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Citation for published version (APA):
Dirksz, D. A., & Scherpen, J. M. A. (2013). Tuning of dynamic feedback control for nonlinear mechanical systems. In *Proceedings of the 2013 European Control Conference (ECC)* (pp. 173-178)

Tuning of dynamic feedback control for nonlinear mechanical systems

Daniel A. Dirksz and Jacquelien M.A. Scherpen

Abstract— Mechanical systems are often only equipped with position measurement encoders and obtain the velocity signal by differentiation. However, differentiation largely amplifies noise. In this paper we look at dynamic feedback control of Euler-Lagrange mechanical systems. Dynamic feedback is often used to avoid velocity measurements in the control feedback. In the Euler-Lagrange literature it is shown that the dynamic extension realizes an approximate differentiator, justifying its application. We show in this paper that the dynamic extension used in the Euler-Lagrange literature actually realizes a lead-compensator, proving that a lead-compensator can also globally asymptotically stabilize a nonlinear mechanical systems. Furthermore, based on the lead-compensator structure it is then possible to offer a frequency approach to tune the controller gains.

I. INTRODUCTION

In motion systems optical encoders are extensively used for feedback control, measuring the position at a fixed sampling rate. The position accuracy is however limited by the quantized position measurement of the encoder, i.e., it is limited by the number of slits on the encoder disk. Velocity (and sometimes acceleration) estimations are often obtained by numerical differentiation, which largely amplifies the quantization errors. Furthermore, differentiation is also known to amplify high frequency noise. The derivative action is however very important in control since it provides the system with damping, which influences the convergence of the system position to a desired value. In classical linear control the derivative action also provides the system with the necessary phase lead in order to have satisfactory stability margins.

In classical linear control [6], [13] a pure differentiator is often avoided by application of lead-compensators or observers. A lead compensator is a control system that differentiates the position only for a specific frequency range. By doing so amplification of high frequency noise is avoided. Another alternative is to use observers. An observer is a dynamic system used to estimate the state of another dynamic system, given knowledge of the system inputs and measurements of the system outputs. Observers have the disadvantage that their design depends on the model of the system, which can make them sensitive to modeling errors or parameter uncertainty. In this paper we look at dynamic feedback as presented in the literature on control of Euler-Lagrange mechanical systems [14], used to avoid velocity

measurements (derivative of position). Dynamic feedback with only position measurements for nonlinear mechanical systems was first presented in [7], for control of robots using approximate differentiation. The dynamic extension is a virtual system, the control system, with dynamics equal to the approximate differentiation filter of [7]. A similar idea was presented in [1] for setpoint control, however, they have a controller consisting of k second order systems (where k are the degrees of freedom). The controller presented in [7] has k first order systems. The idea was later extended for tracking control of robots in [8] and to the case with bounded control inputs in [9]. Many publications on the subject of control with only position feedback have appeared in the last decades. It is however out of the scope of this paper the summarize all of them, we refer to e.g. [1], [2], [3], [4], [7], [8], [9], [10], [12]. Furthermore, the main contribution of this paper is not to present an alternative method to the control with only position feedback problem.

The main contribution of this paper is to show that the dynamic control structure in [1], [8], [14] actually realizes the well-known lead-compensator in classical linear control. By showing the lead-compensator structure we justify the application of lead-compensators for (global) asymptotic stabilization of nonlinear mechanical systems. Furthermore, based on the lead-compensator structure we provide a frequency approach to tune the control parameters. In a similar way as in [5], we also show that we can improve the transient response of the system by also tuning the initial conditions of the controller states. The application of linear controllers for nonlinear systems is usually not trivial, while nonlinear control methods often lack control tuning guidelines. The need for tuning guidelines for control of nonlinear systems is however very high.

In section II we first summarize the basics on passivity-based control as presented in [14] and the application of dynamic feedback for nonlinear systems. In section III we first take a classical control view at derivative feedback. That is, we look at the properties of derivative feedback and its alternatives in the frequency domain. We then show the conditions under which the approximate differentiation structure (also called dirty derivative) of [1], [8], [14] becomes a lead-compensator. A simulation example to confirm our findings is shown in section V. Some final remarks are then given in section VI.

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II. PASSIVITY-BASED CONTROL

In this section we summarize some important results of [14].

A. Stabilization of fully actuated mechanical systems

Consider a mechanical system described by the EL equations

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + \frac{\partial V}{\partial q}(q) + \frac{\partial F}{\partial \dot{q}}(\dot{q}) = Gu \qquad (1)$$

where $q \in \mathbb{R}^n$ is the vector of generalized coordinates, $M(q)$ is the positive definite mass-inertia matrix, $C(q,\dot{q})$ is the matrix of Coriolis and centrifugal forces, $V(q)$ is the potential energy function, $F(\dot{q})$ is the dissipation function, G the input matrix of rank $m \leq n$ and input vector u. For fully actuated system we have direct actuation on each degree of freedom, and we can take $G = I$.

Proposition 1 (Proposition 3.1 in [14]): Consider the system (1) with $G = I$ (fully actuated system). Define the functions $V_a(q), F_d(\dot{q}) \in \mathbb{R}$ which satisfy the following assumptions:

A. 1: The function $V_a(q)$ is such that the potential energy of the closed-loop system

$$V_d = V(q) + V_a(q) \qquad (2)$$

has a unique global minimum at $q = q_d$, with q_d constant, and is radially unbounded with respect to $q - q_d$.

A. 2: The (dissipation) function $F_d(\dot{q})$ satisfies

$$\frac{\partial F_d}{\partial \dot{q}}(0) = 0, \quad \dot{q}^\top \frac{\partial F_d}{\partial \dot{q}}(\dot{q}) > 0, \ \forall \dot{q} \neq 0 \qquad (3)$$

Then, the feedback control input

$$u = -\frac{\partial V_a}{\partial q}(q) - \frac{\partial F_d}{\partial \dot{q}}(\dot{q}) \qquad (4)$$

Globally asymptotically stabilizes system (1) in $(q, \dot{q})^\top = (q_d, 0)^\top$. ◁

Common choices for V_a and F_d are quadratic functions, i.e.,

$$V_a = -V(q) + \frac{1}{2}(q - q_d)^\top K_p (q - q_d) \qquad (5)$$

$$F_d = \frac{1}{2}\dot{q}^\top K_d \dot{q} \qquad (6)$$

with K_p and K_d positive definite constant matrices.

Remark 1: The control input (4) with V_d and F_d as in (5) and (6) is often called PD control plus gravity cancelation in the EL literature. Notice that the V_a term in (4) cancels the original potential energy (often caused by gravitational forces) and adds a proportional feedback term, while the F_d term in (4) adds the derivative action. ◁

B. Dynamic extension and control of underactuated systems

The results of the previous subsection can be extended to underactuated systems and systems for which the feedback control input u can only depend on the measured positions. Control with only position measurements is very useful, since then differentiation of the position measurements is avoided. In [14] the vector q is partitioned into $q = (q_r, q_m)^\top$, where $q_m \in \mathbb{R}^k$ is the vector of measurable/actuated coordinates and q_r is the vector of regulated coordinates with constant desired value q_{rd}. For ease of presentation and without loss of generality we assume that $G = [0 \ I_k]^\top$.

Proposition 2 (Proposition 3.6 in [14]): Define a control system with states $q_c \in \mathbb{R}^k$, potential energy $V_c(q_m, q_c)$ and dissipation function $F_c(\dot{q}_c)$. The control system with dynamics given by

$$\frac{\partial V_c}{\partial q_c}(q_m, q_c) + \frac{\partial F_c}{\partial \dot{q}_c}(\dot{q}_c) = 0 \qquad (7)$$

globally asymptotically stabilizes the system (1) if
1) (Energy shaping). $V(q)$ is proper and has a unique minimum at $q_r = q_{rd}$.
2) (Damping injection). $F_c(\dot{q}_c)$ satisfies

$$\dot{q}_c^\top \frac{\partial F_c}{\partial \dot{q}_c} \geq \alpha \|\dot{q}_c\|^2 \qquad (8)$$

for some constant $\alpha > 0$.
3) (Dissipation propagation). For each trajectory such that $q_c \equiv constant$ and $\frac{\partial V_c}{\partial q_c}(q_m, q_c) = 0$, we have that q is constant. ◁

By describing the dissipation by the Rayleigh dissipation function, i.e.,

$$F_c(\dot{q}_c) = \frac{1}{2}\dot{q}_c^\top R_c \dot{q}_c \qquad (9)$$

with R_c a positive definite constant matrix we satisfy the damping injection condition in Proposition 2 and get the controller dynamics

$$\dot{q}_c = -R_c^{-1}\frac{\partial V_c}{\partial q_c}(q_m, q_c) \qquad (10)$$

The interconnection between system (1) and controller (10) is defined by

$$Gu = -\frac{\partial V_c}{\partial q_m}(q_m, q_c) \qquad (11)$$

It can be verified that (10) and (11) satisfy the conditions of Proposition 2 for fully actuated systems, i.e., systems with $G = I$ and $q_m = q$.

In [1], [8], [14] classes of mechanical systems of the form (1), including both fully and underactuated systems, are described which can be asymptotically stabilized according to Proposition 2. The dynamic control systems used in [1], [8], [14] have the control structure

$$u = \frac{\partial V}{\partial q}(q) \underbrace{-K_p \bar{q}_m - K_d(q_c + B\bar{q}_m)}_{\hat{u}} \qquad (12)$$

$$\dot{q}_c = -A(q_c + B\bar{q}_m) \qquad (13)$$

where $A = \mathrm{diag}\{a_i\}$ and $B = \mathrm{diag}\{b_i\}$ are constant positive definite matrices, $i = 1, ..., k$ and $\bar{q}_m = q_m - q_d$. The controller (13) is then obtained from (10) with

$$V_c = \frac{1}{2}(q_c + B\bar{q}_m)^\top K_d B^{-1}(q_c + B\bar{q}_m) \qquad (14)$$

and dissipation function

$$F_c = \frac{1}{2}\dot{q}_c^\top K_d B^{-1} A^{-1} \dot{q}_c \qquad (15)$$

Remark 2: For tracking control of fully actuated mechanical systems in [8], [14] the control input also uses some kinetic energy shaping terms, i.e., then

$$u = \frac{\partial V}{\partial q}(q) + M(q)\ddot{q}_d + C(q,\dot{q}_d)\dot{q}_d - K_p\bar{q}_m - K_d(q_c + B\bar{q}_m)$$

with desired trajectory $q_d(t) \in \mathcal{C}^2$. For tracking control the dynamic extension part has the same structure as in (13), but with $\bar{q}_m = q - q_d(t)$. In this paper we are only interested in stabilization and thus do not shape the kinetic energy. ◁

Notice that the control input (12) does not depend on velocity measurements, i.e., the derivative of q_m. In [1], [8], [14] the authors explain and justify the control structure by showing that it is the same as applying approximate differentiation. This is shown by taking

$$v = q_c + B\bar{q}_m \qquad (16)$$

We can then define the dynamics

$$\begin{aligned} \dot{v} &= \dot{q}_c + B\dot{\bar{q}}_m \\ &= -Av + B\dot{\bar{q}}_m \qquad (17) \end{aligned}$$

which is a linear state-space system with input $\dot{\bar{q}}_m$. The control input part $\hat{u}$ in (12) can then also be written in the Laplace domain form

$$\begin{aligned} \hat{U}(s) &= -K_p\bar{Q}_m(s) - K_d V(s) \\ &= -K_p\bar{Q}_m(s) - K_d(sI + A)^{-1}Bs\bar{Q}_m(s) \\ &= -K_p\bar{Q}_m(s) - K_d \underbrace{\mathrm{diag}\left\{\frac{b_i s}{s + a_i}\right\}}_{C_i(s)}\bar{Q}_m(s) \quad (18) \end{aligned}$$

where s is the Laplace variable and $\hat{U}(s)$, $V(s)$ and $\bar{Q}_m(s)$ the Laplace transform of $\hat{u}, v$ and $\bar{q}_m$ respectively. We want to emphasize that we are dealing here with a multi-input multi-output (MIMO) system. The controller structure is however diagonal, which makes a single-input single-output (SISO) analysis of the diagonal components sufficient.

Notice that $C_i(s)$ is a transfer function giving the approximate derivative, also called *dirty derivative* in [1], [8], [14]. In the next section we take a closer, frequency-based, look at the dynamic control structure presented above. We show that, under certain conditions, the total control input (12) with (13) realizes a lead-compensator.

III. DERIVATIVE FEEDBACK IN CLASSICAL CONTROL

In the previous section we summarized some results from [1], [8], [14] on dynamic feedback control for EL nonlinear mechanical systems. The main reason for the application of dynamic feedback is to avoid velocity measurements in the control input. We already explained in the Introduction that mechanical systems are often only equipped with position

measurement encoders. Velocity estimations are often obtained by numerical differentiation, which largely amplifies the quantization errors. Furthermore, a pure differentiator with transfer function $C_d(s) = s$ is known to amplify high frequency noise. The gain of the pure differentiator increases as the frequency increases, with an infinite gain at infinite frequencies.

The approximate differentiator described in the previous section is one possible solution to avoid high gains at high frequencies. Figure 1 show the Bode plot for the approximate differentiator

$$C_i(s) = \frac{\kappa_i s}{s + \rho_i} \qquad (19)$$

in (18), with κ_i and ρ_i positive constants. The constant ρ_i
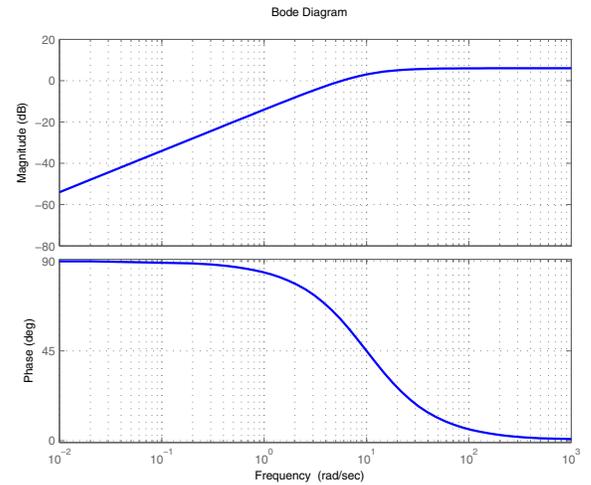

Fig. 1. Bode plot for the tamed differentiator (19) with $\kappa_i = 2$ and $\rho_i = 10$.

determines at which frequency a pole is placed, such that the differentiating action is terminated. Such a controller is also called *tamed differentiator* in [11], since the differentiating action is only limited to the relevant (and relatively) low frequencies. However, such a tamed differentiator has the disadvantage that the gain decreases with lower frequencies. In classical control this can be seen as a proportional decrease of the virtual stiffness, as described in [11]. The direct consequence is then that the capability to suppress errors at the low frequencies is also reduced. This shortcoming can be dealt with by a *lead*-compensator, which has the transfer function

$$C_L(s) = \frac{\frac{1}{\omega_1}s + 1}{\frac{1}{\omega_2}s + 1} \qquad (20)$$

with positive constants $\omega_2 > \omega_1$. A lead-compensator has neutral gain for the lower frequencies, and a bounded gain increase (to be tuned) for the higher frequencies. Figure 2 shows the Bode plot for (20). The lead compensator places a zero at frequency ω_1, which is the frequency at which the differentiation action is started. The differentiating action is
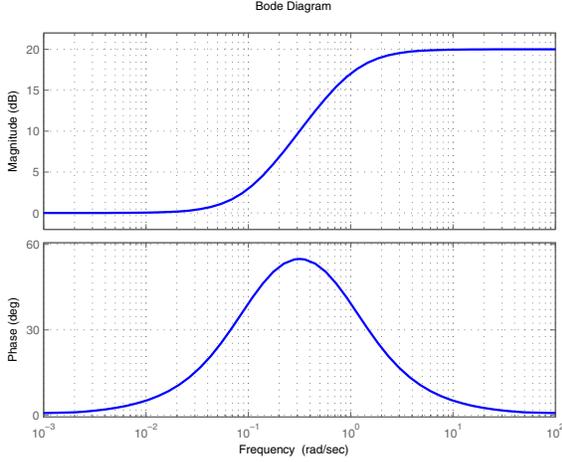
Fig. 2. Bode plot for the lead-compensator (20) with $\omega_1 = 0.1$ and $\omega_2 = 1$ rad/s.

terminated by placing a pole at frequency ω_2. With a lead-compensator (20) approximate differentiation is realized only in a specific frequency range, in order to keep the gain at low frequencies high and low at high frequencies. The gain G_L (in dB) of the lead-compensator after the approximate differentiation action is terminated is equal to

$$G_L = 20 \log(\frac{\omega_2}{\omega_1}) \tag{21}$$

In [1], [8], [14] the application of the controller (13) with control input (12) is justified since it includes the tamed differentiator (19) described above. However, we can show that under mild conditions the controller (13) with control input (12) actually realizes a lead-compensator. Consider again the controller dynamics given by (13) and describe the matrix B by $B = B_1 - B_2$. Define

$$x = q_c + B_1 \bar{q}_m \tag{22}$$

We then have the dynamics

$$\begin{aligned}
\dot{x} &= \dot{q}_c + B_1 \dot{\bar{q}}_m \\
&= -Ax + AB_2 \bar{q}_m + B_1 \dot{\bar{q}}_m \tag{23}
\end{aligned}$$

We now have a linear system with two inputs, i.e., $\bar{q}_m$ and $\dot{\bar{q}}_m$.

Proposition 3: Assume that $K_p = \text{diag}\{k_i\}, K_d = \text{diag}\{d_i\}$, with k_i and d_i positive constants and $i = 1, ..., k$. The control input part $\hat{u}$ in (12) with controller dynamics (13) realizes a lead-compensator control structure as described by (20).

Proof. The control input part $\hat{u}$ in (12) is described in terms of x by

$$\hat{u} = -K_p \bar{q}_m - K_d x + K_d B_2 \bar{q}_m \tag{24}$$

Take $B_2 = K_d^{-1} K_p$, we then have

$$\hat{u} = -K_d x \tag{25}$$

Let $X(s)$ be the Laplace transform of $x(t)$. We can describe the response of (23) in Laplace domain by

$$\begin{aligned}
X(s) &= (sI + A)^{-1} AB_2 \bar{Q}_m(s) + (sI + A)^{-1} B_1 s \bar{Q}_m(s) \\
&= (sI + A)^{-1} (B_1 s + AB_2) \bar{Q}_m(s) \tag{26}
\end{aligned}$$

Taking $B_2 = K_d^{-1} K_p$ implies that $B_1 = B + K_d^{-1} K_p$ and since all the matrices are assumed diagonal we can write for (26)

$$(sI + A)^{-1} (B_1 s + AB_2) \bar{Q}_m(s) = \text{diag}\left\{ \frac{\gamma_i s + \beta_i}{s + a_i} \right\} \bar{Q}_m(s) \tag{27}$$

where $\gamma_i = b_i + \frac{k_i}{d_i}$ and $\beta_i = \frac{a_i k_i}{d_i}$. The control input (25) then becomes

$$\begin{aligned}
\hat{U}(s) &= -K_d \text{diag}\left\{ \frac{\gamma_i s + \beta_i}{s + a_i} \right\} \bar{Q}_m(s) \\
&= -K_d \text{diag}\left\{ \frac{\beta_i}{a_i} \cdot \frac{\frac{\gamma_i}{\beta_i} s + 1}{\frac{1}{a_i} s + 1} \right\} \bar{Q}_m(s) \\
&= -K_p \text{diag}\underbrace{\left\{ \frac{\frac{\gamma_i}{\beta_i} s + 1}{\frac{1}{a_i} s + 1} \right\}}_{\hat{C}_i(s)} \bar{Q}_m(s) \tag{28}
\end{aligned}$$

Notice that $\hat{C}_i(s)$ has the same structure as (20), with

$$\begin{aligned}
\omega_{i,1} &= \frac{\beta_i}{\gamma_i} = \frac{a_i k_i}{d_i b_i + k_i} \tag{29} \\
\omega_{i,2} &= a_i \tag{30}
\end{aligned}$$

$\square$

Notice that besides proving the lead-compensator structure, we also show how ω_1 and ω_2 (which determine the differentiating frequency interval) are related to the control matrices K_p, K_d, A and B in (12) and (13). Relations (29) and (30) can then provide more insight for tuning of these control matrices. Proposition 3 also proves that a lead-compensator asymptotically stabilizes a nonlinear mechanical system. We already mentioned that the application of linear controllers, although very popular, is not trivial for nonlinear systems. Furthermore, we also have the advantage that the lead-compensator structure offers clear tuning guidelines.

We want to emphasize that we have changed neither the input (12) nor the controller dynamics (13) from [1], [8], [14]. While in [1], [8], [14] the coordinate transformation (16) is applied to justify the dynamic extension, here we apply the transformation (22). Coordinate transformations are often used to give a different, however equivalent, representation of a system and usually provides new insights for both the analysis and synthesis of control systems. Such a new insight is described in this paper, where by applying a different coordinate transformation we prove the lead-compensator structure in the results of [1], [8], [14].

Remark 3: From classical linear control theory we know that the controller (20) realizes a *lag*-compensator when we choose $\omega_1 > \omega_2$ (the lead-compensator has $\omega_2 > \omega_1$).

In control applications a lag-compensator is applied as an alternative for the pure integrator to reduce the steady-state errors of a system. While it is straightforward to take $\omega_1 > \omega_2$ in (20), this is not the case for the EL control structure described in section II-B. The proof of Proposition 2 in [14] is based on the candidate Lyapunov function

$$\mathcal{H} = \frac{1}{2}\dot{q}^\top M(q)\dot{q} + V_d(q) + V_c(q, q_c) \tag{31}$$

Given the controller dissipation function (9), we then have

$$\dot{\mathcal{H}} \leq -\frac{\partial V_c}{\partial q_c}^\top R_c^{-1} \frac{\partial V_c}{\partial q_c} \tag{32}$$

and LaSalle's invariance principle is used to prove asymptotic stability. Notice that, whatever we choose for $V_c(q, q_c)$ in Proposition 2, the inequality (32) is always satisfied. Inequality (32) shows that a controller with any function $V_c(q, q_c)$ satisfying the conditions of Proposition 2 is meant to dissipate energy. Energy dissipation or damping in linear control is the same as phase lead. It is then not possible to tune the controller such that it becomes a lag-compensator.◁

IV. IMPROVING TRANSIENT PERFORMANCE

In [5] it is shown that the initial conditions of the controller $q_c(0)$ can be tuned to improve the transient performance of the system. We show here a similar analysis by determining the square L_2-norm of $q_c + B\bar{q}_m$ for the control structure (12), (13). From (32) with (31) and V_c as in (14) we know that

$$\dot{\mathcal{H}} \leq -(q_c + B\bar{q}_m)^\top B^{-1} K_d A(q_c + B\bar{q}_m) \tag{33}$$

Let the constant $\lambda > 0$ be the smallest eigenvalue of the matrix $B^{-1}K_d A$. We then have

$$\dot{\mathcal{H}} \leq -\lambda ||q_c + B\bar{q}_m||^2 \tag{34}$$

For the L_2-norm of $(q_c + B\bar{q}_m)$ we then have

$$\begin{aligned} ||q_c + B\bar{q}_m||_2^2 &= \int_0^\infty ||q_c(\tau) + B\bar{q}_m(\tau)||^2 d\tau \\ &\leq -\frac{1}{\lambda}\int_0^\infty \dot{\mathcal{H}}(\tau)d\tau \\ &\leq \frac{1}{\lambda}\big(\mathcal{H}(0) - \mathcal{H}(\infty)\big) \\ &\leq \frac{1}{\lambda}\mathcal{H}(0) \end{aligned} \tag{35}$$

The square of the L_2-norm of a signal is the energy in the signal. Since $q_c + B\bar{q}_m$ forms part of the control input (12), a lower energy for $q_c + B\bar{q}_m$ means a lower energy in the control input. The bound (35) can be reduced not only by reducing λ, but also by reducing $\mathcal{H}(0)$. In the previous section we showed how to tune the controller matrices A, B, K_d, so we prefer the leave λ unchanged. However, if we take $q_c(0) = -B\bar{q}_m(0)$ we then have $V_c(0) = 0$ and we reduce the bound (35) without changing the controller matrices A, B, K_d. By reducing the bound on $||q_c + B\bar{q}_m||_2$ we can then reduce the initial energy in the control input (the initial input effort). A lower initial control effort can imply

lower overshoots. In the next section we show the effect of tuning $q_c(0)$ with a simulation example.

V. SIMULATION EXAMPLE: CONTROL OF A PLANAR MANIPULATOR

The results of the previous section are now used to design a controller for a two degrees of freedom planar manipulator, shown in figure 3. Dynamic feedback, as described in section
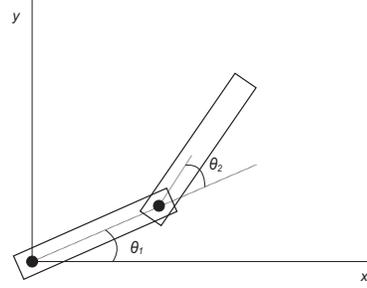

Fig. 3.   Illustration of the planar manipulator.

II-B, is applied to asymptotically stabilize the manipulator without requiring velocity measurements. In practice the motors that actuate the joints are relatively sensitive to noise, which is (easily) amplified when applying a differentiator. By application of dynamic feedback we avoid this problem, while still asymptotically stabilizing the system. Denote the length of link i by l_i, the angle of link i by θ_i such that $q_m = q = (\theta_1, \theta_2)^\top$, the distance from the joint to the center of gravity of the link i by r_i, the mass of link i by m_i and the inertia of link i by I_i. The control input is given by $u = (u_1, u_2)^\top$, with u_i the input torque of joint i. Since the manipulator only works in the horizontal plane, the gravitational forces can be neglected and $V(q) = 0$. The manipulator is then described in the form of (1) by the mass-inertia matrix

$$M(q) = \begin{bmatrix} a_1 + a_2 + 2b\cos\theta_2 & a_2 + b\cos\theta_2 \\ a_2 + b\cos\theta_2 & a_2 \end{bmatrix} \tag{36}$$

with constants

$$a_1 = m_1 r_1^2 + m_2 l_1^2 + I_1, \ a_2 = m_2 r_2^2 + I_2, \ b = m_2 l_1 r_2$$

The Coriolis term is given by

$$C(q, \dot{q})\dot{q} = \dot{M}(q)\dot{q} - \frac{1}{2}\frac{\partial \dot{q}^\top M(q)\dot{q}}{\partial q} \tag{37}$$

and

$$\frac{\partial F}{\partial \dot{q}}(\dot{q}) = D\dot{q} \tag{38}$$

with constant matrices

$$D = \text{diag}\{c_1, c_2\}, \quad c_1, \ c_2 > 0, \qquad G = I \tag{39}$$

The manipulator parameters are shown in table I. The parameters, except for the friction, describe an available experimental manipulator from Quanser. The friction values have not yet been verified on the experimental setup. Assume we want the controller to have a derivative action (for both

TABLE I
2R MANIPULATOR PARAMETER VALUES

Parameter	Value	Parameter	Value	Parameter	Value
m_1	0.5	l_1	0.2	c_1	0.1
m_2	1	l_2	0.25	c_2	0.1
I_1	0.01	r_1	0.1	q_{1d}	0.5π
I_2	0.01	r_2	0.1	q_{2d}	0.25π

joints) between $f_1 = 1$ Hz and $f_2 = 10$ Hz, with a maximum gain of 50 dB. From (21), with $\omega_1 = 2\pi f_1$ and $\omega_2 = 2\pi f_2$ rad/s, we know that $G_L = 20$ dB, which means that the gain k_i should account for $50-20 = 30$ dB, i.e., $k_i = 31.62$. From (30) we know that $a_i = \omega_2$ and from (29) we can compute that $d_i b_i = 284.6$. Figure 4 shows the error trajectories for q_1 and q_2 for different choices of b_i and d_i. Notice that there is
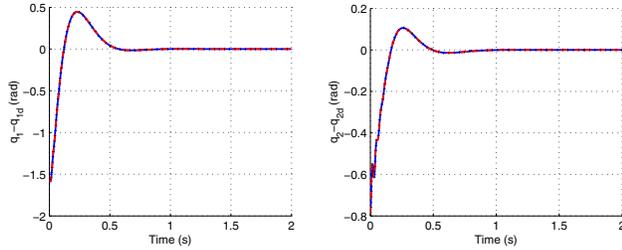

Fig. 4. Error trajectories for the manipulator. Initial conditions $[q_m(0) \ p(0) \ q_c(0)] = [0 \ 0 \ 0 \ 0 \ 0 \ 0]$. Blue solid lines: $a_i = \omega_2$, $b_i = 1$, $d_i = 284.6$. Red dashed lines: $b_i = 10$, $d_i = 28.46$. Black dotted lines: $b_i = 50$, $d_i = 5.69$.

no difference in the transient response for the three different cases, which implies that the choice of b_i and d_i is irrelevant as long as $d_i b_i = 284.6$. How to choose b_i and d_i is not obvious from the EL point of view. However, from (29) it becomes obvious that only the product $d_i b_i$ is relevant, and not the individual values for b_i and d_i. This again emphasizes the tuning insights gained from this paper.

Figure 5 compares the previous results (same values for a_i, k_i and $d_i b_i$) with the results when we tune $q_c(0) = -B\bar{q}_m(0)$. As we expected from the analysis in section IV,
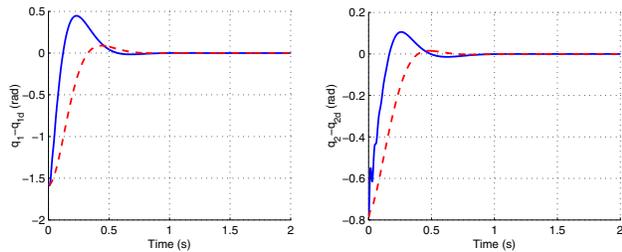

Fig. 5. Error trajectories for the manipulator. Initial conditions $[q_m(0) \ p(0)] = [0 \ 0 \ 0 \ 0]$. Blue solid lines: $q_c(0) = [0 \ 0]$. Red dashed lines: $q_c(0) = -B\bar{q}_m(0)$.

tuning of $q_c(0)$ has reduced the overshoot without changing

the controller matrices A, B, K_p and K_d, since it reduces the energy in the control input part depending on $q_c + B\bar{q}_m$. Notice furthermore that, although the rise time has increased, the settling time is similar to the previous simulation.

VI. CONCLUDING REMARKS

In this paper we show that dynamic feedback control as applied in the Euler-Lagrange literature for nonlinear mechanical systems actually includes the lead-compensator structure. That is, a controller that only takes the derivative in a specified frequency range. Since the lead-compensator is well-known in classical linear control we are able to provide a comprehensive interpretation of the dynamic feedback control applied for nonlinear Euler-Lagrange mechanical systems. Such a comprehensive interpretation of the control structure gives a clear insight in how to tune the control parameters. Contrary to classical linear control, there is a great lack of tuning guidelines for control of nonlinear systems. It is well-known that the application of linear controllers for nonlinear systems is usually not trivial. The results in this paper prove that a nonlinear mechanical system can be globally asymptotically stabilized with a lead-compensator. To conclude, we also show how to tune the controller initial conditions in order to improve the transient response.

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