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Abstract— **In recent years, unfalsified adaptive switching supervisory control (UASSC) has emerged as an effective technique for tackling the problem of controlling uncertain plants only on the basis of the plant I/O data. The aim of this paper is to construct a novel switching logic, which, when combined with appropriate test functions, makes it possible to extend UASSC, so far restricted to time-invariant systems, to the case of systems whose dynamics are subject to infrequent but possibly large variations.**

I. INTRODUCTION

One approach for controlling uncertain plants relies on the introduction of adaptation in the feedback loop. In recent years, *adaptive switching supervisory control* (ASSC) has emerged as an alternative to conventional countinuous adaptation. ASSC resembles an adaptive variant of classic gain-scheduling control, in that it extends gain-scheduling control to applications where the supervisor has only access to I/O data from a time-varying plant P belonging to a, possibly unknown, plant uncertainty set $\mathscr{P}$. More precisely, if P_t represents the time-invariant system that is obtained by freezing the time-varying parameters of the process P, a their values at time $t \in \mathbb{N} \triangleq \{0, 1, \ldots\}$, we assume that each P_t belongs to a class $\mathscr{P}$ of single-input single-output finite-dimensional linear time invariant systems (FDLTI).

In an ASSC system, a data-driven "high-level" unit S, called the supervisor, aims at controlling the uncertain system by switching at any time in feedback with P one controller from a finite family $\mathscr{C}$ of N FDLTI candidate controllers. The scheduling task (when to substitute the acting controller) and the routing task (which controller to switch on) are carried out in real-time by monitoring data-driven test functions (e.g. see [1] for an in-depth overview).

The input u and the output y of the plant are affected by unknown additive disturbances n_u and n_y, respectively (see fig. 1). The resulting noisy closed-loop switched system[1] $\Sigma \triangleq (P/C_{\sigma(\cdot)})$ can be represented as

$$\Sigma : \begin{cases} y(t) = P(u + n_u)(t) + n_y(t) \\ u(t) = C_{\sigma(t)}(r - y)(t) \end{cases}, \quad t \in \mathbb{N} \qquad (1)$$

where r is the reference to be tracked by y and $\sigma(t)$ the index that identifies the candidate controller connected in feedback to P at time t. In this paper, we consider the unfalsified ASSC (UASSC) framework initiated in [2]-[3]. In contrast to model-based schemes (e.g., in [4]-[7]), the UASSC supervisors infer the performance of the alternative candidate controllers without resorting to estimation errors constructed from a collection of nominal process models, making it possible to ensure stability under mild assumptions ([8]).

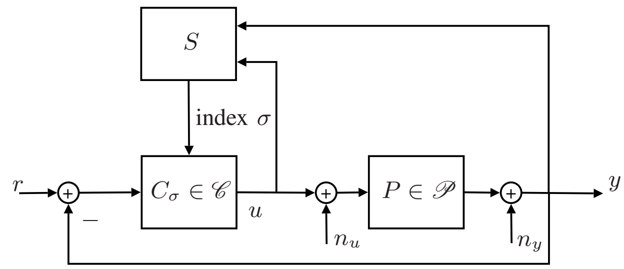

Fig. 1. Typical ASSC scheme.

To date, with the notable exception of [9], UASSC only provide stability guarantees for time-invariant process models, *viz.* $P_t = P^\circ \in \mathscr{P}$, $\forall t \in \mathbb{N}$. The Aim of this paper is to show that it is possible to extend UASSC to cover the case of systems whose dynamics are subject to infrequent but potentially large variations.

II. UNFALSIFIED ASSC BACKGROUND

Let $\mathbb{S}$ be the linear space of all the real-valued sequences on $\mathbb{N}$. Given an infinite sequence $s \in \mathbb{S}$, we denote its truncation up to time t by $s^t \triangleq \{s(0), s(1), \ldots, s(t)\}$. The l_∞ norm of the truncated sequence s^t is defined as

$$\|s^t\|_\infty \triangleq \max_{\tau \in \{0,1,\ldots,t\}} |s(\tau)| ,$$

and the linear space consisting of the uniformly bounded sequences $s \in \mathbb{S}$ is denoted as $l_\infty(\mathbb{N})$. To any sequence s belonging to $l_\infty(\mathbb{N})$ one can associate the norm

$$\|s\|_\infty \triangleq \lim_{t \to \infty} \|s^t\|_\infty .$$

Further, given a positive real $\lambda < 1$, the λ-exponentially weighted l_2 norm of a truncated sequence s^t is defined as

$$\|s^t\|_{2,\lambda}^2 \triangleq \sum_{\tau=0}^{t} \lambda^{t-\tau} s^2(\tau) .$$

Finally, $|w|$ denotes the Euclidean norm of the vector w.

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[1]Hereafter, given a plant Π and a controller C, the notation (Π/C) is used to denote the linear system consisting of the plant Π fed-back by the controller C.

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Denoting by $z \triangleq [u, y]$ the I/O pair of the system P in (1), the following stability notion will be used:

Definition 1: Σ is said to be l_∞ stable if

$$r,\, n_u,\, n_y \in l_\infty(\mathbb{N}) \;\Rightarrow\; z \in l_\infty(\mathbb{N})\,, \tag{2}$$

for every bounded initial condition.

A pre-requisite for an ASSC system is that the set of candidate controllers $\mathscr{C}$ must be adequately chosen relatively to $\mathscr{P}$. In particular, we shall consider the following requirements:

A1 (*Problem Feasibility*): For each frozen model $P^\circ \in \mathscr{P}$, there must exist at least one index $i \in \underline{N}$ such that the spectral radius of (P°/C_i) is smaller than $\sqrt{\lambda} < 1$.

A2 : The reference r and the disturbances n_u and n_y belong to $l_\infty(\mathbb{N})$.

In UASSC, the feedback adaptation task of classic adaptive control is replaced by controller falsification. The active controller can be falsified via a comparative experiment, by resorting to the *virtual reference* concept introduced in [3]. Let each controller C_i, $i \in \underline{N} \triangleq \{1, 2, \cdots, N\}$, be represented by a difference equation of the form

$$C_i:\; R_i(d)u(t) = S_i(d)(r(t) - y(t)), \quad t \in \mathbb{N}\,,$$

with R_i and S_i coprime polynomials in the unit backward shift operator d, and R_i monic, *viz.* $R_i(0) = 1$. In UASSC, one solves in real-time the difference equation

$$R_i(d)u(t) = S_i(d)(v_i(t) - y(t)), \quad t \in \mathbb{N}\,, \tag{3}$$

with respect to the virtual reference v_i, which can be done provided that C_i is causal, and causally stably invertible (CCSI).

The recursion (3) is initialized at time $t = 0$ with zero initial conditions, *viz.* $v_i(t)$, $y(t)$, $u(t) = 0$ for $t < 0$. In words, v_i can be viewed as a fictitious reference that, if injected into the feedback system $\Sigma_i \triangleq (P/C_i)$, would reproduce z, that is, if the closed-loop Σ is intended as a causal transformation (1) mapping (r, n_u, n_y) into z, one has

$$z = \Sigma\left(r, n_u, n_y\right) = \Sigma_i\left(v_i, n_u, n_y\right).$$

A. Controller implementation and switching logic

There are many way for implementing the switching adaptive controller $C_{\sigma(\cdot)}$. At each switching time, the state of $C_{\sigma(t)}$ can be initialized to keep the control output as smooth as possible (bumpless transfer [11]), or a common-state multicontroller scheme can be used (e.g. [1]). For the sake of simplicity, we shall consider the following implementation: when the supervisor switches from C_j to C_i at time t, the state of C_i is chosen as

$$x_i(t) := \left[\varepsilon_i(t-1), \cdots \varepsilon_i(t-p), u(t-1) \cdots u(t-p)\right]'\,,$$

where $\varepsilon := v_i - y$ and p is the maximum order of any controller in $\mathscr{C}$, so that the output of C_i is given by

$$u(t) = \left[s_{i1}, \cdots, s_{ip}, -r_{i1}, \cdots, -r_{ip}\right] x_i(t) + s_{i0}\,\varepsilon(t) \tag{4}$$

where $\varepsilon := r - y$; s_{ij} and r_{ik}, $j+1 \in \underline{p+1}$ and $k \in \underline{p}$, are the coefficients of the polynomials $S_i(d) = \sum_{n=0}^{p} s_{in} d^n$ and, respectively, $R_i(d) = 1 + \sum_{n=1}^{p} r_{in} d^n$. By (3), a direct consequence of this implementation is that $v_{\sigma(t)} = r(t)$ for every $t \in \mathbb{N}$.

This kind of implementation hinges upon the existence of the virtual references v_i as defined in equation (3). Due to lack of space, the extension of UASSC to cover different controllers implementations will not be pursued here.

The choice of the control action to use, among all the available candidate controllers in $\mathscr{C}$, is carried out via the evaluation of N nonnegative *test functions* $\mathfrak{J}_i(t) := \mathscr{J}_i(t, z^t)$, $t \in \mathbb{N}$, each one associated with a specific candidate controller C_i. At every $t \in \mathbb{N}$, the supervisor compares the N test functionals and selects the controller index via the following *Hysteresis Switching Supervisory Logic* (HSSL):

$$\sigma(t+1) = \arg\min_{i \in \underline{N}} \left\{\mathfrak{J}_i(t) - h\,\delta_{i,\sigma(t)}\right\}\,, \tag{5}$$

where $h > 0$ is the hysteresis constant, $\delta_{i,j}$ the Kronecker's index, which is equal to one when $i = j$ and zero otherwise. The recursion (5) is initialized with some $\sigma(0) = \sigma_0$ arbitrarily chosen.

III. Stability results for LTI systems

We analyze first the case of time-invariant plant dynamics. In this case, $P_t \equiv P^\circ$, $\forall t \in \mathbb{N}$, and the switched system Σ reduces to $\Sigma^\circ = (P^\circ/C_{\sigma(\cdot)})$.

To this end, consider the class K of every possible switching functions σ. As a consequence of the results in [12], the next HSSL lemma establishes the limiting behavior of the switching closed-loop Σ° subject to (5).

Lemma 1: For any initial condition, any reference r, and any disturbances n_u, n_y, let $\sigma : \mathbb{N} \mapsto \underline{N}$ be the switching function resulting from (1) and (5). Then, assuming that

- $\mathfrak{J}_i(t)$ is monotone nondecreasing for every $i \in \underline{N}$, and every $\sigma \in K$;
- there exists a finite positive real J such that $\min_{i \in \underline{N}} \mathfrak{J}_i(t) \leq J$, for every $t \in \mathbb{N}$, and every $\sigma \in K$,

there is a finite time t_f beyond which σ is constant. Moreover, $\mathfrak{J}_{\sigma(t_f)}(t) \leq J + h$ for every $t \in \mathbb{N}$, and the total number of switches is bounded by $N\lceil J/h \rceil$.

Proof. We only need to prove the bound on the number of switches, since the other properties follow from [12]. To this end, notice that, by virtue of the HSSL (5) and the monotonicity of the test functions, $\mathfrak{J}_i$ increases at least by h every time the index i is falsified. Since $\min_{i \in \underline{N}} \mathfrak{J}_i(t) \leq J$, $t \in \mathbb{N}$, every index can be selected at most $\lceil J/h \rceil$ times. In fact, if it is selected one more time, its test functional would exceed J at the switch-on time, contradicting (5). ■

A. Plant input-output data behavior

To utilize the convergence properties of HSSL in the context of virtual references one needs to use test functions adapted to a specific type of input-output stability. In this respect, consider the following performance indices

$$\mathfrak{J}_i(t) \triangleq \max\{\mathfrak{L}_i(\tau),\, \tau \in \{0, \cdots, t\}\}, \quad t \in \mathbb{N}, \tag{6}$$

where

$$\mathfrak{L}_i(t) \triangleq \frac{|z(t)|}{\mu + \|v_i^t\|_{2,\lambda}}, \quad t \in \mathbb{N}, \tag{7}$$

and μ is a positive constant. As discussed hereafter, the test function (7) allows one to describe the input-ouput plant data trend obtained from Σ°. More precisely, it is possible to derive an upper-bound on the norm of the plant's state on each time interval over which a controller is not falsified.

To this end, first note that under assumptions **A1** and **A2** there exist indices $i \in \underline{N}$ such that the spectral radius of (P°/C_i) is smaller than $\sqrt{\lambda}$, so that an upper bound on the smallest test function in $\mathfrak{J}$ can be obtained. More specifically, by defining $e \triangleq \max\{\|r\|_\infty, \|n_y\|_\infty, \|n_u\|_\infty\}$ and

$$x(t) \triangleq [y(t-1), \cdots y(t-n), u(t-1) \cdots u(t-n)]'$$

n denoting the maximum among the orders of any process in $\mathscr{P}$ and any controller in $\mathscr{C}$, the next proposition follows:

Proposition 1: Let the plant P be time-invariant, and suppose that **A1** and **A2** hold. Then, there exist positive reals g_1, g_2, g_3 such that, for any $t \in \mathbb{N}$,

$$\min_{i\in\underline{N}} \mathfrak{J}_i(t) \le \frac{1}{\mu}\left(g_0\, |x(0)| + g_1\, e\right) + g_2 =: J^\circ. \tag{8}$$

Proof: The result follows from the fact that for any feedback system (P°/C_i) with spectral radius smaller than $\sqrt{\lambda}$, we have (cf. [10])

$$|z(t)| \le g_0 \lambda^{t/2}\, |x(0)| + g_1 e + g_2\, \|v_i^t\|_{2,\lambda}, \quad t \in \mathbb{N}$$

where the g_k's are the maximum values taken amongst all the stable loops (P°/C_i), $P^\circ \in \mathscr{P}$. ■

Hereafter, w.l.o.g. we assume that $g_0 \ge 1$. Consider now the sequence $\{i_j\}_{j\in\mathbb{N}}$, $i_0 := 0$, of time instants at which the controller changes, so that

$$I_j \triangleq \{i_j, \cdots, i_{j+1} - 1\}, \quad j \in \mathbb{N}$$

represents an interval over which $\sigma(t) = \sigma_j$, $t \in I_j$. The bound J° can be used to establish a bound on the plant's state $x(t)$ over I_j as long as σ_j is not falsified. On the other hand, the performance degradation causing the switch from one controller to another one at time i_{j+1} can be upper bounded as a function of the sets $(\mathscr{P}, \mathscr{C})$. More precisely, by virtue of the controller implementation (4) at the switch-on times, it is an easy matter to show that there exists a finite positive real $\delta \ge 1$ such that, for every $j \in \mathbb{N}$ and every $t \in I_j$

$$|x(t+1)| \le \delta\left(|x(t)| + |x_{\sigma_j}(i_j)| + e\right),$$

In this respect, it is convenient to summarize such a fact as follows:

Proposition 2: For every time-invariant feedback (P_t/C), $P_t \in \mathscr{P}$ and $C \in \mathscr{C}$, there exists a positive real $\delta \ge 1$ such that

$$|x(t+1)| \le \delta\left(|x(t)| + \|v_{\sigma_j}^{i_j - 1}\|_{2,\lambda} + e\right), \tag{9}$$

for every $j \in \mathbb{N}$, and $t \in I_j$. ■

On that basis, the main result of this section can be stated. The proof is omitted due to lack of space.

Theorem 1: Let the plant P be FDLTI, and the HSSL be used along with the test function (6). Provided that **A1** and **A2** hold, there exist constants α, β and γ such that

i) for every $j \in \mathbb{N}$, and $t \in I_j$

$$|x(t)| \le \chi_j, \quad \|v_i^t\|_{2,\lambda} \le A_j,\ i \in \underline{N}$$

where

$$\chi_j := n\, \max\{(J^\circ + h)\,(\gamma + A_{j-1}),\, |x(0)|\} \tag{10}$$

$$A_j := \alpha A_{j-1} + \beta\,(\chi_j + A_{j-1} + e) \tag{11}$$

with $A_{-1} = 0$, h is the hysteresis constant, and δ as taken from (9);

ii) Σ° is l_∞ stable, with $|x(t)| \le \chi_{j^\circ}$ for every $t \in \mathbb{N}$, where $j^\circ := N\, \lceil J^\circ/h \rceil$. ■

For future reference, it is convenient to summarize the results of Theorem 1 by saying that there exist functions φ, ψ (increasing w.r.t. all their arguments) such that, for any initial condition and exogenous inputs,

$$\begin{aligned} |x(t)| &\le \varphi(|x(0)|, A_{-1}, J^\circ, e), \quad t \in \mathbb{N} \\ \|v_i^t\|_{2,\lambda} &\le \psi(|x(0)|, A_{-1}, J^\circ, e), \quad t \in \mathbb{N},\ i \in \underline{N} \end{aligned}$$

The reason for highlighting the dependence of the upper bounds on A_{-1} (even if in the context of Theorem 1 A_{-1} is always initialized at 0) will become clear when we address the case of a time-varying plants.

Remark 1: The proof of Theorem 1 suggests that linearity is not key to this result. In particular, it appears that it is possible to derive analogous results for nonlinear systems having an input-output decription of the type

$$y(t) = g(u(t-1), \cdots, u(t-n), y(t-1), \cdots, y(t-n))$$

where $g : \mathbb{R}^{2n} \mapsto \mathbb{R}$ is a sufficiently smooth nonlinear operator (e.g. globally Lipschitz), with (9) replaced by

$$|x(t+1)| \le f\left(|x(t)|,\, \|v_{\sigma_j}^{i_j-1}\|_{2,\lambda},\, e\right),$$

for some nonlinear function $f : \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \mapsto \mathbb{R}_+$. ♢

IV. A HYSTERESIS SWITCHING SUPERVISORY LOGIC WITH RESET

As in most approaches to ASSC, the monotonicity of the test functions is a key property for the analysis of the switching mechanism (cf. [6]). In the specific context of UASSC, the adoption of the maximum operator is needed to ensure that all the test functions always admit a limit (see Lemma 1). However, in dealing with systems whose dynamics vary with time, one needs to consider switching policies that discount old data, which may not be not representative of the current system behavior.

A simple mechanism that can be used to accomplish this consists of resetting the maximum operator in (6). Unfortunately, this complicates the analysis significantly because one loses the monotonicity of the test functions. The remaining part of this section shows how an appopriate reset rule can be devised to forget old data without compromising stability.. Let $\{t_k\}_{k\in\mathbb{N}}$, $t_0 := 0$, be the sequence of time instants at which the supervisor resets the computation of the maximum in the test functions (6) so that, over each interval

$$T_k \stackrel{\triangle}{=} \{t_k, \cdots, t_{k+1}-1\}\,, \quad k \in \mathbb{N},$$

each test function (6) can be replaced by

$$\mathfrak{J}_i(t) \stackrel{\triangle}{=} \max\{\mathfrak{L}_i(\tau),\, \tau \in \{t_k, \cdots, t\}\}, \quad t \in T_k. \tag{12}$$

To illustrate how the reset instants t_k's are chosen so as to ensure stability to the feedback control system Σ, some preliminary definitions are needed. Let

$$A(t) \stackrel{\triangle}{=} \max\{\|v_i^\tau\|_{2,\lambda},\, i \in \underline{N}, \tau \in \{t_k, \cdots, t\}\} \tag{13}$$

for every $t \in T_k$, and let

$$\begin{aligned} t^* \stackrel{\triangle}{=} \min\{t \in T_k \mid\ & \sigma(t+j) = \sigma(t)\ \& \\ & A(t+j) \le A(t) + h_1,\, j > 0\} \end{aligned} \tag{14}$$

for some positive hysteresis constant h_1. In words, the difference $t - t^*$ provides the magnitude of the elapsed time since i) the maximum operator was last reset, ii) the acting controller was last changed, and iii) $A(t)$ has last increased more than the hysteresis constant h_1. Since the greater the difference $t - t^*$ the longer the time interval during which the ASSC system Σ has shown a "steady" behavior, the following reset rule can be introduced.

Reset Condition: Given t_k, $k \in \mathbb{N}$, and a positive real ρ, $\sqrt{\lambda} < \rho < 1$, one has a reset at time $t_{k+1} > t^*$ provided that

$$\rho^{\,t_{k+1}-t^*} A(t^*) < \epsilon \tag{15}$$

for some finite positive constant ϵ. ■

With this reset mechanism, the HSSL in (5) is replaced by the following *HSSL with Reset* (HSSLR):

Algorithm HSSLR:

1 Set $k := 0$, $t^* := -1$, $t := 0$; choose $\sigma(0)$;
2 Compute $A(t)$;
3 If $\sigma(t) \neq \sigma(t^*)$ OR $A(t) > A(t^*) + h_1$ go to 5;
4 If $\rho^{t-t^*} A(t^*) < \epsilon$ set $k := k+1$, $t_k := t$ and go to 5; otherwise go to 6;
5 Set $t^* := t$;
6 Set $t := t+1$, compute $\sigma(t)$ and go to 2;

A. Properties of the reset rule

A first important observation is that, under suitable conditions, the proposed HSSLR always leads to a reset in finite time. Specifically, the next proposition is a straightforward consequence of the reset rule (15).

Proposition 3: Let the HSSLR be used along with the test functions (12) and suppose that there exist positive reals $\overline{A}_k$ and $\overline{J}_k$ such that, for every $t \in \{t_k, \ldots, t_k + \Delta_k\}$,

$$A(t) < \overline{A}_k\,, \quad \min_{i\in\underline{N}} \mathfrak{J}_i(t) \le \overline{J}_k\,,$$

where

$$\Delta_k := \left\lceil \frac{\overline{J}_k}{h} \right\rceil \left\lceil \frac{\overline{A}_k}{h_1} \right\rceil \log_\rho\left(\frac{\epsilon}{\overline{A}_k}\right). \tag{16}$$

Then, a reset always occurs in a finite time, and, in particular,

$$t_{k+1} - t_k \le \Delta_k\,.$$

■

Eq. (15) can be used to provide a condition that still allows the supervisor to detect instability, even though the reset mechanism "erases" data about the past process state. More precisely, it can be shown that the denominators of the test functions as well as the plant input-output data $x(t)$ are always uniformely bounded when a reset occurs, provided that the following design condition is enforced:

Design Condition: The parameter λ in (7) must be such that, for every $C_i \in \mathscr{C}$, the spectral radius of C_i^{-1} is smaller than $\sqrt{\lambda}$.

Note that this design condition can always be satisfied once the controllers are CCSI. By defining

$$J_k^\circ \stackrel{\triangle}{=} \min_{i\in\underline{N}} \mathfrak{J}_i(t_{k+1}-1)\,, \tag{17}$$

the following result can be stated.

Lemma 2: Let n denote the maximum among the orders of any plant in $\mathscr{P}$ and any controller in $\mathscr{C}$, and let the HSSLR be used along with the test functions (12). Then, if

- assumptions **A1** and **A2** hold and
- the design condition is satisfied,

i) there exist finite positive constants γ_1, γ_2, and η such that, for every $k \in \mathbb{N}$

$$\|v_i^{t_k-1}\|_{2,\lambda} \le A_k^\circ \stackrel{\triangle}{=} \gamma_1 + \gamma_2\, J_{k-1}^\circ, \quad i \in \underline{N} \tag{18}$$

$$|x(t_k)| \le \chi_k^\circ \stackrel{\triangle}{=} \eta \max_{j\in\{-1,\ldots,k-1\}} J_j^\circ \tag{19}$$

where $J_{-1}^\circ := J^\circ$.

ii) whenever $t_k - t_q \ge n$, $k > q$, we have

$$\chi_k^\circ = \eta \max_{j\in\{q,\ldots,k-1\}} J_j^\circ \tag{20}$$

Proof: See the appendix. ■

Thanks to Lemma 2, it is immediate to conclude that, over each interval T_k, the same recursions of Theorem 1 hold. More precisely, for every $I_j \subseteq T_k$

$$\chi_j = n \max\{(J_k^\circ + h)(\gamma + A_{j-1}),\, \chi_k^\circ\}\,,$$

$$A_j = \alpha A_{j-1} + \beta\left(\chi_j + A_{j-1} + e\right),$$

with $A_{-1} := A_k^\circ$. This fact can be summarized as follows:

Proposition 4: Under the assumptions of Lemma 2, for every $t \in T_k$

$$\begin{aligned}|x(t)| &\leq \varphi(\chi_k^\circ, A_k^\circ, J_k^\circ, e),\\ \|v_i^t\|_{2,\lambda} &\leq \psi(\chi_k^\circ, A_k^\circ, J_k^\circ, e), \quad i \in \underline{N},\end{aligned}$$

where the functions φ and ψ are defined as in Section III. ■

By virtue of the previous results, it is simple to show that, when the plant P is FDLTI, the stability of the switched system Σ is preserved under the proposed HSSLR. Indeed, in light of Proposition 4, one has $J_k^\circ \leq J^\circ$ for every $k \in \mathbb{N}$. from which one concludes that for every $t \in \mathbb{N}$,

$$|x(t)| \leq \varphi(\eta\, J^\circ, \gamma_1 + \gamma_2\, J^\circ, J^\circ, e), \tag{21}$$

$$\|v_i^t\|_{2,\lambda} \leq \psi(\eta\, J^\circ, \gamma_1 + \gamma_2\, J^\circ, J^\circ, e), \quad i \in \underline{N}\,. \tag{22}$$

Furthermore, by invoking Proposition 3 with $\overline{J}_k = J^\circ$ and $\overline{A}_k$ equal to the r.h.s. of (22), an upper bound on the maximum interval between consecutive resets is obtained. We shall see next, that similar stability results can be derived also in the case of finite-dimensional linear time-varying (FDLTV) systems subject to potentially large but infrequent jumps in the dynamics.

V. Stability results for time-varying plants

The reset criterion (15) serves as a basis for the analysis of FDLTV plants. In particular, for systems subject to infrequent but sudden potentially large changes in their dynamics. To this end, let $\{\ell_c\}_{c\in\mathbb{N}}$, with $\ell_0 := 0$, be the subsequence of time instants at which changes occur in the plant dynamics, so that

$$L_c \triangleq \{\ell_c, \cdots, \ell_{c+1} - 1\}$$

represents a time interval over which $P_t = P_{\ell_c}$, $t \in L_c$. Due to assumption **A1**, for every $t \in L_c$, there exist indices $i \in \underline{N}$ such that

$$|z(t)| \leq g_0\, \lambda^{(t-\ell_c)/2}\, |x(\ell_c)| + g_1\, e + g_2\, \|v_i^t\|_{2,\lambda}\,. \tag{23}$$

Then, the following proposition can be stated, whose proof is omitted due to lack of space.

Proposition 5: Consider a finite time $\ell_c \in \mathbb{N}$. Then, under the assumptions of Lemma 2,

i) there exist finite positive reals $\overline{A}_c$ and $\overline{J}_c$ such that for every $t < \ell_{c+1}$, and every $i \in \underline{N}$

$$\|v_i^t\|_{2,\lambda} \leq \overline{A}_c\,, \quad \min_{i\in\underline{N}} \mathfrak{J}_i(t) \leq \overline{J}_c\,;$$

ii) for every $k \in \mathbb{N}$ such that $t_{k+1} < \ell_{c+1}$, we have

$$t_{k+1} - t_k \leq \left\lceil \frac{\overline{J}_c}{h} \right\rceil \left\lceil \frac{\overline{A}_c}{h_1} \right\rceil \log_\rho\left(\frac{\epsilon}{\overline{A}_c}\right) =: \Delta_c\,;$$

iii) there exists a finite time

$$\overline{\ell}_c := \Delta_c\left(2 + \left\lceil \frac{n}{\Delta_c} \right\rceil\right),$$

such that, if $\ell_{c+1} > \ell_c + \overline{\ell}_c$, we have

$$|x(t)| \leq \chi_\Sigma, \quad t \in \{\ell_c + \overline{\ell}_c, \cdots, \ell_{c+1}\} \tag{24}$$

$$\mathfrak{J}_i(t) \leq \chi_\Sigma/\mu, \quad t \in \{\ell_c + \overline{\ell}_c, \cdots, \ell_{c+1} - 1\},\ i \in \underline{N}, \tag{25}$$

where $\chi_\Sigma := \varphi(\eta\, J_\Sigma, \gamma_1 + \gamma_2\, J_\Sigma, J_\Sigma, e)$, and

$$J_\Sigma := \frac{1}{\mu}(g_0\, \epsilon + g_1\, e) + g_2\,,$$

where ϵ is the parameter of the reset condition (15). ■

As a consequence of these properties, the l_∞ stability of the ASSC system Σ turns out to be equivalent to the uniform boundedness of the sequences $\left\{\overline{J}_c\right\}_{c\in\mathbb{N}}$ and $\left\{\overline{A}_c\right\}_{c\in\mathbb{N}}$. As shown below, this is guaranteed provided that, for any $c \in \mathbb{N}$, the jumps in the plant dynamics occur each time after a prespecified dwell-time τ_{dwell}, i.e.,

$$\ell_{c+1} - \ell_c > \tau_{\text{dwell}} \;\Rightarrow\; \overline{J}_c \leq J^*\,, \quad \overline{A}_c \leq A^*\,,$$

for some finite positive reals J^* and A^*.

A. Dwell-Time Computation

We stress that the choice $\ell_0 = 0$ amounts to regard $t = 0$ as the first time at which the plant dynamics change. This choice is arbitrary and does not affect the generality of the result. Otherwise, an analogous analysis can be easily carried out. As pointed out in Proposition 5, for every $\ell_c \in \mathbb{N}$, there exists a finite time $\overline{\ell}_c$ after which (24) and (25) hold. In this respect, $\overline{\ell}_c$ represents the time instant after which a jump may occur in a safe way, since J_Σ does not depend on the switching history; being only a function of the sets $\mathscr{P}$ and $\mathscr{C}$. In fact, we can prove that, if $\ell_{c+1} > \ell_c + \overline{\ell}_c$ for any $c \in \mathbb{N}$, the smallest test function can be upper bounded as

$$\overline{J}_c \leq J^* = \frac{1}{\mu}\left(g_0\, \max\{|x(0)|,\, \chi_\Sigma\} + g_1\, e\right) + g_2, \tag{26}$$

and, as a staightforward application of Lemma 2 and Proposition 4, for every $t \in \mathbb{N}$, and $i \in \underline{N}$, one obtains

$$|x(t)| \leq \varphi(\eta J^*, \gamma_1 + \gamma_2 J^*, J^*, e) =: \chi^*, \tag{27}$$

$$\|v_i^t\|_{2,\lambda} \leq \psi(\eta J^*, \gamma_1 + \gamma_2 J^*, J^*, e) =: A^*, \tag{28}$$

(cf. the results derived in Section IV).

Theorem 2: Consider a FDLTV plant P, $P_t \in \mathscr{P}$ for every $t \in \mathbb{N}$. Under the assumptions of Lemma 2, the switched system Σ is l_∞ stable if, for every $c \in \mathbb{N}$,

$$\ell_{c+1} - \ell_c > \Delta\left(2 + \left\lceil \frac{n}{\Delta} \right\rceil\right) \triangleq \tau_{\text{dwell}}\,, \tag{29}$$

where

$$\Delta := \left\lceil \frac{J^*}{h} \right\rceil \left\lceil \frac{A^*}{h_1} \right\rceil \log_\rho\left(\frac{\epsilon}{A^*}\right),$$

where J^* and A^* as in (26), respectively, (28).

Proof. The proof can be constructed by showing that the dwell-time defined in (29) implies that (26) holds for every $c \in \mathbb{N}$. Consider $c = 0$, and notice that $J^* \geq J^\circ$, where J° as in (8) represents the worst-case minimum cost in case no jumps occur after $t = 0$. As a straightforward application of Proposition 5, we have

$$\mathfrak{J}_i(t) \leq \chi_\Sigma / \mu, \quad t \in \{\tau_{\text{dwell}}, \cdots, \ell_1 - 1\},\ i \in \underline{N},$$

and $|x(\ell_1)| \leq \chi_\Sigma$. Consequently, by (23), over the subsequent interval L_1, the worst-case minimum cost is still J^*. Then, it is immediate to conclude that the proof follows along similar lines for $c \geq 1$. ■

VI. Conclusions

In this paper we extend the UASSC approach to systems whose dynamics vary with time. The analysis underscores the potential benefit of UASSC in adaptive switching control. The absence of explicit requirements about the system linearity, indicate that it may be possible to further extend this method to some classes of nonlinear systems. Furthermore, this novel switching logic can also be used in conjunction with model-based test functions, making it possible to improve the closed loop behavior in case a set of nominal plant models be available ([13]).

Appendix

Proof of Lemma 2 i) Consider first $\|v_i^{t_k-1}\|_{2,\lambda}$. The statement is true for $k = 0$, since $\|v_i^{-1}\|_{2,\lambda} = 0$, $i \in \underline{N}$. Consider next $k \geq 1$. In case $t_k - t^* < n$ (cf. (14)), it is easy to check for every $t \in T_{k-1}$ and $i \in \underline{N}$

$$\|v_i^t\|_{2,\lambda} \leq A(t^*) + h_1 \leq \rho^{-n}\,\epsilon + h_1 \tag{30}$$

Consider now the case $t_k - t^* \geq n$. If σ denotes the index generating the reset at t_k, for every $t \in \{t^*, \cdots, t_k\}$

$$\begin{aligned}\|v_\sigma^t\|_{2,\lambda}^2 &= \lambda^{t-t^*+1}\,\|v_\sigma^{t^*-1}\|_{2,\lambda}^2 + \sum_{n=t^*}^{t} \lambda^{t-n}\,r^2(n) \\ &< \rho^{2\,(t-t^*)}\,(A^+)^2 \;+\; (1-\lambda)^{-1}\,\|r\|_\infty^2\end{aligned}$$

where $A^+ := A(t^*) + h_1$. Then, for any $t \in \{t^*, \cdots, t_k\}$

$$|z(t)| \leq J_{k-1}^+\left(\gamma + \rho^{t-t^*}\,A^+\right) \tag{31}$$

with $\gamma := \mu + (1-\lambda)^{-1/2}\,\|r\|_\infty$ and $J_k^+ := J_k^\circ + h$. Further, by virtue of Design Condition, the system (3) has a finite $l_{2,\lambda}$-to-$l_{2,\lambda}$ induced norm. This property implies that, if $\zeta_i(t) := [v_i(t-1), \cdots, v_i(t-n)]$, there exists a positive real c such that, for every $t \geq t^* + n$ and $i \in \underline{N}$

$$\|v_i^t\|_{2,\lambda}^2 \leq c^2 \lambda^{t-t^*} |\zeta_i(t^*+n)|^2 + c^2 \sum_{j=t^*+n}^{t} \lambda^{t-j}\,w_i^2(j)$$

where $w_i(t) := R_i(d)\,u(t) + S_i(d)\,y(t)$. Simple algebraic calculations show that for some positive real ν

$$\sum_{j=t^*+n}^{t} \lambda^{t-j}\,w_i^2(j) \leq \nu^2 \sum_{j=t^*+n}^{t} \lambda^{t-j} \max_{\iota \in \{j-n,\cdots,j\}} |z(\iota)|^2$$

However, thanks to (31), for every $j \geq t^* + n$

$$\max_{\iota \in \{j-n,\cdots,j\}} |z(\iota)| \leq J_{k-1}^+\left(\gamma + \rho^{j-t^*-n}\,A^+\right)$$

Since $\|v_i^t\|_{2,\lambda} \leq A^+$ for any $t \in T_{k-1}$ and $i \in \underline{N}$, we conclude that there exists a positive real κ such that, for every $t \geq t^* + n$

$$\|v_i^t\|_{2,\lambda} \leq \kappa\,\rho^{t-t^*} A^+ + \kappa\,J_{k-1}^+\left(\gamma + \rho^{t-t^*} A^+\right)$$

As a consequence, the fulfillment of (15) implies the existence of positive constants γ_1, γ_2 so that (18) holds.

We focus now the attention on $x(t_k)$. If $t_k - t^* \geq n$, it is immediate to conclude that for any $t \in \{t_k - n, \cdots, t_k\}$

$$|z(t)| \leq J_{k-1}^+\left(\gamma + \rho^{-n}\,\epsilon + h_1\right)$$

On the opposite, in case $t_k - t^* < n$, (30) implies that

$$|z(t)| \leq J_{k-1}^+\left(\mu + \rho^{-n}\,\epsilon + h_1\right), \quad t \in T_{k-1}$$

As a consequence, if t_s denotes the greatest reset time instant such that $t_k - t_s \geq n$, for any $t \in \{t_k - n, \cdots, t_k\}$

$$|z(t)| \leq \max_{j \in \{s,\ldots,k-1\}} J_j^+\left(\gamma + \rho^{-n}\,\epsilon + h_1\right) \tag{32}$$

If $t_k < n$ (hence $t_s < 0$), since $|x(0)| \leq J_{-1}^\circ\,\gamma = J^\circ\,\gamma$, we conclude that (32) holds with $s = -1$, i.e.

$$|x(t_k)| \leq n \max_{j \in \{-1,\ldots,k-1\}} J_j^+\left(\gamma + \rho^{-n}\,\epsilon + h_1\right)$$

which implies that (19) holds for some positive real η.

ii) It is a direct consequence of (32).

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