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AN ENERGY-BALANCING PERSPECTIVE OF  
INTERCONNECTION AND DAMPING  
ASSIGNMENT CONTROL OF NONLINEAR  
SYSTEMS<sup>1</sup>

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Abstract: Stabilization of nonlinear *feedback passive* systems is achieved assigning a storage function with a minimum at the desired equilibrium. For physical systems a natural candidate storage function is the difference between the stored and the supplied energies—leading to the so-called Energy-Balancing control, whose underlying stabilization mechanism is particularly appealing. Unfortunately, energy-balancing stabilization is stymied by the existence of pervasive dissipation, that appears in many engineering applications. To overcome the dissipation obstacle the method of Interconnection and Damping Assignment, that endows the closed-loop system with a special—port-controlled Hamiltonian—structure, has been proposed. If, as in most practical examples, the open-loop system already has this structure, and the damping is not pervasive, both methods are equivalent. In this brief note we show that the methods are also equivalent, with an alternative definition of the supplied energy, when the damping is pervasive. Instrumental for our developments is the observation that, swapping the damping terms in the classical dissipation inequality, we can establish passivity of port-controlled Hamiltonian systems with respect to some new external variables—but with the same storage function. *Copyright © 2003 IFAC*

1. INTRODUCTION AND BACKGROUND  
MATERIAL

It is by now well-understood that equilibria of nonlinear systems of the form<sup>2</sup>

$$\dot{x} = f(x) + g(x)u \quad (1)$$

with  $x \in \mathbb{R}^n$ ,  $u \in \mathbb{R}^m$ , can be easily stabilized if it is possible to find functions  $\alpha(x), h(x) : \mathbb{R}^n \rightarrow \mathbb{R}^m$  and  $\beta(x) : \mathbb{R}^m \rightarrow \mathbb{R}^m$  such that the system

$$\begin{aligned} \dot{x} &= f(x) + g(x)\alpha(x) + g(x)\beta(x)v \\ y &= h(x) \end{aligned}$$

is passive with external variables  $(v, y)$  and a storage function that has a minimum at the desired equilibrium, say  $x_* \in \mathbb{R}^n$ . This class of systems is called *feedback passive* and stabilization is achieved feeding-back the “passive output”  $y$  with a strictly passive operator—a technique that is generically known as Passivity-Based Control (PBC). (See (Byrnes *et al.*, 1991; Schaft, 2000; Isidori, 1995), or (Astolfi *et al.*, 2000) for a recent tutorial that contains most of the background material reviewed in this section). From (Byrnes *et al.*, 1991) it is known that necessary conditions for passification of the system  $(f, g, h)$  are that it has relative degree  $\{1, \dots, 1\}$  and is weakly minimum phase. The process is completed verifying the conditions of the nonlinear Kalman-Yakubovich-Popov lemma. The latter involves the solution of a partial differential equation (PDE)—which is

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<sup>2</sup> Throughout the paper, if not otherwise stated, it is assumed that all functions and mappings are  $C^\infty$ , and all vectors—including the gradient—are column vectors.

difficult to find, in general. An additional complication stems from the minimum requirement on the storage function that imposes some sort of “boundary conditions” on the PDE.

Designing PBCs can be made more systematic for systems belonging to the following class, which contains many physical examples (Ortega *et al.*, 2001):

*Definition 1.* The system (1) with output  $y = h(x)$  is said to satisfy the *energy-balancing* (EB) inequality if, for some function  $H(x) : \mathbb{R}^n \rightarrow \mathbb{R}$ ,

$$H[x(t)] - H[x(0)] \leq \int_0^t u^\top(s)y(s)ds, \quad (2)$$

along all trajectories compatible with  $u : [0, t] \rightarrow \mathbb{R}^m$ .<sup>3</sup>

Typically,  $u, y$  are conjugated variables, in the sense that their product has units of power, and  $H(x)$  is the total stored energy—hence the name EB. The EB inequality reflects a universal property of physical systems and it would be desirable to preserve it in closed-loop. On the other hand, since  $H(x)$  does not have (in general) a minimum at  $x_*$  it is suggested to look for a control action  $u = \alpha(x) + \beta(x)v$  such that the closed-loop system satisfies the new EB inequality

$$H_d[x(t)] - H_d[x(0)] \leq \int_0^t v^\top(s)\tilde{y}(s)ds. \quad (3)$$

for a new output function  $\tilde{y} = \tilde{h}(x)$  (that may be equal to  $y$ ) and some function  $H_d(x) : \mathbb{R}^n \rightarrow \mathbb{R}_+$  that has an isolated local minimum at  $x_*$ . (As discussed in (Astolfi *et al.*, 2000; Ortega *et al.*, 2001), see also below, the inclusion of a new output function adds considerable flexibility to the design procedure without loosing the physical insight.)

A first, natural, approach to solve the problem above is to try to make  $H_d(x)$  equal to the difference between the stored and the supplied energies. For that, we must find a function  $\alpha(x)$  such that the energy supplied by the controller can be expressed as a function of the state. Indeed, from (2) we see that *if* we can find a function  $\alpha(x)$  such that, for some function  $H_a(x) : \mathbb{R}^n \rightarrow \mathbb{R}$  and for all  $x$  and all  $t \geq s$ , we have

$$\begin{aligned} H_a[\phi(x, t)] - H_a[\phi(x, s)] = \\ - \int_s^t \alpha^\top[\phi(x, \tau)]h[\phi(x, \tau)]d\tau, \end{aligned}$$

<sup>3</sup> Notice that no assumption of non-negativity on  $H(x)$  is imposed. Clearly, if it is non-negative, then the system is passive with external variables  $(u, y)$  and storage function  $H(x)$ . Also, notice that (2) implies  $h(x) = g^\top(x)\nabla H(x)$ .

where  $\phi(x, t)$  denotes the trajectory of the system with control  $u = \alpha(x) + v$  starting from the initial condition  $x$  at time  $t$ , then the closed-loop system satisfies (3) with  $y = \tilde{y}$  and *new* energy function

$$H_d(x) = H(x) + H_a(x). \quad (4)$$

Hence,  $x_*$  can be easily stabilized with the desired storage (Lyapunov) function, and we refer to this particularly appealing class of PBCs as EB-PBCs.

The design of EB-PBCs also involves the solution of a PDE, namely,<sup>4</sup>

$$\begin{aligned} [f(x) + g(x)\alpha(x)]^\top \nabla H_a(x) = \\ -\alpha^\top(x)g^\top(x)\nabla H(x) \end{aligned} \quad (5)$$

that results taking the limit of (4), that is,  $\dot{H}_a(x(t)) = -\alpha^\top(x(t))h(x(t))$ , and the fact that  $h(x) = g^\top(x)\nabla H(x)$ . But its applicability is mainly stymied by the presence of *pervasive dissipation* in the system. Indeed, it is clear that a necessary condition for the solvability of the PDE (5) is the implication

$$f(\bar{x}) + g(\bar{x})\alpha(\bar{x}) = 0 \Rightarrow h^\top(\bar{x})\alpha(\bar{x}) = 0.$$

Evaluating, in particular, for  $\bar{x} = x_*$  we see that the power extracted from the controller ( $= h^\top(x)\alpha(x)$ ) should be zero at the equilibrium. (The interested reader is referred to (Ortega *et al.*, 2001) where the effect of pervasive dissipation is illustrated with simple linear time-invariant RLC circuits.)

In order to overcome the dissipation obstacle, the method of Interconnection and Damping Assignment (IDA) PBC, that assigns a special—port-controlled Hamiltonian (PCH)—structure to the closed-loop system, has been proposed in (Ortega *et al.*, 2002a). More specifically, in IDA-PBC we fix the matrices  $J_d(x) = -J_d^\top(x) \in \mathbb{R}^{n \times n}$  and  $R_d(x) = R_d^\top(x) \geq 0 \in \mathbb{R}^{n \times n}$ , that represent the desired interconnection and dissipation structures, respectively, and solve the PDE<sup>5</sup>

$$f(x) + g(x)\alpha(x) = [J_d(x) - R_d(x)]\nabla H_d(x),$$

which can be equivalently expressed in the nicer form

$$g^\perp(x)f(x) = g^\perp(x)[J_d(x) - R_d(x)]\nabla H_d(x) \quad (6)$$

where  $g^\perp(x)$  is a left annihilator of  $g(x)$ , that is,  $g^\perp(x)g(x) = 0$ . The PDE (6) characterizes all energy functions that can be assigned to the closed-loop PCH system with the given interconnection and dissipation matrices, and the control that achieves this objective is

<sup>4</sup> Throughout the paper we denote  $\nabla_p H(p, q) = \frac{\partial H}{\partial p}(p, q)$ . When clear from the context the subindex will be omitted.

<sup>5</sup> IDA-PBC is presented in (Ortega *et al.*, 2002a) only for systems in PCH form, but it is clear that all derivations carry on to general  $(f, g, h)$  systems.

$$\alpha(x) = [g^\top(x)g(x)]^{-1} g^\top(x) \times \{[J_d(x) - R_d(x)]\nabla H_d(x) - f(x)\}.$$

Taking the derivative of  $H_d(x)$  along the closed-loop trajectories we get

$$\dot{H}_d(x) = -\nabla^\top H_d(x)R_d(x)\nabla H_d(x) \leq 0.$$

Again, if we can find a solution for (6) such that  $x_* = \arg \min[H_d(x)]$  then stability of  $x_*$  is ensured. The main interest of IDA-PBC is that, in contrast with EB-PBC, the PDE (6) is still solvable (in principle) when the extracted power is not zero at the equilibrium, hence the method is applicable to systems with pervasive dissipation. Another advantage of IDA-PBC is that the free parameters in the PDE (6),  $J_d(x), R_d(x)$ , have a clear physical interpretation, while there are no simple guidelines for the selection of  $\alpha(x)$  in (5).

Although ‘‘Hamiltonianizing’’ the system may seem like an artifice, there are close connections between IDA-PBC and EB-PBC.<sup>6</sup> Namely, in (Ortega *et al.*, 2002a) conditions on the damping are given so that IDA-PBC is an EB-PBC. More precisely, it is shown that if

- (1) the system is PCH, that is,<sup>7</sup>

$$\begin{aligned} f(x) &= [J(x) - R(x)]\nabla H(x) \\ h(x) &= g^\top(x)\nabla H(x) \end{aligned}$$

for some  $J(x) = -J^\top(x) \in \mathbb{R}^{n \times n}$  and  $R(x) = R^\top(x) \geq 0 \in \mathbb{R}^{n \times n}$ ;

- (2)  $R_d(x) = R(x)$ , that is, no additional damping is injected to the system;
- (3) the assigned energy function  $H_d(x)$  and the natural damping satisfy

$$R(x)[\nabla H_d(x) - \nabla H(x)] = 0, \quad (7)$$

(This property was called ‘‘dissipation obstacle’’ in (Ortega *et al.*, 2001) and, roughly speaking, states that there is no damping in the coordinates where the energy function is shaped.)

then,

$$\dot{H}_d[x(t)] = \dot{H}[x(t)] - \alpha^\top[x(t)]h[x(t)],$$

and the storage function  $H_d(x)$  is equal to the difference between the stored and the supplied energies.

The main contribution of this paper is the establishment of a similar equivalence between IDA-PBC and EB-PBC when the damping is ‘‘not admissible’’, that is when (7) is not satisfied. Specifically, using an alternative definition of the

supplied energy, we prove that the methods are also equivalent when the damping is pervasive. Instrumental for our developments is the observation that, swapping the damping terms in the EB inequality, we can establish passivity of PCH systems with respect to some new external variables. Interestingly, as a kind of partial converse, for mechanical systems and for PCH systems with  $m = 1$ , we also prove that if the damping is admissible then it cannot be swapped and the new passivity property coincides with the classical one.

## 2. A NEW PASSIVITY PROPERTY FOR A CLASS OF PCH SYSTEMS

The following lemma is instrumental for the proof of our main result.

*Lemma 1.* Assume the matrices  $J(x) = -J^\top(x)$  and  $R(x) = R^\top(x) \geq 0$  are such that  $\text{rank}[J(x) - R(x)] = n$ , then

$$z^\top [J(x) - R(x)]^{-1} z \leq 0, \quad (8)$$

for all  $z \in \mathbb{R}^n$ .

*Proof.* The proof is completed with the following simple calculations

$$\begin{aligned} z^\top [J(x) - R(x)]^{-1} z &= \frac{1}{2} z^\top ([J(x) - R(x)]^{-1} + [J(x) - R(x)]^{-\top}) z \\ &= \frac{1}{2} z^\top [J(x) - R(x)]^{-1} ([J(x) - R(x)] \\ &\quad + [J(x) - R(x)]^\top) [J(x) - R(x)]^{-\top} z \\ &= \frac{1}{2} \tilde{z}^\top (J(x) - R(x) + [J(x) - R(x)]^\top) \tilde{z} \\ &= -\tilde{z}^\top R(x) \tilde{z} \leq 0, \end{aligned}$$

where we have defined  $\tilde{z} = [J(x) - R(x)]^{-\top} z$ , with  $(\cdot)^{-\top} = [(\cdot)^{-1}]^\top$ . ■

Notice that, if  $J(x) - R(x)$  is rank deficient then the open-loop system has equilibria at points which are not extrema of the energy function. Hence, the assumption  $\text{rank}[J(x) - R(x)] = n$  does not seem to be restrictive in applications. For this class of PCH systems the proposition below establishes passivity with respect to a new set of external variables.

*Proposition 1.* Consider the PCH system

$$\begin{aligned} \dot{x} &= [J(x) - R(x)]\nabla H(x) + g(x)u \\ y &= g^\top(x)\nabla H(x). \end{aligned} \quad (9)$$

Assume  $J(x) - R(x)$  is full rank. Then, the system satisfies the new EB inequality

$$H[x(t)] - H[x(0)] \leq \int_0^t \tilde{y}^\top(s)u(s)ds, \quad (10)$$

where  $\tilde{y} = \tilde{h}(x, u)$ , with

<sup>6</sup> See (Ortega *et al.*, 2001) for the interpretation of IDA-PBC as control by interconnection (Schaft, 2000).

<sup>7</sup> In (Ortega *et al.*, 2002a) it is shown that all asymptotically stable vector fields admit such a PCH realization.

$$\begin{aligned}\tilde{h}(x, u) &= -g^\top(x)[J(x) - R(x)]^{-\top} \\ &\quad \times \{[J(x) - R(x)]\nabla H(x) + g(x)u\}. \quad (11)\end{aligned}$$

Furthermore, if  $H(x)$  is bounded from below, the system is passive with external variables  $(u, \tilde{y})$  and storage function  $H(x)$ .

*Proof.* Under the assumption that  $\text{rank}[J(x) - R(x)] = n$ , we can rewrite the PCH system (9) in the following form

$$[J(x) - R(x)]^{-1}\dot{x} = \nabla H(x) + [J(x) - R(x)]^{-1}g(x)u. \quad (12)$$

Premultiplying (12) by  $\dot{x}^\top$  we obtain

$$\begin{aligned}\dot{H}(x) &= \dot{x}^\top \nabla H(x) \\ &= \dot{x}^\top [J(x) - R(x)]^{-1}\dot{x} \\ &\quad - \dot{x}^\top [J(x) - R(x)]^{-1}g(x)u \\ &\leq -\dot{x}^\top [J(x) - R(x)]^{-1}g(x)u \\ &= \tilde{y}^\top u,\end{aligned}$$

where we have invoked Lemma 1 to obtain the inequality, and replaced  $\dot{x}$  and used (11) in the last equality. The proof is completed integrating the expression above from 0 to  $t$ . ■

*Remark 1.* From the derivations above we have that

$$\dot{H}(x) = -\tilde{z}^\top R(x)\tilde{z} + \tilde{y}^\top u,$$

where  $\tilde{z} = [J(x) - R(x)]^{-1}\dot{x}$ . Comparing with the classical power balance equation,

$$\dot{H}(x) = -(\nabla H(x))^\top R(x)\nabla H(x) + y^\top u,$$

reveals that the new passivity property is established “swapping the damping”.

*Remark 2.* Proposition 1 lends itself to an alternative interpretation that reveals the close connections with the results reported in (Ortega *et al.*, 2002b), see also (Ortega and Shi, 2002). In these papers a new passivity property for RLC circuits is established and used to propose power-shaping, as an alternative to energy-shaping, to overcome the dissipation obstacle for stabilization of systems with pervasive damping. From the proof of the proposition it is clear that, introducing an input change of coordinates

$$\tilde{u} = [J(x) - R(x)]^{-1}g(x)u, \quad (13)$$

we also have passivity with the external coordinates  $(\tilde{u}, \dot{x})$ —hence, in some respect, we have “added a differentiation” to the port variables as done in (Ortega *et al.*, 2002b; Ortega and Shi, 2002). See Section 4 for an illustrative example.

### 3. IDA-PBC AS AN ENERGY-BALANCING CONTROLLER

As a corollary of Proposition 1 we prove in this section that, even when the damping is pervasive, IDA-PBC is an EB-PBC with the new definition of supplied power  $u^\top \tilde{y}$ .

*Proposition 2.* Consider the PCH system (9), with  $\text{rank}[J(x) - R(x)] = n$ , in closed-loop with an IDA-PBC,  $u = \alpha(x)$ , that transforms the system into

$$\dot{x} = [J(x) - R(x)]\nabla H_d(x). \quad (14)$$

Then,

$$H_d[x(t)] = H[x(t)] - \int_0^t u^\top(s)\tilde{y}(s)ds. \quad (15)$$

where  $\tilde{y} = \tilde{h}(x, u)$ , with  $\tilde{h}(x, u)$  defined in (11).

*Proof.* The PCH system (9), with  $u = \alpha(x)$ , matches (14) if and only if

$$[J(x) - R(x)]^{-1}g(x)\alpha(x) = \nabla H_a(x) \quad (16)$$

where we have used (4) and the assumption  $\text{rank}[J(x) - R(x)] = n$ . Premultiplying the latter equation by  $\dot{x}^\top$  we obtain  $\dot{H}_a(x) = -u^\top \tilde{y}$ , which upon integration yields the desired result. ■

*Remark 3.* The proposition above is restricted to IDA-PBC designs that do not modify the interconnection and damping matrices of the open-loop system, but only shape the energy function. When  $J_d(x) \neq J(x)$  and/or  $R_d(x) \neq R(x)$  the matching condition becomes, see (Ortega *et al.*, 2002a),

$$\begin{aligned}[J_d(x) - R_d(x)]^{-1} \{ [J_a(x) - R_a(x)]\nabla H(x) \\ + g(x)\alpha(x) \} = \nabla H_a(x),\end{aligned}$$

where  $J_d(x) = J(x) + J_a(x)$  and  $R_d(x) = R(x) + R_a(x)$ . Some simple calculations show that a term, that is independent of  $\alpha(x)$ , appears in  $\dot{H}_a$ . Therefore, the latter cannot be made equal to some (suitably defined) supplied power.

### 4. EXAMPLES

In this section we illustrate with several examples the application of Proposition 1. First, we prove that for some practically relevant multi-domain systems, the new passivity property has a natural interpretation in terms of classical concepts from circuit theory. Then, we show that for (simple) mechanical systems and for all single-input single-output systems that do not suffer from the dissipation obstacle the new passive output  $\tilde{y}$  coincides with  $y$ , hence Proposition 1 does not reveal any new property.

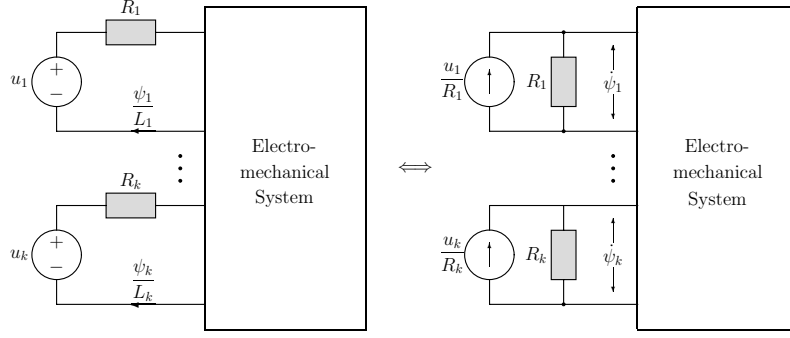


Fig. 1. (left) Thevenin representation of electromechanical systems with passive port variables  $(Bu, L^{-1}\psi)$ ; (right) corresponding Norton equivalent with passive port variables  $(R_e^{-1}Bu, \dot{\psi})$ .

#### 4.1 Connection with Thevenin-Norton equivalence

We will now prove that for electromechanical systems with  $n_e$  unsaturated windings, permanent magnets and one mechanical coordinate<sup>8</sup> the new passivity property appears as a corollary of the well-known Thevenin and Norton equivalence (Desoer and Kuh, 1969). For this class of systems  $x = \text{col}(\psi, \theta, p) \in \mathbb{R}^{n_e+2}$ , with  $\psi \in \mathbb{R}^{n_e}$  the magnetic fluxes,  $\theta, p \in \mathbb{R}$  the mechanical displacement and momenta, respectively, and  $u$  denoting the external voltages applied to some of the windings. The resulting PCH model is defined with the matrices

$$J = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}, \quad g = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}, \quad (17)$$

$$R = \begin{bmatrix} R_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

where  $R_e = R_e^\top > 0 \in \mathbb{R}^{n_e \times n_e}$  represents the resistors (in series with the inductive elements), and  $B \in \mathbb{R}^{(n_e+2) \times m}$  defines the actuated electrical coordinates. We recall at this point that the electrical equations of this system are of the form

$$\dot{\psi} = -R_e i + Bu$$

where  $i = \nabla_\psi H(x)$  are the currents on the inductors, which are related to the fluxes through  $\psi = L(\theta)i$ , with  $L(\theta) = L^\top(\theta) > 0$  the inductance matrix. The dynamics of the system is completed applying Newton's second law to the mechanical subsystem, but this equation is not relevant for the analysis given here.

The natural power port variables are the external voltage sources  $u$  and the corresponding electrical currents  $y = B^\top L^{-1}(\theta)\psi$ . On the other hand, replacing the matrices (17) in (13) we get that

$$\tilde{u}^\top \dot{x} = u^\top B^\top R_e^{-1} \dot{\psi}$$

<sup>8</sup> This class has been thoroughly studied in (Ortega *et al.*, 1998), to which we refer the reader for further details on the model.

where  $R_e^{-1}Bu$  are the current sources obtained from the Norton equivalent of the Thevenin representation of the classical passivity property, with  $\dot{\psi}$  the associated inductor voltages. The equivalence is depicted in Fig. 1 for the case of no mutual couplings between the electrical elements, where to underscore the transformation we have pulled out from the system the resistors associated with the actuated coordinates.

*Remark 4.* Notice that the systems considered in this section are not stabilizable with EB-PBC (using the natural outputs of the system  $y = B^\top \nabla H(x)$ ) due to the fact that all non-trivial equilibria there is a current flowing through the “unactuated resistors”  $B^\perp R_e$ . Hence, the power extracted from the natural ports is nonzero at any nonzero equilibrium point.

#### 4.2 Systems without the dissipation obstacle

In this subsection we explore the implications of Proposition 1 for some systems that are EB stabilizable and, as a partial converse result, prove that in this case the new passivity property exactly coincides with the classical one.

We take as first example position regulation of (simple) mechanical systems, which is the prototypical case study of EB stabilizable systems. For these systems the total energy function is given by

$$H(q, p) = \frac{1}{2} p^\top M^{-1}(q) p + V(q),$$

where  $q, p \in \mathbb{R}^{n/2}$  are the generalized displacements and momenta, respectively,  $M(q) = M^\top(q) > 0$  is the inertia matrix and  $V(q) : \mathbb{R}^{n/2} \rightarrow \mathbb{R}$  is the potential energy. Denoting  $x = \text{col}(q, p) \in \mathbb{R}^n$ , the system is described by the PCH model (9) with the matrices

$$J = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}, \quad R = \begin{bmatrix} 0 & 0 \\ 0 & R_m \end{bmatrix}, \quad g = \begin{bmatrix} 0 \\ B \end{bmatrix},$$

where  $R_m = R_m^\top \geq 0$  are the friction coefficients,  $B \in \mathbb{R}^{n/2 \times m}$ , and  $u \in \mathbb{R}^m$  are the external forces.

Replacing the expressions above in (11), and doing some simple calculations, we get

$$\begin{aligned}\tilde{y} &= -g^\top [J - R]^{-T} ([J - R] \nabla H(x) + gu) \\ &= g^\top \begin{bmatrix} I & 2R_m \\ 0 & I \end{bmatrix} \nabla H(x) - g^\top \begin{bmatrix} -R_m & I \\ -I & 0 \end{bmatrix} gu \\ &= B^\top \nabla_p H(x) \\ &= y\end{aligned}$$

which completes our claim.

Let us move now to our second example. As discussed in Section 1, a necessary condition for an IDA-PBC to be an EB controller is that—for all assignable energy functions  $H_d(x)$ —the damping verifies the condition (7), whose violation is referred as “dissipation obstacle”. The proposition below shows that for single-port PCH systems that do not suffer from the dissipation obstacle we again have  $\tilde{y} = y$ .

*Proposition 3.* Consider a PCH system (9) with  $u, y \in \mathbb{R}^1$  and  $\text{rank}[J(x) - R(x)] = n$ . Assume all energy functions that can be assigned using IDA-PBC with  $J_d(x) = J(x)$  and  $R_d(x) = R(x)$  satisfy (7). Then, the new output  $\tilde{y}$  coincides with the natural output  $y$ .

*Proof.* All energy functions that can be assigned using IDA-PBC with  $J_d(x) = J(x)$  and  $R_d(x) = R(x)$  are given by (4), where  $H_a(x)$  follows from the integration of (16)—for a given  $\alpha(x)$ . For ease of reference we repeat the latter equation here

$$\nabla H_a(x) = [J(x) - R(x)]^{-1} g(x) \alpha(x).$$

Combining (7) and (16) yields  $R(x) \nabla H_a(x) = 0$ , or equivalently

$$R(x)[J(x) - R(x)]^{-1} g(x) = 0. \quad (18)$$

Hence, it remains to show that  $\tilde{y} = y$ . Using (18) we may write

$$\begin{aligned}[J(x) - R(x)][J(x) - R(x)]^{-1} g(x) &= g(x) \\ \Rightarrow J(x)[J(x) - R(x)]^{-1} g(x) &= g(x) \\ \Rightarrow g^\top(x)[J(x) - R(x)]^{-\top} g(x) &= 0,\end{aligned}$$

and by replacing the latter into (11), we obtain

$$\begin{aligned}\tilde{y} &= -g^\top(x)[J(x) - R(x)]^{-\top} [J(x) - R(x)] \nabla H(x) \\ &= g^\top(x) \nabla H(x) \\ &= y.\end{aligned}$$

This completes the proof. ■

## 5. CONCLUDING REMARKS

Summarizing, we have shown that, for the class of systems with pervasive dissipation, the basic IDA-

PBC methodology reduces to an EB-PBC design. Thus, if one accepts a set outputs other than the natural ones, we can give an energy-balancing interpretation of IDA-PBC. Instrumental for our developments is that we swap the damping in the classical power-balance in order to conclude passivity with respect to a different set of external port variables, while using the same storage function. The only necessary condition for swapping the damping is that  $J(x) - R(x)$  needs to be full rank. However, if  $J(x) - R(x)$  is rank deficient then the open-loop system has equilibria at points which are not extrema of the energy function. Therefore, the full rank condition seems not restrictive in physical applications. Interestingly, the new passivity property does not affect systems that do not suffer from pervasive dissipation.

## REFERENCES

- Astolfi, A., R. Ortega and R. Sepulchre (2000). Stabilization and disturbance attenuation of nonlinear systems using dissipativity theory. *Eur. Journal of Contr., Special Issue Devoted to V.M. Popov, (to appear)*.
- Byrnes, C., A. Isidori and J.C. Willems (1991). Passivity, feedback equivalence, and the global stabilization of minimum phase nonlinear systems. *IEEE Trans Aut. Cont.* **36**(11), 1228–1240.
- Desoer, C.A. and E.S. Kuh (1969). *Basic Circuit Theory*. McGraw-Hill, NY.
- Isidori, A. (1995). *Nonlinear Control Systems, Third Edition*. Springer-Verlag.
- Ortega, R., A. Loría, P.J. Nicklasson and H. Sira-Ramírez (1998). *Passivity-Based Control of Euler-Lagrange Systems; Mechanical, Electrical and Electromechanical Applications*. Springer-Verlag.
- Ortega, R., A.J. van der Schaft, B.M. Maschke and G. Escobar (2002a). Interconnection and damping assignment passivity based control of port-controlled Hamiltonian systems. *Automatica* **38**(4), 585–596.
- Ortega, R., A.J. van der Schaft, I. Mareels and B.M. Maschke (2001). Putting energy back in control. *IEEE Control Syst. Magazine* **21**(2), 18–33.
- Ortega, R. and B.E. Shi (2002). A note on passivity of nonlinear RL and RC circuits. *IFAC World Conf. Barcelona, Spain*.
- Ortega, R., D. Jeltsema and J.M.A. Scherpen (2002b). Power shaping: A new paradigm for stabilization of nonlinear RLC circuits. *IFAC Latino-American Control Conference, Guadalajara, Mexico*.
- Schaft, A.J. van der (2000).  *$\mathcal{L}_2$ -Gain and Passivity Techniques in Nonlinear Control*. Springer-Verlag.