, given the adjustment cost parameters of its rival. Note that we assume rational behaviour of this incumbent firm, besides its ability to obtain the relevant information concerning its competitor and the possibility to adapt. Furthermore we assume that firm 1 is able to estimate the amplitude α of the future business cycle. Lets assume that the amplitude equals 0.25 and that firm 2's adjustment cost parameters equal l_2=0.05 and u_2=0.05 (both firms possess the usual production cost parameters c=0.4,d=-0.1). The best strategic choice for firm 1 during the prosperous period (market size $m \geq 1$) is straightforward: firm 1 has to react as flexibly as possible, i.e. u_1=0. And Proposition 4.12 reveals that if $l_1 \geq 0.139$, firm 2's relative total profit compared to its rival over the whole period with decreased economic activity ($m \leq 1$) is minimized. There is a big chance that firm 2 doesn't survive the recession, because Proposition 4.5 (Section 4.4) shows that firm 2 also meets losses during phase 2 (and firm 1's profits even remain positive). Should firm 2 survive the recession waiting for more properous times, the development of the relative total profit over the complete cycle (obtained by computer simulation) speaks volumes. At the moment of (first) recovery of the market (t=1) firm 2's relative total profit in comparison with the total profit of its rival - over the whole period of recession - is somewhat more than 0.5. The relative total profit rises slowly till approximately 70%.

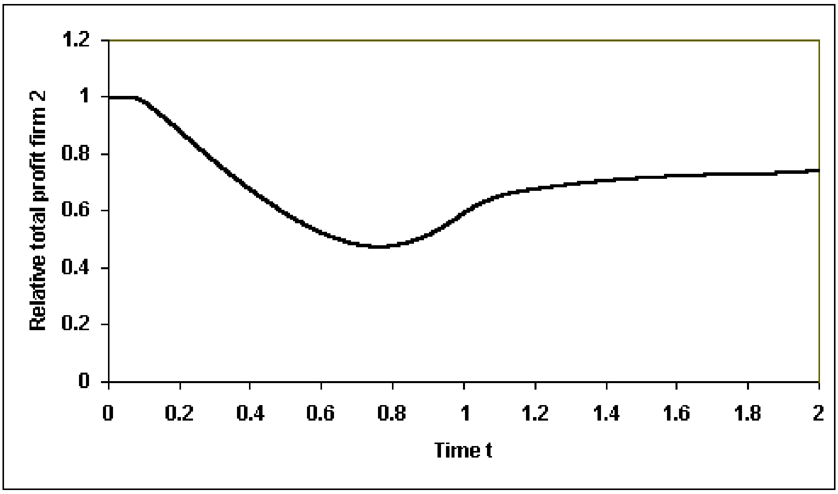


Fig. 4.18 The development of Π^2_{tot}/Π^1_{tot} over time during a complete business cycle with α=0.25 (l_1=0.139, l_2=0.05 and u_1=0, u_2=0.05).

7. Adjustment costs and welfare consequences

The main subject of this section is the analysis of the difference in welfare between classical Cournot competition of two firms without adjustment costs and two competing firms with adjustment costs. In Section 4.4 we already noted that during behavioural phase 1 in the declining market, as a consequence of adjustment costs, total market supply is larger. This implies that the consumer surplus increases in comparison to the classical Cournot case and this observation holds for all three phases during a period with decreased economic activity. During a prosperous period the reverse phenomenon occurs caused by a restricted output level of both competitors. However the analysis of both rivals' profits in comparison to the classical case (Propositions 4.4, 4.6, 4.8,4.10) reveals that several cases have to be distinguished. Important differences occur not only concerning a period of recession and a period of prosperity, but also concerning the two benchmark cases as well. Therefore the analysis in this section is dedicated to social welfare (defined as the sum of consumer surplus and producer surplus). The organization of this section is as follows: the first and second part deal with the examination of the welfare during a period with decreased economic activity (market size $m \leq 1$) and during a prosperous period ($m \geq 1$) respectively. In each part we will distinguish between the symmetrical benchmark case 1 ($l_1=l_2=l>0$ and $u_1=u_2=u>0$) and the asymmetrical benchmark case 2 ($l_1=l>0,\ l_2=0$ and $u_1=u>0,\ u_2=0$). Computer simulations will support the analysis of these two benchmark cases and for general cases ($l_1>l_2>0,\ u_1>u_2>0$) such experiments will clarify the welfare properties as well.

Finally the third part focusses on the relative total welfare in comparison to the total classical welfare over a period of decreased economic activity, a period of increased economic activity and over a complete business cycle. Like in Section 4.6, in cases where the analysis refines the insight (and is feasible), we will make use of integral calculus.

First we consider briefly the expressions and formulas that we will use in this section. In Section 4.2 we already provided the general expression of the consumer surplus. Concerning a linear inverse demand function the consumer surplus CS equals $0.5(p_{\max} - p^*)q^*$, where p_{max}, p^* and q^* equal the maximum price of the good, equilibrium price and total equilibrium supply of both firms respectively. In our normalized model it holds that $p_{max} = m$ (market size) and $p^* = p_{\max} - q^*$, implying that the expression for CS becomes:

$$CS = \tfrac{1}{2}(q^*)^2, \text{ with } q^* = x_1^* + x_2^* \tag{4.34}$$

In this whole section we will use the notations ΔW, ΔCS and $\Delta \Pi$ for the differences in welfare, consumer surplus and total profit between the case with adjustment costs and the classical case respectively. So it holds that

$$\Delta CS = CS - CS_{cl},\ \ \Delta\Pi = \Pi^1 + \Pi^2 - \Pi^1{}_{cl} - \Pi^2{}_{cl},\ \ \Delta W = \Delta CS + \Delta\Pi \tag{4.35}$$

Furthermore, in computing $\Delta\Pi$, we will make use of the expressions for Π^1-Π_{cl} and Π^2-Π_{cl} which are derived concerning both benchmark cases in Sections 4.4 and 4.5.

Part 1: A period with decreased economic activity, i.e. $m \leq 1$

First we consider benchmark case 1 with $l_1=l_2=l>0$.
We present a table with the values of relative differences in welfare $\Delta W/W_{cl}$ for various adjustment cost parameters l. Concerning the production cost parameters we make the (usual) choice c=0.4 and d=-0.1. In each cell also the behavioural phase of both firms is indicated. Note that, as a consequence of symmetry in adjustment costs the "inertia outperforms flexibility" phase 2 doesn't occur. Also note that, during phase 1 ($m \geq 1$-l), the value of $\Delta W/W_{cl}$ increases if the market size m decreases, whereas the relative difference in welfare falls quickly during the "complete flexibility "phase 3.

Table 4.5, the relative difference in welfare for $l > 0$ and various market sizes.

	m=0.95	m=0.90	m=0.85	m=0.8	m=0.75	m=0.70
l=0.05	(1), 0.044	(3), 0.033	(3), 0.016	(3),-0.011	(3), -0.055	(3), -0.130
l=0.10	(1), 0.044	(1), 0.086	(3), 0.057	(3), 0.010	(3), -0.069	(3), -0.205
l=0.15	(1), 0.044	(1), 0.086	(1), 0.123	(3), 0.062	(3), -0.042	(3), -0.224
l=0.20	(1), 0.044	(1), 0.086	(1), 0.123	(1), 0.145	(3), 0.026	(3), -0.187

Quick observation of this table reveals that rather large values of the relative difference in welfare $\Delta W/W_{cl}$ can occur; for instance, corresponding to an adjustment cost parameter of l=0.15 and a market size of m=0.85, this relative difference has risen to more than 12%! It appears to be possible to analyse the development of the difference in welfare during the phases 1 and 3 completely. Proposition 4.13 not only reveals the market sizes for which this difference is positive, but also gives the interval of the market size parameter m for which the difference in welfare rises or falls. Naturally the classical welfare is decreasing with respect to a decreasing market size.Therefore it holds that, if the difference in welfare increases (decreases), the relative difference in welfare even rises (falls) stronger with respect to a decreasing market size.

Proposition 4.13 (difference in welfare, concerning benchmark case 1).

Consider benchmark case 1 with $l_1 = l_2 = l > 0$.

(i) The difference in welfare ΔW between the case with symmetrical adjustment costs and the classical case is positive for $m > 1 - \frac{l}{2} - \frac{(1-c)}{2(2+d)}$ and drops below zero for $m < 1 - \frac{l}{2} - \frac{(1-c)}{2(2+d)}$.

(ii) ΔW (as well as $\Delta W / W_{cl}$) increases if m decreases for $m > 1 - \min\left\{l; \frac{(1-c)}{2(2+d)}\right\}$ and falls for $m < 1 - \min\left\{l; \frac{(1-c)}{2(2+d)}\right\}$.

Proof

During phase 1 the equilibrium outputs of both firms equal $x_1^* = x_2^* =$ (1-c)/(3+2d) (see Section 4.4) and the classical total output equals $q_{cl}^* =$ 2(m-c)/(3+2d). The market size m equals 1 - Δm_1 and for the difference in consumer surplus during phase 1 we obtain

$$\Delta CS = CS - CS_{cl} = \frac{2(1-c)^2}{(3+2d)^2} - 2\left[\frac{(1-c)}{(3+2d)} - \frac{\Delta m_1}{(3+2d)}\right]^2 =$$

$$\Delta m_1 \frac{4(1-c)}{(3+2d)^2} - (\Delta m_1)^2 \frac{2}{(3+2d)^2}$$

Using the expression for Π^i - Π_{cl}, i=1,2 of Section 4.4, we find for the difference in total profits

$$\Delta\Pi = \Pi^1 + \Pi^2 - 2\Pi_{cl} = -\Delta m_1 \frac{2(1-c)}{(3+2d)^2} - (\Delta m_1)^2 \frac{2(1+d)}{(3+2d)^2}$$

The difference in welfare is obtained bij adding the expressions for ΔCS and $\Delta\Pi$.

$$\Delta W = W - W_{cl} = \frac{2\Delta m_1}{(3+2d)^2}\left[(1-c) - (2+d)\Delta m_1\right]$$

So $\Delta W > 0$ during the whole phase 1 for $l < (1\text{-}c)/(2\text{+}d)$ (all reasonable adjustment costs) and rises (with respect to Δm_1) for $\Delta m_1 < (1\text{-}c)/(4\text{+}2d)$.
By deriving an expression for the difference in welfare during phase 3 (phase 2 doesn't occur), we can show that ΔW is a linearly decreasing function if m decreases. Using the equilibrium outputs of both firms during phase 3 and the formulas for the expressions Π^i - Π_{cl}, i=1,2 (Proposition 4.6, Section 4.4) expressions for ΔCS, $\Delta\Pi$ and ΔW can be obtained ($m = 1\text{-}l\text{-}\Delta m_3$):

$$\Delta\Pi = \Pi^1 + \Pi^2 - 2\Pi_{cl} = -l\frac{2(1-c)}{(3+2d)^2} - l^2\frac{2(1+d)}{(3+2d)^2} - \Delta m_3 l\frac{4(1+d)}{(3+2d)^2}$$

$$\Delta CS = CS - CS_{cl} = l\frac{4(1-c)}{(3+2d)^2} - l^2\frac{2}{(3+2d)^2} - \Delta m_3 l\frac{4}{(3+2d)^2}$$

$$\Delta W = \Delta CS + \Delta W = l\frac{2(1-c)}{(3+2d)^2} - l^2\frac{2(2+d)}{(3+2d)^2} - \Delta m_3 l\frac{4(2+d)}{(3+2d)^2}$$

Part (i) is proved by solving $\Delta W = 0$ (with respect to Δm_3) and using $m = 1\text{-}l\text{-}\Delta m_3$.
Part (ii) is proved by realizing that during phase 3 the derivative of ΔW with respect to Δm_3 is always negative.

[End of proof]

Note that the larger the adjustment costs are, the quicker ΔW (and so $\Delta W/W_{cl}$) falls with respect to a decreasing market size m during phase 3. This phenomenon can be observed in Table 4.5. Application of Proposition 4.13 to the (benchmark) case with l_1=l_2=l=0.15 and c=0.4,d=-0.1 leads to (i) the difference and relative difference in welfare are both positive for $m > 0.767$ (and drop below zero for $m < 0.767$) and (ii) ΔW and $\Delta W/W_{cl}$ both rise for $m > 1 - min\{0.15\ ,\ 0.158\} = 0.85$ (and fall for $m < 0.85$ during phase 3). One can compute that at a market size of $m = 0.779$ the profits of both symmetrical firms become zero (use the formula for $(\Delta m_3)^L$, see the proof of Proposition 4.7, Section 4.4 and use $m = 1\text{-}l\text{-}(\Delta m_3)^L$). Then, welfare may drop to zero, due to immediate exit of both competitors. The result of Proposition 4.13, concerning the interval of values of m for which ΔW is positive, leads one to suspect that the larger the adjustment costs are the more beneficial this is for social welfare.

However we have to modify this statement, because too large adjustment costs also sooner lead to nonpositive profits in a declining market and may trigger exit.

The graphical presentation of the relative difference in welfare during a period of decreased economic activity, corresponding to this case, completes the analysis of benchmark case 1.

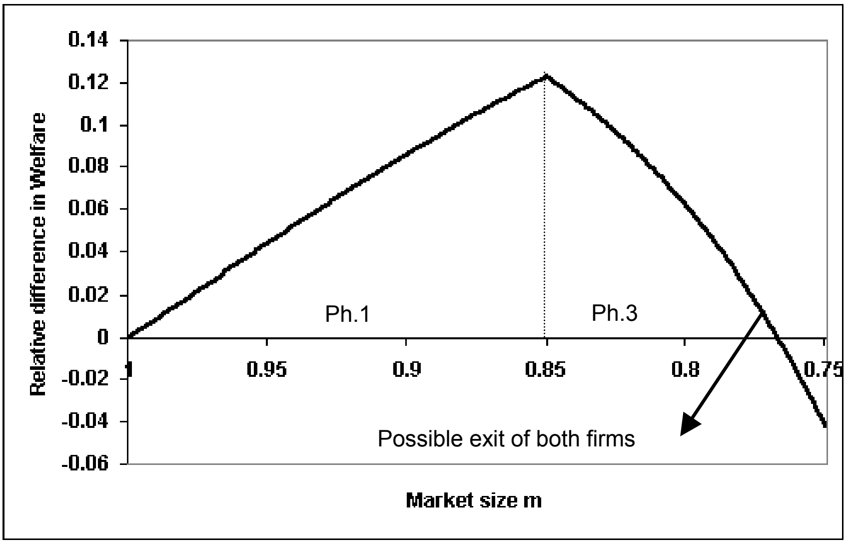


Fig. 4.19 The development of $\Delta W/W_{cl}$ in a declining market .

We now consider benchmark case 2 concerning an asymmetry in adjustment costs between both competitors, i.e. $l_1=l>0$, $l_2=0$. For the production cost parameters we again choose $c=0.4$, $d=-0.1$. To gain a clear insight in the development of the relative difference in welfare, subject to a decreasing market size, we first present a table with some computed values of the fraction $\Delta W/W_{cl}$. For several market sizes m and various values of l the behavioural phase is also indicated. Note that for this asymmetrical case 2 the "complete inertia" phase 1 doesn't occur.

Table 4.6, the relative difference in welfare for $l>0$, $l_2=0$ and various market sizes.

	m=0.95	m=0.90	m=0.85	m=0.8	m=0.75	m=0.70
l=0.05	(2), 0.011	(2), 0.024	(3), 0.023	(3), 0.013	(3), -0.003	(3), -0.032
l=0.10	(2), 0.011	(2), 0.024	(2), 0.040	(2), 0.061	(3), 0.064	(3), 0.031
l=0.15	(2), 0.011	(2), 0.024	(2), 0.040	(2), 0.061	(2), 0.088	(2), 0.125
l=0.20	(2), 0.011	(2), 0.024	(2), 0.040	(2), 0.061	(2), 0.088	(2), 0.125

The results of Table 4.6 suggest some qualitative properties of the welfare

- The relative difference in welfare between benchmark case 2 and the classical case is positive during phase 2 and rises with respect to a decreasing market size.
- The larger firm 1's adjustment costs are, the more beneficial this is for the social welfare.

- During phase 3 the relative difference in welfare falls quickly subject to a further decreasing market size and finally becomes negative.

It appears that these results hold for a wide range of adjustment cost parameters and production cost parameters. Proposition 4.14 contains the precise and analytical formulation of these properties with regard to the occurring phases 2 and 3.
Note that in (the proof of) this proposition again the difference in welfare ΔW is considered. Because the classical welfare is decreasing with respect to a decreasing market size these results also hold for $\Delta W/W_{cl}$.

Proposition 4.14 (difference in welfare, concerning benchmark case 2).

Consider benchmark case 2 with $l_1 = l > 0$ and $l_2 = 0$.
The difference in welfare ΔW as well as the relative difference in welfare $\Delta W / W_{cl}$ between the case with adjustment costs and the classical case are positive during the whole of phase 2, i.e. $m \geq 1 - l\dfrac{(2+2d)}{(1+2d)}$, for

(i) all l if the production cost function is concave with $-0.5 < d \leq -0.2594$
(ii) $l < 0.4(1-c)$ if the production cost function is linear or concave with $-0.2594 < d \leq 0$
(iii) $l < 0.308(1-c)$ if the production cost function is convex with $0 < d \leq 0.5$

In case (i) ΔW and $\Delta W / W_{cl}$ always increase if m decreases and in the cases(ii) and (iii) ΔW and $\Delta W / W_{cl}$ increase for $m \geq 1 - min\left\{ l\dfrac{(2+2d)}{(1+2d)}; 2(1-c)\dfrac{(1+d)(1+2d)}{(8d^3+28d^2+26d+5)} \right\}$.
During phase 3 ΔW decreases linearly with respect to a decreasing market size.

Proof

We use the expressions for the outputs $x_1^{'}$ and $x_2^{'}$ during phase 2 (see Section 4.4) and the classical total output equals $q_{cl}^{'} = 2(m\text{-}c)/(3\text{+}2d)$. Because the market size m equals $m = 1 - \Delta m_2$ (phase 1 doesn't occur) we obtain for the difference in consumer surplus

$$\Delta CS = CS - CS_{cl} = \Delta m_2 \frac{(1-c)(1+2d)}{(1+d)(3+2d)^2} - (\Delta m_2)^2 \frac{(1+2d)(7+6d)}{8(1+d)^2(3+2d)^2}$$

Using the expressions for $\Pi^1\text{-}\Pi_{cl}$ and $\Pi^2\text{-}\Pi_{cl}$ of the proof of Proposition 4.4, Section 4.4, the expression for $\Delta\Pi$ is easily derivable.

$$\Delta\Pi = \Pi^1 + \Pi^2 - 2\Pi_{cl} = -\Delta m_2 \frac{(1-c)(1+2d)}{2(1+d)(3+2d)^2} + (\Delta m_2)^2 \frac{(1-4d-4d^2)}{4(1+d)(3+2d)^2}$$

The difference in welfare is obtained bij adding the expressions for ΔCS and $\Delta\Pi$.

$$\Delta W = W - W_{cl} = \frac{\Delta m_2}{2(1+d)(3+2d)^2}\left\{(1-c)(1+2d) - \Delta m_2 \frac{(8d^3+28d^2+26d+5)}{4(1+d)}\right\}$$

For $d \leq$ -0.2594 the polynomial $P(d)$:=$8d^3+28d^2+26d+5$ is nonpositive and this observation proves case (i).
For the remaining two cases Δm_2 has to satisfy the condition $\Delta m_2 < 4(1\text{-}c)(1+d)(1+2d)/P(d)$.This condition holds during the whole phase 2 if and only if $l(2+2d)/(1+2d) < 4(1\text{-}c)(1+d)(1+2d)/P(d)$. So for the adjustment cost parameter l we obtain the condition

$$l < (1-c)F(d) \text{ with } F(d) = \frac{2(1+2d)^2}{(8d^3+28d^2+26d+5)}$$

The function $F(d)$ is monotonically decreasing for $-0.2594 < d \leq 0.5$ (the function $H(d)$:=$1/F(d)$ can be rewritten as $H(d) = d+5/2+2d/(4d^2+4d+1)$ and is monotonically increasing for $-0.2594 < d \leq 0.5$). So for the cases (ii) and (iii) the conditions for l become $l <(1\text{-}c)F(0)$ and $l <(1\text{-}c)F(0.5)$ respectively.
For case (i) (d<-0.2594) the derivative of ΔW with respect to Δm_2 is always positive and for the cases (ii) and (iii) the quadratic form ΔW is increasing with respect to Δm_2 as long as $\Delta m_2 < 2(1\text{-}c)(1+d)(1+2d)/P(d)$ holds.

The analysis corresponding with phase 3 is similar. Now we use the expressions for the outputs x_1'' and x_2'' during phase 3 (see Section 4.4, with l=l_1, l_2=0) and the classical total output $q_{cl}'' = 2(m\text{-}c)/(3+2d)$ with $m = 1\text{-}l(2+2d)/(1+2d)\text{-}\Delta m_3$. We obtain for the difference in consumer surplus

$$\Delta CS = CS - CS_{cl} = l\frac{2(1-c)}{(3+2d)^2} - l^2\frac{(7+6d)}{2(1+2d)(3+2d)^2} - \Delta m_3 l \frac{2}{(3+2d)^2}$$

Using the expressions for Π^1-Π_{cl} and Π^2-Π_{cl} (see the proof of Proposition 4.6, Section 4.4) and $\Delta W = \Delta CS + \Delta\Pi$, we obtain a linear decreasing expression for the difference in welfare with respect to Δm_3 .

$$\Delta W = W - W_{cl} = l\frac{(1-c)}{(3+2d)^2} - l^2\frac{(8d^3+28d^2+26d+5)}{2(1+2d)^2(3+2d)^2} - \Delta m_3 l \frac{2(2+d)}{(3+2d)^2}$$

[End of proof]

Proposition 4.14 looks rather complicated and this is mainly due to the distinction of the three cases (i), (ii) and (iii) concerning the production cost parameter *d*. For a linear production cost function it holds that d=0 and this choice leads to a simplification of the proposition. Concerning the linear production cost case it holds that the difference and the relative difference in welfare are positive during phase 2 for all $l < 0.4(1\text{-}c)$. Furthermore, for d=0, the relative difference $\Delta W/W_{cl}$ is rising (with respect to a decreasing market size) for $m \geq min\{2l, 0.4(1\text{-}c)\}$. Case (ii) also contains a wide class of concave production cost functions, corresponding to an efficient production technology. Case (iii) corresponds to convex production cost functions ($d > 0$) and we observe that the set of adjustment cost parameters for which the welfare result holds becomes somewhat smaller.

Application of Proposition 4.14 to the benchmark case with $l_1=l=0.15$, $l_2=0$ and $c=0.4$, $d=-0.1$ reveals that $\Delta W/W_{cl}$ remains positive (at least) during phase 2 ($m \geq 0.663$) and is even rising for $m \geq 0.677$.
We conclude the analysis of benchmark case 2 with a graphical display of the relative difference in welfare, during the "inertia outperforms flexibility" phase 2 and the "complete flexibility" phase 3, of this case.

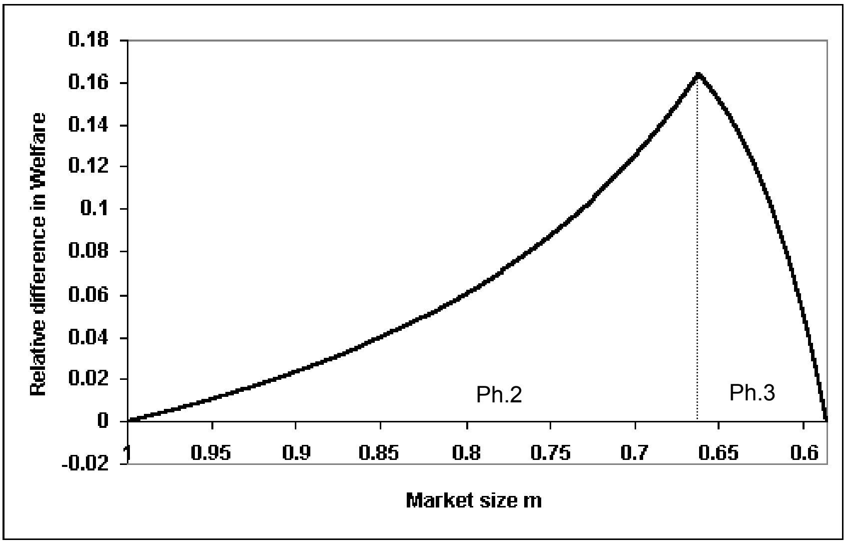


Fig. 4.20 The development of $\Delta W/W_{cl}$ during a declining market .

Note that this relative difference in welfare reaches a value of more than 16% and then falls very quickly. For the general case with $l_1 > l_2 > 0$ the analysis becomes too complicated and computer experiments can be used to compute relative differences in welfare. We conclude part one of this section with a table containing the relative differences in welfare, subject to a decreasing market size. For the adjustment cost parameter of firm 2 we choose $l_2 = 0.05$ and concerning the production cost parameters we make the usual choice $c=0.4$, $d=-0.1$. Behavioural phases are also indicated.

Table 4.7, the relative difference in welfare for $l_1 > l_2 = 0.05$ and various market sizes.

	m=0.95	m=0.90	m=0.85	m=0.8	m=0.75	m=0.70
l_1 =0.10	(1), 0.044	(2), 0.046	(2), 0.046	(3), 0.018	(3), -0.037	(3), -0.134
l_1 =0.15	(1), 0.044	(2), 0.046	(2), 0.046	(2), 0.040	(2), 0.024	(3), -0.043
l_1 =0.20	(1), 0.044	(2), 0.046	(2), 0.046	(2), 0.040	(2), 0.024	(2), -0.011

The table reveals that for large values of l_1 the relative difference in welfare first rises somewhat and then falls during the rest of the behavioural phase 2 and more quickly during phase 3. However in general the relative difference in welfare $\Delta W/W_{cl}$ is positive in the declining market. So we may conclude that the analysis of the two benchmark cases reveals all important qualitative properties of the relative difference in welfare.

Part 2: A period with increased economic activity, i.e. $m \geq 1$

In the introduction of this section we already stated that during a period with increased economic activity, as a consequence of adjustment costs, both firms' supply levels may be restricted (in comparison to the classical output). Therefore it is self-evident that the difference in consumer surplus ΔCS ($=CS\text{-}CS_{cl}$) is negative. In Proposition 4.8, Section 4.5, we proved that the profits of two symmetric firms (with respect to adjustment costs) exceed the classical profit for a long period in the further expanding market. However it will appear that the positive difference in profits $\Delta\Pi$ isn't large enough to compensate the negative difference ΔCS. Therefore the difference in welfare is always negative in a booming market and this result is clearly opposite to the results corresponding to a period of recession. First we focus on benchmark case 1 with u_1=u_2=u>0.

Table 4.8 contains some values of the relative differences in welfare $\Delta W/W_{cl}$ for various values of u, besides the behavioural phase corresponding to the market size (we again choose c=0.4, d=-0.1 for the production cost parameters).

Table 4.8, the relative difference in welfare for u > 0 and various market sizes.

	***m*=1.05**	***m*=1.10**	***m*=1.15**	***m*=1.20**	***m*=1.25**	***m*=1.30**
***u*=0.05**	(1), **-0.043**	(3), **-0.048**	(3), **-0.050**	(3),**-0.052**	(3), **-0.053**	(3), **-0.053**
***u*=0.10**	(1), **-0.043**	(1), **-0.085**	(3), **-0.092**	(3), **-0.096**	(3), **-0.099**	(3), **-0.101**
***u*=0.15**	(1), **-0.043**	(1), **-0.085**	(1), **-0.124**	(3), **-0.133**	(3), **-0.138**	(3), **-0.142**
***u*=0.20**	(1), **-0.043**	(1), **-0.085**	(1), **-0.124**	(1), **-0.161**	(3), **-0.170**	(3), **-0.177**

Clearly the relative difference in welfare is always negative and decreases further with respect to an increasing market size. Furthermore it is obvious that the larger the adjustment costs are the more negative $\Delta W/W_{cl}$ is during phase 3. The analytical results corresponding to this benchmark case are summarized in Proposition 4.15. Because classical welfare is increasing with respect to a increasing market size, the relative difference in welfare falls less strongly than the difference in welfare. But also note that (compare Tables 4.5 and 4.8), because of the increasing classical welfare, the absolute differences in welfare ΔW are even larger than those corresponding with a declining market and a similar deviation Δm from m=1.

Proposition 4.15 (difference in welfare, concerning benchmark case 1).

Consider benchmark case 1 with $u_1 = u_2 = u > 0.$

The difference in welfare ΔW (and $\Delta W/W_{cl}$) between the case with symmetrical adjustment costs and the classical case is always negative during the phases 1 and 3 (phase 2 doesn't occur).

This difference (and also $\Delta W/W_{cl}$) decreases further with respect to an increasing market size.

Proof

Using the equilibrium outputs of the two firms, $x_1^* = x_2^*$= (1-c)/(3+2d), during phase 1 (see Section 4.5) and the expression for the classical total output, q_{cl}^*= 2(m-c)/(3+2d) with m = 1 + Δm_1, we obtain for the difference in consumer surplus during this phase

$$\Delta CS = CS - CS_{cl} = \frac{2(1-c)^2}{(3+2d)^2} - 2\left[\frac{(1-c)}{(3+2d)} + \frac{\Delta m_1}{(3+2d)}\right]^2 =$$
$$-\Delta m_1 \frac{4(1-c)}{(3+2d)^2} - (\Delta m_1)^2 \frac{2}{(3+2d)^2}$$

Using the expression for Π^i - Π_{cl}, i=1,2 of Proposition 4.8, Section 4.5 we obtain for the difference in total profits

$$\Delta\Pi = \Pi^1 + \Pi^2 - 2\Pi_{cl} = \Delta m_1 \frac{2(1-c)}{(3+2d)^2} - (\Delta m_1)^2 \frac{2(1+d)}{(3+2d)^2}$$

The difference in welfare is obtained bij adding the expressions for ΔCS and $\Delta\Pi$ and is clearly always negative and decreasing with respect to Δm_1.

$$\Delta W = W - W_{cl} = -\Delta m_1 \frac{2(1-c)}{(3+2d)^2} - (\Delta m_1)^2 \frac{2(2+d)}{(3+2d)^2}$$

The expression for the difference in welfare during phase 3 (phase 2 doesn't occur), clearly reveals that ΔW is a linearly decreasing function if m increases.
To compute ΔW we use the equilibrium outputs of both firms during phase 3 and the formulas for the expressions Π^i - Π_{cl}, i=1,2, Proposition 4.10, Section 4.5. The expressions for ΔCS, $\Delta\Pi$ and ΔW are :

$$\Delta\Pi = \Pi^1 + \Pi^2 - 2\Pi_{cl} = u\frac{2(1-c)}{(3+2d)^2} - u^2\frac{2(1+d)}{(3+2d)^2} - \Delta m_3 u \frac{4(1+d)}{(3+2d)^2}$$
$$\Delta CS = CS - CS_{cl} = -u\frac{4(1-c)}{(3+2d)^2} - u^2\frac{2}{(3+2d)^2} - \Delta m_3 u \frac{4}{(3+2d)^2}$$
$$\Delta W = \Delta CS + \Delta W = -u\frac{2(1-c)}{(3+2d)^2} - u^2\frac{2(2+d)}{(3+2d)^2} - \Delta m_3 u \frac{4(2+d)}{(3+2d)^2}$$

During phase 3 it holds that ΔW is negative and decreasing with respect to Δm_3.
[End of proof]

We illustrate the analytical results, concerning the first benchmark case, with a graphical presentation of the relative difference in welfare during a period of increased economic activity (with the choice $u_1=u_2=u=0.10$ and $c=0.4$, $d=-0.1$).

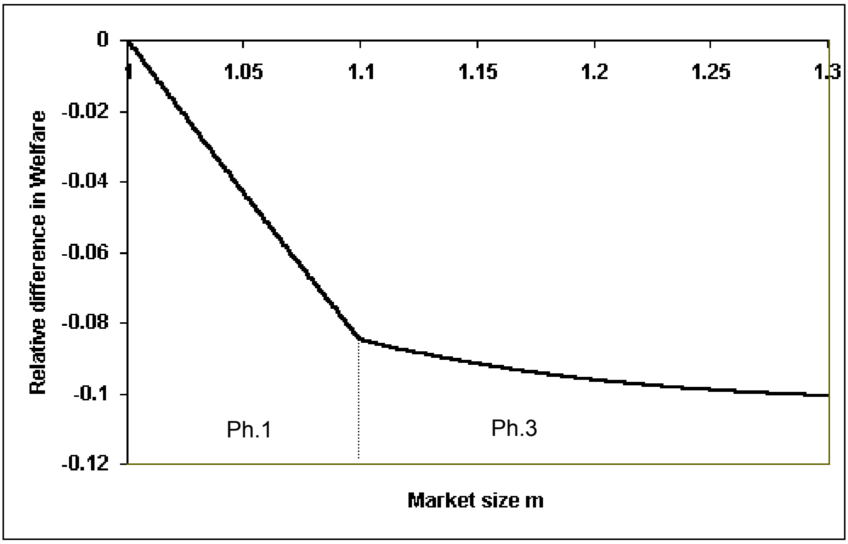


Fig. 4.21 The development of $\Delta W/W_{cl}$ during an expanding market .

Note that this relative difference gradually decreases till roughly -10%. The result of Proposition 4.15 reveals that adjustment costs in a prosperous period are not beneficial to the social welfare. The larger these adjustment costs are the more negative the (relative) difference in welfare will be (see Table 4.8). In the concluding reflections of Section 4.5 we stated that the most flexible firm has strategic benefits over its rival. Besides this fact we may conclude that flexible behaviour also benefits society.

We now continue our discussion concerning the welfare with the analysis of the second benchmark case, i.e. $u_1=u>0$, $u_2=0$. Table 4.9 reveals that the relative difference in welfare $(W-W_{cl})/W_{cl}$ is negative during the behavioural phases 2 and 3 (phase 1 doesn't occur and for the production cost parameters we make the usual choice $c=0.4$, $d=-0.1$).

Table 4.9, the relative difference in welfare for $u>0$, $u_2=0$ and various market sizes.

	m=1.05	m=1.10	m=1.15	m=1.20	m=1.25	m=1.30
u=0.05	(2), -0.009	(2), -0.017	(3), -0.020	(3),-0.021	(3), -0.022	(3), -0.023
u=0.10	(2), -0.009	(2), -0.017	(2), -0.023	(2), -0.029	(3), -0.033	(3), -0.036
u=0.15	(2), -0.009	(2), -0.017	(2), -0.023	(2), -0.029	(2), -0.034	(2), -0.038
u=0.20	(2), -0.009	(2), -0.017	(2), -0.023	(2), -0.029	(2), -0.034	(2), -0.038

These computer simulations suggest some other qualitative properties of the welfare as well.

- The negative relative difference in welfare between benchmark case 2 and the classical case decreases slowly with respect to an increasing market size.
- These negative values of the relative difference are much smaller than those corresponding to symmetrical firms (and equal values of u).

- Under the assumption that phase 3 occurs it holds that the larger the adjustment costs of firm 1 are the less beneficial this is for social welfare.

These results hold for all adjustment cost parameters u and for all reasonable production cost parameters.The analytical conclusions, concerning this second benchmark case, are summarized in Proposition 4.16 and the proof is straightforward for most values of the production cost parameter d.

Proposition 4.16 (difference in welfare, concerning benchmark case 2).

Consider benchmark case 2 with $u_1 = u > 0$ and $u_2 = 0$.

The difference in welfare ΔW as well as the relative difference in welfare $\Delta W / W_{cl}$ between the case with adjustment costs and the classical case are negative during the whole of phase 2, i.e. $m \le 1 + u \dfrac{(2+2d)}{(1+2d)}$, for

(i) all u if the production cost parameter d satisfies $d \ge -0.2594$

(ii) $u < 0.178(1-c)$ if the production cost parameter d satisfies $-0.35 \le d < -0.2594$

In case (i) ΔW and $\Delta W / W_{cl}$ always decrease if m increases.

During phase 3 ΔW decreases linearly with respect to an increasing market size.

Proof

We use the expressions for the outputs $x_1^{'}$ and $x_2^{'}$ during phase 2 (see Section 4.5) and the classical total output equals $q_{cl}^{'} = 2(m\text{-c})/(3+2d)$. Because the market size *m* equals $m = 1 + \Delta m_2$ (phase 1 doesn't occur) we obtain for the difference in consumer surplus

$$\Delta CS = CS - CS_{cl} = -\Delta m_2 \frac{(1-c)(1+2d)}{(1+d)(3+2d)^2} - (\Delta m_2)^2 \frac{(1+2d)(7+6d)}{8(1+d)^2(3+2d)^2}$$

Using the expressions for $\Pi^1\text{-}\Pi_{cl}$ and $\Pi^2\text{-}\Pi_{cl}$ of the proof of Proposition 4.8, Section 4.5, the expression for $\Delta\Pi$ becomes

$$\Delta\Pi = \Pi^1 + \Pi^2 - 2\Pi_{cl} = \Delta m_2 \frac{(1-c)(1+2d)}{2(1+d)(3+2d)^2} + (\Delta m_2)^2 \frac{(1-4d-4d^2)}{4(1+d)(3+2d)^2}$$

The difference in welfare is obtained bij adding the expressions for ΔCS and $\Delta\Pi$.

$$\Delta W = W - W_{cl} = -\frac{\Delta m_2}{2(1+d)(3+2d)^2}\left\{(1-c)(1+2d) + \Delta m_2 \frac{(8d^3+28d^2+26d+5)}{4(1+d)}\right\}$$

For $d \ge -0.2594$, the third degree polynomial $P(d) = 8d^3+28d^2+26d+5$ is nonnegative and then ΔW is always negative and decreasing with respect to Δm_2 ;this proves case (i). For $d < -0.2594$ (case (ii)) this polynomial becomes negative and $\Delta W < 0$ if the following condition is imposed on Δm_2: $\Delta m_2 < -4(1\text{-}c)(1+d)(1+2d)/P(d)$.

This condition holds during the whole of phase 2 if and only if the following holds for the parameter u:

$$u < (1-c)\frac{2(1+2d)^2}{-(8d^3+28d^2+26d+5)} = (1-c)H(d)$$

The function $H(d)$ is monotonically increasing for $-0.5 < d < -0.2594$ and $H(-0.5)=0$. The fact that $H(-0.35)=0.178$ proves case (ii).
The analysis corresponding to phase 3 is similar. Now we use the expressions for the outputs $x_1^{''}$ and $x_2^{''}$ during phase 3 (see Section 5.5, with $u=u_1$, $u_2=0$) and the classical total output $q_{cl}^{''} = 2(m-c)/(3+2d)$ with $m = 1+u(2+2d)/(1+2d)+\Delta m_3$. For the difference in consumer surplus we obtain

$$\Delta CS = CS - CS_{cl} = -u\frac{2(1-c)}{(3+2d)^2} - u^2\frac{(7+6d)}{2(1+2d)(3+2d)^2} - \Delta m_3 u\frac{2}{(3+2d)^2}$$

Using the expressions for Π^1-Π_{cl} and Π^2-Π_{cl} (see the proof of Proposition 4.10, Section 4.5) and $\Delta W = \Delta CS + \Delta\Pi$ we obtain a linearly decreasing expression for the difference in welfare with respect to Δm_3 .

$$\Delta W = W - W_{cl} = -u\frac{(1-c)}{(3+2d)^2} - u^2\frac{(8d^3+28d^2+26d+5)}{2(1+2d)^2(3+2d)^2} - \Delta m_3 u\frac{2(2+d)}{(3+2d)^2}$$

[End of proof]

We emphasize that the results concerning the difference and the relative difference in welfare hold for a wide range of production cost parameters d. If the marginal production costs are constant, the welfare results hold for all adjustment cost parameters u during phase 2 and 3. For this second asymmetrical benchmark case it holds that $u_2=0$, so we have one totally flexible firm and the negative values of the relative difference are much smaller than those corresponding to symmetrical firms (and equal values of u). Apparently the fact that one of both rivals has no adjustment costs, makes a significant difference and naturally the best "advice" for both firms is to be as flexible as possible.
We present a graphical display of the development of $\Delta W/W_{cl}$ subject to an expanding market for the parameter choice $u=0.10$, $c=0.4$ and $d=-0.1$. Comparison of Figures 4.21 and 4.22 reveals the difference between the two benchmark cases.

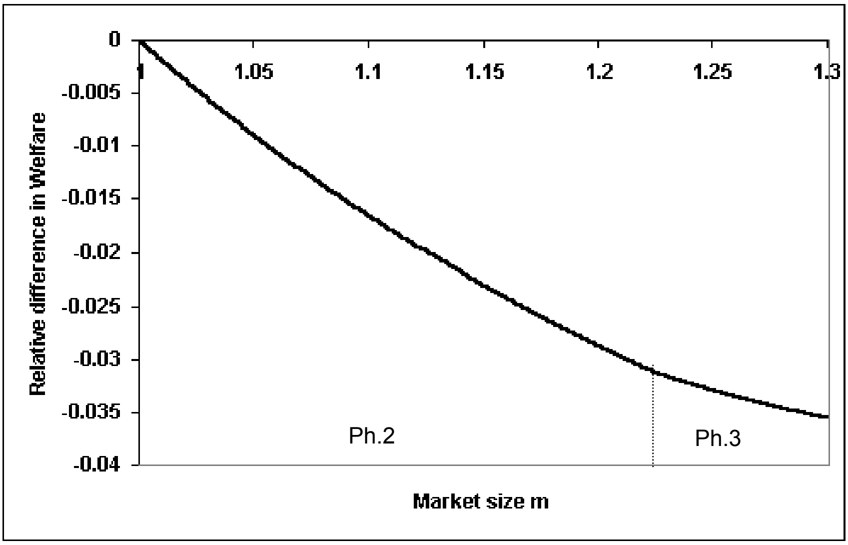


Fig. 4.22 The development of $\Delta W/W_{cl}$ during an expanding market .

To conclude part two of this section we present a table containing the relative differences in welfare in a booming market size for a general case with $u_1 > u_2 > 0$. Firm 2 's adjustment cost parameter equals $u_2 = 0.05$ (and c=0.4, d=-0.1). Behavioural phases are also indicated.

Table 4.10, the relative difference in welfare for $u_1 > u_2 = 0.05$ and various market sizes.

	m=1.05	m=1.10	m=1.15	m=1.20	m=1.25	m=1.30
u_1 =0.10	(1), -0.043	(2), -0.055	(2), -0.065	(3), -0.069	(3), -0.072	(3), -0.073
u_1 =0.15	(1), -0.043	(2), -0.055	(2), -0.065	(2), -0.072	(2), -0.078	(3), -0.083
u_1 =0.20	(1), -0.043	(2), -0.055	(2), -0.065	(2), -0.072	(2), -0.078	(2), -0.083

We observe that the relative difference in welfare $\Delta W/W_{cl}$ is always negative. Again we may conclude that the analysis of the two benchmark cases reveals all important qualitative properties of the relative difference in welfare.

Part 3: Total welfare over (parts of) a business cycle

Clearly social welfare and the difference in welfare (compared to the classical case with no adjustment costs) are influenced by environmental turbulence. Propositions 4.13 - 4.16 and the corresponding tables, concerning both benchmark cases, reveal that (in general) the difference in welfare ΔW is positive during a period with decreased economic activity and is negative during a properous period. Under the assumption that the number of reactions of both competitors, n, is large enough (see also Section 4.6) we can use integral calculus and computer simulations to examine relative total welfare over a period with $m \leq 1$, $m \geq 1$, and over a complete business cycle. By using these mathematical techniques the total effect of the adjustment costs on welfare over a whole period can be examined. Like in Section 4.6 the goniometrical function $m(t)=1-\alpha sin(\pi t)$ with $0 \leq t \leq 2$ can serve as a model for a (part of a) symmetrical business cycle with amplitude α. Again the time interval $0 \leq t \leq 1$ corresponds to a period of decreased economic activity, whereas for $1 \leq t \leq 2$ the market size corresponding to a prosperous period is described. Concerning the following analysis, the quantities F_d, F_e and F express the total welfare (with adjustment costs) as a fraction of the total classical welfare (without adjustment costs) during the periods with $m \leq 1$, $m \geq 1$ and the whole period respectively. The following holds (under the assumption that n is large enough and with the usual notations W and W_{cl} for the welfare and the classical welfare respectively):

$$F_d = \frac{\int_0^1 W(m(t))\mathrm{d}t}{\int_0^1 W_{cl}(m(t))\mathrm{d}t}, \quad F_e = \frac{\int_1^2 W(m(t))\mathrm{d}t}{\int_1^2 W_{cl}(m(t))\mathrm{d}t} \quad \text{and} \quad F = \frac{\int_0^2 W(m(t))\mathrm{d}t}{\int_0^2 W_{cl}(m(t))\mathrm{d}t}$$
$$m(t) = 1-\alpha sin(\pi t) \quad \text{and} \quad W_{cl} = \frac{(m-c)^2(4+2d)}{(3+2d)^2} \tag{4.36}$$

In our examinations both benchmark cases will play a central role and we will use integral calculus to prove some general propositions in cases that only one behavioural phase occurs (during the declining or booming market). In Section 4.6 we already noted that, if several phases occur, several expressions for $W(m(t))$ have to be distinguished, thus complicating the analysis. However computer simulations will be used to support our insight concerning these complicated cases.
We start our discussion with benchmark case 1's analysis and first we prove a Proposition that holds for business cycles with smaller amplitudes, i.e. $\alpha \leq min(l,u)$.

Proposition 4.17 (relative total welfare, concerning benchmark case 1)

Consider the first benchmark case with $l_1 = l_2 = l > 0$ and $u_1 = u_2 = u > 0$.
Assume that the amplitude of the sine function satisfies $\alpha \leq Min\{l;u\}$.
Over the whole period with decreased economic activity ($m \leq 1$) it holds that

$$F_d = \frac{W_{tot}}{W_{cl,tot}} = \frac{(1-c)^2(4+2d) - (4\alpha/\pi)(1-c)(3+2d)}{(1-c)^2(4+2d) - (4\alpha/\pi)(1-c)(4+2d) + (2+d)\alpha^2}.$$

Over the whole period with increased economic activity ($m \geq 1$) it holds that

$$F_e = \frac{W_{tot}}{W_{cl,tot}} = \frac{(1-c)^2(4+2d) + (4\alpha/\pi)(1-c)(3+2d)}{(1-c)^2(4+2d) + (4\alpha/\pi)(1-c)(4+2d) + (2+d)\alpha^2}.$$

And over the whole cycle (with decreased and increased economic activity as well) it holds that $F = \dfrac{W_{tot}}{W_{cl,tot}} = 1 - \dfrac{\alpha^2}{2(1-c)^2 + \alpha^2}$.

Proof

For the whole period with decreased economic activity ($m \leq 1$) we substitute $\Delta m_1 = \alpha sin(\pi t)$ (Δm_1 is positive for $0 \leq t \leq 1$) in the expressions for ΔW and W_{cl} .The (goniometric) expressions during phase 1 equal:

$$\Delta W = \frac{2(1-c)}{(3+2d)^2}\alpha sin(\pi t) - \frac{(4+2d)}{(3+2d)^2}\alpha^2 sin^2(\pi t)$$

$$W_{cl} = (1-c)^2\frac{(4+2d)}{(3+2d)^2} - \frac{2(1-c)(4+2d)}{(3+2d)^2}\alpha sin(\pi t) + \frac{(4+2d)}{(3+2d)^2}\alpha^2 sin^2(\pi t).$$

The fraction F_d can be computed using $W = W_{cl} + \Delta W$ and
$\int_0^1 sin(\pi t)\mathrm{d}t = 2/\pi$, $\int_0^1 sin^2(\pi t)\mathrm{d}t = 1/2$.

For the whole period with $m \geq 1$ we obtain the same expressions for ΔW and W_{cl}, but now for $1 \leq t \leq 2$. Using $\int_1^2 sin(\pi t)\mathrm{d}t = -2/\pi$, $\int_1^2 sin^2(\pi t)\mathrm{d}t = 1/2$ the fraction F_e can be computed. And, concerning the complete business cycle, we can use the goniometric expressions for $0 \leq t \leq 2$. Then the fraction F equals

$F = 1 + \dfrac{\int_0^2 \Delta W(m(t))\mathrm{d}t}{\int_0^2 W_{cl}(m(t))\mathrm{d}t}$. Using $\int_0^2 sin(\pi t)\mathrm{d}t = 0$, $\int_0^2 sin^2(\pi t)\mathrm{d}t = 1$ we obtain F.

[End of proof]

Naturally these expressions for the relative total welfare simplify concerning d=0 , which corresponds to constant marginal production costs.Tables containing computed values of the fractions F_d, F_e and F will support our insight in the meaning of these relative total welfares. First we draw some conclusions from the expressions of Proposition 4.17. Of course we have to realize that these expressions only hold for business cycles with a limited amplitude. In part 1 of this section we showed that the (relative) difference in welfare, corresponding with benchmark case 1 is positive during phase 1 and a part of phase 3 and therefore the following statement will not be surprising

- for all $0 < c < 1$, $d > -1/2$ and $\alpha < (4/\pi)(1-c)/(2+d)$ it holds that F_d, the relative total welfare during the whole period with $m \leq 1$, exceeds 1. So for all reasonable adjustment cost parameters l this property holds (for instance for the production cost parameters c=0.5, d=0 the parameter l has to be smaller than 0.32).

And during a period with a increased economic activity we observed in part 2 that the (relative) difference, corresponding to benchmark case 1 is always negative. Clearly it holds that

- the fraction F_e is less than 1 for all $\alpha \leq min\{l,u\}$, $0 < c < 1$ and $d > -1/2$

The third result, concerning the relative total welfare over a complete business cycle, can be concluded directly from the expression for F and states that

- for all $\alpha \leq min\{l,u\}$, $0 < c < 1$ and $d > -1/2$ the fraction F is less than 1.

Apparently, for the symmetric benchmark case 1 total welfare over a whole business cycle is less than the total welfare corresponding to firms without adjustment costs. However, the property $F_d > 1$ implies that adjustment costs benefit welfare during a whole period of decreased economic activity. We may expect that, in case the business cycle's amplitude is large enough to include behavioural phase 3 (i.e.$\alpha > min\{l,u\}$), relative total welfare always decreases. This is simply due to the fact that, in phase 3, extra adjustment costs occur.

We now present three tables with values of the fractions F_d, F_e and F for several values of α and c. We choose d=0 (constant marginal production costs) and adjustment cost parameters equal to $l_1=l_2=0.10$, $u_1=u_2=0.10$. Proposition 4.17 can be applied for business cycle's amplitude $\alpha = 0.1$, whereas for all larger values of the amplitude we use computer simulations to determine the relevant quotients $W_{tot}/W_{cl,tot}$ over a period with $m\leq1$, $m\geq1$ and a complete business cycle. The fractions are rounded off to two decimals.

Table 4.11, relative total welfare $F_d = W_{tot}/W_{cl,tot}$ over the period with $m\leq1$.

	c=0.2	c=0.4	c=0.6
α=0.10	1.04	1.05	1.07
α=0.20	1.04	1.04	1.00
α=0.30	1.02	0.99	0.79

Table 4.12, relative total welfare $F_e = W_{tot}/W_{cl,tot}$ over the period with $m\geq1$.

	c=0.2	c=0.4	c=0.6
α=0.10	0.96	0.95	0.92
α=0.20	0.94	0.92	0.88
α=0.30	0.93	0.91	0.87

Table 4.13, relative total welfare $F = W_{tot}/W_{cl,tot}$ over the whole cycle.

	c=0.2	c=0.4	c=0.6
α=0.10	0.99	0.99	0.97
α=0.20	0.97	0.96	0.91
α=0.30	0.96	0.93	0.86

We observe that the fraction F is less than 1, not only for $\alpha \leq min\{l,u\}$, but also for larger amplitudes of the cycle. Furthermore all fractions F_d, F_e and F are decreasing with respect to an increasing business cycle's amplitude (for $\alpha \leq min\{l,u\}$ this can be proved).

The first and important conclusion from Proposition 4.17 (which holds for a limited set of α's), that the relative total welfare during a whole period with decreased economic activity exceeds 1 also holds for a wider range of amplitudes.
It strikes the eye that the fraction F is less than 1, which means that the total welfare - over the complete (and symmetrical) business cycle - corresponding to the case with equal adjustment costs ($l = u$) is less than the total welfare concerning the classical case (with no adjustment costs at all). Apparently there exists an asymmetry in the difference in welfare ΔW with respect to the respective periods with $m \leq 1$ and $m \geq 1$. In Section 4.6 we observed that a similar asymmetry exists for the difference in profits $\Delta \Pi$ in comparison to the classical profits which can be explained by the asymmetry of the classical profit (Π^i_{cl} contains the quadratic term $(m\text{-}c)^2$). There we argued that supply's restriction (compared to classical market supply) during the prosperous period results in a larger (and negative) difference in profits than the upholding of the production during the period of recession. This observation served as an explanation for the phenomenon that the totally flexible firm (=the classical case) has an advantage over the whole cycle. The fact that, in spite of a symmetrical business cycle, the fraction F is less than 1 also deserves an explanation. Because it holds that $\Delta W = \Delta \Pi + \Delta CS$, the argumentation concerning the asymmetry in $\Delta \Pi$ explains "half" of the asymmetry in ΔW : total contribution of $\Delta \Pi$ to the difference in welfare over the whole cycle is indeed negative because the disadvantages of the firm with adjustment costs during the period of prosperity exceed the advantages during the period with $m \leq 1$. What about the contribution of ΔCS, the difference in consumer surplus? It holds that $CS = 0.5\,(q^*)^2$, where q^* equals total market supply, so CS_{cl} (like Π_{cl}) displays an asymmetry with respect to the market size $m=1$ as well. And so the larger CS is, the more impact a change in supply level has (the difference in consumer surplus resulting from a (small) change in output equals approximately the product of the two factors "original output" and "price-difference"). Therefore the negative effect on the consumer surplus caused by the restriction of the total output during the period with $m \geq 1$ exceeds the positive effect of supply's upholding on CS during the period of recession. So the total contribution of ΔCS to the difference in welfare over a complete business cycle is also negative implying that the fraction F is less than 1.

We have to realize that Table 4.13 contains the fractions F for benchmark case 1 with $l_1=l_2=l=0.1$ and $u_1=u_2=u=0.1$, i.e. the adjustment costs with respect to a decreasing and an increasing level of production are equal. In case of asymmetrical adjustment costs with respect to a decreasing and an increasing level of production, $l \neq u$, the following holds (and is confirmed by computer simulations):

- If, departing from a case with $l=u$, u decreases, the factor F increases and becomes larger than 1 for $u=0$. For instance the case $c=0.4$ $(d=0)$,$\alpha=0.2$ and $l=0.1$, $u=0.05$ leads to a fraction F equal to 0.98 and the case $c=0.4$,$\alpha=0.2$ and $l=0.1$,$u=0$ leads to $F=1.01$.
- If, departing from a case with $l=u$, l decreases, the factor F decreases (somewhat) further. For instance the case $c=0.4$,$\alpha=0.2$ and $l=0.05$, $u=0.10$ leads to a fraction F equal to 0.95 and the case $c=0.4$,$\alpha=0.2$ and $l=0$,$u=0.10$ leads also to $F=0.95$.

Not only the "shape" of the adjustment costs but also the shape of the business cycle can influence the fraction F. Such an asymmetry in the business cycle can be reflected in the amplitude corresponding to the recession, α_r, and the amplitude α_p concerning the prosperous period. The following holds (and is confirmed by computer simulations)

- If, departing from a symmetrical business cycle (α_r=α_p), α_p decreases, i.e. the recession is deeper than the revival, the factor F increases. The welfare advantages, due to adjustment costs, during the period of decreased economic activity are the decisive factor (for α_p=0, F becomes larger than 1).
- If, departing from a symmetrical business cycle), α_r decreases, i.e. the recession is less deeper than the revival, the factor F decreases. Now welfare disadvantages, due to adjustment costs, during the prosperous period are the decisive factor.

We continue our analysis with the examination of the second benchmark case. If the production behaviour of both firms remains in behavioural phase 2, so again for limited values of the business cycle's amplitude, we are able to derive expressions for the three fractions F_d, F_e and F. We present these results in the following proposition; although these expressions look rather complicated these derivations lead to some general conclusions concerning relative total welfare.

Proposition 4.18 (relative total welfare, concerning benchmark case 2)

Consider the second benchmark case with $l_1 = l > 0, l_2 = 0$ and $u_1 = u > 0, u_2 = 0.$

Assume that the amplitude of the sine function $\alpha \le Min\left\{ l\frac{(2+2d)}{(1+2d)}; u\frac{(2+2d)}{(1+2d)} \right\}$

so that only phase 2 occurs. Let $P(d) = 8d^3 + 28d^2 + 26d + 5.$

Over the whole period with decreased economic activity ($m \le 1$) it holds that

$$F_d = \frac{W_{tot}}{W_{cl,tot}} = 1 + \frac{(\alpha/\pi)(1-c)(1+2d)/(1+d) - \alpha^2 P(d)/(4+4d)^2}{(1-c)^2(4+2d) - (4\alpha/\pi)(1-c)(4+2d) + (2+d)\alpha^2}$$

Over the whole period with increased economic activity ($m \ge 1$) it holds that

$$F_e = \frac{W_{tot}}{W_{cl,tot}} = 1 - \frac{(\alpha/\pi)(1-c)(1+2d)/(1+d) + \alpha^2 P(d)/(4+4d)^2}{(1-c)^2(4+2d) + (4\alpha/\pi)(1-c)(4+2d) + (2+d)\alpha^2}$$

And over the whole cycle (with decreased and increased economic activity as well)

it holds that $F = \frac{W_{tot}}{W_{cl,tot}} = 1 - \frac{P(d)}{(2+d)(4+4d)^2} \cdot \frac{\alpha^2}{2(1-c)^2 + \alpha^2}$

Proof

For the whole period with decreased economic activity ($m \le 1$) we substitute $\Delta m_2 = \alpha sin(\pi t)$ (Δm_2 is positive for $0 \le t \le 1$) in the expressions for ΔW and W_{cl} corresponding to phase 2 (see the proof of Proposition 4.14).These (goniometric) expressions equal:

$$\Delta W = \frac{(1-c)(1+2d)}{2(1+d)(3+2d)^2}\alpha sin(\pi t) - \frac{(8d^3+28d^2+26d+5)}{8(1+d)^2(3+2d)^2}\alpha^2 sin^2(\pi t)$$
$$W_{cl} = (1-c)^2\frac{(4+2d)}{(3+2d)^2} - \frac{2(1-c)(4+2d)}{(3+2d)^2}\alpha sin(\pi t) + \frac{(4+2d)}{(3+2d)^2}\alpha^2 sin^2(\pi t).$$

Then the fraction F_d (using W=W_{cl}+ΔW) equals

$F_d = 1 + \dfrac{\int_0^1 \Delta W(m(t))\mathrm{d}t}{\int_0^1 W_{cl}(m(t))\mathrm{d}t}$. Using $\int_0^1 sin(\pi t)\mathrm{d}t = 2/\pi$, $\int_0^1 sin^2(\pi t)\mathrm{d}t = 1/2$ we obtain F_d.

For the period with $m \geq 1$ and the complete business cycle we can use the same expressions for ΔW and W_{cl} with $1 \leq t \leq 2$ and $0 \leq t \leq 2$ respectively.

[End of proof]

From the expressions of the fractions F_d, F_e and F we can draw some general conclusions which are comparable to the conclusions corresponding to the first benchmark case.

- Clearly the relative total welfare F_e over the whole period with increased economic activity is less than 1. The same holds for the fraction F corresponding to the relative total welfare for the complete business cycle (if the polynomial $P(d) > 0$ i.e. $d >$ -0.2594).These results are similar to the results concerning benchmark case 1.
- For a very wide range of production cost parameters and adjustment cost parameters (l) the relative total welfare F_d over the whole period with decreased economic activity exceeds 1.

This second conclusion needs some further explanation and can be clarified by further analysis of the formula for F_d. It always holds that $F_d > 1$ if the polynomial $P(d)$ is nonpositive, so for $-0.5 < d \leq -0.2594$. And if $P(d)$ is positive, for $d > -0.2594$, the numerator in the expression for F_d is positive if

$\alpha < \dfrac{16(1-c)}{\pi}\cdot\dfrac{(1+d)(1+2d)}{(8d^3+28d^2+26d+5)}$. The condition for the amplitude α holds for the whole of phase 2 if the adjustment cost parameter l satisfies the condition

$l < \dfrac{8(1-c)}{\pi}\cdot\dfrac{(1+2d)^2}{(8d^3+28d^2+26d+5)}$. In this expression the fractional function of d is a monotonically decreasing function (with respect to d) so for $-0.2594 < d \leq 0$ the condition imposed on l equals: $l < 1.6(1\text{-}c)/\pi = 0.51(1\text{-}c)$. This means that for c=0.5 the adjustment cost parameter l has to be smaller than 0.255. Indeed for a wide range of production cost parameters and adjustment cost parameters the property $F_d > 1$ holds.

By computer simulation we examine whether these properties of F_d, F_e and F still hold for larger amplitudes α of the business cycle. The results of these computations for several linear cost functions (d=0) and for the benchmark case with l_1=l=0.10, l_2=0 and u_1=u=0.10, u_2=0 are presented in the following tables. For all values of $\alpha \leq 0.2$ only phase 2 occurs and the corresponding simulation results confirm the complicated formulas for the relative total welfare of Proposition 4.18.

Table 4.14, relative total welfare $F_d = W_{tot}/W_{cl,tot}$ over the period with $m \le 1$.

	c=0.2	c=0.4	c=0.6
α=0.10	1.01	1.01	1.02
α=0.20	1.02	1.03	1.04
α=0.30	1.02	1.02	1.00

Table 4.15, relative total welfare $F_e = W_{tot}/W_{cl,tot}$ over the period with $m \ge 1$.

	c=0.2	c=0.4	c=0.6
α=0.10	0.99	0.99	0.98
α=0.20	0.98	0.98	0.97
α=0.30	0.98	0.97	0.96

Table 4.16, relative total welfare $F = W_{tot}/W_{cl,tot}$ over the whole cycle.

	c=0.2	c=0.4	c=0.6
α=0.10	1.00	1.00	1.00
α=0.20	1.00	0.99	0.98
α=0.30	0.99	0.98	0.96

We have to realize that a value of F_d=1.04 (Table 4.14, c=0.6, α=0.2) indicates that total welfare over such a period with decreased economic activity is 4% more than the total welfare concerning the classical situation (with totally flexible firms).
These simulations show that the results of Proposition 4.18 still hold for larger values of the business cycle's amplitude. If we compare these fractions of benchmark case 2 with the corresponding values (the same c and α) of benchmark case 1 it strikes the eye that in all cases, concerning the fractions F_e and F, the deviations from 1 are smaller than the deviations corresponding to benchmark case 1. This observation also holds for most fractions F_d (except for a few cases with larger c and α). Naturally this is due to the fact that in benchmark case 2 one firm has no adjustment costs so the effects on welfare become smaller. For benchmark case 1 we made some remarks on the fraction F in relation to an asymmetry in adjustment costs and an asymmetry in the business cycle. Concerning the "shape" of the adjustment costs, i.e. an asymmetry between l_1 and u_1, and the shape of the business cycle, i.e. nonequal amplitudes corresponding to the period of recession and the prosperous period, results similar to those concerning benchmark case 1 hold.

For general cases with $l_1 > l_2 > 0$ and $u_1 > u_2 > 0$ simulations confirm that the values of the fractions (for corresponding c and α) lie between the values of both benchmark cases. Because, concerning both benchmark cases, the relative total welfare F_d over a period with decreased economic activity exceeds 1 (except for some cases with large α, so that phase 3 plays a crucial role) it is beneficial for the welfare that both firms possess organizational inertia, reflected by adjustment costs. One of the main conclusions of Section 4.4 is that, in the declining market, the firm with the largest adjustment cost parameter has a strategic advantage over its rival. If both firms (firms' owners) are aware of these strategic benefits (rational adaptation perspective), an adjustment cost-setting game may start; both firms increase their level of organizational inertia in turns. Naturally such a game ends with larger adjustment cost parameters l_1 and l_2 concerning both competitors and as a consequence total welfare benefits from this situation. However, the results of Section 4.5 reveal that, in a booming market, the most flexible firm is the strategically beneficial one, so an adjustment cost-setting game could end with two totally flexible firms, i.e. $u_1 = u_2 = 0$.

Because, for all cases with $u_1>0$ or $u_2>0$, the relative total welfare over a prosperous period is always less than 1, society benefits most from totally flexible firms. We can state that two firms with equal adjustment cost parameters $l>0$ and $u=0$ benefit welfare the most. Apparently there exists an endogeneous drive toward welfare-maximizing adjustment costs, because of rivals' strategic advantages.
We conclude Section 4.7 with a table containing the relative total welfare F (over the whole business cycle) and the relative total welfare F_d over a whole period with decreased economic activity as well. These fractions are presented for several adjustment cost parameters $l_1=l_2=l$ (and $u_1=u_2=0$) and various values of the business cycle's amplitude. Production cost parameters equal $c=0.4$ and $d=0$. The fractions F_d and F both increase with respect to an increasing l.

Table 4.17, relative total welfares F_d and F for various l and α.

	l=0.05	l=0.10	l=0.15	l=0.20
α=0.10	F_d=1.03, F=1.01	F_d=1.05, F=1.02	F_d=1.05, F=1.02	F_d=1.05, F=1.02
α=0.20	F_d=1.02, F=1.00	F_d=1.04, F=1.01	F_d=1.06, F=1.02	F_d=1.08, F=1.02
α=0.30	F_d=0.99, F=1.00	F_d=0.99, F=1.00	F_d=1.00, F=1.00	F_d=1.03, F=1.01

8. Appraisal

In this chapter we examined the effects of linear (asymmetrical) adjustment costs in a declining and expanding market, around the Cournot-Nash equilibrium for a (neutral) market size of m=1. These adjustment costs reflect the level of organizational inertia. We focused on the output levels and profits of both competitors and on the consequences concerning social welfare as well. Because of these adjustment costs the (Cournot) reaction curves of the firms contain an interval of inertia and as an implication of this temporary inertness - in the declining or expanding market - three behavioural (supply) phases can be distinguished. The analysis of Section 4.4, concerning a declining market, revealed that the firm with the largest adjustment costs outperforms its rival. If the difference in adjustment costs is large enough the most flexible firm may face nonpositive profits during the "inertia outperforms flexibility" phase 2 or the "complete flexibility" phase 3 (see Propositions 4.5 and 4.7) and possibly is forced to exit. So during a period with decreased economic activity inertia outperforms flexibility, which confirms Hannan and Freeman's (1984) inertia hypothesis. However, the analysis of Section 4.5 shows that, during a period with increased economic activity, flexibility pays off; the firm with the smallest adjustment costs - with respect to an increasing production level - enjoys increasing advantages over its rival in the further expanding market.

Section 4.7 deals with social welfare and the main macro-economic conclusion is that there exist welfare maximizing adjustment costs. During a period with decreased economic activity (rather) large adjustment costs have a beneficial effect on welfare , whereas during a prosperous period total flexibility is the most advantageous. The awareness of both rivals of the strategic consequences of the adjustment costs may result in an adjustment-cost-setting game. The end result of such a game - large adjustment costs with respect to a decreasing output and total flexibility corresponding to an increasing production level - would precisely lead to the maximizing of the welfare. Of course much more work has to be done to model such an adjustment-cost-setting game and this will be the subject of future research.

Another macro-economic conclusion is that the adjustment costs can serve as a policy instrument to bring peace to the economy, because due to adjustment costs the volatility of total market supply decreases (Section 4.6). One of the limitations of our model is that we only consider (asymmetrical) adjustment costs around the original Cournot-Nash equilibrium. Therefore future research has to be dedicated to the consequences of stepwise adjustment costs around several production levels. Furthermore, concerning this Chapter's analysis, both firms control equally efficient production technologies reflected in equal production cost functions. In Chapter 3 we also examined the consequences of differences in production efficiency and naturally the implications of such a heterogeneity in efficiency will be an interesting subject for further analysis. Another limitation of the Cournot-model of this chapter is that only two competitors have been considered. Many studies deal with n-firm competition and often the main subject of these studies is the stability of the resulting Cournot-game. Therefore we are highly interested in the effects of adjustment costs on stability in the case with 3 or more competitors and the implications of such costs on market supply. So a lot of future work has to be dedicated to these expansions of the research of this Chapter, concerning the effects of adjustment costs.

CHAPTER 5

COURNOT COMPETITION WITH NON-PROFIT-MAXIMIZING OBJECTIVES IN GENERAL AND PREFERENCE FOR MARKET SHARE IN PARTICULAR

1. Introduction

In this chapter our main object of study will be a Cournot duopoly game with a non-profit-maximizing objective function and therefore this research is strongly related to the study of Chapter 3. There we analyzed Cournot duopoly competition with a non-profit-maximizing objective as well and we stated that - supported by the literature on managerial economics - managers are not only motivated by profit maximization, but like to achieve growth (size) as well. In a recent paper Deneffe and Masson (2002) formulate a test for non-profit-maximizing hospitals and apply it to hospitals in Virginia. Both hypotheses, that not-for-profit hospitals maximize profits or maximize pure output, are rejected on the basis of their data. The conclusion of their research is that these hospitals consider both profits and outputs as objectives.

In Chapter 3 the introduction of a double motivation in the Cournot competition model, namely profit and size, combined with habit formation, heterogeneity in production costs and demand turbulence permitted the identification of various equilibria. It appeared that firms may decide to stay in the market notwithstanding the prospect of a negative profit. In Chapter 3 we emphasized the “blueprint” of the firm, reflected in managerial inertia. From the standpoint of Organizational Ecology (OE), managers’ preference for size is determined by a firm’s culture and cannot be changed as quickly as their environments change. Hannan and Freeman (1984) state that: “Since lags in response can be longer than typical environmental fluctuations and longer than the attention spans of decision makers and outside authorities, inertia often blocks structural change completely”. This OE perspective enabled us to study the implications of all sorts of weight combinations (α_1,α_2) (where α_i equals the weight attributed to production size concerning the objective function of firm i).

As already mentioned in Chapter 3, the “delegation games” (Vickers (1985), Fershtman and Judd (1987), Sklivias (1987) and Basu (1995)) provide another explanation for managers’ non-profit-maximizing motives. The analysis of Fershtman and Judd (1987) demonstrates that competing firms’ owners will often distort their managers’ objectives away from strict profit maximization for strategic reasons. However these two-stage games, which lead to fixed weights α, assume highly rational behaviour of firms’ owners. In this chapter we examine, among other topics, the implications of another double motivation in the Cournot competition model, namely profit and market share. Again we analyze the consequences of all sorts of weight combinations (a_1,a_2), where a_i is the weight attributed to market share concerning firm i’s objective function.

We start with the examination of the consequences of more general non-profit-maximizing motives on outputs and profits of two competitors. We deal with the important question whether general extra motives - like the preference for size in Chapter 3 – may imply strategic advantages in direct competition (whether firms are aware of these strategic implications or not). Furthermore the effect on social welfare of such extra motives is investigated. Of course comparative statics ,which leads to some general properties, only holds under the assumption that the Cournot-Nash equilibrium is stable, so general conditions for the stability of the equilibrium of the duopoly Cournot game will be considered.

In particular we will examine the consequence of status motives between two competitors - besides the traditional profit-maximizing motive - in our Cournot model. Such status motives are not new because in modeling consumer behaviour, status motives have been used to describe the additional purpose of consuming a so-called positional good. Rauscher (1992) considers a status game between two neighbours and notes that “Since each person wants to rank as high as possible in society, he or she consumes the more of these positional goods the more the other people are consuming. This may result in a rat race or treadmill in which inefficiently large quantities of positional goods are consumed”. In his example Rauscher defines the status function s by $s=y/Y$, where y and Y equal the respective amounts of the positional good acquired by one family and the “competing” family and he shows that chaotic adjustment paths of consumption are a possible outcome of rational utility-maximizing behaviour.

In line with these models on consumer behaviour we will introduce a status motive as well in this chapter and we will examine the consequences of preference for market share. This very natural and plausible status motive of both competing firms leads to a non-linear Cournot reaction curve and under certain assumptions even to a hill-shaped reaction function. This interesting fact provides a strong link to Chapter 2 where the assumption of a hill-shaped Cournot reaction curve with a nonzero monopoly output leads to possibly complicated dynamics with chaotic output- and profit- patterns. In Chapter 2 we raised the crucial question whether an economic rationale can be provided for the (rather extreme) nonlinear shape of the Cournot reaction curves. Since the publication of the corresponding paper “Chaotic Patterns in Cournot Competition” (1990) several researchers have tried to provide an answer to this essential problem and at least two valuable contributions should be mentioned. The first microeconomic foundation is provided by Puu (1991,1998) who proved that under the assumption of the often used nonlinear inverse demand function (the quantity demanded is reciprocal to the price), and, secondly, constant returns to scale production processes (leading to a linear production cost function) the reaction curves are unimodal. Second Kopel (1996) assumes production cost functions with an interfirm externality (costs are influenced by the output of the rival). In this paper marginal production costs first decrease with respect to an increasing output level of the rival and then increase corresponding to a further rise of the production quantities of the rival. For an example of such an interfirm externality Kopel refers to Poston and Steward (1978) who mention the book-buying habit: “If you start producing books, when no one else is, you will not sell many. There will be no book-buying habit among the public, and no distribution industry to take and display your products to Hull and Halifax. On the other hand, if other people are producing books in huge numbers, yours will be invisible among so many, and again you will sell rather few. Your sales will be best when other output exists but is moderate”. Under this specific assumptions of interfirm externality with respect to production costs Kopel shows that the functional form of the Cournot reaction curve appears to be quadratic (the two dimensional equivalent of the famous quadratic map of May (1976)) and possibly leads to nonstable equilibria and very complicated two dimensional nonlinear dynamics. Although these two microeconomic foundations to explain the nonlinear form of the reaction curves, are well thought-out and are also pleasant from a mathematical point of view - complicated chaotic time-paths concerning the output of the firms occur under certain conditions imposed on the model parameters - these explanations display some shortcomings. First the reaction functions $R(q)$ possess the property that $R(0)=0$, that is: monopoly output is zero. However, as already stated in Chapter 2, it doesn't seem very realistic that the strategic answer to a zero production of the rival is also a zero output. Second one cannot escape the impression that, in order to derive these hill-shaped reaction curves in the paper of Kopel, the assumption of a rather mathematically constructed cost function is needed. And in the papers of Puu a, (for economists) second-favourite demand curve, leads to a zero monopoly output as well.

In this chapter we will present a third and plausible rationale for hill-shaped Cournot reaction curves, which also overcomes the shortcoming of the zero monopoly output. The assumptions that (i) firms consider both profits and market share as objectives and (ii) the inverse demand curve is linear, lead to hill-shaped reaction curves with a positive monopoly output for certain parameter constellations. As we will see the natural assumption that the preference for the market share is included as a part of the maximand (besides the classical profit) unfortunately - but inevitable - leads to a complicated functional form of the reaction curves (a third degree equation has to be solved using Cardan's method). In spite of this analytical complication it is possible to give some examples of complicated dynamical time-paths of output and profits, which will be the subject of Chapter 6.

How is this chapter organized? Section 5.2 contains the model with the non-profit-maximizing objective and some notes on the properties of the non-profit part of the objective function. A general expression for the slope of the reaction function will be considered in relation to these mathematical properties. Section 5.3 focuses on the consequences of the "status function" - the non-profit part of the objective function - concerning production levels and profits of the two firms and social welfare. Using a power series approximation some general propositions can be derived under the assumption that the "weight" attributed to the status function in comparison with the "weight" attributed to the profit part of the objective function is small.

In Section 5.4 this status function is specified; the market share i.e. the fraction of a firm's output in relation to total market supply is concerned. This specific non-profit part leads to several properties of the resulting reaction curves and also leads to a typology of the reaction curves which depends on the weight attributed to market share. If this weight is large enough (reflecting a high level of preference for market share as a blueprint of the firm), the resulting reaction curve appears to be hill-shaped. The consequences for the (stability of) Cournot-Nash outputs and profits for the benchmark case (1) of two completely symmetrical firms - with respect to the attributed weight to market share - will the subject of Section 5.5. There, the stability of the Cournot-Nash equilibrium allows us to examine the influence of this complete symmetry in preference for market share on social welfare as well.

Section 5.6 deals with a second benchmark case - one firm attributes a weight $a_1 = a$ to the status function whereas the other firm only maximizes its profit (classical, $a_2 = 0$). This complete asymmetry leads also to a stable Cournot-Nash equilibrium for $a \leq a_{bif}$ which again makes the analysis of equilibrium profits and social welfare possible. This examination reveals the existence of a specific profit maximizing and an advantage maximizing weight a for the "market share loving" firm (similar to the results of Chapter 3, where preference for size or sales is considered). Section 5.7 contains some concluding remarks on this chapter.

2. The general model and the properties of the non-profit part

Like in Chapter 4 we consider a Cournot competition model, concerning two firms, under the assumption of a homogeneous market and a linearly decreasing inverse demand function (with intercept A and negative slope $-b$, see also Chapter 4, Section 4.2). In this Chapter we don't study the consequences of a business cycle and this is reflected in a neutral market size $m=1$ (using the terminology of Chapter 4). Again we include production processes (functions) with constant, increasing and decreasing returns to scale i.e. production cost functions with constant, decreasing and increasing unit (marginal) costs. Our aim is not to restrict this study to linear cost functions only, but to prove the propositions for a wide class of quadratic cost functions (with nonnegative marginal costs). Similar to Chapter 4 we normalize the model with the general linearly decreasing inverse demand function. Let x_1 and x_2 be the (normalized) outputs of firm 1 and 2 respectively and let $s(x_1,x_2)$ be the non-profit part of the objective function U (the maximand). Under the assumption of myopic expectations of the two rivals ($x_{i,t}^e = x_{i,t-1}$, $i = 1,2$) and the restriction of nonnegative prices we can formulate the following optimization problem for the firms 1 and 2:

$$\begin{aligned}
&\text{Max}\, U^1(x_{1,t}\,;x_{2,t-1}) = x_{1,t}(1-x_{1,t}-x_{2,t-1}) - cx_{1,t} - d(x_{1,t})^2 + a_1 s(x_{1,t},x_{2,t-1})\\
&\text{with respect to } x_{1,t}, \text{and subject to the restriction } 0 \le x_{1,t} + x_{2,t-1} \le 1\\
&\text{Max}\, U^2(x_{2,t}\,;x_{1,t-1}) = x_{2,t}(1-x_{2,t}-x_{1,t-1}) - cx_{2,t} - d(x_{2,t})^2 + a_2 s(x_{2,t},x_{1,t-1})\\
&\text{with respect to } x_{2,t}, \text{and subject to the restriction } 0 \le x_{2,t} + x_{1,t-1} \le 1
\end{aligned} \tag{5.1}$$

Note that we use the symbol U for the maximand because we consider a utility function - adopted from the theory of consumer behaviour - which consists of two parts, the (classical) profit part Π and the non-profit part s (this part will be a specified function in Section 5.4). The nonnegative parameters a_1 and a_2 equal the weights which the firms 1 and 2 attribute to the non-profit part of their objective function respectively. Furthermore note that both firms possess the same functional expression concerning the non-profit parts and that only the role of the outputs is interchanged. So in the modeling we only need one function (with its specific properties) and to clarify this imagine that s equals the market share. Then the expressions for the non-profit parts of firm 1 and 2 equal $x_1/(x_1+x_2)$ and $x_2/(x_2+x_1)$ respectively. It is obvious from this model that, like in Chapter 4, we consider two firms which control equally efficient production processes and for the parameters c and d it holds that (i) $0 < c < 1$ and (ii) $d > (-c/2) > -1/2$. So the only asymmetry between the two competitors is determined by the two weights a_1 and a_2 attributed to the non-profit parts. For the choice $a_1=a_2=0$ we obtain the classical Cournot competition model (the usual textbook benchmark case) resulting in a stable Cournot-Nash equilibrium. We refer to Chapter 4, Section 4.2 for an overview of the equilibrium profits Π_{cl}^{i} , the consumer surplus CS_{cl} and the welfare W_{cl} (choose $m=1$ in these expressions). We reflect on the properties of the non-profit part s and for the sake of brevity we will often use the term "status" for the function s. In his study concerning a status game between two neighbours Rauscher (1992) imposes several natural conditions - widely used in the theory of consumer behaviour - on the utility function (such as quasi-concavity) and keeping in mind these natural conditions we impose the following conditions on the non-profit part s (the brief notations s_{x_1} and s_{x_2} are used to indicate the partial derivatives of the function s with respect to the first and second argument respectively):

(i) $(s_{x_1} =)\ \dfrac{\partial s(x_1,x_2)}{\partial x_1} > 0,$ (ii) $(s_{x_2} =)\ \dfrac{\partial s(x_1,x_2)}{\partial x_2} < 0$ and (iii) $(s_{x_1 x_1} =)\ \dfrac{\partial^2 s(x_1,x_2)}{\partial (x_1)^2} \le 0$ (5.2)

(These three conditions are formulated concerning firm 1 and if we interchange the two output variables we obtain the conditions for firm 2). The first mathematical condition states that the marginal status with respect to one's own output level is positive; if the output of the rival is fixed, the more the firm produces the more status it acquires. Condition (iii) expresses the plausible property that the marginal status decreases (to be precise:is non-increasing) with respect to one's own production (if you already are the happy owner of ten old-timers an extra one will not raise your status very much anymore; the surplus value of an extra car decreases). Condition (ii) expresses the fact that status is decreasing if the output level of the rival increases. Rauscher imposes a fourth condition on the status function namely

(iv) $(s_{x_1 x_2} =)\ \dfrac{\partial^2 s(x_1,x_2)}{\partial x_1 \partial x_2} \le 0$ (5.3)

This fourth inequality states that marginal status is a decreasing (or non-increasing) function of the rival's production level x_2. The more the rival produces, the more difficult it is to raise one's own status by producing additional units of the good. If the non-profit part s equals (only) preference for output - in Chapter 3 we modeled the preference for size combined with habit formation - then $s(x_1,x_2) = x_1$ for firm 1 and the conditions (i),(iii) and (iv) are satisfied with $s_{x_1} = 1, s_{x_1 x_1} = 0$ and $s_{x_1 x_2} = 0$. Note that $s_{x_2} = 0$. As we will show in Section 5.4 the choice $s(x_1,x_2) = x_1/(x_1+x_2)$ (equals the market share of firm 1) satisfies the conditions (i), (ii) and (iii) and the fourth condition is satisfied for $x_1 \le x_2$. These qualitative properties can have important implications for the slope of the reaction curves. The maximization problem of firm 1 leads to the following equation for the marginal utility (apart from the restriction $x_{1,t} + x_{2,t-1} \le 1$):

$$\frac{\partial U^1}{\partial x_{1,t}} = 1 - c - (2+2d)x_{1,t} - x_{2,t-1} + a_1 \frac{\partial s(x_{1,t}, x_{2,t-1})}{\partial x_{1,t}} = 0 \tag{5.4}$$

Note that the marginal utility decreases monotonically with respect to the variable $x_{1,t}$ because it holds that (use condition (iii) for s and the fact that $d > -1/2$)

$$\frac{\partial^2 U^1}{\partial (x_{1,t})^2} = -(2+2d) + a_1 \frac{\partial^2 s(x_{1,t}, x_{2,t-1})}{\partial (x_{1,t})^2} \le -(2+2d) < 0 \tag{5.5}$$

For $a_1=0$ the equation for the marginal utility leads to the classical Cournot reaction curve with a constant negative slope ($= -\ 1/(2+2d)$). However property (i) of the function s implies that, if the weight a_1 is positive, the value of the marginal utility in each point of the classical curve is positive. So we may conclude that, because of the monotonically decreasing marginal utility with respect to $x_{1,t}$, the response of firm

1 to the rival's output $x_{2,t\text{-}1}$ is higher than the corresponding reaction concerning the classical case.

The qualitative property (i) of the non-profit part of the objective function leads to an outward shift of the (graph of the) classical Cournot reaction curve (note that in Chapter three preference for size also led to an outward shift of the reaction curves). Without solving the central equation for the marginal utility we can derive a general expression for the slope of the reaction curve by using the technique of implicit differentiation. Implicit differentiation of the equation $\partial U^1 / \partial x_{1,t} = 0$ with respect to the rival's output in the previous period, $x_{2,t\text{-}1}$, leads to:

$$-(2+2d)\frac{\mathrm{d}x_{1,t}}{\mathrm{d}x_{2,t-1}} - 1 + a_1\left\{\frac{\partial^2 s(x_{1,t},x_{2,t-1})}{\partial (x_{1,t})^2}.\frac{\mathrm{d}x_{1,t}}{\mathrm{d}x_{2,t-1}} + \frac{\partial^2 s(x_{1,t},x_{2,t-1})}{\partial x_{1,t}\partial x_{2,t-1}}\right\} = 0 \tag{5.6}$$

From this equation the general formula for the slope of the reaction curve of firm 1 can be solved:

$$\frac{\mathrm{d}x_{1,t}}{\mathrm{d}x_{2,t-1}} = \frac{1 - a_1\dfrac{\partial^2 s(x_{1,t},x_{2,t-1})}{\partial x_{1,t}\partial x_{2,t-1}}}{-(2+2d) + a_1\dfrac{\partial^2 s(x_{1,t},x_{2,t-1})}{\partial (x_{1,t})^2}} \tag{5.7}$$

For the classical case corresponding with $a_1=0$ we obtain the usual constant negative slope of the reaction curve. And in case that the non-profit part s equals preference for size - $s(x_{1,t},x_{2,t\text{-}1}) = x_{1,t}$ for firm 1 - the fact that all the second order partial derivatives are zero implies that the slope also equals the classical slope $-1/(2+2d)$.

At this point it is important to emphasize that the slope of the reaction curve is completely determined by the properties (iii) and (iv) of the non-profit part of the objective function (the positive slope is associated with strategic complements, whereas the classical negative slope is related to strategic substitutes; note that in general the slope equals $\frac{\mathrm{d}x_{1,t}}{\mathrm{d}x_{2,t-1}} = -\frac{\partial^2 U}{\partial x_{1,t}\partial x_{2,t-1}} / \frac{\partial^2 U}{\partial (x_{1,t})^2}$). To obtain a hill-shaped reaction curve the sign of the slope has to change from positive to negative with respect to an increasing production level of the rival. This mathematical expression provides us with an instrument to examine the possibility of such a sign change in relation with non-profit maximizing motives of firms which forms the main subject of an analysis of van Witteloostuijn and Boone (1997). In this analysis, concerning the plausibility of hill-shaped reaction curves, they also discuss interpretations of such an asymmetrical reaction pattern of a firm and provide an interpretation by considering a situation where one firm produces the monopoly output. They argue that "If a (potential) rival enters the market the monopolist is willing to increase its production initially [this argumentation is based on the theory of entry-deterrence strategies; added]. However when the size of the rival becomes too big this expansion strategy isn't profitable anymore. In other words: In contrast with the usefulness of an aggressive strategy, the benefit of accommodation increases with respect to the size of the competitor (which implies the standard negative slope of the Cournot-reaction curve)".

The third property of the "status" function s - the decrease of marginal status with respect to one's own production - implies that the denominator of the expression of the slope remains negative. So the sign of the numerator has to change from negative to positive, with respect to an increasing production level of the rival, to obtain our desired unimodal reaction curve. If the fourth property of the function s, as proposed by Rauscher, were to hold for every $x_{1,t}$ and $x_{2,t\text{-}1}$ the sign of the reaction curve would always be negative. However, anticipating the results of Section 5.4, it will appear that in the case that s equals the market share the expression $\frac{\partial^2 s(x_{1,t},x_{2,t-1})}{\partial x_{1,t}\partial x_{2,t-1}}$ can be positive indeed for small values of the rival's output.

3. The non-profit part as a strategic instrument and welfare consequences

In this section we will examine the influence of small weights, attributed to the non-profit part s of the maximand, on the profits of two competitors and we will also investigate the effect of these weights on the social welfare. In other words: the parameters a_1 and a_2 of the general Cournot competition model formulated in the previous section are considered to be small (and nonnegative). This essential condition imposed on the weight-parameters allows us to approximate the equilibrium outputs, the equilibrium profits of both firms and the welfare as well using a so called Taylor power series around $a_1=a_2=0$. For $a_1=a_2=0$ this power series solution equals exactly the classical equilibrium output (or profit or welfare) of the duopoly Cournot game and furthermore this approximation provides exact expressions for the derivatives of the equilibrium quantities with respect to the weights a_i . The technique of approximation is often used in the case that a small non-linear disturbance of a set of linear equations occurs. Because the non-profit parts s with small weights can be considered as small disturbances of the classical system of linear equations this technique can be applied adequately in our examination. Naturally the comparative statica which leads to some general properties only holds under the assumption that the Cournot-Nash equilibrium is stable. We start our analysis with the derivation of such a Taylor series for both equilibrium outputs. Then we prove that the Cournot-Nash equilibrium remains stable for small weights a_1 and a_2 attributed to the "status" function s.

The location of the equilibrium depends on the two weights a_1 and a_2 so let $(x_1^*(a_1,a_2);\ x_2^*(a_1,a_2))$ be equal to the Cournot-Nash equilibrium of the two firms. For both weights $a_1=a_2=0$ we obtain $(x_1^*(0,0);x_2^*(0,0)) = ((1-c)/(3+2d);\ (1-c)/(3+2d))$, the classical outcome. From the general optimization model concerning the two firms (see Section 5.2) we obtain a set of nonlinear equations by setting the marginal utility functions of both firms equal to zero. We anticipate the fact that the equilibrium is stable and leave out the subscripts t and $t-1$ of the outputs and we also leave out the weights a_1 and a_2 in the arguments of the partial derivatives for brevity's sake in the notation.The equilibrium $(x_1^*(a_1,a_2);\ x_2^*(a_1,a_2))$ has to satisfy the following equations:

$$\begin{aligned} &1-c-(2+2d)x_1^*(a_1,a_2)-x_2^*(a_1,a_2)+a_1\left\{\frac{\partial s(x_1,x_2)}{\partial x_1}\right\}_{(x_1^*,x_2^*)}=0, \text{ for firm 1} \\ &1-c-x_1^*(a_1,a_2)-(2+2d)x_2^*(a_1,a_2)+a_2\left\{\frac{\partial s(x_2,x_1)}{\partial x_2}\right\}_{(x_1^*,x_2^*)}=0, \text{ for firm 2} \end{aligned} \tag{5.8}$$

Note that in the equation for firm 2 the role of the two (equilibrium) outputs is interchanged. For the expression of the partial derivative of firm 2 we can write

$$\left\{\frac{\partial s(x_2,x_1)}{\partial x_2}\right\}_{(x_1^*,x_2^*)}=\left\{\frac{\partial s(x_1,x_2)}{\partial x_1}\right\}_{(x_2^*,x_1^*)} \tag{5.9}$$

This rewritten expression clarifies that we only need the functional expression of the partial derivative of the "status" function $s(x_1,x_2)$ with respect to x_1 and, to simplify the notation in the forthcoming analysis, we will use the notation s_{x_1} for this derivative. The set of the two nonlinear equations can be rewritten in a matrix-vector form as follows:

$$\text{Using } \begin{bmatrix} 2+2d & 1 \\ 1 & 2+2d \end{bmatrix}^{-1} = \frac{1}{(1+2d)(3+2d)} \begin{bmatrix} 2+2d & -1 \\ -1 & 2+2d \end{bmatrix} \text{ we obtain}$$

$$\begin{bmatrix} x_1^*(a_1,a_2) \\ x_2^*(a_1,a_2) \end{bmatrix} = \frac{1}{(1+2d)(3+2d)} \begin{bmatrix} 2+2d & -1 \\ -1 & 2+2d \end{bmatrix} \cdot \left\{ \begin{bmatrix} 1-c \\ 1-c \end{bmatrix} + \begin{bmatrix} a_1 s_{x_1}(x_1^*,x_2^*) \\ a_2 s_{x_1}(x_2^*,x_1^*) \end{bmatrix} \right\} \tag{5.10}$$

Let a be the maximum of the two weights a_1 and a_2 and let the expansion of the equilibrium-vector in a power-series be given by

$$\begin{bmatrix} x_1^*(a_1,a_2) \\ x_2^*(a_1,a_2) \end{bmatrix} = \begin{bmatrix} x_{1,0} \\ x_{2,0} \end{bmatrix} + a_1 \begin{bmatrix} x_{1,1} \\ x_{2,1} \end{bmatrix} + a_2 \begin{bmatrix} x_{1,2} \\ x_{2,2} \end{bmatrix} + (a_1)^2 \begin{bmatrix} x_{1,3} \\ x_{2,3} \end{bmatrix} + (a_2)^2 \begin{bmatrix} x_{1,4} \\ x_{2,4} \end{bmatrix} + (a_1 a_2) \begin{bmatrix} x_{1,5} \\ x_{2,5} \end{bmatrix} + O(a^3) \tag{5.11}$$

Note that the first index of x on the right side of this expansion indicates the firm number (1 or 2) and that the second index indicates the rank in the power series. The remainder $O(a^3)$ - with Landau's symbol "O" - expresses that the maximum error of this power series in comparison with the exact solution is a constant C times a^3 (this constant can be estimated by an expression containing second and third order partial derivatives of the "status" function s and in many standard textbooks on analysis this error term is referred to as the Taylor remainder). This power series for the equilibrium outputs of both firms enables us to expand the expressions $s_{x_1}(x_1^*(a_1,a_2),x_2^*(a_1,a_2))$ and $s_{x_1}(x_2^*(a_1,a_2),x_1^*(a_1,a_2))$ as well (using second order partial derivatives):

$$\begin{aligned} & s_{x_1}(x_1^*(a_1,a_2),x_2^*(a_1,a_2)) = s_{x_1}(x_{1,0},x_{2,0}) + (a_1 x_{1,1} + a_2 x_{1,2}) s_{x_1 x_1}(x_{1,0},x_{2,0}) + \\ & (a_1 x_{2,1} + a_2 x_{2,2}) s_{x_1 x_2}(x_{1,0},x_{2,0}) + O(a^2), (a \to 0) \quad \text{and} \\ & s_{x_1}(x_2^*(a_1,a_2),x_1^*(a_1,a_2)) = s_{x_1}(x_{2,0},x_{1,0}) + (a_1 x_{2,1} + a_2 x_{2,2}) s_{x_1 x_1}(x_{2,0},x_{1,0}) + \\ & (a_1 x_{1,1} + a_2 x_{1,2}) s_{x_1 x_2}(x_{2,0},x_{1,0}) + O(a^2), (a \to 0) \end{aligned} \tag{5.12}$$

We now substitute the power series concerning the partial derivatives, into the nonlinear matrix-vector equation and use the fact that $x_{1,0} = x_{2,0}$ (the solution for $a_1=a_2=0$ is symmetric!). So the arguments of the partial derivatives of s always both equal $(1-c)/(3+2d)$ and we leave them out in the following expression. This leads to:

$$\begin{bmatrix} x_{1,0} \\ x_{2,0} \end{bmatrix} + a_1 \begin{bmatrix} x_{1,1} \\ x_{2,1} \end{bmatrix} + a_2 \begin{bmatrix} x_{1,2} \\ x_{2,2} \end{bmatrix} + (a_1)^2 \begin{bmatrix} x_{1,3} \\ x_{2,3} \end{bmatrix} + (a_2)^2 \begin{bmatrix} x_{1,4} \\ x_{2,4} \end{bmatrix} + (a_1 a_2) \begin{bmatrix} x_{1,5} \\ x_{2,5} \end{bmatrix} = \begin{bmatrix} {}^{(1-c)}\!/_{(3+2d)} \\ {}^{(1-c)}\!/_{(3+2d)} \end{bmatrix}$$

$$\begin{gathered} a_1 \frac{s_{x_1}}{(1+2d)(3+2d)} \begin{bmatrix} 2+2d \\ -1 \end{bmatrix} + a_2 \frac{s_{x_1}}{(1+2d)(3+2d)} \begin{bmatrix} -1 \\ 2+2d \end{bmatrix} + \\ (a_1)^2 \frac{x_{1,1} s_{x_1 x_1} + x_{2,1} s_{x_1 x_2}}{(1+2d)(3+2d)} \begin{bmatrix} 2+2d \\ -1 \end{bmatrix} + (a_2)^2 \frac{x_{2,2} s_{x_1 x_1} + x_{1,2} s_{x_1 x_2}}{(1+2d)(3+2d)} \begin{bmatrix} -1 \\ 2+2d \end{bmatrix} + \\ \frac{(a_1 a_2)}{(1+2d)(3+2d)} \begin{bmatrix} (2+2d)\{x_{1,2} s_{x_1 x_1} + x_{2,2} s_{x_1 x_2}\} - \{x_{2,1} s_{x_1 x_1} + x_{1,1} s_{x_1 x_2}\} \\ -\{x_{1,2} s_{x_1 x_1} + x_{2,2} s_{x_1 x_2}\} + (2+2d)\{x_{2,1} s_{x_1 x_1} + x_{1,1} s_{x_1 x_2}\} \end{bmatrix} + O(a^3) \end{gathered} \tag{5.13}$$

Setting equal the vectors on the left- and right side of this expression, corresponding to equal powers of a_1, a_2 or a_1a_2, we first note that the solution for $a_1=a_2=0$ equals the classical Cournot-Nash equilibrium (we already used that fact). By comparing the vectors corresponding to a_1 and a_2 we observe that the symmetry in the "zero-solution" leads to $x_{1,1} = x_{2,2}$ and $x_{2,1} = x_{1,2}$. This latter fact can be used to solve the vectors corresponding with $(a_1)^2$, $a_2)^2$ and a_1a_2 and leads to $x_{1,3} = x_{2,4}$ and $x_{2,3} = x_{1,4}$. The following proposition contains the complete expression for the power series approximation of the equilibrium outputs of both firms.

Proposition 5.1.

Let a_1 and a_2 be the weights attributed to the non-profit part s of the maximand by the two firms respectively. Then the second order approximation of the Cournot-Nash equilibrium is given by

$$\begin{gathered} \begin{bmatrix} x_1^*(a_1,a_2) \\ x_2^*(a_1,a_2) \end{bmatrix} = \begin{bmatrix} {}^{(1-c)}\!/_{(3+2d)} \\ {}^{(1-c)}\!/_{(3+2d)} \end{bmatrix} + a_1 \frac{s_{x_1}}{(1+2d)(3+2d)} \begin{bmatrix} 2+2d \\ -1 \end{bmatrix} + a_2 \frac{s_{x_1}}{(1+2d)(3+2d)} \begin{bmatrix} -1 \\ 2+2d \end{bmatrix} + \\ (a_1)^2 \frac{s_{x_1}\{(2+2d)s_{x_1x_1} - s_{x_1x_2}\}}{(1+2d)^2(3+2d)^2} \begin{bmatrix} 2+2d \\ -1 \end{bmatrix} + (a_2)^2 \frac{s_{x_1}\{(2+2d)s_{x_1x_1} - s_{x_1x_2}\}}{(1+2d)^2(3+2d)^2} \begin{bmatrix} -1 \\ 2+2d \end{bmatrix} + \\ (a_1a_2) \frac{s_{x_1}\{-s_{x_1x_1} + (2+2d)s_{x_1x_2}\}}{(1+2d)(3+2d)^2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + O(a^3), (a \to 0) \end{gathered} \tag{5.14}$$

The arguments of the partial derivatives $s_{x_1}, s_{x_1x_1}$ and $s_{x_1x_2}$ in this expression are equal to the classical Cournot-Nash equilibrium values. Naturally this expression plays a central role in computing (equilibrium) profits of both firms, consumer surplus and social welfare. Because of the error term $O(a^3)$ it holds that the smaller the weights are, the better this approximation will be. As already mentioned in the introduction of this section this approximation provides exact expressions for the derivatives of the equilibrium outputs with respect to the weights a_i for $a_i=0$. The first derivative with respect to the weight a_i reveals information about the increase of the output - or size - of firm i if the weight attributed to the non-profit part of the objective function is small and increases somewhat. And if the "status" function s equals the size of firm i, i.e. $s(x_i)=x_i$, the second (and higher order) partial derivatives in the formula for the outputs are zero and the expression provides exactly the equilibrium output. Before we continue with the analysis we have to show that for small weights a_1 and a_2 the Cournot-Nash equilibrium is stable. In the proof we apply the general expression for the slope of the reaction curve of Section 5.2 on the equilibrium. Furthermore we use a theorem on the local stability of equilibria of a system of nonlinear difference equations which can be found in Devaney (1989) .

Proposition 5.2.

The (local) stability of the Cournot - Nash equilibrium $(x_1^*(a_1,a_2),x_2^*(a_1,a_2))$ is guaranteed for small values of the weights a_1 and a_2 and the absolute value of the eigenvalues of the linearized system even decreases if the "status" function s satisfies the condition $-s_{x_1x_2}+ s_{x_1x_1}/(2+2d) < 0$ ($s_{x_1x_2}$ and $s_{x_1x_1}$ at point $(\frac{(1-c)}{(3+2d)},\frac{(1-c)}{(3+2d)})$.

Proof.

The output levels $x_{1,t}$ and $x_{2,t}$ of both firms satisfy the following system of nonlinear first order difference equations:

$$1-c-(2+2d)x_{1,t}-x_{2,t-1}+a_1\frac{\partial s(x_{1,t},x_{2,t-1})}{\partial x_{1,t}}=0$$
$$1-c-(2+2d)x_{2,t}-x_{1,t-1}+a_2\frac{\partial s(x_{2,t},x_{1,t-1})}{\partial x_{2,t}}=0$$

We linearize this system around the Cournot-Nash equilibrium $(x_1^*(a_1,a_2);x_2^*(a_1,a_2))$ and examine the stability of the linearized system in this equilibrium. The eigenvalues equal $\lambda_{1,2}=\pm\sqrt{\frac{\mathrm{d}x_1}{\mathrm{d}x_2}\cdot\frac{\mathrm{d}x_2}{\mathrm{d}x_1}}$ where $\frac{\mathrm{d}x_1}{\mathrm{d}x_2}$ and $\frac{\mathrm{d}x_2}{\mathrm{d}x_1}$ equal the slopes of the reaction curves in $(x_1^*(a_1,a_2);x_2^*(a_1,a_2))$ of firm 1 and firm 2 respectively. Using the general expression for the slope of the reaction curve (eq. 5.7,Section 5.2) - a similar expression holds for the slope concerning firm 2 - we obtain a general formula for the eigenvalues of the linearized system:

$$\lambda_{1,2}=\pm\sqrt{\frac{1-a_1s_{x_1x_2}(x_1^*,x_2^*)}{-(2+2d)+a_1s_{x_1x_1}(x_1^*,x_2^*)}\cdot\frac{1-a_2s_{x_1x_2}(x_2^*,x_1^*)}{-(2+2d)+a_2s_{x_1x_1}(x_2^*,x_1^*)}}$$

(Note that for $a_1=a_2=0$ we obtain the classical eigenvalues $\lambda_{1,2}=\pm 1/(2+2d)$ with absolute values less than 1 for $d>-1/2$ which guarantees even global stability)

Using the following approximating expressions for the partial derivatives (with $a=\max\{a_1,a_2\}$) $s_{x_1x_2}(x_1^*,x_2^*)=s_{x_1x_2}(x_{1,0},x_{2,0})+O(a)$, $s_{x_1x_1}(x_1^*,x_2^*)=s_{x_1x_1}(x_{1,0},x_{2,0})+O(a)$, $s_{x_1x_2}(x_2^*,x_1^*)=s_{x_1x_2}(x_{2,0},x_{1,0})+O(a)$, $s_{x_1x_1}(x_2^*,x_1^*)=s_{x_1x_1}(x_{2,0},x_{1,0})+O(a)$ and the fact that the classical "zero-order" solution is symmetric, i.e. $x_{1,0}=x_{2,0}=(1-c)/(3+2d)$ we can rewrite the expression for the eigenvalues of the linearized system:

$$\lambda_{1,2}=\pm\frac{1}{(2+2d)}\sqrt{1+(a_1+a_2)\{-s_{x_1x_2}+s_{x_1x_1}/(2+2d)\}}+O(a^2),(a\to 0)$$

The arguments of the partial derivatives both equal $(1-c)/(3+2d)$. Clearly the absolute values of these eigenvalues satisfy the necessary and sufficient condition for the stability of linearized system - $|\lambda_{1,2}|<1$ - for small values of a_1 and a_2.
And if the second order partial derivatives of the "status" function satisfy the condtion $-s_{x_1x_2}+s_{x_1x_1}/(2+2d)<0$ at point $((1-c)/(3+2d),(1-c)/(3+2d))$ the absolute value of the eigenvalues even decreases in comparison with the classical case.
And if the linearized system is stable, i.e. the Cournot-Nash equilibrium is a positive attractor, local stability of the nonlinear system is guaranteed.

[End of proof]

If the "status" function s equals the market share - $s(x_1,x_2) = x_1/(x_1+x_2)$ - it holds that $s_{x_1x_2}=\dfrac{(x_1-x_2)}{(x_1+x_2)^3}=0$ and $s_{x_1x_1}=\dfrac{-2x_2}{(x_1+x_2)^3}<0$ in $((1-c)/(3+2d),(1-c)/(3+2d))$ and Proposition 5.2 reveals that, for small weights a_1,a_2, the absolute values of the eigenvalues decrease in comparison with the classical case. This means that the attribution of small weights to the market share has a stabilizing influence on the equilibrium. In the Sections 5.5 and 5.6 we will investigate whether this important property also holds for general weights a_1 and a_2.

Having established the stability of the Cournot-Nash equilibrium for small weights, we now continue the analysis of the profits of both firms with an easy mind. The substitution of both power series of the equilibrium outputs (Proposition 5.1) in the expression for the profit of firm 1 – $\Pi^1(a_1,a_2) = x_1^*(a_1,a_2)\{1-x_1^*(a_1,a_2)-x_2^*(a_1,a_2)\} - cx_1^*(a_1,a_2) - d\{x_1^*(a_1,a_2)\}^2$ - leads to the following approximating power series:

The equilibrium output of firm 1 can be approximated by the power series

$$\Pi^1(a_1,a_2)=\Pi^1(0,0)+a_1\frac{(1-c)s_{x_1}}{(1+2d)(3+2d)^2}-a_2\frac{(2+2d)(1-c)s_{x_1}}{(1+2d)(3+2d)^2}+$$
$$(a_1)^2\frac{s_{x_1}}{(1+2d)^2(3+2d)^2}\left\{-(2+2d)(2d^2+4d+1)s_{x_1}+\frac{(1-c)}{(3+2d)}\left[(2+2d)s_{x_1x_1}-s_{x_1x_2}\right]\right\}+$$
$$(a_2)^2\frac{s_{x_1}}{(1+2d)^2(3+2d)^2}\left\{(1+d)s_{x_1}-\frac{(2+2d)(1-c)}{(3+2d)}\left[(2+2d)s_{x_1x_1}-s_{x_1x_2}\right]\right\}+$$
$$(a_1a_2)\frac{s_{x_1}}{(1+2d)^2(3+2d)^2}\left\{-s_{x_1}-\frac{(1+2d)(1-c)}{(3+2d)}\left[-s_{x_1x_1}+(2+2d)s_{x_1x_2}\right]\right\}+O(a^3),(a\to 0)$$

(5.15)

Here $\Pi^1(0,0)$ equals the classical profit $(=(1-c)^2(1+d)/(3+2d)^2)$ and the arguments of the partial derivatives are both $(1-c)/(3+2d)$. The expression for the profit of firm 2 can be obtained by interchanging the parameters a_1 and a_2.

This expression looks rather complicated, but we will use it to derive some important properties of the welfare as a consequence of the non-profit part of the objective function. The first order part of this formula (only the terms corresponding to a_1, a_2 and the term $\Pi^1(0,0)$ are considered) already reveals some interesting consequences. The following proposition summarizes these implications for firm 1 :

Proposition 5.3.

For small weights a_1 and a_2 attributed to the function s, with $s_{x_1} > 0$, it holds that

(i) The production level (=size) of firm 1 exceeds the production level of its rival if $a_1 > a_2$.
(ii) The profit level of firm 1 exceeds the classical profit (a_1=a_2=0) if $a_1 > (2+2d)a_2$.
(iii) The profit level of firm 1 exceeds the profit level of its rival if $a_1 > a_2$.

(the same conclusions hold for firm 2 by interchanging a_1 and a_2)

Proof.
Using the first order approximation of the Cournot-Nash equilibrium vector (Proposition 5.1) the difference of the equilibrium outputs of firm 1 and firm 2 equals

$$x_1^*(a_1,a_2) - x_2^*(a_1,a_2) = (a_1 - a_2)\frac{s_{x_1}}{(1+2d)} + O(a^2),(a \to 0).$$

This proves part (i). Part (ii) is also proved easily because the first order approximation of the difference $\Pi^1(a_1,a_2) - \Pi^1(0,0)$ (use the first order part of the expression for $\Pi^1(a_1,a_2)$) equals

$$\Pi^1(a_1,a_2) - \Pi^1(0,0) = [a_1 - (2+2d)a_2]\frac{(1-c)s_{x_1}}{(1+2d)(3+2d)^2} + O(a^2),(a \to 0)$$

This difference is positive if $a_1 - (2+2d)a_2 > 0$. Part (iii) is proved by substracting the first order expressions for the profits of firm 1 and firm 2:

$$\Pi^1(a_1,a_2) - \Pi^2(a_1,a_2) = (a_1 - a_2)\frac{(1-c)s_{x_1}}{(1+2d)(3+2d)} + O(a^2),(a \to 0).$$

[End of proof]

We reflect on the results of the Propositions 5.1 and 5.3. Note that the first order expression (Proposition 5.1) for the output of firm 1, $x_1^*(a_1,a_2)$, shows that this output increases in its own weight a_1 - $\frac{\partial x_1^*}{\partial a_1} = \frac{(2+2d)s_{x_1}}{(1+2d)(3+2d)}$ - and decreases in it's rival's weight a_2 because $\frac{\partial x_1^*}{\partial a_2} = -\frac{s_{x_1}}{(1+2d)(3+2d)}$. For linear costs, i.e. d=0, the first force is twice as influential as the second. This property can be considered as a generalisation - for small weights of course - of the results of Chapter 3 where we observed that firm i's equilibrium output increases in its own preference for size and decreases in its rival's preference for size (the habit formation in the model of Chapter 3 has no influence on the location of the equilibrium).The second and third part of Proposition 5.3 reveal that the weight attributed to the non-profit part of the maximand may serve as a strategic instrument (of course this assumes highly rational behaviour of the firm and contradicts managerial inertia). Firm 1 can

outperform its rival by choosing the weight of the "status" function a_1 such that $a_1 > a_2$ (part (ii) also shows that there exists an incentive for raising the weight a_1).
By raising the weight of the non-profit part the profit can rise above the classical level at the expense of the rival. Of course we have to realize that this general conclusion only holds under the condition that both weights are small. For larger weights a_1 and a_2 the particular specification of the "status" function s has to be studied. Because weights are small, a profit-maximizing weight a_i, concerning firm i, cannot be computed using expression 5.15 (compare the two-stage games of Fershtman and Judd (1987) and others, where owners write managers' contracts and "dictate" the weight a). From the viewpoint of managerial inertia, the weights are rather fixed and the firm with the largest weight is favoured in a Darwinian selection process.

We continue our general analysis with an examination of the implications of non-profit maximizing objectives on social welfare and we will consider two benchmark cases. The first benchmark case consists of two symmetric firms, with respect to the weights attributed to the function s, i.e. $a_1 = a_2 = a > 0$.
Because the firms already possess the same production cost function we deal with completely symmetric firms.The choice $a_1 = a > 0$, $a_2 = 0$ determines the second, asymmetric, benchmark case. Our subject of interest consists of two parts. First the influence of an increase of the weight a on the welfare W, indicated by the first derivative ${}^{\mathrm{d}W}\!/\!{}_{\mathrm{d}a}$, is examined. Second also the concavity (or convexity) of the welfare function $W(a)$ with respect to the weight, determined by ${}^{\mathrm{d}^2W}\!/\!{}_{\mathrm{d}a^2}$, is considered. To investigate the concavity of the welfare function, concerning the two benchmark cases, we use second order approximations of the profits $\Pi^i(a_1,a_2)$ (already derived), the consumer surplus $CS(a_1,a_2)$ and the welfare $W(a_1,a_2)$ corresponding with the Cournot-Nash equilibrium (for the symmetrical benchmark case 1 another general approach is also possible - using implicit differentiation - and we will conclude this section with a discussion of this method). The following proposition summarizes the general expressions for the consumer surplus and the welfare. Of course these rather complicated expressions simplify for the two benchmark cases, but the general expression (concerning a_1 and a_2) also makes a general statement on the concavity of the welfare function possible (Appendix 5.1).

Proposition 5.4.

The second order power series for the consumer surplus equals

$$CS(a_1,a_2) = CS(0,0) + a_1\frac{2(1-c)s_{x_1}}{(3+2d)^2} + a_2\frac{2(1-c)s_{x_1}}{(3+2d)^2} +$$

$$(a_1)^2\frac{s_{x_1}}{(1+2d)^2(3+2d)^2}\left\{\tfrac{1}{2}(1+2d)^2 s_{x_1} + \frac{2(1+2d)(1-c)}{(3+2d)}\left[(2+2d)s_{x_1x_1} - s_{x_1x_2}\right]\right\}+$$

$$(a_2)^2\frac{s_{x_1}}{(1+2d)^2(3+2d)^2}\left\{\tfrac{1}{2}(1+2d)^2 s_{x_1} + \frac{2(1+2d)(1-c)}{(3+2d)}\left[(2+2d)s_{x_1x_1} - s_{x_1x_2}\right]\right\}+$$

$$(a_1a_2)\frac{s_{x_1}}{(1+2d)^2(3+2d)^2}\left\{(1+2d)^2 s_{x_1} + \frac{4(1+2d)(1-c)}{(3+2d)}\left[-s_{x_1x_1} + (2+2d)s_{x_1x_2}\right]\right\} + O(a^3),(a\to 0)$$

Here $CS(0,0)$ equals the classical consumer surplus (= ${}^{2(1-c)^2}\!/\!{}_{(3+2d)^2}$).

For the welfare the second order approximation equals

$$W(a_1,a_2) = W(0,0) + a_1 \frac{(1-c)s_{x_1}}{(3+2d)^2} + a_2 \frac{(1-c)s_{x_1}}{(3+2d)^2} +$$

$$(a_1)^2 \frac{s_{x_1}}{(1+2d)^2(3+2d)^2}\left\{-(4d^3+10d^2+7d+\tfrac{1}{2})s_{x_1} + \frac{(1+2d)(1-c)}{(3+2d)}\left[(2+2d)s_{x_1x_1} - s_{x_1x_2}\right]\right\} +$$

$$(a_2)^2 \frac{s_{x_1}}{(1+2d)^2(3+2d)^2}\left\{-(4d^3+10d^2+7d+\tfrac{1}{2})s_{x_1} + \frac{(1+2d)(1-c)}{(3+2d)}\left[(2+2d)s_{x_1x_1} - s_{x_1x_2}\right]\right\} +$$

$$(a_1a_2) \frac{s_{x_1}}{(1+2d)^2(3+2d)^2}\left\{(4d^2+4d-1)s_{x_1} + \frac{2(1+2d)(1-c)}{(3+2d)}\left[-s_{x_1x_1} + (2+2d)s_{x_1x_2}\right]\right\} + O(a^3), (a \to 0)$$

Here $W(0,0)$ equals the classical welfare $\left(= {}^{(1-c)^2(4+2d)}\!\big/_{(3+2d)^2}\right)$.

The arguments of all partial derivatives in these expressions equal the co-ordinates of the classical Cournot-Nash equilibrium ($(1-c)/(3+2d)$, $(1-c)/(3+2d)$).

Proof.
The consumer surplus, for our normalized model, equals $CS = ½(x_1^* + x_2^*)^2$, where x_1^* and x_2^* equal the respective equilibrium outputs of firm 1 and firm 2. Substitution of the second order power series for the outputs (Proposition 5.1) and arranging the terms by a_1, a_2, $(a_1)^2$, $(a_2)^2$ and (a_1a_2) leads to the expression for CS.
To obtain an expression for the welfare we use $W(a_1,a_2) = \Pi^1(a_1,a_2) + \Pi^2(a_1,a_2) + CS(a_1,a_2)$. Substitution of the two power series for the profits (Π^2 is obtained by interchanging the weights a_1 and a_2 in the expression for Π^1) and the power series for the consumer surplus leads to the desired expression for the welfare.
[End of proof]

Benchmark case 1: $a_1=a_2=a>0$
Substitution of $a_1=a_2=a$ into the second order power series for the welfare leads directly to the following proposition:

Proposition 5.5.

Consider two symmetric firms, with respect to the weights a_1 and a_2 attributed to the non-profit part s of the maximand, i.e. $a_1=a_2=a > 0$. Let $s_{x_1} > 0$. Then it holds that (both arguments of the partial derivatives of s equal $(1-c)/(3+2d)$)

(i) $\frac{\mathrm{d}W}{\mathrm{d}a} = \frac{2(1-c)}{(3+2d)^2}s_{x_1}$ (at a=0), so welfare increases if the weight a increases.

(ii) $\frac{\mathrm{d}^2W}{\mathrm{d}a^2} = \frac{4s_{x_1}}{(3+2d)^2}\left[-(1+d)s_{x_1} + \frac{(1-c)}{(3+2d)}\left\{s_{x_1x_1} + s_{x_1x_2}\right\}\right]$ (at a=0), so if the condition $s_{x_1x_1} + s_{x_1x_2} \le 0$ holds the welfare function with respect to the weight is concave for small a, i.e. the increase of the welfare decreases if a increases.

Proof.
If we substitute $a_1=a_2=a$ in the expression for the welfare (Proposition 5.4) we obtain

$$W(a) = \frac{(1-c)^2(4+2d)}{(3+2d)^2} + a\frac{2(1-c)s_{x_1}}{(3+2d)^2} + a^2\frac{2s_{x_1}}{(3+2d)^2}\left[-(1+d)s_{x_1} + \frac{(1-c)}{(3+2d)}\left\{s_{x_1x_1} + s_{x_1x_2}\right\}\right]$$
$$+ O(a^3)$$

Both the arguments of the partial derivatives in this expression equal $(1-c)/(3+2d)$. Computation of the first and second order derivatives of the function $W(a)$ in $a=0$ with respect to the weight a (for the second derivative we assume that the power series is convergent) completes the proof.

[End of proof]

Part (i) of Proposition 5.5 has a clear economic interpretation. If the marginal function s is positive with respect to one's own output (which is very plausible, see eq. 5.2, Section 5.2), social welfare benefits from the weights attributed to the non-profit part. Proposition 5.3 reveals that the weight a may serve as a strategic instrument to outperform the rival. An a-setting game, consisting of the heightening of these weights by turns, would lead to a step by step increase of the welfare under the assumption that a remains small. So society may benefit from a game between the competitors. In general non-profit motives can be beneficial for the welfare, whether firms are aware of the (strategic) implications or not.

Unfortunately part (ii) of Proposition 5.5 indicates that the sky is not always the limit, because under the sufficient condition that $s_{x_1x_1} + s_{x_1x_2} \leq 0$ (at the point $((1-c)/(3+2d), (1-c)/(3+2d))$ the second derivative of the welfare function is negative at $a=0$. And this concavity of the welfare function indicates that the increase of social welfare decreases if the weight a is heightened further. Note that if the conditions (iii) and (iv) (Section 5.2), imposed by Rauscher (1992) on the "status" function, are satisfied this sufficient condition is already fulfilled. If the function s describes preference for size, $s(x_1)=x_1$, the second derivative of the welfare function (for $a=0$) with respect to a equals $-4(1+d)/(3+2d)^2 < 0$. In the concluding part of this section we will apply the general propositions concerning profits and welfare on this specific choice of the "status" function. And if the "status" function s equals $s(x_1,x_2) = x_1/(x_1+x_2)$ (market share) it holds that the sum of the second-order partial derivatives in $((1-c)/(3+2d),(1-c)/(3+2d))$ is negative, which again guarantees the concavity of the welfare function for small a's.

We have to realize that the implications on welfare following from the statements of Proposition 5.5 - because of the use of a power series approximation - are (in general) restricted to small weigths attributed to the non-profit part. The concave character of the welfare function for small a leads one to suspect that the welfare decreases if the weight a becomes large enough and if an a-setting game were to degenerate into a rat race this indeed could happen! Obviously Proposition 5.5 is inadequate to describe the effect on social welfare for those larger weights a. But in the concluding part of this section we show that for symmetric firms, under general and plausible conditions imposed on s, the welfare function first rises, reaches its maximum value and then falls with respect to an increasing a. However a simple workable general statement concerning the concavity of the welfare function for larger weights a can not be given. In Section 5.5 it will appear that, in the special case that s equals the market share, the welfare corresponding with benchmark case 1 is a concave function (with respect to the weight) for large values of a as well. For this specific function s the qualitative properties of Proposition 5.5 also continue to hold for large weights. We now continue our examinations with the analysis of benchmark 2.

Benchmark case 2: $a_1=a>0,\ a_2=0.$
Again we use the general second order power series for the welfare and substitute $a_1=a$ and $a_2=0$ in this expression. The following proposition summarizes the analytical results.

Proposition 5.6.

Consider two completely asymmetric firms, with respect to the weights attributed to the non-profit part s of the maximand, i.e. $a_1=a>0,\ a_2=0$. Let $s_{x_1}>0$. Then it holds that (both arguments of the partial derivatives of *s* equal $(1-c)/(3+2d)$)

(i) $\dfrac{\mathrm{d}W}{\mathrm{d}a}=\dfrac{(1-c)}{(3+2d)^2}s_{x_1}$ (at $a=0$), so welfare increases if the weight *a* increases.

(ii)

$$\frac{\mathrm{d}^2W}{\mathrm{d}a^2}=\frac{2s_{x_1}}{(1+2d)^2(3+2d)^2}\left[-(4d^3+10d^2+7d+½)s_{x_1}+\frac{(1+2d)(1-c)}{(3+2d)}\left\{(2+2d)s_{x_1x_1}-s_{x_1x_2}\right\}\right]$$

(at $a=0$).

Proof.
If we substitute $a_1=a$ and $a_2=0$ in the expression for the welfare (Proposition 5.4) we obtain

$$W(a)=\frac{(1-c)^2(4+2d)}{(3+2d)^2}+a\frac{(1-c)s_{x_1}}{(3+2d)^2}+$$
$$a^2\frac{s_{x_1}}{(1+2d)^2(3+2d)^2}\left[-(4d^3+10d^2+7d+½)s_{x_1}+\frac{(1+2d)(1-c)}{(3+2d)}\left\{(2+2d)s_{x_1x_1}-s_{x_1x_2}\right\}\right]+O(a^3)$$

The two arguments of the partial derivatives in this expression equal $(1\text{-}c)/(3+2d)$. The proof is completed by computing the first and second order derivatives of the function $W(a)$ at $a=0$ with respect to the weight a.

[End of proof]

The economic interpretation of part (i) of Proposition 5.6 is straightforward; social welfare also benefits from this asymmetric situation (assuming that $s_{x_1}>0$). However note that the increase of the welfare resulting from an increase of the weight a (for small a's) equals only half of the increase corresponding to the symmetric benchmark case 1. Apparently the results of the parts (i) of the Propositions 5.5 and 5.6 lead one to suspect that the largest level of welfare is achieved if both firms attribute (equal and not too large) weights to their non-profit motives. Part (ii) of Proposition 5.6 contains a rather complicated expression for the second derivative of the welfare function. One cannot simply draw a conclusion concerning the sign of this second derivative, because first the polynomial $4d^3+10d^2+7d+1/2$ is only positive for $d>-0.0804$ and second the sign of the expression $(2+2d)s_{x_1x_1}-s_{x_1x_2}$ can take both negative and positive values as well. Under the conditions (i) $d>-0.0804$ and (ii) $(2+2d)s_{x_1x_1}-s_{x_1x_2}\le 0$ in the point $((1-c)/(3+2d),\ (1-c)/(3+2d))$ the welfare function is concave in a neighbourhood of $a=0$. And if s equals market share the second condition is satisfied because $s_{x_1x_2}=0$ in $((1-c)/(3+2d),\ (1-c)/(3+2d))$.

Besides the analysis of the two benchmark cases some remarks can be made on the general case with $a_1>0$ and $a_2>0$. The first order approximation for the equilibrium welfare (see Proposition 5.4) equals

$$W(a_1,a_2)=\frac{(1-c)^2(4+2d)}{(3+2d)^2}+(a_1+a_2)\frac{(1-c)s_{x_1}}{(3+2d)^2}+O(a^2),\text{ with } a=\max\{a_1,a_2\} \tag{5.16}$$

It is clear that, for small weights, the welfare is rising with respect to both weights a_1 and a_2 . And for a general statement on the concavity of the welfare function $W(a_1,a_2)$ we refer to the Appendix 5.1, where we use the complete second order approximation of the welfare. We now apply the general propositions, concerning profits and welfare, on the case with two firms with equal preference for size (see also Chapter 3).

Application to "preference for size".

For this interesting case with $a_1=a_2$ and the specified "status" function $s(x_1,x_2)=x_1$ the laborious calculations of this section pay off. Because we assume that both firms attribute the same weights to the non-profit part of the maximand we can use the expression for the equilibrium production levels (Proposition 5.1) with $a_1=a_2=a$ (the equilibrium is stable because the general expression for the eigenvalues in the proof of Proposition 5.2 leads to $\lambda_{1,2}=\pm 1/(2+2d)$).

As already mentioned earlier in this section all second- and higher-order partial derivatives of the function $s(x_1,x_2)=x_1$ equal zero and therefore Proposition 5.1 provides exact expressions for the equilibrium outputs! This implies that the expressions for the profits and the welfare (Proposition 5.5), concerning these symmetrical firms, are no longer approximations but provide exact formulas. Substitution of $a_1=a_2=a$ and $s_{x_1}=1$ in the expressions for the profits and the welfare (benchmark case 1,Proposition 5.5) leads to:

$$\begin{aligned}\Pi^1(a)=\Pi^2(a)&=\frac{(1-c)^2(1+d)}{(3+2d)^2}-a\frac{(1-c)}{(3+2d)^2}-a^2\frac{(2+d)}{(3+2d)^2}\\ W(a)&=\frac{(1-c)^2(4+2d)}{(3+2d)^2}+a\frac{2(1-c)}{(3+2d)^2}-a^2\frac{2(1+d)}{(3+2d)^2}\end{aligned} \tag{5.17}$$

It strikes the eye that the welfare function $W(a)$ is not only rising and concave for small values of a - which neatly illustrates the general result on welfare of Proposition 5.5 - but remains concave for large weights as well. Furthermore quick observation of the expression for the profits reveals that the profits of both firms drop below the classical level (for $a=0$) as soon as some weight is attributed to size (= production level). The profit is decreasing with respect to an increasing weight and it is obvious that the non-profit motive reduces the profit. For large weights, namely for $a > (1-c)(1+d)/(2+d)$, these profits may even become negative. However preference for size is advantageous for the social welfare; the (quadratic) expression for the welfare reveals that the welfare rises above the classical level, reaches its maximum value and then begins to fall with respect to an increasing weight a. Two graphs of the profits Π^i and the relative welfare compared to the classical welfare ($=100\%W(a)/W(0)$) illustrate these properties for the choice $c=0.4$ and $d=-0.1$ concerning the production cost parameters. The Figures 5.1a and 5.1b reveal that the maximum relative welware is reached while profits are negative. Assuming exit of the loss-bearing firms in the long run the (realistic) maximum occurs for that weight which corresponds to zero-profits (a=0.284).

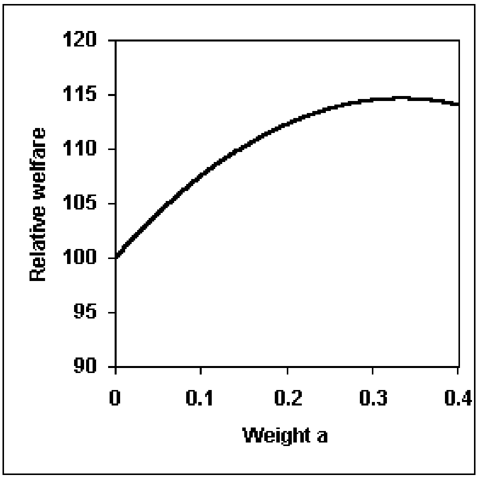


Fig. 5.1a The relative welfare.

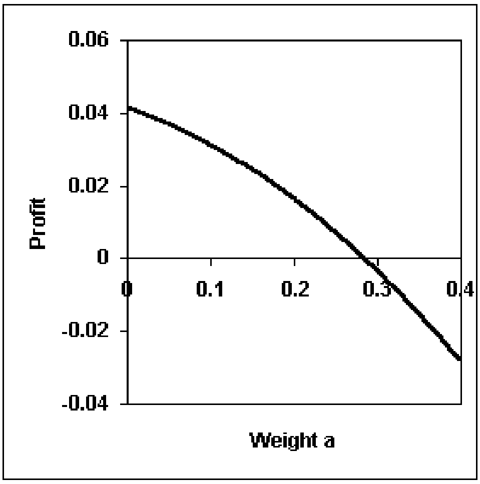


Fig. 5.1b The profit $\Pi^i(a)$.

Proposition 5.7 contains the implications of (symmetric) preference for size, concerning profits and welfare. The proof of this proposition is straightforward by using the expressions for $\Pi^i(a)$ and $W(a)$ and needs no further clarification.

Proposition 5.7.

Consider two symmetric firms, with respect to the weights a_1 and a_2 attributed to the size (as the non-profit part of the maximand) i.e. $a_1=a_2=a > 0$. Then it holds that

(i) the welfare lies above the classical welfare, $W(a)>W(0)$, for $a < (1\text{-}c)/(1+d)$.

(ii) the difference $W(a)$ - $W(0)$ rises for $a < (1- c)/(2+2d)$, is maximized for $a_{s,max} = (1- c)/(2+2d)$ and falls for $a > (1- c)/(2+2d)$.

(iii) for linear production costs the welfare is maximal if $\Pi^i(a) = 0$ for quadratic production costs with $d > 0$ the welfare is maximized if still $\Pi^i(a) > 0$ holds and for quadratic production costs with $d < 0$ the welfare is maximized if already $\Pi^i(a) < 0$ holds. So then the maximum is reached for $a_{s,max}= (1- c)(1+d)/(2+d)$.

The application of the general power series approximations for profits and social welfare developed in this section on the case where s equals the size easily leads to useful expressions and conclusions for those quantities. In fact the second and third part of Proposition 5.7 are related to a general property on welfare for symmetrical firms with respect to the weights attributed to the non-profit part of the maximand. Therefore we conclude this section with some general reflections on these properties concerning welfare. The following analysis only makes sense under the crucial assumption that the equilibrium $(x^*(a),x^*(a))$ is stable. For small a's Proposition 5.2

provides a proof for the required stability of the Cournot-Nash equilibrium. And for general weights a the stability condition equals (see the proof of Proposition 5.2 with $a_1=a_2=a$)

$$\left|\frac{1-as_{x_1x_2}(x^*,x^*)}{-(2+2d)+as_{x_1x_1}(x^*,x^*)}\right|<1$$

Note that this latter condition holds if s equals the market share. First we prove that the equilibrium output $x^*(a)$ of both firms increases with respect to the weight a attributed to an arbitrary non-profit part s under very plausible conditions. The equilibrium outputs of both firms equal $x^*(a)$ and satify the equation (see eq. 5.8 with $a_1=a_2=a$)

$$(1-c)-(3+2d)x^*(a)+as_{x_1}(x^*(a),x^*(a))=0$$

Implicit differentiation of this equation with respect to a leads to

$$\frac{\mathrm{d}x^*}{\mathrm{d}a}=\frac{s_{x_1}(x^*,x^*)}{(3+2d)-a\{s_{x_1x_1}(x^*,x^*)+s_{x_1x_2}(x^*,x^*)\}}$$

If $s_{x_1}(x^*,x^*)>0$ holds - which is very plausible (condition (i) of eq. 5.2, Section 5.2) - and the condition $s_{x_1x_1}(x^*,x^*)+s_{x_1x_2}(x^*,x^*)\le 0$ is satified (conditions (iii) and (iv) of eq. 5.2 and eq. 5.3) then the first derivative of x^* with respect to a is positive.

The two equilibrium profits are equal and can be expressed as

$$\Pi^1(a)=\Pi^2(a)=(1-c)x^*-(2+d)(x^*)^2$$

These profits become zero for $x^*=(1-c)/(2+d)$ (break-even point). The expression for the welfare equals (in both expressions $x^*=x^*(a)$)

$$W(a)=\Pi^1(a)+\Pi^2(a)+CS(a)=2(1-c)x^*-(2+2d)(x^*)^2$$

Using the chain rule for differentiation we easily obtain

$$\frac{\mathrm{d}W}{\mathrm{d}a}=\left[2(1-c)-4(1+d)x^*\right]\cdot\frac{\mathrm{d}x^*}{\mathrm{d}a}\,.$$

For positive $\mathrm{d}x^*/\mathrm{d}a$ the welfare is maximized if the output level (per firm) equals $x^*=(1-c)/(2+2d)$. To find the maximizing value of the weight a attributed to the non-profit part of the maximand we only have to solve this latter equation with respect to a. If $\mathrm{d}x^*/\mathrm{d}a>0$ the function $x^*(a)$ is invertible and the value of a for which the welfare is maximized is either never reached or determined uniquely. We now formulate a generalization of Proposition 5.7:

Proposition 5.8.

Consider two symmetric firms, with respect to the weights a_1 and a_2 attributed to the non-profit part s of the maximand i.e. $a_1=a_2=a > 0$. Under the assumption that
(a) the equilibrium (x^*,x^*) is stable
(b) the function s satisfies $s_{x_1}(x^*,x^*) > 0$
(c) the function s satisfies $s_{x_1x_1}(x^*,x^*) + s_{x_1x_2}(x^*,x^*) \leq 0$

it holds that with respect to the weight a
(i) the equilibrium welfare $W(a)$ rises for $x^*(a) < (1-c)/(2+2d)$, is maximized for $x^*(a_{max}) = (1-c)/(2+2d)$ and falls for $x^*(a) > (1-c)/(2+2d)$.
(ii) for linear production costs the welfare is maximal if $\Pi^i(a)=0$, for quadratic production costs with $d > 0$ the welfare is maximized if still $\Pi^i(a) > 0$ holds and for quadratic production costs with $d < 0$ the welfare is maximized if already $\Pi^i(a) < 0$ holds. So then the (realistic) maximum is reached for $x^*(a_{max}) = (1-c)/(2+d)$.

This concluding reflection, which only uses the technique of (implicit) differentiation, leads to pleasant results for benchmark case 1 and supplements the results of Proposition 5.5 concerning larger values of a. We emphasize that this reflection concerning symmetrical players does not make the detailed expressions for the second order power series approximations of the equilibrium profits and welfare needless. It just provides a different point of view. For benchmark case 2 a general analysis appears to be possible also, which will be used in Section 5.6. Even for the general case it appears to be possible to derive expressions for the partial derivatives of the welfare function using the technique of implicit differentiation. However there this method doesn't require less computations. Furthermore the derived power series in this section provide useful approximations in all cases for which equilibrium quantities cannot be solved analytically for small weights a_i and the approximations clarify general properties of profits and social welfare in the neighbourhood of $a_1=a_2=0$.

4. Market share as the non-profit part of the objective function

In this section we study the implications, concerning the Cournot reaction curve, of a specified non-profit part of the maximand namely market share. Preference for market share - defined as the fraction of the output of a firm and total market supply - provides a natural non-profit part of the utility function of a firm. As we will see maximizing this specific utility function leads to very interesting properties of the reaction curve and also to a classification of these curves. These properties and the typology lay the foundation for a further study of a Cournot duopoly game between firms with symmetric or asymmetric preferences for their market shares, which is the main subject of Sections 5.5 and 5.6. In this section we limit our examination to the reaction curve of one firm, firm 1, which attributes a weight a_1 to its market share. The specification of the "status" function $s(x_1,x_2) = x_1/(x_1+x_2)$ leads to the following optimization problem (with the restriction of nonnegative prices, see also eq. 5.1 Section 5.2, for the general formulation):

$$\text{Max}\, U^1(x_{1,t}\,; x_{2,t-1}) = x_{1,t}(1 - x_{1,t} - x_{2,t-1}) - cx_{1,t} - d(x_{1,t})^2 + a_1 \frac{x_{1,t}}{(x_{1,t} + x_{2,t-1})} \tag{5.18}$$

with respect to $x_{1,t}$, and subject to the restriction $0 \le x_{1,t} + x_{2,t-1} \le 1$

By computation of the relevant partial derivatives of the function $s(x_1,x_2)$ we show that crucial assumptions (see eq. 5.2 and 5.3, Section 5.2) concerning the non-profit part are satisfied.

(i) $s_{x_1} = \dfrac{x_2}{(x_1 + x_2)^2} > 0$ for $x_2 > 0, x_1 \ge 0$, which indicates the plausible property that the marginal market share with respect to one's own output level is positive.

(ii) $s_{x_2} = -\dfrac{x_1}{(x_1 + x_2)^2} < 0$ for $x_1 > 0, x_2 \ge 0$, which expresses the fact that the market share decreases if the output of the rival increases.

(iii) The second order partial derivative $s_{x_1x_1} = \dfrac{-2x_2}{(x_1 + x_2)^3} < 0$ for $x_2 > 0, x_1 \ge 0$ and reveals that the surplus value of market share decreases with respect to an increase of one's own production level.

(iv) The cross-partial derivative $s_{x_1x_2} = \dfrac{(x_1 - x_2)}{(x_1 + x_2)^3}$ can be positive, zero and negative as well. The expression shows that for $x_1 > x_2$ marginal market share is an increasing function of the rival's production level.

Setting the marginal utility equal to zero we obtain the following nonlinear equation which determines the Cournot reaction curve $x_{1,t} = R^1(x_{2,t-1}|a_1)$ (apart from the restriction $x_{1,t} + x_{2,t-1} \le 1$):

$$\frac{\partial U^1}{\partial x_{1,t}} = 1 - c - (2 + 2d)x_{1,t} - x_{2,t-1} + a_1 \frac{x_{2,t-1}}{(x_{1,t} + x_{2,t-1})^2} = 0 \tag{5.19}$$

The fact that the marginal market share (with respect to one's own output level) is positive has important consequences for the reaction curve.

In Section 5.2 we already noted that this qualitative property (i) of the non-profit part of the utility function leads to an outward shift of the (nonlinear) reaction curve in comparison to the classical Cournot curve. The marginal utility decreases monotonically with respect to one's own output $x_{1,t}$ for all nonnegative $x_{1,t}$ and $x_{2,t-1}$ because property (iii) of the "market share" function s holds. Mathematically this latter fact means that the nonlinear equation $\partial U^1 / \partial x_{1,t} = 0$ possesses one real positive solution provided that the marginal utility is positive for $x_{1,t} = 0$ (and if the marginal utility is nonpositive for $x_{1,t} = 0$ the reaction $R^1(x_{2,t-1}|a_1)$ equals zero). The nonlinear equation $\partial U^1 / \partial x_{1,t} = 0$ can be rewritten into a third degree equation with respect to the variable $x_{1,t}$. Using a tranformation of the variable and the coëfficients and applying Cardan's method on the transformed equation, the output $x_{1,t}$ can even be solved analytically (Appendix 5.2).

However in order to give an adequate and complete functional expression for the reaction curve we first need to know for which values of the rival's output, $x_{2,t-1}$, the reaction $x_{1,t}$ equals zero. And secondly we need to know under which condition the reaction curve meets the restriction $x_{1,t} + x_{2,t-1} = 1$ and thus leads to the (perfectly accommodating) reaction $x_{1,t} = 1 - x_{2,t-1}$. Fortunately both conditions can be derived without using the complicated analytical solution of the nonlinear equation. The first condition can be obtained by imposing a condition on the marginal utility at $x_{1,t} = 0$. For those values of the rival's output, $x_{2,t-1}$, for which the marginal utility $\partial U^1 / \partial x_{1,t}$ is already nonpositive at $x_{1,t} = 0$ the reaction $x_{1,t}$ will be $x_{1,t} = 0$, because the marginal utility is a decreasing function with respect to $x_{1,t}$. This argumentation leads to the condition (note that for a_1=0 this condition leads to the classical condition $x_{1,t} = 0$ for $x_{2,t-1} \geq 1 - c$)

$$\left(\frac{\partial U^1}{\partial x_{1,t}}\right)_{x_{1,t}=0} = 1 - c - x_{2,t-1} + \frac{a_1}{x_{2,t-1}} \leq 0 \leftrightarrow x_{1,t} = 0 \tag{5.20}$$

Because differentiation reveals that the marginal utility at $x_{1,t} = 0$ decreases with respect to $x_{2,t-1}$ we may conclude that the smallest value of $x_{2,t-1}$ for which the reaction $x_{1,t}$ equals zero satisfies $1 - c - x_{2,t-1} + a_1/x_{2,t-1} = 0$. This latter equation can be solved and leads to $x_{2,t-1} = \frac{1}{2}(1-c) + \frac{1}{2}\sqrt{(1-c)^2 + 4a_1}$. Note that this smallest value of the rival's output - for which the response equals zero - increases with respect to the weight a_1 attributed to the market share. This smallest value equals 1 for $a_1 = c$ and for this specific weight a_1 the reaction curve precisely intersects the restriction $x_{1,t} + x_{2,t-1} = 1$ at point $(x_{1,t}, x_{2,t-1}) = (0,1)$. This property leads one to suspect that for all values of a_1 with $a_1 \geq c$ the reaction curve $R^1(x_{2,t-1}|a_1)$ meets the restriction which links this first condition for the zero output, to the second condition concerning the intersection with the (nonnegative price) restriction.

The intersection point of the reaction curve $R^1(x_{2,t-1}|a_1)$ with the restriction can be solved by substituting $x_{1,t} + x_{2,t-1} = 1$ into the equation $\partial U^1 / \partial x_{1,t} = 0$. This directly leads to $1 - c - (2+2d)x_{1,t} - (1 - x_{1,t}) + a_1(1 - x_{1,t}) = 0$ from which the co-ordinate $x_{1,t}$ of the intersection point can be solved. This co-ordinate equals $x_{1,t} = (a_1 - c)/(1+2d+a_1)$

(which is nonnegative for $a_1 \geq c$). The derivation of the two properties - concerning $x_{1,t} = 0$ and the perfect accommodation - enables us to summarize the complete functional form of the reaction curve (which depends on the weight parameter a_1, the production cost parameters c and d and the rival's output $x_{2,t\text{-}1}$). The expression $G(x_{2,t\text{-}1}|a_1)$ equals the solution of the nonlinear equation $\partial U^1/\partial x_{1,t} = 0$ (for a_1) and for the derivation of this formula and the complete expression we refer to Appendix 5.2.

Proposition 5.9 (the functional form of the reaction curve).

If the weight a_1 attributed to the market share is less than c the reaction curve $R^1(x_{2,t\text{-}1}|a_1)$ does not intersect the restriction $x_{1,t} + x_{2,t-1} = 1$ and the output $x_{1,t}$ equals zero if the rival's output $x_{2,t-1}$ exceeds $\frac{1}{2}(1\text{-}c) + \frac{1}{2}\sqrt{(1\text{-}c)^2 + 4a_1}$. It holds that (for G see Appendix 5.2)

$R^1(x_{2,t-1}|a_1) = G(x_{2,t-1}|a_1)$ for $0 \leq x_{2,t-1} < \frac{1}{2}(1\text{-}c) + \frac{1}{2}\sqrt{(1\text{-}c)^2 + 4a_1}$

$R^1(x_{2,t-1}|a_1) = 0$ for $\frac{1}{2}(1\text{-}c) + \frac{1}{2}\sqrt{(1\text{-}c)^2 + 4a_1} \leq x_{2,t-1} \leq 1$

If $a_1 \geq c$ the reaction curve intersects the restriction at $(x''_{1,t}, x''_{2,t-1}) = (\dfrac{(a_1 - c)}{(1 + 2d + a_1)}, \dfrac{(1 + c + 2d)}{(1 + 2d + a_1)})$.

It holds that

$R^1(x_{2,t-1}|a_1) = G(x_{2,t-1}|a_1)$ for $0 \leq x_{2,t-1} < \dfrac{(1 + c + 2d)}{(1 + 2d + a_1)}$

$R^1(x_{2,t-1}|a_1) = 1 - x_{2,t-1}$ for $\dfrac{(1 + c + 2d)}{(1 + 2d + a_1)} \leq x_{2,t-1} \leq 1.$

Although the expression for $G(x_{2,t\text{-}1}|a_1)$ is complicated, it will be of great value in the computer simulations of the reaction paths of two asymmetrical firms with respect to the weight attributed to the market share. We present a graph of the reaction curve corresponding to the production cost parameters c=0.4 and d=– 0.1, whereas the level of preference for market share is determined by a_1=0.30.

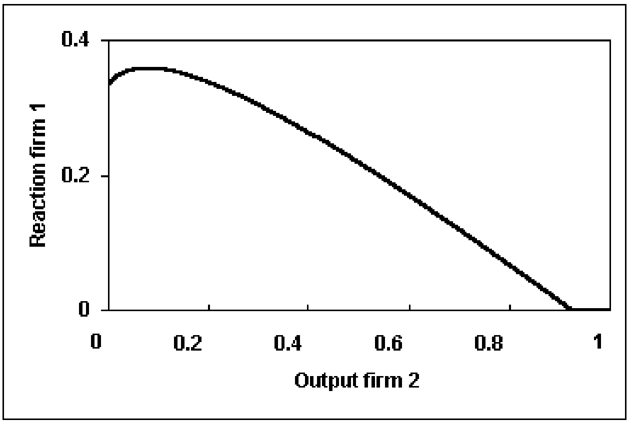


Fig. 5.2 The Cournot-reaction curve of firm 1; *c*=0.4, *d*=–0.1, *a*₁=0.3.

Because $a_1 < c$ it follows from Proposition 5.9 that the output $x_{1,t}$ equals zero for $x_{2,t\text{-}1} \geq 0.924$ (in 3 decimals), which fits with the graphical display.
One observes that the monopoly output $R^1(0|a_1)$ is positive, comparable to the classical case with a_1=0. The strategic answer to a zero production of the rival is a positive output and apparently this production is not negligible. Furthermore it strikes the eye that the slope of the Cournot reaction curve at $x_{2,t\text{-}1} = 0$ is positive, the curve reaches a maximum, and subsequently displays the standard downward slope.
Proposition 5.10 deals with these two important properties of the reaction curve. Although it isn't possible to provide a simple expression for the maximum location (and the corresponding maximum output) of the reaction curve - due to the analytically complicated expression $G(x_{2,t\text{-}1}|a_1)$ - a proof of the existence of a maximum under clear conditions for a_1 can be given. In the proof we use the powerful technique of implicit differentiation, like we did in the general considerations of Section 5.2.

Proposition 5.10 (properties of the reaction function).

(i) For all $a_1 \geq 0$ the monopoly output equals $x_{1,t} = R^1(0|a_1) = \dfrac{(1-c)}{(2+2d)}$.

(ii) For all $a_1 > \dfrac{(1-c)^2}{(2+2d)^2}$ the slope of the reaction curve at $x_{2,t\text{-}1} = 0$ is positive and the reaction curve displays a maximum. The slope at $x_{2,t\text{-}1} = 0$ is equal to $-\dfrac{1}{(2+2d)} + a_1\dfrac{(2+2d)}{(1-c)^2}$ and can be made arbitrarily large by increasing the weight a_1.

(iii) The maximum location (if it exists, see (ii)), $x_{2,max}$ satisfies the equation $a_1\{G(x_{2,max} \mid a_1) - x_{2,max}\} = \{G(x_{2,max} \mid a_1) + x_{2,max}\}^3$.

Proof.
Setting $x_{2,t\text{-}1}$ equal to zero in the equation $\partial U^1 / \partial x_{1,t} = 0$ directly leads to the monopoly output $x_{1,t} = (1-c)/(2+2d)$ for all weights a_1 (one can check that $G(0|a_1)$ leads to the same outcome). This proves part (i). Implicit differentiation of the expression

$$\frac{\partial U^1}{\partial x_{1,t}} = 1 - c - (2+2d)x_{1,t} - x_{2,t-1} + a_1\frac{x_{2,t-1}}{(x_{1,t} + x_{2,t-1})^2} = 0$$

with respect to the variable $x_{2,t\text{-}1}$ leads to a formula for the slope (see also the general expression for the slope in eq. 5.7, Section 5.2):

$$\frac{\mathrm{d}x_{1,t}}{\mathrm{d}x_{2,t-1}} = \frac{-(x_{1,t} + x_{2,t-1})^3 + a_1(x_{1,t} - x_{2,t-1})}{(2+2d)(x_{1,t} + x_{2,t-1})^3 + 2a_1x_{2,t-1}}$$

Substituting $x_{2,t\text{-}1} = 0$ in this expression gives $\left\{\dfrac{\mathrm{d}x_{1,t}}{\mathrm{d}x_{2,t-1}}\right\}_{x_{2,t-1}=0} = -\dfrac{1}{(2+2d)} + a_1\dfrac{(2+2d)}{(1-c)^2}$.

For fixed c and $d > -1/2$ this slope can be made arbitrarily large by increasing the parameter a_1. It immediately follows that for all $a_1 > \frac{(1-c)^2}{(2+2d)^2}$ the slope of the reaction curve at $x_{2,t-1} = 0$ is positive. If $a_1 > c$ the reaction curve meets the restriction (Proposition 5.9) and clearly a change of the sign of the slope from positive to negative occurs. If the reaction curve doesn't intersect the restriction it certainly has to intersect the line $x_{1,t} = x_{2,t-1}$ and if we call this intersection point (x^*,x^*) it follows from the formula of the slope that at (x^*,x^*) the slope equals

$$\left\{\frac{\mathrm{d}x_{1,t}}{\mathrm{d}x_{2,t-1}}\right\}_{(x^*,x^*)} = -\frac{8(x^*)^3}{16(1+d)(x^*)^3 + 2a_1x^*} < 0$$

Again a change of the sign of the slope from positive to negative occurs indicating that the maximum location is reached for some $0 < x_{2,t-1} < x^*$ (Again using implicit differentiation w.r.t. $x_{2,t-1}$, leads to an expression for the second derivative of the reaction function. One can prove that this second derivative is negative if $a_1 > 0$; so the maximum of the reaction function is determined uniquely).

The exact maximum location is reached for that specific value $x_{2,max}$ of $x_{2,t-1}$ which satisfies $\mathrm{d}x_{1,t}/\mathrm{d}x_{2,t-1} = 0$. Part (iii) is proved by using the general formula for the slope and $x_{1,t} = G(x_{2,max}|a_1)$.

[End of proof]

The results of part (i) and part (ii) of Proposition 5.10 reveal that the introduction of preference for market share as the non-profit part of the utility function not only leads to a positive monopoly output - which overcomes the shortcomings of the former studies of Kopel (1996) and Puu (1991,1998) - but provides a plausible rationale for a hill-shaped Cournot reaction curve as well. Part (ii) shows that, if the weight attributed to this specific non-profit part is large enough, an increase in the rival's output - assuming that this output is still small - leads to an increase in one's own production level. Apparently the optimal response to more aggressive play of the rival is also more aggressive play. This behaviour, referred to as the reaction pattern of an imitator or follower in Chapter 2, is fully determined by the sign of the slope, i.e. ${}^{\mathrm{d}x_{1,t}}\!\big/_{\mathrm{d}x_{2,t-1}} > 0$, and of course contradicts the slope of the standard Cournot curve.

However the occurrence of a maximum of the reaction curve at $x_{2,max}$ indicates that, if the rival (2) expands its output beyond a certain level, $x_{2,max}$, firm 1 starts to act as an accommodator. The change of the sign of the slope from positive to negative at the maximum location reveals that the reaction to more aggressive play of the rival - provided that the rival's output exceeds $x_{2,max}$ - is less aggressive play. The fact that the firm's reply can be to imitate as well as to accommodate reflects its dualistic behaviour (see also Chapter 2). Furthermore the results of Proposition 5.10 show that the slope at $x_{2,t-1} = 0$ can be made arbitrarily large and leads one to suspect that for large preferences for market share the reaction curve rises very quickly (with respect to an increasing output of the rival) and meets the restriction. Thus a large preference for market share may result in an unimodal function which can be compared with the well known non-analytical tent map. Such hill-shaped functions open up the possibility for the occurrence of complicated or even chaotic reaction patterns of the two competitors. In Chapter 6 we will need the results of the Propositions 5.9 and 5.10 to apply the Theorem of Li & Yorke (1975) on reaction functions in order to show that chaos is even possible for firms with cost symmetry.

We continue with the analysis of the properties of the reaction curves and introduce a workable classification of these curves which also serves the forthcoming analysis of the Sections 5.5 and 5.6. This typology is based on the value of the weight a_1 attributed to the market share and its purpose is to distinguish the reaction curves in four types with easily recognizable and salient characteristics. We now present the four types in a proposition and after the proof we will illustrate this classification with several graphical displays.

Proposition 5.11 (a typology of the reaction curves).

(i) Type 1, T_1, exclusively possesses a negative slope if the weight a_1 satisfies the condition $0 \le a_1 \le \frac{(1-c)^2}{(2+2d)^2}$ (this curve only intersects with the restriction $x_{1,t} + x_{2,t\text{-}1} = 1$ (for $a_1 \ge c$) if it holds for the production cost parameters c and d that $0 < c \le 1 + 2(1+d)^2 - 2(1+d)\sqrt{(1+d)^2+1}$).

(ii) Type 2, T_2, possesses a positive slope at $x_{2,t\text{-}1} = 0$ and also reaches a differentiable maximum; if this curve intersects the restriction ($x_{1,t} = 1 - x_{2,t\text{-}1}$) then for the co-ordinate $x''_{2,t\text{-}1}$ of the intersection point it holds that $x''_{2,t\text{-}1} \ge 0.5$. The weight a_1 satisfies the condition $\frac{(1-c)^2}{(2+2d)^2} < a_1 \le 1 + 2c + 2d$.

(iii) Type 3, T_3, intersects with the restriction and for the co-ordinate $x''_{2,t\text{-}1}$ of the intersection point it holds that $x''_{2,t\text{-}1} < 0.5$. Furthermore T_3 possesses a differentiable maximum (the maximum location occurs before the restriction is reached, $x_{2,max} < x''_{2,t\text{-}1}$). The weight a_1 satisfies the condition $1 + 2c + 2d < a_1 < 1 + c + d + \sqrt{(1+c+d)^2 + 1 + 2d}$.

(iv) Type 4, T_4, intersects with the restriction and for the co-ordinate $x''_{2,t\text{-}1}$ of the intersection point it holds that $x''_{2,t\text{-}1} \le 0.5$. The function possesses a non-differentiable maximum because this maximum location equals $x''_{2,t\text{-}1}$. The condition for the weight a_1 equals $a_1 \ge 1 + c + d + \sqrt{(1+c+d)^2 + 1 + 2d}$.

Proof.
Part (i) follows directly from the results of Proposition 5.10, part (ii): the slope at $x_{2,t\text{-}1} = 0$ is nonpositive if $\left\{\frac{\mathrm{d}x_{1,t}}{\mathrm{d}x_{2,t-1}}\right\}_{x_{2,t-1}=0} = -\frac{1}{(2+2d)} + a_1\frac{(2+2d)}{(1-c)^2} \le 0$. The T_1-curve already meets the restriction (for $a_1 \ge c$) if it holds that $(1-c)^2/(2+2d)^2 \ge c$ (see Proposition 5.9) and this latter condition can be rewritten as $0 < c \le 1 + 2(1+d)^2 - 2(1+d)\sqrt{(1+d)^2+1}$ (if we include the condition $d > -c/2$ (nonnegative marginal costs) it holds that $d \ge -0.099$, $c \le 0.198$, so the T_1- curve only meets the restriction (for $a_1 \ge c$) if the firm possesses a very efficient production technology).

Part (ii), concerning the T_2-curve, follows from the expression of the co-ordinates of the intersection point of Proposition 5.9. For the co-ordinate $x''_{2,t-1}$ it holds that $x''_{2,t-1} \geq 0.5$ if $x''_{2,t-1} = \dfrac{(1+c+2d)}{(1+2d+a_1)} \geq 0.5 \leftrightarrow a_1 \leq 1+2c+2d$. The condition for a positive slope at $x_{2,t-1} = 0$ leads to the condition for the weight a_1.

We continue with part (iii) concerning the T_3-curve. The left hand side of the inequality in the condition for the weight a_1 directly follows from $x''_{2,t-1} < 0.5$. To prove the right hand side of the inequality we subsitute the two expressions for the co-ordinates of the intersection point with the restriction, $x''_{1,t}$ and $x''_{2,t-1}$, into the general expression for the slope. This leads to a formula for the slope of the reaction curve (the left-derivative to be precise) at the intersection point (before perfect accommodation sets in):

$$\frac{\mathrm{d}x_{1,t}}{\mathrm{d}x_{2,t-1}} = \frac{-(x_{1,t}+x_{2,t-1})^3 + a_1(x_{1,t}-x_{2,t-1})}{(2+2d)(x_{1,t}+x_{2,t-1})^3 + 2a_1x_{2,t-1}} = \frac{a_1(a_1-2-2c-2d)-(1+2d)}{2a_1(2+c+3d)+(1+2d)(2+2d)}$$

Note that this slope can be made arbitrarily large! The reaction curve possesses a differentiable maximum for those weights a_1 for which this slope at the intersection point is still negative. Imposing the condition $a_1(a_1 - 2 - 2c - 2d) - (1+2d) < 0$ leads to the condition for a_1. Finally the condition concerning the fourth type T_4 directly follows from $\mathrm{d}x_{1,t}/\mathrm{d}x_{2,t-1} \geq 0$ at the intersection point. Then the maximum location equals $x''_{2,t-1}$ and the right-derivative equals -1 at the maximum location.

[End of proof]

Application of the results of Proposition 5.11 on a case with production cost parameters c=0.4 and d=– 0.1 leads to a diagram which summarizes the types of the reaction curves corresponding with a rising weight a_1 attributed to the market share.

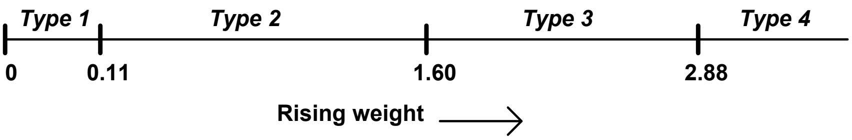


Diagram 5.1. Change of the behavioural types in relation to the weight a_1.

In the Figures 5.3a and 5.3b examples of the types T_1 and T_2 are presented. Corresponding to the first type T_1 the choice $a_1 = 0.05$ has been made and the choice concerning the second type equals $a_1 = 0.5$.

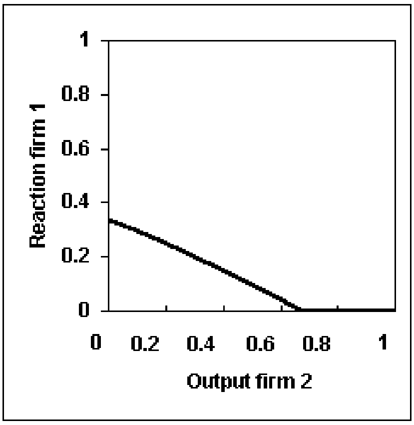


Fig.5.3a Type 1 for $a_1 = 0.05$.

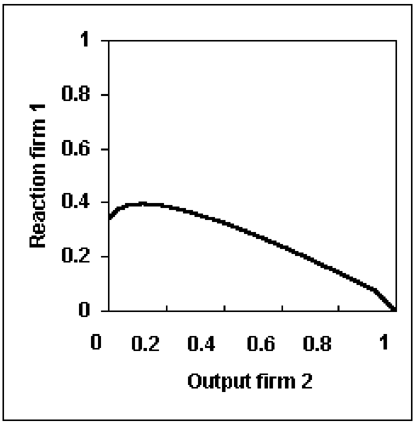


Fig.5.3b Type 2 for $a_1 = 0.5$.

The curve T_1 clearly shows the standard negative slope, although we have to realize that - as a consequence of the (small) positive weight attributed to the market share - this curve is shifted somewhat outwards in comparison with the classical Cournot reaction curve. Note that the output of firm 1 equals zero for $x_{2,t\text{-}1} \geq 0.674$ (Proposition 5.9), which is indeed a larger value for the rival's production than the one corresponding to the classical Cournot curve. The curve of the second type, T_2, shows a dualistic behaviour of the firm, because this curve possesses a (differentiable) maximum. Furthermore it strikes the eye that, whatever the rival's output will be, the reaction never will be equal to a zero output (except for $x_{2,t\text{-}1} = 1$) and that for $x_{2,t\text{-}1} \geq x''_{2,t\text{-}1}$ (= 0.923 here) firm 1 starts to accommodate perfectly because of the nonnegative price restriction.
Two graphs of the respective types T_3 and T_4 are presented in the Figures 5.4a and 5.4b; T_3 corresponds to $a_1 = 1.80$ and corresponding to the graph of T_4 the weight equals $a_1 = 4.00$.

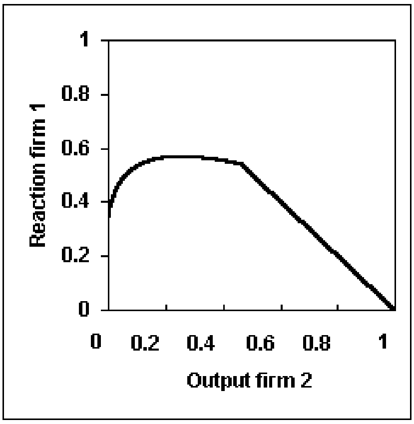


Fig.5.4a Type 3 for $a_1 = 1.80$.

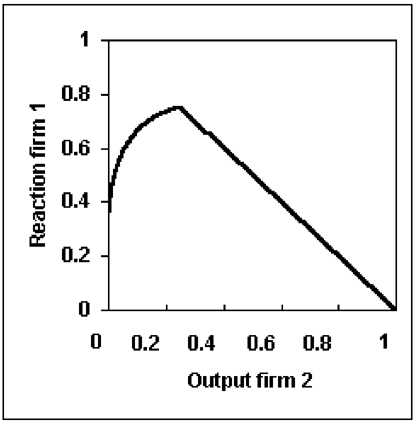


Fig.5.4b Type 4 for $a_1 = 4.00$.

The curve of type 3 still possesses a maximum which is not the consequence of the intersection with the restriction (at the maximum location the function is differentiable). Firm 1 perfectly accommodates for the rival's output $x_{2,t\text{-}1} \geq 0.462$ (Proposition 5.9). The maximum occurring in curve T_4 is caused by the intersection of the reaction curve with the nonnegative price restriction and the co-ordinates of this maximum equal $(x_{2,max}, 1 - x_{2,max}) = (0.25, 0.75)$. Note that the extreme weight of $a_1 = 4.00$ attributed to the market size causes a very aggressive reaction of firm 1 if - starting from a small output level - the competing firm increases its production. However, if the rival's production exceeds 0.25, firm 1's answer is perfect accommodation.The preceding four graphical illustrations clearly demonstrate that managerial inertia, reflected by preference for market share, determines a firm's (production) behaviour in direct competition.

5. Implications of symmetry concerning preference for market share

In this section we study the benchmark case of two symmetrical firms concerning their preferences for market share, i.e. the respective weights a_1 and a_2 attributed to this non-profit part of the maximand are equal. Because both firms control equally efficient production technologies we are dealing with a completely symmetrical case. First we reflect on the question how such complete symmetry can arise. The results of Proposition 5.3 of Section 5.3 provide us with an argument, because this proposition reveals that - for small weights a_i and a_j - firm i outperforms its rival (concerning both the production level and profit level) if a_i exceeds a_j. Moreover there may exist another driving force behind enlarging the weights; if $a_i > (2+2d)a_j$ the profit of firm i even rises above the classical level ($a_i=a_j=0$). So starting from the classical situation an a-setting game may take place. In stage one the two firms determine their level of a, whereas in the second stage of the game Cournot competition takes place. The analysis of a two-stage sequential game will be the subject of future research and is related to the analysis of the "delegation" games of Vickers (1985), Fershtman and Judd (1987), Sklivias (1987) and Basu (1995), where owners and managers are separated and owners "dictate" managers' objective function.

We mention that the results of Proposition 5.3 are even robust with respect to larger weights as the following explanation shows. If (x_1^*,x_2^*) equals the Cournot-Nash equilibrium corresponding with certain weights $a_1 = a_2$, a heightening of the weight a_1 leads to a further outward shift of the reaction curve of firm 1 (e.g. observe the evolution of the curves in Fig. 5.3 a,b and 4 a,b of Section 5.4). The result of such a shift is a (further) increase of firm 1's output level x_1^* at the expense of the rival's production size x_2^* and in general the implication of $a_1 > a_2$ is $x_1^* > x_2^*$. The difference in profits of firm 1 and firm 2 can be expressed in the co-ordinates of the equilibrium by

$$\Pi^1(x_1^*,x_2^*) - \Pi^2(x_1^*,x_2^*) = (x_1^* - x_2^*)[1 - c - (1+d)(x_1^* + x_2^*)] \tag{5.21}$$

(for linear costs, i.e. $d = 0$, this difference equals ($x_1^*- x_2^*$)($p^*- c$) where p^* is the equilibrium price) So this formula clarifies that, if $a_1 > a_2$, firm 1 keeps an advantage in profit over its rival, provided that total supply is restricted. These considerations are also related to Chapter 3 where we studied the implications of preference for size (or growth of size which leads to the same mathematics) and one of the results was that - given the preference for size of the rival - a firm can determine it's own profit-maximizing preference for size. We note that in the case that owners write the incentive contract for their managers, and the managers' objective function is a (weighted) combination of profit and market share, such a "delegation" game leads to a complicated nonlinear analysis. Such a two-stage sequential game would lead to equal weights concerning market share in oligopoly (because of equally efficient production technologies). Of course all these arguments, concerning a-setting games, are related to strategic benefits in a direct-competition setting and assume knowledge of the decision makers of the implications of heightening the weight a and therefore assume also highly rational behaviour.

To motivate the analysis for larger equal weights attributed to market share, we need other arguments, coming from an empirical angle, and concerning the inert behaviour of (top) managers. Cumulative evidence supports the claim that, besides the profit maximization, also (psychological) preference for (growth of) size or market share drives firm's (top-management) behaviour and may imply larger values of a. In their forthcoming paper van Witteloostuijn, Boone & van Lier (2003) relate preference for (growth of) size to inertia at the managerial level (besides inertia on the organization level reflected in adjustment costs). They argue that "Downsizing comes with high political costs through resistance from those organizational participants - work floor and management - who are confronted with retrenchment, whereas expansion tends to be received with applause by those who may grow. ... ...The preference for size or market share implies asymmetric inertia, as the positive utility from sales growth impedes the management's incentive to downsize but increases the management's willingness to expand, even if this implies that profit is sacrificed". The relevance of the preference for market share is supported by a study of Peck (1988) which reports the results of a survey into corporate objectives among 1000 American and 1031 Japanese top managers: two findings are that increasing market share ranks third in the American and second in the Japanese subsample, whereas return on investment is first among American and third among Japanese top managers. If executive bonuses and salaries depend on both profit level and size or market share, naturally managers tend to increase their weight attributed to the preference for size or market share. A large number of studies into managerial compensation (Jensen & Murphy (1990) and Lambert, Larcker & Weigelt (1991)) revealed that the correlation between salaries (bonuses) and size is the stronger one in comparison to the correlation between salaries and profit level.

So the preference for size or market share is encouraged by these practices. Obviously the behaviour of the top management is strongly correlated with the managerial compensation practices. These considerations provide another intuitive argument for a strong symmetry concerning the weights attributed to the market share. If two firms - possessing an equally efficient production technology and producing homogeneous goods - were to differ strongly in their bonuses and salary incentives, a manager would certainly have the inclination to offer his/her experience to the most beneficial firm (in terms of compensation practices). The resulting similarity of managerial compensation practices (otherwise talented managers will go over to the competitor) of both firms probably also leads to more or less equal preferences for the non-profit parts of the maximand reflected in equal weights a_1 and a_2. All these arguments (although requiring more empirical research), concerning the symmetry in preferences, certainly justify a detailed examination of the properties of the equilibrium quantities of two complete symmetrical competitors. An interesting issue will be the implication for the equilibrium profits and welfare corresponding to larger values of the weight a. After all we may not exclude managers' larger (or even extreme) levels of preference for market share.

We start our investigation, concerning two symmetrical firms, with the derivation of the co-ordinates of the Cournot-Nash equilibrium and the proof of stability. For small values of a we refer to the Propositions 5.2 and 5.1, Section 5.3 concerning the stability of the equilibrium and the approximating expression for the equilibrium co-ordinates (choose a_1=a_2=a). In the following proof for arbitrary values of a, we will use the general formula for the eigenvalues of the linearized system of the proof of Proposition 5.2.

Proposition 5.12 (output equilibrium and stability).

Consider two symmetric firms, with respect to the weights a_1 and a_2 attributed to the market share (as the non-profit part of the maximand) i.e. $a_1=a_2=a > 0$.

(i) the output-equilibrium co-ordinates equal $x_1^*(a) = x_2^*(a) = x^*(a)$ with $x^*(a) = \frac{(1-c)}{2(3+2d)} + \frac{\sqrt{(1-c)^2+(3+2d)a}}{2(3+2d)}$

(ii) For $a \leq 1+2c+2d$ the equilibrium is stable and for $a > 1+2c+2d$ there exists a line segment of (neutral) equilibria located on the nonnegative price restriction, namely $x_1^* = 1 - x_2^*$, $(1+c+2d)/(1+2d+a) \leq x_2^* \leq (a-c)/(1+2d+a)$.

Proof.

Substituting $x_{1,t} = x_{2,t-1} = x^*$ in the expression for the marginal utility,

$$\frac{\partial U^1}{\partial x_{1,t}} = 1 - c - (2+2d)x_{1,t} - x_{2,t-1} + a\frac{x_{2,t-1}}{(x_{1,t}+x_{2,t-1})^2} = 0,$$

(which determines the reaction curve of firm 1) leads to $1 - c - (3+2d)x^* + \frac{a}{4x^*} = 0$ from which x^* can be solved directly. This proves part (i) (see also Section 5.3 for the general equation for the equilibrium output of two symmetric players). This equilibrium is unique for $a \leq 1+2c+2d$ whereas for larger values of a the two reaction curves belong to type T_3 or T_4 (Proposition 5.11) and have a (symmetrical) part of the nonnegative price restriction in common. First consider the stability of the unique equilibrium. For $a_1=a_2=a$ the eigenvalues of the linearized system (see the proof of Proposition 5.2, Section 5.3 with s equal to the market share and $x_1^* = x_2^* = x^*(a)$) are real and equal

$$\lambda_{1,2} = \pm\frac{1}{(2+2d) + {a}/{4(x^*)^2}}$$

We used the expressions for the partial derivatives $s_{x_1x_1}$ and $s_{x_1x_2}$ of Section 5.4.

Stability of the linearized system is guaranteed for $d > -1/2$, $a \geq 0$, because then the absolute values of these eigenvalues are less than 1. Because the equilibrium (x^*,x^*) is a positive attractor of the linearized system ($|\lambda_i| < 1$) it is also a (local) positive attractor of the nonlinear system of first order difference equations (Devaney (1989),Theorem 6.3, Chapter 2). This proves the (local) stability for $a \leq 1+2c+2d$.

For $a > 1+2c+2d$ the two reaction curves - which are symmetric with respect to the line $x_1=x_2$ - have a part of the line $x_1+x_2=1$ in common. The description of this line segment follows directly from the co-ordinates of the intersection point of the reaction curve (of firm 1) with the nonnegative price restriction (Proposition 5.9). In each of these equilibria the slopes equal -1 which indicates neutrality.

[End of proof]

Before we illustrate the results of Proposition 5.12 with some graphs, we make several remarks on these results in relation to the power series approximations and stability properties for arbitrary functions s (Section 5.3). For small values of a application of Proposition 5.1, Section 5.3 leads to a power series approximation of the equilibrium outputs $x^*(a)$. The specific choice $s(x_1,x_2) = x_1/(x_1+x_2)$ leads to the required partial derivatives in $((1-c)/(3+2d),(1-c)/(3+2d))$:

$$s_{x_1} = \frac{x_2}{(x_1+x_2)^2} = \frac{(3+2d)}{4(1-c)}, \quad s_{x_1x_2} = \frac{(x_1-x_2)}{(x_1+x_2)^3} = 0 \text{ and } s_{x_1x_1} = \frac{-2x_2}{(x_1+x_2)^3} = -\frac{(3+2d)^2}{4(1-c)^2} \tag{5.22}$$

Choosing $a_1=a_2=a$ in the power series expression we obtain

$$x^*(a) = \frac{(1-c)}{(3+2d)} + \frac{1}{4(1-c)}a - \frac{(3+2d)}{16(1-c)^3}a^2 + O(a^3), \text{for } a \to 0. \tag{5.23}$$

Fortunately we were able to derive an expression for the equilibrium outputs for all values of a and if we expand this expression for $x^*(a)$ into a power series (using the expansion $\sqrt{1+y} = 1 + \frac{1}{2}y - \frac{1}{8}y^2 + O(y^3)$) it fits completely with the expression which follows from the application of Proposition 5.1, Section 5.3. The power series approximation already reveals that (for small values of a) the function $x^*(a)$ is rising and concave with respect to an increasing weight a attributed to market share.

Furthermore note that for $a \leq 1+2c+2d$ only local stability of the Cournot-Nash equilibrium has been proved. Local stability means that there exists a neighbourhood N of the equilibrium with the property that, for all (initial) outputs of both firms in N, supplies converge to the equilibrium under forward iteration. Such a neighbourhood N is often called the "basin of attraction". However observation of two (symmetrical) reaction curves obviously reveals that the basin of attraction consists of all possible (feasible) initial outputs of both firms and therefore the equilibrium is also globally stable. Because of the equilibrium's global stability, comparative statics in this section makes sense.

For small values of a, Proposition 5.2 (Section 5.3) reveals that the preference for the market share has a stabilizing influence on the equilibrium, because the absolute value of the eigenvalues decreases in comparison to the classical case. The exact expression for the absolute value of both eigenvalues (in the proof of Proposition 5.12) allows us to examine this stabilizing property for all values of a. Using the chain rule we obtain

$$\frac{\mathrm{d}|\lambda_{1,2}|}{\mathrm{d}a} = \frac{\mathrm{d}}{\mathrm{d}a}\left\{\frac{1}{(2+2d) + a/4(x^*)^2}\right\} = \frac{-1}{[(2+2d) + a/4(x^*)^2]^2} \cdot \frac{[x^* - 2a\frac{\mathrm{d}x^*}{\mathrm{d}a}]}{4(x^*)^3}. \tag{5.24}$$

$\frac{\mathrm{d}|\lambda_{1,2}|}{\mathrm{d}a}$ is negative if and only if the expression $x^*(a) - 2a\frac{\mathrm{d}x^*}{\mathrm{d}a}$ is positive. Using the formula for $x^*(a)$ of Proposition 5.12 the latter can be proved easily. So now we have proved for arbitrary values of a that the absolute value of both eigenvalues decreases if the weight attributed to preference for market share increases. The smaller the absolute value of the eigenvalues is, the larger the speed will be at which the equilibrium will be approached. In other words: preference for market share has a stabilizing influence on the equilibrium outputs and equilibrium's disturbances will be restored more quickly. This stabilizing property also holds for more than two (symmetrical) competing firms. As we already noted in Chapter 4, Theocharis (1960) examined the stability of the n-firm oligopoly Cournot solutions and found that for the classical case with 3 firms and constant unit production costs cyclical solutions occur. By introducing preference for the market size it is possible to prove that the equilibrium corresponding to the 3-firm case becomes stable and that the endogenous business cycles caused by the (classical) cyclic outputs no longer occur.

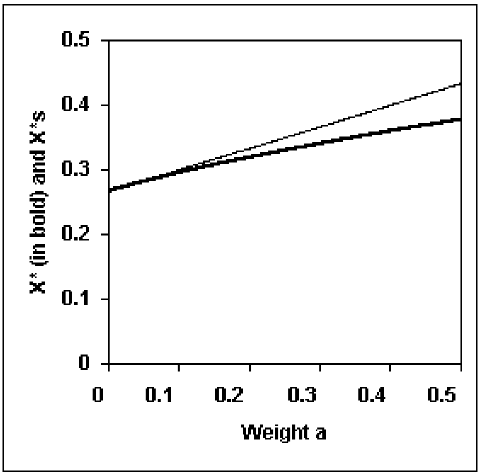
We now illustrate part (i) of Proposition 5.12 with two graphs of the output $x^*(a)$ and in both graphs we combine the output $x^*(a)$ with the output $x_s^*(a)$ which corresponds to the preference for size (so $s(x_1,x_2) = x_1$, see also Chapter 3).
Why this comparison of both outputs is an interesting issue? As already mentioned in the introduction of this section managers like to grow, but dislike to retrench. This managerial (asymmetric) inertia is in general reflected in the weight attributed to the non-profit part of the maximand. The general consideration in Section 5.3 reveales that, if $s_{x_1} > 0$, also the equilibrium output x^* is (for small a) a rising function of the weight a. The use of the function s in the general model - with the property that the more the firm produces, the more status it acquires - nicely shows that the implication of such a non-profit part clearly is an enlargement of the firm's size (and sales). The application of Proposition 5.1 of Section 5.3 gives (choose a_1=a_2=a)

$$\frac{\mathrm{d}x^*}{\mathrm{d}a} = \frac{s_{x_1}\left({}^{(1-c)}\!/\!_{(3+2d)}, {}^{(1-c)}\!/\!_{(3+2d)}\right)}{(3+2d)}, \text{ in } a=0. \tag{5.25}$$

This latter mathematical property illustrates the behaviour of managers - they prefer to be the head of large budgets, organizations or staffs rather than small ones - but obviously the exact specification of the function s determines, besides the weight a, the degree of influence on the size of the firm. Therefore paying attention to the implications of both preference for size and preference for market share is useful because it reveals the different consequences of both preferences.
Note that - by using the equation $1- c- (3+2d)\,x_s^* + a = 0$ (see also the general considerations on equilibria of two symmetric firms at the end of Section 5.3) - the output $x_s^*(a)$, concerning the preference for size, equals

$$x_s^*(a) = \frac{(1-c)}{(3+2d)} + \frac{a}{(3+2d)}. \tag{5.26}$$

(If the nonnegativity condition is imposed on the price, multiple equilibria occur for the weight a > ½ $+c+d$). The first graphical display, Fig. 5.5a, shows how both outputs x^* and x_s^* depend on the weight a; for the production cost parameters the choice c=0.2, d=0 is made. So the first graph deals with an efficient production technology as indicated by the small unit production costs (on a scale from 0 to 1). The graphs of Fig.5.5b allow us to compare the outputs x^* and x_s^* for a less efficient production technology with constant unit costs c=0.6. In both figures the output concerning preference for market share is printed bold.



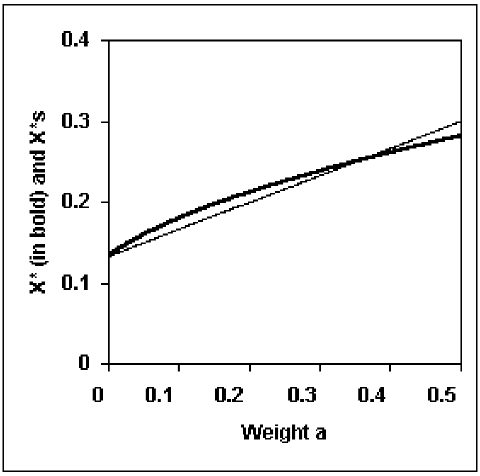


Fig. 5.5a $x^*(a)$ **and** $x^*_s(a)$ **for** c=0.2,d=0.   **Fig. 5.5b** $x^*(a)$ **and** $x^*_s(a)$ **for** c=0.6,d=0.

Clearly both graphs of $x^*(a)$ show that the output - corresponding to the preference for market share - is not only concave for small values of a, but displays an overall concave and rising behaviour with respect to a, whereas $x_s^*(a)$ behaves linearly. And the increase of the output, in terms of percentage, compared with the classical Cournot output ($=(1-c)/(3+2d)$), is substantial: A weight of a=0.1 attributed to the preference for market share raises the classical equilibrium output by 11% (Fig.5.5a) and almost 35% (Fig.5.5b). But another phenomenon strikes the eye. Apparently $x^* < x_s^*$ for all positive values of the weight a if the firm controls an efficient production technology, whereas in the case of the less efficient cost regime there exists an interval (of values of a) where $x^* > x_s^*$. The necessary and sufficient condition for the existence of such an interval is - because of the concavity of $x^*(a)$ - that the slope $\mathrm{d}x^*/\mathrm{d}a$ exceeds the slope $\mathrm{d}x_s^*/\mathrm{d}a$ at a=0. We summarize these considerations in a Proposition.

Proposition 5.13 (properties of the outputs x^* and x_s^* related to the weight a).

Let $x^*(a)$ and $x_s^*(a)$ be the equilibrium outputs of two completely symmetric firms, corresponding to the preference for market share and size respectively.

(i) For all weights a it holds that $\frac{\mathrm{d}x^*}{\mathrm{d}a} > 0$ and $\frac{\mathrm{d}^2x^*}{\mathrm{d}a^2} < 0$, so the output corresponding with preference for market share is a rising and concave function with respect to the weight a (and $x_s^*(a)$ is a linear function) .

(ii) If $c \leq ¼ - ½\, d$ then for all $a > 0$ it holds that $x^* < x_s^*$. If $c > ¼ - ½\, d$ then for all a with $0 < a < -¼ + c + ½\, d$ it holds that $x^* > x_s^*$.

Proof.
Part (i) can be proved easily by differentiation of the expression for the equilibrium output x^* (Proposition 5.12) with respect to a.
There only exists an interval of values of a with $x^* > x_s^*$ if the slope dx^*/da at $a=0$ exceeds the slope dx_s^*/da at $a=0$, i.e. $1/(4-4c) > 1/(3+2d)$. The latter inequality is equivalent with $c > ¼ - ½\,d$ and the interval of values of a for which $x^* > x_s^*$ holds can be obtained by solving

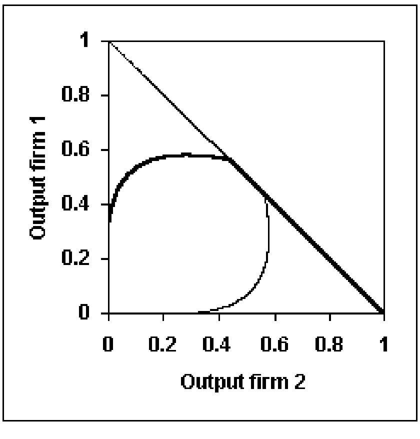
$$\frac{(1-c)}{2(3+2d)}+\frac{\sqrt{(1-c)^2+(3+2d)a}}{2(3+2d)} > \frac{(1-c)}{(3+2d)}+\frac{a}{(3+2d)}$$ with respect to a.

[End of proof]

Note that the application of the result of part (ii) of Proposition 5.13 to the case of Fig.5.5b with $c=0.6$, $d=0$ leads to $x^* > x_s^*$ for $0 < a < 0.35$, which is confirmed by the graphical display. Clearly the output level of a firm - which is heightened by preference for size or market share - determines its profit level. Therefore the differences between x^* and x_s^* under various cost regimes may have important implications for the degree of sacrifice of profits by managerial preferences. We now continue with the analysis of the (equilibrium) profits of both firms and start with a consideration corresponding to larger weights a i.e. $a > 1+2c+2d$. Then both reaction curves belong to the T_3 - or T_4 - type and Proposition 5.12 (ii) reveals that multiple equilibria exist. We deal with a rather large preference for the market share; for $c=0.4$, $d=0$ the weight a has to exceed 1.8 (in the maximand the weight attributed to the profit equals 1). Apparently more importance is attached to market share in comparison to profit.

The case of multiple equilibria.

In the Figures 5.6a and 5.6b both reaction curves are displayed graphically for the production cost function with parameters $c=0.4$ and $d=0$ but for different weights a. Fig. 5.6a corresponds with $a=2.2$ and shows the "T_3 - T_3" case and Fig. 5.6b displays the "T_4 - T_4" case corresponding with $a=3.5$ (firm 1's curve is printed bold).



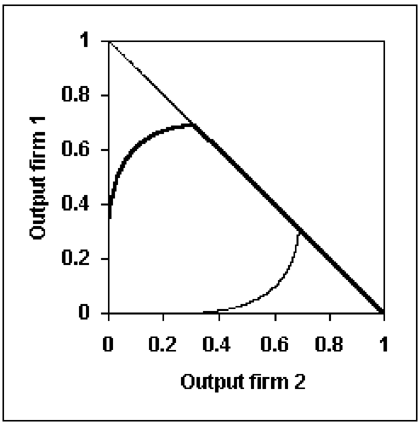


Fig. 5.6a "T_3 - T_3" for $c=0.4, d=0, a=2.2$. Fig. 5.6b "T_4 - T_4" for $c=0.4, d=0, a=3.5$.

Note that for a=2.2 both firms possess a reaction curve with an analytical maximum, whereas for a=3.5 both maxima are non-differentiable (see also Proposition 5.11). All these equilibria located on the common line segment of both curves (for a=2.2 this segment equals $x_1^* = 1 - x_2^*$, $0.4375 \le x_2^* \le 0.5625$) still satisfy the definition of a (strict) Nash-equilibrium provided by Fudenberg and Tirole (1991):

"A Nash equilibrium is a profile of strategies such that each player's strategy is an optimal response to the other players' strategies. Because each player has a unique response to his rivals' strategies the Nash equilibrium is strict".

However in each of these equilibria both firms face losses, because the line segment of multiple equilibria is located on the nonnegative price restriction, i.e. the equilibrium market price p^* equals zero. We have to realize that this uncommon nonbeneficial situation, with give-away prices, is caused by the rather large weight attributed to the market share. The large preference for market share (in comparison to preference for profit) permits the existence of these equilibria and implies total sacrifice of the profit.

Why pay attention to these equilibria if each equilibrium corresponds with losses for both competitors, due to zero-prices? As already mentioned in Chapter 3 there exists empirical evidence that in many cases firms accumulate losses before they are forced to exit. For instance a central issue in the literature on Accounting and Finance is the identification of financial ratios that can predict bankruptcy even a few years before the actual date of this corporate exit. A study of 57 large corporate failures of Hambrick and D'Aveni (1988) showed that, in the five years prior to the date of bankruptcy (say t), the series of the (mean net income/assets) - ratios equals –4.56 (t – 5), –21.79 (t – 4), –21.30 (t – 3), –85.11 (t – 2) and –107.89 (t – 1). These ratios clearly indicate that the decision to exit may be taken after a long period of losses. Now if we assume that firm i can endure losses for some time, this firm has the possibility to choose its strategic actions in such a way that the rival j suffers more severe losses.

An adequate strategy may open the possibility to drive the competitor out of the market, notwithstanding negative profits for some period of time. We have to realize that products are available free and one could think of an advertising stunt with give-away prices as a strategic move. After the exit of the rival in the duopoly game the remaining firm can act as a monopolist. By more detailed examination of the properties of the multiple equilibria we can gain a more clear insight into these beneficial strategies. Additionally some marginal comments have to be made concerning the types (T_3 or T_4) to which the reaction curves belong.

Since the market price p^* is zero in each of the equilibria (x_1^*, x_2^*) on the line segment, the losses of both firms are only determined by their production costs, i.e. $\Pi^i(x_i^*, x_j^*) = -cx_i^* - d(x_i^*)^2$ for firm i. Clearly the equilibrium (on the segment) with the smallest production level x_i^* is the most beneficial "choice" for firm i, because by keeping its output level small this firm can restrict its inevitable losses. The behaviour of the rival j however is dictated by its Cournot reaction curve: firm j accommodates perfectly and the smaller the output of firm i will be the larger firm j's production level will be. So this reaction mechanism leads to more severe losses of firm j. The simple expression of the equilibrium profits on the line segment and the property that $x_1^* + x_2^* = 1$ enable us to obtain an analytical expression for the absolute advantages of the strategically strongest firm.

Proposition 5.14 (multiple equilibria and strategy (advertising stunt)).

If $a > 1{+}2c{+}2d$ multiple Cournot-Nash equilibria occur on the nonnegative price restriction and both firms accumulate losses. The best strategy for firm i is to keep its equilibrium output x_i^* as small as possible i.e. $x_i^* = (1{+}c{+}2d)/(1{+}2d{+}a)$.
By doing so it holds that the absolute advantage over its rival equals

$$\Pi^i - \Pi^j = \frac{(c+d)(a-1-2c-2d)}{(1+2d+a)}.$$

(for linear production costs it holds that $\dfrac{\text{loss firm } j}{\text{loss firm } i} = \dfrac{a-c}{1+c} > 1$ for $a > 1+2c$)

Proof.
From $\Pi^i(x_1^*, x_2^*) = -cx_i^* - d(x_i^*)^2$ and $x_1^* + x_2^* = 1$ it follows that

$$\Delta\Pi = \Pi^i(x_1^*, x_2^*) - \Pi^j(x_1^*, x_2^*) = (c+d)(1-2x_i^*)$$

On the line segment of multiple equilibria this difference is maximized for the smallest value of x_i^* (see Proposition 5.12 (ii)). In general it holds that

$$\frac{\text{loss firm } j}{\text{loss firm } i} = \frac{-\Pi^j}{-\Pi^i} = 1 - \frac{\Delta\Pi}{\Pi^i} = 1 + \frac{(c+d)(1-2x_i^*)}{cx_i^* + d(x_i^*)^2}.$$

For $d{=}0$ this expression simplifies to $-1 + 1/x_i^*$ with $x_i^* = (1{+}c)/(1{+}a)$.

[End of proof]

We apply Proposition 5.14 to example 1: For $c{=}0.4$, $d{=}0$ and $a{=}2.2$ (Fig.5.6a) the equilibrium on the line segment with the smallest output for firm 1 equals (x_1^*, x_2^*) = (0.4375 , 0.5625). This equilibrium leads to losses of 0.175 and 0.225 per period for firm 1 and 2 respectively and the loss of firm 2 equals about 1.3 times the loss of firm 1. Because both reaction curves possess an analytical maximum - for firm 2 (by simulation, Proposition 5.10) the maximum occurs for $x_{1,max} = 0.285$ and the maximum value equals $G(x_{1,max}|2.2) = 0.578$ - firm 1 can start with output 0.285 (in fact firm 1 can start with an output in the interval [0.169 , 0.4375]). The relatively beneficial equilibrium for firm 1 is reached after a few steps and starting with $x_{1,t} = 0.285$ the outputs, obtained by simulation, are $x_{2,t+1} = 0.578$, $x_{1,t+2} = 0.422$ (perfect accommodation), $x_{2,t+3} = 0.565$, $x_{1,t+4} = 0.435$ (perfect accommodation), $x_{2,t+5} = 0.563$. Note that in the first period, consisting of the time periods t and $t{+}1$, the losses of firm 1 and 2 equal 0.075 and 0.152 respectively which is even more beneficial for firm 1. This example clarifies that - due to the analytical maximum of the T_3-type - firm 1 can choose its initial output level in a broader interval and each of these initial values leads to the equilibrium on the segment of multiple equilibria with the smallest output for firm 1. Furthermore the relative advantage for firm 1 is even larger during the first periods.

Example 2 corresponds with Fig. 5.6b and for this larger value of $a{=}3.5$ both curves possess a nonanalytical maximum (type T_4). To reach the smallest equilibrium production level ($x_1^* = 0.311$) firm 1 can only start with this specific output of 0.311. Equilibrium losses of firm 1 and 2 equal 0.124 and 0.276 respectively and the loss of firm 2 is more than two times the loss of its rival. Proposition 5.14 and the two examples reveal that - under the assumption of temporary losses and with the prospect of being a monopolist - there indeed exists an adequate strategic action, with give-away prices, which may drive the rival out of the market.

However some comments have to be made here. Each equilibrium belonging to the set of multiple equilibria is not asymptotically stable but neutral (slope $= -1$) and as a consequence disequilibrium will not be reversed. So once the equilibrium is reached this equilibrium can be shifted along the segment of multiple equilibria. If we assume learning by experience, the firm which suffers the largest losses in equilibrium may shift the equilibrium along the line segment to a more beneficial output location, i.e. a new equilibrium with a smaller output. This learning by doing can only occur if a firm is able to drop its usual (Cournot) reaction pattern. Note that there exists a difference between the "T_3-T_3"- and the "T_4-T_4"-competition (examples 1 and 2). If the reaction curves belong to the T_3-type the neutral equilibrium on the segment is not reached immediately and the firm with an adequate strategic output already benefits from the reaction pattern before its rival is possibly able to drop its former habits and starts to learn by experience.

The previous analysis, concerning the multiple equilibria, shows that, notwithstanding the negative profits of both rivals, some strategic moves are still possible. However such large managerial preference for the market share, indicated by $a > 1+2c+2d$, inevitably leads to a complete sacrifice of the profits, as long as the two firms can stay in the market. After a period of a "war of attrition", one firm may leave the market whereas the other firm becomes a monopolist. For smaller values of a ($a \leq 1+2c+2d$) Proposition 5.12 (ii) reveals the existence of a unique and (asymptotically) stable equilibrium and the reaction curves belong to the T_1- or the T_2-type. We continue our analysis with the examination of the equilibrium profits and equilibrium welfare. Central issue is the sacrifice of profits in relation to managerial inertia, i.e. the weight attributed to the market share (or size).

The case of the unique equilibria.

Using the general expression for the equilibrium profits Π^i in the equilibrium ($x^*(a), x^*(a)$) it follows that - without specifying $x^*(a)$ - the heightening of the weight a always implies a further sacrifice of the profit level.

From $\Pi^i(a) = x^*(a)\left[1-2x^*(a)\right] - cx^*(a) - d\left\{x^*(a)\right\}^2 = (1-c)x^*(a) - (2+d)\left\{x^*(a)\right\}^2$ it follows by differentiation with respect to the weight a that $\dfrac{\mathrm{d}\Pi^i}{\mathrm{d}a} = \left[1-c-(4+2d)x^*(a)\right]\dfrac{\mathrm{d}x^*}{\mathrm{d}a}$.

The latter expression is always negative if $\dfrac{\mathrm{d}x^*}{\mathrm{d}a} > 0$ (clearly $x^*(a) > (1-c)/(3+2d) > (1-c)/(4+2d)$, for $a > 0$). Obviously - under the assumption that $x^*(a)$ is an increasing function of a - the equilibrium profits of both firms decrease with respect to an increasing a, so each heightening of the weight a leads to further decreasing profits (for the general expression for $\mathrm{d}x^*/\mathrm{d}a$ see Section 5.3)

For the specific choice $s(x_1,x_2) = x_1/(x_1+x_2)$ of the non-profit part of the maximand we can use the power series approximation of Π^i of Section 5.3 to obtain a second order approximation of the equilibrium profits. Choosing $a_1=a_2=a$ and substituting the expressions for the partial derivatives $s_{x_1}, s_{x_1x_1}$ and $s_{x_1x_2}$ in the point with co-ordinates $((1-c)/(3+2d), (1-c)/(3+2d))$ leads to the following approximation for small a :

$$\Pi^i(a) = \frac{(1-c)^2(1+d)}{(3+2d)^2} - \frac{1}{4(3+2d)}a - \frac{(1+d)}{16(1-c)^2}a^2 + O(a^3), \text{ for } a \to 0 \qquad (5.27)$$

This expression shows that the profit is not only decreasing but also concave with respect to small weights a. Note that for small weights a the (absolute) amount of sacrificed profit is given by the sum of the terms with a and a^2 of this expression.
The use of general techniques is very useful if the expression for the equilibrium outputs is too complicated, but here we fortunately have a convenient expression for $x^*(a)$ at our disposal (Proposition 5.12 (i)). The exact expression for the equilibrium profits allows us to make a statement on the concavity of the profit function for all values of a and enables us to compute that specific value of a for which profits are sacrified completely. Also a comparison with preference for size as the non-profit part of the objective function can be made.

Proposition 5.15 (sacrifice of profits by managerial inertia).

Consider two completely symmetric firms, with respect to the weight *a* attributed to the preference for market share and let $a \leq 1+2c+2d$. Let $\Pi^i(a)$ be the equilibrium profit of firm i (i=1,2), corresponding with the unique (stable) equilibrium output.

(i) For all $0 \leq a \leq 1+2c+2d$ it holds that $\frac{\mathrm{d}\Pi^i}{\mathrm{d}a} < 0$ and $\frac{\mathrm{d}^2\Pi^i}{\mathrm{d}a^2} < 0$, so the profit corresponding with the preference for the market share is a falling and concave function with respect to the weight a.

(ii) The profit is completely sacrificed, i.e. $\Pi^i(a) = 0$, for $a = \frac{4(1-c)^2(1+d)}{(2+d)^2}$.

(iii) Now consider two symmetric firms with preference for size instead of market share and let $\Pi_s^i(a)$ be the equilibrium profit. This profit is completely sacrificed for $a_s = \frac{(1-c)(1+d)}{(2+d)}$ and $a_s < a$ for $c < ½ - ¼\, d$.

Proof.
Substituting the expression for $x^*(a)$ (Proposition 5.12(i)) in the expression for the equilibrium profits $\Pi^i(a) = (1-c)x^*(a) - (2+d)\{x^*(a)\}^2$ leads to

$$\Pi^i(a) = \frac{(1-c)^2(1+d)}{2(3+2d)^2} + \frac{(1+d)(1-c)}{2(3+2d)^2}\sqrt{(1-c)^2 + (3+2d)a} - \frac{(2+d)}{4(3+2d)}a,$$
$$0 \leq a \leq 1+2c+2d$$

(for a=0 this profit equals the classical profit $(1-c)^2(1+d)/(3+2d)^2$). By differentiating this expression with respect to the variable a we obtain

$$\frac{\mathrm{d}\Pi^i}{\mathrm{d}a} = -\frac{(2+d)}{4(3+2d)} + \frac{(1+d)(1-c)}{4(3+2d)\sqrt{(1-c)^2+(3+2d)a}} < 0 \quad .$$

for all $a \geq 0,\ 0 < c < 1,\ d > -\frac{1}{2}$

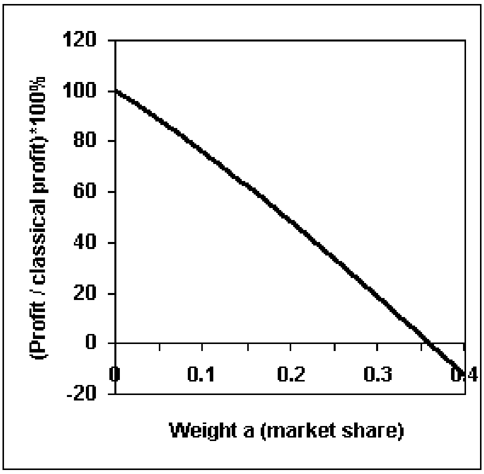
$$\frac{\mathrm{d}^2\Pi^i}{\mathrm{d}a^2} = -\frac{(1+d)(1-c)}{8\left[(1-c)^2+(3+2d)a\right]^{3/2}} < 0 \text{ for all } a \geq 0,\ 0 < c < 1,\ d > -\tfrac{1}{2} .$$

which proves part (i). Part (ii) is proved by using that $\Pi^i(a) = 0$ for $x^*(a) = (1-c)/(2+d)$. Solving the latter equation concerning the equilibrium output gives the formula for a. Using the equilibrium output $x_s^*(a)$, concerning the preference for size, the corresponding profit $\Pi_s^i(a)$ is completely sacrificed for that specific value a_s for which $x_s^*(a) = (1-c)/(2+d)$ holds. This consideration easily leads to an expression for a_s and to the condition (for c and d) for which $a_s < a$ holds.

[End of proof]

The result of part (i) of Proposition 5.15 indicates that profit is sacrificed increasingly if the weight attributed to the market share increases further. In other words: the larger the managerial inertia will be the more extra profit will be sacrificed. This mathematical result confirms the statement in the introduction of this section that "the positive utility from sales (or market share) increases the management willingness to expand, even if this implies that profit is sacrificed". The second part reveals that this managerial inertia can even lead to a complete sacrifice of the firm's profitability if a is large enough. Note that for all $0 < c < 1$ and $d > -1/2$ this specific value of a satisfies $a \leq (1-c)^2 < 1$ (for constant unit costs the weight corresponding to "total sacrifice" equals $a = (1-c)^2$), so the profits drop to zero whereas the weight attributed to the profit in the objective function (=1) is still the largest one. Application of the result of part (iii) of Proposition 5.15 to the case with linear production costs reveals that $a_s < a$ if $c < ½$. Apparently, if both firms control a more efficient production technology, a total sacrifice of firm's profitability occurs at an earlier stage of managerial inertia if preference for size is concerned. Note that Proposition 5.13 shows that, for small values of c, the output level corresponding to preference for market share (x^*) is smaller than the equilibrium production associated with preference for size (x_s^*) .

To gain a more clear insight in the amount of profit that is sacrificed as a consequence of managerial preference for market share we present two graphs of the profits $\Pi^i(a)$ expressed as a percentage of the classical profit Π_{cl} (corresponding with a=0 and equal to the standard expression $(1-c)^2(1+d)/(3+2d)^2$).
Figure 5.7a shows the decline of the equilibrium profits for c=0.4 and d=0 whereas Fig.5.7b graphically displays the falling profit levels for c=0.6 and d=0.



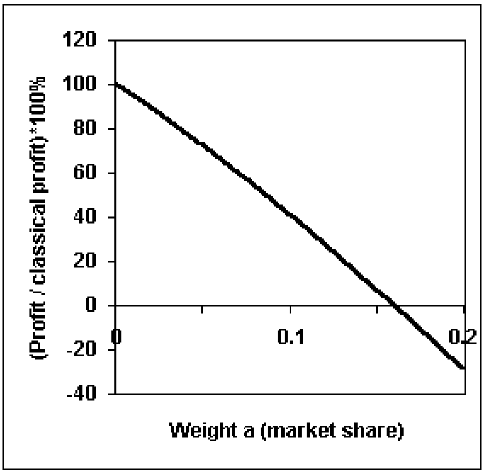


Fig. 5.7a $\Pi^i(a)/\Pi_{cl}$ ***100% for** c**=0.4,**d**=0.** **Fig. 5.7b** $\Pi^i(a)/\Pi_{cl}$ ***100% for** c**=0.6,**d**=0.**

Total profit is sacrificed for the weights $a=0.36$ and $a=0.16$ corresponding with unit costs of 0.4 and 0.6 respectively. Because we do not assume immediate exit related to zero profits beforehand, losses are also displayed. The numerical information of Table 5.1 allows us to compare the percentage of the profits (in comparison with classical profit) that has been sacrificed due to the weight attributed to market share and size respectively; for the production cost parameters we choose $c=0.4$ and $d=0$. Note that in accordance with Proposition 5.15 (part (iii)) $\Pi_s^i(a)$ - the profit corresponding to preference for size - becomes zero for $a_s=0.3$.

Table 5.1, percentage of classical profit that has been sacrificed; c**=0.4,** d**=0.**

	a=0.05	a=0.10	a=0.15	a=0.20	a=0.25	a=0.30	a=0.35	a=0.40
$[\Pi_{cl}-\Pi^i(a)]/\Pi_{cl}$ *100%	11%	24%	37%	52%	66%	81%	97%	losses
$[\Pi_{cl}-\Pi_s^i(a)]/\Pi_{cl}$ *100%	10%	22%	37%	56%	76%	100%	losses	losses

For instance this table reveals that, corresponding with a weight of $a=0.1$ attributed to market share or size, almost a quarter of the (classical) profit has been sacrificed and that equilibrium profits show a decrease of more than 50% for $a=0.2$. Table 5.2 illustrates the differences between the (sacrifice of the) profits concerning preferences for market share and size, but now related to a less efficient production technology ($c=0.6$ and $d=0$).

Table 5.2, percentage of classical profit that has been sacrificed; c**=0.6,** d**=0.**

	a=0.05	a=0.10	a=0.15	a=0.20	a=0.25	a=0.30	a=0.35	a=0.40
$[\Pi_{cl}-\Pi^i(a)]/\Pi_{cl}$ *100%	27%	59%	93%	losses	losses	losses	losses	losses
$[\Pi_{cl}-\Pi_s^i(a)]/\Pi_{cl}$ *100%	16%	37%	66%	100%	losses	losses	losses	losses

As Proposition 5.15, part (iii), predicts, now profit is sacrificed completely for a smaller weight if preference for market share is concerned ($a = 0.16 < a_s = 0.2$). Now for a weight of a=0.1 concerning managerial preference for market share more than half of the (classical) profit has been sacrificed. Clearly the graphs of Fig. 5.7a and 5.7b and both tables show that the managerial "love for market share or sales" has a larger impact on the equilibrium profit levels if both firms control a less efficient production technology reflected in larger unit costs. Before we continue with the (mathematical) analysis of the implications of the behaviour of (top) management on social welfare some brief and intuitive considerations concerning this welfare are in place. Although both firms - due to their "love for market share" - face declining profits or even absolute losses related to an increasing weight attributed to this non-profit part of the managerial objective, the consumer benefits from the situation. Clearly the consumer can acquire products at a low market price, due to the increased production level. This abundant supply at a low market price causes a significant rise of the consumer surplus, which amply compensates both firms' decreased profits. Obviously social welfare benefits from the "love for market share" by a firm's management.

We start the welfare analysis with the application of Proposition 5.5 of Section 5.3, where a general power series expression for the welfare concerning two symmetric firms is given. Choosing a_1=a_2=a and using the expressions for $s_{x_1}, s_{x_1x_1}$ and $s_{x_1x_2}$ in the point ((1– c)/(3+2d), (1– c)/(3+2d)) (see Section 5.4) we obtain the following approximation of the welfare for small a :

$$W(a) = \frac{(1-c)^2(4+2d)}{(3+2d)^2} + \frac{1}{2(3+2d)}a - \frac{(2+d)}{8(1-c)^2}a^2 + O(a^3), \text{ for } a \to 0 \qquad (5.28)$$

Clearly this second order power series reveals that $W(a)$ is rising and concave with respect to small weights a. Although the welfare initially rises with an increasing a the concavity indicates that the sky is not the limit (see also the reflections on Proposition 5.5, Section 5.3 where we provided a link between the concavity of the welfare function and general conditions for $s_{x_1x_1}$ and $s_{x_1x_2}$). Again the expression for $x^*(a)$ of Proposition 5.12 (i) and the general expression for the welfare (Section 5.3) - $W(a) = \Pi^1(a) + \Pi^2(a) + CS(a) = 2(1-c)x^* - (2+2d)(x^*)^2$ - allow us to derive an exact expression for the welfare in equilibrium. The rising and concave character of the welfare function can be proved for arbitrary values of the weight a. Furthermore the welfare maximizing value of the weight a can be derived if we take into account the fact that permanently failing firms may exit in the long run. The exit of the firms means that no products are available anymore and thus implies a complete collapse of social welfare. Proposition 5.16 contains a summary of these general results.

Proposition 5.16 (Properties of the welfare and welfare-maximizing weights).

Consider two completely symmetric firms, with respect to the weight a attributed to the preference for market share and let $a \leq 1+2c+2d$. Let $W(a)$ be the equilibrium welfare, corresponding with the unique (stable) equilibrium output.

(i) $\frac{\mathrm{d}W}{\mathrm{d}a} = 0$ for $a_{max} = \frac{(1\text{-}c)^2}{(1+d)^2}$ and $\frac{\mathrm{d}^2W}{\mathrm{d}a^2} < 0$, so the welfare corresponding with the preference for the market share is a concave function with respect to the weight a and reaches a maximum for a_{max}.

(ii) If $d < 0$ the maximum welfare would be reached for a_{max} corresponding to negative profits of both firms. Assuming that these firms will face bankruptcy in the long run, the maximum welfare is reached for $a_{max} = \frac{4(1-c)^2(1+d)}{(2+d)^2}$ corresponding with a production level at the break-even point. If, however, $d \geq 0$ the maximum welfare is reached for the weight $a_{max} = \frac{(1-c)^2}{(1+d)^2}$ corresponding to nonnegative profits for both firms.

Proof.

From the general expression for the welfare - $W(a) = (2-2c)x^*(a) - (2+2d)\{x^*(a)\}^2$ - it follows that

$$\frac{\mathrm{d}W}{\mathrm{d}a} = \left[2(1-c) - 4(1+d)x^*(a)\right] \cdot \frac{\mathrm{d}x^*}{\mathrm{d}a}.$$

Because $x^*(a)$ is a monotonically rising function with respect to a (Proposition 5.13 (i)) the maximum of the function $W(a)$ is reached for a_{max} satisfying $x^*(a_{max}) = (1-c)/(2+2d)$ (see also Proposition 5.8, Section 5.3). Using the expression for $x^*(a)$ the value a_{max} can be solved easily. The expression for the welfare function equals

$$W(a) = \frac{(1-c)^2(2+d)}{(3+2d)^2} + \frac{(2+d)(1-c)}{(3+2d)^2}\sqrt{(1-c)^2 + (3+2d)a} - \frac{(1+d)}{2(3+2d)}a,$$
$$0 \leq a \leq 1 + 2c + 2d$$

The concavity of the welfare function follows directly from

$$\frac{\mathrm{d}^2W}{\mathrm{d}a^2} = -\frac{(2+d)(1-c)}{4\left[(1-c)^2 + (3+2d)a\right]^{3/2}} < 0 \text{ for all } a \geq 0,\ 0 < c < 1,\ d > -\tfrac{1}{2}$$

This proves part (i). For $d < 0$ the maximum welfare is reached for negative profits of both firms (see also the general result of Proposition 5.8, Section 5.3). Under the assumption that these firms will exit in the long run the (realistic) maximum welfare corresponds with zero profits and a_{max} follows from Proposition 5.15 (ii). Concerning $d \geq 0$ the maximum welfare is reached while the profits are still nonnegative, so no extra restriction has to be imposed on a_{max}.

[End of proof]

We stress that, concerning the period of time for which loss-bearing firms still decide to stay in the market, the maximum welfare is reached for $a_{max} = (1\text{-}c)^2/(1+d)^2$, so the expression for a_{max} for $d < 0$ clearly rests on the assumption of exit of (one or) both firms after some time. We use two graphs to clarify the development of the welfare as a percentage of the classical (equilibrium) welfare, corresponding with a rising weight attributed to the market share. Figure 5.8a shows the relative welfare level concerning a more efficient production technology with constant unit costs c=0.4 whereas the development of the (relative) welfare, corresponding with a less efficient production technology (c=0.6), is displayed in Figure 5.8b. Because the production cost parameter d equals zero the maximum level of (relative) welfare is reached exactly for that specific weight a_{max} corresponding with zero profits, i.e. production at the break-even point. For larger weights the final collapse of the social welfare - due to exit - is displayed in the graph.

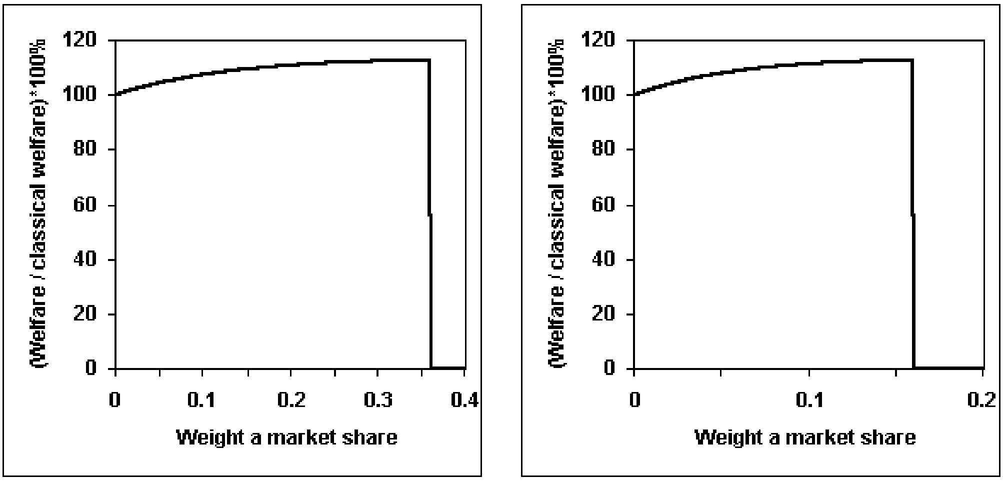


Fig. 5.8a $W(a)/W_{cl}$ *100% for c=0.4,d=0. **Fig. 5.8b $W(a)/W_{cl}$ *100% for c=0.6,d=0.**

Clearly the maximum welfare level (equal to 112.5%) for the efficient production technology is reached for a_{max} = 0.36 for which 100% of the profit is sacrificed (see Fig. 5.7a). Corresponding with the less efficient production technology the maximum welfare (of again 112.5%) is reached for a smaller value a_{max} = 0.16 (also corresponding with firm's production level at the break-even point). Table 5.3 contains numerical information about the relative welfare level for c=0.4 concerning both preference for market share and for size. The relative welfare level corresponding with preference for size is indicated by $W_s(a)/W_{cl}$ *100%. Proposition 5.15, part (iii), predicts that - corresponding to this more efficient production technology - profit is sacrificed completely for a smaller weight if the preference for size is concerned (a_s = 0.30 versus a = 0.36). This observation implies that the maximum welfare is reached also for a (somewhat) smaller weight (indicated by $a_{s,max}$, see Proposition 5.7,Section 5.3) if the firms' love for size is considered.

Table 5.3, welfare as a percentage of the classical welfare; c=0.4, d=0.

	a=0.05	a=0.10	a=0.15	a=0.20	a=0.25	a=0.30	a=0.35	a=0.40
[$W(a)/W_{cl}$] *100%	104%	107%	109%	111%	112%	112%	112%	W=0
[$W_s(a)/W_{cl}$] *100%	104%	107%	109%	111%	112%	113%	W=0	W=0

Table 5.4 allows us to compare the relative welfare levels for both preference for market share and preference for size corresponding with a less efficient production technology reflected by c=0.6. Both maximizing values of a are smaller in comparison to the values related to lower unit production costs. Furthermore now the maximum welfare level is reached at a somewhat earlier stage - a_{max}=0.16 - if preference for market share is concerned in comparison to the welfare maximizing weight - $a_{s,max}$=0.2 - attributed to preference for size.

Table 5.4, welfare as a percentage of the classical welfare; c=0.6, d=0.

	a=0.05	a=0.10	a=0.15	a=0.20	a=0.25	a=0.30	a=0.35	a=0.40
$[W(a)/W_{cl}]$ *100%	108%	111%	112%	W=0	W=0	W=0	W=0	W=0
$[W_s(a)/W_{cl}]$ *100%	105%	109%	112%	113%	W=0	W=0	W=0	W=0

In the previous analysis, concerning the weights a for which profits are sacrificed completely or welfare is maximized, we made several distinctions. First we confronted an efficient production technology with an inefficient technology. And second, we considered preference for market share and preference for size as the non-profit parts of firms' objectives, which provides a link between this chapter and Chapter 3 (concerning symmetric firms with respect to cost regimes and weights a).
Besides these two major distinctions the production cost parameter d also plays a role, because the welfare-maximizing weight is restricted by nonnegative profit levels for $d < 0$. For nonnegative values of d the maximum welfare is reached corresponding to positive profits and the results of Proposition 5.16 and Proposition 5.7, Section 5.3 reveal that $a_{max} = \frac{(1-c)^2}{(1+d)^2}$ and $a_{s,max} = \frac{(1-c)}{(2+2d)}$. Clearly these both maximizing weights decrease if the unit costs c increase, similar to the results concerning $d < 0$. In general we can make the following statement related to firm's technological efficiency:

- The less efficient the production technology which firms control, the lower the weights corresponding to a complete sacrifice of the profits or related to a maximum welfare level. This relation between the firms' efficiency and profit-sacrificing or welfare-maximizing weights holds for both preferences for market share and size.

For $d < 0$ welfare maximizing weights correspond with the weights concerning zero-profits. If we compare both welfare maximizing weights, corresponding with the preference for market share and preference for size, the result of Proposition 5.15 (iii) reveals that $a_{s,max} < a_{max}$ for $c < ½ – ¼\, d$.
And for nonnegative values of the parameter *d* it holds that $a_{s,max} < a_{max}$ if $c < ½ – ½\, d$, this latter relation also indicates a more efficient production technology. Therefore we can formulate the following general qualitative statement:

- Concerning an efficient production technology the welfare maximizing weight corresponding to preference for size is lower than the welfare maximizing weight corresponding to preference for market share. The opposite result holds for an inefficient production technology.

Note that Table 5.3 and Table 5.4 show that the maximum relative welfare level is always about 112% and this phenomenon leads one to suspect that the maximum relative level seems to be independent of the precise nature of the non-profit motive (size or market share or any other motive) under consideration.
Using the general expression for the welfare (Section 5.3) it can be proved that the maximum relative welfare level that can be reached for a certain value of the weight a is independent of the cost parameter c. It can be derived that (taking into account the restriction of nonnegative profits for $d < 0$)

(i) for $d \geq 0$ $\frac{W(a_{max})}{W_{cl}} = \frac{(3+2d)^2}{4(1+d)(2+d)}$, a decreasing function in d.

(ii) for $d < 0$ $\frac{W(a_{max})}{W_{cl}} = \frac{(3+2d)^2}{(2+d)^3}$, also a decreasing function in d.

This interesting result means that for constant unit costs (d=0) the maximum relative welfare level that can be reached equals 112.5%. And if the parameter d decreases to – 0.1 this maximum relative welfare level rises somewhat to 114.3%. However - as the previous analysis reveals - the weight a_{max} for which this maximum possible level can be reached, depends on the nature of the non-profit part of the firms' objective functions and the efficiency of the production technology controlled by the firms.

We conclude this section with some brief remarks. In the introduction we stated that a thorough examination of equilibrium profits and welfare - concerning two symmetric firms with respect to the weight a attributed to the non-profit part of the maximand - is justified by two main arguments. First a two-stage sequential game may lead to a heightening of both weights of the rivals, motivated by strategic advantages (examples of such two-stage games are the "delegation" games of Fershtman and Judd (1987) and others). This first argument assumes awareness of the strategic benefits of heightening the weight (by firms' owners). Or to put it differently, such a game assumes highly rational behaviour. The second argument is supported by empirical findings that (the habit of) preference for (growth of) size or market share directs the firms' (top-management) behaviour and may also lead to equal and higher values of the weight a. It is obvious that bonus practices that attribute significant weight to (growth of) market share or size influence top-management's behaviour. The analytical results of this section reveal that firms sacrifice profits, due to managerial inertia reflected in the level of preference for market share or size (sales). However the social welfare benefits from this behaviour and under the restriction that the weight a is not too large social welfare may reach a maximum level. If the managerial inertia becomes too large firms' profits may be sacrificed completely and this implies a collapse of the welfare level in the long run.

The interesting result that the less efficient the production technology which firms control is, the lower the weights corresponding to a complete sacrifice of the profits or related to a maximum welfare level are, can also be stated in other words. If unit production costs are high, as a result of an old-fashioned or inferior technology, profit (and welfare) may be sacrificed at an early stage of managerial inertia (reflected in the weight a).

If loss-bearing firms stay in the market for some period of time, the analysis of the multiple equilibria in this section reveals that strategic actions can still be useful to expel the rival from the market. Such an action can be looked upon as an advertising stunt with give-away prices with the intention to exhaust the competitor in order to achieve a monopoly position on the market. With the prospect to acquire a monopoly position, a firm may accept losses for some period of time.

6. Implications of asymmetry concerning preference for market share

In this section we examine a second benchmark case of two asymmetrical firms concerning their preferences for market share. Firm 1's preference for market share is reflected in the weight $a_1 = a > 0$ attributed to the market share in the maximand, whereas firm 2 behaves as a classical profit-maximizer, i.e. $a_2 = 0$. From the viewpoint of managers' habits and (psychological) preference for market share, the two weights $a_1 = a > 0$ and $a_2 = 0$ reflect the culture of both firms ("blueprint", Hannan and Freeman (1984)). Concerning "delegation" games (Vickers (1985), Fershtman and Judd (1987), Sklivias (1987) and Basu (1995)) this asymmetrical case would correspond with a firm whose owner hires a manager, whereas the other firm's owner doesn't. Vickers (1985) analyses such an asymmetrical case and Basu (1995) also reflects on this specific case to explain Stackelberg leadership (in stage one of a three-stage sequential game owners may decide to hire a manager or not). Like in Section 5.5, we consider the weight $a_1 = a > 0$ as a measure of firm 1's managerial inertia and we do not limit our analysis to specific small values of a (that would follow from a "delegation" game).

In Section 5.5 the symmetry in preference for market share led to a stable (and symmetrical) Cournot-Nash equilibrium (Proposition 5.12). However the complete asymmetry in "love for market share" leads to an instable equilibrium for larger values of the weight a, which firm 1 attibutes to its market share. It appears that the equilibrium becomes unstable if the weight a exceeds a specific value a_{bif}, whereas the equilibrium is stable for $a < a_{bif}$. The more complex dynamics corresponding to an instable Cournot-Nash equilibrium will be examined in Chapter 6.

For $a < a_{bif}$ the stability of the Cournot-Nash equilibrium - which is now asymmetric - also leads to interesting properties concerning both rivals' profits and social welfare as well. The results of Section 5.5 reveal that complete symmetry in preference for market share of two firms leads to a sacrifice of profits (Proposition 5.15 (i)) and possibly leads to a complete collapse of welfare caused by bankruptcy of the firms. However, Proposition 5.3 of Section 5.3 states that firm 1's profit level exceeds the profit level of its rival if $a_1 > a_2$, for small a_1 and a_2. So the application of the general Proposition 5.3 implies that, if firm 2's behaviour is classical, i.e. profit-maximizing, firm 1 has an advantage in profits over its rival for small values of a. The analysis of this section will show that this advantage of the "market share loving" firm still holds for larger values of a and under broad conditions. Under the assumption that $a_2 = 0$ we will examine the existence of a profit maximizing and an advantage maximizing weight a, concerning firm 1. These considerations provide a link between this Section and Chapter 3 where the existence of a profit maximizing preference for size is studied. In this section the analysis is more general. Under the condition that firm 1's equilibrium output increases with respect to the weight a - implying that the rival's production level decreases - the existence of profit maximizing and advantage maximizing weights can be examined in general. Maximum profit and maximum advantage appear to occur for specific equilibrium supplies of firm 1 and specification of the non-profit part s of the maximand reveals the corresponding levels of preference reflected by the weights. Furthermore we will analyze the existence of a welfare maximizing weight related to concave and convex production cost functions. Anticipating to the analysis we mention that if s equals the market share and the production cost function is linear or concave, social welfare keeps increasing with respect to an increasing weight a for $0 < a < a_{bif}$.

Figure 5.9 supports the analysis of benchmark case 2. Because firm 2's reaction function is linear, the co-ordinates of the Cournot-Nash equilibrium can be computed by using Cardan's Method. For the application of this method we refer to Appendix 5.3 where the functional expression for the equilibrium supply of firm 1, x_1^*, is given for constant unit costs. These formula's facilitate simulation experiments.

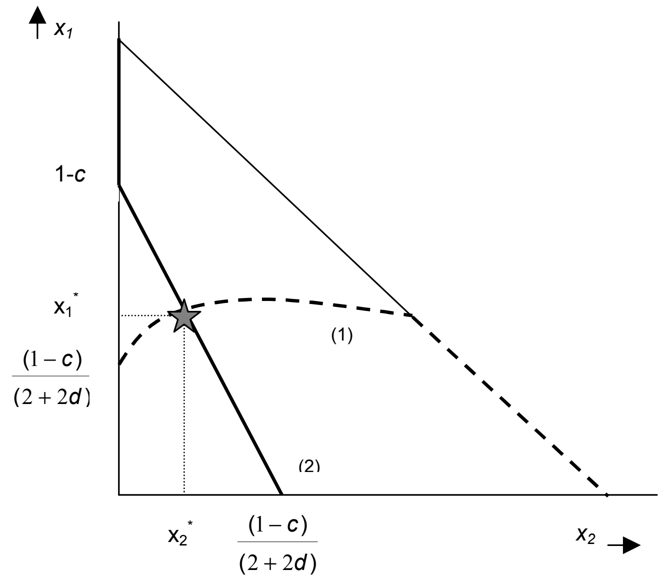


Fig. 5.9 Reaction curves of a "market share loving" and a classical firm.

Because $a_2 = 0$ and $a_1 = a > 0$ we use the notations $x_1^*(a)$ and $x_2^*(a)$ for the respective outputs of firm 1 and 2 in equilibrium. Clearly the graphs of the reaction curves reveal that - if a increases - $x_1^*(a)$ and $x_2^*(a)$ increase and decrease respectively. This monotonicity of $x_1^*(a)$ plays an important role in the forthcoming analysis. For small a the application of Proposition 5.1, Section 5.3 gives

$$x_1^*(a) = \frac{(1-c)}{3+2d} + \frac{(1+d)}{2(1-c)(1+2d)}a - \frac{(1+d)^2(3+2d)}{4(1-c)^3(1+2d)^2}a^2 + O(a^3), \text{ for } a \to 0 \tag{5.29}$$

This expression reveals that, for small a, $x_1^*(a)$ is a monotonically rising (and concave) function of a. The property of monotonicity can also be proved generally. Firm 2's reaction function, $x_2^*(a) = \frac{(1-c)}{(2+2d)} - \frac{1}{(2+2d)}x_1^*(a)$, implies that

$$\frac{\mathrm{d}x_2^*}{\mathrm{d}a} = -\frac{1}{(2+2d)}\frac{\mathrm{d}x_1^*}{\mathrm{d}a}. \tag{5.30}$$

Implicit differentiation with respect to the weight a of the expression

$$(1-c) - (2+2d)x_1^*(a) - x_2^*(a) + a \cdot s_{x_1}(x_1^*(a), x_2^*(a)) = 0 \tag{5.31}$$

leads to:

$$\frac{\mathrm{d}x_1^*}{\mathrm{d}a} = \frac{s_{x_1}(x_1^*(a), x_2^*(a))}{(2+2d) - \dfrac{1}{(2+2d)} - a \cdot s_{x_1x_1}(x_1^*(a), x_2^*(a)) + \dfrac{a}{(2+2d)} s_{x_1x_2}(x_1^*(a), x_2^*(a))} \,. \tag{5.32}$$

This expression determines the conditions - which have to be imposed on the non-profit part of the maximand - necessary for the monotonicity of $x_1^*(a)$ with respect to a. If s equals market share then, for $x_1^* \geq x_2^* > 0$, it holds that $s_{x_1}(x_1^*, x_2^*) > 0$, $s_{x_1x_1}(x_1^*, x_2^*) < 0$ and $s_{x_1x_2}(x_1^*, x_2^*) \geq 0$ (see Section 5.4). So for all $d > -\frac{1}{2}$ it holds that $\frac{\mathrm{d}x_1^*}{\mathrm{d}a} > 0$ if $x_1^* \geq x_2^* > 0$.

The preceding analysis implies that the function $x_1^*(a)$ is invertible. Furthermore firm 1's equilibrium production is restricted by $1-c$ ($\lim_{a\to\infty} x_1^*(a) = 1-c$). If the Cournot-Nash equilibrium is stable ($\frac{\mathrm{d}x_1}{\mathrm{d}x_2} < 2+2d$, where $\frac{\mathrm{d}x_1}{\mathrm{d}x_2}$ equals the slope of firm 1's reaction curve in the equilibrium) the following comparative statics concerning equilibrium profits, difference in profits and equilibrium welfare makes sense.

Using the equilibrium relation $x_2^*(a) = \frac{(1-c)}{(2+2d)} - \frac{1}{(2+2d)} x_1^*(a)$ we obtain the following expressions for the profits of both competitors and for the advantage $\Delta\Pi(a)$ of firm 1 over its rival:

$$\begin{aligned}
\Pi^2(a) &= \frac{1}{4(1+d)}[(1-c) - x_1^*(a)]^2 \\
\Pi^1(a) &= \frac{(1+2d)}{(2+2d)} x_1^*(a)\,[(1-c) - \frac{(2d^2+4d+1)}{(1+2d)} x_1^*(a)] \\
\Delta\Pi(a) &= \frac{1}{4(1+d)}[(3+2d)x_1^*(a) - (1-c)]\,[(1-c) - (1+2d)x_1^*(a)]
\end{aligned} \tag{5.33}$$

Because $x_1^*(a)$ is a monotonically increasing function with respect to the weight a, the profit of the (classical) firm 2 decreases with respect to an increasing a but stays positive.

The fact that these three expressions are quadratic functions of the equilibrium output x_1^* of firm 1 makes it possible to derive general properties such as the existence of a profit-maximizing level of the production x_1^*. Note that application of the general power series solution of the profits of Section 5.3 allows us to derive certain properties as well. If s equals the market share, application of the power series expression for the profit of firm 1 leads to

$$\Pi^1(a) = \frac{(1-c)^2(1+d)}{(3+2d)^2} + \frac{1}{4(1+2d)(3+2d)}a - \frac{(1+d)^3}{4(1-c)^2(1+2d)^2}a^2 + O(a^3), \text{ for } a \to 0 \tag{5.34}$$

This approximation reveals the concave character of the profit function of firm 1 with respect to the variable a and leads one to suspect that there may exist a profit-maximizing weight a. However this expression for the profit only holds for small a's and therefore no conclusion concerning a profit-maximizing weight a can be drawn. The properties of the profit and the advantage of firm 1 over its rival are summarized in Proposition 5.17.

Proposition 5.17 (profit and advantage of firm 1).

Under the assumption that firm 1 attributes a weight a to the non-profit part s of the objective function whereas firm 2 only maximizes its profit the following holds for $a < a_{bif}$ (stable equilibrium):

(i) For $-\frac{1}{2} < d \le -\frac{3}{4} + \frac{1}{4}\sqrt{5}$ Π^1 and $\Delta\Pi$ are always rising with respect to an increasing weight a.

(ii) For $d > -\frac{3}{4} + \frac{1}{4}\sqrt{5}$ Π^1 is maximized for $x_1^* = (1-c)\dfrac{(1+2d)}{2(2d^2+4d+1)}$ corresponding with weight $a_p = (1-c)^2 \dfrac{(1+2d)(8d^2+12d+3)^2}{(4+4d)^2(4d^2+6d+1)(2d^2+4d+1)^2}$ if s equals the market share.

$\Pi^1(a_p) = \dfrac{(1-c)^2(1+2d)^2}{(8+8d)(2d^2+4d+1)}$ and $\Pi^2(a_p) = \dfrac{(1-c)^2(4d^2+6d+1)^2}{(16+16d)(2d^2+4d+1)^2}$.

For $d > 0$ Π^1 becomes negative if $x_1^* > (1-c)\dfrac{(1+2d)}{(2d^2+4d+1)}$.

(iii) For $d > -\frac{3}{4} + \frac{1}{4}\sqrt{5}$ $\Delta\Pi$ is maximized for $x_1^* = (1-c)\dfrac{(2+2d)}{(1+2d)(3+2d)}$ corresponding with weight $a_d = (1-c)^2 \dfrac{(1+2d)(4d+5)^2}{(2+2d)^2(3+2d)(4d^2+6d+1)}$ if s equals the market share.

$\Pi^1(a_d) = \dfrac{(1-c)^2(4d^3+8d^2+4d+1)}{(1+2d)^2(3+2d)^2}$ and $\Pi^2(a_d) = \dfrac{(1-c)^2(4d^2+6d+1)^2}{(4+4d)(1+2d)^2(3+2d)^2}$.

For $d > 0$ $\Delta\Pi$ becomes negative if $x_1^* > \dfrac{(1-c)}{(1+2d)}$, furthermore $a_p < a_d$ $(< a_{bif})$.

Proof

The expression for Π^1 reveals that the profit always rises with respect to an increasing x_1^* if

(a) $2d^2+4d+1 \le 0$, or if

(b) the profit-maximizing $x_1^* = \dfrac{(1+2d)}{2(2d^2+4d+1)}(1-c) \ge 1-c$ (so it is never reached).

Combining the conditions (a) and (b) leads to $-\frac{1}{2} < d \le -\frac{3}{4} + \frac{1}{4}\sqrt{5}$. From the expression for $\Delta\Pi$ it follows that $\Delta\Pi$ is maximized if $x_1^* = (1-c)\dfrac{(2+2d)}{(1+2d)(3+2d)}$ which is never reached if $\dfrac{(2+2d)}{(1+2d)(3+2d)} \geq 1$. This completes the proof of part (i).

For $d > -\frac{3}{4} + \frac{1}{4}\sqrt{5}$ the profit-maximizing x_1^* is reached, because then $\dfrac{(1+2d)}{2(2d^2+4d+1)} < 1$. If s equals the market share the corresponding profit-maximizing weight is obtained by substituting this specific x_1^* (and x_2^*) in the equation

$$1 - c - (2+2d)x_1^* - x_2^* + a\frac{x_2^*}{(x_1^* + x_2^*)^2} = 0 .$$

$\Pi^1(a_p)$ and $\Pi^2(a_p)$ are obtained by substituting the profit-maximizing x_1^* in their respective expressions (in x_1^*).

For $d > 0$ $(1-c)\dfrac{(1+2d)}{(2d^2+4d+1)} < 1-c$, so if a is large enough $\Pi^1 < 0$. This proves part (ii). The advantage-maximizing x_1^* is reached for $\dfrac{(2+2d)}{(1+2d)(3+2d)} < 1$, i.e. $d > -\frac{3}{4} + \frac{1}{4}\sqrt{5}$. Using similar methods as in the proof of part (ii) a_d, $\Pi^1(a_d)$ and $\Pi^2(a_d)$ can be computed. It holds that $a_p < a_d$ because $x_1^*(a_p) < x_1^*(a_d)$. For $d > 0$ $\dfrac{(1-c)}{(1+2d)} < 1-c$, so if a is large enough $\Delta\Pi < 0$.

[End of proof]

The results of Proposition 5.17 are quite detailed, because several intervals for the parameter d of the production cost function have been distinguished. Notwithstanding the fact that the expressions of Proposition 5.17 simplify for constant unit costs, i.e. $d = 0$, it is also useful to reflect on the results for other values of the parameter d. Part (i) reveals that, for $-\frac{1}{2} < d \leq -0.191$, both the profit Π^1 and the advantage over the rival of the “market-share loving” firm keep increasing with a further increasing weight a. Note that this interval of values of the parameter d corresponds with a concave production cost function, or in other words, it reflects a production technology with increasing returns to scale. So if the rival’s objective is to maximize its profit - referred to as “classical” behaviour - the more weight firm 1 attributes to the market share the more beneficial this is in two ways. Not only does the profit of the “market-share loving” firm increase, but the advantage over the competitor increases as well, if a increases. Managerial inertia, reflected in preference for market share pays off, whether this managerially inert firm is aware of these benefits or not. We also note that, from the standpoint of the “delegation” games (rational adaptation perspective in contrast to a firm’s “blueprint”) and in case of a concave production cost function, owners would write managers’ incentive contracts with an infinite weight attributed to market share.

Furthermore if the weight a increases, the production of firm 1, $x_1^*(a)$, increases whereas the output level $x_2^*(a)$ of the rival decreases. By increasing its managerial inertia firm 1 becomes big and profitable (the case that firm 1 actually becomes a

Stackelberg leader will be part of the analysis of Chapter 6). However, the rival still has positive profits in spite of its smallness. The following graphical display with a concave production costs function - c=0.4 and d= – 0.2 - illustrates part (i) of Proposition 5.17. Both profits are expressed as a percentage of the classical profit (which equals $(1-c)^2(1+d)/(3+2d)^2$ in general). Clearly the profit of firm 1 keeps increasing with respect to an increasing weight a for $a < a_{bif}$ (which equals 2.02 in this case) and it reaches a level of about 160% of the classical profit. However rival's profit falls quickly and about 4% of the classical profit is left corresponding to a=2.02.

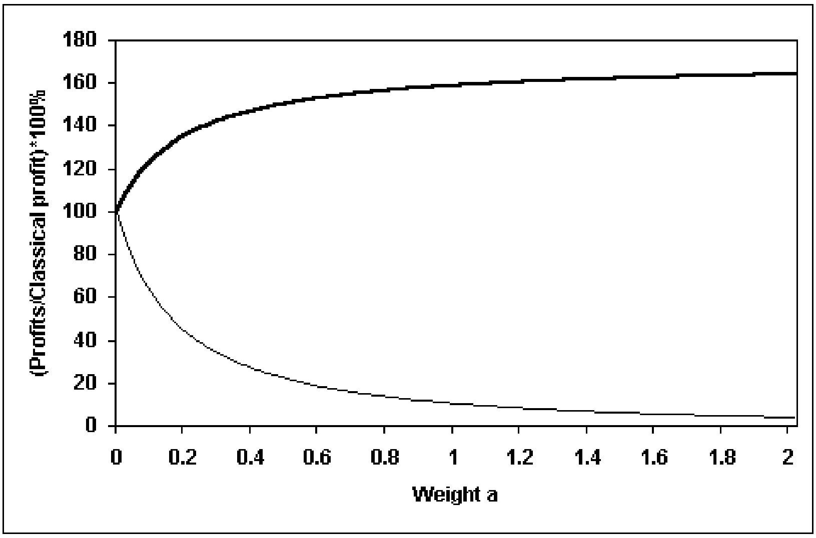


Fig. 5.10 $(\Pi^1(a)/\Pi_{cl})$*100% **and** $(\Pi^2(a)/\Pi_{cl})$*100%, c=0.4, d=-0.2 .

For $d > -0.191$ - so both concave and convex production cost functions are included - firm 1's profit is maximized at the expense of the competitor for the specific weight a_p attributed to the market share. In other words, for a certain level of managerial inertia the profit of the "market-share loving" firm is maximized. If firm 1 were aware of this beneficial outcome, it could choose its level of managerial inertia. We have to realize that such behaviour would be highly rational.

It is very interesting that for a somewhat larger weight the advantage (difference of the profits) of firm 1 over its rival is maximized. Then of course some of the (maximum) profit level of firm 1 is sacrificed. Under the assumption of rational behaviour, only keeping in mind one's own profit may not be the most beneficial strategy. Note that for $d > 0$, corresponding with a production technology with decreasing returns to scale, a further increase of the weight a (beyond a_d) may first lead to a negative $\Delta\Pi$ and then to a negative Π^1, whereas Π^2 always stays positive.

If we assume the production cost parameter d to be zero, i.e. considering constant unit production costs c, the formulas of Proposition 5.17 (parts (ii) and (iii)) reveal a clear pattern in equilibrium outputs, profits and difference in profits. Of course the stability of the Cournot-Nash equilibrium is crucial concerning this comparative statics. Therefore we also include the expression for the bifurcation point a_{bif}, which will be proved in Chapter 6. The value for a_{bif} always exceeds a_d (and a_p), so comparative statics makes sense. For $a > a_{bif}$ the equilibrium is no longer stable but this case will be analyzed separately in Chapter 6. The following table summarizes equilibrium quantities for constant unit costs c.

Table 5.5 The effect of a on equilibrium quantities.

	$a=0$	$a=a_p=\frac{9}{16}(1-c)^2$	$a=a_d=\frac{25}{12}(1-c)^2$	$a=a_{bif}=\frac{1440}{169}(1-c)^2$
x_1^*	$\frac{1}{3}(1-c)$	$\frac{1}{2}(1-c)$	$\frac{2}{3}(1-c)$	$\frac{11}{13}(1-c)$
x_2^*	$\frac{1}{3}(1-c)$	$\frac{1}{4}(1-c)$	$\frac{1}{6}(1-c)$	$\frac{1}{13}(1-c)$
Π^1	$\frac{1}{9}(1-c)^2$	$\frac{1}{8}(1-c)^2$	$\frac{1}{9}(1-c)^2$	$\frac{11}{169}(1-c)^2$
Π^2	$\frac{1}{9}(1-c)^2$	$\frac{1}{16}(1-c)^2$	$\frac{1}{36}(1-c)^2$	$\frac{1}{169}(1-c)^2$
$\Delta\Pi$	0	$\frac{1}{16}(1-c)^2$	$\frac{1}{12}(1-c)^2$	$\frac{10}{169}(1-c)^2$
Π^1/Π^2	1	2	4	11

Before we present graphs of both competitors' profits, expressed as a percentage of the classical profit ($a = 0$), for increasing values of a (and constant unit costs), we make some notes on the results of Table 5.5.

- For the weight $a=\frac{9}{16}(1-c)^2$ the (maximum) profit of firm 1 equals 112.5% of the classical profit. The rival's profit, however, equals only 56.3% of the classical profit. Beside this advantage the profitable firm 1 is twice as large as its rival indicated by x_1^* and x_2^* corresponding to $a=a_p$. Firm 1 may become a market leader. Interesting detail is that firm 1's maximum profit equals the profit (per firm) if both firms were to collude and were to share the monopoly profit of $\frac{1}{4}(1-c)^2$. But now, due to the weight $a=a_p$ attributed to the market share, firm1's profit is twice as large as the rival's profit.
- For the weight $a=\frac{25}{12}(1-c)^2$ the profit of firm 1 again equals the classical profit, but the rival's profit is only 25.0% of the classical profit. By sacrificing some profit firm 1's advantage over its rival has become larger and the market position of firm 1 has become stronger, because now $x_1^*=4x_2^*$. Note that, just before the equilibrium becomes unstable, firm 1's profit is about 59% ($=\frac{99}{169}$) of the classical profit whereas the rival's profit equals 5% of the classical profit. Now $x_1^*=11x_2^*$.

The fact that firm 1 maximizes its profit while its maximand consists of a profit and a non-profit part as well, seems to be quite paradoxical. Apparently the right level of managerial inertia of firm 1 causes the advantages, given the classical, profit maximizing, behaviour of firm 2. Without assuming highly rational behaviour of firm 1, the beneficial position of this firm, due to managerial inertia, increases the probability of surviving in the market.

In their publication "Toward a game theory of organizational ecology" Van Witteloostuijn, Boone and Van Lier (2003) state that:

"From an Organizational Ecology perspective a may be the result of imprinted routines, rules and procedures reflected in an organizational culture that 'urges' management to seek growth [here market-share; added]... Then population dynamics will take care of the selection of firms with fitting a's. So the profit-maximization paradox is solved by Darwinian market selection, without the relatively flexible or inert firms being able to adapt their imprinted a's to environmental contingencies."

From the previous analysis it follows that a firm with preference for market share has benefits over its classical rival. The Darwinian market selection mechanism benefits firms which possess a certain level of managerial inertia, whether these firms display highly rational behaviour or not.

The following graph displays the rivals' profits, for constant unit production costs $c = 0.4$ for an increasing weight a ($a < a_{bif}$). To enable a comparison with the classical profit ($= \frac{1}{9}(1-c)^2$ for constant unit costs), again both profits are expressed as a percentage of this specific profit .

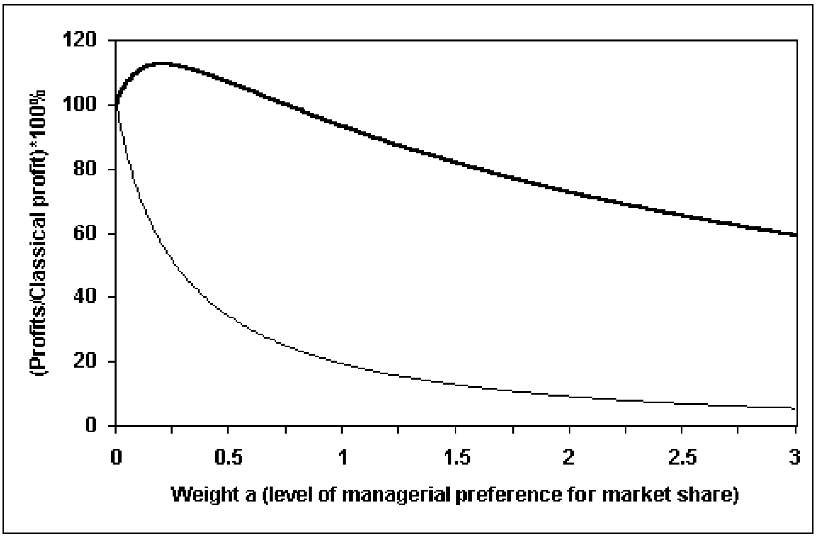


Fig. 5.11 $(\Pi^1(a)/\Pi_{cl})*100\%$ **and** $(\Pi^2(a)/\Pi_{cl})*100\%$, c**=0.4,** d**=0 .**

For $c = 0.4, d = 0$ the profit-maximizing weight of firm 1 equals $a_p = 0.2025$, which is nicely illustrated by the graph. Because Π^2 falls rather strongly with respect to an increasing a, $\Delta\Pi$ increases further until the advantage-maximizing weight $a_p = 0.7500$ is reached (of course this phenomenon cannot be observed easily from the graph). Note that just before the bifurcation occurs ($a_{bif} = 3.067$) the profit of firm 1 is still nearly 60% of the classical profit.

Proposition 5.17 also reveals information concerning the influence of the production cost parameter d on the weights a_p and a_d and on the ratios $\dfrac{\Pi^1(a_p)}{\Pi_{cl}}$

and $\dfrac{\Pi^{1}(a_d)}{\Pi_{cl}}$ (for $d > -\frac{3}{4} + \frac{1}{4}\sqrt{5} \approx -0.191$). It is clear that - if c decreases, reflecting a more efficient production process - the weights a_p and a_d increase. The expressions $\dfrac{\Pi^{1}(a_p)}{\Pi_{cl}}$ and $\dfrac{\Pi^{1}(a_d)}{\Pi_{cl}}$ etc. are independent of c ($\Pi_{cl} = \dfrac{(1-c)^2(1+d)}{(3+2d)^2}$).
Table 5.6 summarizes some results for $d = -0.1$ (a concave production cost function), $d = 0$ (constant unit costs) and $d = 0.1$ (a convex production cost function).

Table 5.6. The influence of d on a_p, a_d and profit-ratios.

	a_p	$\dfrac{\Pi^{1}(a_p)}{\Pi_{cl}}$	$\dfrac{\Pi^{1}(a_p)}{\Pi^{2}(a_p)}$	a_d	$\dfrac{\Pi^{1}(a_d)}{\Pi_{cl}}$	$\dfrac{\Pi^{1}(a_d)}{\Pi^{2}(a_d)}$
$d = -0.1$	$1.29\,(1-c)^2$	1.25	4.10	$4.24\,(1-c)^2$	1.17	12.57
$d = 0$	$0.56\,(1-c)^2$	1.13	2.00	$2.08\,(1-c)^2$	1.00	4.00
$d = 0.1$	$0.34\,(1-c)^2$	1.07	1.52	$1.38\,(1-c)^2$	0.94	2.43

Constants are rounded to two decimals. For instance note that the ratio $\dfrac{\Pi^{1}(a_p)}{\Pi_{cl}}$ increases if d decreases and corresponding to $d = -0.1$ the maximum profit of the "market-share loving" firm occurs for $a_p = 1.29(1-c)^2$ and is 25% higher than the classical profit. In general it holds that

- a_p and a_d increase if c decreases or d decreases, so for a more efficient production technology - reflected in a decreasing c or a more concave cost function - these weights both increase.
- The ratios $\dfrac{\Pi^{1}(a_p)}{\Pi_{cl}}$, $\dfrac{\Pi^{1}(a_p)}{\Pi^{2}(a_p)}$, $\dfrac{\Pi^{1}(a_d)}{\Pi_{cl}}$ and $\dfrac{\Pi^{1}(a_d)}{\Pi^{2}(a_d)}$ are independent of c and increase if d decreases.

To examine the influence of the weight a on the social welfare we have to reflect on both components, total profit of both competitors and consumer surplus. Consideration of the results of Table 5.5, concerning the profits, leads one to suspect that $\Pi^{1}(a) + \Pi^{2}(a)$ decreases if a increases (for $a = 0$ total profits equal $\frac{2}{9}(1-c)^2$ whereas for $a = a_{bif}$ this total profit is $\frac{12}{169}(1-c)^2$). The expression for $\Pi^{1}(a) + \Pi^{2}(a)$ can be written as

$$\Pi^{1}(a) + \Pi^{2}(a) = \frac{1}{4(1+d)}[-(4d^2 + 8d + 1)\{x_1^*(a)\}^2 + 4d(1-c)x_1^*(a) + (1-c)^2] \tag{5.35}$$

and total profit clearly decreases for a linear production cost function if the weight a increases, because the output x_1^* is a monotonically increasing function of the variable a (it can be derived that for $d \geq -\frac{3}{4} + \frac{1}{4}\sqrt{5}$ the total profit always decreases with respect to an increasing weight a, whereas it reaches a minimum for

$-\frac{1}{2} < d < -\frac{3}{4} + \frac{1}{4}\sqrt{5}$). So, generally speaking, the first "component" of the social welfare falls, if the weight which firm 1 attributes to its market share rises. Total output however increases with respect to an increasing a because

$$x_1^*(a) + x_2^*(a) = \frac{1}{(2+2d)}[(1+2d)x_1^*(a)) + (1-c)]. \tag{5.36}$$

Clearly the consumer surplus, $CS = \frac{1}{2}[\{x_1^*(a)\} + \{x_2^*(a)\}]^2$, increases because of an ever more abundant supply of products at a falling price. A detailed examination of the social welfare leads to the following proposition:

Proposition 5.18 (social welfare).

Under the assumption that firm 1 attributes a weight a to its market share whereas firm 2 only maximizes its profit the following holds for $a < a_{bif}$ (stable equilibrium):

(i) For $d \le 0$ (so including linear production costs) the welfare increases for all weights $a \ge 0$, with respect to the weight a.

(ii) For $d > 0$ the welfare is maximized for

$x_1^* = (1-c)\dfrac{(4d^2+6d+1)}{(8d^3+20d^2+14d+1)}$ corresponding with the weight

$a_w = (1-c)^2 \dfrac{(8d^2+10d+1)^2(2d+1)^2}{(4d^2+4d)(8d^3+20d^2+14d+1)^2}$ and

$W(a_w) = \dfrac{(1-c)^2(8d^3+20d^2+13d+1)}{(2+2d)(8d^3+20d^2+14d+1)}$.

Proof

Using the linear relation $x_2^*(a) = \dfrac{(1-c)}{(2+2d)} - \dfrac{1}{(2+2d)}x_1^*(a)$ we obtain the following quadratic form (in x_1^*) for the welfare:

$$W(a) = \frac{1}{8(1+d)^2}[-(8d^3+20d^2+14d+1)\{x_1^*(a)\}^2 + 2(1-c)(4d^2+6d+1)x_1^*(a) + (3+2d)(1-c)^2]$$

.

The coefficient of $(x_1^*)^2$ equals zero for $d^* = -0.08$, is positive for $d < d^*$ and negative for $d > d^*$. The coefficient of x_1^* equals zero for $d^{**} = -\frac{3}{4} + \frac{1}{4}\sqrt{5}$, is positive for $d > d^{**}$ and negative for $d < d^{**}$. Several combinations of the signs of the coefficients of $(x_1^*)^2$ and x_1^* lead to:

- for $-\frac{1}{2} < d < d^{**}$ $W(a)$ reaches a minimum value for $x_1^* = (1-c)\dfrac{(4d^2+6d+1)}{(8d^3+20d^2+14d+1)} < \dfrac{(1-c)}{(3+2d)}$, so $W(a)$ rises.
- for $d^{**} \le d \le d^*$ the coefficients of $(x_1^*)^2$ and x_1^* are both nonnegative which implies an increasing welfare with respect to an increasing a.
- for $d^* < d \le 0$ the maximum location $x_1^* = (1-c)\dfrac{(4d^2+6d+1)}{(8d^3+20d^2+14d+1)} \ge (1-c)$, implying a rising $W(a)$ with respect to a.

This proves part (i).
For $d > 0$ the maximum is reached for the value a_w satisfying
$x_1^*(a_w) = (1-c)\dfrac{(4d^2+6d+1)}{(8d^3+20d^2+14d+1)}$ and this weight can be computed by substituting the welfare-maximizing value of x_1^* (and the corresponding x_2^*) into the equation of the reaction curve of firm 1.

[End of proof]

For linear costs the welfare can be written as $W(a) = \frac{1}{8}[3(1-c) - x_1^*(a)][(1-c) + x_1^*(a)]$ and the proof that W rises with respect to a simplifies by observing that the maximum value of the welfare would occur for $x_1^*(a) = 1-c$ .The result of Proposition 5.18 supports the intuition that with a less efficient production technology - reflected by $d > 0$ (decreasing returns to scale) - welfare may decrease for $a > a_w$. For the sake of completeness we mention that $a_p < a_d < a_w$ for $0 < d < -\frac{1}{2} + \frac{1}{2}\sqrt{2}$ (≈ 0.207).

We conclude this welfare analysis with two graphical illustrations of the welfare function with respect to an increasing weight a. The first graph corresponds to constant unit production costs - $c = 0.4$ - and illustrates part (i) of Proposition 5.18: welfare always increases if the production cost function is linear or concave. We express the welfare as a percentage of the classical welfare (which equals $(1-c)^2(4+2d)/(3+2d)^2$) to make a comparison possible.

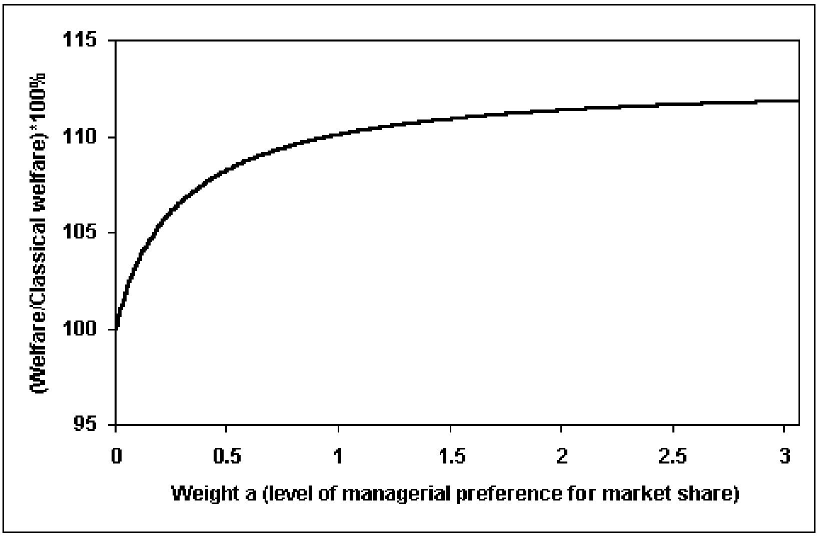


Fig. 5.12 The welfare for $c = 0.4$, $d = 0$ as a function of a

Note that, just before the weight a equals $a_{bif} = 3.07$, welfare has increased to about 112% of the classical welfare. For $d > 0$, as we have seen, the welfare reaches a maximum W_{max} and it appears that W_{max}/W_{cl} decreases if d increases. The following graphical display , with $c = 0.4$ and $d = 0.1$, illustrates both phenomena. Again welfare is expressed as a percentage of the classical welfare. Maximum welfare is reached for $a_w = 0.75$ (in two decimals) and W_{max} equals 106.6% of the classical welfare. For $a > a_w$ welfare decreases slowly but still equals 104.3% of W_{cl} for $a = 3$ ($a_{bif} = 3.66$).

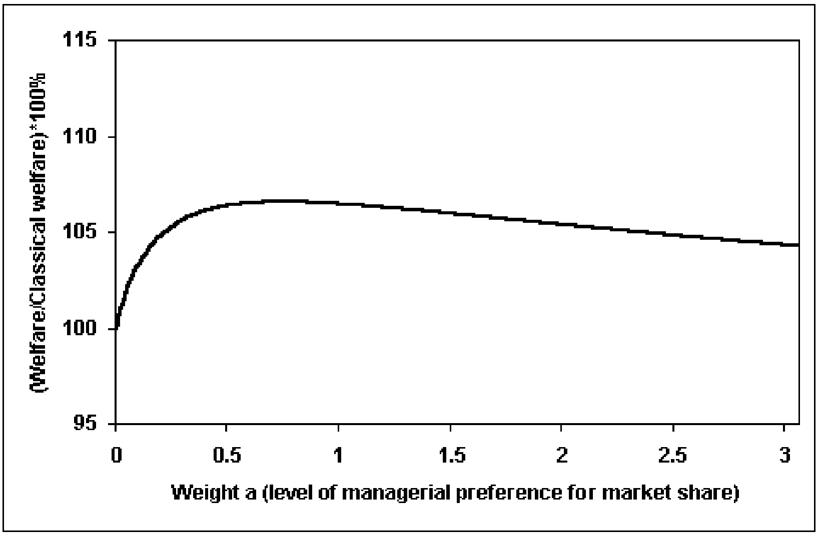


Fig. 5.13 The welfare for $c = 0.4$, $d = 0.1$ as a function of a

The analysis of this section reveals the existence of a profit-maximizing weight a_p and an advantage-maximizing weight a_d for firm 1 and also shows the existence of a welfare-maximizing weight a_w for $d > 0$. These phenomena occur before the bifurcation takes place. More complex dynamics will be the subject of Chapter 6. Benchmark case 2, with one classical profit-maximizer, may lead to interesting dynamical phenomena if the weight a exceeds a_{bif}.

7. Appraisal

One of the main conclusions of this Chapter is that a managerially inert firm - reflected by its weight attributed to market share - is more profitable than its flexible rival. The general analysis of Section 5.3 reveals that this result even holds for a broad class of “status functions” for small values of both competitors’ levels of preference (for the non-profit part of their objective function). If one of the firms is a classical profit-maximizer, there exists a level of preference for the “market share loving” competitor which maximizes the profit of this incumbent firm. We may look upon this paradoxical result - the firm that maximizes a (weighted) combination of profit and market share actually maximizes its profit in a competitive setting - from two poins of view. From the Organizational Ecology perspective the level of managerial inertia may be the result of imprinted routines, rules en procedures in an organizational culture and apparently selection favors firms with a specific level of preference for market share in a Darwinian selection process. Thus our analytical results support Hannan and Freeman’s (1984) inertia hypothesis. However from the standpoint of firms’ stategic consciousness - the perspective of Industrial Organization - a firm may use its managerial inertia as a strategic weapon in direct competition. By manipulating its weight attributed to market share a firm may outcompete its rival.

An a-setting game of the incumbent competitors may result in symmetry of both rivals with respect to their level of preference for market share and the analysis of Section 5.5 shows that this symmetry leads to a sacrifice of profits. Management’s behaviour may even lead to losses and firms’ bancruptcy in this symmetrical case. If the weight attributed to market share is not to heigh - then, both rivals are still profitable - social welfare benefits from managerial inertia. A heigher level of inertia may lead to give away prices of both rivals’ products thus implying a “war of attrition”. Naturally social welfare partially collapses if one of the firms is forced to exit, whereas the other firm becomes a monopolist.

Of course our present Cournot competition model, including preference for market share, doesn’t take into account differences in the efficiency of both firms’ production technologies. Therefore the question whether managerial inertia may compensate the rival’s lower production costs (like in Chapter 3) is an important issue for future research. Also the effect of a business cycle on both firms’ profits and on social welfare - an important part of Chapter 4’s examinations - is yet unanswered. Furthermore our model includes only two competitors, so an extension to 3 or more rivals will be a logical next step, including stability issues. Empirical research reveals that firms may use product differentiation (heterogeneity) to alleviate direct competition (Swaminathan and Delacroix (1991)); the influence of product heterogeneity is yet another refinement of the basic model.

Our basic game theoretical model, including preference for market share, may be considered as an promising first stepping stone to model a more complex market structure. Extensions of these mathematical models may contribute to the insights of Organizational Ecology and Industrial Organization. Furthermore we realize that empirical research concerning the impact of management compensation schemes and incentive systems on the behaviour of (top) management are also of significant importance.

Appendix 5.1
Conditions for the concavity of the welfare function.

In order to make a general statement on the concavity of the welfare function in a neighbourhood of $(a_1,a_2)=(0,0)$ we have to rewrite the second order power series approximation of Proposition 5.4 in a matrix-vector notation.

$$W(a_1,a_2)=W(0,0)+\frac{(1-c)s_{x_1}}{(3+2d)^2}\begin{bmatrix}1 & 1\end{bmatrix}\cdot\begin{bmatrix}a_1\\ a_2\end{bmatrix}+\frac{s_{x_1}}{(1+2d)^2(3+2d)^2}\begin{bmatrix}a_1 & a_2\end{bmatrix}\cdot\begin{bmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{bmatrix}\cdot\begin{bmatrix}a_1\\ a_2\end{bmatrix}+$$

$O(a^3)$ with $W(0,0)=\dfrac{(1\text{-}c)^2(4+2d)}{(3+2d)^2}$, $O(a^3)=O((\max\{a_1,a_2\})^3),(a_1\to 0,a_2\to 0)$,

$$M_{11}=M_{22}=-(4d^3+10d^2+7d+\tfrac{1}{2})s_{x_1}+\frac{(1+2d)(1-c)}{(3+2d)}\{(2+2d)s_{x_1x_1}-s_{x_1x_2}\}\text{and}$$

$$M_{12}=M_{21}=(2d^2+2d-\tfrac{1}{2})s_{x_1}+\frac{(1+2d)(1-c)}{(3+2d)}\{-s_{x_1x_1}+(2+2d)s_{x_1x_2}\}$$

(A1)

The first order part reveals that, if $s_{x_1}>0$ in $((1-c)/(3+2d),(1-c)/(3+2d))$, $\dfrac{\partial W}{\partial a_1}=\dfrac{\partial W}{\partial a_2}>0$ in $a_1=a_2=0$ (as we already used in Section 5.3) which indicates that the welfare increases if a_1 or a_2 increases. And the second order part (with the symmetric matrix) reveals that - under the assumption of convergence of the power series - the second order partial derivatives of the welfare function $\dfrac{\partial^2 W}{\partial(a_1)^2}$, $\dfrac{\partial^2 W}{\partial a_1\partial a_2}$ and $\dfrac{\partial^2 W}{\partial(a_2)^2}$ at $(a_1,a_2)=(0,0)$ equal $2M_{11}$, $2M_{12}$ and $2M_{22}$ respectively. Concavity in a neighbourhood of $(a_1,a_2)=(0,0)$ is guaranteed if and only if the following two conditions hold

$M_{11}<0$ and $M_{11}.M_{22}-\{M_{12}\}^2>0$ (A2)

Because $M_{11}=M_{22}$ the second condition equals $\{M_{11}+M_{12}\}.\{M_{11}-M_{12}\}>0$. Substitution of the expressions for M_{11} and M_{12} leads to the following proposition.

Proposition.

The welfare function $W(a_1,a_2)$ is concave in a neighbourhood of $(a_1,a_2)=(0,0)$ if and only if the following two conditions hold (both arguments of all partial derivatives equal $(1-c)/(3+2d)$:

(i) $-(4d^3+10d^2+7d+\tfrac{1}{2})s_{x_1}+\dfrac{(1+2d)(1-c)}{(3+2d)}\{(2+2d)s_{x_1x_1}-s_{x_1x_2}\}<0$

(ii) $\left[-(1+d)s_{x_1}+\dfrac{(1-c)}{(3+2d)}\{s_{x_1x_1}+s_{x_1x_2}\}\right]\cdot\left[-d(3+2d)^2s_{x_1}+(1-c)(1+2d)\{s_{x_1x_1}-s_{x_1x_2}\}\right]>0$

(A3)

These conditions are satisfied if s equals market share and unit costs are constant.

Appendix 5.2
Derivation of the formula for $x_{1,t} = G(x_{2,t-1}|a_1)$.

We rewrite the expression $\dfrac{\partial U^1}{\partial x_{1,t}} = 1 - c - (2+2d)x_{1,t} - x_{2,t-1} + a_1 \dfrac{x_{2,t-1}}{(x_{1,t}+x_{2,t-1})^2} = 0$ into

$(x_{1,t}+x_{2,t-1})^3 + \Theta(x_{1,t}+x_{2,t-1})^2 + \Omega = 0$ with $\Theta = -\dfrac{1-c+(1+2d)x_{2,t-1}}{(2+2d)}$ and $\Omega = -\dfrac{a_1 x_{2,t-1}}{(2+2d)}$.

The use of the three transformations

(i) $z = (x_{1,t}+x_{2,t-1}) + \dfrac{\Theta}{3} = x_{1,t} - \dfrac{1-c-(5+4d)x_{2,t-1}}{(6+6d)}$

(ii) $p = -\dfrac{\Theta^2}{3} = -\dfrac{\left[1-c+(1+2d)x_{2,t-1}\right]^2}{12(1+d)^2}$

(iii) $q = \tfrac{2}{27}\Theta^3 + \Omega = -2\dfrac{\left[1-c+(1+2d)x_{2,t-1}\right]^3}{(6+6d)^3} - \dfrac{a_1 x_{2,t-1}}{(2+2d)}$

(A4)

leads to the equation $z^3 + pz + q = 0$, from which the variable z has to be solved. This latter equation can be solved using Cardan's Method (see Teller (1965)). If the discriminant D defined as $D = (q/2)^2 + (p/3)^3$ is nonnegative the equation $z^3 + pz + q = 0$ possesses the real solution $z = \sqrt[3]{-q/2 + \sqrt{D}} + \sqrt[3]{-q/2 - \sqrt{D}}$.
Using the tranformation formulas (ii) and (iii) we obtain the following expression for the discriminant D:

$$D = \frac{a_1 x_{2,t-1}}{(4+4d)^2}\left[\frac{\left[1-c+(1+2d)x_{2,t-1}\right]^3}{27(1+d)^2} + a_1 x_{2,t-1}\right] \qquad \text{(A5)}$$

It clearly holds that $D \geq 0$ for $x_{2,t-1} \geq 0$, $a_1 \geq 0$ and $c < 1$, $d > -1/2$. The condition for the real solution is satisfied and the expression for z can be used. Finally using the transformation formulas (i), (ii) and (iii) the expression $x_{1,t} = G(x_{2,t-1}|a_1)$ equals

$$G(x_{2,t-1}|a_1) = \frac{1-c-(5+4d)x_{2,t-1}}{(6+6d)} +$$
$$\sqrt[3]{\frac{\left[1-c+(1+2d)x_{2,t-1}\right]^3}{(6+6d)^3} + \frac{a_1 x_{2,t-1}}{(4+4d)} + \sqrt{\frac{a_1 x_{2,t-1}}{(4+4d)^2}\left[\frac{\left[1-c+(1+2d)x_{2,t-1}\right]^3}{27(1+d)^2} + a_1 x_{2,t-1}\right]}} +$$
$$\sqrt[3]{\frac{\left[1-c+(1+2d)x_{2,t-1}\right]^3}{(6+6d)^3} + \frac{a_1 x_{2,t-1}}{(4+4d)} - \sqrt{\frac{a_1 x_{2,t-1}}{(4+4d)^2}\left[\frac{\left[1-c+(1+2d)x_{2,t-1}\right]^3}{27(1+d)^2} + a_1 x_{2,t-1}\right]}}$$

(A6)

Appendix 5.3
Equilibrium outputs concerning benchmark case 2, i.e. $a_1=a,\ a_2=0.$

Substitution of the linear relation $x_2^* = \frac{(1-c)}{(2+2d)} - \frac{1}{(2+2d)} x_1^*$ into the equation of the reaction function of firm 1, $(1-c)-(2+2d)x_1^* - x_2^* + a\frac{x_2^*}{(x_1^*+x_2^*)^2} = 0$, leads to a third degree equation $(x_1^*)^3 + \Theta(x_1^*)^2 + \Phi(x_1^*) + \Omega = 0$ with coëfficiënts

$$\Theta = \frac{(5+2d)(1-c)}{(1+2d)(3+2d)},\quad \Phi = \frac{(1+2d)(1-2d)(1-c)^2 + a(2+2d)^2}{(1+2d)^3(3+2d)} \quad \text{and}$$
$$\Omega = -\frac{(1+2d)(1-c)^3 + a(2+2d)^2(1-c)}{(1+2d)^3(3+2d)}. \tag{A7}$$

Using Cardan's Method (Teller (1965)) the equilibrium output x_1^* can be solved:

$$x_1^* = -\tfrac{1}{3}\Theta + \sqrt[3]{-\frac{q}{2}+\sqrt{D}} + \sqrt[3]{-\frac{q}{2}-\sqrt{D}},\ \text{with}\ D = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3,\ p = -\tfrac{1}{3}\Theta^2 + \Phi\ \text{and}$$
$$q = \tfrac{2}{27}\Theta^3 - \tfrac{1}{3}\Theta\Phi + \Omega. \tag{A8}$$

(this procedure can be easily implemented in a spread sheet programme)
For linear costs (d=0) the solution x_1^* equals

$$x_1^* = -\tfrac{5}{9}(1-c) + \tfrac{1}{3}\sqrt[3]{\tfrac{64}{27}(1-c)^3 + 28a(1-c) + \tfrac{4}{3}\sqrt{3a[3a+32(1-c)^2][4a+(1-c)^2]}} +$$
$$\tfrac{1}{3}\sqrt[3]{\tfrac{64}{27}(1-c)^3 + 28a(1-c) - \tfrac{4}{3}\sqrt{3a[3a+32(1-c)^2][4a+(1-c)^2]}} \tag{A9}$$

CHAPTER 6

PREFERENCE FOR MARKET SHARE AND COMPLICATED DYNAMICS SOME REFLECTIONS ON MODEL MODIFICATIONS AND STABILITY

1. Introduction

Besides the examination of the properties of the reaction curves related to the weight attributed to market share, the analysis of Chapter 5 focuses on two benchmark cases. Benchmark case 1 (Section 5.5) - concerning complete symmetry with respect to preference for market share - always leads to a stable Cournot-Nash equilibrium, whereas benchmark case 2 (Section 5.6) - one of the incumbent firms is a (classical) profit maximizer and the other is a “market share loving” firm - results in a stable equilibrium for $a < a_{bif}$. The stability of the Cournot-Nash equilibrium is of course crucial in the examination of the implications for profits and social welfare. The comparative statica of Chapter 5 leads to interesting results concerning these two benchmark cases. Yet the analysis of benchmark case 2 is not complete, because the equilibrium is no longer stable for $a > a_{bif}$ under the assumption of naïve (myopic) expectations of both rivals (i.e. $x^e_{i,t} = x_{i,t-1}, i = 1,2$). In the first place this chapter deals with unstable equilibria and the resulting more complicated dynamics. Clearly an asymmetry of the two competing firms, concerning the weight attributed to the market share is needed to obtain unstable equilibria. We will show that benchmark case 2 displays a richness of dynamical phenomena provided that the “market share loving” firm’s preference for market share is large enough. Such large levels of preference for market share may be part of a firm’s “blueprint” and cannot be changed quickly enough in comparison to environmental turbulence. Even if a firm (the owner) would act highly rational (“delegation” games), the analysis of Section 5.6 (p. 191) reveals that, in case of a strongly concave production cost function, the weight a attributed to market share in managers’ incentive contracts may be large. After all, in that specific case, the profits of the “market share loving” firm always increase w.r.t. an increasing weight a.

Computer simulations reveal that under naïve expectations, for $a > a_{bif}$ (see the expression of Table 5.5, which anticipates the analysis of this chapter) cycles with period two occur in the supply of both firms. However for small linear production costs - corresponding with a more efficient production technology - it appears that periodic output cycles with all sorts of periods are also possible These periodic supply paths are related to the Theorem of Sarkovskii (1964), which is not widely known (see also Devaney (1989)). In spite of the analytical complexity of the functional form of the reaction curve (Appendix 5.2) an attempt will be made to clarify these dynamical phenomena. Yet the richness of dynamical phenomena, concerning benchmark case 2, is still not exhausted. Under the assumption of alternately reacting competitors (and myopic expectations) chaotic supply patterns are also possible. Chaotic supply regimes can be defined loosely as completely irregular and aperiodic outputs of both rivals in consecutive time periods. Li and Yorke (1975) provide a theorem to prove the existence of a chaotic set which can be applied to unimodal functions. Because we have the analytical description of the reaction curve at our disposal, the analysis can be supported by simulation experiments to demonstrate some salient features of chaotic regimes.

In this chapter we will also reflect on modifications of the model of alternately reacting rivals. First we will examine the dynamics of the model concerning simultaneously reacting firms (under myopic expectations) and we will demonstrate that this slight modification has a significant influence on supply paths and average profits of both incumbent rivals. Fudenberg and Tirole (1991) note that the condition for stability is the same for two firms, whether they react alternately or simultaneously to their rival's most recent outputs (so-called naïve expectations). However, as we will illustrate, the periodicity of an output path resulting from instability does depend on the assumption of either an alternate or a simultaneous reaction pattern of the competing firms. Even the number of stable cycles changes and simulations reveal that the cycle to which an initial output pair is attracted displays a sensitive dependence on these initial supplies. Second we will reflect on the adjustment process in the supply, because in the case of instability ($a > a_{bif}$) production displays a strongly fluctuating pattern. Here we note that, under the assumption that a change in supply may bring extra adjustment costs (see also Chapter 4), these extra costs may have a stabilizing influence on the equilibrium, because of the existence of an inertia interval in both reaction curves. In this Chapter we will not study these inertia intervals, but leave this issue for future research.

We will examine the stability of the Cournot-Nash equilibrium in case both firms build a weighted average between their previous output $x_{i,t-1}$ and their output resulting from the reaction on the rival $R^i(x_{j,t-1})$ and prove that stability conditions are broader in comparison with the condition $a > a_{bif}$. Bischi and Kopel (2001) examine the complex dynamical structures in the case of two quadratic reaction functions and also use the assumption of such adaptive expectations (this thorough study focuses on equilibrium selection by examining the basins of attraction of two stable equilibria).

Finally we will examine a model concerning Stackelberg leadership of the "market share loving" firm. This leading firm, then, maximizes its utility function using the knowledge that its rival (the follower) reacts classically. The reason that we also examine this model is the fact that the analysis of Chapter 5, Section 5.6 reveals that the size of the firm with preference for market share is much larger than the rival's size.

The main message of this chapter, dedicated to the description and analysis of more complicated dynamical phenomena, is that a relatively simple assumption - namely the "love for market share" - may imply complicated dynamics. Even if one firm still behaves classically and maximizes its profit, the presence of a rival with preference for market share leads to complicated dynamical phenomena which at the very least can be considered thought-provoking.

How is this chapter organized?

Section 6.2 deals with benchmark case 2 concerning alternate reactions of both competitors. First an expression for the specific weight a_{bif}, for which the transition from a (local) stable into an unstable equilibrium takes place, is derived for constant unit costs (and under myopic expectations). Then, using the analytical description of the compound reaction curve - which corresponds with alternately reacting firms - the dynamical behaviour for $a > a_{bif}$ is characterized and clarified. Simulation experiments support the analysis. Section 6.3 deals with small unit costs. Then, periodic output cycles with a higher periodicity are related to Sarkovskii's Theorem (1964) and the Theorem of Li and Yorke (1975).

In Section 6.4 the dynamical behaviour will be examined considering simultaneously reacting firms (and myopic expectations). This section focuses on the differences between simultaneously and alternately reacting competitors including some properties of average profits. Furthermore we examine the stability of a model in which both firms build a weighted average between their previous output quantity $x_{i,t-1}$ and their 'Best Reply' output quantity $R^i(x_{j,t-1})$. We also provide the results of some computer experiments, which illustrates complicated dynamical phenomena.

Section 6.5 deals with Stackelberg leadership of the "market share loving" firm. We reflect on the differences between this model and the Cournot model of Section 5.6, concerning supplies and profits of both competitors. One of the results is that the Stackelberg leader may force its rival to exit and thus acquires a monopolistic market position. Section 6.6 concludes with a short appraisal.

2. Benchmark case 2: dynamical behaviour in case of instability

It is plausible that the Cournot-Nash equilibrium (x_1^*, x_2^*) (under the assumption of naïve expectations i.e. $x_{i,t}^e = x_{i,t-1}, i = 1,2$) loses its stability which is determined by the eigenvalues of the linearized system of the following system of non-linear first-order difference equations in the neighbourhood of (x_1^*, x_2^*)

$$\begin{cases} x_{1,t} = G(x_{2,t-1} \mid a) & \text{(Appendix 5.2)} \\ x_{2,t} = \tfrac{1}{2}(1-c) - \tfrac{1}{2}x_{1,t-1} & (a_2 = 0) \end{cases} \tag{6.1}$$

for linear production costs. These eigenvalues equal $\lambda_{1,2} = \pm\sqrt{\frac{dx_1}{dx_2} \cdot \frac{dx_2}{dx_1}}$, where $\frac{dx_1}{dx_2}$ and $\frac{dx_2}{dx_1}(= -\tfrac{1}{2})$ are the slopes of the reaction curves in the equilibrium of firm 1 and firm 2 respectively. Local stability is guaranteed if and only if the absolute value of both eigenvalues (of the linearized system) is smaller than 1 (see the proof of Proposition 5.2 of Section 5.3). For larger values of the weight a, $\frac{dx_1}{dx_2}$ is positive and it can be made arbitrarily large by increasing the preference for market share. Loosely speaking the slope of the reaction curve of firm 1 in equilibrium lies between the slope $\frac{dx_1}{dx_2}$ at $x_2 = 0$ - which equals $-\tfrac{1}{2} + \frac{2}{(1-c)^2}a$ for linear costs (Proposition 5.10 (ii)) - and the slope at the intersection point with the restriction (for $a \geq c$) - which equals $\frac{a(a-2-2c)-1}{(4+2c)a+2}$ (proof of Proposition 5.11 (iii)). Both slopes can be made arbitrarily large by increasing the weight a.

From the expression for both eigenvalues (which are complex if $\frac{dx_1}{dx_2} > 0$) it follows that the equilibrium is unstable (a so-called negative attractor) for $a > a_{bif}$ corresponding with $\frac{dx_1}{dx_2} > 2$ at the equilibrium (for $d \neq 0$ $\frac{dx_1}{dx_2} > 2 + 2d$ has to hold).
If the firms react alternately on the rival's most recent output we obtain (for linear costs)

$$x_{1,t+1} = G(\tfrac{1}{2}(1-c) - \tfrac{1}{2}x_{1,t-1} \mid a)\,. \tag{6.2}$$

If the slope of this compound curve is smaller than –1 at the fixed point (=equilibrium's first co-ordinate x_1^*), this fixed point is unstable (repelling), which again leads to the condition $\frac{dx_1}{dx_2} > 2$ by using the chain rule (see also Fudenberg and Tirole (1991), p. 25). We mention that, corresponding to simultaneously reactions of both firms, both eigenvalues of the linearized system are complex. The stable or unstable equilibrium is a so-called focal point. In case of instability each (small) disturbance of the equilibrium is (in absolute value) enlarged in each subsequent time period and the output path of both rivals follows an outward spiral movement.

We now examine the case of alternately reacting firms (although the size of the firm with preference for market share may be much larger than its competitor's size, we consider Cournot competition and reserve the case of Stackelberg leadership to Section 6.5). Before we derive an expression for a_{bif} (for linear costs) we display a graph of the outputs of both firms corresponding with an unstable equilbrium. Unit production costs equal $c = 0.4$ and the weight a equals 4.0 ($a > a_{bif} = 3.07$, Table 5.5). This simulation experiment covers 50 periods ($t = 0, 1, ..., 49$) in which firm 1 and its rival react consecutively. Firm 1 (in period 0) starts with an output value of 0.52 close to the (first co-ordinate of the) equilibrium $x_1^* = 0.523$ (in 3 decimals). (The reactions of firm 1 are computed using the expression of the reaction function $x_{1,t} = G(x_{2,t-1})$ of Appendix 5.2 for $c = 0.4$, $d=0$ and $a = 4.0$).

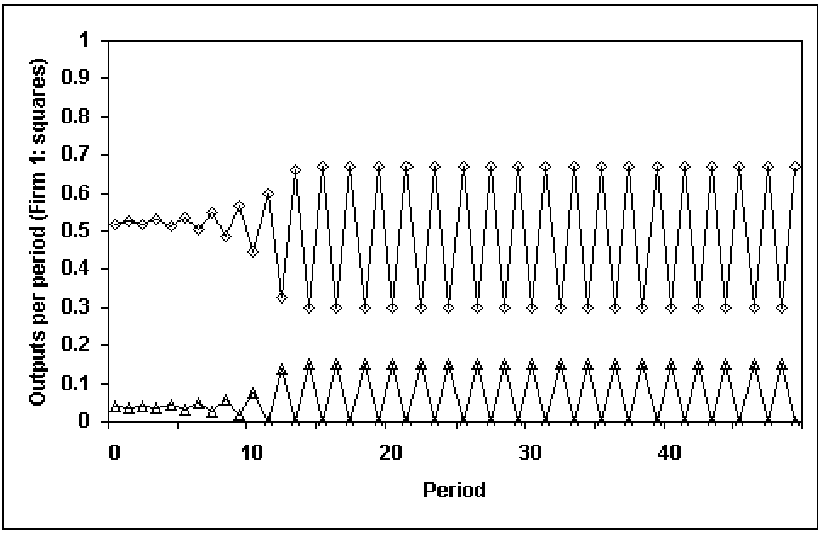


Fig. 6.1 The occurrence of cycles with period 2, c=0.4, a=4.0

We observe that the instability of the equilibrium (=(0.523, 0.038)) leads, after some periods, to a new sort of "steady state", which consists of cycles in the output of both firms of period 2. In period 20 the respective outputs of firm 1 and firm 2 equal 0.3 and 0.15. In fact in all even-numbered periods the supply of firm 1 (market leader) equals the monopoly production, whereas firm 2 reacts on this rival's output. However in the odd-numbered periods firm 1's supply equals 0.671 (in three decimals) - which equals $G(0.15 \mid a = 0.40)$ - and because this supply exceeds 0.6 firm 2 reacts with output zero. Note that the total output per period displays a cycle as well; in even- and odd-numbered periods total supply equals 0.45 and 0.671 respectively. In other words, caused by the more extreme preference for market share of one firm, this system generates a fluctuating output pattern on the market, i.e. an endogenous business cycle. These interesting dynamical phenomena, which are nicely illustrated by Fig. 6.1 appear to hold in general for $a > a_{bif}$ and $c \geq 0.2$. To clarify the occurrence of period-2-cycles the first step is to derive an expression for the so-called bifurcation value a_{bif} of the weight a, corresponding to the transition from a stable into an unstable equilibrium.

Proposition 6.1 (the bifurcation point a_{bif} for linear costs).

Consider benchmark case 2, i.e. $a_1 = a$ and $a_2 = 0$. For linear costs and under myopic expectations the transition from a stable into an unstable equilibrium occurs for $a_{bif} = \frac{1440}{169}(1-c)^2$.

Proof

If we express x_1^* as a fraction of $(1-c)$, i.e. $x_1^* = f \cdot (1-c)$ with $\frac{1}{3} \le f < 1$, it follows that (by the equation of the reaction curve of firm 2) $x_2^* = \frac{1}{2}(1-f)(1-c)$ and total production equals $x_1^* + x_2^* = \frac{1}{2}(1+f)(1-c)$. The corresponding weight a - which is one-to-one related to x_1^* - follows from the equation of the reaction curve of firm 1,

$$1 - c - 2x_1^* - x_2^* + a\frac{x_2^*}{(x_1^* + x_2^*)^2} = 0,$$

and equals $a = \dfrac{(3f-1)(1+f)^2}{4(1-f)}(1-c)^2$.

The slope $\dfrac{\mathrm{d}x_1}{\mathrm{d}x_2}$ at the equilibrium equals (see Proposition 5.10, Section 5.4)

$$\left(\frac{\mathrm{d}x_1}{\mathrm{d}x_2}\right)_{(x_1^*, x_2^*)} = \frac{-(x_1^* + x_2^*)^3 + a(x_1^* - x_2^*)}{2(x_1^* + x_2^*)^3 + 2ax_2^*} = \frac{(5f-3)}{4(1-f)}$$

Note that this slope is independent of the parameter c and increases with respect to an increasing f. Because f determines x_1^* uniquely and a is one-to-one related to x_1^* the f-value of $\frac{11}{13}$ which satisfies $\left(\dfrac{\mathrm{d}x_1}{\mathrm{d}x_2}\right)_{(x_1^*, x_2^*)} = 2$ leads to

$a_{bif} = \frac{1440}{169}(1-c)^2$ (and $x_1^* = \frac{11}{13}(1-c)$)

[End of proof]

Note that a_{bif} increases with respect to a decreasing c, corresponding with a more efficient production technology. Using the same method of proof it can be derived that a_{bif} is also proportional to $(1-c)^2$ for $d \neq 0$, only the factor (which equals about 8.52 for linear production costs) changes. In Table 6.1 some of these factors related to d are presented, rounded to two decimals.

Table 6.1 $\boldsymbol{a_{bif} = f(d) \cdot (1-c)^2}$ **for several values of** $\boldsymbol{d}$

d	-0.2	-0.1	0	0.1	0.2
$f(d)$	5.61	7.00	8.52	10.17	11.95

The simulation experiment illustrated by Figure 6.1 deals with alternately reacting firms. The explanation of the existence of an attracting orbit with period 2 has to be found in the functional form of the compound reaction curve R^c defined by (with a_1=a and a_2=0)

$$R^c(x_1 \mid a,0) := R^1(R^2(x_1 \mid 0) \mid a) \ . \tag{6.3}$$

We used this notation also in Section 5.4 (Proposition 5.9), but for the sake of clarity we now leave out the time subscripts. The definition of R^c reveals its composition: firm 2 reacts on the supply of firm 1 by $x_2 = R^2(x_1 \mid 0)$ and firm 1 reacts on the output x_2 by $x_1 = R^1(x_2 \mid a)$. Because firm 2 is a classical profit-maximizer (a_2=0) its reaction function is simplified by $R^2(x_1) = \max\{\frac{1}{2}(1-c) - \frac{1}{2}x_1;0\}$ (for linear costs). Therefore it suffices to examine the compound curve

$$R^c(x_1 \mid a,0) := R^1(\max\{\tfrac{1}{2}(1-c-x_1);0\} \mid a) \tag{6.4}$$

Two graphs illustrate the development of the curve R^c corresponding to an increasing weight a for constant unit costs c=0.4.

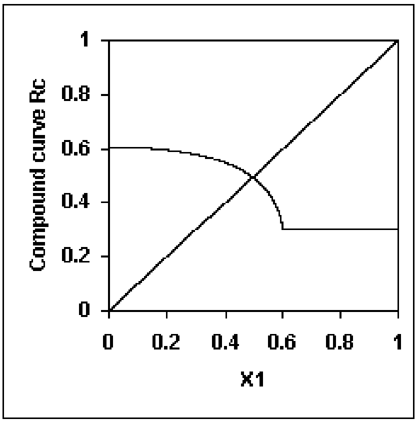


Fig. 6.2a R^c for c=0.4, a=2.5

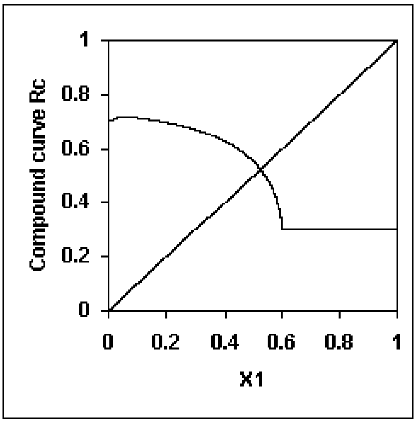


Fig. 6.2b R^c for c=0.4, a=4.0

Computation shows that the fixed point (equilibrium) shifts from $x_1^* = 0.494$ to $x_1^* = 0.523$ for a = 2.5 and 4.0 respectively. The slopes in the fixed points shift from –0.794 to –1.335 indicating the stability and the instability of the fixed point for a = 2.5 and 4.0 respectively (note that now the instability of the fixed point corresponds with a slope smaller than –1). Furthermore we observe that, for a = 4.0, the compound reaction curve consists of three qualitatively differing parts. We now describe the functional form of R^c analytically and support the analysis using Figure 6.3 for linear production costs. The intersection point of the reaction curve with the restriction (see Proposition 5.9, Section 5.4) plays an important role.

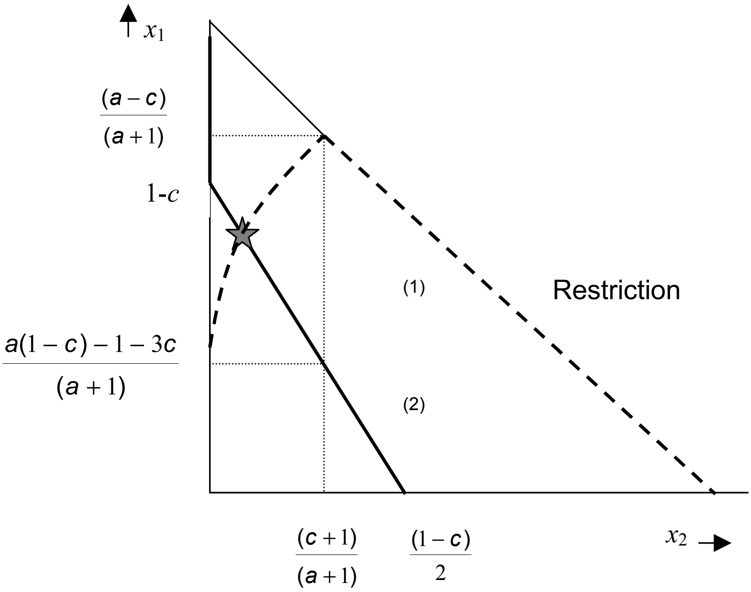


Fig. 6.3 The two reaction curves for $(c+1)/(a+1) < ½(1-c)$.

Proposition 6.2 (description of R^c for linear costs).

For $a > \frac{1+3c}{1-c}$ the compound reaction curve R^c, corresponding with alternate reactions of both firms for $a_1 = a$, $a_2 = 0$, equals

$$R^c(x_1 \mid a,0) = \begin{cases} (i) \quad \tfrac{1}{2}(1+c) + \tfrac{1}{2}x & , \ 0 \le x_1 < \dfrac{a(1-c)-1-3c}{a+1} \\ (ii) \quad G(\tfrac{1}{2}(1-c-x_1) \mid a) & , \ \dfrac{a(1-c)-1-3c}{a+1} \le x_1 < 1-c \\ (iii) \ \tfrac{1}{2}(1-c) & , \ 1-c \le x_1 \le 1 \end{cases} \tag{6.5}$$

If $a \le \frac{1+3c}{1-c}$ the part described in (i) does not exist.

Proof

If $\frac{c+1}{a+1} < \frac{1}{2}(1-c)$ then $R^c(x_1 \mid a,0) = R^1(\frac{1}{2}(1-c-x_1) \mid a)$ is a linear function for $0 \le x_1 < \frac{a(1-c)-1-3c}{a+1}$ (see Figure 6.3) with slope $\frac{1}{2}$ whereas $R^c(0 \mid a,0) = 1 - \frac{1}{2}(1-c) = \frac{1}{2}(1+c)$.

For $\frac{a(1-c)-1-3c}{a+1} \le x_1 < 1-c$, $R^1(\frac{1}{2}(1-c-x_1) \mid a)$ equals the non-linear expression $G(\frac{1}{2}(1-c-x_1) \mid a)$ (see Appendix 5.2), whereas for $1-c \le x_1 \le 1$, firm 2 reacts with output zero whereupon firm 1 reacts with its monopoly output $\frac{1}{2}(1-c)$.

[End of proof]

We refer to the Appendix 6.1 for a description of the compound reaction curve for non-linear production cost functions ($d \neq 0$). Note that the first part of the compound reaction function is linear with a slope equal to ½ and the third part is a constant function equal to the monopoly output of firm 1.
The derivation of the special weight a_{bif}, corresponding to the transition from a stable into an unstable equilibrium and the description of the compound reaction function $R^c(x_1 \mid a,0)$ allow us to clarify the occurrence of a (stable) cycle with period two under certain conditions for the unit production costs. Using Figure 6.2b it is obvious that, starting with a small disturbance from the (unstable) repelling fixed point, after a few iterations (determined by $x_{1,t+1} = R^c(x_{1,t} \mid a,0)$, for $t = 0,1,2,...$) the value $x_{1,t}$ becomes ½(1-c), the monopoly output of firm 1. Clearly firm 2's reaction on this monopoly output equals $\frac{1}{2}[1-c-\frac{1}{2}(1-c)] = \frac{1}{4}(1-c)$. The next reaction of firm 1 then equals $R^1(\frac{1}{4}(1-c) \mid a)$ and if it holds that $R^1(\frac{1}{4}(1-c) \mid a) \geq 1-c$ firm 2's next reaction will be a zero output whereupon firm 1 reacts again with its monopoly output. The whole (stable) cycle starts again and again consisting of the consecutive supplies $x_1 = \frac{1}{2}(1-c), x_2 = \frac{1}{4}(1-c)$ in "period t" and $x_1 = R^1(\frac{1}{4}(1-c) \mid a), x_2 = 0$ in the next period. Such a cycle of period 2 can be observed in Figure 6.1, where after an initial period a stable cycle in outputs occurs. However the condition $R^1(\frac{1}{4}(1-c) \mid a) \geq 1-c$ is not always satisfied for $a > a_{bif}$ and $0 < c < 1$ so this condition has to be examined thoroughly. Before we present Proposition 6.2, concerning the existence of a cycle of period 2 under certain (cost) conditions, we prepare the proof of this proposition by making some notes on the examination of the condition, illustrated by Figure 6.4 (which corresponds to the first note).

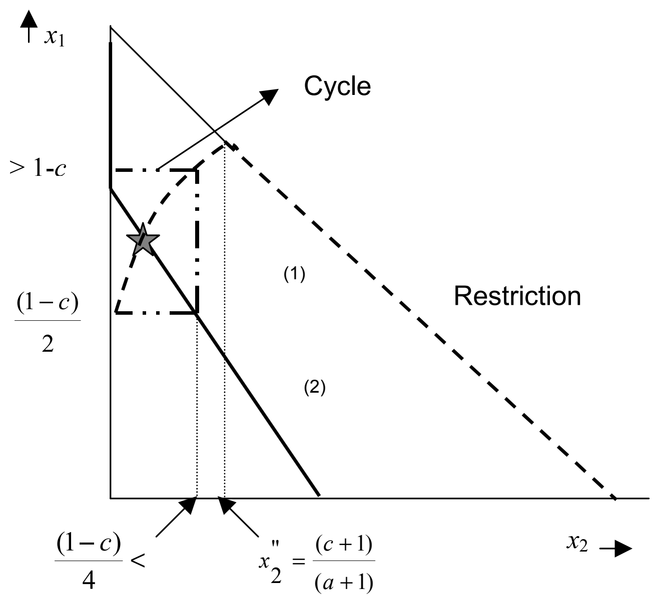


Fig. 6.4 Cycle for $a_{bif} < a < (5c+3)/(1-c)$.

- If the second co-ordinate of the intersection point of firm 1's reaction curve with the restriction - which equals $x_2'' = \dfrac{(c+1)}{(a+1)}$ - exceeds $\frac{1}{4}(1-c)$, the reaction of firm 1 on firm 2's supply of $\frac{1}{4}(1-c)$ equals $G(\frac{1}{4}(1-c)\,|\,a)$ (see Appendix 5.2). The corresponding restriction for the weight a is $a < \dfrac{(5c+3)}{(1-c)}$. Obviously this case is only of importance if $a_{bif} = \frac{1440}{169}(1-c)^2 < \dfrac{(5c+3)}{(1-c)}$ which is equivalent to $0.217 < c < 1$.
- If the second co-ordinate of the intersection point doesn't exceed $\frac{1}{4}(1-c)$ firm 1's reaction will be perfect accommodation i.e. $R^1(\frac{1}{4}(1-c)\,|\,a) = 1 - \frac{1}{4}(1-c) = \frac{1}{4}c + \frac{3}{4}$. The condition for the parameter a equals $a \geq \dfrac{(5c+3)}{(1-c)}$. For $0 < c \leq 0.217$ it holds that $a_{bif} \geq \dfrac{(5c+3)}{(1-c)}$ so for this (smaller) unit costs we only deal with the described (perfect) accommodation of firm 1 if the equilibrium is unstable.

Proposition 6.3 (dynamical phenomena for $a > a_{bif}$ and $0.2 \leq c < 1$*)*

Consider benchmark case 2 for linear production costs, i.e. $d = 0, a_1 = a$ and $a_2 = 0$, with alternately reacting firms. Under the assumption of an unstable equilibrium - $a > a_{bif} = \frac{1440}{169}(1-c)^2$ - and concerning unit production costs with $0.2 \leq c < 1$ the following holds:

There exists a stable (and globally attracting) output cycle with period 2. In two consecutive periods the supplies of both firms equal $x_1 = \frac{1}{2}(1-c), x_2 = \frac{1}{4}(1-c)$ and $x_1 = R^1(\frac{1}{4}(1-c)\,|\,a), x_2 = 0$ respectively. For $a < \dfrac{(5c+3)}{(1-c)}$ $R^1(\frac{1}{4}(1-c)\,|\,a) = G(\frac{1}{4}(1-c)\,|\,a)$ (Appendix 5.2) and otherwise $R^1(\frac{1}{4}(1-c)\,|\,a) = \frac{1}{4}c + \frac{3}{4}$.

For $0 < c < 0.2$ there doesn't exists a stable cycle with period 2 but there exist cycles with all sorts of periods.

Proof

The instability of the fixed point leads - after some iterations $x_{1,t+1} = R^c(x_{1,t}\,|\,a)$ (see Figure 6.2b) - to the reaction path $x_1 = \frac{1}{2}(1-c), x_2 = \frac{1}{4}(1-c)$ and $x_1 = R^1(\frac{1}{4}(1-c)\,|\,a)$.

A stable cycle of period 2 only occurs if $R^1(\frac{1}{4}(1-c)\,|\,a) \geq 1-c$. Under the assumption $0.217 < c < 1$ we have to distinguish two cases:

(a) For $a_{bif} < a < \dfrac{(5c+3)}{(1-c)}$ the function $R^1(\frac{1}{4}(1-c)\,|\,a)$ equals $G(\frac{1}{4}(1-c)\,|\,a)$. Using the expression of Appendix 5.2 for G we can compute $G(\frac{1}{4}(1-c)\,|\,a_{bif}) = 1.028(1-c)$. This implies that $G(\frac{1}{4}(1-c)\,|\,a) > 1.028(1-c) > (1-c)$ for $a > a_{bif}$

(b) For $a \geq \frac{(5c+3)}{(1-c)}$ it holds that $R^1(\frac{1}{4}(1-c)\,|\,a) = \frac{1}{4}c + \frac{3}{4} \geq 1-c$ for $c \geq 0.2$

From (a) and (b) it follows that, for $0.217 < c < 1$, there exists a stable cycle with period 2.

For $0 < c \leq 0.217$ it holds that $a_{bif} \geq \frac{(5c+3)}{(1-c)}$ and so the condition for a stable cycle with period 2, i.e. $R^1(\frac{1}{4}(1-c)\,|\,a) \geq 1-c$, only holds for $0.2 \leq c \leq 0.217$.

Obviously for $0 < c < 0.2$ the condition $R^1(\frac{1}{4}(1-c)\,|\,a) = \frac{1}{4}c + \frac{3}{4} \geq 1-c$ is not satisfied and starting with $x_1 = \frac{1}{2}(1-c)$ the output pattern becomes more complicated. The stable output cycle with period 2 doesn't occur anymore.

[End of proof]

Figure 6.1 provides an illustration of Proposition 6.3 with $a = 4.0 < \frac{(5c+3)}{(1-c)} = 8\frac{1}{3}$ for $c = 0.4$. The supplies of both firms equal $x_1 = \frac{1}{2}(1-c) = 0.3, x_2 = \frac{1}{4}(1-c) = 0.15$ and $x_1 = G(\frac{1}{4}(1-c)\,|\,a) = G(0.15\,|\,4.0) = 0.671, x_2 = 0$ in two consecutive periods.

Note that if $a \geq 8\frac{1}{3}$ the outputs in the first period equal $x_1 = 0.3, x_2 = 0.15$ whereas the outputs in the next period equal $x_1 = 0.85, x_2 = 0$ due to the perfect accommodation of firm 1.

Although the analysis of dynamical phenomena plays the leading part in this chapter, we conclude this section with an analysis of the average profits (per period) over a cycle with period 2 of both competitors. Table 5.5 shows that, just before bifurcation takes place and the Cournot-Nash equilibrium is still stable, the profit of firm 1 equals 11 times the profit of its rival and is about 60% of the classical profit ($\frac{1}{9}(1-c)^2$). However, if firm 1's preference for market share increases further ($a > a_{bif}$), this profitable situation may change completely. Proposition 6.3 states that for $a > a_{bif}$ and $0.2 \leq c < 1$ all initial values are attracted to a stable cycle with period 2; the supplies of both firms can be described precisely. Lets return to Figure 6.1 where the cyclic output is displayed for $c = 0.4, a = 4.0 > a_{bif} = 3.07$. In one period the respective supplies of firm 1 and 2 are $x_1 = 0.3$ and $x_2 = 0.15$ and the profits of both firms equal $\Pi^1 = 0.0450$ and $\Pi^2 = 0.0225$ (rounded to 4 decimals). In the next period however, due to the aggressive play of firm 1, firm 1's supply, 0.6706, exceeds $1-c = 0.6$ whereupon the rival's reaction is zero output. But in this period the unit production costs of 0.4 exceed the market price 0.3294 and firm 1 faces losses whereas firm 2' profit is zero, i.e. $\Pi^1 = -0.0473, \Pi^2 = 0$. The average profits per period of both competitors over the cycle equal $\overline{\Pi^1} = -0.0012$ and $\overline{\Pi^2} = 0.0113$. Apparently firm 1's love for market share (and under myopic expectations) now causes its bankruptcy. We are interested under which conditions the average profit (per period) of firm 1 lies at a lower level than the rival's average profit and also whether there exist conditions for which $\overline{\Pi^1} \leq 0$ holds.

The general analysis deals with this two questions and uses the fact that in the first period - with $x_1 = \frac{1}{2}(1-c), x_2 = \frac{1}{4}(1-c)$ - the respective profits of both competitors equal $\Pi^1 = \frac{1}{8}(1-c)^2$ and $\Pi^2 = \frac{1}{16}(1-c)^2$. Clearly this first period is advantageous for firm 1, but in the next period it holds that - with supplies $x_1 = R^1(\frac{1}{4}(1-c) \mid a), x_2 = 0$ - the profits equal $\Pi^1 = R(1-c-R) \le 0, \Pi^2 = 0$ (with the brief notation R for $R^1(\frac{1}{4}(1-c) \mid a)$). The following proposition summarizes the analytical results:

Proposition 6.4 (profits in case of a cyclic supply with period 2)

Consider benchmark case 2 for linear production costs, i.e. $d = 0, a_1 = a, a_2 = 0$, with alternately reacting firms. For $a > a_{bif}$ and $0.2 \le c < 1$ the (stable) output cycle with period 2 leads to the following properties of the average profits (per period) $\overline{\Pi^1}$ and $\overline{\Pi^2}$ of both firms:

(i) If $0.236 \le c < 1$ there exists a weight a_l ($a_{bif} < a_l \le \frac{(5c+3)}{(1-c)}$) with the property that for all weights $a \ge a_l$ $\overline{\Pi^1} \le \overline{\Pi^2}$, i.e. the average profit level of the "market share loving" firm is lower than the rival's average profit level. The weight a_l is determined by $G(\frac{1}{4}(1-c) \mid a_l) = (\frac{1}{2} + \frac{1}{4}\sqrt{5})(1-c)$ (see Appendix 5.2 for the expression for G).

(ii) If $0.266 \le c < 1$ there exists a weight a_n ($a_{bif} < a_n \le \frac{(5c+3)}{(1-c)}$) with the property that for all weights $a \ge a_n$ $\overline{\Pi^1} \le 0$, i.e. the average profit level of the "market share loving" firm is nonpositive.The weight a_n is determined by $G(\frac{1}{4}(1-c) \mid a_n) = (\frac{1}{2} + \frac{1}{4}\sqrt{6})(1-c)$.

The average profit level (per period) of firm 2 is independent of a and equals $\overline{\Pi^2} = \frac{1}{32}(1-c)^2$

Proof

Using the supplies of both competitors in two consecutive periods, the average profit per period of firm 2 clearly is constant and equals $\overline{\Pi^2} = \frac{1}{2}[\frac{1}{16}(1-c)^2 + 0] = \frac{1}{32}(1-c)^2$.The average profit per period of firm 1 is a decreasing function of the supply R defined by $R = R^1(\frac{1}{4}(1-c) \mid a)$ (and therefore depends on the weight a before perfect accommodation occurs). For $a \ge \frac{(5c+3)}{(1-c)}$ firm 1's reaction on $\frac{1}{4}(1-c)$ equals $R = \frac{1}{4}c + \frac{3}{4}$ because of perfect accommodation and the average profit then equals $\overline{\Pi^1} = \frac{1}{2}[\frac{1}{8}(1-c)^2 + \frac{1}{16}(c+3)(1-5c)]$. Therefore a necessary and sufficient condition for the existence of the weight a_l is $\frac{1}{16}(1-c)^2 + \frac{1}{32}(c+3)(1-5c) \le \frac{1}{32}(1-c)^2$ which leads to $\sqrt{5} - 2 \le c < 1$.
Equating the average profits per period of both firms, before perfect accomodation occurs leads to the following condition concerning the supply R : $\frac{1}{2}[\frac{1}{8}(1-c)^2 + R(1-c-R)] = \frac{1}{32}(1-c)^2 \leftrightarrow R = (\frac{1}{2} + \frac{1}{4}\sqrt{5})(1-c)$. Part (i) is proved by realizing that $R = G(\frac{1}{4}(1-c) \mid a_l)$. The proof of part (ii) is similar.

[End of proof]

We apply the analysis of Proposition 6.4 to the case of two competitors with constant unit production costs $c = 0.4$ and we emphasize that these results hold in general (for $0.266 \le c < 1$. Just before bifurcation occurs ($a_{bif} = 3.067$ in three decimals) the respective profits of firm 1 and 2 expressed as a percentage of the classical profit (for $a = 0$) equal 59% and 5%, clearly indicating the advantage of the "market share loving" firm. For a weight slightly higher than a_{bif} the instability of the equilibrium leads to a cyclic supply pattern with period 2 and also implies a sudden change in profit levels of both rivals. The average profit level (per period) of firm 2 increases and equals $\frac{1}{32}(1-c)^2$ in general which is 28% of classical profit and - using the expression $G(\frac{1}{4}(1-c) \mid a_{bif})$ (see proof of Proposition 6.3) - one can derive that the average profit of firm 1 (just after bifurcation) equals in general

$$\overline{\Pi^1} = \tfrac{1}{2}[\tfrac{1}{8}(1-c)^2 - 0.029(1-c)^2] = 0.048(1-c)^2$$

which is 43% of the classical profit. This sudden change of both profit levels, due to the bifurcation, immediately leads to a more beneficial situation for firm 2. However, caused by a very small behavioural change, firm 1 faces a little catastrophe, indicated by a sudden change of 27% of its profit level (namely from 59% to 43% of Π_{cl}). A further increase of firm 1's preference for market share leads to a quick fall of its average profit over the cycle. Application of part (i) of Proposition 6.4 reveals that for $a > a_l = 3.376$ (rounded to 3 decimals) the average profit level of firm 1 lies below the rival's average profit. Part (ii) of the proposition shows that for $a > a_n = 3.942$ the average profit of firm 1 is even negative and leads to bankruptcy. The larger the preference for market share is the more negative $\overline{\Pi^1}$ becomes, whereas the rival's average profit stays constant.

Proposition 6.4 shows that for higher levels of managerial inertia - indicated by the weight a attributed to market share - the beneficial situation for the "market share loving" firm changes into a catastophe and may even lead to exit.

3. More complicated dynamics and chaos

Proposition 6.3 announces more complicated dynamical phenomena for smaller unit production costs ($0 < c < 0.2$). The fact that a stable orbit with period 2 doesn't occur anymore, doesn't imply that cycles with higher periodicity can't occur. If, during a cycle starting with $x_1 = \frac{1}{2}(1-c)$, at some point firm 1's output exceeds 1-c the reaction of firm 2 is a zero output, whereupon the whole (stable) cycle starts again. The next graphical display corresponds with constant unit production costs $c = 0.15$ and weight $a = 7.3 > a_{bif} = 6.16$ whereas the initial output of firm 1 equals $x_1 = 0.7$ (the first co-ordinate of the equilibrium is $x_1^* = 0.734$ and the slope of the compound reaction curve R^c equals -1.205 indicating the instability of the fixed point).

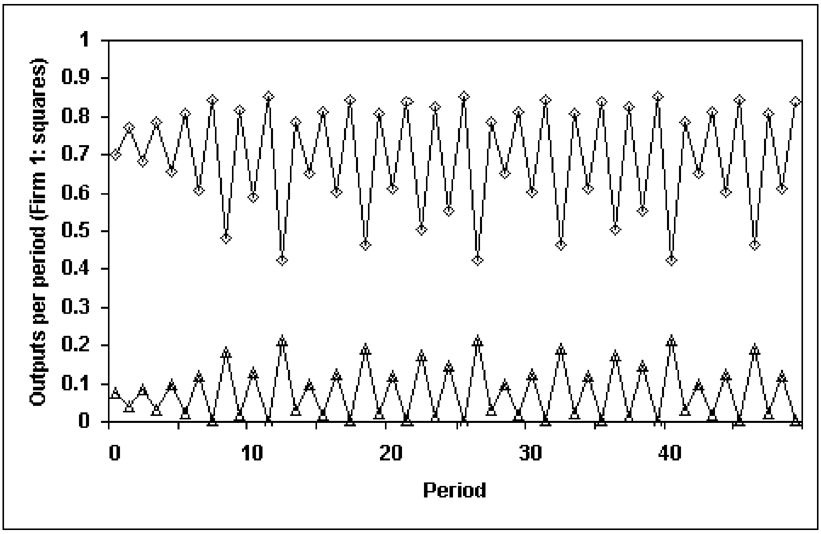


Fig. 6.5 The occurrence of a stable cycles with period 14, *c*=0.15, *a*=7.3

After some initial phenomena firm 1's supply equals its monopoly output and the subsequent cycle appears to possess period 14. Although the output pattern is far from chaotic the supply of boths firms is quite irregular; total output fluctuates between 0.64 and 0.85. The (prime) period of the stable output cycle also depends on the value of a; $a = 7.0$ leads to period 24 whereas $a = 7.5$ corresponds with a period of 6. Apparently the (prime) period displays a sensitive dependence concerning the weight a which firm 1 attributes to its market share. Simulation experiments show that for higher values of a stable cycles with an odd period occur, whereas for lower values of a (like in our examples) the periodicity appears to be even.

The occurrence of stable cycles with even or odd periodicity can be explained by observing the consecutive outputs of firm 1. Starting with its monopoly output of $x_1 = \frac{1}{2}(1-c)$ the next ("second") reaction of firm 1 is determined by perfect accommodation and equals $x_1 = \frac{1}{4}c + \frac{3}{4}$. Because x_1^* (the value of the unstable fixed point) increases with respect to an increasing a, for smaller values of a it holds $\frac{1}{4}c + \frac{3}{4} > x_1^*$. The next outputs of firm 1 are alternately smaller and larger than x_1^*, so the periodicity must be even, because the cycle is completed with an output satisfying $x_1 \geq 1-c > x_1^*$. However, if the weight a increases further, it holds $\frac{1}{4}c+\frac{3}{4} < x_1^*$ and the next outputs of firm 1 are alternately larger and smaller than x_1^* implying that the stable orbit must possess an odd periodicity. The specific weight which marks the transition from even to odd periodicities depends on c and can be solved using $\frac{1}{4}c+\frac{3}{4} = x_1^*$. Application of this condition to $c = 0.15$ leads to stable cycles with even and odd periodicities for a < 16.22 and a > 16.22 respectively. We note that for values of a near 16.22, because $\frac{1}{4}c+\frac{3}{4}$ almost equals x_1^*, the cycle seems to stabilize around the fixed point value, but this is merely appearance.

These periods of stable supply cycles are related to the remarkable and strong Theorem of Sarkovskii (1964), which holds under the minimal assumption of continuity of a map of the real line in itself. A proof of this amazing theorem can be found in Devaney (1989) and to formulate this theorem we first need the following so-called Sarkovskii-ordering of the natural numbers:

$$\begin{aligned} &3 \triangleright 5 \triangleright 7 \triangleright \ldots \triangleright 2.3 \triangleright 2.5 \triangleright 2.7 \triangleright \ldots 2^2.3 \triangleright 2^2.5 \triangleright \ldots \triangleright 2^3.3 \triangleright 2^3.5 \triangleright \ldots \\ &\ldots \triangleright 2^3 \triangleright 2^2 \triangleright 2 \triangleright 1 \end{aligned} \tag{6.6}$$

First all odd numbers except 1 are listed, followed by 2 times the odds, 2^2 times the odds, 2^3 times the odds, etc.. Then all powers of 2 follow with a decreasing exponent. Note that corresponding with this peculiar ordering (not for mathematicians) 3 is the largest Sarkovskii number whereas 1 is the smallest number. Sarkovskii's Theorem states that

> Assume that $f: R \to R$ is continuous and the map f possesses a periodic point of prime period k. If $k \triangleright l$ in the Sarkovskii ordering, then f has also a periodic point of period l.

Clearly our map R^c is continuous and maps the interval $[0,1]$ into itself. Due to the Theorem of Sarkovskii period 14 implies the existence of periodic points with all sorts of periods, for instance all powers of 2. However all these periodic orbits are non-stable and non-attracting, because (almost) each initial value is, after a few iterations, mapped onto $x_1 = \frac{1}{2}(1-c)$ the starting point of a stable cycle with period 14.

Note that the existence of period 3, because 3 is the largest number in the Sarkovskii ordering, implies the occurrence of all other periods. Furthermore, due to the famous Theorem of Li and Yorke (1975), the existence of a periodic point with period 3 implies the existence of a so-called chaotic set $\aleph$, defined as an uncountable set of initial conditions that give rise to completely a-periodic, irregular output paths (the whole output path lies in the chaotic set $\aleph$). The fact that such a chaotic set exists doesn't mean that a member of this special set $\aleph$ can be found. In spite of its uncountability the (Lebesque) measure of the chaotic set can be zero, which in popular terms means that $\aleph$ contains no intervals. If $\aleph$ possesses measure zero the probability to identify a member of this set by a computer is zero. In Proposition 6.5 we apply Li and Yorke's Theorem, using the description of the compound reaction function R^c of Proposition 6.2. Part (i) of Proposition 6.5 deals with the case that the condition of Li and Yorke (period 3) is satisfied implying the existence of a chaotic set. However, if this condition is satisfied, it can be proved that there exists a stable output path with period 3. Part (ii) of Proposition 6.5 deals with very small unit production costs ($0 < c < \frac{1}{13}$) and the formulation of Li and Yorke's condition is more complicated (using the function G in the description of R^c of Proposition 6.2). But now simulations reveal that the attracting orbit with period 3 apparently has vanished and the chaotic set $\aleph$ seems to have a positive measure, which makes the illustration of the properties of the chaotic set possible. To support the proofs we need a graphical display of the compound reaction curve R^c.

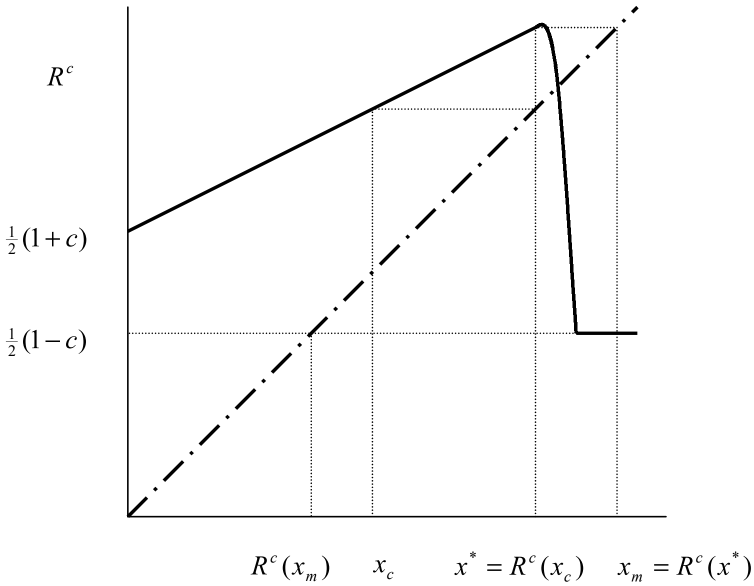


Fig. 6.6 The compound reaction curve for larger values of *a*.

Like in Chapter 2, Section 2.3, Li and Yorke's Theorem can be expressed as

$$R^c(x_m) \le x_c < x^* < x_m \qquad (6.7)$$

with $R^c(x_c) = x^*$ the location of the maximum of R^c and $R^c(x^*) = x_m$ the value of the maximum of R^c. Figure 6.6 shows a case for which these conditions are satisfied. Because we have the description of the compound reaction function at our disposal (Proposition 6.2) the values of x_c, x^*, x_m and $R^c(x_m)$ can be computed which enables us to formulate the Li and Yorke conditions for "period 3".

Proposition 6.5 (dynamical phenomena for $a > a_{bif}$ and $0 < c < 0.2$*)*

Consider benchmark case 2 for linear production costs, i.e. $d = 0, a_1 = a$ and $a_2 = 0$, with alternately reacting firms. Under the assumption of an unstable equilibrium - $a > a_{bif} = \frac{1440}{169}(1-c)^2$ - and concerning unit production costs with $0 < c < 0.2$ the following cases can be distinguished:

(i) For $\frac{1}{13} \leq c < 0.2$ the Li and Yorke conditions for the existence of a chaotic set ℵ are satisfied if $a \geq a_{chaos} = \frac{(13c+7)}{(1-5c)}$. However in this case there exists a stable orbit of period 3 which attracts almost all initial values in $[0,1]$.

(ii) For $0 < c < \frac{1}{13}$ the Li and Yorke conditions are satisfied if $G[\frac{1}{2}(1-c-\frac{(a-c)}{(a+1)})\,|\,a] \leq \frac{a(1-3c)-3-7c}{(a+1)}$ (see Appendix 5.2 for G). Typically chaotic output patterns occur and there exists a sensitive dependency on initial conditions.

Proof

From Proposition 6.2 it follows that $x_c = \frac{a(1-3c)-3-7c}{(a+1)}$, $x^* = \frac{a(1-c)-1-3c}{(a+1)}$ and $x_m = \frac{(a-c)}{(a+1)}$ with $R^c(x_c) = x^*, R^c(x^*) = x_m$. It clearly holds that $x_c < x^* < x_m$ and to satisfy the condition of Li and Yorke we have to impose the condition $R^c(x_m) \leq x_c$ on the parameters a and c. For $a \geq \frac{1}{c}$ $x_m \geq 1-c$ so then $R^c(x_m) = \frac{1}{2}(1-c)$ and $R^c(x_m) \leq x_c$ is equivalent with $a \geq \frac{(13c+7)}{(1-5c)}$. Because it holds that $\frac{(13c+7)}{(1-5c)} \geq \frac{1}{c}$ for $\frac{1}{13} \leq c < 0.2$ the Li and Yorke conditions for the existence of a chaotic set are satisfied for $a \geq a_{chaos} = \frac{(13c+7)}{(1-5c)}$, $\frac{1}{13} \leq c < 0.2$.

But then there exists a stable orbit of period 3 namely (the reactions of firm 2 in each period between brackets): $x_1 = \frac{1}{2}(1-c)$ $[x_2 = \frac{1}{4}(1-c)]$ in period 1, $x_1 = \frac{1}{4}c + \frac{3}{4}$ $[x_2 = \frac{1}{8}(1-5c)]$ in period 2. In the third period, if $a \geq a_{chaos}$, firm 1 accommodates perfectly and its reaction equals $x_1 = \frac{1}{8}(7+5c) \geq (1-c)$ for $\frac{1}{13} \leq c < 0.2$ whereupon firm 2's reaction is zero output.

This proves part (i).

For $0 < c < \frac{1}{13}$ the Li and Yorke conditions are certainly satisfied for $a \geq \frac{1}{c}$ but this condition can be refined by using the fact that for $a < \frac{1}{c}$ it holds that $x_m < 1-c$ so $R^c(x_m) = G[\frac{1}{2}(1-c-\frac{(a-c)}{(a+1)})\,|\,a]$ (see Proposition 6.2). The Li and Yorke conditions are equivalent with $G[\frac{1}{2}(1-c-\frac{(a-c)}{(a+1)}\,|\,a] \leq \frac{a(1-3c)-3-7c}{(a+1)}$.

[End of proof]

In the proof we already mentioned that, for $0 < c < \frac{1}{13}$ and $a \geq \frac{1}{c}$ the Li and Yorke conditions are also satisfied, but – as computations will show – the condition can be refined. To demonstrate the important properties of a chaotic trajectory, we choose the unit production costs equal to $c = 0.05 < \frac{1}{13}$ and using part (ii) of Proposition 6.4 we find that Li and Yorke's condition is satisfied for $a \geq a_{chaos} = 15.92$ (in two decimals). Figure 6.7 corresponds with $c = 0.05, a = 18 > a_{chaos}$ and an initial value of $x_1 = 0.81$; the supplies of both competitors are displayed. This very irregular path seems to stabilize in periods 22-25 (indicated by arrows), to an output equilibrium for firm 1 of about 0.87. However this "equilibrium" is temporary and could be called a "fake equilibrium".

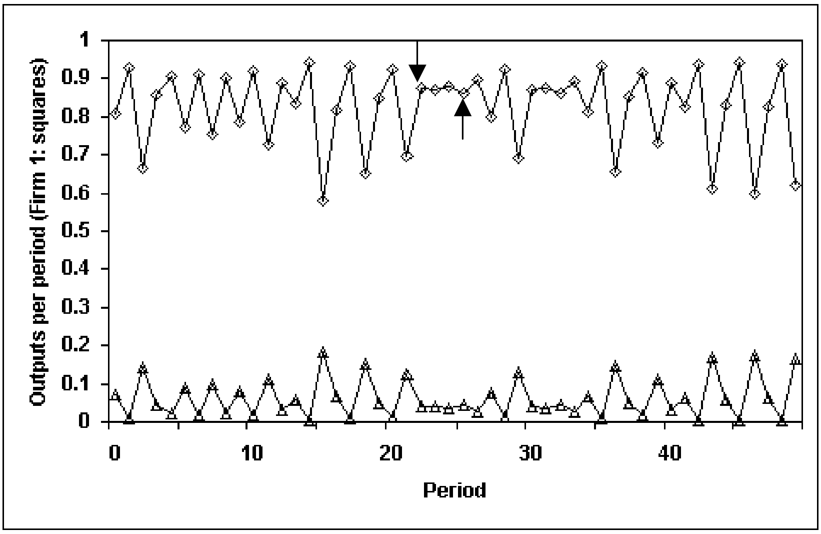


Fig. 6.7 A chaotic output path, c=0.05, a=18, initial value 0.81.

If we change this initial value slightly to 0.82, the chaotic trajectory is quite different:

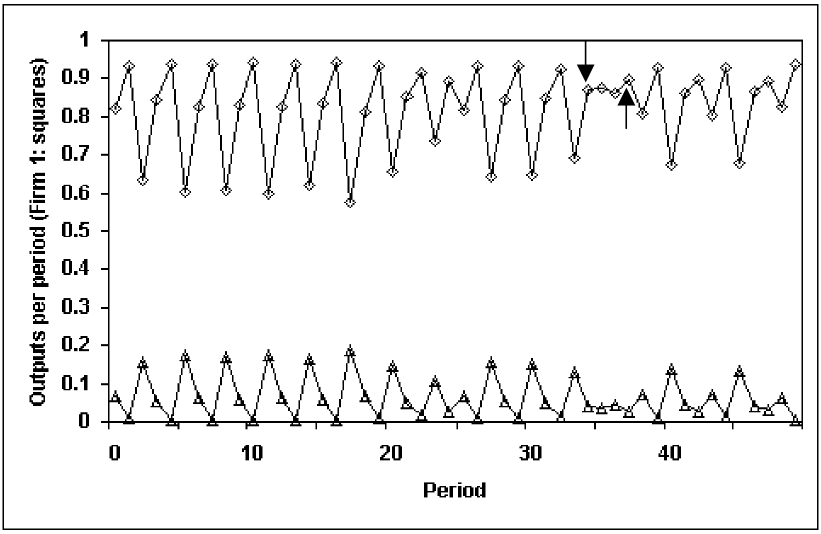


Fig. 6.8 A chaotic output path, c=0.05, a=18, initial value 0.82.

Clearly the "fake equilibrium" now occurs in periods 34-37 and in the periods 2-20 the output shows a temporary cycle wich seems to have period 3. Both the Figures 6.7 and 6.8 clearly demonstrate the first implication of the non-linear reaction curve of the "market-share loving" firm under the assumption that firm 1's preference for market share is large enough. The output pattern of both rivals mimics a random walk and history never repeats albeit that firm 1's supply always lies at a much higher level. The second property of complex (chaotic) dynamics is the extreme sensitivity of the supply path to minor changes of the initial production level, clearly illustrated by both figures. Baumol and Benhabib (1989) state that deterministic chaos poses serious problems to econometric estimation, because an output path which is extremely sensitive to initial conditions, cannot be predicted properly in the long run (like the weather). And it is also difficult to distinguish deterministic chaos from stochastic irregularity. However research focuses on new econometric techniques to test whether deterministic chaos or stochastic randomness (or a combination) underlies such an irregular output path.

Total market supply, determined by both firms' output, is also chaotic. In our example the fact that both individual firms can offer a chaotic series of output levels implies a market supply which is dictated by chaos as well. Figure 6.9, corresponding with constant unit production costs of $c = 0.05$, weight $a = 18$ and an initial total output level of $x_1 + x_2 = 0.81$ demonstrates this property.

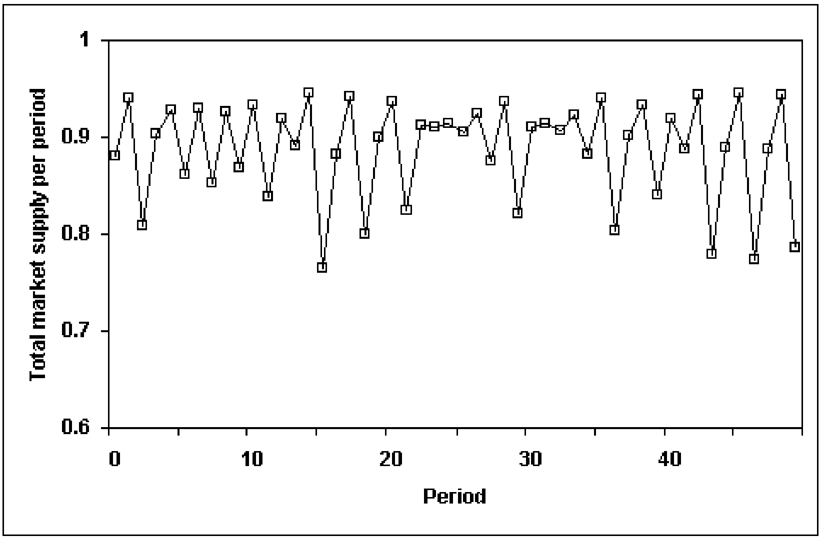


Fig. 6.9 Chaotic market supply, c=0.05, a=18, initial value 0.81.

Total market supply displays a completely irregular pattern and lies between 0.76 and 0.95, which also implies a chaotic pattern of the market price (between 0.05 and 0.24). Note that, due to the fact that the fluctuating market price always exceeds the unit production costs (positive profit margin), the average profit of both firms per period is certainly positive. Clearly the average profit per period of firm 1 is higher than the rival's average profit. Computer simulations reveal that the chaotic output path (for $c = 0.05, a = 18$) leads to positive average profits (per period) of $\overline{\Pi^1} = 0.047$ and $\overline{\Pi^2} = 0.007$ independent of the initial output of firm 1.

Note that the previous analysis and reflection on more complicated dynamics hold under the assumption $0 < c \leq 0.2$ for (constant) unit production costs. Clearly the average profits of both rivals depend on the specific character of the supply cycle, determined by its periodicity or chaotic pattern. However a cycle with a higher periodicity implies that firm 1's output level only in one output period exceeds $1-c$ (whereupon the cycle restarts) and therefore we may state that firm 1's average profit stays positive. To illustrate this statement we used computer simulations to compute the average profits of both firms in case of a period-14-cycle (Fig. 6.5), a period-6-cycle and a chaotic supply pattern (Fig. 6.9). We conclude this section with Table 6.2 containing some parameter constellations and the corresponding average profits.

Table 6.2 Average profits concerning cycles with a high period or chaotic patterns (between brackets the percentage of classical profit)

	$c = 0.15$, $a = 7.3$	$c = 0.15$, $a = 7.5$	$c = 0.05$, $a = 18$
Phenomenon	**Period 14**	**Period 6**	**Chaos**
Average profit firm 1	**0.045 (56%)**	**0.044 (55%)**	**0.047 (47%)**
Average profit firm 2	**0.013 (16%)**	**0.012 (15%)**	**0.007 (7%)**

4. Simultaneously reacting firms and processes of adaptation

The previous section deals with the examination of the dynamical behaviour of alternately reacting firms (under the assumption of myopic expectations and for a weight $a > a_{bif}$). In this section we will first examine the dynamics concerning simultaneously reacting competitors, also assuming naïve expectations (i.e. $x^e_{j,t} = x_{j,t-1}$, $j = 1,2$). As already mentioned in the previous section the condition for (local) asymptotic stability of the Cournot-Nash equilibrium under myopic expectations is the same for alternately and simultaneously reacting firms (note that this statement doesn't hold for three or more competing firms, see Theocharis (1960)). However, as we will demonstrate, simultaneously reacting firms imply output paths with a periodicity that differs from the periodicity resulting from alternately reacting competitors.

Note that the stability of the equilibrium also depends on the assumptions concerning the adjustment processes; the necessary and sufficient conditions for local stability of the equilibrium may be quite different if we deal with other models of adjustment processes. Here we suffice by mentioning briefly two of these models. A model that can be found in Dixit (1986), Varian (1992) and Dastidar (2000) deals with an adjustment of the output of each firm 'in the direction of increasing utility (or pay-off) ':

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = s_i \frac{\partial U^i}{\partial x_i},\ i = 1,2 \tag{6.8}$$

Here U^i equals the utility function of firm i and s_i is a positive parameter that determines the speed of adjustment. In fact we consider a system of first-order, nonlinear, differential equations. By linearizing this system around the equilibrium, we can derive sufficient conditions for stability. In Appendix 6.2 we provide a proof that in the general case that both competitors possess preference for their market share ($a_1 \geq 0, a_2 \geq 0$) the Cournot-Nash equilibrium is always stable.

Another model describes a more sophisticated kind of learning rule with respect to naïve expectations. Firms do not instantaneously offer their 'Best Reply' quantity (which equals their reaction function of the expected rival's supply), but build a weighted average between the previous quantity $x_{i,t-1}$ and the 'new' quantity $x_{i,t}$ (with $x_{i,t} = R^i(x^e_{j,t}),\ i = 1,2$ and $j \neq i$). Kopel (1996) notes that "this tendency is a well-known property of human decision-making behaviour and can be found in the literature on cognitive psychology under terms like 'status quo bias' or 'anchoring and adjustment' ". After examining the consequences of naïve expectations corresponding to simultaneously reacting firms – both firms offer their 'Best Replies' - we will focus on the model where firms build a weighted average between their previous output and their 'Best Reply'.

Simultaneously reacting firms with naïve expectations.

The model is (see also Section 6.2 for stability conditions)

$$\begin{cases} x_{1,t} = R^1(x_{2,t-1} \mid a) \\ x_{2,t} = R^2(x_{1,t-1} \mid 0) = \max\{\tfrac{1}{2}(1-c) - \tfrac{1}{2}x_{1,t-1}, 0\} \end{cases} \tag{6.9}$$

Proposition 6.3 reveals that, for $0.2 \le c < 1$ and alternately reacting firms, each initial output leads to a stable and cyclic supply with period 2. We now reflect on the implications of simultaneously reacting firms (and $0.2 \le c < 1$).
Let the consecutive output vectors corresponding with simultaneously reacting firms be represented by (the first index indicates the firm, whereas the second index indicates the time period)

$$\begin{bmatrix} x_{1,1} \\ x_{2,1} \end{bmatrix}, \begin{bmatrix} x_{1,2} \\ x_{2,2} \end{bmatrix}, \begin{bmatrix} x_{1,3} \\ x_{2,3} \end{bmatrix}, \ldots, \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix}, \ \ldots \tag{6.10}$$

The output paths $x_{1,1}$, $x_{2,2}$, $x_{1,3}$, ... and $x_{2,1}$, $x_{1,2}$, $x_{2,3}$, ... precisely correspond with the consecutive supplies of alternately reacting firms, so these both paths stabilize to a cycle with period 2. Let the path $x_{1,1}$, $x_{2,2}$, $x_{1,3}$, ... stabilize to $R,\ 0,\ \tfrac{1}{2}(1-c),\ \tfrac{1}{4}(1-c)$, et cetera (where we define R as $R^1(\tfrac{1}{4}(1-c) \mid a)$, see Proposition 6.3) then, starting from an arbitrary output vector, simultaneous reactions inevitably lead to the supply path

$$\begin{bmatrix} R \\ \bullet \end{bmatrix}, \searrow \begin{bmatrix} \bullet \\ 0 \end{bmatrix}, \nearrow \begin{bmatrix} \tfrac{1}{2}(1-c) \\ \bullet \end{bmatrix}, \searrow \begin{bmatrix} \bullet \\ \tfrac{1}{4}(1-c) \end{bmatrix}, \nearrow \begin{bmatrix} R \\ \bullet \end{bmatrix}, \searrow \cdots \tag{6.11}$$

Arrows indicate the consecutive reactions of alternately reacting firms. For the output path indicated by the "dots" there exist of course two possibilities: the second co-ordinate of the first output vector (the one with first co-ordinate R) can take the values 0 and $\tfrac{1}{4}(1-c)$ but these values both lead to the same supply path with a periodicity of 4 concerning simultaneously reacting competitors (time periods indicated by I, II, III and IV):

$$\underset{\text{I}}{\begin{bmatrix} R \\ \tfrac{1}{4}(1-c) \end{bmatrix}}, \quad \underset{\text{II}}{\begin{bmatrix} R \\ 0 \end{bmatrix}}, \quad \underset{\text{III}}{\begin{bmatrix} \tfrac{1}{2}(1-c) \\ 0 \end{bmatrix}}, \quad \underset{\text{IV}}{\begin{bmatrix} \tfrac{1}{2}(1-c) \\ \tfrac{1}{4}(1-c) \end{bmatrix}}, \quad \begin{bmatrix} R \\ \tfrac{1}{4}(1-c) \end{bmatrix}, \quad \ldots \tag{6.12}$$

Clearly market supply displays a cycle with a periodicity of 4 and equals $R+\tfrac{1}{4}(1-c),\ R,\ \tfrac{1}{2}(1-c)$ and $\tfrac{3}{4}(1-c)$ in the respective periods I, II, III and IV. In comparison with alternately reacting firms the period of the cycle for $0.2 \le c < 1$ has doubled. Note that, because both firms take into account the restriction $x_{i,t} + x_{j,t-1} \le 1,\ i = 1,2$ and $i \ne j$, it may occur that initially the supply in the actual period exceeds 1, which of course implies a zero market price. However after stabilization in the time period with the largest market supply (=I) the market price is still nonnegative, because even if firm 1's reaction R is perfect accommodation, the market price equals precisely zero. In the periods I and II the profit margin for both firms is nonpositive because $R \geq 1-c$ whereas in the periods III and IV the market price exceeds c and equals $\frac{1}{2}+\frac{1}{2}c$ and $\frac{1}{4}+\frac{3}{4}c$ respectively. We summarize these results in a proposition:

Proposition 6.6 (simultaneous reactions for $a > a_{bif}$ and $0.2 \leq c < 1$)

Consider benchmark case 2 for linear production costs, i.e. $d=0, a_1 = a$ and $a_2 = 0$, with simultaneously reacting firms. Under the assumption of an unstable equilibrium (naïve expectations) - $a > a_{bif} = \frac{1440}{169}(1-c)^2$ - and concerning unit production costs with $0.2 \leq c < 1$ the following holds:

There exists one stable (and globally attracting) output cycle with period 4. In four consecutive periods the supply vectors, where the first co-ordinate equals firm 1's output, are

$$\begin{bmatrix} R \\ \frac{1}{4}(1-c) \end{bmatrix}, \quad \begin{bmatrix} R \\ 0 \end{bmatrix}, \quad \begin{bmatrix} \frac{1}{2}(1-c) \\ 0 \end{bmatrix}, \quad \begin{bmatrix} \frac{1}{2}(1-c) \\ \frac{1}{4}(1-c) \end{bmatrix} \tag{6.13}$$

For $a < \dfrac{(5c+3)}{(1-c)}$ $R = G(\frac{1}{4}(1-c) \mid a)$ (Appendix 5.2) and otherwise $R = \frac{1}{4}c + \frac{3}{4}$.

Under the assumption of simultaneously reacting firms not only the periodicity of the output cycle has doubled, in comparison to alternately reacting rivals, but as we will see the average profits of both competitors over the whole cycle undergo a significant change as well. Apparently a slight change in model selection does matter. We now reflect on these average profits of the rivals and summarize some properties in a proposition.

Proposition 6.7 (profits corresponding to simultaneously reactions)

Consider benchmark case 2 for linear production costs ($d=0, a_1 = a, a_2 = 0$), with simultaneously reacting firms. For $a > a_{bif}$ and $0.2 \leq c < 1$ the stable output cycle with periodicity 4 leads to the following properties of the average profits (per period) $\overline{\Pi^1}$ and $\overline{\Pi^2}$ of both firms:

(i) For $a_{bif} < a < \dfrac{(5c+3)}{(1-c)}$ the average profits of both firms decrease with respect to an increasing weight a; for larger weights both average profits stay constant. The average profit of firm 2 is negative as soon as $a > a_{bif}$ and may lead to exit.

(ii) If $0.233 \le c < 1$ there exists a weight a_n ($a_{bif} < a_n \le \frac{(5c+3)}{(1-c)}$) with the property that for all weights $a \ge a_n$ the average profit of firm 1, $\overline{\Pi^1}$, is negative. The weight a_n can be solved from the following equation: $G(\frac{1}{4}(1-c) \mid a_n) = (\frac{7}{16} + \frac{1}{16}\sqrt{97})(1-c)$.

(iii) If $0.236 \le c < 1$ there exists a weight a_l ($a_n < a_l \le \frac{(5c+3)}{(1-c)}$) with the property that for all weights $a \ge a_l$ it holds that $\overline{\Pi^1} \le \overline{\Pi^2} < 0$. The weight a_l is determined by $G(\frac{1}{4}(1-c) \mid a_l) = (\frac{1}{2} + \frac{1}{4}\sqrt{5})(1-c)$

Proof
Using the supplies of both competitors in four consecutive periods, the average profit per period of firm 2 clearly depends on R, is decreasing with respect to R, and equals $\overline{\Pi^2} = \frac{1}{16}(1-c)^2 - \frac{1}{16}(1-c)R < 0$, because $R > (1-c)$ (with $R = R^1(\frac{1}{4}(1-c) \mid a)$). Average profit over a whole cycle of firm 1 is also a decreasing function of R and therefore depends on the weight a before perfect accommodation occurs: $\overline{\Pi^1} = \frac{1}{4}R[\frac{7}{4}(1-c) - 2R] + \frac{3}{32}(1-c)^2$. This proves part (i). For $a \ge \frac{(5c+3)}{(1-c)}$ it holds that $R = \frac{1}{4}c + \frac{3}{4}$ (perfect accommodation) and the average profit then equals $\overline{\Pi^1} = \frac{1}{64}(-3c^2 - 38c + 9)$. The necessary and sufficient condition for the existence of the weight a_n is $\overline{\Pi^1} \le 0$ which leads to $\frac{1}{6}\sqrt{1552} - \frac{19}{3} \le c < 1$.
Setting the average profits per period of firm 1 equal to zero, before perfect accommodation occurs, now leads to a condition for R : $\frac{1}{4}R[\frac{7}{4}(1-c) - 2R] + \frac{3}{32}(1-c)^2 = 0 \leftrightarrow R = (\frac{7}{16} + \frac{1}{16}\sqrt{97})(1-c)$. Part (ii) is proved by realizing that $R = G(\frac{1}{4}(1-c) \mid a_n)$. The proof of part (iii) is similar.

[End of proof]

To clarify this abstract proposition we apply this analysis to the case of two simultaneously reacting rivals with constant unit production costs $c = 0.4$. Bifurcation occurs for $a = a_{bif} = 3.067$ (in three decimals) similar to the bifurcation weight corresponding to alternately reacting firms. For a weight slightly above bifurcation level now the instability of the equilibrium leads to a stable supply cycle with a periodicity of 4. Part (i) of Proposition 6.7 indicates that, roughly speaking, the model assumption of simultaneously reactions leads to a more disadvantageous market position of both incumbent competitors. The fact that the average profit of the classical firm 2 is always negative for $a > a_{bif}$ leads one to suspect that this firm is forced to exit as soon as the preference for market share of its rival, indicated by a, exceeds a_{bif}. Right after bifurcation both competitors face an even worse catastrophe in comparison to alternately reacting firms. Using $R = G(\frac{1}{4}(1-c) \mid a_{bif}) \approx 1.028(1-c)$ leads to $\overline{\Pi^1} = 0.015(1-c)^2$, which equals only 23% of the profit just before bifurcation. However part (ii) of Proposition 6.7 reveals that firm 1 also faces losses for $a > a_n = 3.316$ (rounded to 3 decimals) and even possesses more average losses than its (classical behaving) rival for $a > a_l = 3.376$.

These two specific weights a_n and a_l lie very close to the bifurcation weight implying that the market position of the "market share loving firm" is very sensitive to the weight a if $a > a_{bif}$. Thus the assumption of simultaneously reacting competitors may even lead to exit of both rivals if the preference for market share of one incumbent firm is large enough. Table 6.3 compares the (general) results concerning alternately and simultaneously reacting firms.

Table 6.3 Comparison of alternately and simultaneously reacting firms

	a **slightly above** a_{bif} **The "catastrophe"**	a **rises,** $a > a_{bif}$	$\overline{\Pi^1} = 0$**, which** a**?**
Alternate reactions	$\overline{\Pi^1}$ **falls by 26%** $\overline{\Pi^2} > 0$	$\overline{\Pi^1}$ **decreases** $\overline{\Pi^2}$ **is constant**	a_n **corr. to** $R = 1.112(1\text{-}c)$
Simultaneous reactions	$\overline{\Pi^1}$ **falls by 77%** $\overline{\Pi^2} < 0$	$\overline{\Pi^1}$ **decreases** $\overline{\Pi^2}$ **decreases**	a_n **corr. to** $R = 1.053(1\text{-}c)$

The table illustrates that model assumptions can make a lot of difference, both qualitatively and quantitatively.

Dynamics corresponding with smaller unit production costs.

Concerning alternately reacting firms more complex dynamics result from a more efficient production technology, i.e. $0 < c < 0.2$. The supply cycles display, dependent on the weight a, all sorts of even or odd periodicities and even visible chaotic patterns (for $0 < c < \frac{1}{13}$,see Proposition 6.5). This dynamical phenomena lead one to suspect that even more complex patterns arise under the assumption of simultaneous reactions. To unravel these patterns we use an example with constant unit costs $c = 0.15$ and a weight $a = 7.5$ which leads to a stable output cycle with periodicity 6 if competitors react alternately. Note that reflections in Section 6.3 show that the case with $\frac{1}{4}c + \frac{3}{4} > x_1^*$ where x_1^* is the value of the unstable equilibrium leads to supply patterns with an even periodicity,so the following considerations hold in general for even periodicities.

Using a notation where the first index indicates the firm, whereas the second index indicates the time period and realizing that the supply path of alternately reacting competitors stabilizes (after some initial phenomena) we obtain (for $c = 0.15$ and $a = 7.5$) the following path corresponding with simultaneous reactions:

$$\begin{bmatrix} x_{1,1} \\ x_{2,1} \end{bmatrix}, \begin{bmatrix} x_{1,2} \\ x_{2,2} \end{bmatrix}, \begin{bmatrix} x_{1,3} \\ x_{2,3} \end{bmatrix}, \begin{bmatrix} x_{1,4} \\ x_{2,4} \end{bmatrix}, \begin{bmatrix} x_{1,5} \\ x_{2,5} \end{bmatrix}, \ \ldots \ , \begin{bmatrix} x_{1,11} \\ x_{2,11} \end{bmatrix}, \begin{bmatrix} x_{1,12} \\ x_{2,12} \end{bmatrix} \tag{6.14}$$

The path A: $x_{1,1},\ x_{2,2},\ x_{1,3},\ x_{2,4},\ \ldots\ , x_{2,10},\ x_{1,11},\ x_{2,12}$ corresponds with the stable supply path of consecutive reactions of both firms and so does the path
B: $x_{2,1},\ x_{1,2},\ x_{2,3},\ x_{1,4},\ \ldots\ , x_{1,10},\ x_{2,11},\ x_{1,12}$. In fact these both paths contain the same values, only shifted.

Clearly now simultaneous reactions lead to an supply cycle with a doubled periodicity of 12. Next question is: how many of these cycles (with periodicity 12) are there? We argue that there exist 3 different supply cycles.

Because both paths A and B, corresponding with alternately reacting firms, contain the same values, the monopoly output of firm 1 ($=\frac{1}{2}(1-c)$) appears twice as a first co-ordinate in each supply cycle of simultaneously reacting competitors but with different second co-ordinates. Like for the first co-ordinates there exist 6 different second co-ordinates, thus implying that there exist exactly 3 different supply cycles with a periodicity of 12. Clearly the previous arguments hold for all cases where the supply path of alternately reacting firms possesses a stable cycle with even periodicity and similar arguments can be given concerning odd periodicities. Proposition 6.8 summarizes these general results together with a statement on sensitive dependence on initial values:

Proposition 6.8 (simultaneously reactions for $a > a_{bif}$ and $0 < c < 0.2$)

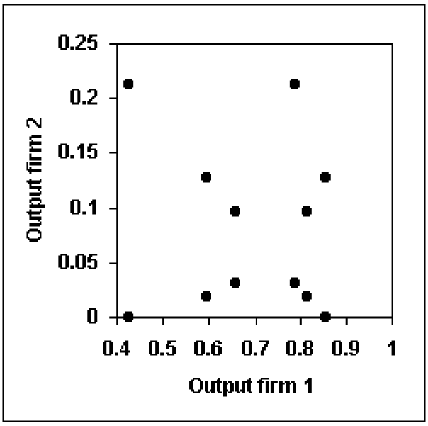
Consider benchmark case 2 for linear production costs ($d = 0, a_1 = a, a_2 = 0$), with simultaneously reacting firms. For $a > a_{bif}$ and $0 < c < 0.2$ the following properties hold:

(i) If the supply path of alternately reacting firms is cyclic with an even periodicity n, there exist $\frac{1}{2}n$ supply paths of simultaneously reacting rivals, each with periodicity $2n$.

(ii) If the supply path of alternately reacting firms is cyclic with an odd periodicity $n \geq 3$, there exist $\frac{1}{2}(n-1)$ supply paths, each with periodicity $2n$ and one supply cycle with period n corresponding with simultaneous reactions.

We note that computer experiments confirm that the cycle to which an initial output vector is attracted depends extremely sensitively on these initial supplies.

Note that we reflected on supply paths with even and odd periodicities, concerning alternately reacting competitors. For the parameter constellation $c = 0.15,\ a = 7.5$ we refer to Appendix 6.3 for the numerical presentation of the 3 cycles (indicated with I, II and III), each with periodicity 12. This numerical example illustrates that in each cycle firm 1's monopoly output (0.425) appears with two different supplies of the rival. The Figures 6.10a and b display the difference between the supply cycles I and III graphically. Note also that total output in one period may exceed 1, because both firms take into account the rival's supply in the previous period (with the corresponding nonnegativity condition for the price). Such case clearly implies a zero market price.



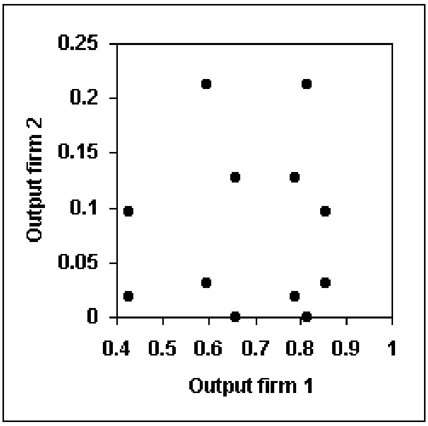


Fig. 6.10a Cycle I, c=0.15, a=7.5. **Fig. 6.10b Cycle III, c=0.15, a=7.5.**

The note of the proposition deals with the so-called basins of attraction (the set of points that is finally attracted to an equilibrium, a cycle or another specified set) of the three different (but locally stable) supply cycles. Simulation experiments reveal that selection of a cycle depends sensitively on the initial outputs of both players. This issue of the topologic structure of these 3 basins of attraction is strongly related to a recent mathematical study of equilibrium selection in a nonlinear duopoly game of Kopel and Bischi (2001), where players build a weighted average between their 'Best Replies' and their previous output. They examine a duopoly game with quadratic reaction functions, adaptive expectations and two locally stable equilibria. The topological structure of the basins of attraction of these two equilibria appears to be highly complex and also leads to an equilibrium selection that depends sensitively on initial supplies. In essence this sensitivity concerning equilibrium selection is caused by the fact that the map under consideration is noninvertible i.e. a point may possess more than one preimage. In the mathematical work of Bischi, Mammana and Gardini (2000) a general technique is developed to unravel the structure of basins of attraction. We will not apply these techniques in this chapter but we suffice with the presentation of the results of computer experiments. Table 6.4 consists of a rather fine-grained set of initial supplies of both rivals and presents the number of the (stable) cycle - indicated by I,II and III - to which these initial values are attracted in the long run.

Table 6.4 Sensitive dependence of cycle selection on initial outputs

	$x_{1,0}$ =0.35	$x_{1,0}$ =0.36	$x_{1,0}$ =0.37	$x_{1,0}$ =0.38	$x_{1,0}$ =0.39	$x_{1,0}$ =0.40
$x_{2,0}$ =0.35	II	II	I	I	III	III
$x_{2,0}$ =0.36	II	II	I	I	III	III
$x_{2,0}$ =0.37	III	III	II	II	I	I
$x_{2,0}$ =0.38	I	I	III	III	II	II
$x_{2,0}$ =0.39	III	III	II	II	I	I
$x_{2,0}$ =0.40	III	III	II	II	I	I

A glance at this table reveals that the selection of a cycle (in this example with periodicity 12) highly depends on the initial output of both rivals in the Cournot game.

Finally we briefly consider the average profits of both competitors with respect to the three different cycles I,II and III also in comparison with the profits corresponding to alternately reacting rivals. Computer simulations show that for cycle I $\overline{\Pi^1} = 0.037$ and $\overline{\Pi^2} = 0.001$, for cycle II $\overline{\Pi^1} = 0.034$ and $\overline{\Pi^2} = 0.002$ and corresponding to cycle III $\overline{\Pi^1} = 0.034$ and $\overline{\Pi^2} = 0.001$ (the average profits are rounded to 3 decimals). Clearly average profits are not significantly different for the three cycles, but they differ somewhat more in comparison to alternately reacting firms (see Table 6.2). For instance the average profit of firm 1 equals 55% and about 43% of the classical profit, concerning alternately and simultaneously reacting rivals.

Do there exist also chaotic patterns? The answer is definitely "yes". Proposition 6.5 provides the sufficient conditions for chaotic output patterns - using the Theorem of Li and Yorke (1975) - corresponding to alternately reacting firms, thus implying that chaotic patterns also exist in the case of simultaneously reacting rivals under the same conditions. For $c = 0.05, a = 18$ Figure 6.11 displays the supply pattern for simultaneously reacting firms with initial outputs $x_{1,0} = 0.35, x_{2,0} = 0.35$.

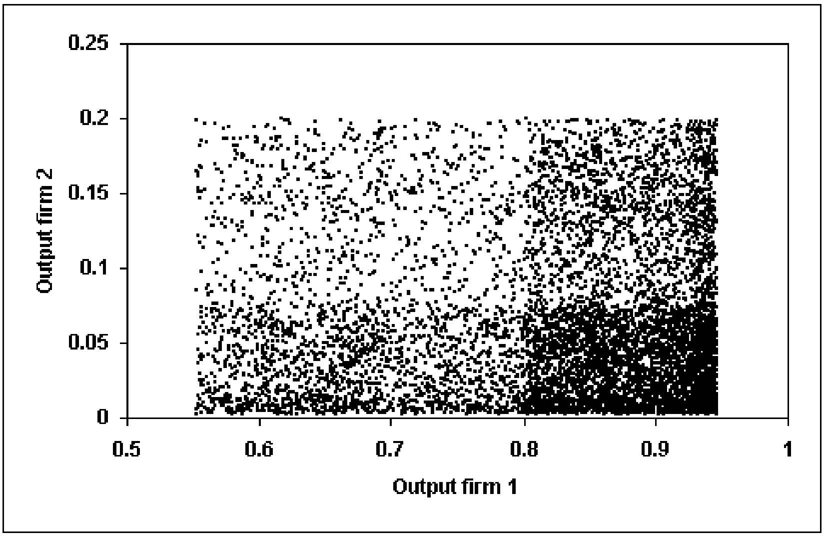


Fig. 6.11 Simultaneous reactions and chaos, c=0.05,a=18, $x_{1,0}$=$x_{2,0}$=0.35.

The repetition of the rectangle suggests a fractal structure of this "strange attractor". Before focusing on a model in which firms' adaptive reactions equal a weighted average between their previous output and their 'Best Reply' we emphasize that the main purpose of the analysis of the case of simultaneous reactions is to show that model selection may influence the quantitative and qualitative results significantly.

Dynamics corresponding to a more sophisticated learning rule.

This model takes into account that both firms build a weighted average between their previous supply and their supply corresponding to their 'Best Reply', i.e. $R^i(x^e_{j,t}),\ i = 1,2$ and $j \neq i$. Because we assume that both firms use $x^e_{j,t} = x_{j,t-1}$ and we have to take into account the (nonnegative price) conditions for both competitors, $x_{1,t} + x_{2,t-1} \le 1,\ x_{2,t} + x_{1,t-1} \le 1$, the model is specified by

$$\begin{cases} x_{1,t} = \min\{1 - x_{2,t-1}; (1-\mu)\cdot x_{1,t-1} + \mu \cdot R^1(x_{2,t-1} \mid a)\} \\ x_{2,t} = \min\{1 - x_{1,t-1}; (1-\mu)\cdot x_{2,t-1} + \mu \cdot R^2(x_{1,t-1})\} \end{cases} \tag{6.15}$$

Note that the 'Best Reply' of firm 2 equals $R^2(x_{1,t-1}) = \max\{\frac{1}{2}(1-c) - \frac{1}{2}x_{1,t-1}; 0\}$. For the so-called adjustment coefficient μ it holds that $0 < \mu \le 1$, and we assume that these coefficients are equal for both rivals. The case $\mu = 0$ corresponds with the (not very exiting) case that both competitors stick to their previous supply, whereas $\mu = 1$ corresponds with full adaptation (both outputs equal 'Best Reply'). We already examined the case of full adaptation and derived that the Cournot-Nash equilibrium is (locally) unstable for $a > a_{bif} = \frac{1440}{169}(1-c)^2$ (for constant marginal costs). If adaptability is not maximal, so $\mu < 1$, one would expect that stability of the equilibrium is maintained longer with respect to a rising weight a (the extreme case with $\mu = 0$ leads to stability for all weights a). To be more precise, the following proposition reveals that the bifurcarion weight $a_{bif,\mu}$, corresponding to a certain μ, is increasing if μ is decreasing. Clearly the equilibrium (x_1^*, x_2^*) of this model with $0 < \mu < 1$ is unique and equals the equilibrium corresponding with full adaptation (determined by $x_1^* = G(x_2^* \mid a),\ x_2^* = \frac{1}{2}(1 - c - x_1^*)$, see Appendix 5.3 for the expression).

Proposition 6.9 (bifurcation corresponding with an adaptation process).

Consider the model

$$\begin{cases} x_{1,t} = \min\{1 - x_{2,t-1}; (1-\mu)\cdot x_{1,t-1} + \mu \cdot R^1(x_{2,t-1} \mid a)\} \\ x_{2,t} = \min\{1 - x_{1,t-1}; (1-\mu)\cdot x_{2,t-1} + \mu \cdot \max\{\frac{1}{2}(1-c) - \frac{1}{2}x_{1,t-1}; 0\}, \qquad 0 < \mu \le 1 \end{cases} \tag{6.16}$$

The transition from a (locally) stable into an unstable equilibrium (x_1^*, x_2^*) occurs for

$$a_{bif,\mu} = H(\mu)\cdot(1-c)^2 = \frac{32(8-3\mu)(4-\mu)^2}{\mu(16-3\mu)^2}\cdot(1-c)^2 \tag{6.17}$$

$\dfrac{\partial a_{bif,\mu}}{\partial \mu} < 0$, i.e. if μ increases $a_{bif,\mu}$ decreases.

Proof

First we note that, in the neighbourhood of the fixed point (x_1^*, x_2^*), the model is

$$\begin{cases} x_{1,t} = (1-\mu)\cdot x_{1,t-1} + \mu \cdot G(x_{2,t-1} \mid a) \\ x_{2,t} = (1-\mu)\cdot x_{2,t-1} + \mu \cdot [\frac{1}{2}(1-c) - \frac{1}{2}x_{1,t-1}] \end{cases}$$

Linearizing this system of nonlinear first-order difference equations around the fixed point (x_1^*, x_2^*) leads to the system

$$\begin{bmatrix} x_{1,t} - x_1^* \\ x_{2,t} - x_2^* \end{bmatrix} = A \cdot \begin{bmatrix} x_{1,t-1} - x_1^* \\ x_{2,t-1} - x_2^* \end{bmatrix} \text{ with } A = \begin{bmatrix} 1-\mu & \mu \cdot \left(\dfrac{\mathrm{d}G}{\mathrm{d}x_{2,t-1}} \right)_{x_2^*} \\ -\frac{1}{2}\mu & 1-\mu \end{bmatrix}.$$

For the sake of brevity we define $S = \left(\dfrac{\mathrm{d}G}{\mathrm{d}x_{2,t-1}} \right)_{x_2^*}$. The eigenvalues of A (for $S > 0$) are $\lambda_{1,2} = (1-\mu) \pm \frac{1}{2}\sqrt{2}\mu\sqrt{S} \cdot i$ and the Cournot-Nash equilibrium is locally stable (unstable) if $|\lambda_{1,2}| < 1$ ($|\lambda_{1,2}| > 1$). If we express the value x_1^* as a fraction of $(1\text{-}c)$, i.e. $x_1^* = f(1-c)$ with $\frac{1}{3} \le f < 1$ we obtain a one-to-one relation between the slope S and the fraction f: $S = \dfrac{(5f-3)}{4(1-f)}$ (see also the proof of Proposition 6.1). The stability condition of the equilibrium leads to the following condition for the fraction f (the fraction f decreases if μ increases):

$$f < \frac{16-5\mu}{16-3\mu}$$

The proof is completed by using the one-to-one relation between the weight a and f;

$$a = \frac{(3f-1)(1+f)^2}{4(1-f)} \cdot (1-c)^2 .$$

[End of proof]

Note that for $\mu = 1$ (firms offer their 'Best Replies') $a_{bif,1} = \frac{1440}{169}(1-c)^2$ which perfectly fits in with the results of Proposition 6.1. The following graphical presentation illustrates the decreasing (increasing) behaviour of the bifurcation weight with respect to an increasing (decreasing) parameter μ for constant unit costs $c = 0.4$. Note that for $\mu = 1$ and $c = 0.4$ the bifurcation weight equals $a_{bif,1} = H(1) \cdot 0.6^2 = 3.067$ (rounded to 3 decimals), whereas for $\mu = 0.5$ this weight equals $a_{bif,0.5} = H(0.5) \cdot 0.6^2 = 8.726$.

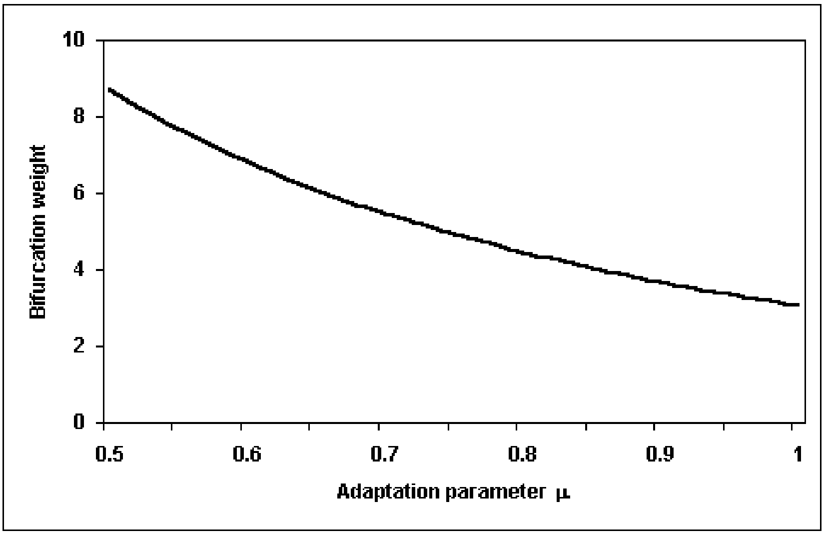


Fig. 6.12 The bifurcation weight $a_{bif,\mu}$ for c=0.4 and $0.5 \leq \mu \leq 1$.

The less firms adapt to their 'Best Replies' the longer stability of the equilibrium is maintained with respect to an increasing preference for market share of one incumbent competitor. The stability of the Cournot-Nash equilibrium clearly depends on assumptions concerning the adjustment process and this latter model - where firms adapt to their 'Best Replies' - provides, together with the model $\frac{\mathrm{d}x_i}{\mathrm{d}t} = s_i \frac{\partial U^i}{\partial x_i},\ i = 1{,}2$ mentioned in the introduction of this section, examples of this phenomenon.

We conclude this section with three examples of complicated dynamics. However we will not provide detailed explanations of the observed phenomena, corresponding with an adjustment coefficient μ smaller than 1, but leave this issue for future research. The first computer experiment corresponds with constant unit production costs $c = 0.4$ and an adjustment coefficient of $\mu = 0.9$ whereas the weight $a = 3.69$ lies slightly above the bifurcation weight $a_{bif,0.9} = 3.686$ (rounded to 3 decimals). The co-ordinates of the instable equilibrium equal $x_1^* = 0.519,\ x_2^* = 0.041$. Initial supplies of $x_{1,0} = 0.522,\ x_{2,0} = 0.040$ in this computer experiment lie very close to this equilibrium.

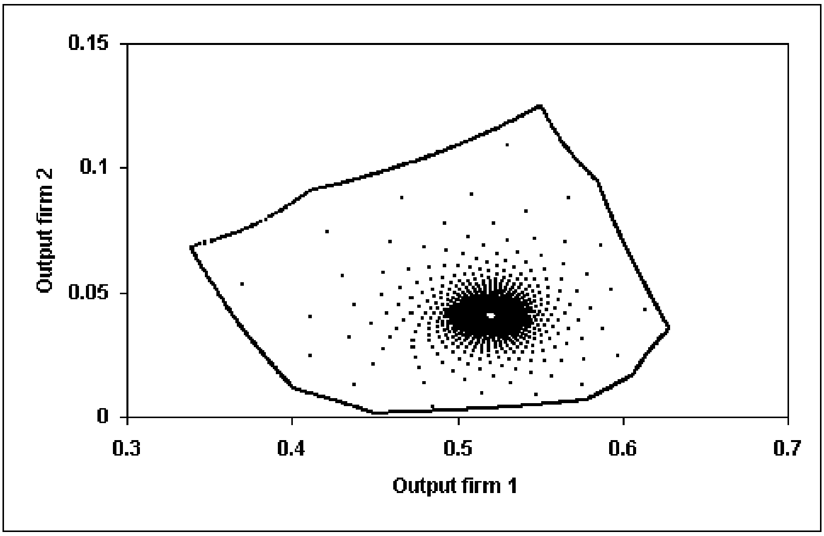


Fig. 6.13 Supply path, c=0.4 (d=0), μ=0.9, a=3.69 and $x_{1,0}$=0.522, $x_{2,0}$=0.040.

Clearly one can observe that, by repeated mappings, the initial supply vector moves in an outward spiral towards an attractor, which looks like a deformed nonagon. The both eigenvalues in this example are so-called complex conjugates and they equal $\lambda_{1,2} = 0.1 \pm 0.996i$. The lengths of these two eigenvalues equal $|\lambda_{1,2}| = 1.001$ (rounded to 3 decimals) indicating the instability of the equilibrium.

The second computer simulation corresponds also with constant unit production costs $c = 0.4$ and an adjustment coefficient of $\mu = 0.9$ whereas the weight increases till $a = 5$. The co-ordinates of the instable equilibrium equal $x_1^* = 0.535,\ x_2^* = 0.032$. Experiments show that, independent of initial supplies, the supply path of both firms is attracted by a cycle with periodicity 17. Apparently the form of the attractor depends on the value of the weight a.

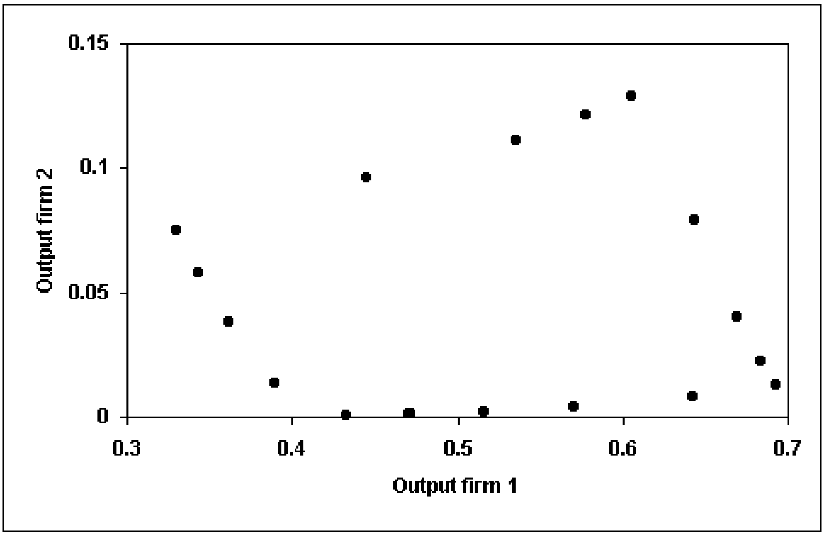


Fig. 6.14 Attractor for c=0.4 (d=0), μ=0.9, a=5.00 and arbitrary initial supplies.

The third computer simulation reveals a “strange attractor”, corresponding with constant unit production costs of $c = 0.05$ and an adjustment coefficient of $\mu = 0.85$. The weight a in this example equals 18.0 and initial supplies are $x_{1,0} = x_{2,0} = 0.25$.

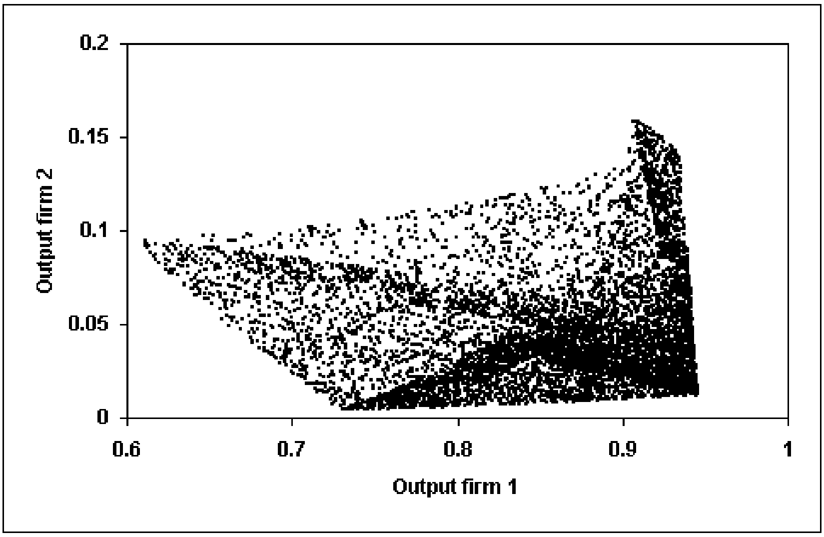


Fig. 6.15 Strange attractor, c=0.05 (d=0), μ=0.85, a=18.0 and $x_{1,0}$=$x_{2,0}$=0.250.

We note that in the general case of two competitors, both with preference for market share, periodic supply cycles and chaotic regimes also occur. However all sorts of interesting dynamical phenomena already occur concerning benchmark case 2. Upon request computer simulations are available for the general case.

5. Stackelberg equilibria

This section deals with Stackelberg leadership and is strongly related to Section 5.6. There we examined the (benchmark) case of completely asymmetrical firms concerning their preference for market share; firm 1 possesses preference for its market share, reflected by the weight $a > 0$, whereas its rival behaves as a classical profit-maximizer. The analysis of Section 5.6 for instance reveals that there exists a profit maximizing weight a_p for firm 1 and also shows that the size of the "market share loving" firm - reflected by its production level - exceeds the competitor's size (see also Table 5.5, if $a = a_p$ then $x_1^* = 2x_2^*$ and if $a = a_{bif}$ $x_1^* = 11x_2^*$) and this size ratio x_1^* / x_2^* increases with respect to an increasing preference for market share of competitor 1. The main subject of this section, Stackelberg equilibria, obviously requires a leader and a follower in the duopoly game under consideration. The leader maximizes its utility function U^1 (which in our study consists of a "profit" part and a weighted "market share" part) under the assumption that the rival's reply corresponding with an output x_1 will equal $R^2(x_1)$. In the words of Fudenberg and Tirole (1991): "Thus, if player 1 knows player 2's payoffs, the argument goes, she should not believe that player 2 would play x_2^c (the Cournot output) no matter what player 1's output. Rather, player 1 should predict that player 2 will play an optimal response to whatever x_1 player 1 actually chooses, so that player 1 should predict that whatever level x_1 she chooses, player 2 will choose the optimal response $R^2(x_1)$. This argument picks out the "Stackelberg equilibrium" as the unique credible outcome".

Note that this model of quantity leadership is in essence a two-stage game in which one firm gets to move first. But why firm 1 would be the Stackelberg leader? Varian (1992) notes that "Which firm actually is the leader would presumably depend on historical factors, e.g., which firm entered the market first, etc." We argue that the organizational "blueprint" of firm 1 reflected by its preference for market share leads to an advantageous size ratio x_1^* / x_2^* in the Cournot-Nash equilibrium and therefore this history may lead to the Stackelberg leadership of this firm.

Therefore in our opinion it makes sense to analyse the case of Stackelberg leadership. The two-stage game corresponding to Stackelberg leadership can be solved by backward reasoning: firm 1 maximizes its utility function under the assumption that the output of the rival equals $x_2 = R^2(x_1)$. We will analyse this game in case that both firms possess an equal production cost function $cx_i + d(x_i)^2,\ i = 1,2$ with our usual assumption that $0 < c < 1, d > -\frac{1}{2}$. The consideration of the general quadratic production cost function allows us to compare the outcomes with the general results of Section 5.6. First Proposition 6.10 deals with the exit of firm 2 if firm 1 is the Stackelberg leader. Note that the analysis of Section 5.6 reveals that, despite the fact that the Cournot-Nash equilibrium implies that fim 1's size and profit grossly exceed the rival's size and profit , firm 2's supply and profit stay positive corresponding with all levels of preference for market share of the largest firm. So in the case of the Cournot-Nash equilibrium firm 2 is not forced to exit by a nonpositive production level, which can be considered as a sufficient condition for exit. Because firm 2's reaction function equals $R^2(x_1) = \max\{\frac{1 - c - x_1}{2 + 2d}; 0\}$ the sufficient exit condition for firm 2 is determined by the condition that the marginal utility of firm 1 is still nonnegative for $x_1 = 1 - c$.

Proposition 6.10 (sufficient conditions for firm 2's exit, the Stackelberg output).

Under the assumption that (i) firm 1 is a Stackelberg leader and (ii) firm 1 attributes a weight a to its market share whereas firm 2 only maximizes its profit the following holds:

(i) For $-\frac{1}{2} < d \le -\frac{3}{4} + \frac{1}{4}\sqrt{5}$ firm 2 is forced to exit for all weights $a \ge 0$.

(ii) For $d > -\frac{3}{4} + \frac{1}{4}\sqrt{5}$ firm 1 forces its rival to exit if firm 1's preference for market share exceeds a certain level, i.e. $a \ge a_f = (4d^2 + 6d + 1)\cdot(1-c)^2$.

(iii) For $d > -\frac{3}{4} + \frac{1}{4}\sqrt{5}$ and $0 \le a < a_f$ there exists an unique solution (x_1^S, x_2^S) for the Stackelberg equilibrium. The output x_1^S (and x_2^S) can be solved analytically and x_1^S increases w.r.t. an increasing weight a.

Proof

Firm 1 maximizes its utility function under the assumption $x_2 = R^2(x_1)$ so it maximizes

$$U^1(x_1, R^2(x_1)) = x_1(1 - x_1 - R^2(x_1)) - cx_1 - d(x_1)^2 + a\frac{x_1}{(x_1 + R^2(x_1))}.$$

Substitution of firm 2's reaction function $R^2(x_1) = \frac{(1-c)}{(2+2d)} - \frac{1}{(2+2d)}x_1$ (for $0 \le x_1 \le 1-c$) leads to the following expression for firm 1's marginal utility:

$$\frac{\mathrm{d}U^1}{\mathrm{d}x_1} = \frac{(1+2d)}{(2+2d)}(1-c) - \frac{(2d^2+4d+1)}{(1+d)}x_1 + a\frac{(2+2d)(1-c)}{[(1+2d)x_1 + (1-c)]^2}$$

For $2d^2 + 4d + 1 \le 0 \leftrightarrow -\frac{1}{2} < d \le -1 + \frac{1}{2}\sqrt{2}$ this marginal utility is always positive, even for $x_1 = 1-c$ and clearly the exit condition for firm 2 is satisfied. For $d > -1 + \frac{1}{2}\sqrt{2}$ the marginal utility decreases monotonously with respect to the variable x_1 and is clearly positive for $x_1 = 0$. Now firm 2 is forced to exit if

$$\left(\frac{\mathrm{d}U^1}{\mathrm{d}x_1}\right)(\text{at } x_1 = 1-c) = -\frac{(4d^2+6d+1)}{(2+2d)}(1-c) + a\frac{1}{(1-c)(2+2d)} \ge 0.$$

For $4d^2 + 6d + 1 \le 0 \leftrightarrow d \le -\frac{3}{4} + \frac{1}{4}\sqrt{5}$ this latter marginal utility at $x_1 = 1-c$ is nonnegative for all weights $a \ge 0$ and again leads to exit of firm 2. This proves part (i). Part (ii) is proved by realizing that for $d > -\frac{3}{4} + \frac{1}{4}\sqrt{5}$ and for all weights a satisfying $a \ge a_f = (4d^2 + 6d + 1)\cdot(1-c)^2$ the marginal utility at $x_1 = 1-c$ is nonnegative.

For $d > -\frac{3}{4} + \frac{1}{4}\sqrt{5}, 0 \le a < a_f$ the monotonous character of the marginal utility w.r.t. x_1 and the fact that the marginal utility at $x_1 = 0$ and $x_1 = 1 - c$ is respectively positive and negative implies that there exists an unique solution x_1^S in the interval $(0, 1-c)$ of the equation $\frac{\mathrm{d}U^1}{\mathrm{d}x_1} = 0$. The output for firm 1 can be solved from this latter equation using Cardan's Method; for this derivation we refer to Appendix 6.4. By implicit differentiation of the equation $\frac{\mathrm{d}U^1}{\mathrm{d}x_1}(x_1^S) = 0$ with respect to a we obtain

$$\frac{\mathrm{d}x_1^S}{\mathrm{d}a} = \frac{(2+2d)(1-c)[(1+2d)x_1^S + 1 - c]}{a(4+4d)(1+2d)(1-c) + \dfrac{(2d^2+4d+1)}{(1+d)}[(1+2d)x_1^S + 1 - c]^3} > 0$$

[End of proof]

Part (i) of Proposition 6.10 reveals that the Stackelberg leader acquires a monopoly position independent of its level of preference for market share if $-\frac{1}{2} < d \le -0.191$. So if the production technology of the leading firm is efficient enough, reflected by the concave character of the production cost function, this firm forces the (equally efficient) competitor to exit. This result is classical, because preference for market share is not required. Part (ii) of the proposition deals with less efficient production technologies and is much more interesting. This part states that a Stackelberg leader with a certain (reasonable) level of managerial inertia, $a \ge a_f$, also expels its rival from the market. For instance, corresponding with constant unit production costs of $c = 0.4$, the required level of inertia equals $a_f = (1-c)^2 = 0.36$, which means that the leading firm attributes a weight of about $\frac{1}{3}$ to its market share.

Note that the specific weight a_f is an increasing function of the parameter d whereas this weight decreases with respect to an increasing c. We also note that from the rational adaptation perspective, which contradicts the "blueprint" of the firm reflected in managerial inertia, the owner of the leading firm may manipulate the weight a in managers' incentive contracts (compare the "delegation" games of Vickers (1985), Fershtman and Judd (1987), Sklivias (1987) and Basu (1995)). Then, by influencing managers' objective functions, the owner acquires strategic benefits (in our case a monopoly position).

Part (iii) of Proposition 6.10 deals with the unique solution (x_1^S, x_2^S) and the increasing character of the supply of the leading firm if a increases for $a < a_f$. Because it holds that $x_2^S(a) = [1 - c - x_1^S(a)]/(2+2d)$ total market supply - which is equal to $x_1^S(a) + x_2^S(a) = [(1+2d)x_1^S(a) + 1 - c]/(2+2d)$ - is also increasing if a increases (note that we also used this relation between the supplies of both competitors in Section 5.6 to obtain expressions for both profits, the difference in profits and the social welfare). Computer simulations reveal that the total "Stackelberg" supply $x_1^S + x_2^S$ exceeds the total market supply corresponding with the Cournot equilibrium ($x_1^* + x_2^*$, see Section 5.6) if it holds that $a < a_f$. However if a exceeds a_f total market supply collapses because then the supply equals firm 1's monopoly output

$x_1^m = \dfrac{(1-c)}{(2+2d)}$. The following graph displays these market supplies, concerning Cournot and Stackelberg equilibria for constant unit production costs $c = 0.4$ as a percentage of the "classical" market supply (Cournot supply for $a = 0$ which equals in general $2(1-c)/(3+2d)$). The weight a increases from $a = 0$ till $a = 0.5$.

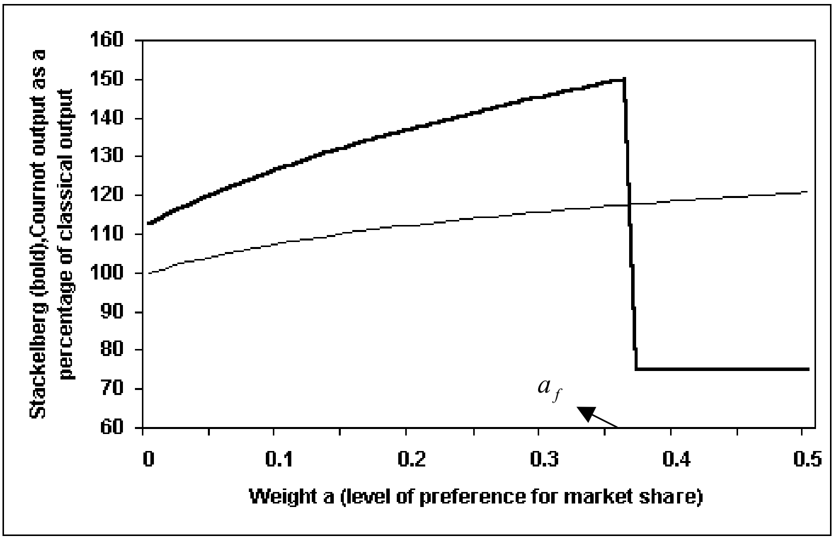


Fig. 6.16 Total "Cournot" and "Stackelberg" supply, c=0.4, d=0.

We observe that, for $a = 0$, the "Stackelberg" supply equals 112.5% of the "classical Cournot" (reference) supply which illustrates this classical property for linear costs (then "Stackelberg" and "Cournot" supply equal respectively $\frac{3}{4}(1-c)$ and $\frac{2}{3}(1-c)$). However the difference between both supplies increases if the weight a increases and for $a = a_f = 0.36$ the "Stackelberg" supply equals 150% of the reference supply. For $a \geq a_f$ exit of the follower leads to a halving of market supply which also influences social welfare strongly.

We now continue with the analysis of the "Stackelberg" profits of both firms, which is strongly related to Proposition 5.17 of Section 5.6. Because $-\frac{1}{2} < d \leq -\frac{3}{4} + \frac{1}{4}\sqrt{5}$ implies exit of firm 2 and a monopoly position of firm 1, we impose the condition $d > -\frac{3}{4} + \frac{1}{4}\sqrt{5}$ on the parameter of the production cost function.The main question here is whether there exist specific weights a_p^S and a_d^S which respectively maximize firm 1's profit and the difference between both rival's profits. Clearly a straightforward answer is that the condition $a \geq a_f = (4d^2 + 6d + 1) \cdot (1-c)^2$ leads to a monopoly position of firm 1 and naturally maximizes both firm 1's profit and the difference in profits. However for smaller levels of managerial inertia of the leading firm, i.e. $a < a_f$, both rivals are still in the market and Proposition 6.11 provides a more precise answer to the main question for this case.

Proposition 6.11 (profit and advantage of the leading firm).

Under the assumption that (i) firm 1 is a Stackelberg leader, (ii) the production cost parameter $d > -\frac{3}{4} + \frac{1}{4}\sqrt{5}$ and (iii) firm 1 attributes a weight $a < a_f$ to its market share whereas firm 2 only maximizes its profit the following holds:

(i) The profit of firm 1, Π^1, decreases if the weight a increases. Therefore this profit is maximized for the weight $a_p^S = 0$.

(ii) The difference in profits, $\Delta\Pi = \Pi^1 - \Pi^2$, is maximized if firm 1's "Stackelberg" supply equals $x_1^S = (1-c)\dfrac{(2+2d)}{(1+2d)(3+2d)}$ (see also Proposition 5.17). This corresponds with the specific weight $a_d^S = (1-c)^2 \dfrac{(5+4d)^2(4d^2+6d+1)}{(1+2d)(2+2d)^2(3+2d)^3}$

Proof

Using the equilibrium relation $x_2^S(a) = \dfrac{1}{(2+2d)}[1-c-x_1^S(a)]$ we obtain expressions for the profit of firm 1, Π^1, and the difference in profits, $\Delta\Pi = \Pi^1 - \Pi^2$:

$$\Pi^1(a) = \frac{(1+2d)}{(2+2d)} x_1^S(a) \cdot [(1-c) - \frac{(2d^2+4d+1)}{(1+2d)} x_1^S(a)]$$

$$\Delta\Pi(a) = \frac{1}{4(1+d)}[(3+2d)x_1^S(a) - (1-c)][(1-c) - (1+2d)x_1^S(a)]$$

The profit of firm 1 is maximized if the "Stackelberg" output equals $x_1^S = \dfrac{(1+2d)}{2(2d^2+4d+1)}(1-c)$. For this special supply firm 1's marginal utility equals

$$\frac{\mathrm{d}U^1}{\mathrm{d}x_1}(x_1^S) = a\frac{4(2+2d)(2d^2+4d+1)^2}{(1-c)(8d^2+12d+3)^2}.$$

For $a = 0$ this marginal utility is zero indicating that the "Stackelberg" output of firm 1 exactly equals the profit maximizing amount. However if the weight a increases, firm 1's supply also increases, exceeds the profit maximizing supply, and the expression for Π^1 reveals that the profit decreases. This proves part (i).

Part (ii) is proved by realizing that the quadratic expression for $\Delta\Pi$ is maximized if firm 1's supply equals $x_1^S = \dfrac{(2+2d)}{(1+2d)(3+2d)}(1-c)$ and subsituting this latter value in the marginal utility of firm 1 we may conclude that this marginal utility equals zero for the specific weight

$$a_d^S = (1-c)^2 \frac{(5+4d)^2(4d^2+6d+1)}{(1+2d)(2+2d)^2(3+2d)^3}.$$

[End of proof]

Figure 6.17 illustrates the "Stackelberg" profits of both firms as a percentage of the classical Cournot profit for $a = 0$ (which in general equals $(1\text{-}c)^2(1+d)/(3+2d)^2$ for both firms), corresponding with constant marginal costs $c = 0.4$ and for weights $0 \le a \le 0.5$. The leader's profit is printed in bold.

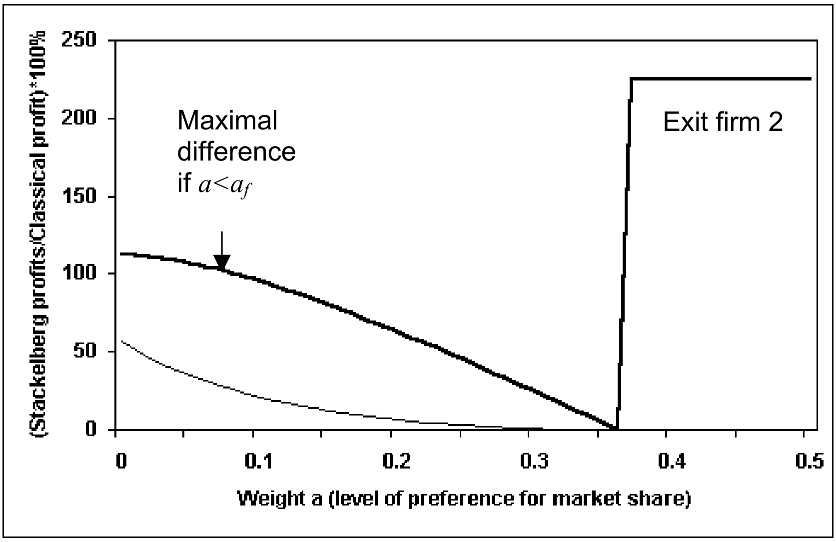


Fig. 6.17 "Stackelberg" profits $(\Pi^i(a)/\Pi_{cl})$*100%, for c=0.4, d=0.

We observe that the profit of the leader is maximal for $a_p^S = 0$ and equals 112.5% of the classical Cournot profit (which equals $\Pi_{cl} = \frac{1}{9}(1-c)^2 = 0.040$), whereas the follower's profit is only 56.25% of the classical profit. Both profits for $a = 0$ are of course well known results of "Stackelberg analysis". The difference in profits, $\Pi^1 - \Pi^2$, is maximized for $a = a_d^S = 0.083$ which is much smaller than the difference-maximizing weight $a = a_d = 0.750$ corresponding with the Cournot case (see Table 5.5, Section 5.6). Using the expressions for a_d and a_d^S this latter property can be proved in general. For $a = 0.083$ the profit of the leader equals four times the follower's profit. For $a \ge a_f = 0.36$ the follower is expelled from the market and the profit of the monopolist now rises to 225% of the (reference) classical profit. Note that for weights slightly smaller than a_f, and concerning constant marginal costs, profits of both leader and follower are nearly zero. However the leading firm dominates strongly in size.

Table 6.5 summarizes the "Stackelberg" equilibrium quantities and combines the main results of Propositions 6.10 and 6.11 for constant unit production costs. This table also makes a comparison with the Cournot equilibria possible (see Table 5.5, Section 5.6). Clearly the main differences are

- The profit maximizing weight of the leader, for $a < a_f$, equals zero and therefore is smaller than the profit maximizing weight concerning the Cournot equilibrium.

- The difference maximizing weight of the leader, for $a < a_f$, is much smaller than the difference maximizing weight concerning the Cournot equilibrium.
- For $a \geq a_f$ the follower is expelled from the market and the leader acquires a monopoly market position.

Table 6.5 The effect of *a* on "Stackelberg" equilibrium quantities.

	$a = a_p^S = 0$	$a = a_d^S = \frac{25}{108}(1-c)^2$	$a = a_f = (1-c)^2$
x_1^S	$\frac{1}{2}(1-c)$	$\frac{2}{3}(1-c)$	$1-c \to \frac{1}{2}(1-c)$, monopolist
x_2^S	$\frac{1}{4}(1-c)$	$\frac{1}{6}(1-c)$	0
Π^1	$\frac{1}{8}(1-c)^2$	$\frac{1}{9}(1-c)^2$	$\frac{1}{4}(1-c)^2$
Π^2	$\frac{1}{16}(1-c)^2$	$\frac{1}{36}(1-c)^2$	0
$\Delta\Pi$	$\frac{1}{16}(1-c)^2$	$\frac{1}{12}(1-c)^2$	$\frac{1}{4}(1-c)^2$
Π^1/Π^2	2	4	--

For $a > 0$, and with respect to an increasing weight a, firm 1's profit decreases under the assumption of Stackelberg leadership, whereas firm 1's profit would rise somewhat (till $a = a_p = \frac{9}{16}(1-c)^2$ for constant marginal costs) in the Cournot case. This leads one to suspect that the "Stackelberg" profit of the leading firm only exceeds the profit corresponding with the "Cournot" case for small weights a. For instance, if unit production costs are constant and equal $c = 0.4$, simulation experiments reveal that this condition for the level of preference for market share is $0 \leq a < 0.050$ (rounded to 3 decimals). For larger weights however (i.e. $0.050 < a < a_f = 0.360$) the "Stackelberg" profit is much lower in comparison to the "Cournot" profit. The following small table illustrates this property by expressing firm 1's "Stackelberg" profit as a percentage of the "Cournot" profit for some weights $a < a_f$.

Table 6.6 Leader's relative "Stackelberg" profits in comparison to "Cournot" profits *c*=0.4, *d*=0 .

	***a* = 0.00**	***a* = 0.05**	***a* = 0.10**	***a* = 0.15**	***a* = 0.20**
(Π^{1S}/Π^{1c})*100%	**112.5 %**	**100.1 %**	**86.5 %**	**71.8 %**	**56.1 %**

This result (which qualitatively holds in general) justifies a reflection on the question whether firm 1 prefers to be a Stackelberg leader. After all, even for small weights a, firm 1 would be better off in the "Cournot" case.

The history of a larger preference for market share may lead to firm 1's leadership, as a possible implication of its size dominance. However if firm 1 actually acquires the desired leading market position, this larger weight lowers its profit substantially. It seems unlikely that this newborn leader can lower its preference for market share in the short run, because from an Organizational Ecology perspective the weight a results from an organizational "blueprint", which is rather fixed. For instance Hannan and Freeman (1984) argue that "The level of structural inertia increases with size for each class of organization (Assumption 5)”. And it is precisely the size of firm 1 that may enhance the possibility to acquire the leading market position. However we have to realize that, if the preference for market share of the leading firm is large enough i.e. $a \geq a_f$, Proposition 6.10 reveals that the rival is expelled from the market and, then, firm 1 acquires the advantageous position of a monopolist.

We conclude this analysis with some brief notes on social welfare, which is defined as the sum of the profits of both competitors and the consumer surplus. Using the equilibrium relation $x_2^S(a) = \frac{1}{(2+2d)}[1-c-x_1^S(a)]$, like in the proof of Proposition 5.18, Section 5.6, we obtain the following expression for the social welfare:

$$W(a) = \frac{1}{8(1+d)^2}[-(8d^3+20d^2+14d+1)\{x_1^S(a)\}^2 + 2(1-c)(4d^2+6d+1)x_1^S(a) + (3+2d)(1-c)^2] \tag{6.18}$$

Using the same techniques as in the proof of Proposition 5.18 we may conclude that for $-\frac{1}{2} < d \leq 0$ welfare always rises with respect to an increasing weight a, but the “Stackelberg” case is more complicated, due to the fact that for $-\frac{1}{2} < d \leq -\frac{3}{4} + \frac{1}{4}\sqrt{5}$ and for $d > -\frac{3}{4} + \frac{1}{4}\sqrt{5},\ a \geq a_f$ the leader acquires a monopoly position (Proposition 6.10). We distinguish the following cases:

- Concave production cost function, $-\frac{1}{2} < d \leq -\frac{3}{4} + \frac{1}{4}\sqrt{5} \approx -0.191$. Then the leading firm becomes a monopolist and market supply becomes $x_1^m = \frac{(1-c)}{(2+2d)}$. Social welfare is independent of firm 1’s preference for market share and equals $W = \frac{(1-c)^2(3+2d)}{8(1+d)^2}$.
- Less concave production cost function and constant unit production costs, $-\frac{3}{4} + \frac{1}{4}\sqrt{5} < d \leq 0$. For $a < a_f$ the welfare rises with respect to an increasing weight a till the maximum level of $W(a_f) = (\frac{1}{2} - d)(1-c)^2$ is reached. For $a \geq a_f$ welfare drops till monopoly level. Note that for constant marginal costs welfare first rises, reaches a maximum for $a_f = (1-c)^2$ of $W = \frac{1}{2}(1-c)^2$ and then drops for larger weights till $W = \frac{3}{8}(1-c)^2$.
- Convex production cost functions, $d > 0$. First welfare rises till it reaches a maximum value for the weight $a_w^S = \frac{(8d^3+12d^2+4d+1)(8d^3+18d^2+11d+1)^2}{(1+d)^2(8d^3+20d^2+14d+1)^3}(1-c)^2$. Then welfare decreases somewhat with respect to an increasing weight a, whereas for $a \geq a_f$ welfare drops till monopoly level.

6. Appraisal

First this chapter deals with the occurrence of complex dynamical phenomena concerning alternately reacting competitors, where one firm attributes a weight a to its market share and its rival is a classical profit-maximizer. This benchmark case leads to various dynamical phenomena, including periodic supply cycles and completely "random walk" supply paths (Li-Yorke chaos). One may conclude that this model of behavioral Cournot competition, which includes preference for market share of one incumbent firm, can result in a diversity of dynamical phenomena. For less efficient production technologies ($0.2 \leq c < 1$) instability of the Cournot-Nash equilibrium leads to supply cycles with a periodicity of 2, thus causing an endogenous business cycle. Concerning very efficient production technologies ($0 < c < 0.2$), supply cycles with a higher periodicity may occur and even chaotic market supply is possible if the level of preference for market share of one incumbent competitor is high enough.

Second this chapter reflects on model selection and its implications for stability of the equilibrium and supply cycles in case of instability. The assumption of simultaneously reacting rivals leads to a change of the periodicity of supply cycles in comparison to alternately reacting competitors. This slight model modification results in significant differences concerning the dynamics. Furthermore the introduction of an "adaptive learning rule", i.e. the simultaneously reacting firms build a weighted average between their previous outputs and their "Best Reply", is examined. Then, the Cournot-Nash equilibrium is stabilized for a larger set of weights a attributed to the market share of one incumbent firm. Computer experiments reveal that the dynamical phenomena become even more complicated and this is one of the issues left for future research. This chapter concludes with another model modification, namely the assumption of Stackelberg leadership. An intriguing contradiction is that, if the leader's level of preference for market share is somewhat higher, this firm faces losses of profit in comparison to the Cournot case.

Of course one of the limitations of the research presented in this chapter is that we only examined dynamical phenomena and model modifications under the assumption of equally efficient production technologies of both rivals. Obviously scale advantages of the largest "market share loving" firm lead to differences in the efficiency of the production technology. This asymmetry in production costs and its implications for the dynamics provide an interesting issue for future research. The question what happens if we deal with 3 competitors remains yet unanswered. Therefore an extension to 3-player models, together with differences in efficiency, capacity, and heterogeneous products of the competitors, also has a high priority on our list of future research issues.

Appendix 6.1
Description of the compound reaction curve $R^c(x_1|\ a,0)$ for quadratic production cost functions.

For $a > \dfrac{c+(1+2d)(1+2c+2d)}{(1-c)}$ the compound reaction curve R^c ,corresponding with alternatively reactions of both firms for $a_1 = a$ and $a_2 = 0$, equals $R^c(x_1|\ a,0) =$

$$\begin{array}{lll}
\text{(i)} & 1-\dfrac{(1-c)}{(2+2d)}+\dfrac{1}{(2+2d)}x_1 & \text{for } 0 \le x_1 < (1-c)-(2+2d)\dfrac{(1+c+2d)}{(1+a+2d)} \\
\text{(ii)} & G(\dfrac{(1-c-x_1)}{(2+2d)} \mid a) & \text{for } (1-c)-(2+2d)\dfrac{(1+c+2d)}{(1+a+2d)} \le x_1 < (1-c) \\
\text{(iii)} & \dfrac{(1-c)}{(2+2d)} & \text{for } (1-c) \le x_1 \le 1
\end{array} \qquad \text{(A1)}$$

If $a \le \dfrac{c+(1+2d)(1+2c+2d)}{(1-c)}$ then the part described in (i) doesn't exist.

For the functional form of the expression G we refer to Appendix 5.2. Note that, for $a > \dfrac{c+(1+2d)(1+2c+2d)}{(1-c)}$, part (i) and (iii) of the compound reaction curve R^c are linear functions.

Appendix 6.2
General stability of the Cournot-Nash equilibrium concerning another adjustment process.

Consider the following adjustment process

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = s_i \frac{\partial U^i}{\partial x_i},\ i = 1,2 \tag{A2}$$

where U^i equals the utility function of firm i and s_i is a positive parameter that determines the speed of adjustment. Linearizing this nonlinear system of first-order differential equations around the equilibrium (x_1^*, x_2^*) leads to

$$\begin{bmatrix} \mathrm{d}x_1/\mathrm{d}t \\ \mathrm{d}x_2/\mathrm{d}t \end{bmatrix} = A \cdot \begin{bmatrix} x_1 - x_1^* \\ x_2 - x_2^* \end{bmatrix} = \begin{bmatrix} s_1 \,.\, \partial^2 U^1 / \partial (x_1)^2 & s_1 \,.\, \partial^2 U^1 / \partial x_1 \partial x_2 \\ s_2 \,.\, \partial^2 U^2 / \partial x_2 \partial x_1 & s_2 \,.\, \partial^2 U^2 / \partial (x_2)^2 \end{bmatrix}_{(x_1^*, x_2^*)} \cdot \begin{bmatrix} x_1 - x_1^* \\ x_2 - x_2^* \end{bmatrix} \tag{A3}$$

Substituting the utility (objective) functions of both firms, which equal

$$U^i(x_1, x_2) = x_i(1 - x_i - x_j) - cx_i - d(x_i)^2 + a_i \frac{x_i}{(x_i + x_j)},\ i = 1,2;\ j \neq i \tag{A4}$$

we obtain

$$A = \begin{bmatrix} s_1\{-(2+2d) - a_1 \dfrac{2x_2^*}{(x_1^* + x_2^*)^3}\} & s_1 \cdot \{-1 + a_1 \dfrac{(x_1^* - x_2^*)}{(x_1^* + x_2^*)^3}\} \\ s_2 \cdot \{-1 + a_2 \dfrac{(x_2^* - x_1^*)}{(x_1^* + x_2^*)^3}\} & s_2 \cdot \{-(2+2d) - a_2 \dfrac{2x_1^*}{(x_1^* + x_2^*)^3}\} \end{bmatrix} \tag{A5}$$

Sufficient conditions for the (local) stability of the nonlinear system of differential equations are (i) $Tr(A) < 0$ and (ii) $Det(A) > 0$. Clearly the first condition is satisfied for all $d > -\frac{1}{2},\ a_1 \geq 0,\ a_2 \geq 0,\ s_1 > 0, s_2 > 0$. The second condition can be written as

$$Det(A) = s_1 s_2 \{(3+2d)(1+2d) + a_1 \frac{(3+4d)x_2^* + x_1^*}{(x_1^* + x_2^*)^3} + a_2 \frac{(3+4d)x_1^* + x_2^*}{(x_1^* + x_2^*)^3} + a_1 a_2 \frac{4x_1^* x_2^* + (x_1^* - x_2^*)^2}{(x_1^* + x_2^*)^6}\} \tag{A6}$$

which is positive for all $d > -\frac{1}{2},\ a_1 \geq 0,\ a_2 \geq 0,\ s_1 > 0, s_2 > 0$.
Therefore this adjustment process obviously leads to a (locally) stable equilibrium.

Appendix 6.3
The 3 supply cycles with periodicity 12, for $c = 0.15,\ a = 7.5$ *and concerning simultaneously reacting rivals.*

<u>Cycle I</u> (all values rounded to three decimals):

$$\begin{bmatrix}0.425\\0.000\end{bmatrix}, \begin{bmatrix}0.425\\0.213\end{bmatrix}, \begin{bmatrix}0.788\\0.213\end{bmatrix}, \begin{bmatrix}0.788\\0.031\end{bmatrix}, \begin{bmatrix}0.657\\0.031\end{bmatrix}, \begin{bmatrix}0.657\\0.097\end{bmatrix},$$
$$\begin{bmatrix}0.814\\0.097\end{bmatrix}, \begin{bmatrix}0.814\\0.018\end{bmatrix}, \begin{bmatrix}0.596\\0.018\end{bmatrix}, \begin{bmatrix}0.596\\0.127\end{bmatrix}, \begin{bmatrix}0.855\\0.127\end{bmatrix}, \begin{bmatrix}0.855\\0.000\end{bmatrix}. \tag{A7}$$

<u>Cycle II</u>:

$$\begin{bmatrix}0.425\\0.031\end{bmatrix}, \begin{bmatrix}0.657\\0.213\end{bmatrix}, \begin{bmatrix}0.788\\0.097\end{bmatrix}, \begin{bmatrix}0.814\\0.031\end{bmatrix}, \begin{bmatrix}0.657\\0.018\end{bmatrix}, \begin{bmatrix}0.596\\0.097\end{bmatrix},$$
$$\begin{bmatrix}0.814\\0.127\end{bmatrix}, \begin{bmatrix}0.855\\0.018\end{bmatrix}, \begin{bmatrix}0.596\\0.000\end{bmatrix}, \begin{bmatrix}0.425\\0.127\end{bmatrix}, \begin{bmatrix}0.855\\0.213\end{bmatrix}, \begin{bmatrix}0.788\\0.000\end{bmatrix}. \tag{A8}$$

<u>Cycle III</u>:

$$\begin{bmatrix}0.425\\0.097\end{bmatrix}, \begin{bmatrix}0.814\\0.213\end{bmatrix}, \begin{bmatrix}0.788\\0.018\end{bmatrix}, \begin{bmatrix}0.596\\0.031\end{bmatrix}, \begin{bmatrix}0.657\\0.127\end{bmatrix}, \begin{bmatrix}0.855\\0.097\end{bmatrix},$$
$$\begin{bmatrix}0.814\\0.000\end{bmatrix}, \begin{bmatrix}0.425\\0.018\end{bmatrix}, \begin{bmatrix}0.596\\0.213\end{bmatrix}, \begin{bmatrix}0.788\\0.127\end{bmatrix}, \begin{bmatrix}0.855\\0.031\end{bmatrix}, \begin{bmatrix}0.657\\0.000\end{bmatrix}, \tag{A9}$$

Note that in some cases total output exceeds 1, because each competitor - while taking into account the nonnegativity condition of the market price - uses the previous output of its rival.

Appendix 6.4
The "Stackelberg output" x_1^S *for* $0 \leq a < (4d^2+6d+1).(1-c)^2$.

Rewriting the equation

$$\frac{\mathrm{d}U^1}{\mathrm{d}x_1}(x_1^S) = \frac{(1+2d)}{(2+2d)}(1-c) - \frac{(2d^2+4d+1)}{(1+d)}x_1^S + a\frac{(2+2d)(1-c)}{[(1+2d)x_1^S+1-c]^2} = 0 \tag{A10}$$

we obtain the equation $[(1+2d)x_1^S+1-c]^3 + \Theta[(1+2d)x_1^S+1-c]^2 + \Omega = 0$ with

$$\Theta = -\tfrac{1}{2}(1-c)\frac{(8d^2+12d+3)}{(2d^2+4d+1)} \text{ and } \Omega = -2a(1-c)\frac{(1+d)^2(1+2d)}{(2d^2+4d+1)}. \tag{A11}$$

Using the transformation $z = (1+2d)x_1^S + (1-c) + \tfrac{1}{3}\Theta$ and

$$p = -\tfrac{1}{3}\Theta^2 = -\tfrac{1}{12}(1-c)^2\frac{(8d^2+12d+3)^2}{(2d^2+4d+1)^2}, \tag{A12}$$

$$q = \tfrac{2}{27}\Theta^3 + \Omega = -\tfrac{1}{108}(1-c)^3\frac{(8d^2+12d+3)^3}{(2d^2+4d+1)^3} - 2a(1-c)\frac{(1+d)^2(1+2d)}{(2d^2+4d+1)} \tag{A13}$$

the equation in the variable z is $z^3 + pz + q = 0$. The so-called discriminant of this latter equation $D = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ is nonnegative, because it equals

$$D = a^2(1-c)^2\frac{(1+d)^4(1+2d)^2}{(2d^2+4d+1)^2} + a(1-c)^4\frac{(1+d)^2(1+2d)(8d^2+12d+3)^3}{108(2d^2+4d+1)^4} \geq 0 \tag{A14}$$

Now Cardan's Method leads to the real solution $z = \sqrt[3]{-q/2+\sqrt{D}} + \sqrt[3]{-q/2-\sqrt{D}}$ and therefore the "Stackelberg" output of firm 1 equals

$$x_1^S = -(1-c)\frac{(4d^2+12d+3)}{(1+2d)(12d^2+24d+6)} + \frac{1}{(1+2d)}\sqrt[3]{-q/2+\sqrt{D}} + \frac{1}{(1+2d)}\sqrt[3]{-q/2-\sqrt{D}} \tag{A15}$$

in which the expressions for q and D can be substituted. For linear costs ($d = 0$) we then obtain

$$x_1^S = -\tfrac{1}{2}(1-c) + \sqrt[3]{\tfrac{1}{8}(1-c)^3 + a(1-c) + (1-c)\sqrt{a^2+\tfrac{1}{4}a(1-c)^2}} + \sqrt[3]{\tfrac{1}{8}(1-c)^3 + a(1-c) - (1-c)\sqrt{a^2+\tfrac{1}{4}a(1-c)^2}} \tag{A16}$$

CHAPTER 7

SUMMARY OF THE MAIN RESULTS AND CONCLUSION

Reflection concerning two main points of view.

As already announced in the introduction of this thesis, we may view the models' analytical consequences in the light of two economical disciplines, namely OE (Organizational Ecology) and IO (Industrial Organization). From the OE perspective, roughly speaking, the inertia hypothesis of Hannan and Freeman (1984) is supported by most of the models' outcomes. Hannan and Freeman (1984) define inertia as follows: "In particular, structures of organizations have high inertia when the speed of reorganization is much lower than the rate at which environmental conditions change". From the viewpoint of OE, managers' behaviour is part of the inert structure of an organization, so for instance managerial preference for size or market share can not be changed quickly enough in response to environmental turbulence. We note that we emphasize the standpoint of OE in this thesis, because we analyse the implications of all sorts of levels of managerial preferences. This standpoint of OE contradicts the standpoint of IO which states that firms may adapt rationally to optimize their benefits and may use their production adjustment costs or weights attributed to their preferences for size or market share as strategic instruments in direct competition. The "delegation" games (there exists an owner-manager relation) are examples of such rational behaviour. If a firm's owner hires a manager he/she may influence manager's objective function for strategic reasons by writing an incentive contract. Fershtman and Judd (1987), Vickers (1985) and Sklivias (1987) show that competing firms' owners will often distort their managers' objectives away from strict profit maximization for strategic reasons. We note that these principal-agent models lead to fixed levels of managerial preferences for size (or market share) and ignores the fact that managers' levels of preferences may represent the "blueprint" of the firm and may differ largely between firms. Nevertheless these principal-agent models - with incentive contracts including market share - will be part of our research in the near future.

In this conclusion we will discuss the main results of each chapter, emphasizing "natural language" instead of formulas and expressions. Besides the OE and IO perspective the consequences for social welfare will be part of our reflections as well. Because Chapters 2 and 6 reveal that even rational decision rules of competing firms may result in turbulent movements of both firms' supplies, we will also pay attention to complicated dynamical phenomena as a result of human behaviour.

Chapter 3.

In this model a form of managerial inertia is introduced, i.e. firms maximize a combination of profits and size (production volume, sales), whereas both competitors use naïve expectations concerning the rival's supply level. The level of preference for size is determined by a parameter α_i for firm i. From the viewpoint of Organizational Ecology, the level of managers' preferences for size is part of the "blueprint" of the firm and we analyse the consequences of all sorts of weight combinations (α_i, α_j) of two rivals in Cournot competition. Concerning Chapter 3's model, the concept of habit formation - habit is determined by the actual size and (geometrically decreasing) weighted previous size - is included. In the examination, concerning multiple equilibria, the assumption is made that firms may accumulate losses before they decide to exit. This assumption is supported by the literature on Accounting and Finance, where results of empirical studies indicate that bankrupt firms are associated with financial ratios that started to deteriorate several years before the year of bankruptcy.

Eigenvalue analysis reveals that habit formation has no influence on both equilibrium's stability and the level of the stationary-state equilibrium supply (Propositions 3.1, 3.5). However, the level of habit in equilibrium does depend on the "depreciation" factor gamma in the model. Concerning equilibrium supplies and profits, in Chapter 3's model, habit formation could also be replaced by size only (albeit that this "habit formation" concept may be interesting in future research). Furthermore we note that including growth of size (instead of size) would leave the mathematics unaffected. Concerning equally efficient competitors, with respect to their production technology, we mention some main results:

- If firm i's preference for size exceeds the rival's preference level ($\alpha_i > \alpha_j$), firm i dominates in sales volume, whether this firm makes losses or not (see Figures 3.1-3.4). A size-motivated firm may survive at the detriment of the (smaller) profit-motivated rival.
- If firm i's preference for size is large enough ($\alpha_i >> \alpha_j$), the rival (j) may be expelled from the market (see for instance the areas B^1,B^2 in Figures 3.2 and 3.3).

These model outcomes indicate that a firm's survival chance, subject to demand turbulence, is enhanced by managerial inertia and therefore support Hannan and Freeman's inertia hypothesis (1984): "Selection within populations of organizations in modern societies favors organizations whose structures have high inertia".
From the (strategical) standpoint of IO we note that there exists a profit-maximizing level of a firm's preference for size (Proposition 3.4) implying that a competitor may use its preference for size as a strategic weapon. So if a firm is able to adapt its level of preference for size, which contradicts the assumption of managerial inertia, this may consolidate its strategic market position and the rival even may be forced to exit (note that a firm's strategic target may be a monopoly position instead of profit-maximizing; this latter target is the usual target in the "delegation" games). In Chapter 3 also the case of asymmetric production costs is examined. A detailed summary of possible equilibrium outcomes is presented in Table 3.1, supported by Figures 3.10-3.12. One of the most intriguing results is that

- (Van Witteloostuijn, Boone and van Lier (2003)) "A cost-efficient (i.e., low-cost) and managerially flexible (i.e., profit-maximizing) firm may well be outcompeted by a cost-inefficient (i.e., high-cost) and managerially inert (nonprofit-maximizing) rival. In the extreme, the latter may even survive at the expense of the former".

Chapter 4.

This model deals with organizational inertia by modeling (linear) asymmetric adjustment costs around a fixed production level, concerning a production decrease and increase as well (rivals also use naïve expectations in determining their actual supply levels). One part of Chapter 4's analysis focuses on the effects of a (partial) business cycle on supply levels and profits of both competitors.

Due to both firms' adjustment costs, three behavioral phases can be distinguished concerning a declining demand. During the first ("complete inertia") phase, both rivals maintain their original supply levels, whereas during the second ("inertia outcompetes flexibility") phase the firm with the largest adjustment costs maintains its output level and its (less inert) competitor reduces its supply. Similar phases can be distinguished in a booming market. In the declining market the flexible rival takes all the burden of environmental decline, by a reduction of output, whereas the inert competitor avoids (during the second behavioral phase) the profit reducing effect of downsizing. Therefore phase 2 is very advantageous for the most inert rival. One of the main results deals with exit and survival conditions of the less inert firm in a declining market:

- If the difference between both competitors' adjustment costs is large enough, and demand declines enough, the more flexible firm faces losses during phase 2, whereas the inert rival still is profitable (Proposition 4.5).
- If the flexible firm survives phase 2, it has fallen behind in profits. However during the next phase 3, this flexible competitor may also face losses (the "exit region" is provided in Proposition 4.7).

If we compare both rivals' total profits during a whole period of decreased economic activity (using integral calculus), Chapter 4's analysis reveals that the most inert rival has relative advantages, even if the flexible rival survives phases 2 and 3. Clearly these outcomes support the inertia hypothesis; selection in a Darwinian selection process favors firms with the highest level of (organizational) inertia. However in a booming market the opposite holds true: flexibility pays off, contradicting the inertia hypothesis. From a strategic point of view (IO perspective), a firm may manipulate its adjustment costs - increasing them in a declining market and decreasing them in a booming market - to outcompete the rival or even force the rival to exit. There exists an optimal level of organizational inertia; Proposition 4.12 deals with the minimization of the rival's relative total profit due to a period of recession. If both competitors adapt strategically, this may result in a two-stage sequential game (in the first stage firms choose their adjustment costs, whereas in the second stage the Cournot game is played). Such game would eventually lead to a higher level of inertia during a period of recession, whereas a period of increased economic activity would be characterized by both rivals' flexibility (we note that rational adaptation of adjustment costs contradicts the OE viewpoint of organizational inertia).

What about social welfare? Social welfare benefits the most from high levels of inertia concerning a declining market. Due to adjustment costs, firms uphold their supply levels for a longer period in a declining market and clearly consumer surplus and social welfare benefit from this phenomenon. In the booming market, however, welfare and consumer surplus both benefit from (total) flexibility. Strategical adaptation of both competitors would be most beneficial for welfare, because such an adjustment cost-setting game would result in high inertia and high flexibility in a respectively declining and booming market (Proposition 4.17 deals with symmetric rivals, concerning adjustment costs). Furthermore, the influence of country's laws and trade union negotiations on human resource management practices may be significant. If country's laws or agreements with trade unions result in beneficial labor contracts for employees, naturally resulting in higher adjustment costs in a declining market, our model predicts that this advantageous situation for employees also stimulates social welfare.

Chapter 5.

In this chapter another form of managerial inertia is modeled; firms maximize a combination of profits and market share (using also naïve expectations). The level of preference for market share is tuned by a parameter a_i for firm i. Clearly this chapter's model is strongly related to Chapter 3's model. Again we study the implications of all sorts of combinations of managerial levels of preference for market share between the two competitors. Firms control equally efficient production technologies (like in Chapter 4). First, preference for market share has important behavioral implications, indicated by properties of firm's reaction curve.

- If the level of preference for market share is large enough, the reaction curve is hill-shaped and firm's behaviour can be characterized as "dualistic". If the rival's supply is below a certain level, an increase of the rival's output level leads to an aggressive response of the firm i.e., an increase of its supply level. However, if the rival's output exceeds a certain level and increases, firm's response is output reduction (even perfect accommodation). Detailed typology of the reaction curve is provided by Proposition 5.11.

Preference for market share provides a microeconomic foundation for hill-shaped reaction curves and implies the possibility of complex dynamics, which is the main subject of Chapter 6. In Chapter 5 two benchmark cases, concerning the weights a_i attributed to market share, are examined thoroughly. Benchmark case 1 deals with complete symmetry of both competitors ($a_1=a_2$) and has important implications for both rivals' profit levels. Apparently, the positive utility from market share (growth) increases management's willingness to expand, even if this implies that profit is sacrificed:

- The equilibrium profit of two firms with equal levels of preference for market share falls if this level increases, till profit is completely sacrificed. If this managerial inertia leads to exit of one or both competitors, social welfare also collapses (details can be found in Proposition 5.15).

This (qualitative) conclusion also holds if managerial inertia is determined by preference for size, although there exist differences (Tables 5.1 and 5.2) related to the efficiency of firms' production technologies. The following statement summarizes the main results and differences.

- Concerning an efficient production technology, total sacrifice of both rivals' profits occurs at a lower level of preference for size (weighted by the parameter α) in comparison to the level of preference for market share (parameter a). The opposite holds true for an inefficient production technology.

Furthermore, for both forms of managerial inertia, it holds that the larger production unit costs are, the lower the level of preference is corresponding with a complete sacrifice of profits. For social welfare a certain level of managerial inertia is beneficial. However, too high levels of inertia may lead to nonpositive profits of both incumbent competitors, implying exit and a collapse of welfare.

- Consider two competitors with equal levels of preference for market share. If this level rises, welfare rises too till it reaches its maximum. For higher levels, however, welfare falls. Concerning constant marginal production costs the maximum welfare is reached corresponding with zero profits of both rivals (details in Proposition 5.16).

Benchmark case 2 deals with total asymmetry, concerning preference for market share ($a_1>0$, $a_2=0$). Firm 1 prefers its market share, whereas its rival behaves classical and maximizes its profit. From the viewpoint of OE these asymmetric levels of preferences reflect the differences between the cultures ("blueprints") of both rivals. Concerning the principal-agent models (rational adaptation perspective), the strict profit-maximizing behaviour of firm 2 corresponds with an owner who hires no manager, whereas firm 1's owner writes an incentive contract for its manager. One of the results of the analysis is that both firms stay in the market, whatever firm 1's level of preference for market share may be, albeit that the "market share loving" firm's size and profits amply exceed its rival's size and profits. Two results are worth mentioning here (details can be found in Propositions 5.17 and 5.18):

- Concerning a (rather) concave production cost function ($-1/2 < d < -0.191$), both the profit and the advantage of the inert competitor over its rival keep increasing with a further increasing level of preference for market share. For constant marginal production costs there exists a level of managerial inertia which maximizes the inert firm's profit.
- Concerning linear production costs, social welfare keeps rising with respect to an increasing level of preference for market share of one incumbent competitor.

From the OE perspective, selection favors firms with certain (higher) levels of inertia (organizational "blueprints") and from the IO standpoint a firm may use its preference for market share as a strategic instrument in direct competition (then production levels and profits of both competitors would be equal to the Stackelberg outcome, see Basu (1995)) . These results are qualitatively similar to the conclusions of Chapters 3 and 4. Interesting is also that, concerning the completely asymmetrical benchmark case and constant marginal production costs, social welfare doesn't collapse if the level of preference increases further, but keeps rising. This implies that the presence of one "market share loving" competitor, whereas the rival's behaviour is pure profit-maximizing, is very beneficial for social welfare. We leave the intriguing question, whether this property also holds for more firms, for future research.

We conclude the reflection on Chapter 5's results with a note on the power-series approach, used in Section 5.3. This section deals with the derivation of rather complicated expressions for firms' outputs, profits and social welfare, for general non-profit parts of the objective functions. This abstract section reveals that, for small levels of preference and certain plausible properties of the non-profit part s, the welfare function is rising and concave with respect to increasing levels of preference. Therefore qualitative results, concerning managerial inertia, are not limited to preference for size or market share, but may be generalized for other (mixed) forms of managerial inertia as well.

Chapters 2 and 6.

Both chapters are strongly related, because the examinations focus on dynamical phenomena. Chapter 2 describes the implications of hill-shaped reaction curves with "tunable steepness" and a (plausible) positive monopoly output. A microeconomic foundation for such a reaction curve is provided in Chapter 5; a certain level of preference for market share leads to the desired (tunable) hill-shape and a positive monopoly output and overcomes the shortcomings of the models of Puu (1991, 1998) and Kopel (1996). In Chapter 2 computer experiments are used to illustrate three properties of chaotic trajectories, namely (i) completely irregular and aperiodic time paths, (ii) sensitive dependencies on initial values and parameter values and (iii) qualitative breaks in the pattern. In Chapter 6, concerning benchmark case 2, the existence of chaotic regimes is proved by using the Theorem of Li and Yorke (1975). The analysis reveals that these chaotic regimes are associated with low marginal production costs.

- If one incumbent firm possesses a high level of preference for market share ($a_1 > a_{chaos}$), whereas its rival only maximizes its profit, and both rivals' unit production costs are low ($0 < c < 0.2$ on a scale from 0 to 1) the Cournot game may result in (visible) chaotic supply paths (see Proposition 6.5).

The simple and plausible assumption of a firm's preference for market share in a direct competitive setting, leads to a fully deterministic decision rule, concerning both firms' supplies. However such a deterministic decision rule may even lead to a totally irregular market output of both rivals. Computer simulations, like in Chapter 2, illustrate the properties of a chaotic trajectory. Of course future research may reveal the influence of the expectation formation on equilibrium's stability and the occurrence of chaos. In the case of chaos there exist significant differences between firms' (naïve) expectations and the actual outcomes. Therefore it is also worthwhile to reflect on patterns in firms' prediction errors (Hommes (1998)). If there exists no pattern in the prediction errors, indicated by zero autocorrelation coëfficients at al lags, firms have no reason to revise their beliefs.

Higher marginal production costs may lead to an output cycle with period 2.

- Consider benchmark case 2 in case of an unstable equilibrium ($a_1 > a_{bif}$, $a_2 = 0$) and somewhat higher constant marginal production costs ($0.2 \leq c < 1$). Then, there exists a stable (and global attracting) supply cycle with period 2 (Proposition 6.3).

An first implication of this phenomenon is that firms' consecutive reactions may cause an endogenous business cycle. Yet another consequence is that the "market share loving" firm may face a catastrophic decrease of its profits, if its level of preference for market share is heightened somewhat and the stability of the Cournot-Nash equilibrium is disturbed (Proposition 6.4). Besides the richness of dynamical phenomena, Chapter 6 contains a section on model reflection. It appears that simultaneously reacting rivals or firms' production adaptation rules influence the model's outcomes significantly. Some computer experiments reveal complex dynamical supply paths concerning such adaptation rules and these phenomena deserve a future explanation. Finally Chapter 6's analysis focuses on the Stackelberg leadership of the "market share loving" firm. One interesting outcome is that this firm may force its competitor to exit, if its preference for market share exceeds a certain level (for details we refer to Proposition 6.10).

- Consider constant marginal production costs. Under the assumption that (i) firm 1 is a Stackelberg leader and (ii) firm 1 attributes a weight a to its market share, whereas firm 2 is a (classical) profit-maximizer, firm 1 forces its rival to exit if $a > a_f = (1-c)^2$.

Future research.

The analysis of this thesis' models reveals that structural inertia may enhance the survival chances of incumbent firms (OE), and that organizational inertia (adjustment costs) or forms of managerial inertia may be used as a strategic weapon in competition (IO). We therefore may state that the game-theoretic models in this thesis contribute to the insights of both Organizational Ecology and Industrial Organization. Game theoretical models may serve as a (mathematical) bridge between IO and OE, because these models allow us to build in environmental turbulence (for example a business cycle), cost- and product- heterogeneities between firms, and forms of structural inertia as well. This latter statement also clarifies the direction of future research, because we realize that the thesis' mathematical models only provide the first stepping-stones in modeling a more complex market structure. And if we just glance at the schematic research presentation in the introduction, we observe many topics that "cry for attention" (indicated by the minus signs). To fill up the holes in this "Emmenthaler cheese" may serve as an adequate description of our future research aim.

In OE research the dual market structure with (larger) generalists and specialists receives much attention. Step by step modeling may allow us to describe this market structure by game-theoretical models, where mathematical analysis is supported by simulation experiments. For instance specialists may enhance their survival chances by product differentiation. In their empirical study of California wineries Swaminathan and Delacroix (1991) show that "Organizations within a population may escape competitive pressures through differentiation". Future research may focuss on

Game-theoretic modeling:

- Product heterogeneity and production cost differences between incumbent firms. Does the outcome support empirical findings?

- Differences in capacity between generalists and specialists.

- The implications of demand turbulence, concerning firms with a certain level of preference for market share. Does preference for market share enhance a firm's survival chance in case of a business cycle?

- Stability issues if more firms are included in a competitive setting. We suspect that structural inertia may have a stabilizing influence if 3 or more rivals are involved.

- Principal-agent models where owners influence the behaviour of their managers (by manipulating the level of preference for market share) for strategic reasons. An owner's (principal's) target may be profit-maximization, maximization of the difference in profits or acquiring a monopoly position.

- Entry deterrence by managerial behaviour, such as preference for market share.

Van Witteloostuijn, Boone and van Lier (2003) argue that “However we believe that game-theoretic work will produce many interesting insights that can then be put to the test in empirical studies”. Propositions following from (mathematical) models can be translated into testable hypothesis. Therefore other future research extensions are

Empirical research:

- The relation between failure rates in a population and levels of managerial inertia.

- Relations between management compensation schemes and manager’s habit formation, concerning preference for size or market share.

In short, there is a good deal of work to do.

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SAMENVATTING

Het (speltheoretisch) model van Augustin Cournot heeft inmiddels de respectabele leeftijd bereikt van 165 jaar en men zou verwachten dat alle varianten en gevolgen van dit model uitputtend zijn bestudeerd. Beknopt geformuleerd beschrijft het klassieke Cournot-model een markt waarin rivaliserende bedrijven ieder hun productiehoeveelheid manipuleren; ieder bedrijf doet dit met het oogmerk van winstmaximalisatie en houdt daarbij rekening met de (verwachte) hoeveelheden van het product die door de rivalen op de markt gebracht (zullen) worden. Deze strategische acties van de bedrijven kunnen leiden tot een evenwicht wat betreft het totale marktaanbod en de marktprijs. Echter: het Cournot-model (en natuurlijk het Bertrand-model, waarbij rivalen juist de prijs van hun heterogene producten manipuleren uit strategisch oogpunt) geniet nog altijd grote belangstelling in het wetenschappelijk onderzoek. Allerlei varianten worden bestudeerd en onderwerpen die in het brandpunt van deze belangstelling staan zijn de existentie van evenwichten, de stabiliteit van evenwichten, de consequenties van twee-stapsspelen (zogenaamde delegatie spelen), leer- en aanpassingsprocessen bij de spelers (bedrijven) en spelen met onvolledige informatie. Vanzelfsprekend hebben ook de verdere ontwikkeling en verfijning van de speltheorie in de laatste decennia bijgedragen tot deze voortdurende belangstelling.

De titel van dit proefschrift, “Gedragsmatige Cournot-concurrentie”, reflecteert het feit dat dit werk gewijd is aan de implicaties van het gedrag van managers in Cournot-concurrentiespellen. Naast de gebruikelijke voorkeur van managers voor de winst van het bedrijf (het klassieke model) kan ook de (productie)grootte of het marktaandeel van de onderneming deel uitmaken van de doelen van managers. Empirisch onderzoek laat duidelijk zien dat managers een voorkeur vertonen voor de vergroting van de onderneming, die vaak direct is weerspiegeld in de omvang van hun eigen departementen. Ook marktaandeelgroei scoort hoog op de lijst van voorkeuren van managers. Deze motieven van managers kunnen van psychologische aard zijn, maar ze worden vanzelfsprekend ook ingegeven door de salarisopbouw; als marktaandeel of productiegrootte zijn opgenomen in het salariscontract van managers (naast winst) en, dus deels het salaris of de bonus bepalen, zullen managers hier ook naar handelen. Naast deze voorkeuren van het management worden in dit proefschrift ook de consequenties van aanpassingskosten van een bedrijf geanalyseerd, die optreden als het productieniveau wijzigt in een krimpende of aantrekkende markt. Dergelijke aanpassingskosten worden (op korte termijn) onder andere beïnvloed door de aard van het personeelsbeleid en de hoogte van salarissen, alsmede eventuele uitkeringen of gouden handdrukken (naast de lange-termijnkosten als gevolg van investering of de-investering). Wanneer bijvoorbeeld vaste aanstellingen voor het personeel behoren tot de bedrijfscultuur, zullen de (aanpassings)kosten bij het inkrimpen van de markt in een periode van economische recessie hoger uitpakken dan bij een personeelsbeleid dat werkt met tijdelijke contracten (uitzendbureaus). Zoals we bij de bespreking van de hoofdresultaten van de hoofdstukken zullen zien, kan het gedrag van het management ingrijpende gevolgen hebben voor de winstgevendheid en de marktpositie van de onderneming enerzijds en de sociale welvaart in ruimere zin anderzijds. De overlevingskansen van een bedrijf kunnen cruciaal afhangen van dit gedrag. Tevens kan, bij wat extremere voorkeuren van de managers, zelfs in een duopolie-marktstructuur de stabiliteit van het markevenwicht verstoord worden, hetgeen kan leiden tot een zogenaamd chaotisch marktaanbod.

Alvorens de belangrijkste resultaten per hoofdstuk op een rij te zetten, merken we hier reeds op dat de analytische consequenties van de gebruikte modellen gezien kunnen worden in het licht van twee subdisciplines, namelijk het perspectief van de Organisatieecologie (OE) uit de sociologie en dat van de Industriële Organisatie (IO) uit de economie. OE bestudeert, kort geformuleerd, populaties van organisaties en concenteert zich op zogenaamde diffuse of indirecte concurrentie. Vanuit dit standpunt concurreren bedrijven (in een populatie) om dezelfde en gelimiteerde bronnen van bestaan, zoals de marktvraag naar hun producten. Bepaalde kenmerken van organisaties in een populatie zijn van invloed op hun overlevingskansen in de context van een Darwiniaans selectieproces. In OE staat het concept inertie centraal; bedrijven creëren standaardroutines en -procedures (die in een moderne maatschappij de betrouwbaarheid van het bedrijf naar buiten toe bevestigen) en de verandering van deze “blauwdruk” stuit op weerstand. Omdat de snelheid waarmee deze “blauwdruk” gewijzigd kan worden vaak lager is dan de snelheid waarmee de omgeving (bijvoorbeeld de marktvraag) verandert, wordt verondersteld dat organisaties relatief inert zijn. Ook in dit proefschrift ligt de nadruk op de inertie van de voorkeuren van managers - dit kan voorkeur voor grootte of marktaandeel zijn. Tevens wordt het begrip “inertheid van de organisatie” gehanteerd betreffende de aanpassingskosten. Vanuit het standpunt van OE kan een organisatie dus niet zomaar de voorkeurspatronen van zijn managers wijzigen of de aanpassingskosten veranderen op korte termijn. Het zijn de bedrijven met de gunstigste eigenschappen (bijvoorbeeld voorkeuren voor omvang of winstgevendheid) die overblijven of gedijen in een bepaalde omgeving als gevolg van een selectieproces. Omdat de nadruk in dit proefschrift ligt op het gezichtspunt van de organisatieecologie, worden allerlei combinaties van de voorkeuren van managers of aanpassingskosten van (twee) concurrerende bedrijven bestudeerd. Vandaar dat ook een extreme voorkeur voor (productie)grootte of marktaandeel onderwerp van analyse kan zijn.

De economische subdiscipline van de industriële organisatie houdt zich bezig met het modelleren van verscheidene vormen van concurrentie, waarbij een breed spectrum van perfecte (diffuse) concurrentie tot oligopolistische (direkte) concurrentie aan bod komt. In tegenstelling tot de inertie die centraal staat in OE, kunnen bedrijven vanuit het gezichtspunt van IO flexibel allerlei strategische acties ondernemen om bijvoorbeeld toetreding van rivalen te belemmeren of een concurrent uit de markt te drijven. Vanuit dit standpunt kunnen (de eigenaren van) ondernemingen de voorkeuren van hun managers of de aanpassingskosten manipuleren, teneinde een gunstige strategische marktpositie te bewerkstelligen. Deze strategische manipulatie van de voorkeuren van managers treffen we bijvoorbeeld aan in de zogenaamde (twee-staps)delegatiespelen, waarbij een eigenaar een manager inhuurt en de doelfunctie van die manager bepaalt met behulp van een “incentive contract”. Op verscheidene plaatsen in dit proefschrift worden ook mogelijke strategische manipulaties belicht onder de voorwaarden dat (i) de onderneming (eigenaar) zich bewust is van de strategische implicaties van een actie, (ii) adequate informatie heeft over de relevante parameters (niveau van voorkeur voor grootte of marktaandeel of grootte van de aanpassingskosten) van de concurrent en (iii) daadwerkelijk in staat is de relevante parameters ten gunste van het bedrijf te wijzigen.

Hoofdstuk 2 neemt (naast hoofdstuk 6) een bijzondere plaats in en is sterk gerelateerd aan hoofdstuk 6 omdat in beide hoofdstukken dynamische verschijnselen worden bestudeerd, die optreden als gevolg van een “unimodale” reactiecurve. In hoofdstuk 2 worden de implicaties van een “beste-antwoord”curve bestudeerd die een maximum vertoont en waarvan de helling kan worden bijgesteld met behulp van een parameter. Essentieel is tevens dat de reactie op een nulproductie van de rivaal een positief productievolume is, de zogenaamde monopolieproductie. Een micro-economische fundering voor het bestaan van zo’n reactiecurve met een maximumlocatie en positieve monopolieproductie wordt in dit hoofdstuk niet gegeven (in hoofdstuk 5 wordt wel een mogelijke micro-economische fundering voorgesteld). Wel wordt de vorm van deze curve in verband gebracht met gedragskenmerken in strategische concurrentie. Startend vanuit een monopolieproductiepositie beschouwt (gezien het stijgende karakter van de curve) de onderneming het productieniveau als een zogenaamd strategisch complement. Als de rivaal de productie ophoogt, zal het bedrijf agressief reageren door eveneens de productie te verhogen. Deze agressieve strategie na toetreding van de rivaal wordt ondersteund in de literatuur betreffende ongebruikte (over)capaciteit als een instrument voor toetredingsbelemmering. Dit agressieve gedrag biedt geen voordelen meer als het productieniveau van de rivaal een zekere hoeveelheid overschrijdt; dan gaan de voordelen van (volledige of gedeeltelijke) inkrimping domineren over het voordeel van nog verdere uitbreiding van de productie. De productie kan nu beschouwd worden als een strategisch substituut. De unimodale reactiecurve correspondeert dus in essentie met dualistisch strategisch gedrag; een agressieve gedragslijn enerzijds en een inschikkend, aanpassend beleid anderzijds, afhankelijk van het productievolume van de concurrent.

In hoofdstuk 2 worden drie scenario's in een duopoliemodel geanalyseerd: ten eerste een situatie waarbij één der rivalen dualistisch gedrag vertoont (unimodale reactiecurve) en de andere concurrent de eerste volledig of gedeeltelijk imiteert, ten tweede een scenario met een dualist en een rivaal die zich perfect aanpast en ten derde een duopolie-spel met twee dualisten. In deze scenario's reageren de concurrenten beurtelings op elkaar en hebben zij naïeve (myopische) verwachtingen ten aanzien van de productie van de rivaal; de bedrijven gaan ervan uit dat hun concurrenten in de huidige periode evenveel zullen produceren als in de vorige. Alledrie de scenario's resulteren, voor een specifiek interval van de "hellingparameter" van de dualist(en), in een volstrekt onregelmatige, zogenoemde chaotische, productievoluminareeks van beide concurrenten. De drie belangrijkste kenmerken van chaos worden in dit hoofdstuk met computersimulaties geïllustreerd: (1) het niet-herhalen van de geschiedenis in een productiereeks in de tijd (m.a.w.: volstrekte onregelmatigheid), (2) gevoelige afhankelijkheid van de waarde van de (helling-)parameter en de startwaarde van de productiereeks, waarbij een minieme verschuiving van één van deze waarden leidt tot een totaal ander productiepad, en (3) kwalitatieve breuken in het productiepad in de tijd, waarbij het bijvoorbeeld lijkt alsof zich een evenwicht of periodiciteit instelt. Het tweede kenmerk van chaotische regimes impliceert dat deterministische chaos ernstige problemen kan opleveren wat betreft de schatting van modelparameters. Bovendien kan het problematisch zijn om chaos die volgt uit een volstrekt deterministisch model te onderscheiden van een stochastisch bepaalde reeks.

In **hoofdstuk 3** wordt de voorkeur van managers voor productiegrootte geïntroduceerd (een vorm van inertie van managers); dit houdt in dat de doelfunctie van de managers een combinatie is van de winst - deze heeft in het model een gewicht 1 - en de productiegrootte, die een gewicht α_i heeft voor onderneming i. Vanuit het gezichtspunt van de organisatie-ecologie maakt dit niveau van voorkeur voor een grotere productieomvang van managers deel uit van de "blauwdruk" van het bedrijf. De analyse van allerlei combinaties van voorkeuren (α_i, α_j) van de beide rivalen in de context van Cournot-concurrentie levert interessante inzichten op. De verzameling van Cournot-evenwichten, waarbij ieder productie-evenwicht wordt bepaald door een unieke combinatie (α_i, α_j) van voorkeuren van beide fbedrijven, kan worden opgedeeld in gebieden met specifieke kenmerken. Ondersteund door empirisch onderzoek nemen we aan dat bedrijven verliezen kunnen accumuleren gedurende verscheidene jaren voorafgaand aan uittreding. Als bedrijven een even efficiënte productietechnologie hebben, dan blijkt dat het bedrijf met het hoogste niveau van voorkeur voor productiegrootte ook daadwerkelijk de concurrent in grootte overheerst, onafhankelijk van winst of verlies. Als het niveau van voorkeur groot genoeg is in vergelijking met de rivaal, dan kan deze concurrent zelfs uit de markt gedreven worden, zodat het bedrijf met dit hoge niveau van voorkeur overblijft als een monopolist met verlies of nulwinst. We dienen ons te realiseren dat hetzelfde niveau van voorkeur dat leidt tot die monopoliepositie, ook verantwoordelijk is voor het winstverlies. Echter: deze voorkeur van de managers maakt deel uit van de “blauwdruk” van het bedrijf en kan derhalve niet op korte termijn worden gewijzigd.

Wanneer de marktvraag zich ongunstig ontwikkelt in relatie tot de productiekosten, laat de analyse verschillende scenario’s zien. Beide rivalen besluiten de markt te verlaten als hun winstmotieven hun voorkeuren voor grootte overheersen. Als de voorkeuren voor grote omvang van beide concurrenten de winstmotieven echter domineren en onderling niet te veel verschillen, dan zullen beide bedrijven in deze ongunstige markt blijven opereren met verlies (de marktprijs is tengevolge van het hoge marktaanbod onder de kostprijs gezakt). Ook in een ongunstige markt kan het bedrijf met een veel hoger niveau van voorkeur voor grootte de concurrent uit de markt drijven. Als de twee rivalen verschillende productiekosten hebben, dan kan een bedrijf met de laagste productiekosten en met uitsluitend winstmotieven toch in het nadeel zijn als de minder efficiënte concurrent een hoge voorkeur voor grootte vertoont. Men kan stellen dat hogere productiekosten (dus lagere efficiëntie) van een ondrneming in concurrentietermen gecompenseerd kunnen worden door een hoger niveau van inertie van de managers, weerspiegeld in een hoog gewicht α voor productieomvang in de doelfunctie. De uitkomsten van het model geven aan dat de overlevingskansen van een bedrijf verhoogd worden als de managers inertie vertonen voor wat betreft hun voorkeur voor grootte. Dit resultaat ondersteunt de “inertiehypothese” van Hannan en Freeman, de gronleggers van OE, die stellen dat organisaties met structuren die een hoge mate van inertie vertonen, in het voordeel zijn bij het (Darwiniaans) selectieproces in een populatie van bedrijven.

Vanuit het perspectief van industriële organisatie kunnen andere interpretaties worden gepropageerd. Gesteld dat (de eigenaren van) bedrijven de voorkeur voor grootte in de doelfunctie zouden kunnen manipuleren (dit is de veronderstelling in de zogenaamde delegatiespelen, waarbij de eigenaar de doelfunctie van de managers manipuleert), bijvoorbeeld via de inrichting van specifieke bonusregelingen, dan zouden er verscheidene mogelijkheden zijn. Het gewicht α_i in de doelfunctie kan zodanig gemanipuleerd worden dat de betreffende onderneming i de winst maximaliseert, gegeven het gewicht van de concurrent α_j. Als beide concurrenten hun gewichten zodanig aanpassen, leidt dit delegatiespel tot vaste en gelijke voorkeuren voor omvang in de beide doelfuncties (bij gelijke productiekosten). De doelstelling van een dergelijke strategische actie kan ook het verdrijven van de concurrent van de markt zijn. Dit motief zou leiden tot veel hogere voorkeuren voor omvang.

De inertie van een organisatie, weerspiegeld in aanpassingskosten, vormt het centrale thema in **hoofdstuk 4**. In dit hoofdstuk worden lineaire, asymmetrische aanpassingskosten beschouwd rondom een gefixeerd productieniveau, zowel als gevolg van een dalend als van een stijgend productievolume. De consequenties van een conjunctuurgolf voor de productiehoeveelheden en de winstgevendheid van twee concurrenten, alsmede de implicaties voor de sociale welvaart, vormen de hoofdthema’s. In een krimpende markt kunnen drie gedragsfasen van de rivalen worden onderscheiden, waarbij de eerste fase, als gevolg van de aanpassingskosten, gekenmerkt wordt door een handhaving van het oorspronkelijke productieniveau door beide bedrijven. De duur van deze eerste fase wordt bepaald door de onderneming met de laagste aanpassingskosten (de meest flexibele). In de tweede fase handhaaft het bedrijf met de hoogste aanpassingskosten (de meest inerte) het productieniveau, terwijl de rivaal reeds inkrimpt. De marktprijs zakt vanzelfsprekend in deze krimpende markt, en de winstgevendheid van beide concurrenten daalt, maar de winst van het bedrijf met de laagste aanpassingskosten zakt veel sneller. Het relatief flexibele bedrijf krimpt de productie in en draagt daarmee alle lasten van de dalende markt. Het blijkt, dat als het verschil in aanpassingskosten tussen de (relatief) inerte onderneming en de flexibele concurrent hoog genoeg is, het flexibele bedrijf verliezen tegemoet kan zien in de steeds verder krimpende markt (gedurende de tweede fase), terwijl de concurrent nog steeds winstgevend is. Dit kan leiden tot een mogelijk faillissement van het bedrijf met de kleinste aanpassingskosten. In de derde fase verlagen beide rivalen hun productieniveau, maar ook dan kan, als de flexibele onderneming de tweede fase overleefd heeft, dit bedrijf verliezen gaan maken in de derde fase terwijl de concurrent met de hogere aanpassingskosten nog steeds winst maakt. De voorwaarden (het verschil in aanpassingskosten tussen het relatief inerte en flexibele bedrijf) worden in hoofdstuk 4 analytisch uitgewerkt; er kunnen gebieden worden aangegeven in het aanpassingskosten-continuüm van beide rivalen corresponderende met het uittreden van de meest flexibele onderneming in de tweede of derde fase.

Zelfs als het meest flexibele bedrijf overleeft gedurende de fase van laagconjunctuur, dan is de winst van dit bedrijf achtergebleven gedurende een langere periode ten opzichte van de inerte concurrent. Met behulp van integraalrekening wordt aangetoond dat, indien de aanpassingskosten van de meest inerte onderneming een specifiek niveau overschrijden, de relatieve winst van het flexibele bedrijf over de gehele periode met laagconjunctuur geminimaliseerd wordt. Als de totale winst van het relatief flexibele bedrijf slechts een klein percentage van de totale winst van de inerte concurrent is in de periode van economische recessie, heeft dit bedrijf minder kunnen investeren, met alle nadelige gevolgen vandien in een weer aantrekkende markt. De analyse van hoofdstuk 4 toont aan dat het bedrijf met het hoogste niveau van inertie, weerspiegeld in aanpassingskosten, in het voordeel is in een krimpende markt. Ook deze bevinding ondersteunt de “inertiehypothese” van de organisatie-ecologen Hannan en Freeman.

In een periode van hoogconjunctuur kunnen er ook weer drie gedragsfasen van de producenten onderscheiden worden. In dit geval is het echter juist het flexibele bedrijf dat in het voordeel is; lage aanpassingskosten bevorderen dan de strategische marktpositie van een bedrijf, hetgeen juist de inertiehypothese tegenspreekt. Gesteld dat een bedrijf zich bewust zou zijn van de strategische implicaties van aanpassingskosten, de aanpassingskosten kan wijzigen en adequate informatie bezit betreffende de aanpassingskosten van de concurrent, dan kan deze onderneming het niveau van aanpassingskosten trachten te optimaliseren als zich een recessie aandient. De manipulatie van aanpassingskosten kan ten minste drie doelen dienen, namelijk het uit de markt drijven van de rivaal in een krimpende markt gedurende fase twee of fase drie of het minimaliseren van de relatieve winst van de concurrent over de gehele periode van laagconjunctuur. In een periode van hoogconjunctuur zouden de aanpassingskosten vanuit strategisch oogpunt daarentegen zo laag mogelijk moeten zijn. Al deze manipulaties zouden leiden tot hogere aanpassingskosten van bedrijven in een krimpende markt en optimale flexibiliteit in een aantrekkende markt. Het manipuleren van aanpassingskosten spreekt uiteraard het standpunt van de organisatie-ecologie tegen; vanuit dit gezichtspunt maken aanpassingskosten deel uit van de inerte cultuur en organisatiestructuur van een bedrijf en is het maar de vraag of deze kosten snel genoeg gewijzigd kunnen worden in reactie op bijvoorbeeld een plotseling intredende economische neergang.

De analyse van hoofdstuk 4 laat zien dat de sociale welvaart het meest gebaat is bij hoge aanpassingskosten in een krimpende markt en zo laag mogelijke aanpassingskosten in een uitbreidende markt. In die zin zou strategische manipulatie van aanpassingskosten van beide rivalen juist leiden tot de meest voordelige situatie voor de welvaart. Immers: hoge aanpassingskosten in een krimpende markt leiden tot een handhaving van het oorspronkelijke productieniveau, en zodoende tot een voor de consument lagere marktprijs van goederen.

In **hoofdstuk 5** bestuderen we de implicaties van weer een andere vorm van inertie van het management, namelijk voorkeur voor een groot marktaandeel naast het traditionele winstmotief. De doelfunctie van de managers, die een lineaire combinatie van winst en marktaandeel is, wordt gekenmerkt door de gewichten 1 en a voor respectievelijk winst en marktaandeel. De gewichten van de marktaandelen van beide bedrijven (a_i en a_j voor bedrijf i en j) worden in hoofdstuk 5 gevarieerd. De analyse van allerlei gewichtscombinaties (a_i, a_j) is wederom gebaseerd op de aanname dat deze voorkeuren van het management deel uit maken van de inerte cultuur en "blauwdruk" van het bedrijf. Hoofdstuk 5 is daarom nauw gerelateerd aan hoofdstuk 3. Het blijkt dat voorkeur voor grote marktaandelen belangrijke consequenties heeft voor de vorm van de reactiecurve, die het gedrag in de concurrentiestrijd weerspiegelt. Als het niveau van voorkeur voor marktaandeel hoog genoeg is, dan is de reactiecurve unimodaal en kan het bijbehorende strategisch gedrag als dualistisch worden gekenschetst (zie de bespreking van hoofdstuk 2). De aanwezigheid van de voorkeur voor marktaandeel (naast het traditionele winstmotief) in de doelfunctie biedt een micro-economische fundering voor het optreden van een "beste-antwoord"curve met een maximum. Het unimodale karakter van de reactiecurve opent de mogelijkheid tot complexe dynamische verschijnselen in een duopolistische marktstructuur, zoals we bij de bespreking van hoofdstuk 6 zullen zien.

In hoofdstuk 5 worden twee speciale situaties betreffende het niveau van voorkeur van beide rivalen onderzocht. Allereerst leidt de casus waarbij de niveau's van voorkeuren van beide concurrenten gelijk zijn ($a_1 = a_2 = a$) (naast gelijke productie-efficiëntie) tot een symmetrisch Cournot-Nash-evenwicht. Omdat het doel van het management deels marktaandeel is, bestaat er een neiging de productie uit te breiden, terwijl inkrimping weerstand oproept. Vergeleken met de klassieke casus, waarbij pure winstmotieven het doel vormen in de concurrentiestrijd, leidt deze symmetrische casus tot het opofferen van de winst. De (evenwichts)winst van beide rivalen daalt met betrekking tot een toenemend niveau van voorkeur voor het marktaandeel, totdat de totale winst opgeofferd is. Omdat, als de voorkeur voor het marktaandeel toeneemt (a stijgend), de producten in steeds grotere hoeveelheden beschikbaar zijn tegen een steeds lagere marktprijs, neemt de welvaart toe (ondanks de dalende winsten). Voor constante marginale productiekosten bereikt de welvaart een maximum voor de specifieke voorkeur voor marktaandeel die correspondeert met een nulwinst van beide bedrijven. Wanneer echter een nog grotere voorkeur van het management voor marktaandeel verliezen tot gevolg heeft, kan dit leiden tot uittreding van beide ondernemimgen en een ineenstorting van de welvaart. Hoe hoger de productiekosten per eenheid product zijn, des te lager is het niveau van voorkeur (het gewicht a terzake marktaandeel) waarbij een totale opoffering van de winst of een maximalisering van de welvaart optreedt. Men kan stellen dat de mate van voorkeur van managers voor het marktaandeel tot een zeker niveau gunstig is voor de sociale welvaart, maar dat extremere voorkeuren kunnen leiden tot faillissementen en verlies van welvaart.

De tweede casus die in hoofdstuk 5 wordt bestudeerd, betreft een duopoliemarktstructuur waarbij de managers van bedrijf 1 naast het winstmotief ook marktaandeel als doel hebben ($a_1 = a > 0$), terwijl de concurrent uitsluitend de maximalisering van de winst voor ogen heeft ($a_2 = 0$; in delegatiespelen correspondeert pure winstmaximalisatie met een eigenaar die geen manager inhuurt en dus ook geen "incentive"contract met een zeker gewicht voor marktaandeel hoeft op te stellen). Uit de analyse blijkt dat, ongeacht het niveau van voorkeur voor marktaandeel van bedrijf 1, beide bedrijven met positieve winsten op de markt kunnen blijven opereren. De winst van het bedrijf met voorkeur voor het marktaandeel ligt echter op een veel hoger niveau dan de winst van de rivaal. Voor constante marginale productiekosten bestaat er een gewicht a_1 waarbij bedrijf 1 zijn winst maximaliseert. Opmerkelijk is het analytische resultaat dat de welvaart blijft toenemen als het niveau van voorkeur voor marktaandeel van bedrijf 1 verder stijgt. De asymmetrische situatie van deze tweede casus is dus klaarblijkelijk gunstig voor de welvaart. Vanuit het gezichtspunt van OE is een bedrijf met een hoger niveau van inertie (een hoger niveau van voorkeur voor marktaandeel) in het voordeel in een selectieproces in een populatie van organisaties. Wederom ondersteunt dit analytische resultaat de inertiehypothese van Hannan en Freeman. Als, vanuit het rationele aanpassingsperspectief van IO, bedrijf 1 zich bewust zou zijn van de consequenties en in staat zou zijn het gewicht a aan te passen, dan zou dit bedrijf het niveau van voorkeur voor marktaandeel kunnen gebruiken als een strategisch instrument in directe concurrentie.

In **hoofdstuk 6** wordt wederom de tweede, asymmetrische, casus beschouwd ($a_1 = a, a_2 = 0$) teneinde complexe dynamische verschijnselen te illustreren, die het gevolg kunnen zijn van een extremere voorkeur voor marktaandeel van bedrijf 1. Als de beide rivalen beurtelings op elkaar reageren en naïeve verwachtingen hanteren t.a.v. het productieniveau van de concurrent, kan aangetoond worden dat het Cournot-Nash-evenwicht instabiel wordt als het niveau van voorkeur voor marktaandeel van bedrijf 1 een bepaalde waarde, a_{bif}, overschrijdt. Als de marginale productiekosten c op een schaal van 0 tot 1 gelegen zijn in het interval $[0{,}2,\ 1)$ en het gewicht a_1 de waarde a_{bif} overstijgt, ontstaat er een cyclus in de individuele productievolumina en het totale marktaanbod met periode 2. Dit betekent dat de extremere voorkeur voor marktaandeel van onderneming 1 tot gevolg kan hebben dat er een endogene conjunctuurgolf gegenereerd wordt. Bovendien leidt deze instabiliteit tot een lagere gemiddelde winst voor het bedrijf met voorkeur voor marktaandeel, terwijl de winst van de onderneming met pure winstmotieven op een gunstiger niveau komt te liggen (dan juist voordat het Cournot-Nash-evenwicht destabiliseerde). In het geval van zeer efficiënte productietechnologieën – de marginale productiekosten liggen op een schaal van 0 tot 1 nu onder de waarde $0{,}2$ – treden er nog interessantere dynamische verschijnselen op. Productiecycli met allerlei perioden kunnen optreden en analyse (met behulp van de Stelling van Li en Yorke) toont aan dat, als het niveau van voorkeur voor marktaandeel van bedrijf 1 boven een specifiek niveau, a_{chaos}, gelegen is, chaotische regimes mogelijk zijn. De eenvoudige en aannemelijke veronderstelling van voorkeur voor marktaandeel in de doelfunctie van een bedrijf leidt tot een deterministische beslissingsregel voor beide concurrenten inzake hun productievolumina. De analyse toont aan dat zo'n heldere beslissingsregel in een situatie met directe concurrentie zelfs kan leiden tot een volstrekt onregelmatige totale marktproductie.

Hoofdstuk 6 bevat ook reflecties op de modelkeuze. Laten we bijvoorbeeld de reacties van beide ondernemingen gelijktijdig plaatvinden, i.p.v. beurtelings zoals in het eerste deel van dit hoofdstuk, dan heeft dit duidelijke gevolgen voor de periodiciteiten van productiecycli (er vindt een verdubbeling van de periode plaats in vergelijking met de periodiciteiten bij beurtelingse reacties) en de gemiddelde winsten van beide rivalen over zo'n cyclus. Het hoofdstuk wordt afgesloten met een analyse waarbij we de aanname maken dat het bedrijf met voorkeur voor marktaandeel een Stackelberg-leider is (en de rivaal pure winstmotieven volgt). De Stackelberg-leider kan in dat geval de rivaal de markt uitdrijven als het niveau van voorkeur voor marktaandeel een voldoend hoge waarde heeft, maar kleiner is dan het gewicht dat in de doelfunctie aan de winst wordt toegekend.

De analytische resultaten, die volgen uit de modellen van dit proefschrift, ondersteunen in veel gevallen de inertiehypothese van Hannan en Freeman: “in een populatie van organisaties zijn bedrijven waarvan de structuren een hoge vorm van inertie vertonen in het voordeel in een (Darwiniaans) selectieproces.” In een krimpende markt zijn de overlevingskansen van bedrijven met hogere aanpassingskosten (inertie van de organisatie genoemd) hoger en ook de voorkeuren van managers voor productiegrootte of marktaandeel naast de gebruikelijke winstmotieven (inertie van het management genoemd) leveren voordelen op in het selectieproces. De speltheoretische modellen dragen dus bij tot de vergroting van inzichten vanuit een organisatie-ecologisch perspectief. Ook blijkt dat aanpassingskosten of voorkeuren voor productiegrootte en marktaandeel goed gebruikt zouden kunnen worden als strategische wapens in de concurrentiestrijd. Hiertoe is het echter wel noodzakelijk dat bedrijven (eigenaars) zich bewust zijn van de gevolgen van aanpassingskosten of de doelfuncties van het management en deze managementvoorkeuren of aanpassingskosten kunnen manipuleren in hun voordeel. Deze twee-stapsspelen waarbij eigenaars de doelfuncties van hun managers of aanpassingskosten strategisch manipuleren (principaal-agent- of delegatiemodellen) zijn in het proefschrift nog onderbelicht gebleven en vormen een aandachtsgebied voor toekomstig onderzoek.

Bovendien kan het onderzoek uitgebreid worden door het aantal concurrerende bedrijven te vergroten in een Cournot- of Bertrand-concurrentiecontext, en door tevens de gevolgen van productheterogeniteiten te beschouwen. Speltheoretische modellering kan inzicht verschaffen in de invloed van productdifferentiatie op de overlevingskansen of de concurrentiepositie van bedrijven. Vanzelfsprekend dient dan ook verder gereflecteerd te worden op de existentie van evenwichten en de stabiliteit ervan.

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