(Super-)Gravity in Three Dimensions

Bergshoeff, Eric A.

Published in:
The Planck Scale, Proceedings of the XXV Max Born Symposium

Publication date:
2009

(Super-)Gravity in Three Dimensions

Eric A. Bergshoeff

Centre for Theoretical Physics, University of Groningen, Nijenborgh 4, 9747 AG Groningen, the Netherlands

Abstract. We present a new unitary model of gravity in three dimensions containing a particular combination of terms of higher-order derivatives in the gravitational field. We discuss some properties of this model. In particular, we present the $N = 1$ supersymmetric extension and discuss the role of the auxiliary scalar in the presence of the higher-derivative terms.

Keywords: three-dimensional gravity, massive supergravity,
PACS: 04.20.Cv, 04.60.-m, 04.60.Kz, 11.30.Pb

INTRODUCTION

It is well-known that Einstein's theory of gravity is a theory of interacting massless spin-two particles around a Minkowski spacetime background. Unfortunately, as it stands, the theory is not renormalizable. One way to try to improve the situation is to add terms to the usual Einstein-Hilbert term which are of higher-order derivatives in the gravitational field. This has the beneficial effect that the behaviour of the propagators improve such that the theory could become renormalizable. However, in the presence of these higher-derivative terms it is not guaranteed that the theory only describes a unitary spin-two particle. In fact, it has been shown a long time ago that this is not the case [1] and that higher-derivative gravity in four spacetime dimensions is perhaps renormalizable but non-unitary.

To tackle the outstanding problem of quantum gravity in four spacetime dimensions it is useful to reconsider this issue in the simpler setting of three spacetime dimensions. Pure Einstein gravity is rather trivial in the sense that it does not describe any propagating degree of freedom [2]. The situation becomes more interesting after a higher-derivative Lorentz Chern-Simon terms has been added [3]. This so-called "Topological Massive Gravity" (TMG) model decribes a massive particle of helicity $+2$ or -2, depending on the sign of the Chern-Simons term. Note that the TMG model breaks parity, due to the presence of the Chern-Simons term.

Recently, we showed that there is another unitary higher-derivative gravity model in in three spacetime dimensions [4]: ghosts are avoided if (i) the EH term appears with the 'wrong-sign' and (ii) the following curvature-squared scalar K is added to the Einstein term

$$K = R_{\mu\nu}R^{\mu\nu} - \frac{3}{8}R^2\,, \tag{1}$$

where $R_{\mu\nu}$ is the Ricci tensor. In its Minkowski vacuum, this "new massive gravity" (NMG) model propagates, unitarily, two massive modes of helicities ± 2 and is power-counting super-renormalizable since the corresponding four-dimensional theory

CP1196, *The Planck Scale, Proceedings of the XXV Max Born Symposium*
edited by J. Kowalski-Glikman, R. Durka, and M. Szczachor

is renormalizable [1]. Unitarity has since been confirmed by other methods [5, 6], as has super-renormalizability [7]. A more general model is obtained by adding both a Lorentz Chern-Simons term and the higher-derivative K term to the Einstein action [4]. This so-called "Generalized Massive Gravity" (GMG) model propagates two spin 2 modes with *different* masses. By taking one of the two masses to infinity one gets the TMG model of [3]. Finally, in order to obtain interesting black hole solutions, a cosmological constant may be added to the above models resulting into cosmological versions of these models. The most general model, containing all other ones by truncation, is obtained by adding a cosmological constant to the GMG model leading to the CGMG model which is the topic of this talk.

A USEFUL ANALOGY: SPIN 1

Before we consider spin 2 in three spacetime dimensions, it is instructive to consider the spin 1 case first. It is well-known that a massive spin 1 particle in $D=3$, with helicities +1 and -1, is described by the Proca Lagrangean

$$\mathscr{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu\,. \tag{2}$$

The equations of motion corresponding to this Lagrangean are given by

$$(\partial^\mu\partial_\mu - m^2)A^\nu = 0\,, \qquad\qquad \partial^\mu A_\mu = 0\,. \tag{3}$$

In three spacetime dimensions the Proca equation can be written in the following factorised form

$$[\mathscr{O}(-m)\mathscr{O}(m)]_\mu{}^\nu A_\nu = 0\,, \qquad\qquad \mathscr{O}_\mu{}^\nu(m) \equiv m\delta_\mu{}^\nu + \varepsilon_\mu{}^{\tau\nu}\partial_\tau\,. \tag{4}$$

Each of the two factors describes a particle of a single helicity:

$$[\mathscr{O}(m)]_\mu{}^\nu A_\nu = 0 \qquad \text{or} \qquad mA_\mu = \varepsilon_\mu{}^{\nu\rho}F_{\nu\rho}\,, \tag{5}$$

where $F_{\nu\rho} \equiv \partial_\nu A_\rho - \partial_\rho A_\nu$ is the curvature tensor. Integrating these equations of motion leads to the following model [8]:

$$S = \frac{1}{2}\int d^3x\,\big(\varepsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho - mA^\mu A_\mu\big)\,, \tag{6}$$

which we dub, for obvious reasons, the $\sqrt{\text{Proca}}$ model.

The above construction suggests the following natural generalization of the Proca model to two particles, one with helicity +1 and mass m_+, and another with helicity -1 and mass m_-:

$$[\mathscr{O}(m_+)\mathscr{O}(m_-)]_\mu{}^\nu A_\nu = 0\,. \tag{7}$$

Integrating these equations of motion leads to the following action

$$S = \int d^3x\,\big(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\mu\,\varepsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho - \frac{1}{2}m^2 A^\mu A_\mu\big)\,, \tag{8}$$

with

$$\mu = 2(m_+ - m_-) \qquad \text{and} \qquad m^2 = m_+ m_- \,. \tag{9}$$

Taking $m_+ = m_-$, i.e. $\mu = 0$, leads to the original Proca model, whereas upon taking the limit $m_+ \to \infty$, one of the two particles decouples and one is left with the $\sqrt{\text{Proca}}$ model.

Starting from the $\sqrt{\text{Proca}}$ model there is a natural way to construct an equivalent action where each term contains one more derivative. This procedure goes as follows. Denoting the basic vector field of the $\sqrt{\text{Proca}}$ model by $\tilde{A}_\mu$ the equations of motion of the $\sqrt{\text{Proca}}$ model are given by

$$m\tilde{A}_\mu = \varepsilon_\mu{}^{\nu\rho}\tilde{F}_{\nu\rho} \,, \qquad \partial^\mu \tilde{A}_\mu = 0 \,. \tag{10}$$

One now solves the constraint in terms of another independent vector field A_μ of different mass dimension [10]:

$$\partial^\mu \tilde{A}_\mu = 0 \qquad \Rightarrow \qquad \tilde{A}_\mu = \varepsilon_\mu{}^{\nu\rho}\partial_\nu A_\rho \,. \tag{11}$$

Substituting this solution back into the equations of motion leads to new equations of motion for the vector fields A_μ. Integrating out these equations of motion leads to the massive topological spin 1 model [9]

$$S = \int d^3x \left(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m\,\varepsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho \right) . \tag{12}$$

Note that we start with a model without gauge symmetry but we end up with one that has such a symmetry.

This concludes our discussion of spin 1 models in three spacetime dimensions. Summarizing we learned two things:

1. Due to a factorization of the Proca model in three spacetime dimensions we can take the square root of that model leading to the so-called $\sqrt{\text{Proca}}$ model.
2. By solving a subsidiary constraint we obtain a duality between the $\sqrt{\text{Proca}}$ model and the higher-derivative topological massive spin 1 model.

Concerning the second point we stress that the outlined procedure in general guarantees on-shell equivalence but it does not guarantee physical equivalence between the two models. In particular, if more helicities are involved, one needs to verify whether in the new dual action all helicities occur with the right sign or whether they are ghosts. For a more detailed discussion of this point, see [11].

In the next section we wish to apply the above lessons to the spin 2 case.

GRAVITY IN THREE SPACETIME DIMENSIONS

For our present purposes it is enough to consider linearized gravity. For each model we have to ask ourselves whether it allows a generalization to non-linear gravity. This will only be the case for a subset of the models we consider. Nevertheless, it is instructive to consider all models here, independent of whether they do or do not allow for an extension to the non-linear case.

We first consider the linearized Einstein-Hilbert action

$$S^{\rm lin}_{\rm EH} = \int d^3x\, h^{\mu\nu} G^{\rm lin}_{\mu\nu}(h)\,, \tag{13}$$

where we have linearized around the Minkowski vacuum, $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$. The linearized Einstein tensor is given by

$$G^{\rm lin}_{\mu\nu}(h) = \frac{1}{2}\varepsilon_\mu{}^{\eta\rho}\varepsilon_\nu{}^{\tau\sigma}\partial_\eta\partial_\tau h_{\rho\sigma}\,. \tag{14}$$

It satisfies the identity $\partial^\mu G^{\rm lin}_{\mu\nu}(h) = 0$. Furthermore, the trace of the linearized Einstein tensor is equal to the the linearized Ricci scalar $R^{\rm lin}(h)$:

$$\eta^{\mu\nu} G^{\rm lin}_{\mu\nu}(h) = -\frac{1}{2}R^{\rm lin}(h)\,. \tag{15}$$

Of course, the linearized Einstein equations $G^{\rm lin}_{\mu\nu}(h) = 0$ do not describe massless propagating gravitons. To obtain (massive) propagating degrees of freedom we must add higher-derivative terms to the Einstein-Hilbert action.

To show how this can be done we first consider the Fierz-Pauli model which is the analogue of the Proca theory describing massive spin-2 particles with helicities ± 2. This theory cannot be extended to the non-linear level but it is suitable for our purposes. The Fierz-Pauli Lagrangean is given by

$$\mathscr{L}_{\rm FP} = -\frac{1}{2}h^{\mu\nu}\mathscr{G}_{\mu\nu}(h) - \frac{1}{2}m^2\left(h^{\mu\nu}h_{\mu\nu} - h^2\right), \tag{16}$$

where $h_{\mu\nu}$ is a symmetric tensor. The two mass terms occur in a precise combination such that the equations of motion do not lead to extra ghost-like particles:

$$\left(\partial^\mu\partial_\mu - m^2\right)h_{\mu\nu} = 0\,, \qquad\qquad \partial^\mu h_{\mu\nu} = 0\,, \quad h = 0\,. \tag{17}$$

Like in the Proca theory, the equations of motion can be written in a factorized form with each of the factors describing a particle with one helicity, +2 or -2. The theory describing only one of these two particles can be dubbed the $\sqrt{\rm Fierz-Pauli}$ model. Its equations of motion are given by

$$m h_{\mu\nu} = \varepsilon_\mu{}^{\rho\sigma}\partial_\rho h_{\sigma\nu}\,, \qquad\qquad h = 0\,. \tag{18}$$

Using a *general* second-rank tensor $h_{\mu\nu}$ these equations of motion can be integrated to the following first-order action

$$S = \int d^3x\left(\varepsilon^{\mu\nu\rho}h_\mu{}^\sigma\partial_\nu h_{\rho\sigma} - m(h^{\nu\mu}h_{\mu\nu} - h^2)\right). \tag{19}$$

The equations of motion corresponding to this action imply (18) together with the algebraic constraint $h_{[\mu\nu]} = 0$.

In the same way that the $\sqrt{\text{Proca}}$ action (6) could be boosted up with one derivative by solving a differential constraint, we will now show that the $\sqrt{\text{Fierz}-\text{Pauli}}$ action (19) can be boosted up with two derivatives by solving a similar constraint. Our starting point is the $\sqrt{\text{Fierz}-\text{Pauli}}$ model where we denominate the fundamental field with $\tilde{h}_{\mu\nu}$. From the equations of motion (18) it follows that the following differential constraint must be solved:

$$\partial^{\mu}\tilde{h}_{\mu\nu} = 0\,. \tag{20}$$

This constraint can be solved in the same way as in the Proca model except that now we are dealing with a two-index tensor field which implies that the solution of the differential constraint involves two Levi-Civita tensors and two spacetime derivatives instead of one. In fact, the solution involves the linearized Einstein tensor and expresses $\tilde{h}_{\mu\nu}$ in terms of a new independent field $h_{\mu\nu}$ of different mass dimension

$$\tilde{h}_{\mu\nu} = G^{\text{lin}}_{\mu\nu}(h)\,. \tag{21}$$

In terms of $h_{\mu\nu}$ the equations of motion for $\tilde{h}_{\mu\nu}$ and the constraint $\tilde{h} = 0$ read

$$\mu G^{\text{lin}}_{\mu\nu}(h) = \varepsilon_{\mu}{}^{\rho\sigma}\partial_{\rho}G^{\text{lin}}_{\sigma\nu}(h)\,, \qquad R^{\text{lin}}(h) = 0\,. \tag{22}$$

One may verify that these are precisely the equations of motion of the linearized TMG model. Therefore, surprisingly, after boosting up the $\sqrt{\text{Fierz}-\text{Pauli}}$ action with two derivatives we obtain an alternative model that can be extended to the non-linear level with an action given by

$$S_{\text{TMG}}[g] = \frac{1}{\kappa^2}\int d^3x\,\big(-\sqrt{-g}R + \frac{1}{\mu}\mathscr{L}_{\text{LCS}}\big)\,. \tag{23}$$

The Lorentz-Chern-Simons Lagrangean is given by the following expression in terms of the Christoffel symbols:

$$\mathscr{L}_{\text{LCS}} = \frac{1}{2}\varepsilon^{\mu\nu\rho}\left[\Gamma^{\alpha}_{\mu\beta}\partial_{\nu}\Gamma^{\beta}_{\rho\alpha} + \frac{2}{3}\Gamma^{\alpha}_{\mu\gamma}\Gamma^{\gamma}_{\nu\beta}\Gamma^{\beta}_{\rho\alpha}\right]. \tag{24}$$

Inspired by this construction we now come to the formulation of the NMG model. The only difference with the previous construction is that we now start from the Fierz-Pauli model, indicating the fundamental field by $\tilde{h}_{\mu\nu}$, instead of the $\sqrt{\text{Fierz}-\text{Pauli}}$ model. We encounter precisely the same constraint (20) which is solved exactly as before, see (21), in terms of a new fundamental field $h_{\mu\nu}$. This leads us to the following equations of motion for $h_{\mu\nu}$:

$$\big(\partial^{\rho}\partial_{\rho} - m^2\big)G^{\text{lin}}_{\mu\nu}(h) = 0\,, \qquad R^{\text{lin}}(h) = 0\,. \tag{25}$$

These equations of motion can be integrated to an action which turns out to be the linearization of the following non-linear action describing the NMG model [4]:

$$S_{\text{NMG}}[g] = \frac{1}{\kappa^2}\int d^3x\,\sqrt{-g}\big(-R + \frac{1}{m^2}K\big)\,, \tag{26}$$

where the curvature-squared terms K are given in (1). Note that at the linearized level the Fierz-Pauli and NMG models are equivalent. However, only one of them, the NMG model, can be extended to the non-linear level. We have verified, by a canonical analysis, that the action (26) describes two equal-mass particles of helicity ± 2 which both are unitary [10].

The TMG and NMG models can easily be combined into the following "generalized" massive gravity (GMG) model:

$$S[g] = \frac{1}{\kappa^2} \int d^3x \sqrt{-g} \left[-R + \frac{1}{m^2} K \right] + \frac{1}{\mu} \mathscr{L}_{\rm LCS} \,. \tag{27}$$

This GMG model describes two particles, one of helicity +2 and mass m_+, and another one of helicity –2 and mass m_-. The mass parameters $m_\pm$ are related to the mass parameters m and μ, occurring in (27), as follows

$$m^2 = m_+ m_- \,, \qquad \mu = -\frac{m_+ m_-}{m_+ - m_-} \,. \tag{28}$$

The special case $m_+ = m_-$ leads to the NMG model with two equal-mass particles of helicity +2 and -2. Taking the limit $m_+ \rightarrow \infty$ one of the particles decouples and we are left with the TMG model which describes a single particle with helicity +2 or –2.

Finally, in order to obtain interesting black hole solutions, we can add a cosmological constant to the Lagrangean containing a new cosmological parameter λ. To stay flexible, when we study other vacua than the Minkowski vacuum, we also keep the sign $\sigma = \pm 1$ in front of the Einstein-Hilbert term free. This leads to the so-called "cosmological" GMG model:

$$S[g] = \frac{1}{\kappa^2} \int d^3x \sqrt{-g} \left[\sigma R + \frac{1}{m^2} K - 2\lambda m^2 \right] + \frac{1}{\mu} \mathscr{L}_{\rm LCS} \,. \tag{29}$$

Restricting to the NMG model its cosmological extension allows besides the Minkowski vacuum also other maximally symmetric de Sitter and anti-de Sitter vacua. Requiring a maximally symmetric vacuum solution $G_{\mu\nu} = -\Lambda g_{\mu\nu}$, with cosmological constant Λ, leads, after substitution into the equations of motion, to the following quadratic equation for Λ [12] :

$$\Lambda^2 + 4m^2 \sigma \Lambda - 4\lambda m^4 = 0 \,. \tag{30}$$

For $\sigma = -1$ and $m^2 > 0$ this leads to the vacua indicated by the parabola in Figure 1. For each choice of the cosmological parameter $\lambda > -1$ in the Lagrangean there are two possible values of the cosmological constant Λ. Note that even for $\lambda = 0$ there exists besides the Minkowski vacuum a de Sitter vacuum solution [13]. For the critical value $\lambda = -1$ the two vacuum backgrounds coincide while for $\lambda < -1$ there are no maximally symmetric solutions.

Considering fluctuations around the different vacuum backgrounds, we encounter the same issues as in the TMG model. There is one region of backgrounds, indicated by the fat line in Figure 1, where the spin-2 particles have positive energy and another disjoint region, indicated by the dashed line, where the BTZ black hole solutions, allowed by

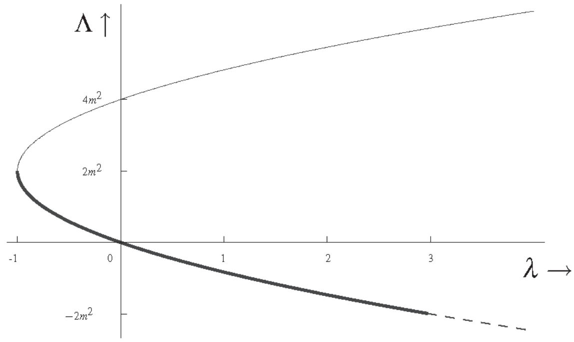

FIGURE 1. The vacuum solutions of the cosmological NMG model indicated in the Figure apply to the case $\sigma = -1, m^2 > 0$. The fat (dashed) line indicates the vacua around which the spin-2 particles (BTZ black holes) have positive energy.

the NMG model, have positive energy. The turn-over point happens at $\lambda = 3$ which is another point where the vacuum solution develops special properties [12].

Turning on the LCS term, corresponding to the GMG model, leads, for a critical value of the parameters, to the extra possibility to decouple the bulk spin-2 particles. This is in complete analogy to the TMG model [14]. However, like in the TMG model, at this critical point new logarithmic solutions develop [15] and the original problem of defining a theory with only positive energy solutions remains [16, 17]. The situation for the cosmological GMG model has been investigated in [18].

Instead of focussing on the properties of the cosmological GMG model we now turn our attention to unitary massive supergravity models. One of the lessons to be drawn from these studies is that the presence of auxiliary fields, which are rather harmless in standard supergravity theories, can have a dramatic impact on the properties of higher-derivative supergravity models.

MASSIVE SUPERGRAVITY

Our starting point is the off-shell $\mathscr{N} = 1$ supergravity multiplet

$$\{e_\mu{}^a\,, \psi_\mu\,, S\}\,, \tag{31}$$

where $e_\mu{}^a$ is the dreibein, ψ_μ the gravitino and S a real scalar auxiliary field. Without higher-derivative terms, the purpose of the auxiliary field is to provide for a off-shell closed supersymmetry algebra, i.e. the commutator of two supersymmetries closes without the use of any equation of motion. Its dynamics is trivial in the sense that it occurs like $\mathscr{L} \sim S^2$ in the Lagrangean leading to the rather trivial equation of motion $S = 0$.

It turns out that in the supersymmetry rules the dreibein and the auxiliary scalar naturally occurs together in the form of a torsionfull spin connection [10]:

$$\Omega_{\pm\mu}{}^{ab} = \omega_\mu{}^{ab}(e) \,\pm\, S\varepsilon_\mu{}^{ab}\,, \tag{32}$$

where $\omega_\mu{}^{ab}(e)$ is the usual spin connection and ε^{abc} is the three-dimensional Levi-Civita symbol. We use here both signs of the torsion since these signs are connected to each other inside the Riemann tensor according to the following identity

$$R_{ab\,cd}(\Omega_+) = R_{cd\,ab}(\Omega_-)\,. \tag{33}$$

It turns out that there exist two possible $\mathcal{N} = 1$ supersymmetric R^2-type invariants. The first one is a supersymmetrization of a Riemann-tensor squared term which in $D = 3$ is a combination of Ricci tensor and Ricci scalar squared terms which is different from the K combination occurring in the NMG model. The explicit construction of this Riemann tensor squared invariant is facilitated by making use of a formal analogy between Poincare supergravity and the off-shell vector multiplet

$$\{A^I_\mu\,, \chi^I\}\,, \tag{34}$$

where I is a Yang-Mills index. A supersymmetric invariant for this vector multiplet, which starts with the kinetic term for the vector field $\mathcal{L} \sim (F^I_{\mu\nu})^2$, is easily constructed. We first make use of the observation that the supersymmetry rule of the torsionfull spin connection $\Omega_{+\mu}{}^{ab}$ is in form precisely the same as the one of the vector field A^I_μ provided we consider the anti-symmetric Lorentz pair ab as a Yang-Mills index I. A supersymmetric action for the Riemnann-tensor squared invariant is now easily constructed by starting from the supersymmetric invariant for the vector multiplet and next making the following substitutions:

$$\{A^I_\mu\,, \chi^I\} \;\; \rightarrow \{\Omega_{+\mu}{}^{ab}\,, \psi^{ab}\}\,, \tag{35}$$

where ψ^{ab} is the gravitino curvature tensor.

For the construction of the second supersymmetric invariant, which starts with the Ricci scalar squared term, we make use of a formal analogy between Poincare supergravity and the off-shell scalar multiplet

$$\{\phi\,, \lambda\,, f\}\,, \tag{36}$$

where ϕ is the physical scalar, λ its supersymmetric partner and f an auxiliary scalar, i.e. $\mathcal{L} \sim (\partial_\mu \phi)^2 + f^2$. In this case we observe that the supersymmetry rules of ϕ and f behave precisely the same as those of S and the super-covariant Ricci scalar $\hat{R}(\Omega)$, respectively. Again, it is a relatively easy matter to construct a supersymmetric invariant for the off-shell scalar multiplet. The supersymmetric Ricci scalar invariant is next obtained by making the following substitutions in the action for the scalar multiplet:

$$\{\phi\,, \lambda\,, f\} \;\rightarrow\; \{S\,, \gamma^{ab}\psi_{ab}\,, \hat{R}(\Omega)\}\,. \tag{37}$$

Note that both supersymmetric invariants contain a kinetic term for the auxiliary field S. However, the combination K occurring in the NMG model is such that the kinetic term drops out leading to an algebraic equation of motion for S.

We are now ready to discuss the existence of supersymmetric AdS vacua solutions. Before doing that we must also embed the cosmological term in a supersymmetric invariant. In this case that is the invariant which starts with a term linear in S, i.e.

$$-2\lambda m^2 \rightarrow MS\,. \tag{38}$$

In pure supergravity this term combines with the $-2\sigma S^2$ term of the supergravity action to yield the equation of motion

$$S = \frac{1}{4}\sigma M\,. \tag{39}$$

Imposing the constraint $\delta_{SUSY}\psi_\mu = 0$ for a supersymmetric AdS vacuum leads to the condition $S^2 = -\Lambda$. This means, in particular, that Λ has to be negative: supersymmetric de Sitter vacua do not exist. Substituting this constraint on Λ into the equations of motion we find that these equations are satisfied provided S is given by the expression given in (39). But this is precisely the expression we got from pure supergravity in the absence of any higher-derivative terms! What has happened is that the non-zero value of the auxiliary scalar S precisely parallelizes the Riemann curvature tensor, i.e.

$$R_{\mu\nu}{}^{ab}(\Omega_+)|_{S^2=-\Lambda} = 0\,. \tag{40}$$

The lesson we draw from this is that auxiliary fields in higher-derivative supergravity theories play a non-trivial role. In the present case, the construction of a supersymmetric AdS vacuum would have been impossible without the help of the auxiliary scalar S.

CONCLUSION

We have constructed the first example of a power counting renormalizable and unitary theory of gravity, be it a massive gravity theory in three spacetime dimensions. It remains to be seen whether an explicit quantum calculation confirms this picture. It would be interesting to see whether there is a relation between NMG and the bigravity model of [19].

We briefly discussed the supersymmetrization of the NMG model and concluded that the role of the auxiliary scalar is non-trivial: it makes the construction of a supersymmetric AdS vacuum possible. It would be interesting to extend this result to $\mathcal{N} > 1$ supersymmetry. In particular, the $\mathcal{N} = 8$ case is interesting since the $\mathcal{N} = 8$ new massive supergravity model is expected to provide for a realization of the $\mathcal{N} = 8$ massive supermultiplet that underlies the famous $\mathcal{N} = 8$ supergravity theory in four spacetime dimensions.

One way to investigate further the NMG model is via the so-called AdS_3/CFT_2 correspondence, along the lines of [20]. Even if the outcome would be that the two-dimensional boundary conformal field theory is non-unitary, like it is the case for the TMG model, such a classical gravity description of a two-dimensional CFT is interesting in view of possible condensed matter applications. With this application in mind, recently black hole solutions have been constructed that do not asympote to AdS but to a Lifshitz metric [21].

As a final note we wish to mention that it would be interesting to see whether the NMG model is related to the recently proposed Horăva-Lifshitz model of quantum gravity in four spacetime dimensions [22].

ACKNOWLEDGMENTS

The work presented in this talk is based on references [4, 12, 10] and on work in progress [11]. I wish to thank my collaborators Roel Andringa, Mees de Roo, Olaf Hohm, Ergin Sezgin and Paul Townsend for many insightful discussions. I also wish to thank the organizers of the XXV Max Born Symposium for a very stimulating conference.

REFERENCES

1. K. S. Stelle, "Renormalization Of Higher Derivative Quantum Gravity," Phys. Rev. D **16** (1977) 953.
2. S. Deser, R. Jackiw and G. 't Hooft, "Three-Dimensional Einstein Gravity: Dynamics Of Flat Space," Annals Phys. **152** (1984) 220.
3. S. Deser, R. Jackiw and S. Templeton, "Topologically massive gauge theories," Annals Phys. **140** (1982) 372 [Erratum-ibid. **185** (1988 APNYA,281,409-449.2000) 406.1988 APNYA,281,409].
4. E. A. Bergshoeff, O. Hohm and P. K. Townsend, "Massive Gravity in Three Dimensions," Phys. Rev. Lett. **102** (2009) 201301 [arXiv:0901.1766 [hep-th]].
5. M. Nakasone and I. Oda, "On Unitarity of Massive Gravity in Three Dimensions," arXiv:0902.3531 [hep-th], M. Nakasone and I. Oda, "Massive Gravity with Mass Term in Three Dimensions," arXiv:0903.1459 [hep-th].
6. S. Deser, "Ghost-free, finite, fourth order D=3 (alas) gravity," arXiv:0904.4473 [hep-th].
7. I. Oda, "Renormalizability of Massive Gravity in Three Dimensions," JHEP **0905** (2009) 064 [arXiv:0904.2833 [hep-th]].
8. P. K. Townsend, K. Pilch and P. van Nieuwenhuizen, "Selfduality In Odd Dimensions," Phys. Lett. **136B** (1984) 38 [Addendum-ibid. **137B** (1984) 443].
9. S. Deser, R. Jackiw and S. Templeton, "Three-Dimensional Massive Gauge Theories," Phys. Rev. Lett. **48** (1982) 975.
10. R. Andringa, E. A. Bergshoeff, M. de Roo, O. Hohm, E. Sezgin and P. K. Townsend, "Massive 3D Supergravity," arXiv:0907.4658 [hep-th].
11. E. A. Bergshoeff, O. Hohm and P. K. Townsend, "in preparation".
12. E. A. Bergshoeff, O. Hohm and P. K. Townsend, "More on Massive 3D Gravity," Phys. Rev. D **79** (2009) 124042 [arXiv:0905.1259 [hep-th]].
13. D. G. Boulware and S. Deser, "String Generated Gravity Models," Phys. Rev. Lett. **55** (1985) 2656.
14. W. Li, W. Song and A. Strominger, "Chiral Gravity in Three Dimensions," JHEP **0804** (2008) 082 [arXiv:0801.4566 [hep-th]].
15. D. Grumiller and N. Johansson, "Instability in cosmological topologically massive gravity at the chiral JHEP **0807** (2008) 134 [arXiv:0805.2610 [hep-th]].
16. S. Carlip, S. Deser, A. Waldron and D. K. Wise, "Cosmological Topologically Massive Gravitons and Photons," Class. Quant. Grav. **26** (2009) 075008 [arXiv:0803.3998 [hep-th]]; "Topologically Massive AdS Gravity," Phys. Lett. B **666** (2008) 272 [arXiv:0807.0486 [hep-th]].
17. A. Maloney, W. Song and A. Strominger, "Chiral Gravity, Log Gravity and Extremal CFT," arXiv:0903.4573 [hep-th].
18. Y. Liu and Y. W. Sun, "Consistent Boundary Conditions for New Massive Gravity in AdS_3," JHEP **0905** (2009) 039 [arXiv:0903.2933 [hep-th]]; "On the Generalized Massive Gravity in AdS_3," Phys. Rev. D **79** (2009) 126001 [arXiv:0904.0403 [hep-th]].
19. M. Banados and S. Theisen, "Three-dimensional massive gravity and the bigravity black hole," arXiv:0909.1163 [hep-th].
20. K. Skenderis, M. Taylor and B. C. van Rees, "Topologically Massive Gravity and the AdS/CFT Correspondence," arXiv:0906.4926 [hep-th].
21. E. Ayon-Beato, A. Garbarz, G. Giribet and M. Hassaine, "Lifshitz Black Hole in Three Dimensions," arXiv:0909.1347 [hep-th].
22. P. Hořava, "Quantum Gravity at a Lifshitz Point," Phys. Rev. D **79** (2009) 084008 [arXiv:0901.3775 [hep-th]].