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A Distributed Reconfigurable Control Law for Escorting and Patrolling Missions using Teams of Unicycles

Ying Lan¹ Zhiyun Lin¹ Ming Cao² Gangfeng Yan¹

Abstract—Recent years have seen rapidly growing interest in the development of networks of vehicles for which adaptive cooperation and autonomous execution become a necessity. In the paper, we develop a distributed reconfigurable control law to distribute unicycle-type vehicles evenly on a circle surrounding a moving target for the escorting and patrolling missions. The even distribution of the vehicles provides the best overall coverage of the target in its surroundings. It is shown that as the target moves, the group formation moves and rotates around the target to keep the target around the formation centroid. When some vehicles in the group are lost due to faults, the remaining vehicles recognize the loss and adaptively reconfigure themselves to a new evenly distributed formation.

Keywords: Cooperative control; unicycles; escorting; patrolling

I. INTRODUCTION

Over the last few years, cooperation in multi-robot systems has received increasing attention. A network of relative simple and inexpensive agents, equipped with sensing and control devices, are invoked to fulfill complex tasks in a robust, fault-tolerant, and distributed manner. In distributed multi-agent systems, there is usually no centralized controller and each agent has to act based on its local sensing information. Up till now, the available literature has addressed a wide range of topics, e.g., consensus [18], formation control [19], coordinated path following [1], and cooperative target tracking [5], [9]. In this direction, the cooperative escorting and patrolling mission, which arises from security services, is also a fairly important research problem. Recently, [2] and [17] explicitly address the escorting and patrolling task, for which a group of vehicles are asked to surround and maintain close to a target whose motion is unknown a priori but can be measured in real time, and meanwhile collisions between each other are avoided. The problem is closely related to the problem of circular formation control and target-enclosing, but it has its own distinct features since the target object being escorted and patrolled works collaboratively with the autonomous vehicles rather than competing with them as in the target-enclosing problem. The target may feed back certain measured information to the vehicles via communication though it does not participate in coordinating its motion with others.

One can distinguish several control design approaches for the escorting/patrolling problem and related problems. By far, most approaches rely on simple models (single integrators) [6], [8], [9], centralized schemes [2], [17], or the knowledge of global information in the presence of a common reference frame [7], [11]. On one hand, the approaches developed for single-integrator model ([6], [8], [9]) have had limited success when applied to teams of unicycles due to nonlinearity and nonholonomic constraints which give rise to more challenges in control synthesis. On the other hand, the unicycle model is a common and practical model for mobile robots and unmanned aerial vehicles (UAVs). To overcome the difficulties appeared in unicycle-type vehicles, it is often assumed in some literature that absolute orientations and absolute positions with respect to a common reference frame are available (e.g., [7], [11]) so that the model can be transformed into a simple one via a global coordinate transformation. But from the practical point of view, it is more desirable not using global information as it is difficult or costly to acquire. Without allowing to utilize global information, the escorting and patrolling problem using teams of unicycle-type vehicles becomes even more challenging. Within the context of local control, several different control strategies have been explored. Some work makes use of the acceleration information of the target object [3], some work can only deal with three agents for the task [13], [14] and the problem for more than three vehicles remains an open problem, and some work obtains only experimental results without rigorous analysis [2], [17]. Moreover, some of these control approaches rely heavily on the total number of vehicles in coordination, which makes the group behavior not reconfigurable autonomously when the number of functioning vehicles changes.

To date, for the escorting and patrolling problem using teams of unicycles, none of the various control approaches have produced a satisfying reconfigurable control law based on only local sensing information with provable stability properties. Developing such a reconfigurable control law based on only local sensing information and proving stability are the primary goal of this paper. In a related problem, a hybrid control approach is proposed in [15] to enclose and rotate around a stationary target object with an evenly spaced distribution. The approach results in a reconfigurable control law for which when some vehicles become malfunctioning, the remaining vehicles autonomously reconfigure themselves to a new evenly spaced distribution. The paper aims at continuing this idea and developing control approaches for the escorting and patrolling mission, in which the target

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object is moving. However, when a target object has a varying velocity profile, even though the velocity of the target object is available to all the escorting and patrolling vehicles, it is still difficult to eliminate the translation. One of the objectives and one of the main contributions of the present work are to derive a control law that can nearly eliminate the translation and can be designed in a way that greatly decreases the amount of measurements necessary for the control. In the paper, a control scheme is developed where each vehicle utilizes the local information of distance and bearing angle about the target and its two neighbors called *pre-neighbor* and *next-neighbor*, which can be measured by onboard sensors, and utilizes the information of linear speed and bearing angle measured by the target object, which is available to the vehicles via communication from the target object. With the proposed control scheme, we show that a group of vehicles are able to follow and rotate around the smoothly moving target to keep the target around their centroid if initially the vehicles are ready for escorting and patrolling. From theoretic point of view, if some other control laws can be designed to steer the vehicles near the target with appropriate postures, a switching control law can always be derived based on our results to solve the problem globally. Also, we show that with our proposed control law, collisions among the group of cooperating vehicles and between the vehicles and the target object are assured not to occur. In addition, the control law presented in the paper is distributed and scalable. When some vehicles in the group become malfunctioning due to faults, the remaining vehicles can recognize and adaptively reconfigure themselves into a new formation to accomplish the task.

II. PROBLEM FORMULATION

The mission of escorting and patrolling a target is the task to maintain a formation around a given target. As the target moves, the formation moves (or furthermore, rotates around the target) to keep the target at its centroid, maintains the desired distances from the formation vehicles to the target, and distributes the formation vehicles around the target evenly in angle [2], [17]. The equal-angle spacing gives the team the best chance to track the target in the presence of occlusions and minimizes the intruding possibilities of an external agent.

In the following, we formulate the escorting and patrolling problem for networks of unicycle-type vehicles, which can also be applied to unmanned air vehicle (UAV) systems. Consider a group of n unicycles labeled 1 through n . For any unicycle i ($i = 1, \dots, n$), its posture is described by $q_i = (x_i, y_i, \theta_i)^T \in \mathbb{R}^2 \times [-\pi, \pi)$, where (x_i, y_i) denotes its representing point defined in an inertia coordinate frame \mathcal{W} , and the angle θ_i is its orientation with respect to the x -axis. Each unicycle has the following dynamics

$$\dot{q}_i = \begin{pmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{pmatrix} = \begin{pmatrix} v_i \cos \theta_i \\ v_i \sin \theta_i \\ \omega_i \end{pmatrix} \quad (1)$$

where the linear velocity v_i and angular velocity ω_i are the

control inputs which are subject to physical constraints, i.e., $0 < v_i \leq v_{max}$ and $|\omega_i| \leq \omega_{max}$. The target is labeled 0, whose position is denoted by (x_0, y_0) , whose orientation is denoted by θ_0 , and whose forward speed is denoted by v_0 . It is assumed that it can move freely in the plane, its linear speed v_0 and angular speed ω_0 are smooth, and v_0 is upper-bounded, i.e., $|v_0| \leq v_{0,max}$ (where $v_{0,max} < v_{max}$).

For each vehicle i (including the target 0), we construct a moving frame, the Frenet-Serret frame, that is fixed on the vehicle with its origin at the representing point and x -axis coincident with the orientation of the vehicle. Denote d_{ij} the distance from vehicle i to vehicle j and α_{ij} the bearing angle of vehicle j in vehicle i 's Frenet-Serret frame (see Fig. 1).

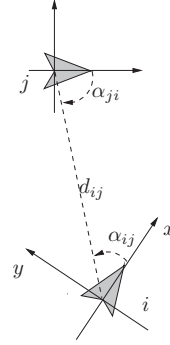


Fig. 1. The distance d_{ij} as well as the bearing angles α_{ij} and α_{ji} .

With respect to the target object, the dynamics of unicycle i ($i = 1, \dots, n$), which is not collocated with the target object, can be written as

$$\begin{cases} \dot{d}_{i0} = -v_i \cos \alpha_{i0} - v_0 \cos \alpha_{0i}, \\ \dot{\alpha}_{i0} = -\omega_i + \eta_i(v_i), \end{cases} \quad (2)$$

where $\eta_i : \mathbb{R} \rightarrow \mathbb{R}$ is defined as

$$\eta_i(\varsigma) = \frac{1}{d_{i0}} (\varsigma \sin \alpha_{i0} + v_0 \sin \alpha_{0i}). \quad (3)$$

Note that when $\sin \alpha_{i0} \neq 0$, the inverse function exists and has the following form: $\eta_i^{-1}(\varrho) = \frac{\varrho d_{i0} - v_0 \sin \alpha_{0i}}{\sin \alpha_{i0}}$.

Next, in order to formulate the evolution of the group formation, we evaluate the angle between the line through the target and vehicle i and the line through the target and vehicle j . More formally, we let ψ_{ij} be the angle formed by rotating the ray (originating at the target and pointing towards vehicle i) counterclockwise until meeting vehicle j . The angle ψ_{ij} is called the *separation angle from vehicle i to j* , which belongs to $[0, 2\pi)$ by our definition (see Fig. 2). Moreover, a vehicle, that is first met by rotating counterclockwise the ray originating at the target and pointing towards vehicle i , is called a *next-neighbor* of vehicle i . In a mathematical way, the *next-neighbor set* \mathcal{N}_i is defined as $\mathcal{N}_i := \{j | \psi_{ij} = \min_{k \neq i} \psi_{ik}\}$. Similarly, we define the *pre-neighbor set* \mathcal{P}_i of vehicle i as $\mathcal{P}_i := \{j | \psi_{ij} = \max_{k \neq i} \psi_{ik}\}$. A member in the pre-neighbor set \mathcal{P}_i is called a *pre-neighbor* of vehicle i . From the definitions, we can see that if there is a next-neighbor for vehicle i then there must be a pre-neighbor, and vice versa. Next, let $\psi_i^- := \psi_{ij} |_{j \in \mathcal{N}_i}$ and $\psi_i^+ := \psi_{ji} |_{j \in \mathcal{P}_i}$.

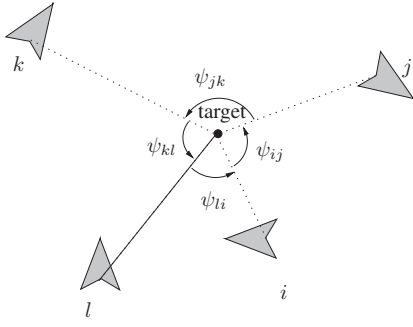


Fig. 2. The separation angles.

That is, ψ_i^- and ψ_i^+ are the separation angle from vehicle i to its next-neighbor and the separation angle from its pre-neighbor to vehicle i , respectively. With these notations, one obtains that for any vehicle $i = 1, \dots, n$, if there is only one next-neighbor, say $\mathcal{N}_i = \{j\}$, then the evolution of the separation angle ψ_i^- is governed by

$$\dot{\psi}_i^- = \eta_j(v_j) - \eta_i(v_i). \quad (4)$$

If there are more than one next-neighbors, then the dynamics of ψ_i^- can be derived as $\dot{\psi}_i^- = \min_{j \in \mathcal{N}_i} (\eta_j(v_j) - \eta_i(v_i))$.

Now we are ready to give the formal problem statement for the escorting and patrolling problem studied in the paper.

The Cooperative Escorting and Patrolling Problem: Devise v_i and ω_i for each vehicle $i = 1, \dots, n$ such that

- (i) the error evolution $(d_{i0}(t) - R, \alpha_{i0}(t) + \pi/2)$ for $i = 1, \dots, n$ is uniformly bounded, where R is the desired radius of the enclosing circle and $-\pi/2$ indicates the desired rotation direction;
- (ii) $\lim_{t \rightarrow \infty} \psi_i^-(t) = \dots = \lim_{t \rightarrow \infty} \psi_n^-(t) \geq 0$.

Instead of making $d_{i0}(t)$ and $\alpha_{i0}(t)$ converge to R and $-\pi/2$ exactly, condition (i) ensures that $d_{i0}(t)$ and $\alpha_{i0}(t)$ are kept remaining close to the desired values R and $-\pi/2$ if the bound is small, which represents a relaxed and practical objective for the escorting and patrolling mission. To solve the problem, we assume that

- 1) each vehicle i can measure the distance to the target object d_{i0} and the bearing angle α_{i0} ;
- 2) each vehicle i can obtain the separation angles ψ_i^- and ψ_i^+ if it has a next-neighbor and a pre-neighbor;
- 3) the target object communicates its linear speed v_0 and measured bearing angle α_{0i} to each agent i .

Remark 2.1: For the escorting and patrolling mission, the target object and the patrolling vehicles are collaborative rather than competitive. Hence, α_{0i} and v_0 can be available by vehicle i for the control purpose via communication or other collaboration manners. The reason to have these information is to overcome the difficulties caused by time-varying speed and possible rotations of the target object as each vehicle is hard or incapable of measuring the orientation and the moving speed of the target by onboard sensors. But compared to some work (e.g., [3], [7], [11]) requiring to know all information of the target object including both forward and angular speed and acceleration, or global infor-

mation such as absolute position and absolute orientation of the target object, the assumption is mild and greatly decreases the amount of required information.

III. MAIN RESULTS

In this section, we synthesize control laws for the cooperative escorting and patrolling problem described in the paper. That is, a group of vehicles follow the target, at the same time maintain a formation and rotate around the target to keep the target at its centroid approximately. In addition, we expect that no collision happens between formation vehicles in the cooperative escorting and patrolling process, which is a key issue in motion coordination for multi-agent systems.

A. Our control law

In this subsection, we propose a distributed reconfigurable control law. First, we introduce a set of states d_{i0} and α_{i0} , which corresponds to the situation that the vehicles locate near the enclosing circle and their orientations are close to the tangent direction of the enclosing circle.

Define a set

$$\mathcal{S} = \left\{ (d, \alpha) \mid \begin{array}{l} -a \leq \alpha + \pi/2 \leq a \\ b(\alpha + \pi/2) - a \leq d - R \leq b(\alpha + \pi/2) + a \end{array} \right\}$$

where a and b are constant values satisfying $0 < a < \pi/2$ and $b > 0$. In geometry $1/b$ is the slope of the line shown in Fig. 3. Describe the boundary of \mathcal{S} by

$$\begin{aligned} \mu_1 &= \{(d, \alpha) \in \mathcal{S} : \alpha + \pi/2 = a\}, \\ \mu_2 &= \{(d, \alpha) \in \mathcal{S} : d - R = b(\alpha + \pi/2) + a\}, \\ \mu_3 &= \{(d, \alpha) \in \mathcal{S} : \alpha + \pi/2 = -a\}, \\ \mu_4 &= \{(d, \alpha) \in \mathcal{S} : d - R = b(\alpha + \pi/2) - a\}. \end{aligned}$$

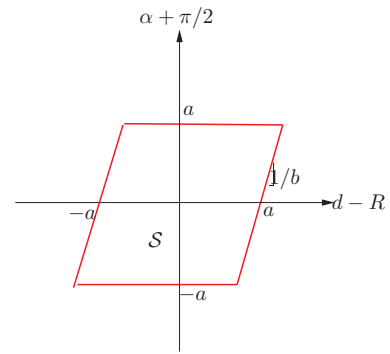


Fig. 3. Set \mathcal{S} .

Let n_j , $j = 1, \dots, 4$, be a normal vector of μ_j pointing outside of \mathcal{S} . Without loss of generality, select $n_1 = [0, 1]^T$ and $n_2 = [1, -b]^T$. we construct a distributed control law:

$$\begin{cases} v_i = \eta_i^{-1} [-(c_1 + \gamma_i \cdot (\psi_i^+ - \psi_i^-))] \\ \omega_i = \frac{1}{b} (v_i \cos \alpha_{i0} + v_0 \cos \alpha_{0i}) + \eta_i(v_i) \\ \quad - c_2 v_i [d_{i0} - R - (1+b)(\alpha_{i0} + \pi/2)] \end{cases} \quad (5)$$

where $\eta_i(\cdot)$ is the function defined in (3), b is the parameter used in defining the set \mathcal{S} , $c_1, c_2 > 0$ are constants of suitable values, and γ_i is a coefficient depending on the sign of $(\psi_i^+ - \psi_i^-)$, i.e., $\gamma_i = c_3$ (a positive constant) if $(\psi_i^+ - \psi_i^-) > 0$

and $\gamma_i = 0$ otherwise. It is worth pointing out that at this stage, the pre-neighbor and next-neighbor of each vehicle are those as defined in Section II, but their states must be in \mathcal{S} . Otherwise, they are treated as malfunctioning vehicles. When a vehicle i has neither pre-neighbor nor next-neighbor, we let $\gamma_i = 0$ so that it accomplishes the task by itself only.

Remark 3.1: Note that for states in \mathcal{S} , $\sin \alpha_{i0} \neq 0$, so the inverse function $\eta_i^{-1}(\cdot)$ exists and for each $i = 1, \dots, n$,

$$\begin{aligned} v_i &= \eta_i^{-1}[-(c_1 + \gamma_i \cdot (\psi_i^+ - \psi_i^-))] \\ &= -\frac{(c_1 + \gamma_i \cdot (\psi_i^+ - \psi_i^-))d_{i0} + v_0 \sin \alpha_{i0}}{\sin \alpha_{i0}} \end{aligned}$$

with d_{i0}, v_0, α_{i0} and α_{0i} are smooth functions. Denote $\beta_i = \psi_i^+ - \psi_i^-$. Clearly, $\beta_i \in [-2\pi, 2\pi]$ and by definition $\gamma_i \cdot \beta_i = \begin{cases} c_3 \beta_i, & \beta_i > 0, \\ 0, & \beta_i \leq 0. \end{cases}$ In addition, $\lim_{\beta_i \rightarrow 0^-} \gamma_i \cdot \beta_i = \lim_{\beta_i \rightarrow 0^+} \gamma_i \cdot \beta_i = 0$. It means that $\gamma_i \cdot \beta_i$ is continuous in the neighborhood of $\beta_i = 0$ and continuous for states in \mathcal{S} . Moreover, $\gamma_i \cdot \beta_i$ is Lipschitz with respect to β_i and the Lipschitz constant is c_3 . Therefore, the solution of the resulting closed-loop system locally exists and is unique for any initial state in \mathcal{S} .

B. Ensuring circling motion

In this subsection, we show that our proposed control law (5) can make the formation vehicles follow and rotate around the moving target approximately. In other words, if the vehicles are initially with states in \mathcal{S} , under control law (5), the states can be maintained in \mathcal{S} for smooth motion of the target. Denote $\xi_i = [d_{i0}, \alpha_{i0}]^T$. Substituting the control law ω_i in (5), we obtain the following closed-loop system:

$$\begin{aligned} \dot{\xi}_i &= f(\xi_i) \\ &:= \begin{bmatrix} -v_i \cos \alpha_{i0} - v_0 \cos \alpha_{0i} \\ \frac{1}{b} \dot{d}_{i0} + c_2 v_i [d_{i0} - R - (1+b)(\alpha_{i0} + \pi/2)] \end{bmatrix}. \end{aligned}$$

Let v_{min} be the minimum speed of v_i in the control law (5) when the state ξ_i lies in \mathcal{S} . It can be seen later that with suitable choices of parameters in the control law, v_{min} is positive. Next, we present our main result.

Theorem 3.1: If $v_{min} \sin a \geq v_{0max}$, then $\xi_i(0) \in \mathcal{S}$ implies $\xi_i(t) \in \mathcal{S}$ under the control law (5).

Proof: We prove the theorem using Nagumo's Theorem [4]. Recall that from Remark 3.1, v_i is continuous and locally Lipschitz for states in \mathcal{S} . Thus, $f(\xi_i)$ is continuous and locally Lipschitz.

On the boundary μ_1 , we check

$$n_1^T f(\xi_i) = -\frac{1}{b} (v_i \sin a + v_0 \cos \alpha_{0i}) + c_2 v_i [d_{i0} - R - (1+b)a].$$

Note that $\frac{1}{b} (v_i \sin a + v_0 \cos \alpha_{0i}) \geq \frac{1}{b} (v_i \sin a - v_{0max}) \geq 0$ due to the condition $v_{min} \sin a \geq v_{0max}$ in the theorem. Also, on the boundary μ_1 ,

$$c_2 v_i [d_{i0} - R - (1+b)a] \leq c_2 v_i (a - a) = 0.$$

So it follows that $n_1^T f(\xi_i) \leq 0$.

On the boundary μ_2 , we check

$$\begin{aligned} n_2^T f(\xi_i) &= -bc_2 v_i [d_{i0} - R - (1+b)(\alpha_{i0} + \pi/2)] \\ &= -bc_2 v_i [a - (\alpha_{i0} + \pi/2)]. \end{aligned}$$

Since on μ_2 , $(\alpha_{i0} + \pi/2) \leq a$, it follows $n_2^T f(\xi_i) \leq 0$.

Similarly, we can show that $n_3^T f(\xi_i) \leq 0$ on the boundary μ_3 and $n_4^T f(\xi_i) \leq 0$ on the boundary μ_4 . Thus, the set \mathcal{S} is positively invariant for the closed-loop system. In other words, if $\xi_i(0) \in \mathcal{S}$ then $\xi_i(t) \in \mathcal{S}$ for all time. ■

Remark 3.2: In Theorem 3.1, it is shown that the set \mathcal{S} is positively invariant. That means, the error evolution $(d_{i0}(t) - R, \alpha_{i0}(t) + \pi/2)$ for $i = 1, \dots, n$ is uniformly bounded, which addresses the condition (i) of our formulated problem.

Finally, we exploit the conditions for the existence of control parameters such that the control law meets the physical constraints.

Theorem 3.2: If the following conditions hold:

$$c_1 [R - (1+b)a] \geq \left(1 + \frac{1}{\sin a}\right) v_{0max}, \quad (6)$$

$$c_1 [R + (1+b)a] < v_{max} \cos a - v_{0max}, \quad (7)$$

$$\frac{1}{b^2} + \frac{1}{[R - (1+b)a]^2} < \frac{\omega_{max}^2}{(v_{max} + v_{0max})^2}, \quad (8)$$

then the control law (5) exists satisfying the physical constraints.

Proof: From Theorem 3.1, we know that in order to make $\xi_i(t) \in \mathcal{S}$ for all t , it should hold that $v_{min} \sin a \geq v_{0max}$. That is, the following should hold: $v_i \sin a \geq v_{0max}$ for all states in \mathcal{S} . Note that when $\xi_i \in \mathcal{S}$,

$$\begin{aligned} v_i &= \eta_i^{-1}[-(c_1 + \gamma_i \cdot (\psi_i^+ - \psi_i^-))] \\ &= -\frac{(c_1 + \gamma_i \cdot (\psi_i^+ - \psi_i^-))d_{i0} + v_0 \sin \alpha_{0i}}{\sin \alpha_{i0}} \\ &\geq c_1 d_{i0} - v_{0max} \geq c_1 [R - (1+b)a] - v_{0max}. \end{aligned}$$

This leads to the following condition $c_1 [R - (1+b)a] - v_{0max} \geq \frac{v_{0max}}{\sin a}$, which is equivalent to (6).

Moreover, we have to assure that $|v_i| \leq v_{max}$ and $|\omega_i| \leq \omega_{max}$. When $\xi_i \in \mathcal{S}$, it can be checked that

$$v_i \leq \frac{(c_1 + 2\pi c_3)[R + (1+b)a] + v_{0max}}{\cos a}.$$

Hence, the following should hold:

$$\frac{(c_1 + 2\pi c_3)[R + (1+b)a] + v_{0max}}{\cos a} \leq v_{max}.$$

Notice that $c_3 > 0$ can be arbitrarily small, so the condition (7) ensures the existence of c_3 such that the above inequality holds. On the other hand, when $\xi_i \in \mathcal{S}$, it is obtained that

$$|\omega_i| \leq (v_{max} + v_{0max}) \sqrt{\frac{1}{b^2} + \frac{1}{[R - (1+b)a]^2}} + 2ac_2 v_{max}.$$

Thus, (8) ensures $c_2 > 0$ such that $|\omega_i| \leq \omega_{max}$. ■

C. Ensuring equal spacing

In this subsection, we show that our proposed control law (5) also ensures that the formation vehicles are distributed evenly around the target and no collision occurs between them (condition (ii) of our formulated problem). In other words, if all vehicles initially have their states in \mathcal{S} and if the separation angle of any two vehicles is more than ψ_{min} (a constant depending on the size of the vehicle), then the group of vehicles encloses the target with equal separation

angular between each other. In addition, the separation angle can never become less than ψ_{min} as the system evolves under the control law (5). Thus, no collision would occur and the neighbor relationship does not change at all.

Since the labels of vehicles do not affect the control strategy, for notation simplicity, we renumber the vehicles in a counterclockwise order around the target. Thus, for vehicle i , its next-neighbor is vehicle $i + 1$ and its pre-neighbor is $i - 1$. In the paper, we use circular indices. That is, we use the same notation $i + 1$ for all $i = 1, \dots, n$, but when $i = n$, the index $i + 1$ means 1.

Theorem 3.3: Suppose that $\xi_i(0) \in \mathcal{S}$ and $\psi_i^-(0) \geq \psi_{min}$ for all $i = 1, \dots, n$. Then the control law (5) ensures $\psi_i^-(t) \geq \psi_{min}$ for all i .

Proof: For notation simplicity, we drop the superscript $-$ of ψ_i^- in the proof. Considering control law (5), we get that

$$\begin{aligned}\dot{\psi}_i &= \eta_{i+1}(v_{i+1}) - \eta_i(v_i) \\ &= \eta_{i+1}\eta_{i+1}^{-1}[-(c_1 + \gamma_{i+1} \cdot (\psi_i - \psi_{i+1}))] \\ &\quad - \eta_i\eta_i^{-1}[-(c_1 + \gamma_i \cdot (\psi_{i-1} - \psi_i))] \\ &= -\gamma_{i+1} \cdot (\psi_i - \psi_{i+1}) + \gamma_i \cdot (\psi_{i-1} - \psi_i).\end{aligned}$$

Writing in the vector form, we have $\dot{\psi} = A_1\psi$ where $\psi = [\psi_1, \dots, \psi_n]^T$ and A_1 is the resulting system matrix. Since γ_i is either 0 or a positive coefficient c_3 , it is clear that the matrix A_1 is a generator matrix [16] (i.e., the off-diagonal entries are nonnegative and all row sums equal to 0). Since A_1 is a generator matrix, we know that $\exp(A_1 t)$ is a row stochastic matrix [16], which means that each $\psi_i(t)$ lies in the convex combination of $\psi_1(0), \dots, \psi_n(0)$. Hence, $\psi_i(t) \geq \min(\psi_i(0)) \geq \psi_{min}$. ■

Next, we show that the vehicles are evenly spaced around the target equal in angle.

Theorem 3.4: Suppose $\xi_i(0) \in \mathcal{S}$ and $\psi_i^-(0) \geq \psi_{min}$ for all i . Then the control law (5) ensures $\lim_{t \rightarrow \infty} \psi_i^-(t) = \dots = \lim_{t \rightarrow \infty} \psi_n^-(t) \geq 0$.

Sketch of proof: Denote $\zeta_i = \psi_i^- - \psi_{i-1}^-$. On one hand, we can show that for any j , when $\zeta_j(0) < 0$ then $\zeta_j(t) < 0$ all the time. Moreover, it can be proved that for this j , $\zeta_j(t)$ converges to 0 asymptotically. On the other hand, if for some j , $\zeta_j(0) \geq 0$, then $\zeta_j(t)$ either becomes negative at some time instant or remains nonnegative all the time. For the first case, it remains negative after that time instant and then converges to 0 asymptotically. For the second case, it must converge to 0 asymptotically since $\zeta_1(t) + \dots + \zeta_n(t) = 0$ and all the negative terms in the left-hand-side remain negative and asymptotically converge to 0. ■

IV. SIMULATION

In this section, we simulate five unicycles for the mission of escorting and patrolling a moving target using our proposed control law. It is assumed that the vehicles are subject to the following physical constraints: $|v_i| \leq v_{max} = 2.6$ and $|\omega_i| \leq \omega_{max} = 5.9$.

Suppose the target object moves in the plane with its maximal speed $v_{0,max} = 0.25$, and its angular speed $\omega_0 = 0.5 \sin(t)$ is time-varying. Let the desired enclosing circle have radius $R = 3$. We consider the set \mathcal{S} with the parameters

$a = \frac{\pi}{4}$ and $b = 1$. The initial postures of the five unicycles are set to satisfy $\xi_i(0) \in \mathcal{S}$ and $\psi_i^-(0) \geq \psi_{min} = 0.3$, which are $(-4, -2, 0.5\pi)$, $(1, -2, -0.85\pi)$, $(0, 4.5, -0.2\pi)$, $(-2, 3, 0.5)$, and $(-4, 1, 1.4)$, respectively.

The control parameters of (5) are chosen as follows: $c_1 = 0.32$, $c_2 = 0.5$, and $c_3 = 0.004$. The simulated trajectories of the five unicycles for the mission of escorting and patrolling are shown in Fig. 4 and Fig. 5.

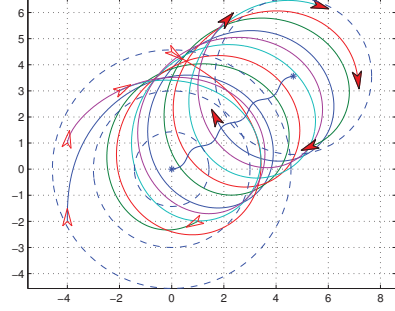


Fig. 4. Trajectories of the five unicycles in the plane for $t \in [0, 25]$.

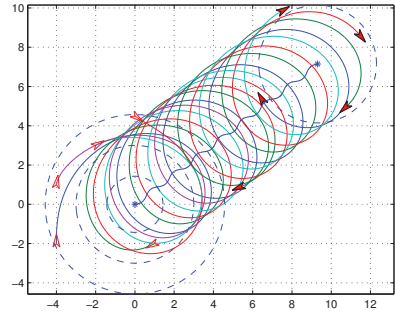


Fig. 5. Trajectories of the five unicycles in the plane for $t \in [0, 50]$.

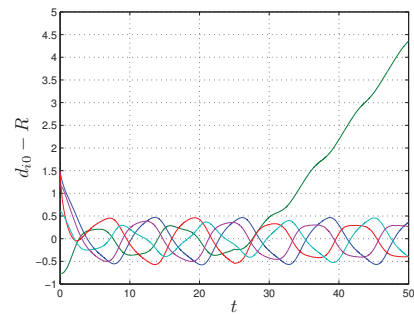


Fig. 6. The evolution of $d_{i0}(t) - R$ ($i = 1, \dots, 5$).

In the figures, the blank wedges represent the initial postures and the filled wedges represent the current postures. Shown in Fig. 4, the five unicycles converge asymptotically to the neighborhood of the desired enclosing circle and rotate around the target with equal angular distances between each other. From Fig. 5, we can see that when a malfunction occurs for a vehicle that stops, the remaining four vehicles can adaptively reconfigure themselves and enclose the moving target with a new formation that is evenly spaced again.

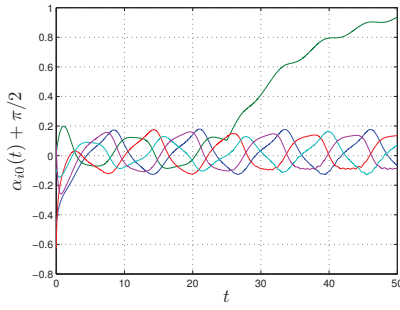


Fig. 7. The evolution of $\alpha_{i0}(t) + \pi/2 (i = 1, \dots, 5)$.

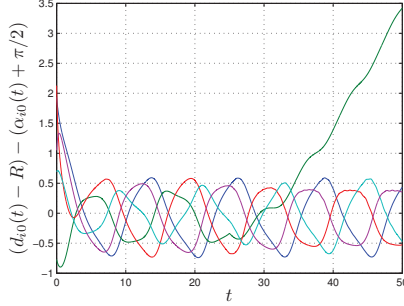


Fig. 8. The evolution of $(d_{i0}(t) - R) - (\alpha_{i0}(t) + \pi/2) (i = 1, \dots, 5)$.

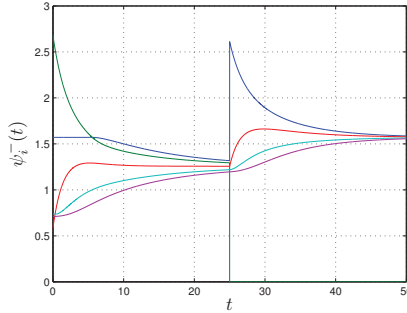


Fig. 9. The evolution of $\psi_i^-(t) (i = 1, \dots, 5)$.

Fig. 6 and Fig. 7 show the evolution curves of the states $d_{i0}(t)$ and $\alpha_{i0}(t)$ of all the vehicles. It can be seen that they converge to the neighborhood of the desired values and remain there, but are not able to converge to the desired values exactly. From Fig. 8, it can be seen that all the vehicles are ensured in \mathcal{S} all time. In addition, the evolution curves of the separation angles between every two neighbor vehicles are depicted in Fig. 9, from which we can see that the separation angles are greater than ψ_{min} all the time and converge to $2\pi/5$ when the five vehicles are all live. When a vehicle becomes malfunctioning, the remaining four vehicles re-achieve a new equal spacing formation with their separation angles converging to $\pi/2$.

V. CONCLUSION AND FUTURE WORK

The paper addresses the cooperative escorting and patrolling problem for a moving target with multiple unicycle-type robots. A distributed reconfigurable control law is proposed so that a group of vehicles can follow and surround the moving target in a formation that is evenly spaced,

providing the best overall coverage of the target and its surrounding. When some vehicles become malfunctioning, the remaining vehicles can reconfigure and achieve a new evenly-spaced formation for the task, which shows that the system is fault tolerant. In the ongoing work, we try to address several challenges. For example, we are interested in carrying out global convergence analysis and synthesizing new control laws using only distance information rather than relative position information.

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