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ATTRIBUTE-SPACE CONNECTED FILTERS

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Abstract In this paper connected operators from mathematical morphology are extended to a wider class of operators, which are based on connectivities in higher dimension spaces, similar to scale spaces which will be called attribute spaces. Though some properties of connected filters are lost, granulometries can be defined under certain conditions, and pattern spectra in most cases. The advantage of this approach is that regions can be split into constituent parts before filtering more naturally than by using partitioning connectivities.

Keywords: Mathematical morphology, multi-scale analysis, connected filters, perceptual grouping.

1. Introduction

Semantic analysis of images always involves grouping of pixels in some way. The simplest form of grouping is modelled in digital image processing by connectivity [4], which allows us to group pixels into connected components or flat-zones in the grey-scale case. In mathematical morphology, *connected operators* have been developed which perform filtering based on these kinds of groupings [7][8][9]. However, the human observer may either interpret a single connected component of a binary image as multiple visual entities, or group multiple connected components into a single visual entity. These properties have to some extent been encoded in second-order connectivities, which can be either partitioning or clustering [1] [3][12].

In this paper I will demonstrate a problem with partitioning connectivities when used for second-order connected attribute filters, due to the large numbers of singletons they produce in the image. This *over-segmentation* effect is shown in Fig. 1. It will be shown that these attribute filters reduce to performing e.g. an opening with ball B followed by an application of the attribute filter using the normal (4 or 8) connectivity. The approach presented here is is different from second-order connectivities, in that it restates the connectiv-

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(a) (b) (c) (d) (e)

Figure 1. Attribute-space compared to regular attribute filtering: (a) original image X; (b) the connected components of X according to $\mathcal{C}^\psi$, with ψ an opening by a 3×3 structuring element (see Section 3); (d) partitioning of X by attribute space method of Section 4; (e) regular attribute thinning $\Psi_\psi^T(X)$ with $T(C) = (I(C)/A^2(C) < 0.5)$; (f) attribute-space connected attribute thinning $\Psi_A^T(X)$ with the same T. T is designed to remove elongated structures. Note that only the attribute-space method removes the elongated bridge.

ity relationships in an image in terms of connectivity in higher-dimensional spaces, which I will call *attribute spaces* . As can be seen in Fig. 1, this leads to a more natural partitioning of the connected component into two squares and a single bridge. This effect is also shown in a practical application in Fig. 7.

This paper is organized as follows. First connected filters are described formally in Section 2, followed by second-order connectivities in Section 3. Problems with attribute filters using partitioning connectivities are dealt with in detail in this section. After this, attribute spaces are presented in section 4.

2. Connectivity and Connected Filters

As is common in mathematical morphology binary images X are subsets of some universal set E (usually $E = \mathbb{Z}^n$). Let $\mathcal{P}(E)$ be the set of all subsets of E. Connectivity in E can be defined using *connectivity classes* [10].

DEFINITION 1 *A connectivity class $\mathcal{C} \subseteq \mathcal{P}(E)$ is a set of sets with the following three properties:*

1 $\emptyset \in \mathcal{C}$

2 $\{x\} \in \mathcal{C}$

3 for each family $\{C_i\} \subset \mathcal{C}$, $\cap C_i \neq \emptyset$ *implies* $\cup C_i \in \mathcal{C}$.

This means that both the empty set and singleton sets are connected, and any union of connected sets which have a nonempty intersection is connected.

Any image X is composed of a number of connected components or *grains* $C_i \in \mathcal{C}$, with i from some index set I. For each C_i there is no set $C \supset C_i$ such that $C \subseteq X$ and $C \in \mathcal{C}$. If a set C is a grain of X we denote this as $C \lessdot X$.

An alternative way to define connectivity is through *connected openings*, sometimes referred to as connectivity openings [1].

Figure 2. Binary attribute filters applied to an image of bacteria: (left) original; (middle) area opening using area threshold $\lambda = 150$; (right) elongation thinning using attribute $I/A^2 > 0.5$.

DEFINITION 2 *The binary connected opening* Γ_x *of* X *at point* $x \in \mathbf{M}$ *is given by*

$$\Gamma_x(X) = \begin{cases} C_i : x \in C_i \wedge C_i \lessdot X & \textit{if } x \in X \\ \emptyset & \textit{otherwise}. \end{cases} \quad (1)$$

Thus Γ_x extracts the grain C_i to which x belongs, discarding all others.

Attribute filters

Binary attribute openings are based on binary connected openings and *trivial openings*. A trivial opening Γ_T uses an increasing criterion T to accept or reject connected sets. A criterion T is increasing if the fact that C satisfies T implies that D satisfies T for all $D \supseteq C$. Usually T is of the form

$$T(C) = (\mathit{Attr}(C) \geq \lambda), \quad (2)$$

with $\mathit{Attr}(C)$ some real-valued attribute of C, and λ the attribute threshold. A trivial opening is defined as follows $\Gamma_T : \mathcal{C} \to \mathcal{C}$ operating on $C \in \mathcal{C}$ yields C if $T(C)$ is true, and $\emptyset$ otherwise. Note that $\Gamma_T(\emptyset) = \emptyset$. *Trivial thinnings* differ from trivial openings only in that the criterion T in non-increasing instead of increasing. An example is the scale-invariant elongation criterion of the form (2), in which $\mathit{Attr}(C) = I(C)/A^2(C)$, with $I(C)$ the moment of inertia of C and $A(C)$ the area [13]. The binary attribute opening is defined as follows.

DEFINITION 3 *The binary attribute opening* Γ^T *of set* X *with increasing criterion* T *is given by*

$$\Gamma^T(X) = \bigcup_{x \in X} \Gamma_T(\Gamma_x(X)) \quad (3)$$

The attribute opening is equivalent to performing a trivial opening on all grains in the image. Note that if the attribute T is non-increasing, we have an attribute thinning rather than an attribute opening [2, 8]. The grey-scale case can be derived through threshold decomposition [6]. An example in the binary case is shown in Figure 2.

3. Second-Order Connectivities

Second-order connectivities are usually defined using an operator ψ which modifies X, and a base connectivity class $\mathcal{C}$ (4 or 8 connectivity)[1, 10]. The resulting connectivity class is referred to as $\mathcal{C}^\psi$. If ψ is extensive $\mathcal{C}^\psi$ is said to be *clustering*, if ψ is anti-extensive $\mathcal{C}^\psi$ is *partitioning* . In the general case, for any $x \in E$ three cases must be considered: (i) $x \in X \cap \psi(X)$, (ii) $x \in X \setminus \psi(X)$, and (iii) $x \notin X$. In the first case, the grain to which x belongs in $\psi(X)$ is computed according to $\mathcal{C}$, after which the intersection with X is taken to ensure that all grains $C_i \subseteq X$. In the second case, the x is considered to be a singleton grain. In the third case the connected opening returns $\emptyset$ as before.

DEFINITION 4 *The connected opening Γ_x^ψ for a second-order connectivity based on ψ of image X is*

$$\Gamma_x^\psi(X) = \begin{cases} \Gamma_x(\psi(X)) \cap X & \text{if } x \in X \cap \psi(X) \\ \{x\} & \text{if } x \in X \setminus \psi(X) \\ \emptyset & \text{otherwise,} \end{cases} \tag{4}$$

in which Γ_x is the connected opening based on $\mathcal{C}$.

If $X \subset \psi(X)$ the second case of (4) never occurs. Conversely, if $\psi(X) \subset X$ we have $\psi(X) \cap X = \psi(X)$, simplifying the first condition in (4). An extensive discussion is given in [1, 10].

Attribute operators

Attribute operators can readily be defined for second-order connectivities by replacing the standard connected opening Γ_x by Γ_x^ψ in Definition 3.

DEFINITION 5 *The binary attribute opening Γ_ψ^T of set X with increasing criterion T, and connectivity class $\mathcal{C}^\psi$ is given by*

$$\Gamma_\psi^T(X) = \bigcup_{x \in X} \Gamma_T(\Gamma_x^\psi(X)) \tag{5}$$

Though useful filters can be constructed in clustering case, and partition of grains in soil samples for computation of area pattern spectra has been used [12, 11], a problem emerges in the partitioning case.

PROPOSITION 1 *For partitioning connectivities based on ψ the attribute opening Γ_ψ^T with increasing, shift invariant criterion T is*

$$\Gamma_\psi^T(X) = \begin{cases} X & \text{if } T(\{x\}) \text{ is true} \\ \Gamma^T(\psi(X)) & \text{otherwise} \end{cases} \tag{6}$$

with Γ^T the underlying attribute opening from Definition 3.

Proof If $T(\{x\})$ is true for any x, all $x \in X \setminus \psi(X)$ are preserved by Γ_ψ^T, because $\Gamma_x^\psi(X) = \{x\}$ for those pixels. Because T is increasing we have that $T(\{x\}) \Rightarrow T(C)$ for any $C \in \mathcal{C}$ with $C \neq \emptyset$. Thus, if $T(\{x\})$ is true for any x, all $x \in \psi(X)$ are also preserved, because $\Gamma_x(\psi(X)) \in \mathcal{C}$ and $\Gamma_x(\psi(X)) \neq \emptyset$ for those x. In other words if $T(\{x\})$ is true,

$$\Gamma_\psi^T(X) = X, \tag{7}$$

which proves (6) in the case that $T(\{x\})$ is true.

Conversely, if $T(\{x\})$ is false for any x, all $x \in X \setminus \psi(X)$ are rejected, i.e. $\Gamma_T(\{x\}) = \emptyset$. Therefore, if $T(\{x\})$ is false

$$\Gamma_\psi^T(X) = \bigcup_{x \in \psi(X)} \Gamma_T(\Gamma_x^\psi(X)). \tag{8}$$

Because all $x \in X \setminus \psi(X)$ are rejected, $\Gamma_x^\psi(X)$ can be rewritten as $\Gamma_x(\psi(X))$, and we have

$$\Gamma_\psi^T(X) = \bigcup_{x \in \psi(X)} \Gamma_T(\Gamma_x(\psi(X))) = \Gamma^T(\psi(X)). \tag{9}$$

The right-hand equality derives from Definition 3. □

Proposition 1 means that an attribute opening using a partitioning connectivity boils down to performing the standard attribute opening on $\psi(X)$, unless the criterion has been set such that Γ^T is the identity operator. The reason for this is the fact that the grains of $X \setminus \psi(X)$ according to the original connectivity are split up into singletons by Γ_x^ψ. Even if non-increasing criteria are used, singleton sets carry so little information that setting up meaningful filter criteria is not readily done. In Section 4 a comparison with the attribute-space alternative is given and illustrated in Figure 1.

4. Attribute Spaces and Attribute-Space Filters

As was seen above, connectivities based on partitioning operators yield rather poor results in the attribute-filter case. To avoid this, I propose to transform the binary image image $X \subset E$ into a higher-dimensional *attribute-space* $E \times A$. Scale spaces are an examples of attribute spaces, but other attribute spaces will be explored here. Thus we can devise an operator $\Omega : \mathcal{P}(E) \to \mathcal{P}(E \times A)$. Thus $\Omega(X)$ is a binary image in $E \times A$. Typically $A \subseteq \mathbb{R}$ or $\mathbb{Z}$, although the theory presented here extends to cases such as $A \subseteq \mathbb{R}^n$. The inverse operator $\Omega^{-1} : \mathcal{P}(E \times A) \to \mathcal{P}(E)$, projects $\Omega(X)$ back onto X, i.e. $\Omega^{-1}(\Omega(X)) = X$ for all $X \in \mathcal{P}(E)$. Furthermore, Ω^{-1} must be increasing: $Y_1 \subseteq Y_2 \Rightarrow \Omega^{-1}(Y_1) \subseteq \Omega^{-1}(Y_2)$ for all $Y_1, Y_2 \in \mathcal{P}(E \times A)$. Attribute-space connected filters can now be defined as follows.

DEFINITION 6 *An attribute-space connected filter* $\Psi_A : \mathcal{P}(E) \to \mathcal{P}(E)$ *is defined as*

$$\Psi^A(X) = \Omega^{-1}(\Psi(\Omega(X))) \tag{10}$$

with $X \in \mathcal{P}(E)$ *and* $\Psi : \mathcal{P}(E \times A) \to \mathcal{P}(E \times A)$ *a connected filter.*

Thus attribute-space connected filters work by first mapping the image to a higher dimensional space, applying a connected filter and projecting the result back. Note that the connected filter Ψ may use second-order connectivity rather the underlying connectivity in $E \times A$ (e.g. 26-connectivity in 3D). Note that if Ψ is anti-extensive (or extensive), so is Ψ^A due to the increasingness of Ω^{-1}. However, if Ψ is increasing, this property does not necessarily hold for Ψ^A, as will be shown on page 91 and following and Fig. 5. Similarly, idempotence of Ψ does not imply idempotence of Ψ^A. However, if

$$\Psi(\Omega(X)) = \Omega(\Psi^A(X)) = \Omega(\Omega^{-1}(\Psi(\Omega(X)))), \tag{11}$$

for all $X \in \mathcal{P}(E)$, idempotence of Ψ does imply idempotence of Ψ^A, because Ω maps $\Psi^A(X)$ exactly back onto $\Psi(\Omega(X))$. Eqn. (11) obviously holds when $\Omega(\Omega^{-1}(Y)) = Y$ for all $Y \in \mathcal{P}(E \times A)$, but (11) is slightly more general.

We can also define attribute-space shape or size granulometries and spectra in analogy to connected shape or size granulometries [2, 13]. Let $\{\alpha_r\}$ be a granulometry, with each $\alpha_r : \mathcal{P}(E \times A) \to \mathcal{P}(E \times A)$ a connected filter, with r from some ordered set Λ. The set of attribute-space connected filters $\{\alpha_r^A\}$ defined as

$$\alpha_r^A = \Omega^{-1}(\alpha_r(\Omega(X))), \tag{12}$$

has the following properties

$$\alpha_r^A(X) \subseteq X, \tag{13}$$

$$s \leq r \Rightarrow \alpha_r^A(X) \subseteq \alpha_s^A(X) \tag{14}$$

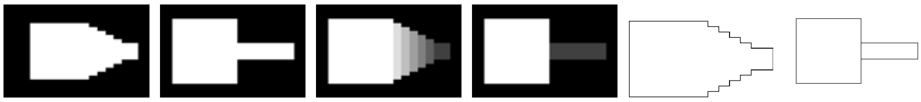
for all $X \subseteq E$. However, the stronger nesting property of granulometries, i.e.

$$\alpha_r^A(\alpha_s^A(X)) = \alpha_{\max(r,s)}^A(X) \tag{15}$$

only holds if the condition on idempotence in (11) is true for all α_r in the granulometry. However, property (14) does lead to a nesting of the resulting images $\alpha_r^A(X)$ as a function of r, so a pattern spectra f_X^A based on these filters can be defined as

$$f_X^A(r) = \begin{cases} A(X \setminus \alpha_r^A(X)) & \text{if } r = 1 \\ A(\alpha_{r-1}(X) \setminus \alpha_r^A(X)) & \text{if } r > 1 \end{cases} \tag{16}$$

with A the Lebesgue measure in E (area in 2-D), and $\Lambda = 1, 2, \ldots, N$, similar to [5]. Finally, note that connected filters form a special case of attribute-space connected filters, in which $\Omega = \Omega^{-1} = I$, with I the identity operator.



(a) X (b) Y (c) Ω_X (d) Ω_Y (e) (f)

Figure 3. Attribute-space partitioning of two binary sets: (a) and (b) binary images X and Y each containing a single (classical) connected component (c) and (d) their respective opening transforms; (e) and (f) partitioning of X and Y using edge strength threshold $r = 1$. X is considered as one component due to the slow change in attribute value, whereas the abrupt change in width causes a split in Y.

Width-based attribute spaces

In the following $E = \mathbb{Z}^2$. As an example of mapping of a binary image $X \in \mathcal{P}(E)$ to binary image $Y \in \mathcal{P}(E \times A)$ we can use local width as an attribute to be assigned to each pixel $x \in X$, using an opening transform defined by granulometry $\{\beta_r\}$, in which each operator $\beta_r : E \to E$ is an opening with a structuring elements B_r. An opening transform is defined as

DEFINITION 7 *The opening transform Ω_X of a binary image X for a granulometry $\{\beta_r\}$ is* $\Omega_X(x) = \max\{r \in \Lambda | x \in \beta_r(X)\}$. (17)

In the case that $\beta_r(X) = X \circ B_r$ with $\circ$ denoting structural openings and B_r ball-shaped structuring elements of radius r, an opening transform assigns the radius of the largest ball such that $x \in X \circ B_r$. An example is shown in Fig. 3. We can now devise a width-based attribute space by the mapping $\Omega_w : \mathcal{P}(E) \to \mathcal{P}(E \times \mathbb{Z})$ as

$$\Omega_w(X) = \{(x, \Omega_X(x)) | x \in X\} \tag{18}$$

The inverse is simply

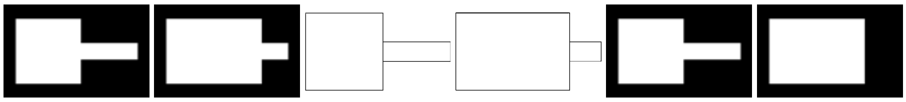
$$\Omega_w^{-1}(Y) = \{x \in E | (x, y) \in Y\} \tag{19}$$

with $Y \in \mathcal{P}(E \times \mathbb{Z})$.

Let $C_i \subset E \times \mathbb{R}$ be the connected components of $\Omega_w(X)$ with i from some index set. Because a single attribute value is assigned to each pixel by Ω_X, it is obvious that the projections onto E of these sets $C_i^w = \Omega_w^{-1}(C_i)$ are disjoint as well. Thus they form a partition of the image plane in much the same way as classical connected components would do, as can be seen in Fig. 3. In this example we can work in a 2-D grey-scale image, rather than a 3-D binary image, for convenience. Connectivity in the attribute space is now partly encoded in the grey-level differences of adjacent flat zones in these images. In the simplest case, corresponding to 26-connectivity in the 3-D binary image, a grey-level difference of 1 means adjacent flat-zones are

(a) X (b) X_1 (c) X_2 (d) (e)

Figure 4. Attribute-space connectivity is not connectivity: (a) binary image X c is the union of two overlapping sets X_1 (b) and X_2 (c) each of which are considered connected in attribute space; however, X is partitioned into two sets (d) by the same attribute-space connectivity; (e) any partitioning connectivity which separates the square from the elongated part of X splits the elongated part into 14 singletons.



(a) X (b) Y (c) (d) (e) $\Psi^A(X)$ (f) $\Psi^A(Y)$

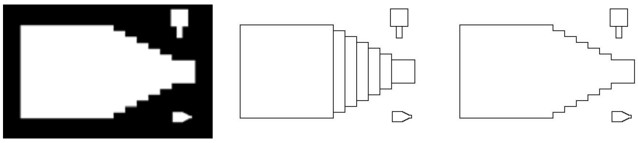
Figure 5. Non-increasingness of Ψ^A for increasing Ψ: (a) and (b) binary images X and Y, with $X \subseteq Y$; (c) and (d) partitions of X and Y in attribute space projection of Ω_w; (e) and (f) $Psi^A(X)$ and $Psi^A(Y)$, using for Psi an area opening with area threshold 10. Clearly $Psi^A(X) \not\subseteq Psi^A(Y)$, even though Ψ is increasing

connected in attribute space. More generally, we can use some threshold r on the grey level difference between adjacent flat zones. This corresponds to a second-order connectivity $\mathcal{C}^{\psi_r}$ with ψ_r a dilation in $\mathbb{Z}^3$, with structuring element $\{(0,0,-r),(0,0,-r+1),\ldots,(0,0,r)\}$. The effect of this can be seen in Fig. 3(f), in which abrupt changes in width lead to splitting of a connected component into two parts. Fig. 4 demonstrates that this splitting is different from caused by a partitioning connectivity . Fig. 5 shows the non-increasingness of an attribute-space area operator Ψ^A based on an area opening Ψ in $E \times A$. This effect occurs due to the fact that overlap of X_1 and X_2 in E does not imply overlap of $\Omega_w(X_1)$ and $\Omega_w(X_2)$ in $E \times A$.

A slightly different partitioning is obtained if we change (18)

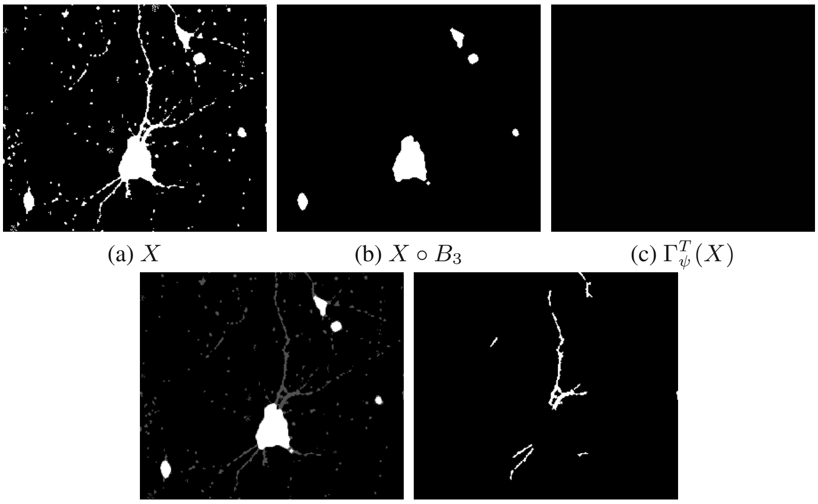
$$\Omega_{\log w}(X) = \{(x, 1 + \log(\Omega_X(x)))|x \in X\} \tag{20}$$

with $\Omega^{-1}_{\log w} = \Omega^{-1}_w$. Note that one is added to the logarithm of the width to separate bridges of unity width from the background. Though very similar in behaviour to the attribute-space connectivity using Ω_w, attribute-space connectivity based on $\mathcal{C}^{\psi_r}$ is now scale-invariant, as is shown in Fig. 6. No second-order connectivity in E can achieve this, because they are all based on increasing operators [1][10], and scale-invariance and increasingness are incompatible [13].



(a) (b) (c)

Figure 6. Scale invariant partitioning using 26-connectivity in 3-D: (a) Binary image in which the large and the bottom small connected component have identical shapes; (b) partitioning using Ω_w; (c) scale-invariant partitioning using $\Omega_{\log w}$, which splits the top small connected component, but regards the other two as single entities.



(a) X (b) $X \circ B_3$ (c) $\Gamma^T_\psi(X)$

(d)Ω^t_X (e) $\Psi^A(X)$

Figure 7. Elongation filtering of neurons: (a) binary image of neuron; (b) opening by B_3 to separate cell body from dendrites; (c) second-order connected attribute thinning *preserving* elongated features with $I(C)/A^2 > 0.5$; (e) Classification of pixels by thresholding Ω_X at the same value of $t = 3$; attribute-space connected filter result using same attribute as Γ^T_ψ.

Any nonlinear transformation on the attribute can be used to obtain different results, depending on the application. A simple method is to threshold the opening transform Ω_X assigning foreground pixels to different classes, denoted by Ω^t_X, allowing connectivity only within a class. A simple two-class classification is shown in Fig. 7, in which a second-order connected attribute filter is compared to the corresponding two-class pixel classification method. Only the attribute-space method recovers dendrites.

The first two attribute-space connectivities have a scale parameter, or rather a *scale-difference* or *scale-ratio* parameter. This means we can develop multiscale or perhaps more properly *multi-level* visual groupings in analogy to the well-defined multi-scale connectivities [1, 12]. Increasing r in the attribute-space connectivities generated by Ω_w or $\Omega_{\log w}$ combined with $\mathcal{C}^{\psi_r}$ yields a hierarchy, in which the partitioning becomes coarser as r is increased.

5. Discussion

Attribute-space morphology solves the problems with attribute filters using partitioning connectivities as noted in Proposition 1. The fragmentation caused by splitting parts of connected components into singletons is absent. This means that attribute-space attribute filters are more than just applying a standard attribute filter to a preprocessed image. The price we pay for this is loss of the increasingness property, and increased computational complexity. In return we may achieve scale invariance, combined with a more intuitive response to, e.g., elongation-based attribute filters, as is seen in Fig. 1. Future research will focus on grey-scale generalizations, efficient algorithms for these operators, and on the possibilities of dealing with overlap in this framework.

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